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1. Introduction and Motivation

Natural resources have never been as crucial for the development of the world's economies as they have been during the last two centuries. The "cheap" energy derived from non-renewable resources like fossil fuels, especially from coal and crude oil, and the possibility to extract minerals at low costs enabled miraculous growth rates of per-capita income and the creation of a food supply chain that can sustain the ever faster growing population of our planet. This development would have been inconceivable before the dawn of the industrial revolution.

Today, inhabitants of industrialised countries have a high standard of living that most take for granted, but which once would have been unimaginable. Due to low costs of resource extraction and transportation, a citizen of an OECD country can enjoy a degree of mobility that was restricted to kings and nobles two centuries ago, and has to pay relatively low prices for food and energy, amongst many other benefits. Of course, there is also a shady side to all of this. Disparities in world income have become larger over the course of the 20th century and the extraction of the non-renewable resources of our planet and their usage for energy generation and transportation have led to negative externalities like pollution and to a lasting impairment of the biosphere.

At the latest after the renowned study "The Limits to Growth" (Meadows et al. 1972) was released by the Club of Rome in 1972, a very basic but nonetheless important realization has expanded into public awareness: Once the stock of a non-renewable resource is fully exhausted, the resource stock will not regenerate in human timeframes. According to "The Limits to Growth", the global economy will collapse and exhibit a "Malthusian disaster", once all important non-renewable resources are depleted. Due to technological progress in terms of higher energy efficiency and better possibilities of resource recycling, the prophecies of the study have yet to become true, but still the world's economies are highly dependent on non-renewable resources. Hence, an important question, which will also serve as our research question, remains:

If and under what conditions is long-run economic growth sustainable in an economy that is dependent on a non-renewable resource?

The economic literature on this subject consists of two major approaches that try to provide an answer to this question. After the release of "The Limits to Growth" at the beginning of the 1970s, famous economists like Robert Solow and Josef Stiglitz analysed the effects of non-renewable resource extraction on economic growth and tried to find optimal paths of resource extraction in Solow- and Ramsey-type frameworks.[1] They found that long-run growth is feasible under certain technological conditions.

This analysis was repeated from the 1990s onwards by Aghion and Howitt (1992 and 1998), Schou (1996), Barbier (1999), and Grimaud and Rougê (2004 and 2005), amongst others, in models that

[1] Cf. Solow (1974a and 1974b), Stiglitz (1974a, 1974b and 1976), and Dasgupta and Heal (1974), amongst others;

feature endogenous technological progress. Their results are much more diverse, as the respective authors put their emphasis on different aspects in modelling. For example, Aghion and Howitt (1998) find that positive long-run growth is sustainable, while Grimaud and Rougê (2004) find that long-run growth is feasible under certain conditions, but that the equilibrium paths are non-optimal.

As this is a very broad topic, we will have to put some limitations in place under which we will conduct our analysis. Foremost, this thesis will focus on a theoretical approach, though we will provide some empirics towards the end to check the statistical plausibility of key model predictions. Further, to limit the scope of the thesis, we will completely ignore pollution and other externalities, but will instead focus on analysing two major economic growth models in terms of their implications for long-run per-capita output growth, extraction rates and resource prices. It seems convenient that we choose models that represent the two main approaches in the literature, a Solow model with a non-renewable resource and an endogenous growth model.

The thesis is structured as follows. In Chapter 2, we introduce the models and theorems we will use for our analysis. We give a short overview of Hotelling's theorem (Hotelling 1931), as it was the first notable attempt to formally model the optimizing behaviour of the extraction sector. Hotelling's rule implies that non-renewable resource prices should exhibit exponential growth in the long run. However, the results of many empirical studies contradict this implication. We will investigate this in more detail in the empirical part of this thesis. Further, we consider two variants of a Solow model (Dyerberg Greisen and Hvelplund 2012), in which a non-renewable resource is essential in production, and introduce an endogenous growth model (Nguyen and Nguyen-Van 2008) that is based upon a modern Schumpeterian framework and adds a renewable resource into the mix.

After we have given a detailed description of our models and determined the steady-state growth rates, Chapter 3 contains an analysis of the long-run equilibria and of the transitional dynamics. Thereby we get a better grasp on the underlying mechanisms and we find that, under certain conditions, positive growth rates of per-capita output are feasible in the long run. Further, we observe that modelling the extraction sector endogenously allows for (limited) policy measures and provides a deeper understanding of the economic mechanisms.

In Chapter 4, we present empirical data. We investigate the growth rates of non-renewable resource prices and find that resource prices have been constant over long periods. This stands in contradiction to the implications of Hotelling's theorem. Further, we look at the econometric estimation of the endogenous growth model by Nguyen and Nguyen-Van (2008) to examine how the theoretical implications their model provides hold against empirical evidence. In Chapter 5, we critically assess the models discussed in this thesis and summarize our findings. Chapter 6 concludes.

2. Model Descriptions and Theorems

In this chapter, we first provide a short overview of Hotelling's rule. Then, we introduce two variants of an augmented Solow model as depicted in Dyerberg Greisen and Hvelplund (2012). After that, we investigate an endogenous growth model by Nguyen and Nguyen-Van (2008), which adds a renewable resource into the mix.

We determine the long-run growth rates in all of the models, thereby laying the groundwork for the next chapter, where we will conduct an analysis on the long-run equilibria and on the models' implications concerning the transitional dynamics and convergence patterns.

2.1. Hotelling's theorem

In 1931, Harold Hotelling released his paper "The Economics of Exhaustible Resources", which later got renowned as one of the most influential articles in the field of resource economics.

Hotelling wrote the paper in reaction to the demands of the conservation movement, which was active in the United States in the early 20th century and advocated a more sustainable approach regarding the extraction of natural resources. In his article, he developed a framework in which he tried to determine the optimal paths of resource extraction under different market conditions. The novelty was that he regarded a natural resource as an asset, which its owner, who wishes to maximize the present value of all his future profits, compares to any other interest-bearing capital asset. (Hotelling 1931, 140)

However, Hotelling's work did not attract much attention in the beginning, as the world's economies were still suffering from the after-effects of the Great Depression at that time. Therefore, most contemporary economists disregarded the topic of resource exploitation. Further, the complex mathematical formalisms used in the paper were ahead of the times and difficult to grasp. (Gaudet 2007, 2)

After the end of World War II in 1945, the topic of natural resource extraction became more important, especially concerning future economic planning. For example, the Paley commission, a think-tank based in Washington with the goal to secure resource supply by developing long-term geopolitical strategies, was founded in 1952.

After the release of the study "The Limits to Growth" (Meadows et.al. 1972) on the authority of the notable Club of Rome in 1972, a global public discourse sprouted and the topic finally got into the focus of economic research. At this time, Hotelling's theorem was "rediscovered" and his theory used to model the extraction of non-renewable resources and to find the optimal growth paths.

2.1.1. Formulation of Hotelling's rule

An economic agent who owns a non-renewable resource has two options at hand: Either he holds the resource *in situ* (in the ground) to sell it later at a higher price or he can extract the resource today and reinvest his profits. This implies that a natural resource can be treated as an interest-bearing asset. (Gaudet 2007, 2)

If the agent keeps the resource in the ground, then it can be compared to any other physical capital asset. The yield of such an asset can be decomposed into the marginal productivity of holding it, its depreciation rate and its market value. For a non-renewable resource only an increase in its market value can contribute to its appreciation, as it neither contributes with a marginal product (i.e. a dividend), nor does the resource stock depreciate while held *in situ*. (ibid. 3)

We define π_t as the net resource price (or *in-situ* value) in period t. It is equal to the nominal resource price (u_t) minus the marginal cost of extraction (c_t). (Hotelling 1931, 141)

$$\pi_t = u_t - c_t \tag{1.1}$$

As an equilibrium condition in the assets market, we have that the growth rate of the net resource price must be equal to the real interest rate (*ρ*), which is equal to the nominal interest rate (*r*) minus the capital depreciation rate (*δ*).[2] (Gaudet 2007, 5)

$$\gamma_{\pi_t} = \frac{\dot{\pi}_t}{\pi_t} = \rho = r - \delta \tag{1.2}$$

Throughout this thesis, the letter *γ* denotes the growth rate of a variable, while a dot over a variable denotes the first derivative w.r.t. time.

We make a further simplification by assuming that the marginal costs of extraction are equal to zero.[3] The resource price is then equal to the nominal price (i.e. the *in-situ* value). This yields the final definition of Hotelling's rule, which is given by

$$\gamma_{u_t} = \frac{\dot{u}_t}{u_t} = \rho = r - \delta \tag{1.3}$$

We will use Hotelling's rule later on to model the extraction behaviour of a non-renewable resource endogenously.

2.1.2. Critique of Hotelling's rule

The major critique of Hotelling's theorem derives from the fact that the rule implies that non-renewable resource prices should exhibit exponential growth over long periods, i.e. that there should be a significant positive trend in price evolution. (Krautkraemer 1998, 2067) However, many empirical studies on this subject conclude that such a trend does not seem to exist neither for the resource prices nor for the *in-situ* values.[4] Contrariwise, the results of these studies suggest that the growth rates are either centred on a mean of zero or that there is a slightly positive, but statistically insignificant, trend.

[2] Hotelling assumes that the interest rate is constant. (Hotelling 1931, 140)

[3] Dyerberg Greisen and Hvelplund (2012) make this assumption, arguing that the empirics suggest that the marginal costs of extracting a non-renewable resource are in comparison to its price relatively small.

[4] Cf. The empirical studies by Barnett and Morse (1963), Smith (1978), Slade (1982), Berck and Roberts (1996), Halvorsen and Smith (1996), and Farrow (1985), amongst others;

In the literature there are many suggestions on theoretical extensions concerning exploration, cost modelling, quality and durability of ore, and market structures, amongst others, which could improve the basic Hotelling framework and help to close the gap between theoretical implications and empirical evidence.[5]

In spite of this critique, Hotelling's rule is still widely used in the literature and is a key part of one of the models we want to investigate in the following sections. Therefore, we will use the rule as defined above, but we will give a short review on the critique and look at the growth rates of non-renewable resource prices in Chapter 4.

2.2. The Solow model with a non-renewable resource

In this section, we introduce two adapted variants of the standard Solow model (Solow 1956) as depicted in Dyerberg Greisen and Hvelplund (2012). Josef Stiglitz's works on resource economics heavily influenced their model.[6]

In the first variant, the extraction rate of the non-renewable resource will be an exogenously given constant. Then we will utilize Hotelling's rule to determine the extraction rate endogenously.

2.2.1. The Solow model with a non-renewable resource and an exogenous extraction rate

We consider a Solow economy where output is produced according to a Cobb-Douglas production function, which satisfies all the typical neoclassical properties: CRS, positive and diminishing returns to the production inputs and the Inada conditions. This model is formulated in continuous time, but we leave out time indices to simplify the notation.

Two of the three input factors that are used for producing the final output (Y) (by using technology (A)), Capital (K) and labour (L), are already known from the standard Solow model. However, additionally, production now requires an energy input (*E*) that can be obtained by extraction from a non-renewable resource stock (*R*), which is thereby depleting over time.[7] Hence, the production function is given by

$$Y = F\left(A, K, L, E\right) = K^{\alpha}\left(AL\right)^{\beta} E^{\varepsilon}, \text{ with } \alpha, \beta, \varepsilon > 0 \text{ and } \alpha + \beta + \varepsilon = 1 \tag{2.1}$$

where the parameters α, β and ε denote the income shares of capital, labour and of the energy input, respectively. Energy is essential in production, i.e. $F(K, L, 0) = 0$. Therefore, if there is no energy input left, i.e. the non-renewable resource stock gets exhausted, production cannot commence.[8] The factor inputs are earning their marginal products, the capital rental rate (*r*), the wage rate (*w*) and the non-renewable resource price (*u*), respectively.

[5] Cf. Krautkraemer (1998, 2070 - 2078) and Gaudet (2007, 6 - 21);
[6] Cf. Stiglitz (1974b);
[7] Dyerberg Greisen and Hvelplund (2012) assume that technological progress is of Harrod-neutral (labour-augmenting) form.
[8] Capital (*K*) and labour (*L*) are also essential in production. This is an additional but non-essential assumption of a neoclassical production function. (Barro and Sala-i-Martin, 1999, 28)

Capital accumulation follows the same rule as in the standard Solow model. The capital stock depreciates at the constant rate δ and is replaced with the part of the output that is saved at the exogenously given saving rate s.

$$\dot{K} = sY - \delta K, \text{ with } \delta \in (0,1),\ s \in (0,1) \text{ and } K_0 \text{ given} \tag{2.2}$$

Population (*L*) (i.e. the labour force) and knowledge (*A*) grow at the exogenously given constant rates *n* and *g*.

$$\gamma_L = n,\ L_0 \text{ given} \tag{2.3}$$

$$\gamma_A = g,\ A_0 \text{ given} \tag{2.4}$$

Energy can be obtained from the resource stock at the exogenous extraction rate s_E. The following equations depict the evolution of the resource stock (*R*) and of energy utilization (*E*).

$$\dot{R} = -E = -s_E R, \text{ with } s_E \in (0,1) \text{ and } R_0 \text{ given} \tag{2.5}$$

$$E = s_E R \tag{2.6}$$

Taking logarithms of (2.6), differentiating w.r.t. time and substituting (2.5) into the resulting equation yields the growth rate of the energy input

$$\gamma_E = \frac{\dot{E}}{E} = \frac{-E}{R} = -s_E \tag{2.7}$$

We want to analyse this model in terms of the capital-output ratio. Hence, we define the capital-output ratio *z* from which we can derive the law of motion for this economy.

$$z \equiv \frac{K}{Y} = \frac{K}{K^{\alpha}\left(AL\right)^{\beta} E^{\varepsilon}} = K^{1-\alpha}\left(AL\right)^{-\beta} E^{-\varepsilon} \tag{2.8}$$

By taking logarithms, differentiating w.r.t. time, and by substituting (2.7) and (2.2) into the resulting equation we obtain the law of motion for the capital-output ratio.[9]

$$\dot{z} = \left(\beta + \varepsilon\right)s + \left(\varepsilon\left(s_E - \delta\right) - \beta\left(n + g + \delta\right)\right)z \tag{2.9}$$

The capital-output ratio is constant ($\dot{z} = 0$) in steady state. Hence, by setting (2.9) equal to zero and rearranging terms, we get the long-run value of the capital-output ratio.

$$z^* = \frac{\left(\beta + \varepsilon\right)s}{\beta\left(n + g + \delta\right) - \varepsilon\left(s_E - \delta\right)} \tag{2.10}$$

[9] Cf. Appendix A.1.1 for a full derivation;

Now we can derive the long-run growth rate of aggregate output. In the long run, capital and output must be growing at the same rate, as the capital-output ratio is constant in steady state.

Hence, we have that

$$\gamma_Y^{\,*} = \gamma_K^{\,*} \Leftrightarrow \frac{\dot{z}}{z} = \frac{\dot{K}}{K} - \frac{\dot{Y}}{Y} = 0 \tag{2.11}$$

Using that $\gamma_Y^* = \gamma_K^*$ from (2.11) and rearranging terms yields the long-run growth rate of aggregate output

$$\gamma_Y^{\,*} = \frac{\beta}{\beta+\varepsilon}(g+n) - \frac{\varepsilon}{\beta+\varepsilon}s_E \tag{2.12}$$

By using that $\gamma_y = \frac{\dot{y}}{y} = \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = \gamma_Y - \gamma_L$, we determine the steady-state growth rate of output per worker

$$\gamma_y^{\,*} = \gamma_Y - \gamma_L = \gamma_Y - n = \frac{\beta g - \varepsilon n}{\beta+\varepsilon} - \frac{\varepsilon}{\beta+\varepsilon}s_E \tag{2.13}$$

The steady-state value of the non-renewable resource price is given by

$$u^* = \varepsilon K^{\alpha}(AL)^{\beta} E^{\varepsilon-1} = \varepsilon \frac{Y}{E} \tag{2.14}$$

From (2.14) the growth rate of the resource price can be computed by taking logarithms, differentiating w.r.t. time, and by substituting (2.7) and (2.12) into the resulting equation and by rearranging terms.[10]

$$\gamma_u^{\,*} = \left(\frac{\dot{u}}{u}\right)^* = \frac{\beta}{\beta+\varepsilon}(g+n+s_E) \tag{2.15}$$

We can easily check from Equations (2.12) - (2.15) that all variables grow at a constant rate in steady state. Hence, the economy follows a balanced growth path in the long run, whilst the constant steady-state growth rate of the resource price implies that the price of the non-renewable resource exhibits exponential growth in the long run.

2.2.2. The Solow model with an endogenous extraction rate

Now we want to introduce another variant of the Solow model (Dyerberg Greisen and Hvelplund 2012, 16-20) that utilizes Hotelling's rule to model the extraction of the non-renewable resource endogenously.

Most equations in this model are the same as defined in Section 2.2.1. Output is produced according to a Cobb-Douglas production function as in (2.1), input factors are earning their marginal products,

[10] Cf. Appendix A.1.2 for a full derivation;

capital is evolving as defined in (2.2), and population and knowledge grow at the same constant rates as in Equations (2.3) and (2.4). However, as the extraction rate of the non-renewable resource will now be determined endogenously, we need four additional equations to model the (optimizing) extraction sector.

Hotelling's rule as defined in Equation (1.3) states that the growth rate of the non-renewable resource price must be equal to the real interest rate (*ρ*), which is equal to the nominal interest rate (*r*) minus the capital deprecation rate (*δ*).

$$\gamma_u = \frac{\dot{u}}{u} = \rho = r - \delta = \frac{\alpha}{z} - \delta \tag{2.16}$$

The endogenous extraction rate is defined as the amount of energy derived from the non-renewable resource stock (*E*), divided by the proved reserves (*R*) of the resource that are still *in situ*. [11]

$$q = \frac{E}{R} \tag{2.17}$$

Further, we define a resource constraint, which states that the amount of energy derived from the resource stock is upper-bounded by the initial resource stock, as an efficiency condition.

$$\int_0^\infty E(t)dt \leq R_0 \tag{2.18}$$

The initial resource stock is exhausted over time by energy utilization. This equation is identical to (2.5) in the previous section.

$$\dot{R} = -E,\ R_0 \text{ given} \tag{2.19}$$

As in the last section, we first define the capital-output ratio (*z*), which is given by

$$z \equiv \frac{K}{Y} = \frac{K}{K^\alpha (\mathrm{AL})^\beta E^\varepsilon} = K^{1-\alpha}(AL)^{-\beta} E^{-\varepsilon} \tag{2.20}$$

By taking logarithms and differentiating w.r.t. time we get

$$\frac{\dot{z}}{z} = (1-\alpha)\frac{\dot{K}}{K} - \beta(g+n) - \varepsilon\frac{\dot{E}}{E} \tag{2.21}$$

To determine the law of motion for the capital-output ratio, we first have to determine the growth rate of the energy input. To achieve this, we compute the growth rate of the resource price γ_u and substitute it into (2.16).

[11] Equations (2.17) - (2.19) are inspired by Stiglitz. (Stiglitz 1974b, 142)

We rearrange terms and obtain the growth rate of the energy input (γ_E).[12]

$$\gamma_E = \frac{\dot{E}}{E} = \frac{1}{\alpha+\beta}\left(\alpha\frac{s-1}{z} + \beta(g+n+\delta) + \varepsilon\delta\right) \tag{2.22}$$

By substituting Equation (2.22) and $\gamma_K = \frac{\dot{K}}{K} = \frac{sY - \delta K}{K} = s\frac{Y}{K} - \delta = \frac{s}{z} - \delta$ into (2.21) we obtain the law of motion for the capital-output ratio

$$\dot{z} = \frac{\beta s + \alpha\varepsilon}{\alpha+\beta} - \frac{\beta(g+n+\delta)+\varepsilon\delta}{\alpha+\beta}z \tag{2.23}$$

The capital-output ratio does not change ($\dot{z} = 0$) in steady state. Hence, we set Equation (2.23) equal to zero and rearrange terms to get the long-run value of the capital-output ratio

$$z^* = \frac{\beta s + \alpha\varepsilon}{\beta(g+n+\delta)+\varepsilon\delta} \tag{2.24}$$

To obtain the long-run energy growth rate γ_E^*, we insert Equation (2.24) into (2.22)

$$\gamma_E^* = \left(\frac{\dot{E}}{E}\right)^* = \frac{s-\alpha}{\beta s + \alpha\varepsilon}(\beta(g+n+\delta)+\varepsilon\delta) \tag{2.25}$$

Substituting (2.24) into (2.16) yields growth rate of the resource price in steady state

$$\gamma_u^* = \left(\frac{\dot{u}}{u}\right)^* = \frac{\alpha(\beta(g+n+\delta)+\varepsilon\delta)}{\beta s + \alpha\varepsilon} - \delta \tag{2.26}$$

Next, we determine the law of motion for the endogenous extraction rate (*q)* by taking logarithms of (2.17), differentiating w.r.t. time and by substituting (2.22) into the resulting equation.[13]

$$\dot{q} = \frac{1}{\alpha+\beta}\left(\alpha\frac{s-1}{z} + \beta(g+n+\delta) + \varepsilon\delta\right)q + q^2 \tag{2.27}$$

The extraction rate is constant ($\dot{q} = 0$) in the long run. Hence, by setting (2.27) equal to zero and substituting (2.24) into the resulting equation we obtain the steady-state value of the extraction rate.[14]

$$q^* = -\left(\frac{\dot{E}}{E}\right)^* = \frac{\alpha-s}{\beta s + \alpha\varepsilon}(\beta(g+n+\delta)+\varepsilon\delta) \tag{2.28}$$

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[12] Cf. Appendix A.2.1;

[13] Cf. Appendix A.2.2;

[14] Dyerberg Greisen and Hvelplund (2012) assume that the income share of capital is larger than the saving rate. ($\alpha > s$)

At last, the long-run growth rates of output (γ_Y) and output per-worker (γ_y) can be computed the same way as in Section 2.2.1.[15]

$$\gamma_Y^* = \frac{\beta}{\beta+\varepsilon}(g+n) - \frac{\varepsilon}{\beta+\varepsilon}\frac{\alpha-s}{\beta s+\alpha\varepsilon}(\beta(g+n+\delta)+\varepsilon\delta) = \frac{\beta}{\beta+\varepsilon}(g+n) - \frac{\varepsilon}{\beta+\varepsilon}q^* \quad (2.29)$$

$$\gamma_y^* = \frac{\beta g-\varepsilon n}{\beta+\varepsilon} - \frac{\varepsilon}{\beta+\varepsilon}\frac{\alpha-s}{\beta s+\alpha\varepsilon}(\beta(g+n+\delta)+\varepsilon\delta) = \frac{\beta g-\varepsilon n}{\beta+\varepsilon} - \frac{\varepsilon}{\beta+\varepsilon}q^* \quad (2.30)$$

Again, the economy follows a balanced growth path, as all variables grow at a constant rate in steady state. The constant growth rate of the non-renewable resource price suggests that the resource price exhibits exponential growth in the long run.

Now that we have derived the long-run growth rates in our Solow economies, we want to introduce the last model that we will discuss in this thesis.

2.3. An endogenous growth model with renewable and non-renewable resources

The Solow model provides a good benchmark as it is a well-established framework that is simple to analyse. However, as we also intent to investigate what implications we can derive from a more recent model, we will now introduce an endogenous growth model by Nguyen and Nguyen-Van (2008) which is based upon a modern Schumpeterian framework.[16]

This framework features endogenous technological progress and further the model does include renewables in addition to non-renewable resources. Note that in this model the behaviour of the extraction sector is not endogenously modelled, but that the extraction rates of the natural resources will be determined as part of the solution to the social planner's problem.

In this model, final output (*Y*) is produced with technology (*A*) by employing physical capital (*K*), labour used in production (L_Y), a non-renewable (*Q*) and a renewable resource (*R*) as input factors.[17]

$$Y = F(A,K,L_Y,Q,R) = A^{\theta} f(L_Y,K,Q,R),\ L_{Y0} \text{ given} \quad (3.1)$$

Nguyen and Nguyen-Van (2008) assume that $f(L_Y,K,Q,R)$ denotes the Cobb-Douglas production function

$$Y = F(A,K,L_Y,Q,R) = \mathrm{A}^{\theta} L_Y^{\psi} K^{\xi} Q^{\alpha} R^{\beta},\ \text{with } \psi,\xi,\alpha,\beta > 0 \text{ and } \psi+\xi+\alpha+\beta = 1 \quad (3.2)$$

where the parameters α, β, ξ and ψ denote the income shares of the non-renewable and renewable resource inputs, physical capital and labour used in production, respectively.

[15] Again, we use that capital and output must be growing at the same rate due to the capital-output ratio being constant in steady state. (Cf. Equation (2.11))

[16] Cf. Aghion and Howitt (1998), Ch. 5;

[17] In this model, knowledge is not embodied inside intermediate goods.

A special feature of this model is that by setting the parameter θ equal to one of the other input parameters, technological progress can be used to "augment" any of the input factors, including the resource inputs. (Nguyen and Nguyen-Van (2008, 4)) Again, the factor inputs are earning their marginal products, the capital rental rate, the wage rate, and the non-renewable (p_Q) and renewable resource prices (p_R), respectively.

As opposed to the Solow models in the last section, technological progress is now determined endogenously. The evolution of knowledge depends on the initial stock of technology (*A*) and on the amount of labour devoted to the R&D sector (L_A). The value of the parameter ϕ determines the influence of the existing stock of knowledge on future technological progress.[18] (Aghion and Howitt (1998, 154))

$$\dot{A} = bA^{\phi}L_A, \text{ with } 0 < \phi \leq 1 \text{ and } b \text{ const.} \tag{3.3}$$

It is assumed that population is normalized to one and constant. Each individual is endowed with one unit of labour, which it can use either to produce the final output or to undertake research. The labour constraint is thus given by

$$L_Y + L_A = 1 \tag{3.4}$$

Final output (*Y*) can be either used for private consumption (*C*) or can be invested to form new capital (*I*=*Y*-*C*). The existing capital stock depreciates at the constant rate δ.[19]

$$\dot{K} = Y - C - \delta K, \text{ with } \delta \in (0,1) \text{ and } K_0 \text{ given} \tag{3.5}$$

The resource stocks (*S*) of non-renewable (*Q*) and renewable resources (*R*) are evolving according to the following equations. Note that the renewable resource stock is regenerating over time at the constant rate *m*.

$$\dot{S}_Q = -Q, \ S_{Q0} \text{ given} \tag{3.6}$$

$$\dot{S}_R = mS_R - R, \text{ with } m > 0 \text{ and } S_{R0} \text{ given} \tag{3.7}$$

The representative consumer maximises his utility according to the instantaneous utility function

$$U = \int_{t=0}^{\infty} u(C_t)e^{-\rho t}dt, \text{ with } \rho > 0 \tag{3.8}$$

where the parameter ρ denotes the rate of time preference, which determines how the representative consumer values current against future consumption. The higher ρ is, the more the consumer prefers today's consumption over tomorrows.

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[18] If the parameter Φ would be equal to one, then the evolution of technology would fully depend on the existing stock of knowledge.

[19] As before, time indices are left out to keep the notation simple.

We derive the optimal growth rates as part of the solution to the following social planner's problem. [20]

$$\max \int_{t=0}^{\infty} \mathrm{u}(C_t) e^{-\rho t} dt \tag{3.9}$$

Subject to (3.1) - (3.7)

Six key variables are defined to encapsulate the equilibrium: The output-capital ratio, the consumption-capital ratio, and the extraction rates of non-renewable and renewable resources, respectively. The other growth rates can be derived from these variables. (Nguyen and Nguyen-Van 2008, 9)

$$x = Y/K,\ y = C/K,\ z = Q/S_Q,\ u = R/S_R,\ q = L_Y A^{\phi-1} \text{ and } r = A^{\phi-1}$$

The solution to the social planner's problem yields the optimal growth rates as depicted below. [21]

$$\gamma_{S_Q} = -z \tag{3.10}$$

$$\gamma_{S_R} = m - u \tag{3.11}$$

$$\gamma_A = b(r-q) \tag{3.12}$$

$$\gamma_K = x - y - \delta \tag{3.13}$$

$$\gamma_C = \frac{\xi x - \delta - \rho}{\varepsilon} \tag{3.14}$$

$$\gamma_Q = -y + \frac{b\theta r + m\beta + (1-\xi)\delta}{\xi} \tag{3.15}$$

$$\gamma_R = -y + \frac{b\theta r + m(\beta + \xi) + (1-\xi)\delta}{\xi} \tag{3.16}$$

$$\gamma_{L_Y} = -y + \frac{b\theta}{\psi} q + \frac{b\theta r + m\beta + (1-\xi)\delta}{\xi} \tag{3.17}$$

$$\gamma_Y = \xi x - y + \frac{b\theta r + m\beta + \delta(1-\xi)}{\xi} - \delta \tag{3.18}$$

$$\gamma_{L_A} = \frac{q}{q-r} \gamma_{L_Y} \tag{3.19}$$

Equations (3.10) - (3.19) denote the optimal growth rates of the non-renewable and renewable resource stocks, technology, capital, non-renewable and renewable resource utilization, labour used in the production sector, aggregated output, and labour used in the R&D sector, respectively.

[20] Cf. Appendix A.3.1, and Nguyen and Nguyen-Van (2008, 9, 21 - 23) for a full derivation;
[21] Cf. Nguyen and Nguyen-Van (2008, 5 - 8), who provide a detailed proof that an optimal solution exists.

Next, we take a look at what happens in steady state. The output-capital ratio (x^*) and the consumption-capital ratio (y^*) are constant in the long run, due to the constant growth rates of capital (γ_K^*) and consumption (γ_C^*). Further, the growth rates of the renewable (γ_R^*) and non-renewable (γ_Q^*) resource inputs are constant. Therefore, the variables r^*, q^*, z^* and u^* are also constant. In turn, the labour inputs given by $L_Y^* = q^*/r^*$ and $L_A^* = L_Y^* - 1$ are constant in steady state.

The growth rates of the renewable and non-renewable resource stocks are equal to the growth rates of the respective resource inputs, since $\gamma_{S_Q}^*$ and $\gamma_{S_R}^*$ are constant.

$$\gamma_Q^* = \gamma_{S_Q}^* = -y^* + \frac{b\theta r^* + m\beta + (1-\xi)\delta}{\xi} \tag{3.20}$$

$$\gamma_R^* = \gamma_{S_R}^* = -y^* + \frac{b\theta r^* + m(\beta+\xi) + (1-\xi)\delta}{\xi} \tag{3.21}$$

Aggregate output, physical capital and private consumption grow at the same constant rate in the long run.

$$\gamma_Y^* = \gamma_K^* = \gamma_C^* = \frac{\xi x^* - \delta - \rho}{\varepsilon} \tag{3.22}$$

In steady state, both labour devoted to the production sector and to the R&D sector exhibit zero growth.

$$\gamma_{L_Y}^* = \gamma_{L_A}^* = 0 \tag{3.23}$$

The long-run rate of technological progress is given by

$$\gamma_A^* = b(r^* - q^*) \tag{3.24}$$

The long-run extraction rates of the non-renewable (*z*) and renewable resources (*u*) are

$$z^* = -\gamma_{S_Q}^* = y^* - \frac{b\theta r^* - m\beta + (1-\xi)\delta}{\xi} \tag{3.25}$$

$$u^* = m - \gamma_{S_R}^* = m + y^* - \frac{b\theta r^* - m(\beta+\xi) + (1-\xi)\delta}{\xi} = y^* - \frac{b\theta r^* - m\beta + (1-\xi)\delta}{\xi} \tag{3.26}$$

Moreover, the long-run resource prices are given by

$$p_Q{}^* = \frac{\delta Y}{\delta Q} = \alpha A^\theta L_Y^\psi K^\xi Q^{\alpha-1} R^\beta = \alpha \frac{Y}{Q} \tag{3.27}$$

$$p_R{}^* = \frac{\delta Y}{\delta R} = \beta A^\theta L_Y^\psi K^\xi Q^\alpha R^{\beta-1} = \beta \frac{Y}{R} \tag{3.28}$$

By logarithmic differentiation of (3.27) and (3.28) we obtain the long-run growth rates of the resource prices.

$$\gamma_{P_Q}^* = \gamma_Y^* - \gamma_Q^* \tag{3.29}$$

$$\gamma_{P_R}^* = \gamma_Y^* - \gamma_R^* \tag{3.30}$$

The steady-state values of *x*, *y*, *q* and *r* depend on the value of the technology parameter ϕ. Using that $(\phi-1)\gamma_A^* = 0$, and setting $\gamma_C = \gamma_K$, $\gamma_Y = \gamma_K$ and $\gamma_{L_Y} = 0$, yields a system of equations from which x^*, y^*, q^* and r^* can be derived. (Nguyen and Nguyen-Van 2008, 24)

<u>Case 1:</u> $\phi = 1 \Rightarrow r^* = A^{\phi-1} = A^0 = 1$

$$x^* = \frac{b\theta + m\beta + \delta(1-\xi)}{\xi(1-\xi)} \tag{3.31}$$

$$y^* = \frac{(\varepsilon-\xi)(b\theta + m\beta + \delta(1-\xi)) + \xi(1-\varepsilon)\delta + \rho}{\varepsilon\xi(1-\xi)} \tag{3.32}$$

$$q^* = \left[y^* - \frac{b\theta + m\beta + (1-\xi)\delta}{\xi} \right] \frac{\psi}{\theta b} \tag{3.33}$$

<u>Case 2:</u> $\phi \neq 1 \Rightarrow r^* = q^*$

$$x^* = \frac{b\theta q^* + m\beta + \delta(1-\xi)}{\xi(1-\xi)} \tag{3.34}$$

$$y^* = \frac{(\varepsilon-\xi)(b\theta q^* + m\beta + \delta(1-\xi)) + \xi(1-\varepsilon)\delta + \rho}{\varepsilon\xi(1-\xi)} \tag{3.35}$$

$$q^* = \left[y^* - \frac{m\beta + (1-\xi)\delta}{\xi} \right] \frac{\xi\psi}{\theta b(\xi+\psi)} \tag{3.36}$$

Again, we observe that the economy follows a balanced growth path in the long run, as all variables grow at a constant rate in steady state.

Now that we have given detailed descriptions of all of the models and have derived the steady-state growth rates, we want to explore what implications the models provide concerning the feasibility of long-run growth and further we want to conduct an analysis on the transitional dynamics.

3. Model Analysis

In the last chapter, we introduced two augmented variants of the Solow model, with a non-renewable resource that is essential in production, and an endogenous growth model, which features both renewable and non-renewable resources. We have described the models formally and derived the long-run growth rates.

In this chapter, we will first devote to the steady-state analysis of our models. Our main interest lies in answering if and under what conditions long-run growth is sustainable. Further, we want to investigate the models in terms of their implications concerning the transitional dynamics and convergence patterns.

3.1. Steady-state analysis

3.1.1. The long-run equilibrium in the Solow models with a non-renewable resource

For convenience, we restate the steady-state growth rates of aggregated and per-capita output in the adapted Solow models and compare them to the long-run growth rates in the standard Solow model. (Cf. Table 1)

Table 1: The growth rates of output in the standard and augmented Solow models

Model type	Aggregated output	Output per-worker
Standard Solow model	$\gamma^*_{Y,SOLOW} = g + n$	$\gamma^*_{y,SOLOW} = g$
Solow model with exogenous extraction	$\gamma^*_{Y,EXO} = \frac{\beta}{\beta+\varepsilon}(g+n) - \frac{\varepsilon}{\beta+\varepsilon}s_E$	$\gamma^*_{y,EXO} = \frac{\beta g - \varepsilon n}{\beta+\varepsilon} - \frac{\varepsilon}{\beta+\varepsilon}s_E$
Solow model with endogenous extraction	$\gamma^*_{Y,ENDO} = \frac{\beta}{\beta+\varepsilon}(g+n) - \frac{\varepsilon}{\beta+\varepsilon}q^*$	$\gamma^*_{y,ENDO} = \frac{\beta g - \varepsilon n}{\beta+\varepsilon} - \frac{\varepsilon}{\beta+\varepsilon}q^*$

Notes: Here, the standard Solow model used for comparison is the version with constant population growth *n* and exogenously given technological progress *g*. The parameters *α*, *β* and *ε* denote the income shares of capital, labour and of the non-renewable resource input, respectively, while s_E and *q* denote the exogenous and endogenous extraction rate, respectively.

In the standard Solow model (with constant and exogenously given technological progress and population growth), all aggregate variables (including physical capital, private consumption and aggregate output) grow at the same constant rate, which equals the rate of technological progress (*g*) plus the population growth rate (*n*). The per-capita variables grow at the rate of technological progress. Policy measures, e.g. changes in the saving rate (*s*), only have a (one-time) levelling effect, but no growth effect. Hence, only technological progress contributes to long-run per-capita output growth.

By comparing these growth rates with those in the augmented Solow models as stated in Table 1, we see that the long-run growth rates in these two models will always be lower than the growth rates in the standard Solow model. This is because by definition for the input shares of capital, labour and energy (*α*, *β* and *ε*) the conditions $\alpha, \beta, \varepsilon > 0$ and $\alpha + \beta + \varepsilon = 1$ must hold.

The larger the income share of the non-renewable resource ε, the larger the negative influence on the long-run growth rates of per-capita output becomes.[22] The same intuition applies for the (exogenous and endogenous) extraction rate. Further, we observe that population growth has a negative impact on long-run growth. How significant the negative influence of n becomes also depends on the income share of the resource. Table 2 summarizes how a marginal change in one of the parameters would influence the long-run growth rates of per-capita output in the adapted Solow models.

Table 2: The effects of parameter variations on the long-run growth rates of per-capita output

	$\xi=\alpha$	$\xi=\beta$	$\xi=\varepsilon$	$\xi=g$	$\xi=n$	$\xi=s$	$\xi=s_E$	$\xi=q$
$\frac{\delta\gamma^*_{y,EXO}}{\delta\xi}$	n/a	>0	<0	>0	<0	n/a	<0	n/a
$\frac{\delta\gamma^*_{y,ENDO}}{\delta\xi}$	>0	>0	<0	>0	<0	>0	n/a	<0

Notes: The signs of the partial derivatives show whether a marginal change in one of the parameters / variables, ceteris paribus, has a positive or a negative effect (colour code: green = positive; red = negative) on long-run per-capita output growth. The letters g, n, s, s_E and q denote the rate of technological progress, the population growth rate, the saving rate, and the exogenous and endogenous extraction rate, respectively. The parameters α, β and ε denote the income shares of capital, labour and of the non-renewable resource input, respectively.

It is obvious that a non-renewable resource that is essential in production has a negative influence on the long-run growth rates.

Next, we want to investigate which of the two augmented Solow models generates higher long-run growth rates. Dyerberg Greisen and Hvelplund (2012, 25) compute the steady-state growth rates (Cf. Table 3) by calibrating the models with parameter estimates provided by Sørensen and Whitta-Jacobsen (2010). Further, based on their findings, Dyerberg Greisen and Hvelplund (2012, 7) estimate the exogenous extraction rate to be approximately 2.25 percent.

Table 3: Parameter estimates and the long-run growth rates

Parameter	α	β	ε	s	g	n	δ	s_E
Estimated value	0.30	0.60	0.10	0.15	0.02	0.01	0.05	0.0225
Variable	z^*	q^*	r^*	$\gamma^*_{u,EXO}$	$\gamma^*_{u,ENDO}$	$\gamma^*_{y,SOLOW}$	$\gamma^*_{y,EXO}$	$\gamma^*_{y,ENDO}$
Steady-State	2.264	0.066	0.133	0.045	0.083	0.020	0.013	0.006

Sources: Parameter estimates by Sørensen and Whitta-Jacobsen (2010); Table adapted from Dyerberg Greisen and Hvelplund 2012, 25;
Notes: The letters g, n, r, s, s_E and q denote the rate of technological progress, the population growth rate, the real rental rate, the saving rate, and the exogenous and endogenous extraction rate, respectively. The parameters α, β and ε denote the income shares of capital, labour and of the non-renewable resource input, respectively.

[22] In the case where the parameter ε is equal to zero, the long-run growth rates of aggregate and per-capita output are equal to their values in the standard Solow model.

As suggested above theoretically, we can check from Table 3 that the adapted Solow models generate growth rates of per-capita output that lie below the long-run growth rate of output in the standard Solow model.

A rather surprising finding is that the exogenous model generates higher long-run growth rates of per-capita output (0.013) than the model with the endogenously determined extraction rate (0.006). One would assume that the model with endogenous extraction should perform better, due to the optimizing behaviour of the extraction sector. Dyerberg Greisen and Hvelplund (2012, 26) provide two possible explanations for this: Either the profit-maximizing behaviour of the extraction sector ignores the "growth drag" of the essential non-renewable resource, or the cause lies in the lack of an infinite perfect futures market (as suggested by Stiglitz (1974, 151)), which would prohibit that the resource stocks could be exploited too quickly.

Further, we find that the endogenous extraction rate *q** (0.066) is three times as high as the exogenous growth rate s_E (0.0225) in steady state. This might be explained by the fact that the mechanisms of the endogenous model imply that a higher growth rate of the resource price (0.083) would lead to an increase in the extraction rate. This in turn would lead to a faster depletion of the resource stock and thereby to a lower growth rate of per-capita output in the long run.

The resource price (*u*) exhibits exponential growth in the long run, due to its positive constant growth rate in steady state. This is in accordance to Hotelling's rule but might not be in line with empirical observations, as suggested in Section 2.1. We will further look into this issue in Chapter 4.

Finally, we want to determine how large the rate of technological progress has to be to facilitate positive growth rates in the long run. By substituting the parameter estimates from Table 3 into the respective per-capita output growth rates we get

$$\gamma^*_{y,EXO} = \frac{\beta g - \varepsilon n}{\beta + \varepsilon} - \frac{\varepsilon}{\beta + \varepsilon} s_E > 0 \quad \text{if} \quad g > 0.0054$$

$$\gamma^*_{y,ENDO} = \frac{\beta g - \varepsilon n}{\beta + \varepsilon} - \frac{\varepsilon}{\beta + \varepsilon} q^* > 0 \quad \text{if} \quad g > 0.012$$

To exhibit positive per-capita growth rates, the rate of technological progress must be larger than 0.54 percent in the model with exogenous extraction and 1.2 percent in the Solow model with an endogenous extraction rate. This is in accordance with our theoretical observation that the higher the resource extraction rate is, the larger the rate of technological progress has to be to compensate.

In general, it must hold that

$$g > \frac{\varepsilon(n + s_E)}{\beta} \qquad \text{in the Solow model with exogenous extraction}$$

$$g > \frac{\varepsilon((n+\delta)(\alpha\beta + \alpha\varepsilon))}{\beta s(1+\varepsilon)} - \varepsilon\delta \qquad \text{in the Solow model with endogenous extraction}$$

in order that per-capita output exhibits positive growth rates in the long run.

Apart from the rate of technological progress, the two other possibilities to positively influence long-run growth would be by enforcing population control (thereby lowering the exogenously given population growth rate *n*) or by substituting the resource input with capital. Using capital as a substitute for the resource input is possible to a certain degree, due to the properties of the Cobb-Douglas production function but is limited by the essentiality of the non-renewable resource in production.

In the Solow model with an endogenous extraction rate there also exists another possibility for a policy measure, as the saving rate (*s*) can be used to influence the growth rate of per-capita output directly. The equations below depict the extraction rate (*q*) and the capital-output ratio (*z*) in steady state.

$$q^* = \alpha \frac{1-s}{\alpha+\beta} \frac{1}{z} - \frac{\beta(g+n+\delta)+\varepsilon\delta}{\alpha+\beta}$$

$$z^* = \frac{\beta s + \alpha\varepsilon}{\beta\left(n+g+\delta\right)+\varepsilon\delta}$$

By looking at these equations we see that the extraction rate negatively depends on the capital-output ratio and that both the extraction rate and the capital-output ratio depend on the (exogenously given) saving rate *s*. Therefore, an increase in *s* would lead to a rise in the capital-output ratio. This in turn would lead to a decreasing extraction rate and to a higher rate of per-capita output growth in the long run.[23]

We conclude that our analysis suggests that positive long-run growth is sustainable in a Solow economy after the introduction of an essential non-renewable resource, but only if the rate of technological progress is sufficiently high.

3.1.2. The long-run equilibrium in the endogenous growth model

Now we want to conduct an analysis on the long-run equilibrium in the endogenous growth model by Nguyen and Nguyen-Van (2008). As we have already shown in Section 2.3, the parameter ϕ determines how large the influence of the existing technology stock is on the future evolution of knowledge. We find that in the case where $\phi \neq 1$ the model becomes very unintuitive to analyse. Hence, we will focus our analysis on the case where $\phi = 1$.[24]

If the technology parameter ϕ is equal to one, then *r** is also equal to one. By substituting Equations (3.31) - (3.33) into the growth rates given by the Equations (3.20) - (3.24) and (3.29) - (3.30), we obtain the steady-state growth rates of aggregated output, technology, non-renewable and renewable resource consumption/utilization, the long-run extraction rates, and the growth rates of the non-renewable and renewable resource prices, respectively.

[23] As mentioned above, we have that the income share of capital must be larger than the saving rate. Therefore, the usage of the saving rate as a policy instrument is limited by this condition.
[24] We find that in the case where $\phi \neq 1$ the rate of technological progress is zero in steady state. As long-run output growth is dependent on the rate of innovations (*b*), it seems unlikely that in this case long-run growth would be sustainable.

$$\gamma_Y^* = \frac{b\theta + m\beta - (1-\xi)\rho}{\varepsilon(1-\xi)} \tag{4.1}$$

$$\gamma_A^* = \mathrm{b}\left[1 - \left(\frac{\xi(\varepsilon-1)(b\theta + m\beta + (1-\xi)\delta) + \xi(1-\varepsilon)\delta + \rho}{\varepsilon\xi(1-\xi)}\right)\frac{\psi}{\theta b}\right] \tag{4.2}$$

$$\gamma_Q^* = \gamma_{S_Q}^* = \frac{\xi(1-\varepsilon)(b\theta + m\beta - \xi\delta) - \rho}{\varepsilon\xi(1-\xi)} \tag{4.3}$$

$$\gamma_R^* = \gamma_{S_R}^* = \frac{\xi(1-\varepsilon)(b\theta + m\beta - \xi\delta) - \rho}{\varepsilon\xi(1-\xi)} + m \tag{4.4}$$

$$z^* = -\gamma_{S_Q}^* = -\frac{\left[\xi(1-\varepsilon)(b\theta + m\beta - \xi\delta) - \rho\right]}{\varepsilon\xi(1-\xi)} \tag{4.5}$$

$$u^* = m - \gamma_{S_R}^* = -\frac{\left[\xi(1-\varepsilon)(b\theta + m\beta - \xi\delta) - \rho\right]}{\varepsilon\xi(1-\xi)} \tag{4.6}$$

$$\gamma_{P_Q}^* = \frac{\varepsilon\xi(b\theta + m\beta) + \xi^2(1-\varepsilon)\delta - \rho(\xi(1-\xi) - 1)}{\varepsilon\xi(1-\xi)} \tag{4.7}$$

$$\gamma_{P_R}^* = \frac{\varepsilon\xi(b\theta + m\beta) + \xi^2(1-\varepsilon)\delta - \rho(\xi(1-\xi) - 1)}{\varepsilon\xi(1-\xi)} - m \tag{4.8}$$

The parameters ξ, ψ, α, and β denote the income shares of capital, labour and of the non-renewable and renewable resource inputs, respectively. The letters δ, ρ, θ, m and ε denote the capital depreciation rate, the rate of time preference, the "input-augmenting" parameter, the rate of renewable resource regeneration and the elasticity of marginal utility, respectively. [25]

We observe that the extraction rates of the non-renewable and renewable resources are equal in steady state, hence we have $z^* = u^*$. Further, the growth rate of renewable energy utilization will always be higher than the growth rate of non-renewable resource consumption, $\gamma_R^* > \gamma_Q^*$, while the growth rate of the renewable resource price will always be lower than the growth rate of the non-renewable resource price, $\gamma_{P_R}^* < \gamma_{P_Q}^*$. This is due to the renewable resource constantly regenerating at rate m.

We now want to investigate what effects a marginal change in one of the parameters would induce on the long-run growth rates. Our results are as follows and summarized in Table 4 below:

- An increase in the rate of innovations (*b*), i.e. a higher productivity in the R&D sector, would lead to an increase in the growth rate of technology. Due to the growth rate of output being dependent on the growth rate of technology, this would in turn lead to an increase in the long-

[25] Remember that the parameter θ can be chosen to be any of the four input shares α, β, ξ and ψ.

run growth rate of output. If the elasticity of marginal utility (ε) is smaller than one, an increase in *b* would also increase the (negative) steady-state growth rate of (renewable and non-renewable) energy utilization, i.e. fewer resources would be needed in production due to a higher frequency of technological innovations. Thereby the growth rates of the resource prices would decrease, which would make it more profitable for the resource owner to keep the resource *in situ*. Hence, the extraction rates would be lower in the long run. Vice-versa holds in the case where the elasticity of marginal utility is larger than one. The same intuition applies in the case where there is an increase in the regeneration rate of the renewable resource (*m*).

- An increase in the rate of time preference (ρ) implies that the representative consumer would obtain higher utility from current consumption relative to future consumption. Intuitively, a consumer who prefers to consume more today does not care what happens in the long run. Such a behaviour would lead to a lower growth rate of knowledge and in turn to a lower growth rate of output. Further, this implies that the growth rates of renewable and non-renewable resource consumption would decrease in the long run. Consequently, this would lead to higher resource prices and thereby to higher extraction rates in equilibrium.

- In the case where the long-run growth rate of output is larger than zero, $\gamma_Y^* > 0$, a positive change in the elasticity of marginal utility (ε) would have the same effect as an increase in the time preference rate. In this case, the growth rates of technology, output and resource consumption would decrease, while the extraction rates and resource prices would be higher in steady state. Vice-versa occurs if the growth rate of output is smaller than zero in the long run.

Table 4: The effects of parameter variations on the long-run growth rates

	$\frac{\delta\gamma_Y^*}{\delta\chi}$	$\frac{\delta\gamma_A^*}{\delta\chi}$	$\frac{\delta\gamma_Q^*}{\delta\chi}$	$\frac{\delta\gamma_R^*}{\delta\chi}$	$\frac{\delta z^*}{\delta\chi}$	$\frac{\delta u^*}{\delta\chi}$	$\frac{\delta\gamma_{P_Q}^*}{\delta\chi}$	$\frac{\delta\gamma_{P_R}^*}{\delta\chi}$
$\chi = b$	>0	>0	>0 if $\varepsilon < 1$	>0 if $\varepsilon < 1$	<0 if $\varepsilon < 1$	<0 if $\varepsilon < 1$	<0 if $\varepsilon < 1$	<0 if $\varepsilon < 1$
$\chi = m$	>0	>0	>0 if $\varepsilon < 1$	>0 if $\varepsilon < 1$	<0 if $\varepsilon < 1$	<0 if $\varepsilon < 1$	<0 if $\varepsilon < 1$	<0 if $\varepsilon < 1$
$\chi = \varepsilon$	<0 if $\gamma_Y^* > 0$	<0 if $\gamma_Y^* > 0$	<0 if $\gamma_Y^* > 0$	<0 if $\gamma_Y^* > 0$	>0 if $\gamma_Y^* > 0$	>0 if $\gamma_Y^* > 0$	>0 if $\gamma_Y^* > 0$	>0 if $\gamma_Y^* > 0$
$\chi = \rho$	<0	<0	<0	<0	>0	>0	>0	>0

Notes: The signs of the partial derivatives show whether a marginal change in one of the parameters / variables, ceteris paribus, has a positive or a negative effect (colour code: green = positive; yellow = dependent on a condition; red = negative) on the long-run growth rates. The letters *b*, *m*, ε and ρ denote the rate of technological innovations, the rate of renewable resource regeneration, the elasticity of marginal utility and the rate of time preference, respectively.

We still need to check which of the respective growth rates will be positive and which will exhibit negative growth in the long run. First, we consider the growth rates of resource consumption, the growth rates of resource prices and the extraction rates. From one of the transversality conditions given by

$$\lim_{t\to\infty} \mu S_Q^* e^{-\rho t} = 0, \text{ where } \mu = \mu(0)e^{-\rho t}$$

and

$$S_Q^*(t) = S_Q^*(0)\,\mathrm{e}^{\gamma_Q^* t} = 0$$

we get

$$\lim_{t\to\infty} \mu(0) S_Q^*(0)\,\mathrm{e}^{\gamma_Q^* t} = 0, \text{ which implies } \gamma_Q^* < 0 \text{ , } \gamma_{P_Q}^* > 0 \text{ and } z^* > 0$$

Similarly, from the transversality condition given by $\lim_{t\to\infty} \lambda S_R^* e^{-\rho t} = 0, \text{ where } \lambda = \lambda(0)e^{(\rho - m)t}$, we obtain

$$\lim_{t\to\infty} \lambda(0) S_R^*(0)\,\mathrm{e}^{(\gamma_R^* - m)t} = 0, \text{ which implies } \gamma_R^* - m < 0 \text{ , } \gamma_{P_R}^* > 0 \text{ and } u^* > 0$$

In words this means, that the growth rates of renewable and non-renewable resource utilization exhibit negative growth rates, while resource prices and the extraction rates grow at positive constant rates in the long run.

By definition we have $r = A^{\phi-1}$ and $q = L_Y A^{\phi-1}$. If $\phi = 1$, then $r = 1$ and because the population is normalized to one we have that $L_Y < 1$ and therefore it must hold that $r^* > q^*$. This implies that

$$\gamma_A^* = b(1 - q^*) > 0$$

Hence, in the case where ϕ is equal to one, knowledge grows at a positive constant rate in steady state.

Finally, we want to examine under which conditions positive growth rates of output are sustainable in equilibrium. From Equation (4.1) we get that

$$\gamma_Y^* > 0 \quad \text{if} \quad \frac{b\theta + m\beta}{(1-\xi)} > \rho$$

We conclude that whether positive long-run growth is feasible, depends on the rate of time preference (ρ), i.e. on how patient consumers are, on the rate the renewable resource stock regenerates (m) and on the rate at which the R&D sector can push out innovations (b). Further, a higher income share of capital (ξ) and of the renewable resource input (β) also has a positive effect, due to the possibility to use these inputs as partial substitutes for the non-renewable resource input in production. So, similar to the result in the last section, positive long run growth is sustainable in this endogenous growth model, but again, only under certain conditions.

3.2. Transitional dynamics

3.2.1. Transitional dynamics in the Solow model with endogenous extraction

Next, we analyse what happens in the short-run, i.e. we look at the transitional dynamics. We only consider the Solow model with endogenous extraction, because the implications it provides are more interesting due to the optimizing behaviour of the extraction sector.

For convenience, we restate the law of motions for the capital-output ratio (*z*) and the endogenous extraction rate (*q*).

$$\dot{z} = \frac{\beta s + \alpha\varepsilon}{\alpha+\beta} - \left(\frac{\beta(g+n+\delta)+\varepsilon\delta}{\alpha+\beta}\right) z$$

$$\dot{q} = \frac{1}{\alpha+\beta}\left(\alpha\frac{s-1}{z} + \beta(g+n+\delta)+\varepsilon\delta\right) q + q^2$$

In steady state, the capital-output ratio and the extraction rate are given by the following equations.

$\underline{\dot{z}=0:}$

$$z^* = \frac{\beta s + \alpha\varepsilon}{\beta(n+g+\delta)+\varepsilon\delta}$$

$\underline{\dot{q}=0:}$

$$q^* = \alpha\frac{1-s}{\alpha+\beta}\frac{1}{z} - \frac{\beta(g+n+\delta)+\varepsilon\delta}{\alpha+\beta}$$

Figure 1 and Figure 2 depict the law of motions for the capital-output ratio and for the extraction rate respectively. By looking at Figure 1 we can see that if the initial capital-output ratio initially lies below its steady-state value (*z**) that *z* would exhibit positive growth and would start converging towards *z**. The contrary would happen if the initial *z* lies above *z**. This constitutes a stable system, in which the capital-output ratio ultimately converges towards its equilibrium value.

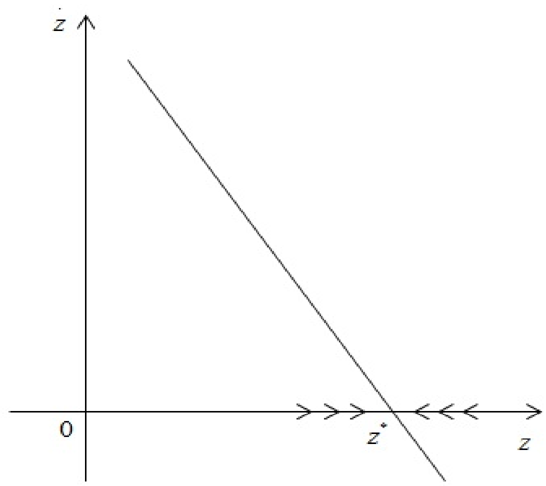


Figure 1: The law of motion for the capital-output ratio (*z*)

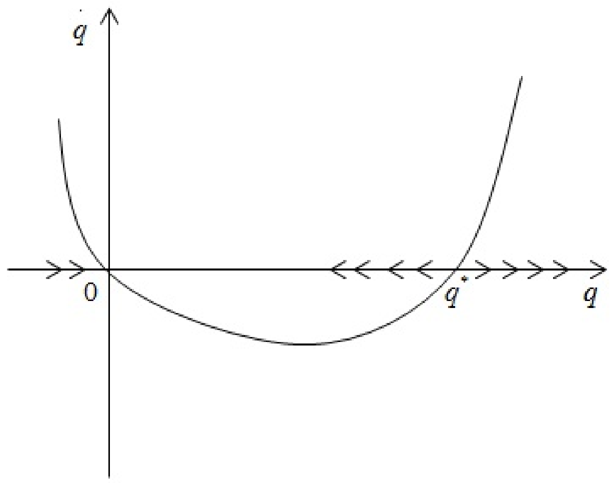


Figure 2: The law of motion for the endogenous extraction rate (*q*)

We can easily check by looking at Figure 2 that this stabilizing mechanism does not exist for the extraction rate (*q*). If the initial *q* lies above *q**, the growth rate of *q* would be larger than zero and the extraction rate would start diverging towards infinity, while if the initial *q* lies below its long-run value, the growth rate would become negative and *q* would start converging towards zero.

To analyse the dynamics of this system, we utilize a phase diagram as depicted in Figure 3 below. The stable system of *z* and the diverging system of *q* entail that a unique stable saddle path exists in this model.[26] An economy, with initial values of *z* and *q* that lie on this saddle path, would ultimately converge towards the long-run equilibrium point E, independently of whether the economy starts on the left-hand or on the right-hand side of the saddle point path in relation to E.

The economic intuition behind this can be explained as follows. If the capital-output ratio initially lies below *z**, *z* would exhibit positive growth rates. Therefore, capital (*K*) would grow faster compared to the other input factors. Due to the rising capital input, the nominal capital rental rate (*r*) and subsequently the real interest rate (ρ) would decrease. The decreasing real interest rate and the rising capital-output ratio would in turn lead to lower or negative growth rates of the resource price (*u*), which is negatively dependent on *z* and positively dependent on ρ , in accordance to Hotelling's rule.

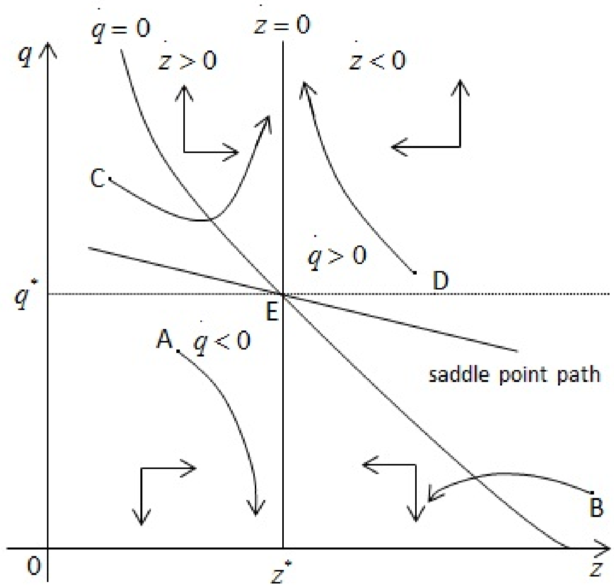


Figure 3: The transitional dynamics in the Solow model with endogenous extraction

As the resource price decreases, it would become more profitable for the owner of the resource stock to keep the resource *in situ*, which in turn would lead to a decreasing resource extraction rate. This development would go on until the capital-output ratio reaches its equilibrium value *z*,* where the growth rate of the capital-output ratio is equal to zero. At this stage the extraction rate would have also converged to its long-run value *q** and would exhibit zero growth. Vice-versa holds for an economy that starts on the right-hand side of the saddle path.

[26] Cf. Appendix A.2.3 for the proof that a stable saddle point path exists in this model.

However, what happens in the case where the economy is starting off the equilibrium path? Again, we look at Figure 3 to find an answer to this question:

- If the economy starts at point A, then z would grow at a positive rate and q would converge towards zero over time, due to exhibiting negative growth rates. If the economy is starting at point B, then the extraction rate would first grow at a positive rate, but because of z converging towards z^*, the growth rate of q would become negative at a certain point and q would once again converge towards zero.

- If the economy starts at points C or D, then q would exhibit positive growth rates and the extraction rate would diverge towards infinity. Note that this is not possible due to the resource constraint defined in Equation (2.18). (Cf. Section 2.2.2)

- Further, Dyerberg Greisen and Hvelplund (2012) analyse a “knife-edge” case, where the capital-output ratio is equal to z^* and q lies just below the saddle path in the beginning. They argue that this does not make sense economically, because in this case the economy would not exhaust the whole resource stock. As this would in turn lead to excess profits, this behaviour would be non-optimal and will therefore not commence. (Dyerberg Greisen and Hvelplund 2012, 22, 35-39)

Due to the optimizing behaviour of the extraction sector, it seems plausible to assume that an owner of a non-renewable resource would choose the extraction rate (with z given) s.t. the economy would be perched on the stable saddle-point path in the beginning.[27]

Concluding, we observe that an economy that is starting farther away from its long-run equilibrium will converge faster towards steady state than an economy that starts near the equilibrium point. Hence, the convergence pattern is the same as in the standard Solow model.

3.2.2. Transitional dynamics in the endogenous growth model

In Section 2.3, we determined the steady-state growth rates in the endogenous growth model. Further, we defined six variables from which the other growth rates can be derived: the output-capital ratio (x), the consumption-capital ratio (y), the extraction rate of the non-renewable resource (z), the extraction rate of the renewable resource (u), and r and q. To analyse the transitional dynamics in this model, we first have to determine the law of motions for these variables.

We first look at the case where the technology parameter Φ is equal to one. In this case, r is also equal to one and we only have to derive the law of motions for the other five variables. We achieve this by differentiating logarithmically and using the optimal growth rates as stated in Equations (3.10) - (3.19).

[27] The empirics (Cf. Maddison 1991, 67) suggest that the capital-output ratio has been increasing for the Netherlands, France, Japan and the United Kingdom during the period 1890 – 1987. This implies that economies would start on the left-hand side of the equilibrium path. (Dyerberg Greisen and Hvelplund 2012, 24)

Table 5 below depicts the law of motions for the output-capital ratio (x), the consumption-capital ratio (y), the non-renewable (z) and renewable resource extraction rates (u), and labour used in production (*q*). The law of motions describe the dynamic system of this model. To find out how this system behaves around the steady state, we need to determine the *eigenvalues* (characteristic roots) of the corresponding Jacobian matrix.[28] We find that one of the characteristic roots is negative, while the other four *eigenvalues* are positive. This suggests that a unique stable saddle point exists in the case where ϕ is equal to one.[29]

In the case where the technology parameter ϕ is not equal to one, Nguyen and Nguyen-Van (2008) proof that two of the characteristic roots are negative, while the remaining four are positive. Their result suggests that in this case also a unique saddle point of stability exists.[30]

Unfortunately, we cannot derive any meaningful insights on how the economy converges towards the long-run equilibrium, as the dynamic system of this model is quite complex and very unintuitive to analyse.

Table 5: The law of motions for the key variables in the endogenous growth model (*Φ*=1 and *r*=1)

Name	Variable	Law of motion
Output-capital ratio	$x=\frac{Y}{K}$	$\dot{x}=\left((\xi-1)x+\frac{b\theta+m\beta+\delta(1-\xi)}{\xi}\right)x$
Consumption-capital ratio	$y=\frac{C}{K}$	$\dot{y}=\left(y+\frac{(\xi-\varepsilon)x+(\varepsilon-1)\delta-\rho}{\varepsilon}\right)y$
Labour used in production	$q=L_Y$	$\dot{q}=\left(-y+\frac{b\theta}{\psi}q+\frac{b\theta+m\beta+(1-\xi)\delta}{\xi}\right)q$
Extraction rate (Non-renew.)	$z=\frac{Q}{S_Q}$	$\dot{z}=\left(-y+z+\frac{b\theta+m\beta+(1-\xi)\delta}{\xi}\right)z$
Extraction rate (Renewable)	$u=\frac{R}{S_R}$	$\dot{u}=\left(-y+u+\frac{b\theta+m\beta+(1-\xi)\delta}{\xi}\right)u$

Notes: The parameters *ξ*, *ψ*, *α*, and *β* denote the income shares of capital, labour and of the non-renewable and renewable resource inputs, respectively. The letters *δ*, *ρ*, *θ*, *m* and *ε* denote the capital depreciation rate, the rate of time preference, the "input-augmenting" parameter, the rate of renewable resource regeneration and the elasticity of marginal utility, respectively.

[28] Cf. Appendix A.3.2, and Nguyen and Nguyen-Van (2008, 11 - 14);
[29] Cf. Barro and Sala-i-Martin (1999, 587);
[30] Cf. Nguyen and Nguyen-Van (2008, 12 - 14);

4. Empirics

In the last chapter, we observed that the predicted prices of natural resources exhibit exponential growth in the long run. This occurs in both the Solow models with an exogenous and endogenous extraction rate, and in the endogenous growth model. We will show in this chapter that this pattern cannot be supported by empirical evidence.

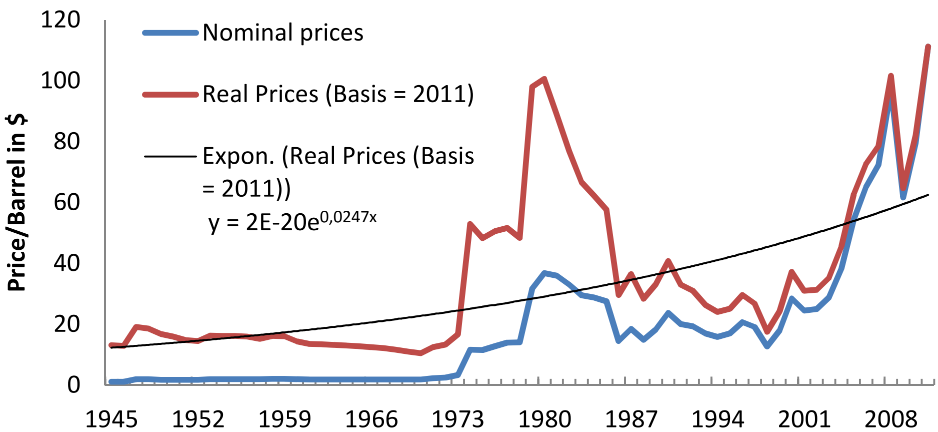
Further, Nguyen and Nguyen-Van (2008) provide an econometric estimation of their endogenous growth model. Therefore, we give a short summary of their findings and investigate how the implications of their theoretical model hold against the empirics.

4.1. Growth rates of non-renewable resource prices

We already mentioned in Section 2.1 that Hotelling's rule implies that resource prices should exhibit exponential growth in the long run, but that this implication is contradicted by the results of many empirical studies. These studies suggest that the prices of most non-renewable resources have been constant over long periods. In this section, we want to investigate this critique further and give a short review on some of our empirical findings.

We first look at the absolute price levels of one of the most important non-renewable resources - crude oil. Figure 4 depicts the evolution of crude oil prices during the period 1945 – 2011, where the blue line depicts nominal prices and the red line depicts real prices (with the base year 2011). We observe that prices of crude oil have been relatively stable between the end of WWII and the early 1970s. There is one huge spike in the year 1973, when the first oil crisis occurred, and another spike around the year 1980, which coincides with the Iran-Iraq war. After that period, the price has successively fallen over the course of the 1980s and remained relatively stable until around the year 2000. Since then the crude oil price has been increasing steadily and reached an all-time-high in 2011, even considering real prices. The evolution of the crude oil price over this period suggests that there might be a positive (non-linear) trend in the price levels.

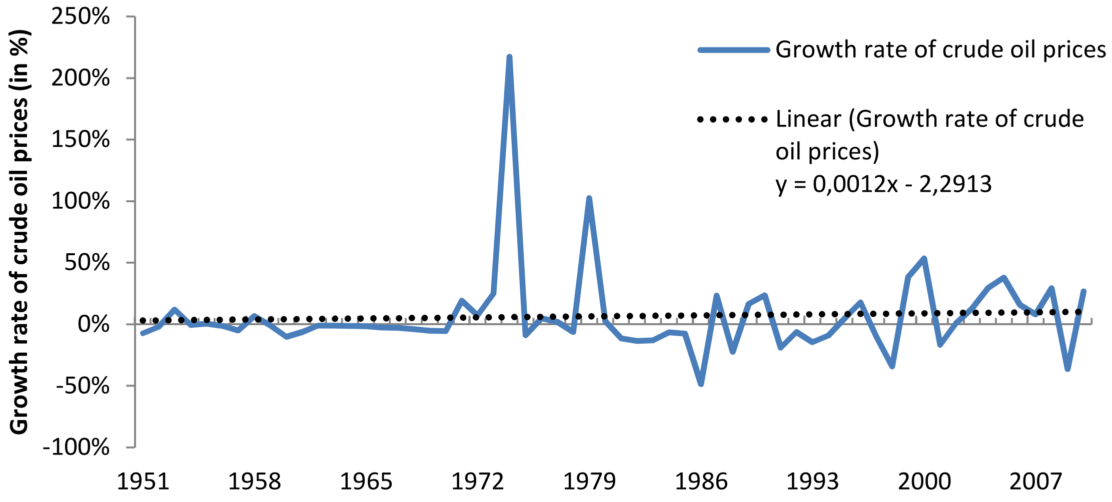
Figure 4: Crude oil prices per barrel in USD (1945 - 2011)



Source: Data from BP Statistical Review of World Energy 2011

However, as Hotelling's rule is not formulated in terms of absolute price levels, we want to look at the growth rates of prices instead. Therefore, Figure 5 depicts the growth rates in crude oil prices during the period 1951 – 2011. We observe that, even though there are very high fluctuations in the growth rates, that there might be a slightly positive trend in the data as depicted by the dotted line.

Figure 5: Growth rates in real crude oil prices (2011 USD) (1951 - 2010)



Source: Data from BP Statistical Review of World Energy 2011

This suggests that the growth rates might be either centred on a mean of zero, as suggested by the empirical studies on this subject, or that there is a very low positive trend in crude oil prices. We cannot provide a definitive conclusion on this matter without conducting a deeper analysis. As this would be out of scope of this thesis, due to its emphasis on economic theory, we want to give a summary of two findings in the literature.

Dyerberg Greisen and Hvelplund (2012) also perform an analysis on the evolution of crude oil prices, but on a longer timescale. (Cf. Table 6) They consider the development of the price levels and of the corresponding growth rates during the period 1900 – 2010. They find that on the one hand the null hypothesis that the growth rates in crude oil prices have been constant over that period ($\beta_0 = 0$) cannot be rejected, but that on the other hand the null hypothesis that the price levels have been constant ($\gamma_1 = 0$) can be rejected. Hence, they cannot draw a definitive conclusion from their findings either.[31]

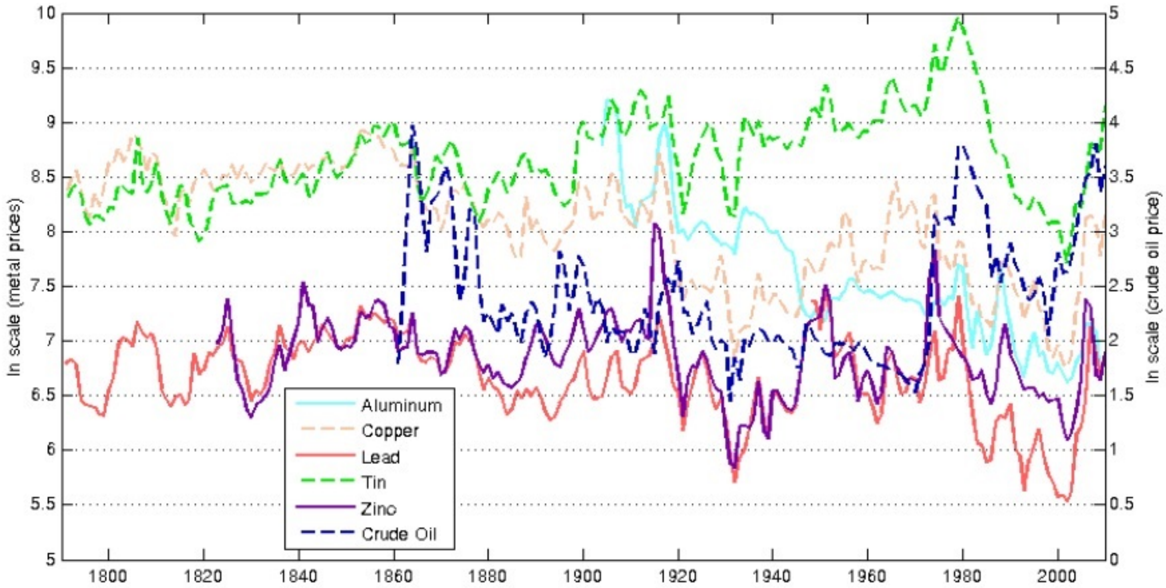
Table 6: Testing for trendless growth rates and constant price levels in crude oil prices (1900 – 2010)

Oil price	Parameter	Estimate	Std. error	t-value	p-value
Growth rate	Intercept β_0	0.054	0.028	1.950	0.053
Price level	Intercept γ_1	0.038	0.028	1.340	0.182

Sources: Data from BP Statistical Review 2010; Table adapted from Dyerberg Greisen and Hvelplund 2012, 5-6;
Notes: Dyerberg Greisen and Hvelplund (2012) use heteroskedasticity consistent standard errors.

[31] Cf. Dyerberg Greisen and Hvelplund (2012, 4 - 6);

Figure 6: Prices of various non-renewable resources in constant 1982 USD (in log)



Sources: Data from Schmitz (1979), BP plc. (2010), US Bureau of Mines (1980) and Federal Institute for Geosciences and Natural Resources (2011); Data collected by Schwerhoff and Stürmer (2012); Figure as depicted in Schwerhoff 2012, 24;
Notes: "All prices, except for the price for crude oil, are prices of the London Metal Exchange and its predecessors. The oil price is the US price, as assembled by BP plc. (2010). As the price of the London Metal Exchange used to be denominated in Sterling in earlier times, [Schwerhoff and Stürmer (2012)] converted these prices to US-Dollar by using historical exchange rates from Officer (2011b). [Schwerhoff and Stürmer (2012)] use the US-Consumer Price Index provided by Officer (2011a) and the U.S. Bureau of Labour Statistics (2010) for deflating prices. The secondary y-axis relates to the price of crude oil." (Schwerhoff and Stürmer 2012, 24)

Schwerhoff and Stürmer (2012, 4) conduct a deeper analysis on this subject, using data on real crude oil prices and prices of five base metals - Aluminium, Copper, Lead, Tin and Zinc. They conclude that, for the periods they consider, the growth rates of the resource prices are centred on a mean of zero, i.e. that there is no statistically significant positive trend in price evolution. (Cf. Figure 6 and Table 7) As mentioned above, their findings are in line with many renowned empirical studies on this subject, which either come to the same conclusion or find that there is a low positive, but statistically insignificant, trend in the evolution of non-renewable resource prices. [32] Hence, our findings suggest that the implications of Hotelling's rule do not hold against empirical evidence. However, as mentioned in Section 2.1.2, the literature offers some recommendations on how Hotelling's rule could be extended to improve on its shortcomings.

Table 7: Testing for trendless growth rates in non-renewable resource prices

		Aluminium	Copper	Lead	Tin	Zinc	Crude Oil
Range		1905 - 2009	1792 - 2009			1824 - 2009	1862 - 2009
Constant	Coeff.	-1.764	0.184	0.109	1.668	0.702	7.236
	t-stat.	-0.181	0.073	0.4	0.73	0.148	0.79
Lin. Trend	Coeff.	0.008	0.011	0.006	-0.001	0.013	-0.017
	t-stat.	0.138	0.533	0.259	-0.079	0.378	-0.276

Sources: Data from Schmitz (1979), BP plc. (2010) and Federal Institute for Geosciences and Natural Resources (2011); Data collected by Schwerhoff and Stürmer (2012); Table adapted from Schwerhoff and Stürmer 2012, 28;
Notes: "The table presents coefficients and t-statistics for regressions of the growth rates on a constant and a linear trend. ***, **, and * indicate significance at the 1%, 2.5% and 5% level, respectively." (Schwerhoff and Stürmer 2012, 28)

[32] Cf. Barnett and Morse (1963), Smith (1978), Slade (1982), Berck and Roberts (1996), Halvorsen and Smith (1996), and Farrow (1985), amongst others;

4.2. Estimating the endogenous growth model

Nguyen and Nguyen-Van (2008) conduct an empirical analysis on their endogenous growth model, in which they focus on the growth rates of the output-capital ratio ($\gamma_{Y/K}$) and of renewable (γ_R) and non-renewable resource utilization (γ_Q). [33]

Further, they only consider the case where the technology parameter ϕ is equal to one, and the three growth rates are therefore only dependent on the capital-output ratio ($x = Y/K$) and on the consumption-capital ratio ($y = C/K$). In the case where ϕ is not equal to one, the model is not well defined and an econometric estimation is therefore impossible without putting further restrictions in place. (Nguyen and Nguyen-Van 2008, 14)

Nguyen and Nguyen-Van (2008) transform the model into a panel structure by defining the following equations.

<u>The growth rate of output-capital consumption:</u>

$$\gamma_{(Y/K)_{it}} = \alpha_0 + \alpha_1 Y_{i,t-1} / K_{i,t-1} + \varepsilon_{(Y/K)_{it}}$$

$$\text{with } \alpha_0 = \frac{b\theta + m\beta + \delta(1-\xi)}{\xi} + (\xi - 1)(1 - e^{\lambda_1 T})\, x^*$$

$$\text{and } \quad \alpha_1 = (\xi - 1) e^{\lambda_1 T}$$

<u>The growth rate of non-renewable energy consumption:</u>

$$\gamma_{Q_{it}} = \beta_0 + \beta_1 C_{i,t-1} / K_{i,t-1} + \varepsilon_{Q_{it}}$$

$$\text{with } \beta_0 = \frac{b\theta + m\beta + \delta(1-\xi)}{\xi} - e^{\lambda_1 T} y_0 - (1 - e^{\lambda_1 T}) y^*$$

$$\text{and } \quad \beta_1 = -e^{\lambda_1 T}$$

<u>The growth rate of renewable energy consumption:</u>

$$\gamma_{R_{it}} = \gamma_0 + \gamma_1 C_{i,t-1} / K_{i,t-1} + \varepsilon_{R_{it}}$$

$$\text{with } \gamma_0 = \frac{b\theta + m(\beta + \xi) + \delta(1-\xi)}{\xi} - (1 - e^{\lambda_1 T}) y^*$$

$$\text{and } \quad \gamma_1 = -e^{\lambda_1 T}$$

where $i = 1, \ldots, N$ denotes the index of countries, $t = 1, \ldots, T$ denotes the time index and ε denotes the respective error terms, which include country and time effects.

[33] Cf. Nguyen and Nguyen-Van (2008, 14 - 19);

Their model predicts that the coefficients (α_1, β_1 and γ_1) are negative. Due to possible correlations of the regressors with unobserved factors, they estimate their model using Generalized Methods of Moments (GMM), because this method accounts for such occurrences. (Nguyen and Nguyen-Van 2008, 17)

Their data sample includes 108 observations from twenty-seven OECD countries (N=27), using data sets corresponding to the five year interval ranging from 1977 to 1997 (T=4). They obtained the data for production (*Y*), physical capital (*K*) and private consumption (*C*) from the Penn World Table (per-capita variables, 1996 prices, PPP), while they collected data on renewable (*R*) and non-renewable energy consumption (*Q*) from the International Energy Agency (IEA).[34] (ibid. 17)

The results of their econometric estimation are depicted below in Table 8. We observe that, as predicted by their model, the coefficients α_1, β_1 and γ_1 are negative. Further, the coefficients α_1 and γ_1 are statistically significant at the 5% and 10% levels respectively, while the value for β_1 is insignificant. Nguyen and Nguyen-Van (2008) further state, "the Wald test confirms the existence of time effects in all regressions" (the test results are either significant on the 5% or on the 10% level) and that "the Sargan test for over-identifying restrictions […] is always satisfied". (ibid. 19) They conclude that their model provides a relatively good fit to the data.

Table 8: Estimation results for the endogenous growth model (Nguyen and Nguyen-Van 2008)

Equation	Variable	Coeff.	Sargan	AR(1)	AR(2)	Wald-Test
$\gamma_{(Y/K)t}$	$(Y/K)_{t-1}$	-0.174** (0.060)	0.229	-3.08**	-1.15	17.4**
$\gamma_{Q(t)}$	$(C/K)_{t-1}$	-0.015 (0.067)	0.883	-2.86**	1.41*	17.8**
$\gamma_{R(t)}$	$(C/K)_{t-1}$	-0.275* (0.166)	5.7	-2.37**	-0.553	6.52*

Source: Table adapted from Nguyen and Nguyen-Van 2012, 28;
Notes: "Regressions include country effects and year effects. Over-identifying restrictions are tested by the Sargan test. AR (1) and AR (2) tests are the Arellano and Bond (1991) tests for serial correlation of order 1 and 2 respectively. The Wald test is for significance of year dummies. Estimation results are obtained by GMM with robust standard error *â la White* given in parentheses. * and ** represent significance levels of 10% and 5% respectively." (Nguyen and Nguyen-Van 2008, 28)

[34] Nguyen and Nguyen-Van (2008) measure non-renewable energy consumption as the sum of fossil fuels (coal, gas and crude oil), and renewable energy consumption as the sum of nuclear energy, hydroelectricity, geothermal energy, renewable fuels and waste, solar energy, wind energy, and hydraulic energy. (Nguyen and Nguyen-Van 2008, 17)

5. Model Comparison and Critical Assessment

At last, we want to critically assess the models discussed in this thesis and further we want to summarize and compare our findings from the last chapters.

In Chapter 3, we have analysed the long-run equilibria in our models and found that, in both the adapted Solow models and in the endogenous growth model, long-run growth is sustainable under certain conditions. In the two augmented Solow models long-run economic growth is feasible if the rate of technological progress is sufficiently high, while in the endogenous growth model the sustainability of long-run growth mainly depends on the rate of innovations, the regeneration rate of the non-renewable resource stock and on the patience of consumers (i.e. on the rate of time preference). From these findings, we can immediately spot the main differences between the adapted Solow models and the endogenous growth model.

The first difference is the most obvious one. In the endogenous growth model, technological progress is not an exogenously given constant, but determined endogenously. This kind of modelling accounts for the major downside of classical models like Solow and Ramsey-type frameworks, as technological progress does not fall like "*manna from heaven*", but is instead determined in an endogenously modelled R&D sector. This not only enabled us to examine how long-run output growth is influenced by technological progress, but also allowed us to investigate how a higher productivity in the R&D sector affects the equilibrium prices, resource utilization and the extraction rates. Still, we have to admit that the implications the Solow models and the endogenous growth model provide, concerning the role of technological progress in economic growth, are basically the same.

Another major difference is that the endogenous growth model includes consumer optimization. We have seen that the feasibility of long-run economic growth depends on the patience of the representative consumer. This intuition is very plausible, as we would expect that a society that pursues a more sustainable way of living would have fewer problems to economize their scarce resources. We think that the inclusion of consumer optimization is one of the major advantages of the endogenous growth model over the adapted Solow models.

The third difference is that the endogenous growth model does also include renewable resources. As in all three models the final output is produced according to a Cobb-Douglas production function, the factor inputs can be partially substituted for one another. However, in the endogenous growth model not only can capital be used to substitute for the non-renewable resource input, but the renewable resource can also be used for substitution. We have seen that a higher regeneration rate of the renewable resource stock acts as a kind of "indirect" technological progress as it reduces the "pressure" on the economies and allows for higher growth rates of output in equilibrium.

Further, we found that the models provide the same implications concerning the extraction rates and the resource prices in equilibrium. In both models, the resource prices grow at constant positive rates in steady state. In the Solow model with an endogenous extraction rate this happens in accordance to Hotelling's rule, while in the Solow model with exogenous extraction and in the endogenous growth model the lack of realistic cost modelling generates the exponential growth rates in resource prices. As we have shown in the last chapter, this behaviour is not supported by the empirics and would need to be accounted for by different modelling strategies. In the literature, many possible theoretical extensions can be found, concerning the endogenous modelling of exploration, extraction costs, quality and durability of ore, and market structures, amongst others. [35]

Apart from that, we find that the transitional dynamics provide a very plausible explanation of the economic behaviour, at least in the Solow model with an endogenous extraction rate. The transitional dynamics in the endogenous growth model are far more complex and therefore much more difficult to analyse. We found that a unique saddle point of stability exists in the endogenous growth model, but we were not able to get any insights on how the economy converges towards equilibrium due to the complexity of the dynamic system.

To conclude, we think that both types of frameworks are useful in their own way. The Solow model, even though it has the disadvantage that it does not explain where technological progress comes from, still provides interesting conclusions and is very accessible due to its simplicity. The more complex endogenous growth model has the advantage that it provides deeper insights into the mechanisms that are affecting the long-run equilibrium growth rates. However, the downside is that it basically gives the same implications as the Solow models concerning the role of technological progress in economic growth, while it is at the same time much more difficult to analyse in terms of the transitional dynamics.

[35] Cf. Krautkraemer (1998, 2070 - 2078) and Gaudet (2007, 6 - 21);

6. Conclusion

In this thesis, we have discussed three different types of economic growth models, two variants of an augmented Solow model and an endogenous growth model, and conducted an analysis on the long-run equilibrium and on the transitional dynamics in the respective models. We have deliberately chosen these models as they cater to the two major modelling approaches used in the field of resource economics. We have seen that utilizing Hotelling's rule to model the extraction of a non-renewable resource endogenously provides a better understanding of resource pricing and allows us to examine the optimizing behaviour of the extraction sector directly. Further, we found that the endogenous growth model provides even deeper insights into the underlying economic mechanisms, as it explains where technological progress comes from and does include consumer optimization. The major downside of this model is that it becomes very unintuitive to analyse in some cases and that the transitional dynamics are overly complicated. Therefore, it is quite difficult to get any insights from them on how the economy is converging towards the long-run equilibrium.

As we stated in the beginning, we wanted to find out if and under what conditions long-run economic growth is sustainable in an economy that is dependent on a non-renewable resource. Our results suggest that, under certain conditions, long-run economic growth is feasible in the models discussed in this thesis, but we also found that the empirics do not support the theoretical implications concerning the evolution of resource prices. As mentioned in the last chapter, these shortcomings would have to be accounted for by developing different modelling strategies.[36]

Even though the models include many simplifying assumptions, we still find that the models provide intuition that describes a kind of behaviour that we would also expect in a real economy. For further research, it would be appealing to consider an endogenous growth model that also includes population growth, to examine whether in such a model long-run economic growth would still be sustainable. Further, it would be interesting to explore the impact of negative externalities like pollution on the behaviour of the optimizing representative consumer and on long-run growth. The economic literature on this topic already provides some examples of such models.[37]

To conclude, we think this has been a very important subject to work on, as securing the resource and energy supply is and will be one of the most pressing issues mankind faces throughout the 21st century. In the end, the question is not only whether economic growth is sustainable or not, but also what our lives and the world in its entirety will be like in the future.

[36] Cf. Krautkraemer (1998, 2070 - 2078) and Gaudet (2007, 6 - 21);
[37] Cf. Grimaud and Rougê (2004 and 2005);

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Appendix

A.1. The Solow model with a non-renewable resource and an exogenous extraction rate

A.1.1. Deriving the law of motion for the capital-output ratio

We define the capital-output ratio (z) as being

$$z \equiv \frac{K}{Y} = \frac{K}{K^{\alpha}(AL)^{\beta}E^{\varepsilon}} = K^{1-\alpha}(AL)^{-\beta}E^{-\varepsilon} \tag{1.1}$$

By taking logarithms and differentiating w.r.t. time, we obtain the growth rate of the capital-output ratio

$$\gamma_z = \frac{\dot{z}}{z} = (1-\alpha)\frac{\dot{K}}{K} - \beta\left(\frac{\dot{A}}{A} + \frac{\dot{L}}{L}\right) - \varepsilon\frac{\dot{E}}{E} = (1-\alpha)\frac{\dot{K}}{K} - \beta(g+n) - \varepsilon\gamma_E \tag{1.2}$$

Next, we have to determine the growth rate of the energy input. We know that

$$E = s_E R \tag{1.3}$$

and that
$$\dot{R} = -E = -s_E R \tag{1.4}$$

By differentiating (1.3) logarithmically, we get

$$\gamma_E = \frac{\dot{E}}{E} = \frac{\dot{s}_E}{s_E} + \frac{\dot{R}}{R} = 0 + \frac{\dot{R}}{R} = \frac{\dot{R}}{R} \tag{1.5}$$

Substituting (1.4) into (1.5) yields the growth rate of the energy input

$$\gamma_E = \frac{\dot{E}}{E} = -\frac{E}{R} = -\frac{s_E R}{R} = -s_E \tag{1.6}$$

Further, we know that capital evolves according to the equation

$$\dot{K} = sY - \delta K \tag{1.7}$$

Finally, we derive the law of motion for the capital-output ratio by substituting (1.6) and (1.7) into (1.2)

$$\dot{z} = (\beta+\varepsilon)s + (\varepsilon(s_E - \delta) - \beta(n+g+\delta))\,z \tag{1.8}$$

A.1.2. Deriving the growth rate of the non-renewable resource price

Like the other two input factors, the factor energy earns its marginal product, the non-renewable resource price (u).

$$u = \varepsilon K^{\alpha}(AL)^{\beta}E^{\varepsilon-1} = \varepsilon\,(Y/E) \tag{1.9}$$

By taking logarithms and by differentiating w.r.t. time we get

$$\gamma_u = \frac{\dot{u}}{u} = \frac{\dot{\varepsilon}}{\varepsilon} + \frac{\dot{Y}}{Y} - \frac{\dot{E}}{E} = 0 + \gamma_Y - \gamma_E \tag{1.10}$$

We know that

$$\gamma_Y^* = \frac{\beta}{\beta+\varepsilon}(g+n) - \frac{\varepsilon}{\beta+\varepsilon} s_E \qquad (1.11)$$

By substituting (1.6) and (1.11) into (1.10) we obtain the growth rate of the resource price

$$\gamma_u^* = \left(\frac{\dot{u}}{u}\right)^* = \frac{\beta}{\beta+\varepsilon}(g+n+s_E) \qquad (1.12)$$

A.2. The Solow model with an endogenous extraction rate

A.2.1. Deriving the law of motion for the capital-output ratio

Again, the capital-output ratio is given by

$$z \equiv \frac{K}{Y} = \frac{K}{K^{\alpha}(AL)^{\beta} E^{\varepsilon}} = K^{1-\alpha}(AL)^{-\beta} E^{-\varepsilon} \qquad (1.13)$$

Taking logarithms, differentiating w.r.t. time and using that $\gamma_K = \frac{\dot{K}}{K} = \frac{sY - \delta K}{K} = s\frac{Y}{K} - \delta = \frac{s}{z} - \delta$ yields

$$\frac{\dot{z}}{z} = (\beta+\varepsilon)\frac{s}{z} - \varepsilon\gamma_E - \beta(g+n+\delta) - \varepsilon\delta \qquad (1.14)$$

To compute the growth rate of the energy input (γ_E) we first have to derive the growth rate of the non-renewable resource price (*u*). We know that factor inputs earn their marginal products. Hence, we have that

$$u = \frac{\delta Y}{\delta E} = \varepsilon K^{\alpha}(AL)^{\beta} E^{\varepsilon-1} \qquad (1.15)$$

Taking logarithms and differentiating w.r.t. time yields the growth rate of the resource price

$$\gamma_u = \frac{\dot{u}}{u} = \alpha\frac{\dot{K}}{K} + \beta(g+n) + (\varepsilon-1)\frac{\dot{E}}{E} \qquad (1.16)$$

Next, we substitute Equation (1.16) into Hotelling's rule. Hence, we get

$$\alpha\frac{\dot{K}}{K} + \beta(g+n) + (\varepsilon-1)\frac{\dot{E}}{E} = r - \delta$$

For the wage rate (*r*) we can substitute $r = \frac{\delta Y}{\delta K} = \alpha K^{\alpha-1}(AL)^{\beta} E^{\varepsilon} = \alpha\frac{Y}{K} = \frac{\alpha}{z}$ and for the growth rate of capital $\gamma_K = \frac{\dot{K}}{K} = \frac{sY - \delta K}{K} = s\frac{Y}{K} - \delta = \frac{s}{z} - \delta$.

Thereby, we obtain

$$\alpha\left(\frac{s}{z}-\delta\right)+\beta(g+n)+(\varepsilon-1)\frac{\dot{E}}{E}=\frac{\alpha}{z}-\delta$$

Rearranging terms yields the growth rate of the energy input

$$\gamma_E=\frac{\dot{E}}{E}=\frac{1}{\alpha+\beta}\left(\alpha\frac{s-1}{z}+\beta(g+n+\delta)+\varepsilon\delta\right) \tag{1.17}$$

Finally, by substituting (1.17) into (1.16) we derive the law of motion for the capital-output ratio

$$\dot{z}=\frac{\beta s+\alpha\varepsilon}{\alpha+\beta}-\frac{\beta(g+n+\delta)+\varepsilon\delta}{\alpha+\beta}z \tag{1.18}$$

A.2.2. Deriving the law of motion for the endogenous extraction rate

The endogenous extraction rate is given by the equation $q=E/R$. We take logarithms and differentiate w.r.t. time to obtain the growth rate of the extraction rate (γ_q). We also use that $\dot{R}=-E$.

$$\gamma_q=\frac{\dot{q}}{q}=\frac{\dot{E}}{E}-\frac{\dot{R}}{R}=\frac{\dot{E}}{E}+\frac{E}{R}=\gamma_E+q \tag{1.19}$$

Substituting Equation (1.17) into (1.19) then yields the law of motion for the extraction rate

$$\dot{q}=\frac{1}{\alpha+\beta}\left(\alpha\frac{s-1}{z}+\beta(g+n+\delta)+\varepsilon\delta\right)q+q^2 \tag{1.20}$$

A.2.3. Proofing the existence of a unique stable saddle path

For convenience, we restate the law of motions for the capital-output ratio (*z*) and the endogenous extraction rate (*q*).

$$\dot{z}=\frac{\beta s+\alpha\varepsilon}{\alpha+\beta}-\left(\frac{\beta(g+n+\delta)+\varepsilon\delta}{\alpha+\beta}\right)z$$

$$\dot{q}=\frac{1}{\alpha+\beta}\left(\alpha\frac{s-1}{z}+\beta(g+n+\delta)+\varepsilon\delta\right)q+q^2$$

We determine the Jacobian matrix (*J*) which describes the dynamics of the linear system around the steady state.

$$J=\begin{pmatrix}\frac{\delta\dot{z}}{\delta z} & \frac{\delta\dot{z}}{\delta q}\\ \frac{\delta\dot{q}}{\delta z} & \frac{\delta\dot{q}}{\delta q}\end{pmatrix}=\begin{pmatrix}-\frac{\beta(g+n+\delta)+\varepsilon\delta}{\alpha+\beta} & 0\\ -\frac{\alpha}{\alpha+\beta}\frac{s-1}{z^2}q & \frac{1}{\alpha+\beta}\left(\alpha\frac{s-1}{z}+\beta(g+n+\delta)+\varepsilon\delta\right)+2q\end{pmatrix}$$

This Jacobian matrix has two *eigenvalues* (characteristic roots) denoted by the letters λ_1 and λ_2 .

The stability property of the dynamic system, which is described by the law of motions for *z* and *q*, depends on the sign of these *eigenvalues*. We obtain the characteristic roots as the solution to the equation $|J-\lambda U|=0$, where U denotes the 2x2 unit matrix.

$$\det|J-\lambda U|=0 \quad\Rightarrow\quad \left(-\frac{\beta(g+n+\delta)+\varepsilon\delta}{\alpha+\beta}-\lambda_1\right)\left(\frac{1}{\alpha+\beta}\left(\alpha\frac{s-1}{z}+(\beta+n+\delta)+\varepsilon\delta\right)+2q-\lambda_2\right)=0$$

We can easily check that $\lambda_1<0$ and $\lambda_2>0$. The fact that these two eigenvalues have opposite signs proofs that for the dynamic system a unique stable saddle path exists.[38]

A.3. An endogenous growth model with non-renewable and renewable resources

A.3.1 Deriving the optimal growth rates

In this section, we derive the optimal growth rates in the endogenous growth model by solving the social planner's problem.[39] The representative consumer maximizes his utility according to an instantaneous utility function, subject to several optimality conditions.

$$\max \int_{t=0}^{\infty} u(\mathrm{C}_t)\,\mathrm{e}^{-\rho t}\,dt \tag{2.1}$$

subject to

$$Y=F(L_Y,K,Q,R)=A^{\theta}L_Y^{\psi}K^{\xi}Q^{\alpha}R^{\beta},\ L_{Y0}\ \text{given} \tag{2.2}$$

$$\dot{S}_Q=-Q,\ S_{Q0}\ \text{given} \tag{2.3}$$

$$\dot{S}_R=mS_R-R,\ \text{with } m>0 \text{ and } S_{R0}\ \text{given} \tag{2.4}$$

$$\dot{K}=Y-C-\delta K,\ \text{with } \delta\in(0,1) \text{ and } K_0\ \text{given} \tag{2.5}$$

$$\dot{A}=bA^{\phi}L_A,\ \text{with } 0<\phi\le 1 \text{ and } b \text{ const.} \tag{2.6}$$

$$L_Y+L_A=1 \tag{2.7}$$

The instantaneous utility function is defined as being

$$u(C)=\begin{cases}\dfrac{C^{1-\varepsilon}-1}{1-\varepsilon} & if\ \varepsilon\neq 1\\ \ln C & if\ \varepsilon=1\end{cases} \tag{2.8}$$

where ε denotes the elasticity of marginal utility.

The transversality conditions are given by

$$\lim_{t\to+\infty}\lambda S_R e^{-\rho t}=\lim_{t\to+\infty}\mu S_Q e^{-\rho t}=\lim_{t\to+\infty}vKe^{-\rho t}=\lim_{t\to+\infty}\omega Ae^{-\rho t}=0$$

[38] Cf. Barro and Sala-i-Martin (1999, 587);
[39] Cf. Nguyen and Nguyen-Van (2008, 21 - 23);

Equilibrium is summarized by the following six key variables from which the other growth rates can be derived. (Nguyen and Nguyen-Van 2008, 9)

$$x = Y/K\,,\ y = C/K,\ z = Q/S_Q,\ u = R/S_R,\ q = L_Y A^{\phi-1} \text{ and } r = A^{\phi-1}$$

Reformulating (2.7) yields the equation $L_A = 1 - L_Y$, which we substitute into (2.6) to get

$$\dot{A} = bA^{\phi}(1 - L_Y) \tag{2.9}$$

To determine the optimal growth rates we first construct the current-value Hamiltonian and derive its first order conditions.[40]

$$H(C, K, Q, R, L_Y, A) = u(\mathrm{C}) + \lambda(\mathrm{mS}_R - R) - \mu\mathrm{Q} + v(\mathrm{Y} - \mathrm{C} - \delta\mathrm{K}) + \omega\mathrm{bA}^{\theta}(1 - L_Y) =$$
$$= u(\mathrm{C}) + \lambda(\mathrm{mS}_R - R) - \mu\mathrm{Q} + v(A^{\theta}L_Y^{\psi}K^{\xi}Q^{\alpha}R^{\beta} - \mathrm{C} - \delta\mathrm{K}) + \omega\mathrm{bA}^{\theta}(1 - L_Y)$$

<u>FOC's:</u>

$\frac{\delta H}{\delta C} = 0:$ $$v = u'(\mathrm{c}) = \frac{\delta u}{\delta c} = U_c \tag{2.10}$$

$\frac{\delta H}{\delta K} = \rho v - \dot{v}:$ $$\frac{\dot{v}}{v} = \rho - \frac{\delta Y}{\delta K} + \delta = \rho - F_K + \delta \tag{2.11}$$

$\frac{\delta H}{\delta Q} = 0:$ $$\mu = v\frac{\delta Y}{\delta Q} = vF_Q \tag{2.12}$$

$\frac{\delta H}{\delta S_R} = \rho\lambda - \dot{\lambda}:$ $$\frac{\dot{\lambda}}{\lambda} = \rho - m \tag{2.13}$$

$\frac{\delta H}{\delta R} = 0:$ $$\lambda = v\frac{\delta Y}{\delta R} = vF_R \tag{2.14}$$

$\frac{\delta H}{\delta S_Q} = \rho\mu - \dot{\mu}:$ $$\frac{\dot{\mu}}{\mu} = \rho \tag{2.15}$$

$\frac{\delta H}{\delta L_Y} = 0:$ $$\omega = \frac{v\frac{\delta Y}{\delta L_Y}}{bA^{\phi}} = \frac{vF_{Y_L}}{bA^{\phi}} \tag{2.16}$$

$\frac{\delta H}{\delta A} = \rho\omega - \dot{\omega}:$ $$\dot{\omega} = \omega(\rho - \phi bA^{\phi-1}(1 - L_Y) - vF_A \tag{2.17}$$

By substituting (2.10) into (2.17) we get

$$\dot{\omega} = \omega(\rho - \phi bA^{\phi-1}(1 - L_Y)) - U_C F_A \tag{2.18}$$

From Equations (2.3) - (2.5) we derive the growth rates of the non-renewable and renewable resource stocks, physical capital and technology, respectively.

$$\gamma_{S_Q} = -\frac{Q}{S_Q} = -z \tag{2.19}$$

$$\gamma_{S_R} = \frac{\dot{S}_R}{S_R} = m - \frac{R}{S_R} = m - u \tag{2.20}$$

[40] Here, the letters λ, μ, υ and ω denote costate-variables.

$$\gamma_K = \frac{\dot{K}}{K} = \frac{Y}{K} - \frac{C}{K} - \frac{\delta K}{K} = x - y - \delta \tag{2.21}$$

$$\gamma_A = \frac{\dot{A}}{A} = bA^{\phi-1}(1-L_Y) = bA^{\phi-1} - bA^{\phi-1}L_Y = br - bq = b(r-q) \tag{2.22}$$

From (2.2) we get that $\frac{\delta Y}{\delta K} = F_K = \xi \frac{Y}{K} = \xi x$

Further, we define the following three equations.

$$\frac{\dot{F}_Q}{F_Q} = \theta\gamma_A + \psi\gamma_{L_Y} + \xi\gamma_K + (\alpha-1)\gamma_Q + \beta\gamma_R \tag{2.23}$$

$$\frac{\dot{F}_R}{F_R} = \theta\gamma_A + \psi\gamma_{L_Y} + \xi\gamma_K + \alpha\gamma_Q + (\beta-1)\gamma_R \tag{2.24}$$

$$\frac{\dot{F}_{L_Y}}{F_{L_Y}} = \theta\gamma_A + (\psi-1)\gamma_{L_Y} + \xi\gamma_K + \alpha\gamma_Q + \beta\gamma_R \tag{2.25}$$

The goal is to determine values that we can substitute for $\dot{F}_Q / F_Q$, $\dot{F}_R / F_R$ and $\dot{F}_{L_Y} / F_{L_Y}$. Thereby we obtain a system of equations from which we can derive the other growth rates.

Substituting (2.10) into (2.11) yields the Euler equation

$$\rho - \frac{\dot{U}_C}{U_C} = F_K - \delta = \xi x - \delta \tag{2.26}$$

From the instantaneous utility function (2.8) we get

$$\frac{\dot{U}_C}{U_C} = -\varepsilon \frac{\dot{C}}{C} = -\varepsilon\gamma_c \tag{2.27}$$

By substituting the Euler equation (2.26) into (2.27) and by rearranging terms we obtain the growth rate of private consumption

$$\gamma_c = \frac{\xi x - \delta - \rho}{\varepsilon} \tag{2.28}$$

By taking logarithms of (2.12) and differentiating w.r.t. time we get

$$\frac{\dot{\mu}}{\mu} = \frac{\dot{v}}{v} + \frac{\dot{F}_Q}{F_Q} \tag{2.29}$$

We insert (2.10) and (2.15) into (2.29) and obtain

$$\frac{\dot{F}_Q}{F_Q} = \rho - \frac{\dot{U}_C}{U_C} \tag{2.30}$$

Substituting (2.26) into (2.29) yields

$$\frac{\dot{F}_Q}{F_Q} = \xi x - \delta \tag{2.31}$$

Similarly, by differentiating logarithmically (2.14) and by substituting (2.10), (2.13) and (2.26) into the resulting equation we obtain

$$\frac{\dot{F}_R}{F_R} = \xi x - \delta - m \tag{2.32}$$

By differentiating (2.16) logarithmically, we get

$$\frac{\dot{\omega}}{\omega} = \frac{\dot{v}}{v} + \frac{\dot{F}_{L_Y}}{F_{L_Y}} - \phi \frac{\dot{A}}{A} \tag{2.33}$$

By substituting (2.10) and (2.17) into (2.33), we derive that

$$\frac{\dot{F}_{L_Y}}{F_{L_Y}} = \rho - \frac{\dot{U}_C}{U_C} - \frac{b\theta}{\psi} q \tag{2.34}$$

Substituting (2.26) into (2.34) yields

$$\frac{\dot{F}_{L_Y}}{F_{L_Y}} = \xi x - \delta - \frac{b\theta}{\psi} q \tag{2.35}$$

Finally, by substituting (2.21), (2.22), (2.31), (2.32) and (2.35) into (2.23), (2.24) and (2.25), respectively, and by rearranging terms, we obtain a system consisting of three equations and three unknowns.

$$\psi \gamma_{Y_L} + (\alpha - 1)\gamma_Q + \beta \gamma_R = \xi y - b\theta (r - q) + (\xi - 1)\delta = T1 \tag{2.36}$$

$$\psi \gamma_{Y_L} + \alpha \gamma_Q + (\beta - 1)\gamma_R = \xi y - m - b\theta (r - q) + (\xi - 1)\delta \tag{2.37}$$

$$(\psi - 1)\gamma_{Y_L} + \alpha \gamma_Q + \beta \gamma_R = \xi y - \frac{b\theta}{\psi}(r - q) + (\xi - 1)\delta \tag{2.38}$$

By subtracting (2.37) from (2.36), we get

$$-\gamma_Q + \gamma_R = m \ \rightarrow \gamma_R = \gamma_Q + m \tag{2.39}$$

In addition, from Equations (2.36) - (2.38) we obtain

$$\psi\gamma_{L_Y} = b\theta q + \psi\gamma_Q \;\rightarrow \gamma_{L_Y} = \frac{b\theta}{\psi} q + \gamma_Q \tag{2.40}$$

By substituting (2.39) and (2.40) into (2.36) we derive the growth rates of the resource inputs and of labour used in the production sector.

$$\gamma_Q = \frac{m\beta + b\theta q - T1}{\xi} = \frac{b\theta r - \xi y + m\beta + (1-\xi)\delta}{\xi} = -y + \frac{b\theta r + m\beta + (1-\xi)\delta}{\xi} \tag{2.41}$$

$$\gamma_R = \gamma_Q + m = \frac{b\theta r - \xi y + m(\beta + \xi) + (1-\xi)\delta}{\xi} = -y + \frac{b\theta r + m(\beta + \xi) + (1-\xi)\delta}{\xi} \tag{2.42}$$

$$\gamma_{L_Y} = \frac{b\theta}{\psi} q + \gamma_Q = \frac{b\theta r - \xi y + m\beta + (1-\xi)\delta}{\xi} + \frac{b\theta}{\psi} = -y + \frac{b\theta}{\psi} + \frac{b\theta r + m\beta + (1-\xi)\delta}{\xi} \tag{2.43}$$

We derive the growth rate of labour used in the R&D sector by using that $L_Y = q/r$

$$\gamma_{L_A} = \frac{\dot{L}_A}{L_A} = -\frac{\dot{L}_Y}{1-L_Y} = \frac{L_Y}{L_Y - 1}\gamma_{L_Y} = \frac{q}{q-r}\gamma_{L_Y} \tag{2.44}$$

We take logarithms of (2.2) and differentiate w.r.t. time to get

$$\gamma_Y = \theta\gamma_A + \psi\gamma_{L_Y} + \xi\gamma_K + \alpha\gamma_Q + \beta\gamma_R \tag{2.45}$$

Finally, substituting (2.21), (2.22), (2.41), (2.42), (2.43) into (2.45) yields the growth rate of output

$$\gamma_Y = \xi x - y + \frac{b\theta r}{\xi} + \frac{m\beta + \delta(\alpha + \beta + \psi)}{\xi} - \delta \tag{2.46}$$

A.3.2. Transitional dynamics in the endogenous growth model

The law of motions for *x*, *y*, *z*, *u*, *q* and *r* constitute a dynamic system, which is denominated by the vector h $\left[h = (x, y, z, u, q, r)\right]$. Near the steady state the dynamic behaviour of the non-linear system is characterized by the behaviour of the linearized system around the steady state $\left[\dot{h} = J(h - h^*)\right]$, where h^* is a vector that is containing the long-run values of the key variables $\left[h^* = (x^*, y^*, z^*, u^*, q^*, r^*)\right]$ and the letter J denotes the Jacobian matrix evaluated at the steady state. (Nguyen and Nguyen-Van 2008, 11)

The Jacobian matrix is consisting of all partial derivatives. To find out how the dynamic system behaves around the steady state, we need to determine the *eigenvalues* (characteristic roots) of this matrix.

Case 1: $\phi = 1$

The Jacobian matrix (*J*) is given by

$$J = \begin{pmatrix} \delta\dot{x}/\delta x & \delta\dot{x}/\delta y & \delta\dot{x}/\delta z & \delta\dot{x}/\delta u & \delta\dot{x}/\delta q \\ \delta\dot{y}/\delta x & \delta\dot{y}/\delta y & \delta\dot{y}/\delta z & \delta\dot{y}/\delta u & \delta\dot{y}/\delta q \\ \delta\dot{z}/\delta x & \delta\dot{z}/\delta y & \delta\dot{z}/\delta z & \delta\dot{z}/\delta u & \delta\dot{z}/\delta q \\ \delta\dot{u}/\delta x & \delta\dot{u}/\delta y & \delta\dot{u}/\delta z & \delta\dot{u}/\delta u & \delta\dot{u}/\delta q \\ \delta\dot{q}/\delta x & \delta\dot{q}/\delta y & \delta\dot{q}/\delta z & \delta\dot{q}/\delta u & \delta\dot{q}/\delta q \end{pmatrix} = \begin{pmatrix} (\xi-1)x^* & 0 & 0 & 0 & 0 \\ \frac{(\xi-\varepsilon)}{\varepsilon} y^* & y^* & 0 & 0 & 0 \\ 0 & -z^* & z^* & 0 & 0 \\ 0 & -u^* & 0 & u^* & 0 \\ 0 & -q^* & 0 & 0 & \frac{b\theta}{\psi} q^* \end{pmatrix}$$

This Jacobian matrix has five *eigenvalues*. The stability property of the dynamic system depends on the sign of these *eigenvalues*, which we obtain as the solution to the characteristic equation $|J - \lambda U| = 0$, where U denotes the 5x5 unit matrix. By computing the determinant of the resulting 5x5 matrix we get

$$\det|J - \lambda U| = 0 \quad \Rightarrow \quad \left[\frac{b\theta}{\psi} q^* - \lambda_5\right]\left[u^* - \lambda_4\right]\left[z^* - \lambda_3\right]\left[y^* - \lambda_2\right]\left[(\xi-1)x^* - \lambda_1\right] = 0$$

We can easily check that only one of the *eigenvalues* will be negative.

$$\lambda_1 = (\xi-1)x^* < 0$$

$$\lambda_2 = y^* > 0$$

$$\lambda_3 = z^* > 0$$

$$\lambda_4 = u^* > 0$$

$$\lambda_2 = \frac{b\theta}{\psi} q^* > 0$$

The fact that only one *eigenvalue* has a negative sign, while the other four characteristic roots are larger than zero, suggests that for this dynamic system a unique saddle point of stability exists.[41]

Case 2: $\phi \neq 1$

Cf. Nguyen and Nguyen-Van (2008, 12-14), who provide the proof that in this case also a unique saddle point of stability exists.

[41] Cf. Barro and Sala-i-Martin (1999, 587);

Abstract (English)

The role of non-renewable resources is a recurring topic in economic growth theory that originates back to the early thirties of the last century. In the literature, the focus has lately shifted more towards research on the influence of environmental effects and externalities on long-run growth. However, as we investigate if and under what conditions positive long-run growth is feasible in an economy that is dependent on a non-renewable resource, we intent to keep the scope of this thesis as limited as possible. Therefore, we focus on the analysis of growth models that feature an essential non-renewable resource, but which do not include pollution and other externalities.

In the theoretical part, we first introduce Hotelling's rule, which we will use later on as an optimality condition to determine the extraction rate of a non-renewable resource endogenously. Further, we introduce two variants of an adapted Solow model and an endogenous growth model with Schumpeterian characteristics. We conduct an analysis on the long-run equilibria and investigate the transitional dynamics. In accordance with the literature, we find that, under certain conditions, positive growth rates of per-capita output are sustainable in the long run. Further, we find that, in both the Solow models and in the endogenous growth model, a unique stable saddle path exists on which the economy converges towards the long-run equilibrium.

In the empirical part, we give a short review on the evolution of non-renewable resource prices and summarize the results the econometric estimation of the endogenous growth model provides. We conclude with a critical assessment of the models discussed in this thesis.

Keywords: Economic growth; Non-renewable resources; Hotelling's rule, Solow model; Endogenous growth model;

Abstract (German)

Welche Bedeutung nicht erneuerbare Ressourcen für die Erhaltung von langfristigem Wirtschaftswachstum haben, ist ein oft behandeltes Thema in der ökonomischen Theorie. In der Literatur hat sich der Fokus in den letzten zwei Jahrzehnten auf die Erforschung des Einflusses von externalen Effekten auf das Wirtschaftswachstum verschoben. In dieser Arbeit soll hingegen anhand ausgewählter Wachstumsmodelle untersucht werden, ob und unter welchen Bedingungen positive Wachstumsraten in einer Wirtschaft, welche von einer nicht erneuerbaren Ressource abhängig ist, langfristig möglich sind. Deshalb konzentrieren wir uns in dieser Arbeit auf die Analyse von Modellen in denen eine nicht erneuerbare Ressource essentiell für den Produktionssektor ist, aber Umweltverschmutzung und andere negative Externalitäten keine Rolle spielen.

Im theoretischen Teil stellen wir die Hotelling-Regel, zwei unterschiedliche Varianten des Solow-Modells und ein endogenes Wachstumsmodell mit Schumpeter'schen Merkmalen vor, und führen eine Analyse des langfristigen Gleichgewichts und der Übergangsdynamiken durch. Wir stellen fest, dass unter bestimmten Bedingungen positives Wachstum auch langfristig möglich ist und dass in allen Modellen ein stabiler Sattelpfad existiert, auf dem sich die Wirtschaft in Richtung des langfristigen Gleichgewichts bewegt.

Im empirischen Teil geben wir eine Übersicht über die Preisentwicklung von nicht erneuerbaren Ressourcen und fassen die Resultate der ökonometrischen Analyse des endogenen Wachstumsmodells zusammen. Abschließend führen wir eine kurze kritische Beurteilung der in dieser Arbeit vorgestellten Modelle durch.

Schlüsselwörter: Wirtschaftswachstum; Nicht erneuerbare Ressourcen; Hotelling-Regel; Solow-Modell; Endogenes Wachstumsmodell;

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