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Vorwort

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Introduction

The recent economic crisis have led the European Central Bank (ECB) as well as the Federal Reserve (FED) to commit to low interest rates over a certain period in order to stabilize the economy. We think that announcing such a commitment is quite unusual, since it may lower the central bank's possibility to react to unforeseen shocks. We are interested in the effects on the economy of such a commitment in contrast to a discretionary policy.

By leaving the gold standard and moving towards floating exchange rates, policymakers in virtually every economy faced the problem of managing economic measures in the interest of the public. In doing so, policymakers can behave in basically two ways: (i) Either the policymaker sets his decisions according to commitment, which implies sticking to past promises about future actions, although there may be rising incentives over time to deviate from the committed plan. (ii) On the other hand, the policymaker may follow a discretionary policy, which allows for the option to freely choose a decision considered optimal given the current state, independent of previous actions or promises.

The crucial difference between commitment and discretion is whether an optimal plan constrains future policy choices in any plausible way. Both approaches are, in some way, feedback rules to the current state of the economy. But they differ in the link between policy intentions and private agent's expectations. In the discretion approach, a private agent accounts for the possibility of a policy readjustment in every period. Thus a rational expectations equilibrium can only be realised if the policymaker does not alter the policy in an unexpected way - although the discretionary approach would allow him to deviate. In contrast, a commitment policy leaves no room for uncertainty. Though an optimal plan may force the policymaker to adapt the policy instrument, the size and the timing of such an adjustment are common knowledge.

Concerning the measures of inflation and output gap, economists refer to this problem as the average inflation bias. This problem was first observed by Simons [15], who concluded that the optimal rule for the policymaker is price level stability. Later on, Friedman and Schwartz [6] suggested the rule of a constant growth rate of money. Furthermore, they indicated that

discretion could destabilize monetary policy, since a discretionary policymaker could concede to political pressures. Up to this time, the analysis of this topic has suffered from the one shortcoming that discretion always dominates the rule: a discretionary policymaker could always behave in the sense of a certain rule that would stabilize the economy, but has the advantage to react discretionary if required.

A new approach was proposed by Kydland and Prescott [8] in their seminal work *Rules Rather than Discretion: The Inconsistency of Optimal Plans*. They first analyse the inconsistency problem in a dynamic framework and explain it by the example of unemployment and inflation:

“Even if there is an agreed-upon, fixed social objective function and policymakers know the timing and magnitude of the effects of their actions, discretionary policy, namely, the selection of that decision which is best, given the current situation and a correct evaluation of the end-of-period position, does not result in the social objective function being maximized. The reason for this apparent paradox is that economic planning is not a game against nature but, rather, a game against rational economic agents.” (Kydland and Prescott 1977 [8], p.473)

Later on, Barro and Gordon [1] pick up this example and analyse it in a natural rate model. These two articles represent the background of our analysis. Throughout this thesis we will have to concentrate on results that are robust over a wide range of macroeconomic models. As our aim is not to develop a specific policy rule that could be used in real life monetary policy, we will rather attempt to provide input for optimal policy design.

In chapter 2 we will discuss the example of Kydland and Prescott [8] and relate our analysis to the one of Barro and Gordon [1]. In contrast to the historical example, we define a natural rate model of output gap and inflation, in order to be consistent with recent literature. We find that a discretionary policy leads to a time-consistent but inefficient equilibrium, while a policy that commits to a rule ends in a time-inconsistent but efficient equilibrium. The source of this inconsistency lies in the fact that the natural output of the economy is lower than the optimal output. A policymaker, though possibly restricted to a rule, is always seduced to reach the optimal output by inducing surprise inflation and pushing output above the natural level.

In chapter 3 we will analyse the average inflation bias in a new Keynesian framework. In this framework, nominal price rigidities build the basis for non-neutral effects of monetary policy. We first derive the optimal policy under discretion which allows us to illustrate the trade-off between output gap and inflation. Then we show the gains from commitment in the classical version of

the inflationary bias, where the source of the bias lies in the incentive of the policymaker to push output above its potential. In the end of chapter 3 we will show that gains from commitment may arise even if a policymaker understands the classical inflationary bias problem and avoids increasing output artificially. Here the dominance of commitment over discretion comes from the definition of inflation and from the positive impact of a commitment on private agent's expectations.

In chapter 4 we extend the new Keynesian framework and introduce a theory of partial commitment. This gives a whole range of equilibria between the two extremes of discretion and commitment. We model partial commitment as a commitment device that allows for changes of the optimal plan. In our model, alterations to the optimal plan come along with a change of the policymaker. Every new policymaker re-optimizes and installs the plan that matches the current state of the economy. By doing so, he tends to foil the promises of his predecessor. The frequency of these policy changes is arbitrary and we link them to a measure of credibility. Fewer policy changes increase the credibility of a central bank, which benefits its position when fighting inflationary pressures. We find that credible policymakers can lower inflation with lower costs of output reductions.

In chapter 5 we relate our results to real world examples. First, we consider the different economic situations of the oil price shocks in the US in 1970s and 2000s. After the first shock, the FED acted discretionarily, leading to extremely high levels of inflation. Noticing that this is not sustainable over the long run, the FED started to follow a commitment policy towards low and stable inflation rates. Continuing this long-lasting process allowed the FED to gain credibility and anchor private agents' expectations. Thus, the effects of the second oil price shock in 2000 were dampened, which in turn simplified the FED's decision to continue its committed policy. The second example discusses the effects of the recent economic crisis starting in 2008. The size and scale of this disturbance altered the economic variables substantially so that traditional actions were not enough to stabilize the economy. Thus a broad range of new discretionary policy actions were invented. We focus on the challenges provided by these measures on commitment strategies in the future. Finally, the main results are summarized in chapter 6.

Model of Output Gap and Inflation

An extensive part of this chapter is based on the work of Barro and Gordon [1]. For the sake of readability, specific references to this source have been reduced.

Kydland and Prescott [8] are the first to develop the concept of an average inflation bias in the context of rules versus discretion in a dynamic framework. This chapter should give a basic overview and serve as a background for a more sophisticated analysis in the following chapters.

2.1 The Economic Environment

In line with the work of Barro and Gordon [1], we use a natural rate model, comparing the output gap, $(Y_t - Y_t^n)$, to the difference of actual and expected inflation, $(\pi_t - \pi_t^e)$. The output gap is the difference between actual output, Y_t , and the natural level of output, Y_t^n . Hence the output gap is an appropriate measure for overall real activity in an economy. We model this relationship with an expectational Phillips curve,

$$Y_t = Y_t^n + \alpha(\pi_t - \pi_t^e), \quad \text{or} \quad \pi_t = \pi_t^e + \frac{1}{\alpha}(Y_t - Y_t^n), \quad (2.1)$$

with $\alpha > 0$ a positive constant, reflecting the slope of the Phillips curve. The theory of forming expected inflation, π_t^e , is shown in section 2.2. The way we write the Phillips curve (2.1) we assume that Y_t depends only on this period's unexpected inflation, $(\pi_t - \pi_t^e)$. Barro and Gordon [1] show that this can be extended to a model including lagged values of unexpected inflation, without changing the main results.

Concerning the natural level of output, we assume that this period's natural level of output is a convex combination of last period's natural level of output and the long-term mean of the natural level of output, $\bar{Y}^n$, plus a term, ε_t , that allows for single autonomous real shocks. We assume these shocks to be independently and identically distributed with zero mean. They can have a persisting influence on the natural output of the economy, though this influence decreases

over time. Hence the long-term mean of the natural level of output, $\overline{Y^n}$, is a constant,

$$Y_t^n = \lambda Y_{t-1}^n + (1 - \lambda) \overline{Y^n} + \varepsilon_t, \quad 0 \leq \lambda \leq 1. \quad (2.2)$$

The single period objective function reflects the society's preferences to maximize utility, or in other words, to minimize the loss function, L_t . It has the form of a quadratic approximation,

$$L_t = a(Y_t - kY_t^n)^2 + b(\pi_t)^2, \quad a, b > 0, \quad k > 1, \quad (2.3)$$

whose first term represents the loss in utility that arises when the discrepancy between actual and target output, kY_t^n , is large. A value $k > 1$ reflects the tendency of a policymaker to set the target level of output above its natural level. This enables an activist policy satisfying fiscal interests. The second term reflects that private agents consider inflation as a tax on reserves and currency, hence low inflation is preferred.

The policymaker chooses the inflation rate, π_t , in order to minimize the expected present discounted value of the loss function (2.3),

$$E \left(\sum_{t=1}^{\infty} \frac{L_t}{(1+r)^t} \middle| \mathcal{I}_0 \right), \quad (2.4)$$

where $\mathcal{I}_0$ is the information at the initial period ($t = 0$) and r is a constant exogenous real discount rate.

The optimal monetary policy is determined in a game between the policymaker and private agents. In period t the policymaker uses his information set $\mathcal{I}_{t-1}$ and sets actual inflation, π_t , in order to minimize costs (2.4). Private agents understand the cost minimizing process (2.4), on which the policymaker's decisions are based. Hence they determine inflation expectations, π_t^e , based on the same information set $\mathcal{I}_{t-1}$. Finally, equations (2.1), (2.2) and (2.3) determine actual output, Y_t , and the cost, L_t .

2.2 Expectations Mechanism

Private agents need to understand the optimization process of the policymaker in order to form their expectations on inflation. In this model, the policymaker has only one choice variable, π_t . There is no way the policymaker can influence current and future expected inflation, π_t^e and π_{t+i}^e .

The choice of current inflation, π_t , only affects current actual output, Y_t , through the Phillips curve. Future values of actual output, Y_{t+i} , are unaffected. Furthermore, the choice of current inflation, π_t , does not constrain any future choices of inflation, π_{t+i} . Thus the policymaker's optimization problem relates to a one period trade-off between costs of high inflation and an increased output gap. Thus he chooses π_t in order to minimize $E_{t-1}L_t$.

An interesting aspect of the problem is the relation between the expected inflation, π_t^e , and the actual inflation, π_t . Private agents form their expectations based on information available at the start of period t , $\mathcal{I}_{t-1}$. In addition, they understand that the policymaker accounts for their inflation expectations, π_t^e , when he determines the inflation rate. Hence they can indirectly influence the policymaker's decision.

According to previous discussion, an equilibrium in the problem could be characterized as follows: Private agents perceive the actions of the policymaker as a reaction to available information, $\mathcal{I}_{t-1}$. Let this reaction be a function $h^e(\mathcal{I}_{t-1})$. Thus private agents form their expectations according to expected reaction,

$$\pi_t^e = h^e(\mathcal{I}_{t-1}). \quad (2.5)$$

Furthermore, $\pi_t = h^e(\mathcal{I}_{t-1})$, the reaction of the policymaker to the current state, has to appear as a solution to the cost minimization problem (2.4), given that private agents expect a reaction $\pi_t^e = h^e(\mathcal{I}_{t-1})$. The absence of lagged values leads to an equilibrium with $\frac{\partial \pi_t^e}{\partial \pi_{t-i}} = \frac{\partial h^e(\mathcal{I}_{t-1})}{\partial \pi_{t-i}} = 0$ for all $i > 0$. In addition, the policymaker knows that inflation expectations are formed in line with this equation (2.5).

2.2.1 Formal Discussion

We receive actual output from equation (2.1) together with equations (2.2) and (2.5) as

$$Y_t = \lambda Y_{t-1}^n + (1 - \lambda) \overline{Y}^n + \varepsilon_t + \alpha (\pi_t - h^e(\mathcal{I}_{t-1})). \quad (2.6)$$

We transform the loss function (2.3) with equations (2.5) and (2.6) to

$$\begin{aligned}
L_t &= a \left(Y_t^n + \alpha(\pi_t - \pi_t^e) - kY_t^n \right)^2 + b(\pi_t)^2 \\
&= a \left((1-k)Y_t^n + \alpha(\pi_t - \pi_t^e) \right)^2 + b(\pi_t)^2 \\
&= a \left((1-k) \left(\lambda Y_{t-1}^n + (1-\lambda)\bar{Y}^n + \varepsilon_t \right) + \alpha(\pi_t - h^e(\mathcal{I}_{t-1})) \right)^2 + b(\pi_t)^2.
\end{aligned} \tag{2.7}$$

If the private agents' expected inflation is $\pi_t^e = h^e(\mathcal{I}_{t-1})$, the policymaker minimizes $E_{t-1}L_t$ with L_t from equation (2.7).

Reformulating L_t allows us to easily take expectations.

$$\begin{aligned}
L_t &= a \left((1-k)Y_t^n + \alpha(\pi_t - \pi_t^e) \right)^2 + b(\pi_t)^2 \\
&= a \left((1-k)^2(Y_t^n)^2 + 2(1-k)\alpha Y_t^n(\pi_t - \pi_t^e) + \alpha^2(\pi_t - \pi_t^e)^2 \right) + b(\pi_t)^2 \\
&= a \left((1-k)^2(Y_t^n)^2 + 2(1-k)\alpha Y_t^n(\pi_t - \pi_t^e) + \alpha^2(\pi_t^2 - 2\pi_t\pi_t^e + \pi_t^{e2}) \right) + b(\pi_t)^2
\end{aligned} \tag{2.8}$$

$$\begin{aligned}
E_{t-1}L_t &= a \left((1-k)^2 E_{t-1}(Y_t^n)^2 + 2(1-k)\alpha E_{t-1}(Y_t^n(\pi_t - \pi_t^e)) \right. \\
&\quad \left. + \alpha^2 E_{t-1}(\pi_t^2 - 2\pi_t\pi_t^e + \pi_t^{e2}) \right) + b E_{t-1}\pi_t^2
\end{aligned} \tag{2.9}$$

For the resulting solution to be a minimum, we need to calculate the first and second derivatives according to standard theory. The first order conditions are:

$$\begin{aligned}
\frac{\partial}{\partial \pi_t} E_{t-1}L_t &= 0 \\
0 &= a\alpha \left(2(1-k)E_{t-1}Y_t^n + 2\alpha E_{t-1}\pi_t - 2\alpha E_{t-1}\pi_t^e \right) + 2bE_{t-1}\pi_t \\
-2bE_{t-1}\pi_t &= a\alpha \left(2(1-k)E_{t-1}Y_t^n + 2\alpha(E_{t-1}\pi_t - E_{t-1}\pi_t^e) \right) \\
E_{t-1}\pi_t &= -\frac{a\alpha}{b} \left((1-k)E_{t-1}Y_t^n + \alpha(E_{t-1}\pi_t - E_{t-1}\pi_t^e) \right) \\
\hat{\pi}_t &= \frac{a\alpha}{b} \left(-\alpha(\hat{\pi}_t - h^e(\mathcal{I}_{t-1})) + (k-1) \left(\lambda Y_{t-1}^n + (1-\lambda)\bar{Y}^n \right) \right),
\end{aligned} \tag{2.10}$$

where $\hat{\pi}_t = E_{t-1}\pi_t$ is the actual choice of inflation the policymaker takes in response to expected economic conditions. Here the term $\hat{\pi}_t - h^e(\mathcal{I}_{t-1})$ accounts for unexpected inflation. Finally we show that the resulting policy choice $\hat{\pi}_t$ indeed leads to an optimum.

Thus we calculate the second order condition:

$$\begin{aligned}
\frac{\partial^2}{\partial \pi_t^2} E_{t-1} L_t &> 0 \\
\frac{\partial^2}{\partial \pi_t^2} E_{t-1} L_t &= 2a\alpha^2 E_{t-1} 1 + 2b E_{t-1} 1 \\
&= 2a\alpha^2 + 2b > 0.
\end{aligned} \tag{2.11}$$

Though the policymaker may have incentives not to choose the reaction $h^e(\mathcal{I}_{t-1})$, private agents know about these incentives and attend to them when forming their expectations. Hence rational private agents use equation (2.10) to calculate their expectations in equation (2.5). A consistent equilibrium requires that actual inflation equals expected inflation, $\hat{\pi}_t = h^e(\mathcal{I}_{t-1})$. Hence the unexpected inflation term drops out of equation (2.10). Private agents' expectations on inflation thus relate to,

$$\begin{aligned}
\pi_t^e = h^e(\mathcal{I}_{t-1}) &= \frac{a\alpha}{b}(k-1) \left(\lambda Y_{t-1}^n + (1-\lambda) \overline{Y}^n \right) \\
&= \frac{a\alpha}{b}(k-1) E_{t-1} Y_t^n.
\end{aligned} \tag{2.12}$$

2.3 Equilibrium Policy

A solution to the problem can be found in two ways. In the first case, the policymaker minimizes $E_{t-1} L_t$ in each period, subject to private agents' rational equilibrium expectations equal some reaction function, $\pi_t^e = h^e(\cdot)$. Equations (2.10) and (2.12) motivate the policymaker to set $\hat{\pi}_t = h^e(\cdot)$ in each period. Hence agents have no incentives to deviate from their expectations, since these expectations are correct given the choice of the policymaker,

$$\boxed{\hat{\pi}_t \equiv \pi_t^d = \frac{a\alpha}{b}(k-1) E_{t-1} Y_t^n = \pi_t^e}, \tag{2.13}$$

where π_t^d is the policy choice under discretion. The first equilibrium implies an inflation rate that equals rational expectations and from the Phillips curve (2.1) we learn that actual output equals natural output, $Y_t = Y_t^n$. We will refer to this as the discretionary equilibrium.

In the second case, the policymaker takes a once-and-for-all decision. He chooses today that policy $\hat{\pi}_t = h(\cdot)$, which minimizes the loss function (2.4), and he will not deviate from that policy in future periods. Thus rational expectations are equal for each upcoming period, $\pi_t^e =$

$h^e(\cdot) = h(\cdot)$, and for all possible reactions $h(\cdot)$. Like in Sargent and Wallace [13], the choice of $h(\cdot)$ affects both π_t^e and π_t in each period, hence they are strongly correlated in each period. Thus $\pi_t - \pi_t^e = \pi_t - h^e(\mathcal{I}_{t-1}) = 0$ is a constraint in the once-and-for-all framing of the problem. With $\pi_t = \pi_t^e$ for every period, the policymaker realizes that his decision on $h(\cdot)$ does not change actual output $Y_t = Y_t^n$. Since the social objective function (2.3) penalizes deviations of π_t from 0, the once-and-for-all equilibrium inflation rate is,

$$\boxed{\hat{\pi}_t \equiv \pi_t^c = h(\mathcal{I}_{t-1}) = 0}, \quad (2.14)$$

where $Y_t = Y_t^n$ and π_t^c is the policy choice under commitment. We will refer to this as the commitment equilibrium.

A crucial aspect of the once-and-for-all equilibrium is the question of how to enforce a current decision on inflation in future periods. That is the question of an adequate commitment technology that penalizes deviations from the rule. In the absence of such a technology, the policymaker will have an incentive to raise current output above the natural level by surprise inflation in each upcoming period. In equilibrium, the output, Y_t , depends on $\pi_t - \pi_t^e = \pi_t - h^e(\mathcal{I}_{t-1})$. If the policymaker induces surprise inflation, he sets $\pi_t > \pi_t^e = h^e(\mathcal{I}_{t-1})$ in period t . Since it is only optional to set $\pi_t = \pi_t^e$, he will raise inflation in each future period. Thus we end up in the discretionary equilibrium, where the policymaker optimizes for given initial conditions taking the private agents' expectations into account. Private agents realize this and form their expectations accordingly. Hence the discretionary equilibrium cannot end up in equation (2.14).

2.4 Time-consistent Inefficient vs

Time-inconsistent Efficient Equilibrium

Comparing the discretionary equilibrium π_t^d with the commitment equilibrium $\pi_t^c = 0$, we see that both end up with the same level of output but with different levels of inflation at each period. Those under discretion will always exceed those under commitment. Hence the corresponding equilibrium costs fulfill $L_t^d > L_t^c$. Here we omit all labelling costs which arise through high inflation as well as costs of enforcing a commitment technology.

Next we want to examine the discretionary equilibrium in more detail and point out that an inflation $\pi_t = 0$ is infeasible in this case. Assuming that $\pi_t^e = h^e(\mathcal{I}_{t-1}) = 0$, actual inflation $\pi_t > 0$ would increase this period's output Y_t . The policymaker faces a trade-off between gains

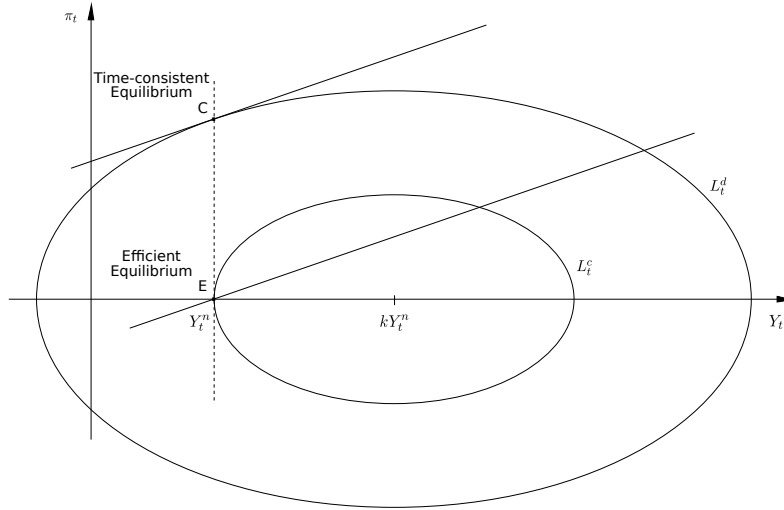


Figure 2.1: Consistent and Efficient Equilibrium

in reducing the output gap and increased losses from inflation. The policymaker faces this trade-off by regarding equation (2.10). Due to the form of the loss function (2.3), the resulting inflation rate will be positive. Since private agents understand this trade-off, a choice of $\pi_t > 0$ is inconsistent, given the expectation $\pi_t^e = 0$. Hence $\pi_t^e = 0$ is not a feasible expectation for private agents.

If inflation expectations are slightly positive, $\pi_t^e > 0$, but still too low to prohibit the policymaker's incentives of surprise inflation $\pi_t > \pi_t^e$, the outcome would still not be an equilibrium - and that for the same reason. Only if inflation expectations are high enough to change the policymaker's reaction and let him choose $\pi_t = \pi_t^e$, no one has incentives to deviate from his decision. At this point marginal losses from inflation equal marginal gains from reducing the output gap. The corresponding inflation rate that reflects this equilibrium is the one from equation (2.13) - point C in figure 2.1.

In other words, one can address this issue as follows: Assume the economy enters period t at point E . There are no commitment technologies that prevent the policymaker from cheating on the public. The policymaker takes the level of the Phillips curve as given. His decision on the actual rate of inflation only shifts the resulting outcome along the Phillips curve. Thus the policymaker acts in a discretionary regime and faces incentives to increase inflation. In the short run the policymaker knows that the economy will stay on the original Phillips curve through the point E . As explained above, the policymaker follows these incentives and chooses a rate of inflation that leads the economy to a point that lies on the original Phillips curve, but is closer to the target output kY_t^n . Thus the economy would shift to a lower loss curve with a

slightly positive rate of inflation and a small output gap. However the policymaker plays the game against rational economic agents. Without an adequate commitment technology, a rational private agent understands the incentives of the policymaker. He will expect the actual rate of inflation to be positive, $\pi_t^e > 0$. On the other hand, a positive expectational inflation term shifts the Phillips curve upwards immediately. Thus it is impossible for the economy to stay at a point E without a sufficient commitment technology that prevents private agents from expecting positive rates of inflation. So we see that a point E is not sustainable for a discretionary regime. But which outcome is likely to be reached by a discretionary regime? Private agents will increase their expectations until they are able to forecast the policymaker's choice. Thus the Phillips curve that corresponds to these expectations also shifts upwards. We will have to bear in mind that private agents expect the output of the economy to stay at its natural level. Thus the relevant loss function is the one that crosses the Phillips curve at the natural level of output. Hence, as the Phillips curve shifts upwards, the value of the loss function increases. Private agents know that, as long as the Phillips curve crosses the loss function, the policymaker will always be able to choose a point on the Phillips curve that is closer to the target than the point at the natural level of inflation. Thus private agents choose expected inflation in such a way that the resulting Phillips curve is tangent to the loss function at the natural level of output. This corresponds to point C in figure 2.1. At C the policymaker is no longer able to find a point on the Phillips curve that is closer to the target point kY_t^n and tends to reduce losses. However, if there exists a commitment technology, the policymaker is not able to choose the rate of inflation in period t . He is forced to follow a rule that has been set before. Such a rule minimizes losses over all periods by definition. Thus the only feasible rule enforces the rate of inflation $\pi_t = 0$. The policymaker has no possibility to manipulate, thus private agents have no reason to expect a rate of inflation different from zero. The economy will stay at point E in figure 2.1, although the Phillips curve is not tangent to the loss function.

In the relevant literature, the commitment equilibrium is often referred to as efficient ($\pi_t = 0$, $Y_t = Y_t^n$) but time-inconsistent - point E in figure 2.1. In the absence of an adequate commitment technology that constraints the policymaker to compliance with the rule in upcoming periods, he always has incentives to mislead the public and deviate from the rule. Contrary, the discretionary equilibrium is referred to as inefficient ($\pi_t > 0$, $Y_t = Y_t^n$) but time-consistent - point C in figure 2.1.

We should point out that these two results are actually equilibria of different games. Though the rules of the games are the same - Phillips curve and loss function - they differ in the possible

reaction sets. In the first case, future policy choices are independent from the current choice. Thus an inflation $\pi_t = 0$ cannot be an equilibrium. In the second case a pre-committed rule is enforceable. Hence incentives to cheat on the public and deviate from the rule are prohibited.

2.5 Reputational Equilibrium

In order for the commitment to be feasible, we mentioned before that there has to exist an adequate commitment technology. Such a technology can be fixed by law, or, more effectively, by constitution. One difficulty inherited with such a legislative commitment technology is that it considerably impairs the independency of the policymaker. A commonly used strategy of policymakers, in order to undergo this problem, is to build up reputation or credibility. The literature shows many different ways to model credibility. In this section we use a simple game theoretic approach according to Friedman [5] in order to understand the idea. In section 4 we present a more complex idea of modelling credibility. Here the idea is that the policymaker may not take the opportunity to misguide on the public and build up a reputation. This reputation causes private agents' inflationary expectations to be well-anchored at $\pi_t^e = 0$.

Private agents expect the commitment solution, $\pi_t^e = 0$, to hold in every period, $\pi_t^e = 0$, until the policymaker gives them a reason to develop misgivings. Once the policymaker misleads the public about the level of inflation and private agents observe inflation rates $\pi_t > 0$, expectations are not well-anchored any more. Private agents learn about policymaker's behaviour and loose trust, hence $\pi_t^e > 0$ for all upcoming periods. Confronted with these expectations, the policymaker has two options: Either, (i) to set $\pi_1 = \pi_1^d > 0$ in the first period. Since expectations for the first period are $\pi_1^e = 0$, this generates a profitable outcome in period 1. Output is above its natural level while staying on the same loss function isoquant. Hence, for the first period, the discretionary solution dominates the commitment outcome. In future periods the policymaker has lost his reputation and private agents expect high rates of inflation, $\pi_t^e = \pi_t^d > 0$. Thus the best response of the policymaker to high expected inflation rates is to set a high inflation rate, $\pi_t = \pi_t^d$. This leads to the discretionary equilibrium for all upcoming periods, with losses possibly higher than the short-term gain of period 1. Or, (ii) the policymaker aims for a good reputation and understands how public expectations are formed. Hence he sets $\pi_t = 0$ in all periods. Thus expectations are well-anchored and the public trusts in the policymaker, $\pi_t^e = 0$. Here the policymaker abstains from short-term benefits in order to gain credibility. In this scenario, the economy reaches the commitment equilibrium in every period. We refer to this

outcome as the reputational equilibrium.

In contrast to the formation of expectations in 2.2, the policymaker needs to understand that the action he sets in this period effects the expectations of future periods. Thus well-anchored expectations, $\pi_t^e = 0$, can only prevail if inflation in all previous periods remain equal to zero. Which of the two options dominate in the long run, depends on the particular weights the policymaker, or in principal the public, assigns to a short-term gain from high output compared to the discounted present value of losses from high inflation in all upcoming periods.

Though the reputational equilibrium seems to be preferable, it is often difficult to reach. First, consider only a finite number of periods. The reputational equilibrium cannot be sustained in the last period and backward induction shows that it is impossible to hold in any period. If the game ends with a certain probability, a higher probability of termination may lead to high discount rates, which make a short-term benefit more interesting. This makes the reputational equilibrium unlikely to appear, though it is not impossible. Second, there may be a change in the natural level of output, which causes the short-term benefit of an increased output to exceed the discounted present value of losses.

Although it is not completely clear how reputation is built up in a more complex setting than the one of the model, we think that some results can be generalized. A policymaker has strong incentives not to misinform the public in any period. Once he had done so, he would find himself on the inefficient path, increasing economic losses. Hence he learns and wishes to come back to the state of the first period which would allow him to choose honesty. Unfortunately, this is a long and exhausting way. The policymaker has to tighten the economy in order to disinflate. This may lead to a recession. At the same time, the policymaker must convince the public that he will not be tempted by the option for short-term benefits again. By doing so, he may be able to regain credibility, which allows him to bring down inflationary expectations.

A New Keynesian Perspective

An extensive part of this chapter is based on the article of Clarida, Gali and Gertler [2]. Again, for the sake of readability, we will refrain from individual references to this source.

Analysing the average inflation bias in a more complex framework, we define a dynamic general equilibrium model of the monetary transmission mechanism. In this setting, nominal price rigidities form the basis of frictions that lead to interesting effects of monetary policy. This modern approach for studying the qualitative behaviour of monetary policy allows to further develop the ideas of Kydland and Prescott [8].

3.1 The Economic Environment

As the model presented in the previous chapter, this describes an economy with a large number of competitive, forward-looking private agents and a policymaker. The policymaker's objective is to maximize welfare of the representative consumer. In contrast to the natural rate model of the previous section, the instrument of monetary policy is a short-term interest rate and not directly the rate of inflation. By choosing the interest rate, the policymaker determines, together with the decisions of private agents, the rate of inflation and real output.

First, we will have to define the economy. Let y_t and y_t^n be the logarithm of output and the logarithm of the natural level of output, which is the level of output under perfectly flexible prices and wages. Thus the output gap, $x_t = y_t - y_t^n$, is the difference of actual output from its potential. Let π_t be the period t inflation rate and i_t the short-term nominal interest rate controlled by the policymaker. For technical reasons, we interpret π_t and x_t as deviations from their associated trend. Then the model is defined by two equations: a dynamic IS curve (3.1) representing the demand side that relates the output gap, x_t , to the real interest rate, which is the difference between the nominal interest rate, i_t , and expected inflation for the next period, $E_t\pi_{t+1}$; a Phillips curve (3.2) representing the supply side that relates inflation, π_t , and the output gap, x_t .

$$x_t = E_t x_{t+1} - \varphi(i_t - E_t \pi_{t+1}) + g_t \quad (3.1)$$

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t \quad (3.2)$$

Here $\varphi > 0$ is the inter-temporal elasticity of substitution in consumption, $\kappa > 0$ is a positive parameter and β is the discount factor of the representative agent. The disturbance terms g_t and u_t are stationary $AR(1)$ processes,

$$\begin{aligned} g_t &= \iota g_{t-1} + \acute{g}_t \\ u_t &= \rho u_{t-1} + \acute{u}_t, \end{aligned} \quad (3.3)$$

with $0 \leq \iota, \rho \leq 1$ and both $\acute{g}_t$ and $\acute{u}_t$ are independent identically distributed random variables with zero mean and time constant variances σ_g^2 and σ_u^2 .

The demand side is determined by the household's optimal savings decision. Equation (3.1), is a log-linearized version of the Euler equation that differs from the traditional IS curve mainly by expectations. Current output depends both on expected future outputs and on the real interest rate. In equilibrium, consumption of the representative agent equals generated output minus governmental spending. We argue that people prefer small consumption differences to big jumps from one period to the other. This means a person that consumes one unit today will not surprisingly change its standard of living and consume 10 units the next day. In other words, an agent that expects an increase in consumption next period, will consume more in the current period. Hence this period's demand for output increases. In contrast, current output depends negatively on the real interest rate, $i_t - E_t \pi_{t+1}$. This reflects inter-temporal substitution. The coefficient φ can be interpreted as the inter-temporal elasticity of substitution. The disturbance term g_t can shift the IS curve, which allows to induce demand shocks. After iterating equation (3.1),

$$x_t = E_t \sum_{i=0}^{\infty} (-\varphi(i_{t+i} - \pi_{t+1+i}) + g_{t+i}),$$

we observe how expectations effect current aggregate activity in the model. Current output depends on the expected future paths of the real interest rate as well as on the expected development of shocks. Here, monetary policy enters the model. In the presence of nominal rigidities, a policymaker can influence the short-term real interest rate.

The supply side of the market evolves from the price setting behaviour of optimizing monopo-

listically competitive business enterprises. Each business enterprise sets prices to maximize profits according to Calvo pricing. Equation (3.2) is a log-linearised approximation of the business enterprises' optimal pricing decision. In difference to the traditional Phillips curve we add expectations on future inflation in order to eliminate lagged dependence of inflation. Thus we see after iterating,

$$\pi_t = E_t \sum_{i=0}^{\infty} \beta^i (\kappa x_{t+i} + u_{t+i}).$$

Business enterprises set nominal prices in accordance with current and expected future economic conditions. The path of x_{t+i} can be interpreted as the development in marginal costs, while u_{t+i} expresses cost push shocks.

In contrast to the natural rate model in the previous section, where we assume that the policy-maker directly sets the rate of inflation, in this model the instrument of monetary policy is the nominal interest rate. This reflects the instrument of a modern central bank. The central bank sets the key interest rate which, in turn, affects the economy through the monetary transmission mechanism. In the presence of nominal price rigidities, the policymaker can influence the short-term real interest rate. But forward-looking rational households and business enterprises anticipate these influences and react accordingly in the current period. Thus monetary reactions to short-term disturbances are not a trivial issue. Keep in mind the statement of Kydland and Prescott [8] that monetary policy is a game against rational economic agents.

The objective function of the policymaker that relates economic variables of inflation and output gap to a welfare measure is given by

$$\max -\frac{1}{2} E_t \left(\sum_{i=0}^{\infty} \beta^i \left(a(x_{t+i} - k)^2 + \pi_{t+i}^2 \right) \right), \quad (3.4)$$

where a is the relative weight on deviations of the output gap from its trend. Implicitly a punishes deviations of actual output, y_t , from its target, y_t^n . Including a factor k allows the policymaker to set an output gap target greater than zero. Concerning inflation we take a target rate of zero, but again π_t is defined as the deviation from the trend. In the literature there is big concern on how to rationalize policy objectives. The representative agent approach is criticized for not capturing the cost of uncertainty in financial planning with high inflation fluctuations. In order to be consistent with the previous chapter, we use the traditional approach of a loss function

that minimizes the squared of inflation and output gap over all periods. As target values we use the natural level of the output. If there are frictions in the model, like imperfect markets, the natural level may not optimize welfare and the policymaker has incentive to set a target above the natural level. This issue arises in the context of credibility. Nowadays most central banks agree that the main target should be price level stability. While the ECB announces this as the only goal, the FED also incorporates a maximum of employment and moderate long-term interest rates. Some economists argue that, with the nominal interest rate as the only instrument, a central bank should also have only one target. Price stability is defined as a rate of inflation that causes no public concern. The ECB sets this goal to an inflation rate of 2 percent over the medium term, which is in line with the concern about overshooting measurement errors.

3.2 The Policy Problem

The policymaker uses its instrument, the nominal interest rate, i_t , to guide the economy. The nominal interest rate determines the time path of the variables x_t and π_t through the IS curve (3.1) and the Phillips curve (3.2). These paths should develop optimal with respect to the objective function (3.4). Thus the value of i_t should, in some sense, reflect the current situation of an economy. However, in contrast to classical problems, the optimal paths of x_t and π_t depend not only on current information but also on expectations about the development of these paths. The output gap reacts to the expected real interest rate and inflation reacts to the output gap. As shown before, this is the crucial point for all actions by private agents. Their decisions on how much to supply and demand depend on expectations of inflation and output gap. Hence, in order to manage the economy in the intended way, the policymaker must be able to influence these expectations. In other words, a policymaker with the possibility to make credible commitments about future policy actions can influence private agents' expectations at lower cost in terms of a reduced output than a policymaker without credibility. Hence, a credible policymaker can maintain price level stability with less effort.

3.3 Optimal Monetary Policy without any Incentive Problems

We start our analysis of the new Keynesian model with the situation where the policymaker can freely choose the level of the policy instrument. This framing is very close to the discretionary setting in the previous chapter although here we omit the policymaker's option to set the target

level of the output gap above zero. Thus we set $k = 0$. Note here that the k in equation (3.4) equals the logarithm of the factor k in chapter 2, since here all variables are defined in logarithmic terms. We will use the resulting outcome as a basis for our analysis.

3.3.1 Formal Discussion

Under discretion, the policymaker chooses the nominal interest rate, i_t , in each period in order to influence the level of target variables, x_t and π_t , which, in turn, should maximize the objective function (3.4). Constraints are defined by the equations (3.1) and (3.2). For simplicity, we divide the problem into two steps: (i) first the policymaker optimizes the policy objective (3.4) subject to the Phillips curve (3.2). This step results in optimal values of output gap, x_t , and inflation, π_t ; (ii) then he plugs the optimal values of output gap and inflation in the IS curve (3.1) and receives the optimal level of the nominal interest rate, i_t . The resulting value of i_t supports the values of x_t and π_t , which maximize the objective. According to private agents' expectations, we use the same assumptions as in the natural rate model. Under discretion, a policymaker cannot influence expectations, thus he takes expectations as given. On the other hand private agents understand the decision-making process of the policymaker and form their expectations accordingly.

The optimal policy problem under discretion can be formulated as the following static optimization problem: In each period the policymaker re-optimizes. In the first step he chooses values of x_t and π_t to fulfil

$$\max -\frac{1}{2} \left(ax_t^2 + \pi_t^2 \right) + F_t \quad \text{subject to} \quad \pi_t = \kappa x_t + f_t, \quad (3.5)$$

where

$$F_t \equiv -\frac{1}{2} E_t \left(\sum_{i=1}^{\infty} \beta^i (ax_{t+i}^2 + \pi_{t+i}^2) \right) \quad \text{and} \quad f_t \equiv \beta E_t \pi_{t+1} + u_t.$$

Equation (3.5) simply reformulates equations (3.4) and (3.2). This allows us to reflect the discussion on the expectations mechanism in chapter 2.2: This period's actions do not constrain future periods' output gap and inflation; the policymaker cannot influence private agents' expectations. We analyse this optimization problem using the Lagrangian

$$\mathcal{L} = -\frac{1}{2} \left(ax_t^2 + \pi_t^2 \right) + F_t - \phi(\kappa x_t + f_t - \pi_t).$$

This leads us to the following first order conditions:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \pi_t} &= -\pi_t + \phi = 0 \\ \pi_t &= \phi \end{aligned} \tag{3.6}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial x_t} &= -ax_t - \phi\kappa = 0 \\ x_t &= -\frac{\phi\kappa}{a}. \end{aligned} \tag{3.7}$$

The appropriate Hessian Matrix for second order conditions is negative definite, hence the outcome accounts for a local maximum. Thus, combining (3.6) and (3.7), we get the optimality condition for the first step

$$\boxed{x_t = -\frac{\kappa}{a}\pi_t} \quad \text{or} \quad \boxed{\pi_t = -\frac{a}{\kappa}x_t}. \tag{3.8}$$

The negative relation between x_t and π_t in the optimality condition implies that the policymaker should follow a counter-cyclical policy. If inflation is above target, optimal monetary policy suggests reducing the output gap. The size of adjustment depends on the value of $\frac{\kappa}{a}$, where κ reflects the gains from lower inflation compared to a unit loss in output and a reflects the relative weight on the output gap in the objective function (3.4).

In order to get reduced form expressions of x_t and π_t , we take the optimality condition (3.8) and plug in the Phillips curve (3.2),

$$x_t = -\frac{\kappa}{a}(\kappa x_t + \beta E_t \pi_{t+1} + u_t).$$

At this point we concentrate on a class of policy rules that assume a negative linear relation of the output gap and the supply shocks, $x_t = -\omega u_t$. In chapter 4 we will give an analysis that works without this restriction. However, this linear relation leads to $\pi_t = \frac{a\omega}{\kappa}u_t$, which simplifies expected inflation, $E_t \pi_{t+1} = \frac{a\omega\rho}{\kappa}u_t = \rho\pi_t$,

$$\begin{aligned}
ax_t &= -\kappa^2 x_t - \kappa\beta\rho\pi_t - \kappa u_t \\
(\kappa^2 + a)x_t &= -\kappa(\beta\rho\pi_t + u_t) \\
-\frac{\kappa^2 + a}{\kappa}x_t &= \beta\rho\pi_t + u_t.
\end{aligned}$$

We use once again the optimality condition (3.8) in order to replace π_t ,

$$\begin{aligned}
-\frac{\kappa^2 + a}{\kappa}x_t &= -\beta\rho\frac{a}{\kappa}x_t + u_t \\
-\frac{\kappa^2 + a(1 - \beta\rho)}{\kappa}x_t &= u_t \\
x_t &= -\frac{\kappa}{\kappa^2 + a(1 - \beta\rho)}u_t.
\end{aligned}$$

Finally, we get the reduced form expressions,

$$\boxed{x_t = -\kappa qu_t} \quad \text{and} \quad \boxed{\pi_t = a qu_t}, \quad (3.9)$$

where $q = \frac{1}{\kappa^2 + a(1 - \beta\rho)}$ and $\omega = \kappa q$.

Next, we develop the optimal feedback policy for the nominal interest rate, i_t , in the second step.

Thus we insert the optimal value of $x_t = -\kappa qu_t$ in a transformed version of the IS curve (3.1).

$$\begin{aligned}
i_t &= \left(1 - \frac{\kappa}{a\varphi}\right) E_t \pi_{t+1} - \frac{1}{\varphi} x_t + \frac{1}{\varphi} g_t \\
i_t &= \left(1 - \frac{\kappa}{a\varphi}\right) \rho a qu_t + \frac{\kappa}{\varphi} qu_t + \frac{1}{\varphi} g_t \\
i_t &= \frac{a\varphi\rho + (1 - \rho)\kappa}{\varphi} qu_t + \frac{1}{\varphi} g_t \\
i_t &= \frac{a^2\varphi\rho^2 + (1 - \rho)a\kappa\rho}{a\varphi\rho} qu_t + \frac{1}{\varphi} g_t \\
i_t &= \left(1 + \frac{(1 - \rho)\kappa}{a\varphi\rho}\right) \rho a qu_t + \frac{1}{\varphi} g_t
\end{aligned}$$

This leads to the optimal choice of the nominal interest rate

$$\boxed{i_t = \gamma_\pi E_t \pi_{t+1} + \frac{1}{\varphi} g_t} \quad \text{with} \quad \gamma_\pi = 1 + \frac{(1 - \rho)\kappa}{a\varphi\rho} > 0. \quad (3.10)$$

Here we should stress once again that above derivations work only under the assumption that

$$x_t = -\omega u_t.$$

3.3.2 Implications on Monetary Policy

From the reduced form expressions (3.9) we can conclude some interesting facts that help us to evaluate monetary policy in general. Taylor [16] finds that, in the presence of cost push shocks, the policymaker faces a trade-off between the variability of inflation and the variability of the output gap. We make the trade-off more transparent by defining the efficient policy frontier. This frontier is a set of points that shows how a policymaker's preferences, a , change the unconditional standard deviations of output, σ_x , and inflation, σ_π , under the optimal policy.

$$\begin{aligned}
 \sigma_x &= \sqrt{\text{Var}(x_t)} \\
 &= \sqrt{\text{Var}(-\kappa q u_t)} \\
 &= \sqrt{\kappa^2 q^2 \text{Var}(u_t)} \\
 &= \sqrt{\kappa^2 q^2 \sigma_u^2} \\
 &= \kappa q \sigma_u
 \end{aligned}$$

$$\begin{aligned}
 \sigma_\pi &= \sqrt{\text{Var}(\pi_t)} \\
 &= \sqrt{\text{Var}(a q u_t)} \\
 &= \sqrt{a^2 q^2 \text{Var}(u_t)} \\
 &= \sqrt{a^2 q^2 \sigma_u^2} \\
 &= a q \sigma_u
 \end{aligned}$$

A policymaker who tends towards output stability, increases a . Thus as a increases, σ_x decreases, since a appears in the denominator of q , but σ_π increases. Combining these equations directly displays the relation between inflation variability and output variability.

$$\begin{aligned}
 \sigma_x &= \kappa q \sigma_u \\
 &= \kappa q \frac{\sigma_\pi}{a q} \\
 &= \frac{\kappa}{a} \sigma_\pi
 \end{aligned}$$

Although this relation is independent of the variability of the cost push shock σ_u , we should bear in mind that this relation only holds in the presence of cost push shocks. Further interesting situations are the limiting cases:

$$\begin{aligned}
\lim_{a \rightarrow 0} \sigma_x &= \lim_{a \rightarrow 0} \kappa q \sigma_u \\
&= \lim_{a \rightarrow 0} \frac{\kappa \sigma_u}{\kappa^2 + a(1 - \beta\rho)} \\
&= \frac{\sigma_u}{\kappa}
\end{aligned} \tag{3.11}$$

$$\begin{aligned}
\lim_{a \rightarrow 0} \sigma_\pi &= \lim_{a \rightarrow 0} a q \sigma_u \\
&= \lim_{a \rightarrow 0} \frac{a \sigma_u}{\kappa^2 + a(1 - \beta\rho)} \\
&= 0
\end{aligned} \tag{3.12}$$

$$\begin{aligned}
\lim_{a \rightarrow \infty} \sigma_x &= \lim_{a \rightarrow \infty} \kappa q \sigma_u \\
&= \lim_{a \rightarrow \infty} \frac{\frac{\kappa \sigma_u}{a}}{\frac{\kappa^2}{a} + (1 - \beta\rho)} \\
&= 0
\end{aligned} \tag{3.13}$$

$$\begin{aligned}
\lim_{a \rightarrow \infty} \sigma_\pi &= \lim_{a \rightarrow \infty} a q \sigma_u \\
&= \lim_{a \rightarrow \infty} \frac{\sigma_u}{\frac{\kappa^2}{a} + (1 - \beta\rho)} \\
&= \frac{\sigma_u}{1 - \beta\rho}.
\end{aligned} \tag{3.14}$$

In the case of $\sigma_u = 0$, the Phillips curve (3.2) relates current inflation only to current and future demand. Thus with an output gap of zero over all periods, $x_t = 0 \forall t$, a policymaker can achieve both optimal output gap and optimal inflation. However, if cost push inflation is present, a decrease in demand lowers inflation only in the short run. In order to show this, we start with the general case, where a central bank accounts for output deviations, $a > 0$, and cost push shocks exist, $\sigma_u > 0$. From equations (3.9) and (3.3) we see that optimal policy implicitly leads inflation to its target in the long run,

$$\lim_{i \rightarrow \infty} E_t(\pi_{t+i}) = \lim_{i \rightarrow \infty} a q \rho^i u_t = 0.$$

In our setting, the long-term target of inflation is zero, since we can interpret π_t as the deviation of inflation from its long-term trend in period t . Policy measures to increase the speed of convergence in order to reach the inflation target earlier are only optimal if either there is no cost push inflation, or the policymaker omits the costs related to output deviations (i.e., $a = 0$).

In order to model the case of extreme inflation targeting, we look at equations (3.11), (3.12), (3.13) and (3.14). Without cost push inflation, $\sigma_u = 0$, there is no concern about the preferences of the policymaker. There is no trade-off between output variability and inflation variability. Thus extreme inflation targeting, especially manipulating the rate of inflation in order to immediately reach the target, is not costly in terms of output. In turn, if inflation is the only interest of a policymaker, he omits his preferences on output. Hence, $a = 0$, which leads us to the case of (3.11) and (3.12), where only the value of inflation variability is positive.

Next we will focus on implications affecting the policy instrument, the nominal interest rate, i_t . From equation (3.10) we get an intuition about how to react to changes in the given economic conditions. If expected inflation rises, an optimal policy should increase the nominal interest rate as much as needed to increase the real interest rate. For the model this means $\gamma_\pi > 1$. If expected inflation exceeds target inflation, economic theory suggests contracting demand. An optimal policy choice raises nominal interest rates sufficiently high, in order to make savings more interesting for private agents. This rise in nominal interest rates should exceed the rate of inflation, because otherwise the real interest rate will not raise private agents' incentives to save more. If the nominal interest rate is high enough, private agents spend less on consumption, which, in turn, lowers demand.

In contrast to changes in expectations, the optimal reaction to shocks is more difficult. The optimal reaction to demand shocks, g_t , differs from the one to shocks to potential output, y_t^n . Thus it is important to identify the source of the shock. From equation (3.10), we see immediately that an optimal policy should counter demand shocks in order to offset their negative effects for the economy. A shock in demand shifts both, output and inflation, off the long-term trend. In order to bring output and inflation back on the right track, a policymaker should adjust the nominal rate of inflation in a way that offsets the demand shock. A shock that shrinks demand should be followed by a policy reaction that increases the nominal interest rate. Here it is important to notice that a demand shock does not cause a short-term trade-off between output and inflation.

The case of a shock to potential output is different. A permanent rise in output causes a positive income effect which, in turn, is followed by an increase in output demand. Thus, since output and demand increase at the same scale, there is no effect on the output gap. Furthermore, there is no effect on prices and inflation remains unchanged. In fact, there is no reason for the policymaker to react on shocks in output.

3.4 The Classic Inflationary Bias Problem

The source of the inflationary bias problem first defined by Kydland and Prescott [8] lies in the ability of the policymaker to set the target level of output above its natural level. Here, we extend the discussion in the new Keynesian model and allow for a factor k larger than zero. This relates directly to the discussion on monetary policy under discretion in the natural rate model. The associated problem changes to

$$\max -\frac{1}{2}E_t \left(\sum_{i=0}^{\infty} \beta^i \left(a(x_{t+i} - k)^2 + \pi_{t+i}^2 \right) \right).$$

3.4.1 Formal Discussion

As before, we reformulate the problem into a static optimization problem

$$\max -\frac{1}{2} \left(a(x_t - k)^2 + \pi_t^2 \right) + F_t \quad \text{subject to} \quad \pi_t = \kappa x_t + f_t, \quad (3.15)$$

where

$$F_t \equiv -\frac{1}{2}E_t \left(\sum_{i=1}^{\infty} \beta^i \left(a(x_{t+i} - k)^2 + \pi_{t+i}^2 \right) \right) \quad \text{and} \quad f_t \equiv \beta E_t \pi_{t+1} + u_t.$$

We should always consider that this period's actions do not constrain future periods' output gap and inflation; under discretion, the policymaker cannot influence the private agents' expectations.

We analyse this optimization problem using the Lagrangian

$$\mathcal{L} = -\frac{1}{2} \left(a(x_t - k)^2 + \pi_t^2 \right) + F_t - \phi(\kappa x_t + f_t - \pi_t).$$

This leads us to the following first order conditions:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \pi_t} &= -\pi_t + \phi = 0 \\ \pi_t &= \phi \end{aligned} \quad (3.16)$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial x_t} &= -ax_t + ak - \phi\kappa = 0 \\ x_t &= -\frac{\phi\kappa}{a} + k.\end{aligned}\tag{3.17}$$

The appropriate Hessian Matrix for second order conditions is negative definite, hence the outcome accounts for a local maximum. Thus combining (3.16) and (3.17) we get the optimality condition for the first step

$$\boxed{x_t^d = -\frac{\kappa}{a}\pi_t^d + k} \quad \text{or} \quad \boxed{\pi_t^d = \frac{a}{\kappa}(-x_t^d + k)}.\tag{3.18}$$

Here the superscript d labels the optimal rules under discretion. In order to get reduced form expressions of x_t^d and π_t^d , we take the optimality condition (3.18) and plug in the Phillips curve (3.2),

$$x_t^d = -\frac{\kappa}{a}(\kappa x_t^d + \beta E_t \pi_{t+1}^d + u_t) + k.$$

As in section 3.3 we restrict our analysis to the case $x_t^d = -\omega u_t$. This leads to a rate of inflation of the form $\pi_t^d = \frac{a}{\kappa}(\omega u_t + k)$, which simplifies expected inflation $E_t \pi_{t+1}^d = \frac{a\omega\rho}{\kappa}u_t + \frac{a}{\kappa}k = \rho\pi_t^d - \frac{a\rho}{\kappa}k + \frac{a}{\kappa}k$.

$$ax_t^d = -\kappa^2 x_t^d - \beta\rho\kappa\pi_t^d + a\beta\rho k - a\beta k - \kappa u_t + ak$$

We use once again the optimality condition (3.18) in order to replace π_t^d ,

$$\begin{aligned}(\kappa^2 + a)x_t^d &= a\beta\rho x_t^d - a\beta\rho k + a\beta\rho k - a\beta k - \kappa u_t + ak \\ \kappa^2 + a(1 - \beta\rho)x_t^d &= -\kappa u_t + a(1 - \beta\rho)k \\ x_t^d &= -\frac{\kappa}{\kappa^2 + a(1 - \beta\rho)}u_t + \frac{a(1 - \beta\rho)}{\kappa^2 + a(1 - \beta\rho)}k.\end{aligned}$$

Finally we get the reduced form expressions,

$$x_t^d = -\kappa qu_t + aq(1 - \beta\rho)k \quad \text{and} \quad \pi_t^d = aqu_t - \frac{a}{\kappa}(aq(1 - \beta\rho) - 1)k,$$

$$\boxed{x_t^d = x_t + aq(1 - \beta\rho)k} \quad \text{and} \quad \boxed{\pi_t^d = \pi_t + \frac{a}{\kappa}k - \frac{a^2q(1 - \beta\rho)}{\kappa}k},$$

where $q = \frac{1}{\kappa^2 + a(1 - \beta\rho)}$. From the fact that

$$\begin{aligned} aq(1 - \beta\rho) &= \frac{a(1 - \beta\rho)}{\kappa^2 + a(1 - \beta\rho)} < 1 \\ a(1 - \beta\rho) &< \kappa^2 + a(1 - \beta\rho) \\ 0 &< \kappa^2 \end{aligned}$$

we see that a discretionary policy together with the attempt to reach an output gap target higher than the natural level results in an output gap that is below target while inflation is systematically increased

$$\boxed{x_t^d < x_t + k} \quad \text{and} \quad \boxed{\pi_t^d > \pi_t + \frac{a}{\kappa}k - ak}. \quad (3.19)$$

3.4.2 Implications on Monetary Policy

The analysis of the new Keynesian model gives us a similar intuition as the result in chapter 2.2. The central bank announces that it keeps future inflation rates low in order to influence expected inflation in the intended way. But, as $k > 0$ appears in the optimality condition (3.18), a central bank is tempted to raise current demand in order to raise output. However, rational private agents will recognize these incentives and incorporate them in their expectations formation. In the model of full information, private agents know exactly the form of equation (3.18), thus the policymaker cannot misguide them without getting punished. This ends up with inflation rates too high to allow for further increases of demand. As we have seen before, there is no long-term trade-off between output and inflation. Thus, although x_t converges to zero in the long-term, the equilibrium rate of inflation lies systematically over the long-term target.

If we interpret this result in a normative way, we see some arguments for making binding commitments. Such commitments should force a policymaker to act as if $k = 0$ in equation (3.18). We see a clear argument that such commitments increase economic welfare. Previous analyses have clearly shown that a commitment could keep inflation rates at its target, because of its positive impact on the expectations formation of private agents, without any impact on the output.

Theoretically, a policymaker should try to make binding commitments on future policy choices. In the model we can define and incorporate such a commitment quite simply, however in real life, defining a commitment is complicated. An interesting approach is the one proposed by Rogoff [12], who would assign a conservative policymaker in order to reduce the costs of the

inflationary bias that arises under discretion when $k > 0$. In this context, conservative means that the policymaker is more reluctant to accept inflation than the majority. This means that the value of a for the policymaker is lower than the one of the total economy. We get an intention of this idea from equation (3.19). If the policymaker assigns a relative cost to inflation which is smaller than the relative cost on inflation of the society, a , the inflationary bias becomes smaller. Although this idea seems reasonable, it has some shortcomings too. From previous discussion we know that a reduction in inflation variability may increase the variability in output. Another argument against the idea of Rogoff [12] is that a policymaker who has a clear distaste against inflation with an a close to zero may shrink economic welfare.

Today the problem of the inflationary bias seems quite under control in western economies. Most of the central banks follow the idea of Rogoff [12] and install central bankers who are rather disinclined towards inflation. We can see this development also in the goals of central banks which somehow include a rule that inflation rates should be close to a certain low but positive level. In addition, many economists argue that today no central bank would give in to the traditional source of the inflationary bias. On the whole, modern central banks seem to accept that short-term increases in output lead to long-term costs that by far offset any short-term gains. Thus misguiding the public and pushing output above its natural level are not really options for a rational central banker. This raises the question why most central banks, like the ECB, retain this commitment to stable inflation rates?

We argue that committing to a policy rule may have positive impacts on policymaking even if $k = 0$ and there are no intentions to push the output above potential. Previous analysis shows that the main power of a central bank is its credibility. A central bank which acts discretionary loses its credibility and hence causes a substantial amount of effort and considerable additional costs to regain it, besides the economic losses incurred by digressing from the optimal path. We have argued that one way to ensure credibility is by installing a commitment technology that prohibits the policymaker from misguiding the public. Moreover, in the next section we will point out that a central bank benefits from credibility and commitment even if there are no risks of the inflationary bias, $k = 0$. Today price setting depends virtually on expectations about future economic conditions, especially on the future development of the rate of inflation. A central bank that can credibly commit on stable inflation rates may benefit from a better output/inflation trade-off in the short run.

3.5 Improving the Short-term Output/Inflation Trade-off: Gains from Commitment with $k = 0$

The goal of this chapter is to show that a commitment solution always dominates the discretionary solution. In order to achieve this, we will start from the solution under discretion and restrict our analysis to equilibria of this form. Within this class of policy rules we will assess the optimum. Then we will look for the commitment solution within this class of policy rules which turns out to dominate the optimal rule under discretion. The main difference to the previous analysis is that the policymaker takes private agents' expectations no longer as given. He understands that his actions affect private agents' decisions. The superscript c labels the optimal rules under commitment.

We define the class of policy rules according to the analysis of sections 3.3 and 3.4,

$$x_t^c = -\omega u_t, \quad \forall t, \quad (3.20)$$

where $\omega > 0$ is the coefficient of the feedback rule and a high ω means that the central bank fosters a tough policy. Note that this class of policy rules includes the optimal rule under discretion if we set $\omega = \kappa q$. Using the rule (3.20) in the original Phillips curve (3.2), we see that this class of rules also imply a linear relation of inflation and the cost push shock.

$$\begin{aligned} \pi_t^c &= \kappa x_t^c + \beta E_t \pi_{t+1}^c + u_t \\ &= E_t \sum_{i=0}^{\infty} \beta^i (\kappa x_{t+i}^c + u_{t+i}) \\ &= E_t \sum_{i=0}^{\infty} \beta^i (-\kappa \omega u_{t+i} + u_{t+i}) \\ &= \frac{1 - \kappa \omega}{1 - \beta \rho} u_t \end{aligned} \quad (3.21)$$

The task of the policymaker is to set an optimal ω . While doing this, the policymaker now faces an improved short-term trade-off between output gap and inflation. This becomes obvious if we plug in (3.20) in (3.21).

$$\begin{aligned} \pi_t^c &= \frac{1}{1 - \beta \rho} u_t - \frac{\kappa}{1 - \beta \rho} \omega u_t \\ &= \frac{\kappa}{1 - \beta \rho} x_t^c + \frac{1}{1 - \beta \rho} u_t \end{aligned} \quad (3.22)$$

Equation (3.22) shows that, under commitment, a reduction of the output gap, x_t^c , by one percent lowers inflation, π_t^c , by a factor $\frac{\kappa}{1-\beta\rho}$. In contrast, the same shift of the output gap under discretion shifts inflation only by a factor $\kappa < \frac{\kappa}{1-\beta\rho}$, which is less than under commitment. The advantage under commitment comes from the possibility to influence private agents expectations about the future path of the output gap, x_{t+i}^c , $i = 1, 2, \dots$. Rational private agents would expect $E_t x_{t+i}^c = -\omega u_t$. We should notice the impact of this form of expectations. For example, a high value for ω allows the policymaker to credibly announce strict reactions to a persistent supply shock. In addition, a commitment to a policy rule with high ω leads to a bigger drop in inflation per reduced unit of output gap compared to discretion.

Now that we understand the positive impact of commitment on the short-term output/inflation trade-off, we are interested in the optimal value of ω . For this purpose we transform the objective function (3.4) to a function of period t loss. We can do so since we interpret x_{t+i}^c and π_{t+i}^c as functions of the period $(t+i)$ cost push shock, u_{t+i} ,

$$\max -\frac{1}{2} E_t \left(\sum_{i=0}^{\infty} \beta^i \left(a(x_{t+i}^c)^2 + (\pi_{t+i}^c)^2 \right) \right) \longleftrightarrow \max -\frac{1}{2} \left(a(x_t^c)^2 + (\pi_t^c)^2 \right) J_t, \quad (3.23)$$

with $J_t \equiv E_t \left(\sum_{i=0}^{\infty} \beta^i \left(\frac{u_{t+i}}{u_t} \right)^2 \right) > 0$. The optimization task here is to find the value of ω that maximizes (3.23) subject to equation (3.22).

In a first step, we analyse this optimization problem using the Lagrangian

$$\mathcal{L} = -\frac{1}{2} \left(a(x_t^c)^2 + (\pi_t^c)^2 \right) J_t - \phi \left(\frac{\kappa}{1-\beta\rho} x_t^c + \frac{1}{1-\beta\rho} u_t - \pi_t^c \right).$$

This leads us to the following first order conditions:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \pi_t^c} &= -\pi_t^c J_t + \phi = 0 \\ \pi_t^c J_t &= \phi \end{aligned} \quad (3.24)$$

$$\begin{aligned}
 \frac{\partial \mathcal{L}}{\partial x_t^c} &= -ax_t^c J_t - \frac{\phi\kappa}{1 - \beta\rho} = 0 \\
 x_t^c &= -\frac{\phi\kappa}{aJ_t(1 - \beta\rho)} \\
 x_t^c &= -\frac{\pi_t^c J_t \kappa}{aJ_t(1 - \beta\rho)} \\
 x_t^c &= -\frac{\kappa}{a^c} \pi_t^c,
 \end{aligned} \tag{3.25}$$

with $a^c \equiv a(1 - \beta\rho) < a$.

The appropriate Hessian Matrix for second order conditions is negative definite, hence the outcome accounts for a local maximum. Thus, combining (3.24) and (3.25), we get the optimality condition for the first step optimization problem

$$\boxed{x_t^c = -\frac{\kappa}{a^c} \pi_t^c} \quad \text{or} \quad \boxed{\pi_t^c = -\frac{a^c}{\kappa} x_t^c}. \tag{3.26}$$

We see immediately that a^c , the cost of lowering inflation under commitment, is less than a . Lowering inflation a certain amount under commitment costs only a fraction of $(1 - \beta\rho)$ in terms of output loss compared to the discretionary case. In other words, a policymaker who commits to a rule can fight inflation more aggressively with the same economic costs required under discretion. This issue is a consequence of the improved output/inflation trade-off under commitment. Technically, we can express this by comparing the optimality conditions under commitment (3.26) and discretion (3.8).

As in the discretionary case, we are not only interested in the relation between output gap and inflation, but also in their dependence on shocks. Thus we transform the optimality conditions (3.26) into reduced form expressions of the cost push shock u_t . For this purpose, we take the optimality condition (3.26) and plug in the Phillips curve (3.2),

$$x_t^c = -\frac{\kappa}{a^c} (\kappa x_t^c + \beta E_t \pi_{t+1}^c + u_t).$$

We make use of the assumption that private agents' expectations are rational, $E_t \pi_{t+1}^c = \rho \pi_t^c$, where ρ is the autoregressive component of the cost push shock in equation (3.3),

$$\begin{aligned}
a^c x_t^c &= -\kappa^2 x_t^c - \kappa \beta \rho \pi_t^c - \kappa u_t \\
(\kappa^2 + a^c) x_t^c &= -\kappa(\beta \rho \pi_t^c + u_t) \\
-\frac{\kappa^2 + a^c}{\kappa} x_t^c &= \beta \rho \pi_t^c + u_t.
\end{aligned}$$

Once again, we use the optimality condition (3.26) in order to replace π_t^c ,

$$\begin{aligned}
-\frac{\kappa^2 + a^c}{\kappa} x_t^c &= -\beta \rho \frac{a^c}{\kappa} x_t^c + u_t \\
-\frac{\kappa^2 + a^c(1 - \beta \rho)}{\kappa} x_t^c &= u_t \\
x_t^c &= -\frac{\kappa}{\kappa^2 + a^c(1 - \beta \rho)} u_t.
\end{aligned}$$

Finally, we get the reduced form expressions,

$$\boxed{x_t^c = -\kappa q^c u_t} \quad \text{and} \quad \boxed{\pi_t^c = a^c q^c u_t}, \quad (3.27)$$

where $q^c = \frac{1}{\kappa^2 + a^c(1 - \beta \rho)}$.

Comparing the results to those under discretion and omitting the influence of u_t , we see that inflation is closer to the target under commitment and the output gap is further away. In addition, we can see that the equilibrium conditions are identical to those under discretion if we replace a^c against a .

Next we will attempt to show that the solution under commitment improves economic welfare. For this purpose, we plug in the optimal values under discretion (3.9) and commitment (3.27) in the policy objective function and compare the outcomes.

$$\begin{aligned}
[discretion] &= \frac{1}{2} E_t \left(\sum_{i=0}^{\infty} \beta^i \left(a(-\kappa q u_{t+i})^2 + (a q u_{t+i})^2 \right) \right) \\
[commitment] &= \frac{1}{2} E_t \left(\sum_{i=0}^{\infty} \beta^i \left(a^c(-\kappa q^c u_{t+i})^2 + (a^c q^c u_{t+i})^2 \right) \right)
\end{aligned}$$

Next, we look at period $(t + i)$ for both cases and assume that the outcome under commitment dominates the outcome under discretion in every period.

$$\begin{aligned} -\frac{1}{2}\beta^i \left(a^c(-\kappa q^c u_{t+i})^2 + (a^c q^c u_{t+i})^2 \right) &> -\frac{1}{2}\beta^i \left(a(-\kappa q u_{t+i})^2 + (a q u_{t+i})^2 \right) \\ a^c \kappa^2 (q^c)^2 u_{t+i}^2 + (a^c)^2 (q^c)^2 u_{t+i}^2 &< a \kappa^2 q^2 u_{t+i}^2 + a^2 q^2 u_{t+i}^2 \\ a^c (q^c)^2 u_{t+i}^2 (\kappa^2 + a^c) &< a q^2 u_{t+i}^2 (\kappa^2 + a) \\ a^c (q^c)^2 (\kappa^2 + a^c) &< a q^2 (\kappa^2 + a) \end{aligned}$$

Further, we plug in the corresponding rules for q and q^c .

$$\begin{aligned} \frac{a^c(\kappa^2 + a^c)}{(\kappa^2 + a^c(1 - \beta\rho))^2} &< \frac{a(\kappa^2 + a)}{(\kappa^2 + a(1 - \beta\rho))^2} \\ (a^c \kappa^2 + (a^c)^2) (\kappa^2 + a(1 - \beta\rho))^2 &< (a \kappa^2 + a^2) (\kappa^2 + a^c(1 - \beta\rho))^2 \end{aligned}$$

Calculating the binomial formula and multiplying according to standard theory, we get

$$\begin{aligned} a^c \kappa^6 + (a^c)^2 \kappa^4 + 2a a^c \kappa^4 (1 - \beta\rho) + 2a (a^c)^2 \kappa^2 (1 - \beta\rho) + a^2 a^c \kappa^2 (1 - \beta\rho)^2 + a^2 (a^c)^2 (1 - \beta\rho)^2 &< \\ < a \kappa^6 + a^2 \kappa^4 + 2a a^c \kappa^4 (1 - \beta\rho) + 2a^2 a^c \kappa^2 (1 - \beta\rho) + a (a^c)^2 \kappa^2 (1 - \beta\rho)^2 + a^2 (a^c)^2 (1 - \beta\rho)^2. \end{aligned}$$

Since $a^c < a$, we see that $a^c \kappa^6 < a \kappa^6$ and $(a^c)^2 \kappa^4 < a^2 \kappa^4$. Thus, dropping these terms and those which are equal on both sides does not affect the generality of the statement. This leads us to the following inequality:

$$\begin{aligned} 2a(a^c)^2 \kappa^2 (1 - \beta\rho) + a^2 a^c \kappa^2 (1 - \beta\rho)^2 &< 2a^2 a^c \kappa^2 (1 - \beta\rho) + a(a^c)^2 \kappa^2 (1 - \beta\rho)^2 \\ a a^c (2a^c + a(1 - \beta\rho)) &< a a^c (2a + a^c(1 - \beta\rho)) \\ -a - a\beta\rho &< -a^c - a^c\beta\rho \\ a &> a^c. \end{aligned}$$

These results suggest that commitment dominates discretion in every period. Hence, if we sum up all periods, we see that commitment distinctly and undoubtedly benefits economic welfare more than discretion. Besides this proof, we can see the dominance of commitment in the definition of the optimization problem. When looking for the optimal rule under commitment, we include, in the set of possible rules, the one under discretion. Hence the resulting rule must be better or equal to the rule under discretion. Since we find a rule that is slightly different, we know that

this rule must, at least locally, optimize economic welfare.

The dominance of commitment over discretion suggests interesting implications for real-world policymaking even if a central bank does not try to push output above potential. Since expectations on future output gaps influence today's rate of inflation, a central bank tends to guide these expectations. On the other hand, a central bank prefers a smooth development of economic indicators. Thus it will always aim to convince private agents of a strict policy course in the future which loosens the pressure for harsh reactions today. However, as time goes by, there are rising incentives of the central bank to delay the switch to a tough policy and instead keep the present policy. Imagine, for instance, that a positive cost push shock hits the economy.

A policymaker without any restrictions on future policy choices tends to re-optimize, which guides him to choose the optimal policy under discretion. Following economic theory, this policy implies a decrease in output. However, the discretionary policy demands less contraction in output than the commitment policy. Rational private agents understand the incentives of the policymaker. Due to the fact that the policymaker has no constraints on future policy choices, they will not expect any big contractions of output in the future. Even if the policymaker announces such a measure, private agents will not believe him unless his announcements are credible. As an announcement is only credible if it cannot be altered in every upcoming period, the policymaker is not able to face the cost push shock in the best possible way, although he may think he is doing so. This setting leads to an unnecessarily high inflation. Important here is that the policymaker never tries to push output above the natural level. In this example, the policymaker simply tries to guide the economy through a difficult time where he needs to incorporate private agents' expectations in his analysis. The model we use accounts for this through the forward looking Phillips curve (3.2).

Concerning the second step of the optimization problem, where we try to obtain an optimal rule for the nominal interest rate under commitment, we will start our analysis with the optimal interest rate rule under discretion. Since all results are quite similar under discretion and under commitment, we shall abstain from going through all the calculations again which would lead us to the following rule where we just replace a^c instead of a in equation (3.10).

$$\boxed{i_t = \gamma_\pi^c E_t \pi_{t+1} + \frac{1}{\varphi} g_t} \quad \text{with} \quad \gamma_\pi^c \equiv 1 + \frac{(1-\rho)\kappa}{a^c \varphi \rho} > 1 + \frac{(1-\rho)\kappa}{a \varphi \rho} \equiv \gamma_\pi \quad (3.28)$$

Compared to discretion, a central bank following the commitment solution would increase the nominal interest rate more in response to increases of expected inflation.

Indeed, our analysis of commitment in a new Keynesian model reflects some of the eight principles of the *new neoclassical synthesis* presented by Goodfriend and King [7]. We find that expectations take a prominent role in the game against rational private agents; the real interest rate should rise in response to increases of inflation, which is in line with the *Taylor Principle*; an optimal monetary policy needs to account for the time-inconsistency problem; and commitment to an announced rule improves the short-term output gap/inflation trade-off and thus the efficiency in the economy.

Partial Commitment in a New Keynesian Model

An extensive part of this chapter is based on the article of Schaumburg and Tambalotti [14], thus specific references to this source have been omitted.

So far the discussion on the average inflation bias has focused on one main objective of a central bank. This is how to make a credible policy announcement in order to guide private agents' expectations. In the previous chapters we divided the actions of a central bank in two classes. A central bank can either commit to a certain policy rule, with the positive effect that private agents will believe in its actions, or they can follow a discretionary policy, re-optimize in every period and exclude the possibility that private agents believe in any pre-announced plans. From the private agents' view, the problem relates to the question if they trust in the policymaker or if they should be cautious and expect the policymaker to cheat. In this chapter we will add a third option to the optimal policy problem. Instead of either re-optimizing in every period or committing to a rule for all future periods, we allow the policymaker to commit to an optimal plan over his whole tenure.

Up to now we have assumed that the optimal policy decision is not related to the person which is in charge of the central bank. In the commitment case, each policymaker sticks to the rule that has been announced in the past. Now we model a commitment technology that allows for retaining an optimal plan for a certain undetermined time. Thus we assume that in every period there is a certain positive probability $\alpha \in [0, 1]$ that the current policymaker will be in office for another period. Hence, there is a chance of $(1 - \alpha)$ that a new policymaker is appointed. For technical reasons we assume that α is exogenous and constant over time. When a new policymaker is installed in period j , he first breaks with the policy of his predecessor and re-optimizes. He commits to the resulting optimal plan as long as he is in charge. Private agents know about the possibility, $(1 - \alpha)$, of a regime change and adopt their expectations accordingly. Thus private agents will always be doubtful about the promises on future policy although they know that the policymaker will not change the policy himself. Schaumburg and Tambalotti [14] refer to the idea of modelling a commitment technology, where a policymaker

can only commit over a certain undetermined time horizon, as a quasi-commitment technology. A policymaker can guarantee for his policy but cannot influence or restrict the policy choice of his successors. Private agents know this and assume a new policymaker to take an optimal policy decision independently of past promises. Contrarily, given these expectations of private agents, it is optimal for a new policymaker to re-optimize.

Here we should stress that although there may be re-optimizations in every period, under partial commitment a policymaker tries not to gain benefit by short-term increases in inflation as is the case under discretion. A new central banker always optimizes with the knowledge that he could stay in charge for an unlimited period. (In every period there is a positive probability that the policymaker stays until the next period). Thus the partial commitment solution prohibits the traditional source of the average inflation bias where a policymaker tries to manipulate the short-term output/inflation trade-off and pushes output above potential. The source of gains from commitment comes from the positive influence on private agents' expectations, as in section 3.3. We see also that the global optimum is still the outcome under full commitment. However, a partial commitment equilibrium can reach the global optimum only if we can guarantee that a policymaker stays in charge forever, $\alpha = 1$. Thus, partial commitment with a policy turnover probability $\alpha \in (0, 1)$ can achieve suboptimal outcomes only.

Partial commitment, as we define it, builds a link between the two extreme policy modes. Under partial commitment, we get a continuum of policy rules according to different values of α . Thus we can rank the resulting equilibria from close to discretion, where α is close to zero, up to commitment, where the corresponding rule reflects a value for α close to one. If we think about the principles of the problem once again, we see that the commitment equilibrium is reachable only if the policymaker is able to make credible announcements. If the policymaker lacks credibility, private agents' expectations will not be well-anchored and the resulting outcome tends to the discretionary solution. Thus we interpret α , the probability that a policymaker stays in office in the next period, as a measurement of credibility for the central bank. A central bank with a high turnover rate of policymakers, a low value of α , will have more changes in their policy. If policy changes happen frequently, private agents will trust less in the policy plans of a new policymaker because they know that he will not be in charge for a long time. Private agents will incorporate this knowledge in their expectations. However, as the probability of a regime change is high, the mismatch between private agents' expectations and the announced optimal plan increases. Thus we consider a central bank credible if a policymaker is in charge for a long time and is able to make durable plans. This, in turn, means that the value of α is high.

Referring to the former argument, we can rank the partial commitment equilibria according to their credibility α . We should mention that this interpretation of credibility is quite different from the one we analysed in the first chapter for the natural rate model. Here we somehow assume credibility and analyse what would be the outcome for a certain level of credibility. Before, the aim of analysis was to explain the advantages of a credible policymaker. In addition, here we see credibility as an attribute of the central bank and not of a single policy plan. As we expect a central bank to follow the best possible plans, we can rank central banks according to their available commitment technology if we link the level of credibility to the set of possible policy choices.

In addition, we should stress that under partial commitment deviations from a pre-announced optimal plan are part of the equilibrium. However, since the turnover probability of a policymaker is common knowledge, such deviations do not surprise private agents.

4.1 The Economic Environment

If we consider the economy as a long-term project with one major authority, the central bank that tries to guide the economy towards the best possible outcome, these decisions are taken by one policymaker, the governor of the central bank. Over time there are many governors that follow each other, with a common policy objective. Thus we can think of a sequence of policymakers with a random duration of their tenure. Each policymaker commits to his plan during his tenure but he cannot restrict the decisions of his successors. The tenure of a policymaker is divided in a random number of periods with equal duration. At the beginning of each period, the economy receives a perfectly observable signal $\eta_t \in \{0, 1\}$, where the process $\{\eta_t\}_{t \geq 0}$ is assumed to be a sequence of independent identically distributed Bernoulli draws. A signal $\eta_t = 1$ tells all agents in the economy that a new policymaker will be installed at the beginning of period t . The probability of signal $\eta_t = 1$ is equal to $(1 - \alpha)$. At the beginning of his tenure, a new policymaker reneges on the plans of his predecessor and optimizes over the common policy objective subject to the current state of the economy. He formulates an optimal plan that he commits to from period t onwards. If, with probability α , the signal $\eta_t = 0$ occurs, the current policymaker stays in the office and he follows the plan announced at the start of his tenure.

As in chapter 3, the current state of the economy is described by the rate of inflation and an exogenous shock process,

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t \quad \text{with} \quad u_t = \rho u_{t-1} + \dot{u}_t. \quad (4.1)$$

4.2 Optimal Policy under Partial Commitment

In order to get rules for optimal decisions, the policymaker needs to solve the following optimization problem.

$$\max -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\alpha^t \left(a(x_t - k)^2 + \pi_t^2 \right) + (1 - \alpha) \sum_{s=0}^{t-1} \alpha^s \left(a(\tilde{x}_t - k)^2 + \tilde{\pi}_t^2 \right) \right) \quad (4.2)$$

That is the expected discounted sum of the loss function taking into account the possibility that the optimizing policymaker may be replaced in some upcoming period t . Compare to equation (3.4). Since we assume that the regime change probability is constant over time, α^t is the probability that the policymaker's tenure still goes on in period t . The second term depicts the probability that the policymaker under consideration is exchanged in some period ($t - s \leq t$), i.e. there must have been a regime change with probability $(1 - \alpha)$ and afterwards the new regime lasts for the following s periods. Thus the probability that the policymaker's tenure in period t lasts exactly s periods, is $(1 - \alpha)\alpha^s$.

Note here that including the probability of a regime change has three obvious effects: (i) The discount rate includes the chance of a tenure of t periods and is modified to $(\alpha\beta)^t$; (ii) the probability of a regime change in period $(t - s)$ is included by a second term multiplied by $(1 - \alpha)\alpha^s$; (iii) rational private agents now expect a regime change with a positive probability in every period.

However, the second term is independent of any decisions of the current policymaker. Thus this term is irrelevant for the optimization task of the current policymaker and we can drop this term out of the optimization problem, which results in

$$\max -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \alpha^t \beta^t \left(a(x_t - k)^2 + \pi_t^2 \right). \quad (4.3)$$

The possibility of regime changes in a model of partial commitment influences the expectations formation of private agents. Thus we need to modify the Phillips curve constraint.

First, we assume that the equilibrium rate of inflation is linear of the form

$$\pi_{t+1} = h_0 + h_1 u_{t+1} + h_2 \phi_{t+1},$$

where ϕ_{t+1} is the Lagrange multiplier. Second, private agents form their expectations on inflation for period $t+1$ based on the information set $\mathcal{I}_t = \{u_i, \eta_i\}_{i \leq t}$ and the common known parameters of the model,

$$\begin{aligned} E_t \pi_{t+1} &= \alpha E_t^0 \pi_{t+1} + (1 - \alpha) E_t^1 \pi_{t+1} \\ &= \alpha E_t^0 \pi_{t+1} + (1 - \alpha) \tilde{\pi}_0, \end{aligned} \tag{4.4}$$

where $E_t^i \pi_{t+1} = E_t(\pi_{t+1} | \eta_{t+1} = i)$, $i \in \{0, 1\}$. Note here that $E_t^1 \pi_{t+1}$ is the expected inflation for the first period of a new regime. Thus $\tilde{\pi}_0 = h_0 + \rho h_1 u_t$ denotes the choice of inflation of a new policymaker for the initial period of his regime. Consider that the Phillips curve constraint is not binding in the first period of a new regime, hence $E_t^1 \phi_{t+1} = 0$.

With probability α , the current policymaker's tenure goes on and the level of inflation will be π_{t+1} , which is derived in line with the optimal plan of this policymaker. With probability $(1 - \alpha)$, a new policymaker is appointed and chooses a rate of inflation which is optimal according to the re-optimized plan, h_0 . Based on these inflation expectations, private agents decide how much output they produce in period t . The corresponding Phillips curve that accounts for these expectations is

$$\pi_t = \kappa x_t + \alpha \beta E_t^0 \pi_{t+1} + (1 - \alpha) \beta \tilde{\pi}_0 + u_t. \tag{4.5}$$

4.2.1 Formal Discussion

We analyse this optimization problem using the Lagrangian. Note, however, that under the partial commitment approach we need to assume that the policymaker does influence private agents' expectations or at least the regime turnover probability $(1 - \alpha)$ does so. Hence, compared to sections 3.1 and 3.2 we do not have a static optimization problem. Furthermore, we will have to solve a non-homogeneous second order stochastic difference equation,

$$\begin{aligned}
\mathcal{L} &= -\frac{1}{2} \sum_{t=0}^{\infty} (\alpha\beta)^t \left(a(x_t - k)^2 + \pi_t^2 + \phi_t \left(\kappa x_t + \alpha\beta E_t^0 \pi_{t+1} + (1 - \alpha)\beta \tilde{\pi}_0 + u_t - \pi_t \right) \right) \\
&= -\frac{1}{2} \left(a(x_0 - k)^2 + \pi_0^2 + \phi_0(\kappa x_0 + u_0 - \pi_0) \right) \\
&\quad - \frac{1}{2} \sum_{t=1}^{\infty} (\alpha\beta)^t \left(a(x_t - k)^2 + \pi_t^2 + \phi_t(\kappa x_t + u_t - \pi_t) + \phi_{t-1} E_{t-1}^0 \pi_t \right) + F,
\end{aligned}$$

where $F = -\frac{1}{2} \sum_{t=0}^{\infty} (\alpha\beta)^t \phi_t (1 - \alpha)\beta \tilde{\pi}_0$, which is independent of any decisions of the current policymaker. The corresponding first order conditions for period t are:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial x_t} &= -(\alpha\beta)^t a(x_t - k) - \frac{1}{2}(\alpha\beta)^t \phi_t \kappa = 0 \\
\phi_t &= -\frac{2a}{\kappa}(x_t - k)
\end{aligned} \tag{4.6}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \pi_t} &= -(\alpha\beta)^t \pi_t + \frac{1}{2}(\alpha\beta)^t \phi_t - \frac{1}{2}(\alpha\beta)^t \phi_{t-1} = 0 \\
\pi_t &= \frac{1}{2}\phi_t - \frac{1}{2}\phi_{t-1} \\
\pi_t &= -\frac{a}{\kappa}(x_t - k) + \frac{a}{\kappa}(x_{t-1} - k) \\
\pi_t &= -\frac{a}{\kappa}(x_t - x_{t-1})
\end{aligned} \tag{4.7}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \phi_t} &= -\frac{1}{2}(\alpha\beta)^t \left(\kappa x_t + \alpha\beta E_t^0 \pi_{t+1} + (1 - \alpha)\beta \tilde{\pi}_0 + u_t - \pi_t \right) = 0 \\
\alpha\beta E_t^0 \pi_{t+1} + (1 - \alpha)\beta \tilde{\pi}_0 &= \pi_t - \kappa x_t - u_t.
\end{aligned} \tag{4.8}$$

Since the first order conditions for period t include values of the output gap of two consecutive periods, we need to transform them into a difference equation.

$$\begin{aligned}
-\alpha\beta \frac{a}{\kappa} E_t^0 (x_{t+1} - x_t) + (1 - \alpha)\beta \tilde{\pi}_0 &= -\frac{a}{\kappa}(x_t - x_{t-1}) - \kappa x_t - u_t \\
-\alpha\beta \frac{a}{\kappa} E_t^0 x_{t+1} + \alpha\beta \frac{a}{\kappa} \underbrace{E_t^0 x_t}_{x_t} + (1 - \alpha)\beta \tilde{\pi}_0 &= -\frac{a}{\kappa} x_t + \frac{a}{\kappa} x_{t-1} - \kappa x_t - u_t \\
-\alpha\beta \frac{a}{\kappa} E_t^0 x_{t+1} + \left(\alpha\beta \frac{a}{\kappa} + \frac{a}{\kappa} + \kappa \right) x_t - \frac{a}{\kappa} x_{t-1} &= -u_t - (1 - \alpha)\beta \tilde{\pi}_0 \\
\alpha\beta E_t^0 x_{t+1} - \left(\alpha\beta + 1 + \frac{\kappa^2}{a} \right) x_t + x_{t-1} &= \frac{\kappa}{a} (u_t + (1 - \alpha)\beta \tilde{\pi}_0)
\end{aligned}$$

Dividing by $\alpha\beta$ and using the definition of $\tilde{\pi}_0$ transforms this non-homogeneous second order stochastic difference equation into

$$E_t^0 x_{t+1} = \frac{a\alpha\beta + a + \kappa^2}{a\alpha\beta} x_t - \frac{1}{\alpha\beta} x_{t-1} + \frac{\kappa}{a\alpha\beta} \left((1 + (1 - \alpha)\beta\rho h_1) u_t + (1 - \alpha)\beta h_0 \right). \quad (4.9)$$

Qualitative Behaviour

Before solving this difference equation analytically we will search for a qualitative interpretation. We are interested in stable solutions; hence we will search for eigenvalues inside the unit circle. In order to make use of the *Trace, Determinant* or $(\mathcal{T}, \mathcal{D})$ approach, we transform equation (4.9) into a first order difference equation,

$$\begin{pmatrix} E_t^0 x_{t+1} \\ x_t \end{pmatrix} = \begin{pmatrix} \frac{a\alpha\beta + a + \kappa^2}{a\alpha\beta} & -\frac{1}{\alpha\beta} \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_t \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} \frac{\kappa}{a\alpha\beta} \left((1 + (1 - \alpha)\beta\rho h_1) u_t + (1 - \alpha)\beta h_0 \right) \\ 0 \end{pmatrix}.$$

Next we display the trace and the determinant,

$$\begin{aligned} \mathcal{T} &= \frac{a\alpha\beta + a + \kappa^2}{a\alpha\beta} = 1 + \frac{1}{\alpha\beta} + \frac{\kappa^2}{a\alpha\beta} \\ \mathcal{D} &= \frac{1}{\alpha\beta}. \end{aligned}$$

The eigenvalues are of the form $\mu_1 = \frac{\mathcal{T}}{2} - \sqrt{\frac{\mathcal{T}^2}{4} - \mathcal{D}}$ and $\mu_2 = \frac{\mathcal{T}}{2} + \sqrt{\frac{\mathcal{T}^2}{4} - \mathcal{D}}$. We show that one eigenvalue is inside the unit circle, while the other is not ($0 < \mu_1 < 1$ and $1 < \mu_2$).

$$\begin{aligned} \mu_1 &> 0 \\ \frac{\mathcal{T}}{2} - \sqrt{\frac{\mathcal{T}^2}{4} - \mathcal{D}} &> 0 \\ \frac{\mathcal{T}^2}{4} &> \frac{\mathcal{T}^2}{4} - \mathcal{D} \\ 0 &> -\mathcal{D} \\ 0 &> -\frac{1}{\alpha\beta} \end{aligned}$$

$$\begin{aligned}
\mu_1 &< 1 \\
\frac{\mathcal{T}}{2} - \sqrt{\frac{\mathcal{T}^2}{4} - \mathcal{D}} &< 1 \\
\frac{\mathcal{T}}{2} - 1 &< \sqrt{\frac{\mathcal{T}^2}{4} - \mathcal{D}} \\
\frac{\mathcal{T}^2}{4} - \mathcal{T} + 1 &< \frac{\mathcal{T}^2}{4} - \mathcal{D} \\
-\mathcal{T} + 1 &< -\mathcal{D} \\
-1 - \frac{1}{\alpha\beta} - \frac{\kappa^2}{a\alpha\beta} + 1 &< -\frac{1}{\alpha\beta} \\
-\frac{\kappa^2}{a\alpha\beta} &< 0
\end{aligned}$$

This shows $0 < \mu_1 < 1$. Since the determinant must always equal the product of the eigenvalues, we see from $\mathcal{D} = \frac{1}{\alpha\beta} > 1$ that the second eigenvalue μ_2 must be greater than unity. Thus we have one stable eigenvalue, μ_1 , and one unstable eigenvalue, μ_2 . In order to be able to make predictions, we need to omit the unstable eigenvalue in the solution to the difference equation. For convenience, we will, from now on, denote the remaining stable eigenvalue μ_1 as μ_α , which should remind us of its dependence on the probability of continuing the current regime, α .

$$\begin{aligned}
\mu_1 = \mu_\alpha &= \frac{\mathcal{T}}{2} - \sqrt{\frac{\mathcal{T}^2}{4} - \mathcal{D}} \\
&= \frac{a\alpha\beta + a + \kappa^2}{2a\alpha\beta} - \sqrt{\frac{\left(\frac{a\alpha\beta + a + \kappa^2}{a\alpha\beta}\right)^2}{4} - \frac{1}{\alpha\beta}} \\
&= \frac{a\alpha\beta + a + \kappa^2 - \sqrt{a^2\alpha^2\beta^2 + 2a\alpha\beta(\kappa^2 - a) + (a + \kappa^2)^2}}{2a\alpha\beta}
\end{aligned} \tag{4.10}$$

In order to be able to interpret the results later on, we need to analyse the reaction of μ_α on changes in α .

$$\frac{\partial \mu_\alpha}{\partial \alpha} = - \frac{(a + \kappa^2)\sqrt{a^2\alpha^2\beta^2 + 2a\alpha\beta(\kappa^2 - a) + (a + \kappa^2)^2} - a\alpha\beta(\kappa^2 - a) - (a + \kappa^2)^2}{2a\alpha^2\beta\sqrt{a^2\alpha^2\beta^2 + 2a\alpha\beta(\kappa^2 - a) + (a + \kappa^2)^2}}$$

We will show that this reaction is negative, $\frac{\partial \mu_\alpha}{\partial \alpha} < 0$. Since the characteristic polynomial of the system matrix has two real roots, the discriminant must be positive. Thus multiplying both sides of the inequality by the denominator and by (-1) gives

$$(a + \kappa^2)\sqrt{a^2\alpha^2\beta^2 + 2a\alpha\beta(\kappa^2 - a) + (a + \kappa^2)^2} > a\alpha\beta(\kappa^2 - a) + (a + \kappa^2)^2,$$

where the right side is positive because of $a^2 - a^2\alpha\beta > 0$. Squaring both sides gives

$$(a + \kappa^2)^2 \left(a^2\alpha^2\beta^2 + 2a\alpha\beta(\kappa^2 - a) + (a + \kappa^2)^2 \right) > \left(a\alpha\beta(\kappa^2 - a) + (a + \kappa^2)^2 \right)^2$$

$$4a^3\alpha^2\beta^2\kappa^2 > 0.$$

This is correct since all variables are positive. Thus the stable root μ_α decreases in α .

Analytical Solution

In order to find the stable solution of the stochastic difference equation (4.9), we will use the method of undetermined coefficients,

$$x_t = Ax_{t-1} + Bu_t + C.$$

Note here that $E_t^0 x_t = x_t$ and $E_t^0 u_{t+1} = \rho u_t$.

$$\begin{aligned} E_t^0 x_{t+1} &= \frac{a\alpha\beta + a + \kappa^2}{a\alpha\beta} x_t - \frac{1}{\alpha\beta} x_{t-1} + \\ &\quad + \frac{\kappa}{a\alpha\beta} \left((1 + (1 - \alpha)\beta\rho h_1) u_t + (1 - \alpha)\beta h_0 \right) \\ Ax_t + \rho Bu_t + C &= \frac{a\alpha\beta + a + \kappa^2}{a\alpha\beta} x_t - \frac{1}{\alpha\beta} x_{t-1} + \\ &\quad + \frac{\kappa}{a\alpha\beta} \left((1 + (1 - \alpha)\beta\rho h_1) u_t + (1 - \alpha)\beta h_0 \right) \\ A^2 x_{t-1} + ABu_t + AC + \rho Bu_t + C &= \frac{a\alpha\beta + a + \kappa^2}{a\alpha\beta} (Ax_{t-1} + Bu_t + C) - \frac{1}{\alpha\beta} x_{t-1} + \\ &\quad + \frac{\kappa}{a\alpha\beta} \left((1 + (1 - \alpha)\beta\rho h_1) u_t + (1 - \alpha)\beta h_0 \right). \end{aligned}$$

Collecting terms yields

$$\begin{aligned} 0 &= \left(\frac{a\alpha\beta + a + \kappa^2}{a\alpha\beta} A - \frac{1}{\alpha\beta} - A^2 \right) x_{t-1} + \\ &\quad + \left(\frac{a\alpha\beta + a + \kappa^2}{a\alpha\beta} B + \frac{\kappa(1 + (1 - \alpha)\beta\rho h_1)}{a\alpha\beta} - AB - \rho B \right) u_t + \\ &\quad + \frac{a\alpha\beta + a + \kappa^2}{a\alpha\beta} C + \frac{(1 - \alpha)\kappa}{a\alpha} h_0 - AC - C \end{aligned}$$

Next we compare the coefficients:

$$x_{t-1} : \quad A^2 - \frac{a\alpha\beta + a + \kappa^2}{a\alpha\beta}A + \frac{1}{\alpha\beta} = 0$$

Comparing with equation (4.10), we see that the root of this quadratic equation that lies inside the unit circle equals the stable eigenvalue of the difference equation, hence $A = \mu_\alpha$.

$$\begin{aligned} u_t : \quad & B \left(\frac{a\alpha\beta + a + \kappa^2}{a\alpha\beta} - \mu_\alpha - \rho \right) = -\frac{\kappa(1 + (1 - \alpha)\beta\rho h_1)}{a\alpha\beta} \\ & B \left(\frac{a\alpha\beta + a + \kappa^2 - a\alpha\beta\mu_\alpha - a\alpha\beta\rho}{a\alpha\beta} \right) = -\frac{\kappa(1 + (1 - \alpha)\beta\rho h_1)}{a\alpha\beta} \\ & B = -\frac{\kappa(1 + (1 - \alpha)\beta\rho h_1)}{a\alpha\beta(1 - \mu_\alpha - \rho) + a + \kappa^2} \\ \text{constant} : \quad & C \left(\frac{a\alpha\beta + a + \kappa^2}{a\alpha\beta} - \mu_\alpha - 1 \right) = -\frac{(1 - \alpha)\kappa}{a\alpha}h_0 \\ & C \left(\frac{a\alpha\beta + a + \kappa^2 - a\alpha\beta\mu_\alpha - a\alpha\beta}{a\alpha\beta} \right) = -\frac{(1 - \alpha)\kappa}{a\alpha}h_0 \\ & C = \frac{(1 - \alpha)\beta\kappa}{a\alpha\beta\mu_\alpha - a - \kappa^2}h_0. \end{aligned}$$

Hence, the solution to the stochastic difference equation (4.9) is

$$\boxed{x_t = \mu_\alpha x_{t-1} - \frac{\kappa(1 + (1 - \alpha)\beta\rho h_1)}{a\alpha\beta(1 - \mu_\alpha - \rho) + a + \kappa^2}u_t + \frac{(1 - \alpha)\beta\kappa}{a\alpha\beta\mu_\alpha - a - \kappa^2}h_0.} \quad (4.11)$$

Determining the Unknowns h_0 and h_1

In order to determine the unknown coefficients h_0 and h_1 , we transform (4.7) into $x_t = -\frac{\kappa}{a}\pi_t + x_{t-1}$ and plug it in the solution of the difference equation (4.11)

$$\begin{aligned} -\frac{\kappa}{a}\pi_t &= (\mu_\alpha - 1)x_{t-1} - \frac{\kappa(1 + (1 - \alpha)\beta\rho h_1)}{a\alpha\beta(1 - \mu_\alpha - \rho) + a + \kappa^2}u_t + \frac{(1 - \alpha)\beta\kappa}{a\alpha\beta\mu_\alpha - a - \kappa^2}h_0 \\ \pi_t &= \frac{a}{\kappa}(1 - \mu_\alpha)x_{t-1} + \frac{a(1 + (1 - \alpha)\beta\rho h_1)}{a\alpha\beta(1 - \mu_\alpha - \rho) + a + \kappa^2}u_t - \frac{a(1 - \alpha)\beta}{a\alpha\beta\mu_\alpha - a - \kappa^2}h_0 \end{aligned}$$

Then we derive an expression for $E_t^1\pi_{t+1}$, the expectations on inflation if the policymaker is replaced within period t . Remember that a new policymaker skips the optimal plan of his predecessor. For technical reasons we assume that at the end of a policymaker's tenure, the level

of the output gap is at the target, $E_t^1 x_t = k$. We will verify this argument when we derive the equilibrium.

$$E_t^1 \pi_{t+1} = \frac{a}{\kappa}(1 - \mu_\alpha)k + \frac{a\rho(1 + (1 - \alpha)\beta\rho h_1)}{a\alpha\beta(1 - \mu_\alpha - \rho) + a + \kappa^2}u_t - \frac{a(1 - \alpha)\beta}{a\alpha\beta\mu_\alpha - a - \kappa^2}h_0$$

Comparing the coefficients with $E_t^1 \pi_{t+1} = \tilde{\pi}_0 = h_0 + \rho h_1 u_t$ yields for h_0 ,

$$\begin{aligned} \frac{a}{\kappa}(1 - \mu_\alpha)k - \frac{a(1 - \alpha)\beta}{a\alpha\beta\mu_\alpha - a - \kappa^2}h_0 &= h_0 \\ \frac{a}{\kappa}(1 - \mu_\alpha)k &= \left(1 + \frac{a(1 - \alpha)\beta}{a\alpha\beta\mu_\alpha - a - \kappa^2}\right)h_0 \\ \frac{a}{\kappa}(1 - \mu_\alpha)k &= \frac{a\alpha\beta(\mu_\alpha - 1) - a(1 - \beta) - \kappa^2}{a\alpha\beta\mu_\alpha - a - \kappa^2}h_0 \\ \frac{a(1 - \mu_\alpha)k}{\kappa} \frac{a\alpha\beta\mu_\alpha - a - \kappa^2}{a\alpha\beta(\mu_\alpha - 1) - a(1 - \beta) - \kappa^2} &= h_0, \end{aligned} \tag{4.12}$$

and for h_1 ,

$$\begin{aligned} \frac{a\rho(1 + (1 - \alpha)\beta\rho h_1)}{a\alpha\beta(1 - \mu_\alpha - \rho) + a + \kappa^2}u_t &= \rho h_1 u_t \\ \frac{a}{a\alpha\beta(1 - \mu_\alpha - \rho) + a + \kappa^2} &= \left(1 - \frac{a(1 - \alpha)\beta\rho}{a\alpha\beta(1 - \mu_\alpha - \rho) + a + \kappa^2}\right)h_1 \\ \frac{a}{a\alpha\beta(1 - \mu_\alpha - \rho) + a + \kappa^2} &= \frac{a\alpha\beta(1 - \mu_\alpha - \rho) + a + \kappa^2 - a(1 - \alpha)\beta\rho}{a\alpha\beta(1 - \mu_\alpha - \rho) + a + \kappa^2}h_1 \\ \frac{a}{a\alpha\beta(1 - \mu_\alpha) + a + \kappa^2 - a\beta\rho} &= h_1. \end{aligned} \tag{4.13}$$

Modifying the solution

First we concentrate on the constant of the solution (4.11). Substituting equation (4.12) for h_0 yields

$$\frac{(1 - \alpha)\beta\kappa}{a\alpha\beta\mu_\alpha - a - \kappa^2} \frac{a(1 - \mu_\alpha)k}{\kappa} \frac{a\alpha\beta\mu_\alpha - a - \kappa^2}{a\alpha\beta(\mu_\alpha - 1) - a(1 - \beta) - \kappa^2} = \frac{a(1 - \alpha)\beta(1 - \mu_\alpha)}{a\alpha\beta(\mu_\alpha - 1) - a(1 - \beta) - \kappa^2}k.$$

In order to simplify this expression, we show

$$\begin{aligned} \frac{a(1 - \alpha)\beta(1 - \mu_\alpha)}{a\alpha\beta(\mu_\alpha - 1) - a(1 - \beta) - \kappa^2}k &= -(1 - \mu_\alpha) \frac{(1 - \alpha)\beta\mu_\alpha}{1 - \beta\mu_\alpha}k \\ a - a\beta\mu_\alpha &= -\mu_\alpha \left(a\alpha\beta(\mu_\alpha - 1) - a(1 - \beta) - \kappa^2\right). \end{aligned}$$

Using (4.10), the left hand side of this equation yields

$$\frac{4a^2\alpha\beta - 2a^2\alpha\beta^2 - 2a^2\beta - 2a\beta\kappa^2 + 2a\beta\sqrt{(\dots)}}{4a\alpha\beta},$$

and the right hand side yields

$$\begin{aligned} & \frac{\left(-a\alpha\beta - a - \kappa^2 + \sqrt{(\dots)}\right) \left(a\alpha\beta + a + \kappa^2 - \sqrt{(\dots)} - 2a\alpha\beta - 2a + 2a\beta - 2\kappa^2\right)}{4a\alpha\beta} = \\ & = \frac{4a^2\alpha\beta - 2a^2\alpha\beta^2 - 2a^2\beta - 2a\beta\kappa^2 + 2a\beta\sqrt{(\dots)}}{4a\alpha\beta}, \end{aligned}$$

where $\sqrt{(\dots)} = \sqrt{a^2\alpha^2\beta^2 + 2a\alpha\beta(\kappa^2 - a) + (a + \kappa^2)^2}$. Hence, both sides are equal and we can write the constant of the solution (4.11)

$$-(1 - \mu_\alpha) \frac{(1 - \alpha)\beta\mu_\alpha}{1 - \beta\mu_\alpha} k.$$

Next we concentrate on the coefficient of u_t of the solution (4.11). Substituting equation (4.13) for h_1 yields

$$\begin{aligned} & -\frac{\kappa \left(1 + \frac{a(1-\alpha)\beta\rho}{a\alpha\beta(1-\mu_\alpha)+a+\kappa^2-a\beta\rho}\right)}{a\alpha\beta(1-\mu_\alpha-\rho)+a+\kappa^2} = -\frac{\kappa \left(\frac{a\alpha\beta(1-\mu_\alpha-\rho)+a+\kappa^2}{a\alpha\beta(1-\mu_\alpha)+a+\kappa^2-a\beta\rho}\right)}{a\alpha\beta(1-\mu_\alpha-\rho)+a+\kappa^2} = \\ & = -\frac{\kappa}{a\alpha\beta(1-\mu_\alpha)+a+\kappa^2-a\beta\rho}. \end{aligned}$$

Finally, we can write the solution of the difference equation (4.11) as

$$x_t = \mu_\alpha x_{t-1} - \frac{\kappa}{a\alpha\beta(1-\mu_\alpha)+a+\kappa^2-a\beta\rho} u_t - (1 - \mu_\alpha) \frac{(1 - \alpha)\beta\mu_\alpha}{1 - \beta\mu_\alpha} k. \quad (4.14)$$

Determining the Equilibrium

In order to find the equilibrium values of the output gap and inflation, we need to solve the recursive solution of the difference equation (4.14) explicitly. Thus we need to define the initial value of the output gap x_0 . So far we have analysed the optimal policy problem straightforward, thereby omitting one of the main aspects of partial commitment. A policymaker skips the optimal plan of his predecessor before he re-optimizes. Technically, this means that he omits the Phillips curve constraint for the initial period, $\phi_0 = 0$. Thus, from the first order condition (4.6) it follows

that the initial value for the output gap $x_0 = k$. This implies that the resulting policy plan is independent of any past economic conditions.

For convenience we define $\mathcal{A} \equiv \frac{\kappa}{a\alpha\beta(1-\mu_\alpha)+a+\kappa^2-a\beta\rho}$ and $\mathcal{B} \equiv \frac{(1-\alpha)\beta\mu_\alpha}{1-\beta\mu_\alpha}$. The explicit form of the solution (4.14) is

$$\begin{aligned} x_t &= \mu_\alpha^t x_0 - (1 - \mu_\alpha) \mathcal{B} k \sum_{i=0}^{t-1} \mu_\alpha^i - \mathcal{A} \sum_{i=1}^t \mu_\alpha^{t-i} u_i \\ &= \mu_\alpha^t x_0 - (1 - \mu_\alpha^t) \mathcal{B} k - \mathcal{A} \sum_{i=1}^t \mu_\alpha^{t-i} u_i. \end{aligned}$$

Using the initial condition, this yields for the equilibrium output gap,

$$\boxed{x_t = \left((1 + \mathcal{B}) \mu_\alpha^t - \mathcal{B} \right) k - \mathcal{A} \sum_{i=1}^t \mu_\alpha^{t-i} u_i.} \quad (4.15)$$

For the equilibrium rate of inflation we use equation (4.7). First, we derive an expression for $(x_t - x_{t-1})$,

$$\begin{aligned} x_t - x_{t-1} &= \left((1 + \mathcal{B}) \mu_\alpha^t - \mathcal{B} \right) k - \mathcal{A} \sum_{i=1}^t \mu_\alpha^{t-i} u_i - \\ &\quad - \left((1 + \mathcal{B}) \mu_\alpha^{t-1} - \mathcal{B} \right) k + \mathcal{A} \sum_{i=1}^{t-1} \mu_\alpha^{t-1-i} u_i \\ &= \left(\mu_\alpha^t - \mu_\alpha^{t-1} \right) (1 + \mathcal{B}) k - \mathcal{A} \sum_{i=1}^{t-1} \left(\mu_\alpha^{t-i} - \mu_\alpha^{t-1-i} \right) u_i - \mathcal{A} u_t \\ &= -(1 - \mu_\alpha) \left(\mu_\alpha^{t-1} (1 + \mathcal{B}) k - \mathcal{A} \sum_{i=1}^{t-1} \mu_\alpha^{t-1-i} u_i \right) - \mathcal{A} u_t. \end{aligned} \quad (4.16)$$

Hence, in equilibrium the rate of inflation yields

$$\boxed{\pi_t = \frac{a(1 - \mu_\alpha)}{\kappa} \left((1 + \mathcal{B}) \mu_\alpha^{t-1} k - \mathcal{A} \sum_{i=1}^{t-1} \mu_\alpha^{t-1-i} u_i \right) + \frac{a}{\kappa} \mathcal{A} u_t.} \quad (4.17)$$

In the end, we derive an optimal rule for the nominal interest rate using the *IS* curve (3.1) and two times (4.7),

$$\begin{aligned}
 i_t &= \frac{1}{\varphi} (-x_t + E_t x_{t+1} + \varphi E_t \pi_{t+1} + g_t) \\
 &= \frac{1}{\varphi} \left(-x_t + E_t x_{t+1} - \frac{a\varphi}{\kappa} (E_t x_{t+1} - x_t) + g_t \right) \\
 &= \frac{1}{\varphi} \left(\left(1 - \frac{a\varphi}{\kappa} \right) (E_t x_{t+1} - x_t) + g_t \right)
 \end{aligned}$$

$i_t = \gamma_\pi^p E_t \pi_{t+1} + \frac{1}{\varphi} g_t$

with

$\gamma_\pi^p = \frac{a\varphi - \kappa}{\varphi\kappa}.$

(4.18)

From (4.16) we see that the term $E_t x_{t+1} - x_t$ yields

$$E_t x_{t+1} - x_t = -(1 - \mu_\alpha) \left(\mu_\alpha^t (x_0 + \mathcal{B}k) - \mathcal{A} \sum_{i=1}^t \mu_\alpha^{t-i} u_i \right) - \mathcal{A} E_t u_{t+1}.$$

Thus the optimal interest rate yields

$$i_t = \frac{a\varphi - \kappa}{\varphi\kappa} \left((1 - \mu_\alpha) \left(\mu_\alpha^t (1 + \mathcal{B})k - \mathcal{A} \sum_{i=1}^t \mu_\alpha^{t-i} u_i \right) + \rho \mathcal{A} u_t \right) + \frac{1}{\varphi} g_t.$$

(4.19)

The long-term values of the Output Gap and Inflation

Finally, we will present a perhaps surprising result on the expected value of the output gap conditional on no further regime change in the future. Which output gap level would be achievable if the society follows the current optimal plan forever? Thus we consider the expected behaviour of the single terms of the equilibrium output gap (4.15) as t approaching infinity. The first term includes no stochastic input, thus it yields in the long run

$$\lim_{t \rightarrow \infty} \left((1 + \mathcal{B}) \mu_\alpha^t - \mathcal{B} \right) k = -\mathcal{B}k.$$

Since the second term includes a stochastic part u_t , we need to calculate the expression

$$\begin{aligned}
\lim_{t \rightarrow \infty} E_0 \left(-\mathcal{A} \sum_{i=1}^t \mu_\alpha^{t-i} u_i \right) &= -\mathcal{A} \lim_{t \rightarrow \infty} \sum_{i=1}^t \mu_\alpha^{t-i} E_0 u_i \\
&= -\mathcal{A} \lim_{t \rightarrow \infty} \sum_{i=1}^t \mu_\alpha^{t-i} \rho^i u_0 \\
&= -\mathcal{A} \lim_{t \rightarrow \infty} \frac{\rho \mu_\alpha^{t-1} + \rho^{t+1} \mu_\alpha^{-1}}{1 - \rho \mu_\alpha^{-1}} u_0 \\
&= 0.
\end{aligned}$$

Hence, the within regime expected long-term output gap is

$$\boxed{\bar{x} = -\mathcal{B}k} \tag{4.20}$$

Concerning long-term value of inflation we proceed in a similar way:

$$\begin{aligned}
\lim_{t \rightarrow \infty} \frac{a(1 - \mu_\alpha)}{\kappa} \mu_\alpha^{t-1} (1 + \mathcal{B})k &= 0 \\
\lim_{t \rightarrow \infty} E_0 \left(-\frac{a(1 - \mu_\alpha)}{\kappa} \mathcal{A} \sum_{i=1}^{t-1} \mu_\alpha^{t-1-i} u_i \right) &= 0 \\
\lim_{t \rightarrow \infty} E_0 \left(\frac{a}{\kappa} \mathcal{A} u_t \right) &= \frac{a}{\kappa} \mathcal{A} \lim_{t \rightarrow \infty} E_0 u_t \\
&= \frac{a}{\kappa} \mathcal{A} \lim_{t \rightarrow \infty} \rho^t u_0 \\
&= 0.
\end{aligned}$$

Hence, the within regime expected long-term value of inflation is

$$\boxed{\bar{\pi} = 0}. \tag{4.21}$$

4.2.2 Implications on Monetary Policy

So far we have mentioned one difference to the analysis of chapter 3 with regard to contents. The framework of partial commitment allows finding equilibria that include changes in the optimal plan. Another difference affects the way we analyse the optimization problem. In chapter 3 we

restrict the set of possible outcomes of the output gap to the form $x_t = -\omega u_t$. This restriction lead to a special class of policy rules, where the rate of inflation depends linearly on the supply shock plus a constant. Within this class of policy rules, we are able to find only locally optimal outcomes. In contrast, the analysis of the current chapter abstains from this restriction. This has two consequences: First, the analysis is more complex in mathematical terms. Second, the resulting equilibria are globally optimal.

Generally an equilibrium of a new Keynesian model includes rules for the optimal output gap, the optimal rate of inflation and the nominal interest rate. However, our analysis depicts some additional features. Recalling the first order condition (4.6), we can interpret the Lagrange multiplier, ϕ_t .

$$\phi_t = -\frac{2a}{\kappa}(x_t - k)$$

In the context of partial commitment, ϕ_t depicts the incentives of a policymaker to abandon the current optimal plan. These incentives increase with the distance of the output gap from its target. Similar to section 3.3, we see that even if the output gap target $k = 0$ holds, this temptation still exists. Consider the following example: Optimal policy calls for a protracted recession in response to an inflationary cost push shock. Such a reaction has moderating effects on the output gap/inflation trade-off as we have seen in chapter 3. However, as inflationary pressures vanish, incentives rise to abandon this contractionary policy in order to reach the output gap target. These incentives are captured by the positive value of the Lagrange multiplier for missing the output gap target. The second first order condition (4.7) shows that inflation depends negatively on changes in the output gap.

Regarding the equilibrium output gap, we identify benefits for credible central banks. We see that higher credibility dampens both, the initial impact and the influence over time of a shock. Thus the economy reacts less to supply shocks in general.

$$x_t = \left((1 + \mathcal{B})\mu_\alpha^t - \mathcal{B} \right) k - \mathcal{A} \sum_{i=1}^t \mu_\alpha^{t-i} u_i$$

The reaction of q_α with respect to changes in α has positive impacts on the evolution of the output gap, too. Since q_α is smaller for a credible policymaker, he faces a smaller output gap in every period compared to a less credible one. This effect is increased by the evolution of μ_α^t .

Recalling these results, we see that through the beneficial effects of a credible central bank on

private agents' expectations, the economy reacts less to shocks and attains a higher level of output in every period. In addition, the economy stabilizes earlier. We should bear in mind that we observe these results only if the tenure of the central banker is long enough. However, this is, in fact, the case if we link credibility directly to the level of α . If the level of α is high, the probability that the tenure of the current central banker lasts long, is high, too. This implies that private agents believe in keeping up the current policy for future periods. Hence, they believe in the promises of the central bank. Thus, the central bank is credible.

The equilibrium rate of inflation behaves similarly. Again credibility improves the reactions on inflationary pressures. Inflation increases less after an inflationary shock in an economy whose central bank is credible. In addition, inflation decreases faster towards the long-term target. This implies that credible central banks can fight inflation with lower costs in terms of changes in the output gap.

$$\pi_t = \frac{a(1 - \mu_\alpha)}{\kappa} \left((1 + \mathcal{B})\mu_\alpha^{t-1}k - \mathcal{A} \sum_{i=1}^{t-1} \mu_\alpha^{t-1-i} u_i \right) + \frac{a}{\kappa} \mathcal{A} u_t$$

Concerning the optimal interest rate rule, we see that it takes a similar form as the ones of chapter 3 (cf. equations (3.10) and (3.28)).

$$i_t = \gamma E_t \pi_{t+1} + \frac{1}{\varphi} g_t \quad (4.22)$$

The optimal nominal interest depends positively on inflation expected for one period ahead plus some uncertainty. Compared to the interest rate rule of the restricted analysis of chapter 3, we see differences only in the value of the coefficient γ . Thus, we think that the restrictions on the form of the output gap do not limit the generality of the results.

What's left is the discussion on the perhaps surprising negativity of the long-term output gap.

$$\bar{x} = -\frac{(1 - \alpha)\beta\mu_\alpha}{1 - \beta\mu_\alpha} k$$

Intuitively one would expect the output gap to be zero over the complete time horizon, which becomes clear in the context of the definition of the output gap $x_t = y_t - y_t^n$. An output gap of zero would mean that the economy produces at its potential in long-term. Producing more than potential is not sustainable since resources in the economy are limited. A negative output gap means that the economy produces less than the potential output over the long run. This, in turn,

implies that private agents have resources left, which would induce them to decrease their prices in order to generate demand. These price reductions would lead to deflation in the economy, which is a contradiction to the equation (4.21). However, we can find some arguments that verify a negative output gap in the long run, at least in the framework of partial commitment: (i) There is a constant positive probability of a regime change in every period and even if there is no regime change observed for a long time, private agents still do not adapt their expectations; (ii) a new policymaker only needs to re-optimize if the rate of inflation is positive at the beginning of his tenure; and (iii) without a regime change, the policymaker follows his optimal plan, which leads to the long-term rate of inflation $\bar{\pi} = 0$. But since private agents still expect a regime change with a positive probability, expected inflation is greater than zero, as we see in equation (4.4). Combining these two observations in the Phillips curve (4.1), we see that the long-term output gap $\bar{x}$ must be less than zero.

$$0 = \kappa \bar{x} + \beta E\bar{\pi} + \bar{u}$$

$$-\underbrace{\bar{x}}_{<0} = \frac{\beta}{\kappa} \underbrace{E\bar{\pi}}_{>0} + \frac{1}{\kappa} \bar{u}$$

The long-term values of the output gap and inflation give two implications for policymaking. No matter how credible a policymaker is, if his tenure is long enough, he should bring inflation close to zero. However, a credible policymaker with a high level of α , achieves this with less reductions in output. Since μ_α is decreasing in α , increasing the credibility, which means increasing α , brings the output gap closer to zero in the long run. Remember here that in the model credibility means that the probability of regime changes is low.

Finally, we consider the dynamic inconsistency problem intensively discussed in chapter 2 in the context of partial commitment. Under partial commitment, the optimal plan alters with every policymaker. Each policymaker chooses the plan which is optimal from the perspective of his tenure's first period. He optimizes independently of any decisions taken by his predecessors. He only accounts for recent economic conditions and ignores the ones in the past. Under the assumption that all policymakers are identical, every policymaker chooses the same optimal policy, but at different points in time. Thus every policymaker must skip the optimal plan of his predecessor that is inconsistent with the economic conditions at the time of the regime change. This shows that dynamic inconsistency of optimal plans is a basic assumption of partial commitment. Unless an optimal plan defined in the past was inconsistent with the economic conditions today, there would not be any incentives to skip this plan. Thus every new appointed

policy maker would follow the plan of his predecessor. Hence, there would be only one optimal plan and the discussion on partial commitment would not be relevant.

Commitment in Monetary Policy - a brief Case Study

In order to relate the theoretical discussion of the previous chapters to real world situations we will look at two examples of monetary policy in the past. First we will analyse the different reactions of the US economy to the oil-price shocks in the 1970s and 2000s. Although there might be aspects of the reactions that cannot be related to our topic, we intend to look at it from the background of the discussion on commitment versus discretion. In a latter section we will try to analyse the situation after the financial crisis in 2008 that led to the recent economic crisis. Here we focus on the effects of newly invented policies in the context of commitment. We should stress that most of the information available has not yet been published in economic journals, thus we extract the information from speeches and working papers.

5.1 Oil Price Shocks in the USA

In the 1960s the US economy faced inflation rates of about 4 to 5 percent and it was not clear whether the FED really wanted to decrease these. There were some arrangements to stabilize inflation at lower levels, but they were not credible and thus ineffective. Hence the public was unsure about the goal of the FED concerning the rate of inflation. In the year 1973 the first Arab oil embargo hit the US economy and the rate of inflation more than doubled to 9 percent within one year. The earlier history of unclear policy measures led to a lack of credibility of the FED. Thus people did not believe in the FED's intention to maintain a low inflation rate and expected it to rise. The FED then had two options: Either to follow the discretionary path and ease monetary policy satisfying people's expectations, thus easing economic tensions caused by the oil embargo, or to commit to the pre-announced but incredible goal of low inflation, thus tightening the economy with the risk of increased output losses. The FED decided to act discretionarily ending with an inflation rate of about 15 percent in 1980 without any gains in

economic output. As a consequence, the economy had to go through a recession in 1981-82 in order to lower inflation expectations to an affordable level. Then the economy experienced two decades of strict and sometimes unpopular arrangements towards a low and stable inflation rate, with the result that the FED gained credibility. These arrangements lowered private agents' expectations of inflation. Even during the oil price shock in 2003, where oil prices more than doubled, people expected the FED to tighten the economy. This allowed the FED to credibly commit to the rule, thus the inflation rate stayed constant without any significant effect on economic output. In line with Plosser [10], we believe that the difference in the outcomes of the shocks is due to increased credibility, which, in turn, enables the policymakers to follow the commitment path.

5.2 The Economic Crisis after 2008

At the beginning of the new century central banks as well as most economists believed that economic models were quite well-developed. At least since the 1980s most central banks followed a commitment policy leading to low and stable inflation rates. This indeed helped them to build up credibility. In addition, their behaviour in former times of economic tensions built up a good reputation, which also improved credibility. In line with this reputation, it seemed that the range of policy measures was enough to guide the economy. Unfortunately, at the beginning of the financial crisis, demand decreased substantially. As a reaction, central banks lowered the interest rates, which was still in line with their commitments. As a consequence of the low inflation policy during the last decades, the interest rates had already been at low levels. Hence they soon reached the zero lower bound, but the economy still was in serious conditions. Thus central banks noticed that traditional measures were not enough to bear these crisis and invented some innovative policy measures. Though some of these had already been known, they were not expected to be used. For example no renowned economist expected the FED to actively manage their balance sheet through large asset purchases. This implies that such actions, taken by the central banks, which were indeed surprising to private agents, are discretionary. These newly invented policies came in line with some significant challenges for policymakers in the time after the crisis.

The increased set of policy actions will make it more difficult for policymakers to commit to a certain policy rule in the future. From our analysis we know that a commitment policy leads to better economic outcomes. However, the positive experiences with these newly developed policy

measures have increased the incentives to act discretionary in the future. This effect could be increased by political pressures, which may claim their use even under ordinary economic conditions. Besides the increased risk of political influence on monetary policy decisions, a central bank may take more risk in the size and composition of the balance sheet. Plosser [11] stresses the importance for mechanisms that limit the use of these discretionary tools while increasing the commitment to reach long-term policy objectives. Although the newly invented policy actions have benefits in times of economic crisis, their use in normal times would come with enormous expenses. Cœuré [3] suggests differentiating between long-term commitments and short-term conducts in defining a policy objective. An optimal monetary policy should include commitment to a systematic policy rule that allows for some well-defined discretion in the short-term conduct of the economy.

“Financial and macroeconomic disturbances are sometimes of a scale and complexity that they alter the underlying structural relationships between key economic variables. This, in turn, challenges monetary policy-makers to temporarily adapt their established strategies to a new environment without undermining their inflation-fighting credentials.” (Cœuré 2013 [3], p.1)

The recent financial crisis depict besides this that actual macroeconomic models were to some extent misleading. While they serve well to describe the economy under normal conditions, they have missed to show the effects of a crisis at such a substantial scale. Mishkin [9] argues that the eight principles of the *new neoclassical synthesis* remain intact. However, the financial sector plays a bigger role in the economy than assumed before. In addition, disturbances in the financial sector are not normally distributed. They appear to be heavy tailed instead. Hence, even if there is still agreement towards commitment in monetary policy, the underlying models should be extended in order to define more adequate rules.

Conclusions

Starting the analysis with a simple framework, we have found results that are in line with the one of Barro and Gordon [1]. A discretionary policy destabilizes the economy. Inflation rates increase as long as the policymaker has incentives to increase the output. This leads to high levels of inflation without any gains in the output. On the other hand, a commitment policy allows the policymaker to reach inflation targets, thus maximizing economic welfare. The reason for this effect lies in the nature of the game against rational private agents, first named in Kydland and Prescott [8]. Only a policymaker who commits to an optimal rule is able to guide private agents' expectations. Thus they expect low inflation rates, which, in turn, allows the policymaker to set a low inflation rate. Technically this is emphasized by the forward-looking nature of the Phillips curve. Moreover, we find that after choosing a discretionary policy, the economy can never reach the optimal outcome, at least not in our simple natural rate model.

Replicating these results in a new Keynesian framework, we find that commitment dominates discretion even if there are no incentives to increase output above its potential. Even if a policymaker understands the effects of the inflationary bias and does not target towards artificial output increases, he experiences benefits of a commitment policy. These benefits lie in the improved short-term output gap/inflation trade-off. In particular our analysis resembles some of the eight principles of the *new neoclassical synthesis*, presented by Goodfriend and King [7]. We find that expectations take a prominent role in the game against rational private agents; the real interest rate should rise in response to increases of inflation, which is in line with the *Taylor Principle*; an optimal monetary policy needs to account for the time-inconsistency problem; and commitment to an announced rule improves the short-term output gap/inflation trade-off and thus the efficiency in the economy.

The forward-looking characteristics of the new Keynesian model allow us to extend the analysis of one policymaker towards a sequence of policymakers, each with a random duration of his tenure. Each change of a policymaker comes along with a re-optimization, thus a change of the optimal plan. This increases the uncertainty in the model which allows for developing a theory of

equilibria that lie between the extreme cases of full commitment and discretion. Interpreting the average length of a policymaker's tenure as the level of credibility of a central bank enables us to rank the equilibria. Doing so, we find that those central banks with a high level of credibility reach better levels of economic welfare compared to those with less credibility.

It seems particularly interesting that the way we increase uncertainty in the model leads to a negative output gap in the long run. We think this effect, though not realistic, is based on the re-optimization process. Private agents expect a re-optimization with constant probability in every period. The length of the current tenure does not influence their expectations. Furthermore, after a re-optimization, the optimal plan only changes if inflation is above target. When inflation is at the target, the re-optimization leads to the optimal plan that is currently traced.

In the final stage we considered two examples where the theory of time-inconsistency of optimal plans or the average inflation bias influence real world policy decisions. We find that commitment towards stable inflation rates in the past indeed improves a central banks credibility. This, on the other hand, dampens the negative effect of economic shocks on expectations, which simplifies the decision of the policymaker to stick to the committed plan. However, the recent economic crisis teach us that sometimes a shock is of such a size that the relation of economic measurements is fundamentally changed. Thus traditional measures for stabilizing the economy do not work in the intended way. Hence central banks take discretionary steps to conquer the shock. These steps, although maybe not in line with the commitment, so far effect the economy in a positive way. However, based on the theory presented in this thesis, there are rising discussions on how to prevent a central bank to take such discretionary steps in times of economic stability. The most prominent statements of the recent discussions are: *(i)* The current economic models need to be extended. They work well in times of economic stability, whereas the current crisis have drastically exposed their weaknesses. Especially, they do not fully account for the influences of the financial sector on the economy. *(ii)* Optimal rules should include a commitment to long-term objectives while allowing for some discretion in the short-term conduct when necessary.

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Abstract

English

Based on the analysis of Kydland and Prescott [8] and Barro and Gordon [1] on the time-inconsistency of optimal plans we show the dominance of commitment over discretion thanks to the positive impact on expectations. First we reproduce the results of Barro and Gordon [1] in a natural rate model of output gap and inflation. Then we strengthen our results in detail in a new Keynesian framework. Afterwards we consider a new modelling technique of a central bank's credibility according to Schaumburg and Tambalotti [14]. This gives a whole range of equilibria in between the extremes of commitment and discretion. In the end we discuss two real world examples of monetary policy in the context of our analysis.

Deutsch

Anhand der Arbeit von Kydland und Prescott [8] und Barro und Gordon [1] zur zeitlichen Inkonsistenz optimaler Pläne zeigen wir die Dominanz von Bekenntnissen gegenüber Diskretion dank des positiven Einflusses auf Erwartungen. Zuerst bilden wir die Ergebnisse von Barro und Gordon [1] in einem natürlichen Quotenmodell von Produktionslücke und Inflation nach. Danach bestätigen wir unsere Ergebnisse im Detail anhand eines neukeynesianischen Modells. Anschließend betrachten wir einen neuen Modellierungsansatz von Schaumburg und Tambalotti [14] zu Glaubwürdigkeit einer Zentralbank. Dies führt zu einer ganzen Bandbreite von Equilibria zwischen den Extremen Bekenntnis und Diskretion. Am Ende diskutieren wir zwei reale Beispiele der Geldpolitik unter dem Gesichtspunkt unserer Analysen.

Zusammenfassung

Ziel der Masterarbeit ist es zu zeigen, dass sich in der Geldpolitik Bekenntnisse von Entscheidungsträgern positiv auf die Entwicklung der Volkswirtschaft auswirken. Als Entscheidungsträger sehen wir hier eine Institution, welche alle geldpolitischen Entscheidungen autonom trifft. Dabei sind zwei Aspekte wesentlich. Zum einen wird die Entwicklung einer Volkswirtschaft nicht vom Entscheidungsträger allein bestimmt, sondern gemeinsam mit einem rational denkenden privaten Sektor, der seine Entscheidungen aufgrund von Erfahrungen und Erwartungen trifft. Zum anderen liegt das effiziente Produktionsniveau einer Volkswirtschaft über dem natürlichen Produktionsniveau, was den Entscheidungsträger dazu verführt, Maßnahmen zu tätigen, um das Produktionsniveau zu erhöhen. Dieses Problem wurde erstmals von Kydland und Prescott [8] formuliert.

Basierend auf der Arbeit Barro und Gordon [1] zur zeitlichen Inkonsistenz optimaler Pläne analysieren wir das Problem in einem natürlichen Quotenmodell von Produktionslücke und Inflation. Dabei werden die Erwartungen des privaten Sektors mit Hilfe einer vorausschauenden Phillipskurve modelliert.

Eine diskretionäre Geldpolitik destabilisiert eine Volkswirtschaft; während der Entscheidungsträger versucht das Produktionsniveau zu erhöhen, steigt die Inflationsrate. Dies jedoch führt zu erhöhter Inflation, wobei das Produktionsniveau unverändert bleibt. Andererseits führt ein Bekenntnis zu einer bestimmten Geldpolitik dazu, dass die Ziele erreicht werden können, wobei die Wohlfahrtsgewinne der Volkswirtschaft maximiert werden. Der Grund dafür liegt in der Natur der Spieler. Einerseits der geldpolitische Entscheidungsträger, welcher versucht die Volkswirtschaft bestmöglich, im Sinne möglichst großer Wohlfahrtsgewinne, zu lenken. Andererseits der private Sektor, bestehend aus rational denkenden Individuen, welche versuchen, ihrerseits den Profit zu maximieren. Wesentlich dabei ist, dass ein Entscheidungsträger die Wohlfahrtsgewinne nur maximieren kann, wenn er in der Lage ist, die Erwartungen des privaten Sektors zu beeinflussen. Dies wiederum ist nur möglich, wenn sich der Entscheidungsträger zu niedriger Inflation bekennt und keine Handlungen setzt, die den privaten Sektor daran zweifeln

lassen würden. Solch ein Bekenntnis führt dazu, dass der private Sektor in der Zukunft niedrige Inflationsraten erwartet, was es wiederum dem Entscheidungsträger ermöglicht, niedrige Inflationsraten zu erreichen. Darüber hinaus finden wir, dass eine Volkswirtschaft, die sich einmal auf dem Pfad, welcher durch eine diskretionäre Geldpolitik erreicht wird, befindet, nie mehr die bestmöglichen Wohlfahrtsgewinne erzielen kann.

Danach reproduzieren wir diese Ergebnisse anhand eines neukeynesianischen Modells. Dabei sehen wir, dass ein Bekenntnis des Entscheidungsträgers die Wohlfahrtsgewinne gegenüber einer diskreten Geldpolitik auch dann steigert, wenn der Entscheidungsträger keiner Versuchung, das Produktionsniveau zu erhöhen, ausgesetzt ist. Dies liegt am positiven Einfluss eines Bekenntnisses auf das Kurzzeitverhalten von Produktionslücke und Inflation.

Einige unserer Ergebnisse finden sich auch unter den acht Prinzipien der neuen neoklassizistischen Synthese nach Goodfriend und King [7]. Erwartungen haben großen Einfluss in einem Spiel gegen rational denkende Mitspieler; wenn die Inflation steigt, sollte sich der Realzinssatz erhöhen, was dem Taylorprinzip entspricht; eine optimale Geldpolitik muss sich mit dem Problem der zeitlichen Inkonsistenz beschäftigen; und ein Bekenntnis beeinflusst das Kurzzeitverhalten von Produktionslücke und Inflation positiv, was wiederum die Effizienz einer Volkswirtschaft steigert.

Danach erweitern wir unser Modell und erlauben dem Entscheidungsträger die Möglichkeit eines bedingten Bekenntnisses. Dieser Ansatz wird erstmals von Schaumburg und Tambalotti [14] präsentiert. War bisher von einer Institution als Entscheidungsträger die Rede, denken wir nun an eine Person, die dieser Institution vorsteht. Diese Person kann für die Einhaltung ihres Bekenntnisses garantieren, solange sie der Institution vorsteht. Sie kann keinen Einfluss auf das Bekenntnis ihres Nachfolgers nehmen. Mit jedem Tausch dieser Person wird erneut optimiert und die neue Führung bekennt sich zu jener Regel, die aufgrund der derzeitigen Entwicklung der Volkswirtschaft optimal ist. Wir finden, dass die Ergebnisse, welche mittels bedingtem Bekenntnis erreicht werden können, zwischen jenen von Diskretion und Bekenntnis liegen. Dabei nähert man sich dem optimalen Ergebnis, welches mittels Bekenntnis erreicht wird, an, wenn der Zeitraum, welchem der Entscheidungsträger der Institution vorsteht, länger wird.

Im letzten Kapitel betrachten wir den Einfluss der Theorie der zeitlichen Inkonsistenz optimaler Pläne auf das Verhalten von Entscheidungsträgern in Zeiten realer volkswirtschaftlicher Krisensituationen.

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