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„Expectations, Precautionary Savings and Unemployment"

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Abstract

In the absence of a centralized goods market and perfect credit markets, precautionary savings, driven by pessimistic expectations, can lead to a lack of aggregate demand and involuntary unemployment in steady state, whereas optimistic expectations would keep the economy in a steady state with full employment. While the steady state with full employment is efficient, the steady state with unemployment is not. Once the economy is in the inefficient steady state, under certain conditions there is no equilibrium path which leads back to the efficient steady state. This result was shown by Chamley (2014), assuming that agents face idiosyncratic preference shocks of a certain probability. I show that this result is robust to changes in the probability of preference shocks. It holds for nearly all probabilities of these shocks. In addition, I show that rare preference shocks lead to higher unemployment in the inefficient steady state than frequent shocks. This is because the higher the probability of idiosyncratic shocks, the higher the fraction of agents with a high propensity to consume. A higher fraction of agents with a high propensity to consume leads to less unemployment in the inefficient steady state.

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1 Introduction

What influence do beliefs have on macroeconomic aggregates? Can pessimistic or optimistic beliefs influence output, unemployment and consumption? These are the questions that I address in my thesis. More precisely, I focus on the influence of expectations on precautionary savings and aggregate demand. The intuitive mechanism I examine is the following: suppose agents in an economy expect that there is a risk of being unemployed. Due to these expectations, agents accumulate precautionary savings to insure themselves against this risk. Increased savings lead to less consumption, which leads to a lack of aggregate demand. The lack of aggregate demand causes the expected unemployment. Thus, pessimistic expectations are self-fulfilling and depress the economy. This intuition is a very popular way to think about business cycles and depressions. Arguably, many economists and policy makers have it in mind when they think about business cycles, particularly since Keynes published his General Theory (1936).

Whether pessimistic expectations are responsible for lower economic activity is a highly topical question. After the Great Recession we have seen a very slow recovery in the US and in Europe. Six years after the financial crisis struck, unemployment and labor market participation have still not returned to pre-crisis levels. During the crisis, we have seen an increasing demand for precautionary savings and lower consumption. The Great Depression and Japan's Lost Decade displayed similar developments. The mechanism described above could be a reasonable explanation, especially for these types of long lasting recessions.

My work is a generalization of the paper: "When Demand Creates its Own Supply: Saving Traps", by Christophe Chamley (2014). Chamley shows that the mechanism described earlier is consistent with optimizing behavior of agents with rational expectations. In order to have involuntary unemployment one has to depart from some of the standard assumptions of general equilibrium theory. Chamley drops the assumption of a centralized goods market and instead, he introduces a decentralized trading mechanism in his model. In addition, he assumes the absence of perfect credit markets. The absence of these markets is justified by the well-known literature on imperfect information (Jaffee and Russell 1976; Stiglitz and Weiss 1982; see Clemenz and Ritthaler 1992 for a survey). A further important element of Chamley's model are idiosyncratic preference shocks. An agent affected by a preference shock faces a higher disutility from not consuming. Chamley shows the existence of an efficient steady state with full employment and the existence of a second inefficient steady state with unemployment. Both steady states are equilibria of the economy. Assuming a specific set of parameter values, he can show that an equilibrium transition from the efficient to the inefficient steady state is possible. Once the economy is in the inefficient steady state, there is no equilibrium path back to full employment, and the economy is caught in a trap. Savings are the driving force behind the whole mechanism. Whereas pessimistic expectations lead to convergence towards the inefficient steady state, optimistic expectations do not lead to convergence back to the efficient steady state. This could explain why some recessions are very persistent. In order to prove his results, Chamley needs to make specific assumptions about the probability of the preference shocks in his model. I generalize his results and show that they hold for most probabilities of idiosyncratic shocks. I show that Chamley's results are robust to changes in the probability of preference shocks. This generalization of the model allows me to analyze which influence the probability of preference shocks has on equilibrium outcomes in the economy. Whereas the efficient steady state is not affected by changes in the probability of shocks, unemployment in the inefficient steady state is. Economies where there is a high probability of idiosyncratic shocks face less unemployment in the inefficient steady state than economies with rare shocks. This is true if the respective shocks have the right intensity. A high probability of idiosyncratic preference shocks leads to a large fraction of agents affected by the shock. Agents affected by the shock have a high propensity to consume and if there is a large fraction of them this stabilizes aggregate demand and leads to less unemployment. My findings support the "Keynesian" claim that a large fraction of agents with a high propensity to consume stabilizes recessions.

In order to show that beliefs can have an influence on macroeconomic aggregates, one has to focus on models with multiple equilibria. The beliefs of agents matter because rational expectations are not unique and the prevailing equilibrium depends on beliefs. These models are dynamic versions of coordination games, and as occurs in these games questions regarding equilibrium selection arise. Equilibria are typically Pareto-rankable and from a policy perspective there is a role for government interventions in coordinating beliefs towards the efficient equilibrium. Business cycles can be explained by a collective shock in agents' beliefs. These models provide an explanation for long lasting recessions. Economies in such recessions are possibly following a different, Pareto-inferior equilibrium path as compared to the path in the pre-recession period. Of course the literature on multiple equilibria in macroeconomics is not new. Cooper (1999) gives a good overview. Multiple equilibria can arise from all sorts of complementarities. In my work I focus on complementarities arising from frictions in the trading process. If one drops the assumption of a central market and a Walrasian auctioneer, one needs to introduce a decentralized trading mechanism which specifies how trades are organized. Again, this literature is not new. The idea of involuntary unemployment caused by frictions in trading was first formalized by Diamond (1982, 1984). In Diamond's first paper on this topic (1982) trading frictions are modeled assuming random matching of agents who directly trade good by good. As it is simpler to find a trading partner when aggregate productivity is high and a large number of agents are trying to trade, multiple equilibria arise because of thick market effects. Similar results can be obtained by introducing money into the model as shown by Diamond (1984). As Chamley (2014) points out, aggregate supply and aggregate demand effects cannot be separated in Diamond's models. Although Chamley's work is in Diamond's tradition he can show that trading frictions lead to aggregate demand effects due to a precautionary savings motive. As this is exactly the effect I want to examine, I work on Chamley's model in my analysis.

In section 2 I introduce Chamley's model. Section 3 describes the steady state with full employment. Section 4 presents the steady state with unemployment. In section 5 I show under which conditions transitions between the two steady states are possible and under which conditions a savings trap emerges. Section 6 offers a conclusion. The appendix contains most of the proofs.

2 The Model

2.1 Model Specifications

In the following section I describe the model used by Chamley. Throughout the whole section I simply replicate his results. Goods and agents live on the unit interval and are indexed by $i \in [0, 1)$. Agents have an overall mass of one. Each agent i produces good i which he cannot consume himself. Time is discrete. Idiosyncratic preference shocks are modeled in the following way: each period each agent is randomly assigned to one of two types which derive different utility from consumption. The type is a random variable $\theta_{i,t} \in \{0, 1\}$ that is independently distributed across agents and periods. The idiosyncratic probability of being the high type ($\theta_{i,t} = 1$) is equal to α each period, whereas the probability of being the low type ($\theta_{i,t} = 0$) is $1 - \alpha$:

$$P(\theta_{i,t} = 1) = \alpha,$$
$$P(\theta_{i,t} = 0) = 1 - \alpha, \quad \alpha \in (0, 1).$$

As there are infinitely many agents with an overall mass of one, the fraction of high type agents in each period is equal to α and the fraction of low type agents is equal to $1 - \alpha$. The type-assigning process is known by all agents and the type is revealed to each agent at the beginning of every period. High types derive a higher utility from consumption than low types. The overall utility of each agent i is the expected discounted sum of per period utility:

$$u_{i,t}(x_{i,t}, \theta_{i,t}) = (1 + \theta_{i,t}c)x_{i,t} - \theta_{i,t}c, \quad c > 0, \tag{2.1}$$
$$U_i = E[\sum_{t=0}^{\infty} \beta^t u_{i,t}], \quad \beta \in (0, 1). \tag{2.2}$$

$x_{i,t}$ is the consumption of an agent in a certain period and β is the discount factor. c is a penalty for not consuming. It only concerns high type agents. In each period each agent i can produce one unit of a good i at no cost. One can interpret the production process as agents being self employed and deriving no utility from leisure. The good is assumed to be indivisible. As there is no centralized market, Chamley introduces a random matching process between agents. Each agent represents a household with two heads, a buyer and a seller. In each period a household is randomly matched to another household where it can buy a consumption good. Equivalently each household is matched to a customer who can buy the good the household produces. The buyer leaves the house in the morning if he decides to consume in a certain period. The seller stays at home and waits for a customer to arrive. It is assumed that the two heads cannot communicate during the day which forces them to make a consumption decision without knowing if they can sell later during the day. Agents meet for a trade not knowing the type or the amount of bond holdings of their trading partner. I introduce bond holdings in the next paragraph. Note that because of the indivisibility of the consumption goods, agents consume an amount of one or zero, $x_{i,t} \in \{0, 1\}$. The choice variable of an agent is a binary choice between consuming and not consuming. This leads to a simplification of the utility function. Both types derive a utility of 1 if they consume, whereas low types derive a utility of 0 if they do not consume and high types receive a penalty of -c if they do not consume:

$$\begin{aligned}
u_{i,t}(x_{i,t} = 1, \theta_{i,t} = 1) &= 1, \\
u_{i,t}(x_{i,t} = 1, \theta_{i,t} = 0) &= 1, \\
u_{i,t}(x_{i,t} = 0, \theta_{i,t} = 1) &= -c, \\
u_{i,t}(x_{i,t} = 0, \theta_{i,t} = 0) &= 0.
\end{aligned}$$

Households can save using bonds as the consumption goods are not storable. The net supply of bonds is zero and bonds can be seen as inside money. The debt of some agents is the savings of others. There is a credit constraint which can be justified by standard arguments about imperfect credit markets. The credit limit is assumed to be one unit of the consumption good. Agents which are at the credit limit at the beginning of a period cannot consume. Households which cannot sell in a certain period are called the unemployed.

2.2 Prices

Next I analyze how prices are determined. I only consider equilibria with symmetric prices across agents, which is reasonable as agents are symmetric. The price of a consumption good in bonds in a certain period is p_t for all i. I restrict prices further and focus on equilibria where prices are constant over time, $p_t = p$ for all t. There may be other equilibria where the price level varies across periods as in the Samuelson model of overlapping generations (1958). As Chamley points out, these equilibria have already been extensively studied. He does not consider them and they are not the focus of this paper either. In the next paragraph I show why p is an equilibrium price for goods in bonds and can be normalized to 1.

Trades are assumed to be organized as in Green and Zhou (2002): a buyer and a seller meet and each of them posts a price without bargaining. Suppose all buyers and sellers post price p. Consider the deviation of one seller. If she posts a lower price she makes less revenue and she cannot attract additional buyers as she is randomly matched to one buyer. To post a lower price is a strictly dominated strategy. If she posts a higher price she will not sell the good as all buyers post price p. To post a higher price is again a strictly dominated strategy. Now consider the deviation of one buyer. Posting a lower price is strictly dominated as it would result in not getting the good, whereas posting a higher price is strictly dominated as she would pay more than needed. Buyers and sellers posting p is a Nash equilibrium. Trade will occur at price p and I normalize it to $p = 1$, where 1 means one unit of bonds. Upward and downward rigidity of prices is not assumed, but is a result of the decentralized trading mechanism. The absence of a centralized market causes price rigidity. Of course it is the rather extreme form of decentralized trading assumed in the model that causes completely rigid prices.

2.3 The First Best Solution

The first best solution maximizes the utility of each agent in each period. The first best solution is the maximum of the following function with respect to x:

$$\max_{x\in\{0,1\}} \int_0^1 \alpha u(x,1) + (1-\alpha)u(x,0)di.$$

$\alpha u(x,1)$ is the utility of the high type agents, $(1-\alpha)u(x,0)$ the utility of the low type agents. It is easy to see that the function is maximized when both low and high types consume, $x = 1$. Everybody consumes, therefore everybody can produce and sell and there is no unemployment. Note that this is the only Pareto-efficient outcome.

2.4 The Distribution of Bond Holdings

Each period has a specific bond distribution. The bond distribution is discrete as all transactions in the economy are 0 or 1. $\gamma_k(t)$ is the fraction of individuals holding an amount of k bonds in period t. As the credit limit is -1 it follows that $k \in \mathbb{N}_0 \cup \{-1\}$. From now on I refer to an agent holding the amount of k bonds as an agent in state k. All fractions $\gamma_k(t)$ sum up to one as it is the mass of the entire population:

$$\sum_k \gamma_k = 1. \tag{2.3}$$

The net supply of bonds is zero:

$$\sum_k \gamma_k k = 0. \tag{2.4}$$

The vector $\Gamma(t) = (\gamma_{-1}(t), \gamma_0(t), ...)$ denotes the bond distribution in each period.

Households matched to a non-consuming household do not produce as they cannot sell. π_t denotes the fraction of households not producing, the unemployed. The fraction of households not consuming is equal to the fraction of households not producing. The fraction of households producing is equal to $1 - \pi_t$ and is equal to the fraction of households consuming:

$$1 - \pi_t = \int x_{i,t}di. \tag{2.5}$$

As the overall mass of households is one and households are randomly assigned to a customer, $1-\pi_t$ is the probability that a household is matched to a consuming household. It is therefore the probability that the household can sell in period t. Equivalently, π_t is the probability that a household cannot sell.

$k_{i,t}$ denotes the number of bonds household i holds at the beginning of period t. How are they related to bond holdings at the beginning of the next period? If a household consumes and cannot sell, its bond holdings decrease by one. If a household does not consume and cannot sell, or consumes and can sell, its bond holdings stay the same. If a household does not consume and can sell, its bond holdings increase by one. The following equation summarizes this behavior:

$$k_{i,t+1} = \begin{cases} 1 + k_{i,t} - x_{i,t} & \text{with a probability of } 1 - \pi_t, \\ k_{i,t} - x_{i,t} & \text{with a probability of } \pi_t. \end{cases} \tag{2.6}$$

Consumers maximize expected lifetime utility in each period by making a consumption decision. The consumption decision of a household i maximizes the following optimization problem in each period t:

$$\max_{x_{i,\tau} \in \{0,1\}} \quad \mathrm{E} \sum_{\tau \geq t} \beta^{\tau - t} u(x_{i,\tau}, \theta_{i,\tau}), \tag{2.7}$$

- for given bond holdings $k_{i,t}$ and type $\theta_{i,t}$ at the beginning of a period t,
- subject to the evolution of bond holdings according to Equation (2.6) and the credit constrained $k_{i,t} \geq -1$,
- and given perfect foresight on the path of unemployment rates from period t onwards, which I denote by $\hat{\pi}^t = \{\pi_\tau\}_{\tau \geq t}^{\infty}$.

Suppose perfect foresight on the path of unemployment rates $\hat{\pi}^t$. The consumption function of an agent in period t, which is the solution of the maximization problem (2.7), depends only on his state $k_{i,t}$, his type $\theta_{i,t}$ and the path of unemployment rates $\hat{\pi}^t$ from period t onwards. The consumption function x_t and the unemployment rate π_t in period t determine the bond distribution in period $t+1$. The bond distribution determines the consumption function in period $t+1$, which determines the unemployment rate π_{t+1}. The path of unemployment rates is deterministic and can be calculated by rational agents if they know the model. The assumption of perfect foresight is therefore justified.

I can define an equilibrium of the economy as:

Definition 1 *A sequence of bond distributions $\{\Gamma(t)\}_{t=0}^{\infty}$, a sequence of unemployment rates $\hat{\pi}^t$ and a consumption function $x_t = x(k_t, \theta_t, \hat{\pi}^t)$ is an equilibrium if the bond distribution fulfills Equation (2.3) and (2.4) in every period and evolves according to Equation (2.6), if the unemployment rates fulfill Equation (2.5), and if the consumption function is a solution to the maximization problem (2.7).*

2.5 Optimal Household Behavior

Credit-constrained agents cannot consume. The consumption function for $k_t = -1$ is equal to zero:

$$x_t = x(k_t, \theta_t, \hat{\pi}^t) = 0 \text{ if } k_t = -1 \ \forall \theta_t \text{ and } \forall t.$$

High type agents always consume if they can. It is easy to see why this is optimal. Assume a high type agent saves in a certain period. As a consequence, his utility in that period is $-c$. The optimal use of the additional unit of savings is to avoid the penalty c in the future. As he discounts the future, the agent can avoid a discounted c. The benefit from saving (avoiding $-c$ in the future) is smaller than the cost ($-c$ now) and therefore the agent does not save. A rigorous proof of this argument can be found in the appendix of Chamley (2014). The consumption function for $\theta_t = 1$ and $k_t \geq 0$ is one:

$$x_t = x(k_t, \theta_t, \hat{\pi}^t) = 1 \text{ if } \theta_t = 1 \text{ and } k_t \geq 0 \ \forall t.$$

The behavior of low type unconstrained agents is more difficult to analyze.

Define $V_k(t)$ as the utility of an agent with bond holdings k at the end of period t after all transactions have occurred.

$$\begin{aligned} V_{-1}(t) &= \beta[\mathrm{E}[u(0,\theta)] + \pi_{t+1}V_{-1}(t+1) + (1-\pi_{t+1})V_0(t+1)] \\ &= \beta[-\alpha c + \pi_{t+1}V_{-1}(t+1) + (1-\pi_{t+1})V_0(t+1)] \end{aligned} \tag{2.8}$$

The Bellman equation above gives V for a constrained agent. He cannot consume and his utility in the next period is $E[u(0,\theta)]$. With a probability of π_{t+1} his bond holdings will remain at -1 as this is the probability of making no sale. It is therefore the probability that his expected utility at the end of next period is $V_{-1}(t+1)$. With a probability of $(1-\pi_{t+1})$ his expected utility is $V_0(t+1)$, as this is the probability to sell which increases his bond holdings. The next Bellman equation refers to unconstrained agents:

$$V_k(t) = \beta\mathrm{E}[\max_{x\in\{0,1\}}(u(x,\theta) + \pi_{t+1}V_{k-x}(t+1) + (1-\pi_{t+1})V_{k-x+1}(t+1))], \ k \geq 0. \tag{2.9}$$

Expectations are taken with regard to θ in period $t+1$.

Consider a low type agent above the credit constraint at the beginning of period t. He knows his type and has to make a decision. One can calculate his expected utility of saving at the end of the period t minus his expected utility of consuming at the end of the period t. I call this variable the marginal utility of saving, $\zeta_k(t)$:

$$\zeta_k(t) = \underbrace{[\pi_t V_k(t) + (1-\pi_t)V_{k+1}(t)]}_{\text{expected utility if an agent saves}} - \underbrace{[\pi_t V_{k-1}(t) + (1-\pi_t)V_k(t)]}_{\text{expected utility if an agent consumes}}$$

$$\zeta_k(t) = \pi_t(V_k(t) - V_{k-1}(t)) + (1-\pi_t)(V_{k+1}(t) - V_k(t)). \tag{2.10}$$

It is optimal for low type agents in state k and above the credit constraint ($k \geq 0$) to consume if $\zeta_k(t) \leq 1$. To consume gives a utility of one. If saving gives less utility it is optimal to consume. It is optimal to save if $\zeta_k(t) \geq 1$. Any consumption function x_t which is optimal must fulfill these two conditions at any point in time.

One can propose a certain consumption function which leads to a certain behavior of the economy. If the above conditions are satisfied at each point in time, the proposed function fulfills the equilibrium conditions and the behavior it produces is an equilibrium path of the economy. I define two consumption functions and two corresponding regimes of the economy:

Definition 2 *Consider the following two regimes.*
The High Regime: in the high regime, unconstrained low type agents always consume. The consumption function is equal to

$$x_t = x(k_t, \theta_t, \hat{\pi}^t) = 1 \text{ if } \theta_t = 0 \text{ and } k_t \geq 0 \ \forall t.$$

The Low Regime: in the low regime, low type agents with zero bond holdings do not consume and low type agents with positive bond holdings always consume. The consumption function is equal to

$$x_t = x(k_t, \theta_t, \hat{\pi}^t) = 0 \text{ if } \theta_t = 0 \text{ and } k_t = 0 \ \forall t,$$
$$x_t = x(k_t, \theta_t, \hat{\pi}^t) = 1 \text{ if } \theta_t = 0 \text{ and } k_t \geq 1 \ \forall t.$$

High type agents and credit constrained agents follow the behavior derived above.

If the economy follows one of these two regimes bond holdings never exceed 1: $k \in \{-1, 0, 1\}$. $\Gamma(t)$ is the state variable of the economy. It is reduced to

$$\Gamma(t) = (\gamma_{-1}(t), \gamma_0(t), \gamma_1(t)).$$

Define B_t as the amount of debt in the economy. It is true that $B_t = \gamma_{-1}(t)$, as agents in state -1 hold the whole amount of debt. Agents in state 1 hold the whole amount of savings, as bond holdings do not exceed one. This follows from Equation (2.4). Therefore

$$\gamma_1(t) = \gamma_{-1}(t) = B_t. \tag{2.11}$$

From Equation (2.3) it follows that all fractions of agents sum up to zero. Therefore

$$\gamma_0(t) = 1 - \gamma_{-1}(t) - \gamma_1(t) = 1 - 2\gamma_1(t) = 1 - 2B_t. \tag{2.12}$$

The vector $\Gamma(t)$ is fully determined by B_t and one can use the scalar B_t instead of the vector $\Gamma(t)$ as the state variable of the economy.

3 The High Regime

In the high regime, only agents in state -1 do not consume. As the level of unemployment is equal to the fraction of agents not consuming

$$\pi_t = \gamma_{-1}(t) = B_t. \tag{3.1}$$

The following list describes how agents move from state to state over time:

- Agents in state -1 at the beginning of period t:
  They do not consume. With a probability of π_t they do not sell, and as their bond holdings stay the same they stay in state -1. With a probability of $1 - \pi_t$ they sell, their bond holdings increase by one and they move to state 0 in the next period.

- Agents in state 0 at the beginning of period t:
  They always consume. With a probability of π_t they do not sell and move to state -1 next period, with a probability of $1-\pi_t$ they sell and stay in state 0.

- Agents in state 1 at the beginning of period t:
  They follow the same behavior as agents in state 0. With a probability of π_t they move to state 0, with a probability of $1-\pi_t$ they stay in state 1.

What is the unemployment rate next period? It is the fraction of state -1 agents staying in state -1 plus the fraction of state 0 agents moving to state -1:

$$\pi_{t+1} = \gamma_{-1}(t)\pi_t + \gamma_0(t)\pi_t. \tag{3.2}$$

Use (2.11), (2.12) and (3.1) in (3.2) to obtain that

$$B_{t+1} = O(B_t) = B_t^2 + (1-2B_t)B_t = B_t(1-B_t). \tag{3.3}$$

Equation (3.3) describes the dynamic behavior of B_t. The initial debt level in period 0 is B_0. $0 \le B_t \le 1/2$, as the net supply of bonds is zero. From Equation (3.3) it follows that $B_{t+1} < B_t$ if $B_t \in [0, 1/2]$. B_t converges as it is strictly decreasing in t and bounded from below. Omitting the time argument in (3.3) yields the limit of B_t: $\lim_{t\to\infty} B_t = 0$. Figure 1 provides graphical proof of the dynamic behavior of B. The same dynamic properties hold for the unemployment rate as $\pi_t = B_t$.

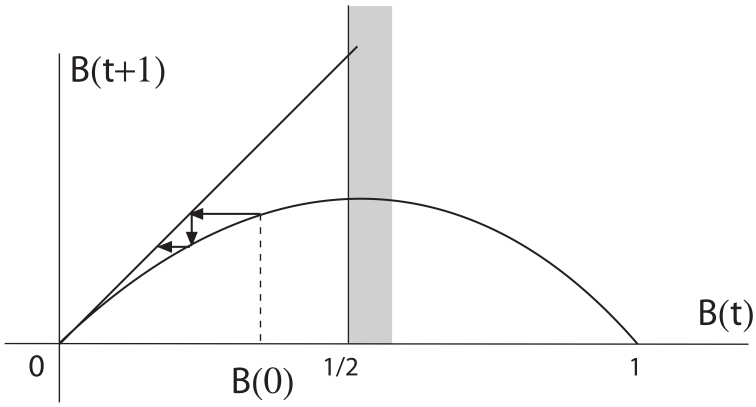

Figure 1: The Dynamics of B (Chamley 2014)

One can see from Figure 1 that the rate of convergence to the steady state is asymptotically zero. This is because the 45° line is a tangent to the graph of $O(B_t)$ at $B_t = 0$. Convergence to the steady state is therefore slow.

Result 1 *In the high regime, the economy converges to the steady state with zero debt and zero unemployment. The rate of convergence is asymptotically zero.*

Next I show that the derived behavior of the economy is an equilibrium. The only part of the equilibrium conditions that remains to be proven is the optimality of the proposed consumption function for low type agents in state 0 and 1. Given the proposed consumption function, the Bellman equations are as follows:

$$\begin{aligned}
V_{-1}(t) &= \beta[-\alpha c + \pi_{t+1}V_{-1}(t+1) + (1-\pi_{t+1})V_0(t+1)], && (3.4)\\
V_0(t) &= \beta[1 + \pi_{t+1}V_{-1}(t+1) + (1-\pi_{t+1})V_0(t+1)], && (3.5)\\
V_k(t) &= \beta[1 + \pi_{t+1}V_{k-1}(t+1) + (1-\pi_{t+1})V_k(t+1)], \quad \forall k \geq 1. && (3.6)
\end{aligned}$$

I use the marginal utility of saving for low type agents in state 0 and 1, defined in Equation (2.10), to examine optimality.

$$\begin{aligned}
\zeta_0(t) &= \pi_t(V_0(t) - V_{-1}(t)) + (1-\pi_t)(V_1(t) - V_0(t))\\
&= \pi_t\beta(1+\alpha c) +\\
&\quad (1-\pi_t)\beta[\pi_{t+1}(V_0(t+1) - V_{-1}(t+1)) + (1-\pi_{t+1})(V_1(t+1) - V_0(t+1))]\\
\zeta_0(t) &= \pi_t\beta(1+\alpha c) + (1-\pi_t)\beta\zeta_0(t+1) && (3.7)
\end{aligned}$$

$$\begin{aligned}
\zeta_1(t) &= \pi_t(V_1(t) - V_0(t)) + (1-\pi_t)(V_2(t) - V_1(t))\\
&= \pi_t\beta[\pi_{t+1}(V_0(t+1) - V_{-1}(t+1)) + (1-\pi_{t+1})(V_1(t+1) - V_0(t+1))] +\\
&\quad (1-\pi_t)\beta[\pi_{t+1}(V_1(t+1) - V_0(t+1)) + (1-\pi_{t+1})(V_2(t+1) - V_1(t+1))]\\
\zeta_1(t) &= \pi_t\beta\zeta_0(t+1) + (1-\pi_t)\beta\zeta_1(t+1) && (3.8)
\end{aligned}$$

Use $\pi_t = B_t$ in Equation (3.7) to obtain that

$$\zeta_0(t) = (1+\alpha c)\beta B_t + \beta(1-B_t)\zeta_0(t+1). \tag{3.9}$$

By repeated iterations of (3.9) it follows that

$$\zeta_0(0) = (1+\alpha c)\beta[B_0 + \sum_{i=0}^{\infty}(B_{i+1}\beta^{i+1}\prod_{j=0}^{i}(1-B_j))]. \tag{3.10}$$

Use $B_{t+1} = B_t(1-B_t)$ to see that $\zeta_0(0)$ is a function of B_0 and c. One can take period 0 as an arbitrary period and define $\hat{\zeta_0}(B,c)$ as the marginal utility of saving, when the aggregate debt level is equal to B. One can show that $\hat{\zeta_0}(B,c)$ is continuous in both arguments and strictly increasing for $B \in [0, 1/2]$ and in c. Proof of this statement can be found in the appendix of Chamley (2014).

Define c^* as $\hat{\zeta_0}(1/2, c^*) = 1$. If $c \leq c^*$ it holds that $\zeta_0(t,c) \leq 1 \ \forall t$, as $B_t \leq 1/2 \ \forall t$. This follows from the monotonic properties of $\hat{\zeta_0}$. If $c > c^*$, there exists a $\hat{B}(c)$ such that $\hat{\zeta_0}(\hat{B}, c) = 1$. It follows that for all $B \leq \hat{B}(c)$, $\hat{\zeta_0}(B,c) \leq 1$. If $B_0 \leq \hat{B}(c)$ it holds that $\zeta_0(t,c) \leq 1 \ \forall t$, as $B_t \leq B_0 \ \forall t$. Remember that the proposed consumption function is optimal if $\zeta_0(t) \leq 1 \ \forall t$. If $c \leq c^*$ the proposed consumption function for agents in state 0 is optimal for any initial level of debt. If $c > c^*$ the proposed consumption function is optimal if and only if $B_0 \leq \hat{B}(c)$.

B_t is decreasing in t and as $\hat{\zeta}_0(B, c)$ is increasing in B it follows that $\zeta_0(t)$ is decreasing in t. It is true that $\zeta_0(t) \geq 0$ as all parts in (3.10) are positive. $\zeta_0(t)$ is decreasing and has a lower bound, therefore $\zeta_0(t)$ converges. From Equation (3.10) it follows that $\zeta_0(t)$ converges to zero as B_t converges to zero.

Next I have to prove the optimality of the high regime consumption function for agents in state 1. I deviate from Chamley's analysis now, as his proof of this point is not correct. He states that the marginal utility of saving for agents in state 1 fulfills the following equation (Chamley 2014, p. 662):

$$\zeta_1(t) = \frac{\beta \pi_t}{1 - \beta(1 - \pi_t)} \zeta_0(t). \tag{3.11}$$

I will show that (3.11) should be an inequality instead of an equality. Chamley uses Equation (3.11) for his proof, nevertheless his results are still correct.

To prove the optimality of the high regime consumption function for agents in state 1 I examine the dynamic behavior of Equation (3.8). I analyze the equation with the help of two theorems I derived. The proofs are in the appendix.

Theorem 1 *Convergent Solutions of Difference Equations:*
Consider the difference equation

$$\zeta(t) = v(t) + w(t)\zeta(t+1) \tag{3.12}$$

with $\lim_{t\to\infty} v(t) = v^*$ *and* $\lim_{t\to\infty} w(t) = w^*$*. If it also holds that* $0 < w(t) < 1$*,* $v(t) \geq 0$ *and* $0 < w^* < 1$*, then there exists a unique convergent solution of Equation (3.12) with*

$$\lim_{t\to\infty} \zeta(t) = \zeta^* = v^*/(1 - w^*).$$

The convergent solution is equal to

$$\zeta(t) = v(t) + \lim_{s\to\infty} \sum_{i=t+1}^{s-1} v(i) \prod_{j=t}^{i-1} w(j).$$

I need this theorem to show that the difference equations I use have a convergent solution. It provides a formula for the convergent solution and for the respective limit ζ^*.

I use the following notation from now on:

$$br(t) = \frac{v(t)}{1 - w(t)}.$$

One can use this notation for any difference equation of the same form as Equation (3.12).

Theorem 2 *Monotonicity of Difference Equations:*
Consider Equation (3.12). All premises of Theorem 1 are assumed to hold and Equation (3.12) has a unique convergent solution. Consider only the convergent solution.
a.i) If $br(t) > br(t+1)$ $\forall t$ it follows that $\zeta(t) > \zeta(t+1) > \zeta^*$ $\forall t$.
a.ii) If $\zeta(t) > \zeta(t+1)$ it follows that $\zeta(t) < br(t)$.
b.i) If $br(t) < br(t+1)$ $\forall t$ it follows that $\zeta(t) < \zeta(t+1) < \zeta^*$ $\forall t$.
b.ii) If $\zeta(t) < \zeta(t+1)$ it follows that $\zeta(t) > br(t)$.

This theorem is very helpful. One can derive the monotonic behavior of the convergent solution from the monotonic behavior of br. This is helpful as $br(t)$ does not depend on variables from the next period $t+1$. $\zeta(t)$ depends on $\zeta(t+1)$ which makes a direct analysis difficult. You will see later that it is this theorem, which allows me to generalize Chamley's results.

Consider Equation (3.8):

$$\zeta_1(t) = \underbrace{\pi_t \beta \zeta_0(t+1)}_{v(t)} + \underbrace{(1-\pi_t)\beta}_{w(t)} \zeta_1(t+1).$$

In order to use Theorem 1 I have to show that the premises hold for Equation (3.8). I only consider cases where ζ_0 fulfills the equilibrium requirements. $0 \leq \zeta_0(t) \leq 1$, it is decreasing in t and it converges to zero. $0 \leq \pi_t \leq 0.5$, it is decreasing in t and it also converges to zero. I can use Theorem 1, as $v(t) = \pi_t \beta \zeta_0(t+1) \geq 0$, $\lim_{t\to\infty} v(t) = 0$, $w(t) = (1-\pi_t)\beta > 0$, $w(t) < 1$, $\lim_{t\to\infty} w(t) = \beta$ and $\beta \in (0,1)$. The premises hold and Equation (3.8) has a unique convergent solution. I only consider the convergent solution, as the other solutions are not relevant. Use the definition of $br(t)$ to obtain the following inequality:

$$br(t) = \frac{\beta \pi_t \zeta_0(t+1)}{1-\beta(1-\pi_t)} > \frac{\beta \pi_{t+1} \zeta_0(t+2)}{1-\beta(1-\pi_{t+1})} = br(t+1). \tag{3.13}$$

It is easy to check that the above is true, as $\zeta_0(t)$ is decreasing t and π_t is strictly decreasing in t. As $br(t) > br(t+1)$, it follows from Theorem 2 that $\zeta_1(t)$ is strictly decreasing in t. Use the monotonicity of ζ_0 and ζ_1 to derive that

$$\zeta_1(t) = \pi_t \beta \zeta_0(t+1) + (1-\pi_t)\beta\zeta_1(t+1) \quad < \quad \pi_t \beta \zeta_0(t) + (1-\pi_t)\beta\zeta_1(t).$$

Move $\zeta_1(t)$ to the left to obtain that

$$\zeta_1(t) \quad < \quad \underbrace{\frac{\beta \pi_t}{1-\beta(1-\pi_t)}}_{<1} \zeta_0(t) \tag{3.14}$$

$$\Rightarrow \zeta_1(t) \quad < \quad \zeta_0(t). \tag{3.15}$$

Compare (3.14) with Equation (3.11) to see that the formula used by Chamley is not correct. It should be an inequality instead of an equality.

As $\zeta_1(t) < \zeta_0(t)$ it holds that $\zeta_1(t) \leq 1$ if $\zeta_0(t) \leq 1$. If the consumption function for agents in state 0 is optimal, the consumption function for agents in state 1 is optimal and the high regime is an equilibrium. Let me summarize these findings:

Result 2 *The high regime is an equilibrium for any initial level of debt if $c \leq c^*$. If $c > c^*$ there exists a $\hat{B}(c)$ and the high regime is an equilibrium if and only if $B_0 \leq \hat{B}(c)$.*

4 The Low Regime

I come back to Chamley's analysis and examine the behavior and the equilibrium conditions in the low regime. In the low regime, low type agents in state 0 do not consume. The following list describes how agents move from state to state over time:

- Agents in state -1 at the beginning of period t:
  They do not consume. With a probability of π_t they do not sell and stay in state -1. With a probability of $1 - \pi_t$ they can sell and move to state 0 in the next period.
- Agents in state 0 at the beginning of period t:
  With a probability of $\alpha\pi_t$ they move to state -1 in the next period, as this is the probability that they are of the high type, consume and do not sell.
  With a probability of $a_t = (1-\alpha)\pi_t + \alpha(1-\pi_t)$ they stay in state 0. This is the probability they are of the high type, consume and make a sale, or that they are of the low type, do not consume and make no sale. In these two cases their bond holdings do not change.
  With a probability of $b_t = (1-\pi_t)(1-\alpha)$ they move to state 1 in the next period as this is the probability that they are of the low type, do not consume, make a sale and bond holdings increase by 1.
- Agents in state 1 at the beginning of period t:
  They always consume. With a probability of π_t they do not sell and move to state 0. With a probability of $1 - \pi_t$ they sell and stay in state 1.

New definitions used above:

$$a_t = (1-\alpha)\pi_t + \alpha(1-\pi_t), \tag{4.1}$$

$$b_t = (1-\pi_t)(1-\alpha). \tag{4.2}$$

The fraction of agents not consuming, which is the same as the fraction of unemployed agents, is the fraction of agents in state -1 plus the fraction of low type agents in state 0:

$$\pi_t = \gamma_{-1}(t) + (1-\alpha)\gamma_0(t) \tag{4.3}$$

Use Equation (2.11) and (2.12) to express the unemployment rate as a function of debt:

$$\pi_t = 1 - \alpha - (1 - 2\alpha)B_t \tag{4.4}$$

Debt in period $t+1$ is the same as the fraction of agents in state 1 in period $t+1$. How is it related to the period t? The fraction $\gamma_1(t+1)$ is the same as the fraction of agents in state 1 in period t staying in state 1 plus the fraction of agents in state 0 in period t moving to state 1 in period $t+1$:

$$B_{t+1} = \gamma_1(t+1) = b_t\gamma_0(t) + (1 - \pi_t)\gamma_1(1). \tag{4.5}$$

Use (2.11), (2.12), (4.2) and (4.4) in (4.5) to obtain that

$$B_{t+1} = P(B_t) = -(1-2\alpha)^2 B_t^2 + (1-2\alpha)^2 B_t + \alpha(1-\alpha). \tag{4.6}$$

As the net supply of bonds is zero it holds that $0 \leq B_t \leq 1/2$. Note that $P(B)$ is increasing for $B \in [0, 1/2]$. It is true that $P(1/2) \leq 1/4$ and $P(0) > 0$. Therefore there exists a unique value $B^* \in (0, 1/4]$ s.t. $P(B^*) = B^*$. It is easy to see that from Figure 2:

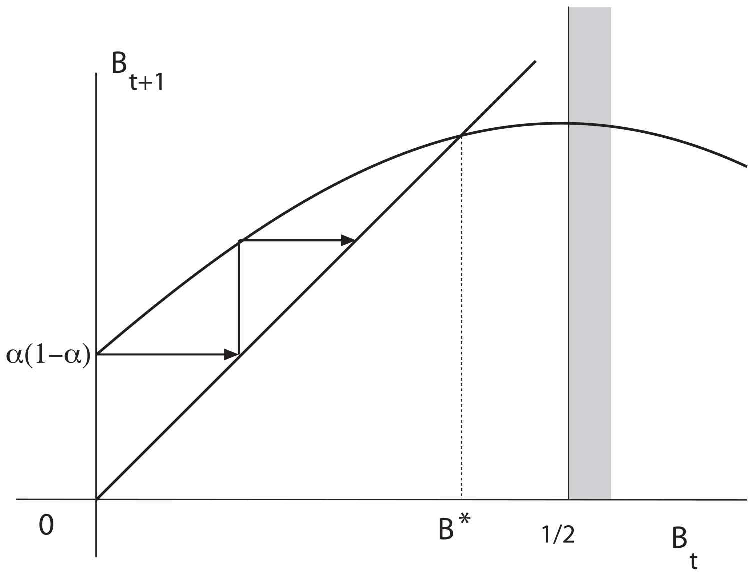

Figure 2: The Dynamics of B in the Low Regime (Chamley 2014)

There is an important difference to the dynamic behavior of B in the high regime. One can see from Figure 2 that the 45° line is not a tangent to $P(B)$ in B^*. The rate of convergence is asymptotically strictly positive. Convergence towards the low regime steady state is fast.
If $B_0 < B^*$ the sequence B_t is strictly increasing in t and converges to B^*. If $B_0 > B^*$

it is strictly decreasing and converges to B^*. Omit the time argument in (4.6) to obtain B^* as a function of α:

$$B^* = \frac{-2\alpha + 2\alpha^2 + \sqrt{\alpha - \alpha^2}}{1 - 4\alpha + 4\alpha^2} \quad \text{if } \alpha \neq 1/2 \tag{4.7}$$

$\alpha = 1/2$ is a special case. If $\alpha = 1/2$ it is true that $P(B) = \alpha(1-\alpha) = 1/4$. For any initial level of debt B_0 it holds that $B_1 = 1/4$. Therefore also $B^* = 1/4$. If $\alpha = 1/2$ convergence to the steady state level of debt takes one period. Figure 3 shows B^* as a function of α.

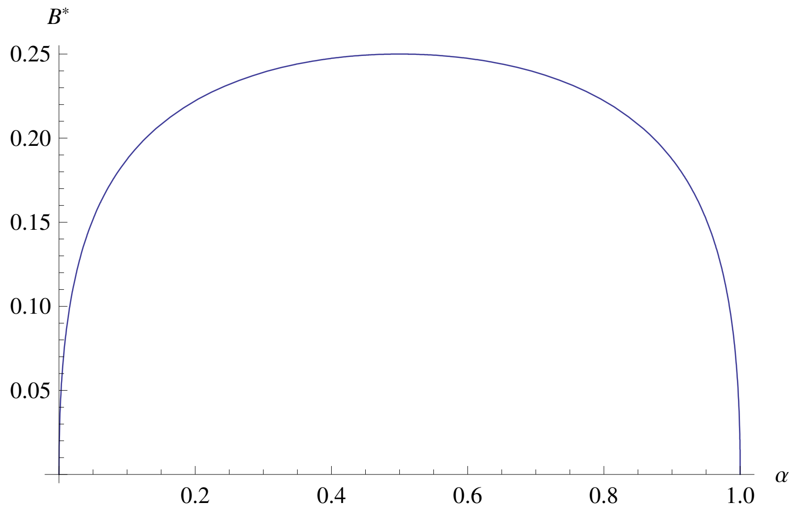

Figure 3: B^* as a Function of α

Recall Equation (4.4):

$$\pi_t = 1 - \alpha - (1 - 2\alpha)B_t.$$

Note that if $\alpha < 1/2$ the unemployment rate decreases if B_t increases t. If $\alpha = 1/2$, π_t does not depend on debt and if $\alpha > 1/2$ unemployment increases if debt increases. What is the intuition behind that? Suppose the economy is in the high regime steady state with zero debt. A sudden regime switch occurs and all low type agents in state 0 do not consume. At this point in time all agents are in state 0 and the unemployment rate jumps up to $1 - \alpha$. Over time B_t increases as some agents accumulate savings and others become indebted. The fraction of agents in state 0 decreases by $2B_t$ whereas the fraction of agents in state -1 increases by B_t. The fraction of agents not consuming decreases by $(1-\alpha)2B_t$, due to the decrease of agents in state 0, and increases by B_t, due to the increase of agents in state -1. If $\alpha < 1/2$ the net effect is negative and unemployment decreases over time. If $\alpha = 1/2$ the two effects cancel each other out and unemployment does not change. If $\alpha > 1/2$ the net effect is positive and unemployment increases if debt increases.

I already mentioned that if $\alpha = 1/2$, π_t does not depend on debt. It holds that $\pi_t = \pi^* = 1/2 \; \forall t$. This is also true in period 0. Unemployment converges to its steady state level instantaneously.

Let me summarize the dynamic behavior of π_t in the low regime. It follows from Equation (4.4) and the dynamic behavior of B_t. Remember, B_t is increasing in t if $B_0 < B^*$ and B_t is decreasing in t if $B_0 > B^*$. B_t converges to B^*.

Result 3 *π_t shows the following dynamic behavior in the low regime:*

- *Case 1: If $\alpha < 1/2$ and $B_0 < B^*$, π_t is decreasing in t.*
- *Case 2: If $\alpha > 1/2$ and $B_0 > B^*$, π_t is decreasing in t.*
- *Case 3: If $\alpha < 1/2$ and $B_0 > B^*$, π_t is increasing in t.*
- *Case 4: If $\alpha > 1/2$ and $B_0 < B^*$, π_t is increasing in t.*
- *Case 5: If $\alpha = 1/2$, π_t is equal to π^* for any initial B_0. π_t is constant over time.*

In all five cases π_t converges to π^*.

Use Equation (4.4) and (4.7) to obtain the steady state level of unemployment as a function of α:

$$\pi^* = \begin{cases} \frac{1-\alpha-\sqrt{\alpha-\alpha^2}}{1-2\alpha} & \text{if } \alpha \neq 1/2 \\ 1/2 & \text{if } \alpha = 1/2 \end{cases} \tag{4.8}$$

Note that $\pi^* \in (0,1) \; \forall \alpha \in (0,1)$. The unemployment rate in the low regime steady state is always positive. Figure 4 shows π^* as a function of α:

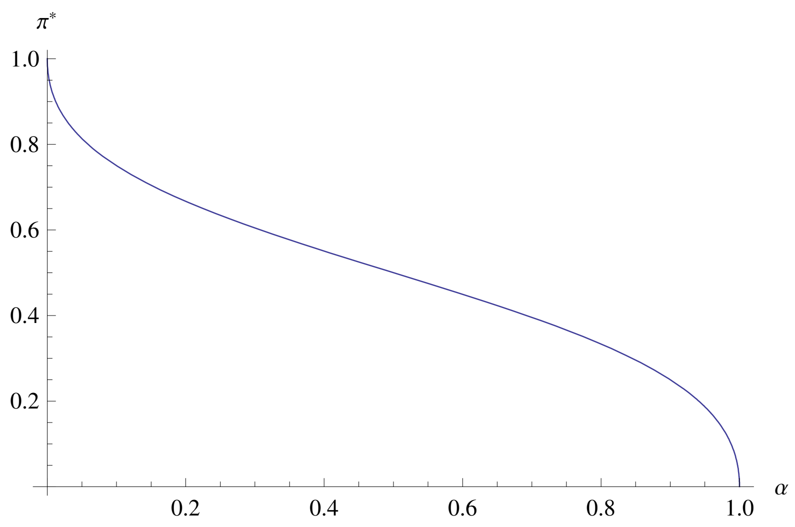

Figure 4: π^* as a Function of α

The behavior of π^* is surprising. If the probability α of being a high type is low, steady state unemployment is high. The fear of ending up as a high type with no possibility to consume induces low type agents in state 0 to not consume. If the fraction of low type agents, $1-\alpha$, is very large, almost nobody consumes. Only high type agents in state 0 and agents in state 1 consume, but there are only very few high type agents in state 0 as α is small. There are also very few agents in state 1. It is very unlikely to move to state 1 as only low type agents in state 0 who can sell move up to state 1. The probability of selling is very low, so the probability of accumulating savings is very low. On the other hand it is very likely that agents in state 1 will move to state 0. They do consume and they only stay in state 1 if they can sell, which is very unlikely. If the low regime is an equilibrium, and I will show that it is under certain conditions, a low probability of negative preference shocks can shut down most of the economy. Remember, this is very inefficient. There are no production costs and the first best solution is everybody consuming and producing.

On the other hand, a large fraction α of high type agents stabilizes the economy. It is optimal for them to consume whenever they can. Low type agents, who do not consume, do not affect the economy that much when α is high, as there are not so many of them. However, unemployment is always positive in the low regime.

The following result summarizes the behavior of the economy in the low regime.

Result 4 *If the low regime is an equilibrium the economy converges to a steady state with positive debt and unemployment. The rate of convergence is asymptotically strictly positive. Unemployment in steady state is a decreasing function of α.*

In the next subsection I show under which conditions the low regime is an equilibrium. The part that remains to be proven is the optimality of the consumption function.

4.1 Equilibrium in the Low Regime

The procedure for proving optimality is the same as for the high regime. Given the proposed consumption function the Bellman equations are as follows:

$$
\begin{aligned}
V_{-1}(t) &= \beta[-\alpha c + \pi_{t+1}V_{-1}(t+1) + (1-\pi_{t+1})V_0(t+1)], && (4.9)\\
V_0(t) &= \beta[\alpha(1+\pi_{t+1}V_{-1}(t+1) + (1-\pi_{t+1})V_0(t+1)) \\
&\quad +(1-\alpha)(\pi_{t+1}V_0(t+1) + (1-\pi_{t+1})V_1(t+1))], && (4.10)\\
V_k(t) &= \beta[1+\pi_{t+1}V_{k-1}(t+1) + (1-\pi_{t+1})V_k(t+1)], \quad \forall k \geq 1. && (4.11)
\end{aligned}
$$

Use the marginal utility of saving defined in Equation (2.10) to derive ζ_0 and ζ_1 in the low regime:

$$\begin{aligned}
\zeta_0(t) &= \pi_t(V_0(t) - V_{-1}(t)) + (1-\pi_t)(V_1(t) - V_0(t)) \\
&= \beta[\pi_t\alpha + \alpha\pi_{t+1}\pi_t V_{-1}(t+1) + (1-\pi_{t+1})\pi_t\alpha V_0(t+1) \\
&\quad +(1-\alpha)\pi_{t+1}\pi_t V_0(t+1) + (1-\alpha)(1-\pi_{t+1})\pi_t V_1(t+1) \\
&\quad +\alpha c\pi_t - \pi_{t+1}\pi_t V_{-1}(t+1) - (1-\pi_{t+1})\pi_t V_0(t+1) \\
&\quad +(1-\pi_t) + (1-\pi_t)\pi_{t+1}V_0(t+1) + (1-\pi_{t+1})(1-\pi_t)V_1(t+1) \\
&\quad -(1-\pi_t)\alpha - (1-\pi_t)\alpha\pi_{t+1}V_{-1}(t+1) - (1-\pi_{t+1})(1-\pi_t)\alpha V_0(t+1) \\
&\quad -(1-\alpha)(1-\pi_t)\pi_{t+1}V_0(t+1) - (1-\pi_{t+1})(1-\pi_t)(1-\alpha)V_1(t+1)] \\
&= \beta[\pi_t\alpha(1+c) + \underbrace{(1-\pi_t)(1-\alpha)}_{b_t}] \\
&\quad +\beta\underbrace{[(1-\alpha)\pi_t + \alpha(1-\pi_t)]}_{a_t}[\pi_{t+1}(V_0(t+1) - V_{-1}(t+1)) \\
&\quad +(1-\pi_{t+1})(V_1(t+1) - V_0(t+1))] \\
\zeta_0(t) &= \beta(\pi_t\alpha(1+c) + b_t) + \beta a_t\zeta_0(t+1) \qquad (4.12)
\end{aligned}$$

$$\begin{aligned}
\zeta_1(t) &= \pi_t(V_1(t) - V_0(t)) + (1-\pi_t)(V_2(t) - V_1(t)) \\
&= \beta[\pi_t + \pi_{t+1}\pi_t V_0(t+1) + \pi_t(1-\pi_{t+1})V_1(t+1) \\
&\quad -\alpha\pi_t - \alpha\pi_t\pi_{t+1}V_{-1}(t+1) - \pi_t\alpha(1-\pi_{t+1})V_0(t+1) \\
&\quad -(1-\alpha)\pi_t\pi_{t+1}V_0(t+1) - (1-\alpha)(1-\pi_{t+1})\pi_t V_1(t+1) \\
&\quad +(1-\pi_t)[1 + \pi_{t+1}V_1(t+1) + (1-\pi_{t+1})V_2(t+1) \\
&\quad -1 - \pi_{t+1}V_0(t+1) - (1-\pi_{t+1})V_1(t+1)]] \\
&= \beta[\pi_t(1-\alpha) + \pi_t\alpha[\pi_{t+1}(V_0(t+1) - V_{-1}(t+1)) \\
&\quad +(1-\pi_{t+1})(V_1(t+1) - V_0(t+1))] \\
&\quad +(1-\pi_t)[\pi_{t+1}V_1(t+1) + (1-\pi_{t+1})V_2(t+1) \\
&\quad -\pi_{t+1}V_0(t+1) - (1-\pi_{t+1})V_1(t+1)]] \\
\zeta_1(t) &= \beta(\pi_t(1-\alpha) + \pi_t\alpha\zeta_0(t+1)) + \beta(1-\pi_t)\zeta_1(t+1) \qquad (4.13)
\end{aligned}$$

The low regime is an equilibrium if the low regime consumption function is optimal. The low regime consumption function is optimal if and only if $\zeta_0(t) \geq 1$ and $\zeta_1(t) \leq 1$ for all t.

Following Chamley, I first show under which conditions the steady state is an equilibrium. One can derive the marginal utility of saving in steady state by omitting the time argument in (4.12) and (4.13):

$$\zeta_0^* = \beta\frac{\pi^*\alpha(1+c) + b^*}{1-\beta a^*}, \qquad (4.14)$$

$$\zeta_1^* = \beta\frac{(1-\alpha)\pi^* + \alpha\pi^*\zeta_0^*}{1-\beta(1-\pi^*)}, \qquad (4.15)$$

where

$$a^* = \lim_{t\to\infty} a_t = (1-\alpha)\pi^* + \alpha(1-\pi^*),$$
$$b^* = \lim_{t\to\infty} b_t = (1-\pi^*)(1-\alpha).$$

ζ_0^* and ζ_1^* are both linear functions of c and increasing in c. Therefore there exists a lower bound for c, $\underline{c}$ s.t. $\forall c \geq \underline{c}$, $\zeta_0^*(c) \geq 1$. Solve $\zeta_0^*(c) = 1$ for c to obtain $\underline{c}$:

$$\underline{c} = \underbrace{\frac{1-\beta}{\beta}}_{\rho} \frac{1}{\pi^*\alpha} = \frac{\rho}{\pi^*\alpha}. \tag{4.16}$$

Equivalently there exists an upper bound for c, $\overline{c}$ s.t. $\forall c \leq \overline{c}$, $\zeta_1^*(c) \leq 1$. Solve $\zeta_1^*(c) = 1$ for c to obtain $\overline{c}$:

$$\overline{c} = \frac{(1-\beta)(1-\alpha\beta+(-1+3\alpha)\beta\pi^*)}{\alpha^2\beta^2\pi^{*2}}. \tag{4.17}$$

As $\zeta_1^* < \zeta_0^*$ if $\zeta_0^* \geq 1$ it follows that $\underline{c} < \overline{c}$. Therefore there exists an interval $C = (\underline{c}, \overline{c})$ s.t. for $c \in C$ $\zeta_0^* > 1 > \zeta_1^*$. Note that $\underline{c}$ and $\overline{c}$ are functions of α and β, as π^* is a function of α.

Result 5 *Consider the steady state of the low regime with debt equal to B^* and unemployment equal to π^*:*

1. *The steady state of the low regime is an equilibrium if $c \in [\underline{c}, \overline{c}]$.*
2. *If $c \in (\underline{c}, \overline{c})$, there exists an open interval around B^* s.t. for all B_0 in that interval the consumption function is optimal, the economy is in equilibrium and it converges to the steady state.*

Part 2 holds by the following argument. If $c \in (\underline{c}, \overline{c})$ and B_0 is close to B^*, it is true that $\zeta_0(t) > 1 > \zeta_1(t)$ as $\zeta_0(t)$ and $\zeta_1(t)$ are continuous in B_0. The steady state is locally stable. In the next subsection I show under which conditions the low regime is globally stable.

4.2 Global Stability of the Low Regime

Chamley shows under which conditions the low regime is globally stable if $\alpha = 1/2$. I extend his analysis and show under which conditions the low regime is globally stable for most α.

I examine under which conditions the low regime consumption function is optimal outside the steady state. In order to do that I have to show under which conditions

$\zeta_0(t) \geq 1$ and $\zeta_1(t) \leq 1$ $\forall t$. I need the following equations from above for my analysis:

$$\text{Eq. (4.12) revisited:} \quad \zeta_0(t) = \underbrace{\beta(\pi_t\alpha(1+c)+b_t)}_{v_0(t)} + \underbrace{\beta a_t}_{w_0(t)} \zeta_0(t+1)$$

$$\text{Eq. (4.13) revisited:} \quad \zeta_1(t) = \underbrace{\beta(\pi_t(1-\alpha)+\pi_t\alpha\zeta_0(t+1))}_{v_1(t)} + \underbrace{\beta(1-\pi_t)}_{w_1(t)} \zeta_1(t+1)$$

$$\text{Eq. (4.1) revisited:} \quad a_t = (1-\alpha)\pi_t + \alpha(1-\pi_t)$$

$$\text{Eq. (4.2) revisited:} \quad b_t = (1-\pi_t)(1-\alpha)$$

$$\text{Eq. (4.4) revisited:} \quad \pi_t = 1-\alpha-(1-2\alpha)B_t$$

I use Theorem 1 to show that Equation (4.12) and (4.13) have a unique convergent solution. Let me examine the premises of Theorem 1. Note the new definitions of $v_0(t)$, $w_0(t)$, $v_1(t)$ and $w_1(t)$ from above. Define v_0^* as $v_0^* = \lim_{t\to\infty} v_0(t)$. Define w_0^*, v_1^* and w_1^* equivalently. From Equation (4.4) it follows that $\pi_t \in (0,1)$ as $1/2 \geq B_t \geq 0$ and $\alpha \in (0,1)$. Therefore it holds that $a_t \in (0,1)$ and $b_t \in (0,1)$. As $\beta \in (0,1)$ and $c \geq 0$ it follows that $v_0(t) \geq 0$. It also follows that $w_0(t)$ is in $(0,1)$. v_0^* and w_0^* both exist and as $\pi^* \in (0,1)$ it follows that $a^* \in (0,1)$ and therefore w_0^* is in $(0,1)$. Equation (4.12) fulfills the premises of Theorem 1 and it is true that the equation has a unique convergent solution equal to

$$\zeta_0(t) = v_0(t) + \lim_{s\to\infty} \sum_{i=t+1}^{s-1} v_0(i) \prod_{j=t}^{i-1} w_0(j). \tag{4.18}$$

$\zeta_0(t)$ denotes the convergent solution. Note that $\zeta_0(t) \geq 0$ as $v_0(i)$ and $w_0(j)$ are positive for all i and j. The limit of $\zeta_0(t)$ for t to infinity is ζ_0^* from Equation (4.14).

Use the definition of a_t and b_t to see that $v_0(t)$ and $w_0(t)$ are functions of π_t, c, α and β. Use Equation (4.4) to see that π_t is a function of α and B_t, which by repeated iterations of Equation (4.6) is a function of B_0 and α. $v_0(t)$ and $w_0(t)$ are functions of B_0, c, α and β. This holds for all t. Therefore, from Equation (4.18) it follows that $\zeta_0(t)$ is a function of B_0, c, α and β. I denote $\zeta_0(t)$ as $\zeta_0(t, c, B_0, \alpha, \beta)$. To keep the notation simple I will denote only the arguments I explicitly need in a respective step.

Use the convergent solution $\zeta_0(t)$ in Equation (4.13) to obtain that $v_1(t) \geq 0$ and v_1^* exists. It holds that $w_1(t) \in (0,1)$, w_1^* exists and $w_1^* \in (0,1)$. The premises of Theorem 1 hold and Equation (4.13) has a unique convergent solution

$$\zeta_1(t) = v_1(t) + \lim_{s\to\infty} \sum_{i=t+1}^{s-1} v_1(i) \prod_{j=t}^{i-1} w_1(j), \tag{4.19}$$

which converges to ζ_1^* in Equation (4.15). Due to the analogous arguments as in the case of $\zeta_0(t)$, $\zeta_1(t)$ is a function of c, B_0, α and β. Again I will use the simplest notation in every step. Consider only the convergent solutions denoted by $\zeta_0(t)$ and $\zeta_1(t)$ from now on.

4.2.1 The Monotonic Behavior of $\zeta_0(t)$ and $\zeta_1(t)$

Next I examine the monotonic behavior of $\zeta_0(t)$ and $\zeta_1(t)$ over time with the help of Theorem 2. Define $br_0(t)$ and $br_1(t)$ as

$$br_0(t) = \frac{v_0(t)}{1 - w_0(t)},$$
$$br_1(t) = \frac{v_1(t)}{1 - w_1(t)}.$$

As $v_0(t)$ and $w_0(t)$ are functions of B_0, c, α and β, $br_0(t)$ is a function of B_0, c, α and β. Again I will use the simplest notation in every step. The same is true for $br_1(t)$.

Theorem 2 says that the monotonic behavior of $\zeta_0(t)$ is the same as the monotonic behavior of $br_0(t)$. I use the difference $br_0(t+1) - br_0(t)$ to examine the monotonic behavior of $br_0(t)$:

$$\begin{aligned} br_0(t+1) - br_0(t) &= \frac{v_0(t+1)}{1 - w_0(t+1)} - \frac{v_0(t)}{1 - w_0(t)} \\ &= \frac{v_0(t+1)(1 - w_0(t)) - v_0(t)(1 - w_0(t+1))}{(1 - w_0(t))(1 - w_0(t+1))} \end{aligned} \tag{4.20}$$

If the difference is positive for all t, $br_0(t)$ is strictly increasing, if it is negative $br_0(t)$ is strictly decreasing. As the two factors in the denominator are positive one can omit them. They do not influence the sign of the difference. Use the definitions of $v_0(t)$, $v_1(t)$, a_t and b_t to obtain that:

$$\begin{aligned} v_0(t+1)(1 - w_0(t)) - v_0(t)(1 - w_0(t+1)) &= \\ \beta(\pi_{t+1}\alpha(1+c) + b_{t+1})(1 - \beta a_t) - \beta(\pi_t\alpha(1+c) + b_t)(1 - \beta a_{t+1}) &= \\ \beta[\pi_{t+1}\alpha(1+c) + (1-\alpha)(1-\pi_{t+1})][1 - \beta[(1-\alpha)\pi_t + \alpha(1-\pi_t)]] & \\ -\beta[\pi_t\alpha(1+c) + (1-\alpha)(1-\pi_t)][1 - \beta[(1-\alpha)\pi_{t+1} + \alpha(1-\pi_{t+1})]] &= \\ \beta(\pi_t - 2\alpha\pi_t - c\alpha\pi_t - \beta\pi_t + 2\alpha\beta\pi_t + c\alpha^2\beta\pi_t & \\ -(\pi_{t+1} - 2\alpha\pi_{t+1} - c\alpha\pi_{t+1} - \beta\pi_{t+1} + 2\alpha\beta\pi_{t+1} + c\alpha^2\beta\pi_{t+1})) &= \\ (\pi_t - \pi_{t+1})(1 - 2\alpha - c\alpha - \beta + 2\alpha\beta + \alpha^2\beta c)\beta & \end{aligned} \tag{4.21}$$

If $br_0(t)$ is decreasing or increasing follows from the sign of (4.21). The sign of (4.21) follows from the sign of each of its factors. The last factor β is positive. The second factor is negative if c is large enough:

$$\begin{aligned} (1 - 2\alpha - \beta + 2\alpha\beta - \alpha c + \alpha^2\beta c) < 0 \Leftrightarrow c > \tilde{c} &= \frac{1 - 2\alpha - \beta + 2\alpha\beta}{\alpha - \alpha^2\beta} \\ &= \frac{(1-2\alpha)(1-\beta)}{\alpha(1-\alpha\beta)} \end{aligned} \tag{4.22}$$

If $\alpha > 1/2$ it holds that $\tilde{c} < 0$. As $c \geq 0$ by definition, the second factor is always negative in this case. In order to have global stability, the steady state must be an equilibrium.

I therefore only consider cases where the steady state is an equilibrium and $c \geq \underline{c}$. If $\underline{c} > \tilde{c}$ the second factor in (4.21) is negative for all relevant cases. Use the definition of π^* from Equation (4.8) in the definition of $\underline{c}$ from Equation (4.16) to obtain that

$$\underline{c} = \frac{(1-\beta)}{\beta\pi^*\alpha} = \frac{(1-2\alpha)(1-\beta)}{\alpha\beta(1-\alpha-\sqrt{\alpha-\alpha^2})}.$$

Next I prove that $\underline{c} > \tilde{c}$ if $\alpha < 1/2$:

$$\begin{aligned}
\underline{c} &> \tilde{c} \\
&\Leftrightarrow \\
\frac{(1-2\alpha)(1-\beta)}{\alpha\beta(1-\alpha-\sqrt{\alpha-\alpha^2})} &> \frac{(1-2\alpha)(1-\beta)}{\alpha(1-\alpha\beta)} \\
&\Leftrightarrow \quad \text{as } (1-2\alpha), (1-\beta) \text{ and } \alpha \\
&\quad\quad \text{are all positive (remember } \alpha < 1/2) \\
\frac{1}{\beta(1-\alpha-\sqrt{\alpha-\alpha^2})} &> \frac{1}{1-\alpha\beta} \\
&\Leftrightarrow \quad \text{as both denominators are positive} \\
\beta(1-\alpha-\sqrt{\alpha-\alpha^2}) &< 1-\alpha\beta \\
&\Leftrightarrow \\
\beta-\alpha\beta-\beta\sqrt{\alpha-\alpha^2} &< 1-\alpha\beta \\
&\Leftrightarrow \\
\beta(1-\sqrt{\alpha-\alpha^2}) &< 1 \qquad \text{which is true as } \beta \text{ and } \alpha \in (0,1).
\end{aligned}$$

The second factor in (4.21) is negative for all relevant cases. The sign of (4.21) depends on the monotonic behavior of the unemployment rate. If π_t is strictly decreasing in t, $(\pi_t - \pi_{t+1})$ is positive, (4.21) is negative, $br_0(t)$ is strictly decreasing in t and from Theorem 2 it follows that $\zeta_0(t)$ is strictly decreasing in t. By the same argument $\zeta_0(t)$ is strictly increasing in t if π_t is strictly increasing in t. $\zeta_0(t)$ has the same monotonic behavior as π_t.

Next I examine the monotonic behavior of $\zeta_1(t)$ in the same way. I consider two cases. In case one $\zeta_0(t)$ and π_t are decreasing in t. In case two $\zeta_0(t)$ and π_t are increasing in t.

Case 1, $\zeta_0(t)$ and π_t are decreasing in t: I will show that in this case also $\zeta_1(t)$ is decreasing in t. In order to prove that I have to show that $br_1(t) - br_1(t+1) > 0$:

$$\begin{aligned}
br_1(t) - br_1(t+1) &> 0 \\
&\Leftrightarrow \quad \text{by def. of } br_1 \\
\frac{v_1(t)}{1-w_1(t)} - \frac{v_1(t+1)}{1-w_1(t+1)} &> 0
\end{aligned}$$

$$
\begin{aligned}
& \Leftrightarrow \\
\frac{v_1(t)(1-w_1(t+1)) - v_1(t+1)(1-w_1(t))}{(1-w_1(t))(1-w_1(t+1))} &> 0 \\
& \Leftrightarrow \text{ as } (1-w_1(t))(1-w_1(t+1)) > 0 \\
v_1(t)(1-w_1(t+1)) - v_1(t+1)(1-w_1(t)) &> 0 \\
& \Leftrightarrow \text{ by def. of } v_1 \text{ and } w_1 \\
\beta(\zeta_0(t+1)(\alpha\pi_t - \alpha\beta\pi_t + \alpha\beta\pi_t\pi_{t+1}) & \\
-\zeta_0(t+2)(\alpha\pi_{t+1} - \alpha\beta\pi_{t+1} + \alpha\beta\pi_t\pi_{t+1}) & \\
+\pi_t(1-\alpha+\alpha\beta-\beta) - \pi_{t+1}(1-\alpha+\alpha\beta-\beta)) &> 0 \\
& \Leftrightarrow \text{ as } \beta > 0 \\
\zeta_0(t+1)(\alpha\pi_t - \alpha\beta\pi_t + \alpha\beta\pi_t\pi_{t+1}) & \\
-\zeta_0(t+2)(\alpha\pi_{t+1} - \alpha\beta\pi_{t+1} + \alpha\beta\pi_t\pi_{t+1}) & \\
+\pi_t(1-\alpha+\alpha\beta-\beta) - \pi_{t+1}(1-\alpha+\alpha\beta-\beta) &> 0 \\
& \Leftrightarrow \text{ rearranging yields} \\
\zeta_0(t+1)\alpha\pi_t \underbrace{(1-\beta+\beta\pi_{t+1})}_{>0} & \\
-\zeta_0(t+2)\alpha\pi_{t+1}(1-\beta+\beta\pi_t) & \\
+(\pi_t - \pi_{t+1})(1-\alpha+\alpha\beta-\beta) &> 0 \\
& \Leftarrow \text{ as } \zeta_0(t+1) > \zeta_0(t+2) \\
\zeta_0(t+2)\alpha\pi_t(1-\beta+\beta\pi_{t+1}) & \\
-\zeta_0(t+2)\alpha\pi_{t+1}(1-\beta+\beta\pi_t) & \\
+(\pi_t - \pi_{t+1})(1-\alpha+\alpha\beta-\beta) &> 0 \\
& \Leftrightarrow \text{ rearranging yields} \\
\zeta_0(t+2)\alpha(1-\beta)(\pi_t - \pi_{t+1}) & \\
+(\pi_t - \pi_{t+1})(1-\alpha)(1-\beta) &> 0 \\
& \Leftrightarrow \text{ as } \pi_t \text{ is decreasing and } \beta < 1 \\
\zeta_0(t+2)\alpha + (1-\alpha) &> 0, \quad \text{which is true.}
\end{aligned}
$$

Of course the correct logical order of the proof is from the last line to the first.

Case 2, $\zeta_0(t)$ and π_t are increasing in t: in this case $br_1(t) - br_1(t+1) < 0$ and therefore $\zeta_1(t)$ is increasing in t. The proof is analogous to the case above.

The marginal utility of saving for low type agents in state 0 and 1 has the same monotonic behavior as the unemployment rate. This is not surprising. The higher the unemployment rate, the higher is the marginal utility of saving, as it is more likely that agents cannot sell. Use Result 3, which summarizes the monotonic behavior of π_t together with the findings of this section to obtain the following result:

Result 6 *$\zeta_0(t)$ and $\zeta_1(t)$ are strictly increasing in t if $B_0 < B^*$ and $\alpha > 1/2$ or $B_0 > B^*$ and $\alpha < 1/2$. $\zeta_0(t)$ and $\zeta_1(t)$ are strictly decreasing in t if $B_0 < B^*$ and $\alpha < 1/2$ or $B_0 > B^*$ and $\alpha > 1/2$.*

Next I examine how $\zeta_0(t)$ and $\zeta_1(t)$ depend on c. Consider Equation (4.18):

$$\zeta_0(t) = v_0(t) + \lim_{s\to\infty} \sum_{i=t+1}^{s-1} v_0(i) \prod_{j=t}^{i-1} w_0(j).$$

Use the definitions of $v_0(t)$ and $w_0(t)$ to see that $\zeta_0(t)$ is a continuous and strictly monotone function of c:

$$\zeta_0(t) = (1+c)\beta\alpha(\pi_t + \lim_{s\to\infty} \sum_{i=t+1}^{s-1} \pi_i \prod_{j=t}^{i-1} w_0(j)) + \beta(b_t + \lim_{s\to\infty} \sum_{i=t+1}^{s-1} b_i \prod_{j=t}^{i-1} w_0(j)).$$

The same can be shown for $\zeta_1(t)$ in an analogous way.

Result 7 *$\zeta_0(t,c)$ and $\zeta_1(t,c)$ are strictly increasing and continuous in c.*

4.2.2 Global Stability

With the help of Result 6 and 7 I examine the global stability of the low regime. Remember, in order to have global stability the consumption function needs to be optimal at any point in time. The low regime consumption function is optimal if $\zeta_0(t,c) \geq 1$ and $\zeta_1(t,c) \leq 1$. I consider five distinct cases:

Case 1, $\alpha = 1/2$: This is also the case which Chamley considers. If $\alpha = 1/2$ it holds that $\pi_t = \pi^*$ $\forall t$. Unemployment does not depend on debt in this case. As the only time dependent variable in the marginal utilities of saving ζ_0 and ζ_1 is unemployment, it holds that $\zeta_0(t) = \zeta_0^*$ and $\zeta_1(t) = \zeta_1^*$ for all t. The regime is globally stable if the steady state is an equilibrium. Debt converges to its steady state level within one period. The steady state is an equilibrium if $c \in [\underline{c}, \overline{c}]$. If $\alpha = 1/2$ it is true that $\underline{c} = 4\rho$ and $\overline{c} = 4\rho(3+4\rho)$ with $\rho = (1-\beta)/\beta$. This follows from Equation (4.16) and (4.17). The low regime is an equilibrium for any initial level of debt if $\alpha = 1/2$ and $c \in [4\rho, 4\rho(3+4\rho)]$. As $\zeta_0(t)$ and $\zeta_1(t)$ are continuous in α, this result also holds if α is in an open interval around $1/2$. If c is in the open interval $(4\rho, 4\rho(3+4\rho))$ there exists an open interval A around $1/2$, s.t. for all $\alpha \in A$ the low regime is an equilibrium for any initial level of debt. However Chamley does not show how large this open interval is. In the following four cases I show under which conditions the low regime is globally stable if $\alpha \neq 1/2$. This allows me to generalize Chamley's results and is my main contribution to this model.

Case 2, $\alpha < 1/2$ and $0 \leq B_0 < B^*$: From Result 6 it follows that $\zeta_0(t,c)$ and $\zeta_1(t,c)$ are decreasing in t. Suppose $c \in [\underline{c}, \overline{c}]$. The steady state is stable, a minimum condition for global stability, and $\zeta_0^*(c) \geq 1$. As $\zeta_0(t,c)$ is decreasing in t, $\zeta_0(t,c) > 1$ $\forall t$ and the low regime consumption function for low type agents in state 0 is optimal. The same does not hold for $\zeta_1(t,c)$. Suppose $c = \overline{c}$. As $\zeta_1^*(\overline{c}) = 1$ and $\zeta_1(t,\overline{c})$ is strictly decreasing in t it follows that $\zeta_1(0,\overline{c}) > 1$. The low regime consumption function for low type agents in state 1 is not optimal for all $c \in [\underline{c},\overline{c}]$. Define $\overline{c}'$ as $\zeta_1(0,\overline{c}') = 1$. As $\zeta_1(t,c)$ is decreasing in t and increasing in c, $\zeta_1(t,c) \leq 1$ for all t and for all $c \leq \overline{c}'$. If $c \leq \overline{c}'$, the low regime consumption function for low type agents in state 1 is optimal. Note that $\overline{c}' < \overline{c}$ as $\zeta_1(0,\overline{c}) > 1$, $\zeta_1(0,\overline{c}') = 1$ and $\zeta_1(0,c)$ is strictly increasing in c. But is $\overline{c}' > \underline{c}$? Only then there exists an interval $[\underline{c},\overline{c}']$ s.t. for all c in that interval, the low regime consumption function is optimal for low type agents in state 1 and in state 0. If $\zeta_1(0,\underline{c}) < 1$ it holds that $\overline{c}' > \underline{c}$ as $\zeta_1(0,c)$ is strictly increasing in c. Unfortunately $\zeta_1(0,\underline{c})$ cannot be expressed analytically. In order to show that $\zeta_1(0,\underline{c}) < 1$ I use an analytically solvable upper bound for $\zeta_1(0,c)$. From Theorem 2 it follows that $\zeta_1(0,c) < br_1(0,c)$:

$$
\begin{aligned}
\zeta_1(0,c) &< br_1(0,c) \\
&\quad \text{use the definition of } br_1 \\
\zeta_1(0,c) &< \beta\frac{\pi_0(1-\alpha) + \alpha\pi_0\zeta_0(1,c)}{1-w_1(0)} \\
&\quad \text{use } \zeta_0(1,c) < br_0(1,c) < br_0(0,c) \text{ from Theorem 2} \\
\zeta_1(0,c) &< \beta\frac{\pi_0(1-\alpha) + \alpha\pi_0 br_0(0,c)}{1-w_1(0)} \\
&\quad \text{define the right hand side as } Ub(c) \\
Ub(c) &= \beta\frac{\pi_0(1-\alpha) + \alpha\pi_0 br_0(0,c)}{1-w_1(0)}
\end{aligned}
$$

Use the definitions for br_0, $w_1(0)$, π_0 and $\underline{c}$ to express $Ub(\underline{c})$ as a function of α, β and B_0. With the help of Mathematica I can show that the inequality $Ub(\underline{c}) < 1$ holds if $\alpha \in (0,1/2)$, $\beta \in (0,1)$ and $B_0 < B^*$. As $Ub(\underline{c}) < 1$ it holds that $\zeta_1(0,\underline{c}) < 1$ and it therefore holds that $\overline{c}' > \underline{c}$.

If $B_0 < B^*$ and $\alpha < 1/2$ there exists an interval $[\underline{c},\overline{c}']$ s.t. for all c in that interval the low regime is an equilibrium. $\overline{c}'$ is defined as $\zeta_1(0,\overline{c}',B_0,\alpha,\beta) = 1$ and can be obtained numerically for any B_0, α and β of interest. Note that $\overline{c}'$ depends on B_0, α and β.

Case 3, $\alpha < 1/2$ and $1/2 \geq B_0 > B^*$: in this case $\zeta_0(t,c)$ and $\zeta_1(t,c)$ are increasing in t. Suppose $c \in [\underline{c},\overline{c}]$, so the steady state is stable. As $\zeta_1^*(c) \leq 1$ and $\zeta_1(t,c)$ is increasing in t, $\zeta_1(t,c) < 1$ $\forall t$. Optimality for low type agents in state 1 is fulfilled. Define $\underline{c}'$ as $\zeta_0(0,\underline{c}') = 1$. If $c \geq \underline{c}'$ optimality for low type agents in state 0 is fulfilled as $\zeta_0(t,c)$ is increasing in t and c. Note that $\underline{c}' > \underline{c}$ as $\zeta_0(0,\underline{c}) < 1$, $\zeta_0(0,\underline{c}') = 1$ and $\zeta_0(0,c)$ is strictly increasing in c. Analogously to case 1, I need to show that $\underline{c}' < \overline{c}$. $\underline{c}' < \overline{c}$ is true if $\zeta_0(0,\overline{c}) > 1$. Again there is no analytical expression for $\zeta_0(0,\overline{c})$. From Theorem 2 it follows that $\zeta_0(0,\overline{c}) > br_0(0,\overline{c})$. Use the definition of br_0, $v_0(0)$, $w_0(0)$, π_0 and $\overline{c}$ to see that $br_0(0,\overline{c})$ is a function of α, β and B_0. Using Mathematica I can show that $br_0(0,\overline{c}) > 1$ if $\alpha \in (0,1/2)$, $\beta \in (0,1)$ and $B_0 > B^*$. Therefore $\zeta_0(0,\overline{c}) > 1$ and it holds that $\underline{c}' < \overline{c}$. There exists an interval $[\underline{c}',\overline{c}]$, s.t. for all c in that interval, $B_0 > B^*$ and $\alpha < 1/2$, the low regime is an equilibrium. $\underline{c}'$ can be obtained numerically, solving

$\zeta_0(0, \underline{c}', B_0, \alpha, \beta) = 1$ for any B_0, α and β of interest. $\underline{c}'$ depends on B_0, α and β.

Case 4, $\alpha > 1/2$ and $0 \leq B_0 < B^*$: this case is analogous to case 3 as $\zeta_0(t, c)$ and $\zeta_1(t, c)$ are increasing in t. If $c \in [\underline{c}', \overline{c}]$ the consumption function is optimal for agents in states 1 and 0. $\underline{c}'$ is again defined as $\zeta_0(0, \underline{c}') = 1$. There is an important difference to case 3, however. It does not hold that $br_0(0, \overline{c}) > 1$ for all $\alpha \in (1/2, 1)$, $\beta \in (0, 1)$ and $B_0 < B^*$. $br_0(0, \overline{c}) < 1$ for some α, β and B_0. Therefore I cannot prove that $\underline{c}' < \overline{c}$ in the same way as in case 3. Note that even if $br_0(0, \overline{c}) < 1$ for some parameter values, it may still hold that $\zeta_0(0, \overline{c}) > 1$ for all parameter values, as $br_0(0, \overline{c}) < \zeta_0(0, \overline{c})$. Remember, one cannot examine the inequality $\zeta_0(0, \overline{c}) > 1$ directly as it has no closed analytical form. I need an analytically solvable lower bound for $\zeta_0(0, \overline{c})$ to examine if $\zeta_0(0, \overline{c}) > 1$. $br_0(0, \overline{c})$ is maybe just a lower bound which is too small. In fact if I approximate $\zeta_0(0, \overline{c})$ numerically, it is larger than one for all parameter values which I considered. I construct a larger lower bound in order to show that $\underline{c}' < \overline{c}$ for as many parameter values as possible. By Equation (4.12), $\zeta_0(t)$ is equal to

$$\zeta_0(t) = v_0(t) + w_0(t)\zeta_0(t+1). \tag{4.23}$$

By repeated iterations of (4.23) it follows that

$$\zeta_0(t) = v_0(t) + \sum_{i=t+1}^{s-1} v_0(i) \prod_{j=t}^{i-1} w_0(j) + \zeta_0(s) \prod_{j=t}^{s-1} w_0(j), \qquad \forall s > t.$$

For $t = 0$ this yields

$$\zeta_0(0) = v_0(0) + \sum_{i=1}^{s-1} v_0(i) \prod_{j=0}^{i-1} w_0(j) + \zeta_0(s) \prod_{j=0}^{s-1} w_0(j), \qquad \forall s \geq 1.$$

Use $br_0(t) < \zeta_0(t)$ to obtain that

$$\zeta_0(0) > \underbrace{v_0(0) + \sum_{i=1}^{s-1} v_0(i) \prod_{j=0}^{i-1} w_0(j) + br_0(s) \prod_{j=0}^{s-1} w_0(j)}_{Lb(s)}, \qquad \forall s \geq 1.$$

Define the right hand side as $Lb(s)$. It is a lower bound for $\zeta_0(0)$ depending on s. $Lb(s)$ has a closed analytical form for every s. The higher s is, the closer $Lb(s)$ is to $\zeta_0(0)$. But the higher s is, the more complicated the expression becomes. Use the definition of $v_0(t)$, $w_0(t)$, π_t and B_t to see that $Lb(s)$ is a function of c, α, β and B_0. I denote it by $Lb(s, c, \alpha, \beta, B_0)$. Consider the following proposition. The proof is in the appendix.

Proposition 1 *Let α be in $(1/2, 1)$. If $B_0 < B_0'$ it follows that $\zeta_0(t, c, B_0) \leq \zeta_0(0, c, B_0')$.*

$\zeta_0(0, c, 0)$ is the marginal utility of saving in period 0 if initial debt is equal to 0. If $\zeta_0(0, \overline{c}, 0) > 1$, $\zeta_0(0, \overline{c}, B_0) > 1$ for all B_0 as $\zeta_0(0, \overline{c}, B_0)$ is increasing in B_0. By the definition of Lb it holds that $\zeta_0(0, \overline{c}, 0) > Lb(s, \overline{c}, \alpha, \beta, 0)$. $\overline{c}$ is a function of α and β and therefore $Lb(s, \overline{c}, \alpha, \beta, 0)$ is reduced to a function of s, α and β. I denote it by $Lb'(s, \alpha, \beta)$. I use Mathematica to examine the inequality $Lb'(s, \alpha, \beta) > 1$. The largest s the computer can handle is $s = 4$. Even with this sophisticated lower bound, I cannot show that $Lb'(4, \alpha, \beta) > 1$ for all α. The program gives exact conditions for which the inequality holds. They are solutions of high order polynomials. However, $\alpha < 0.988$ is a sufficient condition for the inequality to hold. This is sufficient for our purposes. If $\alpha \in (0.5, 0.988)$ it holds that $Lb'(4, \alpha, \beta) > 1$ and therefore $\zeta_0(0, \overline{c}, 0) > 1$. From Proposition 1 it follows that $\zeta_0(0, \overline{c}, B_0) > 1 \ \forall B_0$ and as $\zeta_0(0, c)$ is increasing in c, it follows that $\underline{c}' < \overline{c}$. If $\alpha \in (0.5, 0.988)$ and $B_0 < B^*$, there exists an interval $[\underline{c}', \overline{c}]$, s.t. for all c in the interval, the low regime is an equilibrium. If $\alpha \in (0.988, 1)$ such an interval may exist. It exists for several cases for which I computed it numerically. However I can not prove analytically that it exists for all α. This is not so important, as 0.988 is already very close to one.

Case 5, $\alpha > 1/2$ and $1/2 \geq B_0 > B^*$: This case is analogous to case 2 as $\zeta_0(t, c)$ and $\zeta_1(t, c)$ are decreasing in t. If $c \in [\underline{c}, \overline{c}']$ the consumption function is optimal for agents in states 1 and 0. $\overline{c}'$ is again defined as $\zeta_1(0, \overline{c}') = 1$. Similarly to case 4, it is again difficult to prove that $\overline{c}' > \underline{c}$. Remember, if $\zeta_1(0, \underline{c}) < 1$ it follows that $\overline{c}' > \underline{c}$. In case 2 I used $Ub(\underline{c}) < 1$ to show that $\zeta_1(0, \underline{c}) < 1$. In this case $Ub(\underline{c}) < 1$ does not hold for all $\alpha \in (1/2, 1)$, $\beta \in (0, 1)$ and $B_0 > B^*$. Again, if I approximate $\zeta_1(0, \underline{c})$ numerically it is smaller than one for all parameter values which I considered. A smaller upper bound than Ub is needed in order to prove $\underline{c} < \overline{c}'$ for as many parameter values as possible. Analogously to case 4 I rewrite $\zeta_1(0)$ as:

$$\zeta_1(0) = v_1(0) + \sum_{i=1}^{s-1} v_1(i) \prod_{j=0}^{i-1} w_1(j) + \zeta_1(s) \prod_{j=0}^{s-1} w_1(j), \qquad \forall s \geq 1. \tag{4.24}$$

As $\zeta_1(t)$ is decreasing in t, from Theorem 2 it follows that $\zeta_1(t) < br_1(t)$. Use the definition of $br_1(t)$ to obtain that:

$$\zeta_1(t) < br_1(t) = \frac{\beta((\pi_t(1-\alpha) + \pi_t \alpha \zeta_0(t+1))}{1 - \beta(1 - \pi_t)}.$$

As $\zeta_0(t)$ is decreasing, it follows from Theorem 2 that $\zeta_0(t) < br_0(t)$. Substitute $\zeta_0(t+1)$ by $br_0(t+1)$ in the last line above to obtain

$$\zeta_1(t) < br_1(t) < \frac{\beta((\pi_t(1-\alpha) + \pi_t \alpha br_0(t+1))}{1 - \beta(1 - \pi_t)}.$$

Define the right hand side above as $Br_1(t)$. It holds that $Br_1(t) > \zeta_1(t)$. Remember the definition of $v_1(t)$:

$$v_1(t) = \beta(\pi_t(1-\alpha) + \pi_t \alpha \zeta_0(t+1)).$$

Substitute $\zeta_0(t+1)$ by $br_0(t+1)$ to obtain that

$$v_1(t) < \beta(\pi_t(1-\alpha) + \pi_t\alpha br_0(t+1)).$$

Define the right hand side as $m_1(t)$. Substitute $v_1(t)$ by $m_1(t)$ and $\zeta_1(s)$ by $Br_1(s)$ in Equation (4.24) to obtain that

$$\zeta_1(0) < \underbrace{m_1(0) + \sum_{i=1}^{s-1} m_1(i) \prod_{j=0}^{i-1} w_1(j) + Br_1(s) \prod_{j=0}^{s-1} w_1(j)}_{Ub_2(s)}, \qquad \forall s \geq 1. \tag{4.25}$$

Define the right hand side as $Ub_2(s)$. $Ub_2(s)$ is an upper bound of $\zeta_1(t)$ depending on s. The higher s is, the closer $Ub_2(s)$ is to $\zeta_1(0)$, but the more complicated it becomes. Use the definitions of m_1, w_1, Br_1, br_0, π_t and B_t to see that $Ub_2(s)$ is a function of c, α, β and B_0. I denote it by $Ub_2(s, c, \alpha, \beta, B_0)$. Consider the following proposition. The proof is in the appendix.

Proposition 2 *Let α be in $(1/2, 1)$ and $B_0 > B^*$. If $B_0 < B_0'$ it follows that $\zeta_1(0, c, B_0) \leq \zeta_1(0, c, B_0')$.*

By Proposition 2, it is sufficient to show that $\zeta_1(0, \underline{c}, B_0 = 1/2) < 1$. As ζ_1 is increasing in B_0 and $B_0 \leq 1/2$, from $\zeta_1(0, \underline{c}, 1/2) < 1$ it follows that $\zeta_1(0, \underline{c}, B_0) < 1 \; \forall \; B_0 > B^*$. Ub_2 is an upper bound of $\zeta_1(0)$ and it therefore holds that $\zeta_1(0, \underline{c}, 1/2) < Ub_2(s, \underline{c}, \alpha, \beta, 1/2)$. As $\underline{c}$ is a function of α and β, $Ub_2(s, \underline{c}, \alpha, \beta, 1/2)$ is reduced to a function of s, α and β. I denote it by $Ub_2'(s, \alpha, \beta)$. I use Mathematica to evaluate the inequality $Ub_2'(s, \alpha, \beta) < 1$. The most complex case the computer can handle is $s = 3$. I obtain the result that for $\alpha \in (0.5, 0.983)$ the inequality holds. Therefore $\zeta_1(0, \underline{c}, 1/2) < 1$ and from Proposition 2 it follows that $\zeta_1(0, \underline{c}, B_0) < 1 \; \forall B_0 > B^*$. As $\zeta_1(0, c)$ is increasing in c, $\underline{c} < \overline{c}'$. If $\alpha \in (0.5, 0.983)$ and $B_0 > B^*$ there exists an interval $[\underline{c}, \overline{c}']$ s.t. for all c in that interval the low regime is an equilibrium.

Let me summarize the results:

Result 8 *The low regime is an equilibrium and converges to π^* if and only if:*

- *Case 1: $\alpha = 1/2$ and $c \in [4\rho, 4\rho(3 + 4\rho)]$ for all B_0.*
- *Case 2: $0 \leq B_0 < B^*$, $0 < \alpha < 1/2$ and $c \in C = [\underline{c}, \overline{c}']$. C exists, $\underline{c} < \overline{c}' < \overline{c}$ and $\overline{c}'$ can be obtained numerically by solving $\zeta_1(0, \overline{c}') = 1$.*
- *Case 3: $1/2 \geq B_0 > B^*$, $0 < \alpha < 1/2$ and $c \in C = [\underline{c}', \overline{c}]$. C exists, $\underline{c} < \underline{c}' < \overline{c}$ and $\underline{c}'$ can be obtained numerically by solving $\zeta_0(0, \underline{c}') = 1$.*

The low regime is an equilibrium and converges to π^* if but not only if:

- *Case 4: $0 \leq B_0 < B^*$, $0.988 > \alpha > 1/2$ and $c \in C = [\underline{c}', \overline{c}]$. C exists, $\underline{c} < \underline{c}' < \overline{c}$ and $\underline{c}'$ can be obtained numerically by solving $\zeta_0(0, \underline{c}') = 1$.*

- *Case 5:* $1/2 \geq B_0 > B^*$, $0.983 > \alpha > 1/2$ *and* $c \in C = [\underline{c}, \overline{c}']$*.* C *exists,* $\underline{c} < \overline{c}' < \overline{c}$ *and* $\overline{c}'$ *can be obtained numerically by solving* $\zeta_1(0, \overline{c}') = 1$*.*

For all other α*, I cannot prove analytically that* C *exists. However, it exists for several cases which I computed numerically. Given a special* α *and* β *of interest not covered above,* $\underline{c}'$ *or* $\overline{c}'$ *can be obtained numerically to check whether* C *exists. If it exists, the low regime is an equilibrium if and only if* $c \in C$*.*

As $\underline{c}$*,* $\overline{c}$*,* $\underline{c}'$ *and* $\overline{c}'$ *depend on* B_0*,* α *and* β*,* C *depends on* B_0*,* α *and* β*.*

5 The Transition Between the High and Low Regime

5.1 From the High to the Low Regime

Suppose we are in the steady state of the high regime. Debt and unemployment are equal to 0. Define the actual debt level as $B_0 = 0$. Use case 1, case 2 and case 4 from Result 8 to obtain the following result:

Result 9 *If* $\alpha \in (0, 0.988)$ *there exists a respective interval* $C(\alpha)$ *depending on* α*, such that for all* c *in that interval, there exists an equilibrium path from the high regime steady state to the low regime steady state.*

From Result 2 it follows that there exists a second equilibrium path on which full employment is maintained forever. Which path the economy follows depends on the expectations of the agents. If all agents expect the full employment equilibrium to continue, it will continue, but if all agents expect convergence to the unemployment steady state, there will actually be convergence to the unemployment steady state. Chamley derives the same result in his paper, but only for $\alpha = 1/2$ or α close to $1/2$. The existence of an equilibrium path from the high to the low regime steady state does not depend on this special assumption. The path exists for most α. Chamley's result is robust to changes in α. Remember, α is the idiosyncratic probability of a negative preference shock. This result allows me to compare economies with different probabilities of preference shocks. Take two economies with different values of α. Suppose c in each of the two economies is in the respective interval $C(\alpha)$, such that the transition from high to low is an equilibrium. The high regime steady state is the same in both economies. The unemployment rate in the low regime steady state depends on α and is therefore not the same in the two economies. A very small α has severe consequences if preference shocks have the right intensity c. It leads to very high levels of unemployment in steady state as you can see in Figure 4. If α goes to zero, π^* goes to one. If α is low, the fraction of agents who consume in the low regime steady state is low. The loss in demand caused by pessimistic expectations is substantial, which leads to high unemployment rates. On the other hand, the higher α is, the lower the level of unemployment in steady state. Although preference shocks are responsible for the existence of an equilibrium with unemployment in the first place, a high probability of such shocks leads to less unemployment in steady state. Remember, saving is very costly for high type agents, which induces them to consume whenever possible. High type agents have a high propensity to consume. If there is a high fraction α of high types, there is a high fraction of agents consuming in the low regime steady state. The loss in aggregate demand is not so great, and unemployment, although always positive, is not so high. What is harmful is if there is a high fraction of agents for whom saving is not very costly. This result supports the well known "Keynesian" claim that a high fraction of households with a high propensity to consume stabilizes recessions. Rare preference shocks can lead to deeper recessions than frequent preference shocks. Note that the comparison between two such economies is not ceteris paribus, as the interval C varies with α.

A ceteris paribus comparison is also possible. Consider an economy with a probability of individual preference shocks equal to $\tilde{\alpha}$. Suppose the economy is in the high regime steady state. Debt is equal to zero. For all c in a certain interval $C(\tilde{\alpha})$ there exists an equilibrium path to the low regime steady state. Consider a certain $\tilde{c}$ in the interior of $C(\tilde{\alpha})$. It holds that $\zeta_0(t) > 1$ and $\zeta_1(t) < 1$ for all t. $\zeta_0(t)$ and $\zeta_1(t)$ are continuous functions of α. Therefore, holding $\tilde{c}$ constant, there exists an open interval I around $\tilde{\alpha}$ such that for all α in I, $\zeta_0(t) > 1$ and $\zeta_1(t) < 1$. The equilibrium conditions are still fulfilled for all α in I. One can compare two economies with the same $\tilde{c}$ and different α, but both α in I, ceteris paribus. The results from above still hold. Higher values of α lead to higher steady state unemployment in the low regime. The interval I can be very large as I will show in the next figure. Figure 5 shows an approximation of $C(\alpha)$ for different α and a certain β, which I set to $\beta = 0.99$. The interval you see in the figure, which I call $C_A(\alpha)$, is not the whole interval $C(\alpha)$, but an approximation I derived with the help of the upper and lower bounds stated above. It is not possible to draw $C(\alpha)$ as it has no closed analytical form. $C_A(\alpha)$ is a subset of $C(\alpha)$: $C_A(\alpha) \subset C(\alpha)$. It therefore holds that the transition is possible for all c in $C_A(\alpha)$.

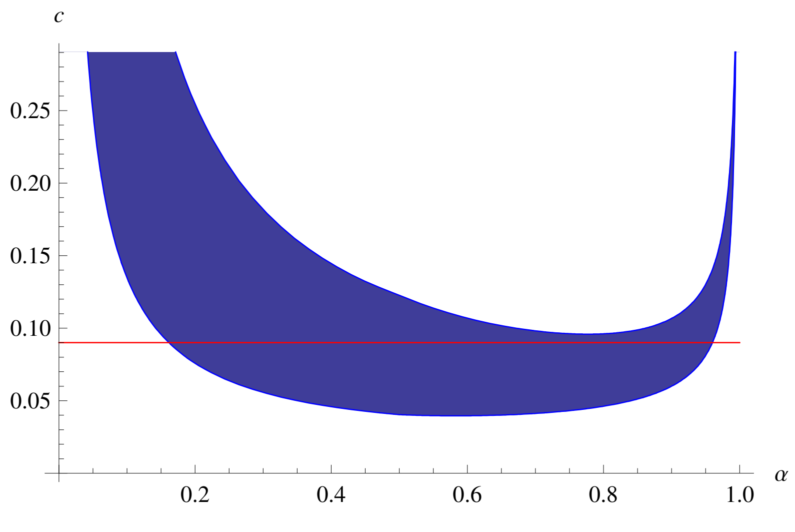

Figure 5: C_A as a Correspondence of α

The correspondence $C_A(\alpha)$ is the blue area. The red line shows $\tilde{c} = 0.09$. As you can see, $\tilde{c} = 0.09$ is in C_A for a large interval of α. The interval is approximately $I = (0.15, 0.95)$. Suppose we have two economies. Preference shocks have the same intensity $\tilde{c} = 0.09$ in both economies. One economy faces rare shocks of probability $\alpha = 0.15$, the other faces frequent shocks of probability $\alpha = 0.95$. In both economies there exists an equilibrium path from high to low as $c \in C(\alpha)$ respectively. The low regime steady state in the economy with rare shocks is characterized by a substantially higher unemployment rate.

5.2 From the Low to the High Regime: Savings Traps

In this section I will show under which conditions a transition from the low to the high regime is not possible. Suppose therefore the economy started in the steady state of the high regime. B_0 was equal to zero, c fulfills the conditions stated in case 1, 2 or 4 from Result 8, and by a sudden switch in expectations the economy ended up in the steady state of the low regime. I will now show that under certain conditions there is no equilibrium path back to the full employment steady state. The economy can be caught in a savings trap.

From Result 2 it follows that there exists a certain c^* and $\hat{B}$, such that for $c > c^*$ and $B_0 > \hat{B}$ the high regime consumption function is not an equilibrium. Take B^*, the steady state level of debt in the low regime, as the new B_0. I will show that for such a B_0 there exists a c_T, s.t. for all $c > c_T$ the high regime consumption function is not optimal. This means that from the steady state of the low regime there exists no equilibrium path to the full employment steady state. Recall Equation (3.10), the marginal utility of saving in the high regime. From now on I will use the subscripts H and L to distinguish between the marginal utility of saving in the high and low regime:

$$\zeta_0^H(0) = (1 + \alpha c)\beta[\sum_{i=0}^{\infty}(B_i\beta^i \prod_{j=0}^{i-1}(1 - B_j))].$$

Note that I use $\prod_j^i = 1$ if $i < j$.
Recall Equation (3.3), the evolution of debt in the high regime:

$$B_{t+1} = B_t(1 - B_t).$$

By repeated iterations of (3.3) one can express B_t as

$$B_t = B_0 \prod_{j=0}^{t-1}(1 - B_j). \tag{5.1}$$

Use (5.1) in (3.10) to obtain that

$$\zeta_0^H(0) = (1 + \alpha c)\beta[\sum_{i=0}^{\infty}(B_0\beta^i \prod_{j=0}^{i-1}(1 - B_j) \prod_{j=0}^{i-1}(1 - B_j))]$$

$$= (1+\alpha c)\beta B_0[\sum_{i=0}^{\infty}(\beta^i \prod_{j=0}^{i-1}(1-B_j)^2)].$$

Note that $(1-B_0) < (1-B_j)\ \forall j$ as B_t is decreasing in t in the high regime, and therefore it follows that

$$\begin{aligned}\zeta_0^H(0) &> (1+\alpha c)\beta B_0[\sum_{i=0}^{\infty}(\beta^i \prod_{j=0}^{i-1}(1-B_0)^2)] \\ &= (1+\alpha c)\beta B_0[\sum_{i=0}^{\infty}(\beta^i(1-B_0)^{2i})] \\ &= \frac{(1+\alpha c)\beta B_0}{1-\beta(1-B_0)^2}. \end{aligned} \tag{5.2}$$

If expression (5.2) is greater than one, $\zeta_0^H(0)$ is greater than one. (5.2) greater than one is equivalent to:

$$\frac{(1+\alpha c)\beta B_0}{1-\beta(1-B_0)^2} > 1$$

$$\Leftrightarrow$$

$$c > \underbrace{\frac{1-\beta+B_0\beta-B_0^2\beta}{B_0\alpha\beta}}_{c_T}.$$

Define the last expression as c_T. For $c > c_T$ it holds that $\zeta_0^H(0, c, B_0) > 1$. If $\zeta_0^H(0) > 1$, the high regime consumption function is not optimal. For all $c > c_T$, the high regime consumption function is not optimal and no equilibrium path from B_0 to the full employment steady state exists.

I consider the following three distinct cases to examine the transition:

Case 1, $\alpha = 1/2$: this is the case which Chamley also examines. Transition from high to low is possible if $c \in [4\rho, 4\rho(3+4\rho)]$, with $\rho = (1-\beta)/\beta$, as shown in Result 8. Chamley shows that if $c \in [6+8\rho, 4\rho(3+4\rho)]$ the transition from high to low is an equilibrium, but no equilibrium path back to full employment emerges (Chamley 2014 p. 667). The interval $[6+8\rho, 4\rho(3+4\rho)]$ only exists if $\beta < 2/3$, as only then $6+8\rho < 4\rho(3+4\rho)$.

I can generalize this result slightly. If $\alpha = 1/2$ it is true that $B^* = 1/4$. Set $B_0 = 1/4$ to obtain c_T: $c_T = 8\rho + 3/2$. Therefore, a transition back to full employment is not possible if $c \in [8\rho + 3/2, 4\rho(3+4\rho)]$. This interval exists for $\beta < 0.82$. I obtain the following result: if $\alpha = 1/2$ and $\beta < 0.82$ there exists an interval $[8\rho + 3/2, 4\rho(3+4\rho)]$, s.t. for all c in that interval, there is an equilibrium path from the full employment steady state to the unemployment steady state, but no equilibrium path back to full employment.

Case 2, $\alpha \in (0, 1/2)$: transition from high to low is possible if $c \in [\underline{c}, \overline{c}']$. Suppose we are in the low regime steady state. Recall (4.7) to obtain the level of debt $B^*(\alpha)$. Set $B_0 = B^*(\alpha)$ and to obtain c_T. c_T is a complicated expression depending on α and β. With the help of Mathematica I can show that $c_T > \underline{c}$. Remember, an analytical expression for $\underline{c}$ is available from Equation (4.16). But is $c_T < \overline{c}'$? There is no analytical expression for $\overline{c}'$. Remember $\overline{c}'$ is defined as $\zeta_1^L(0, \overline{c}', 0) = 1$. If $\zeta_1^L(0, c_T, 0) < 1$ it follows that $c_T < \overline{c}'$. I show that $\zeta_1^L(0, c_T, B_0 = 0) < 1$ with the help of Ub_2 defined in Equation (4.25), the upper bound for ζ_1^L I used in case 5 of chapter 4.2.2. Remember that $\zeta_1^L(0, c, B_0) < Ub_2(s, c, \alpha, \beta, B_0)$, therefore it holds that $\zeta_1^L(0, c_T, 0) < Ub_2(s, c_T, \alpha, \beta, 0)$. I evaluate the inequality $Ub_2(s, c_T, \alpha, \beta, 0) < 1$ with the help of Mathematica. The highest s the computer can handle in this case is $s = 1$. I can show that $Ub_2(1, c_T, \alpha, \beta, 0) < 1$, if $\beta < 0.78$. Therefore $c_T < \overline{c}'$ if $\beta < 0.78$. If $\alpha < 1/2$ and $\beta < 0.78$ there exists an interval $[c_T, \overline{c}']$, s.t. for all c in that interval, there is an equilibrium path from the full employment steady state to the unemployment steady state, but no equilibrium path back to full employment.

Case 3, $\alpha \in (1/2, 0.988)$: transition from high to low is possible if $c \in [\underline{c}', \overline{c}]$. Compute again c_T for $B_0 = B^*(\alpha)$. With the help of Mathematica I can show that $c_T < \overline{c}$ if $\beta < 0.82$. Remember that an analytical expression of $\overline{c}$ is available from Equation (4.17). If $\alpha \in (1/2, 0.988)$ and $\beta < 0.82$ there exists an interval $[\underline{c}', \overline{c}] \cap [c_T, \overline{c}]$, s.t. for all c in that interval there is an equilibrium path from the full employment steady state to the unemployment steady state, but no equilibrium path back to full employment.

Let me summarize the three cases:

Result 10 *If $\alpha \in (0, 0.988)$ and $\beta < 0.78$ there exists a respective interval $C_T(\alpha)$ depending on α, such that for all c in that interval there exists an equilibrium path from the full employment steady state to the unemployment steady state but no equilibrium path back to the full employment steady state.*

I can extend Chamley's result about the possible existence of savings traps for cases where $\alpha \neq 1/2$. Again this shows that his findings are robust. Under certain conditions precautionary savings are responsible for the existence of an equilibrium path with unemployment. In addition to that, they can lead to a trap which makes the transition back to full employment impossible. As I have shown, this is true for nearly all possible values of α. What is the intuition behind the savings trap? Consider the case where the economy is in the steady state of the low regime. If the regime changes, low type agents in state zero consume. Why is this behavior not optimal under certain conditions even though it would bring the economy back to full employment? After a regime change there are still credit constrained agents in state -1 who cannot consume. The consumption of state zero agents might not create enough aggregate demand in order to make consuming optimal for themselves. This is especially true if agents discount the future. It will take some time to bring debt down, even if all state 0 agents consume.

During this time there are still some credit constrained agents who are not consuming. Therefore, in the near future agents in state 0 face the risk of making no sale, moving to state -1 and receiving the penalty c. If this near future is valuable (β is not to high) it is not optimal to follow this path, even though it is clear that following the path long enough would lead back to full employment.

The comparison of economies with different α is the same as in the previous section. The results of such a comparison are the same. Ceteris paribus comparisons are again possible among certain α in an open interval. This follows from the same argument as above.

6 Conclusion

Decentralized markets in the form of a random matching process, imperfect credit markets and idiosyncratic preference shocks can lead to multiple equilibrium paths of an economy. One path is characterized by full employment, no aggregate debt and no precautionary savings. The other path is characterized by equilibrium unemployment, precautionary savings and debt. If preference shocks have the right intensity there is an equilibrium path from the full employment to the unemployment steady state, but the reverse is not true. The economy can shift into the inefficient steady state by a switch in agents' expectations. These findings, already shown by Chamley, are robust to changes in the probability of idiosyncratic shocks. The results are true for almost all possible probabilities of shocks. Economies where shocks are frequent face less unemployment in the inefficient steady state than economies where shocks are rare.

A Appendix

A.1 Convergent Solutions of Difference Equations

Theorem 1 *Consider a difference equation of the form*

$$\zeta(t) = v(t) + w(t)\zeta(t+1)$$

with $\lim_{t\to\infty} v(t) = v^*$ *and* $\lim_{t\to\infty} w(t) = w^*$*. If it holds that* $0 < w(t) < 1$*,* $v(t) \geq 0$ $\forall t$ *and* $0 < w^* < 1$*, then there exists a unique convergent solution* $\zeta(t)$ *with*

$$\lim_{t\to\infty} \zeta(t) = \zeta^* = v^*/(1-w^*).$$

The convergent solution is equal to

$$\zeta(t) = v(t) + \lim_{s\to\infty} \sum_{i=t+1}^{s-1} v(i) \prod_{j=t}^{i-1} w(j).$$

Proof:
Consider the difference equation

$$\zeta(t) = v(t) + w(t)\zeta(t+1). \tag{A.1}$$

By repeated iterations of (A.1), $\zeta(t)$ is equal to

$$\zeta(t) = v(t) + \sum_{i=t+1}^{s-1} v(i) \prod_{j=t}^{i-1} w(j) + \zeta(s) \prod_{j=t}^{s-1} w(j), \qquad \forall s > t. \tag{A.2}$$

The above holds especially for $s \to \infty$:

$$\zeta(t) = v(t) + \lim_{s\to\infty} \sum_{i=t+1}^{s-1} v(i) \prod_{j=t}^{i-1} w(j) + \lim_{s\to\infty} \zeta(s) \prod_{j=t}^{s-1} w(j). \tag{A.3}$$

All solutions of (A.1) must fulfill Equation (A.3).

Consider the following guess at a solution:

$$\zeta'(t) = v(t) + \lim_{s\to\infty} \sum_{i=t+1}^{s-1} v(i) \prod_{j=t}^{i-1} w(j). \tag{A.4}$$

I will first show that the guess $\zeta'(t)$ exists. This is not clear, as the formula for $\zeta'(t)$ contains a limit where s goes to infinity. Then I will show that $\zeta'(t)$ converges for $t \to \infty$. Finally I will show that $\zeta'(t)$ fulfills Equation (A.3) and is therefore a solution of the difference equation (A.1).

Consider the following set: $V_t = \{v(i)|i \geq t\}$. This set contains all elements of the sequence $v(i)$ that follow a certain element $v(t)$. The set V_t has an infimum and a supremum, as the sequence $v(i)$ converges. I use the following notation: $v_{\text{sup}}(t) = \sup V_t$ and $v_{\text{inf}}(t) = \inf V_t$. Define $w_{\text{sup}}(t)$ and $w_{\text{inf}}(t)$ equivalently for the sequence $w(i)$. Note that $\lim_{t\to\infty} v_{\text{inf}}(t) = \lim_{t\to\infty} v_{\text{sup}}(t) = v^*$. The equivalent holds for the limit of $w_{\text{sup}}(t)$ and $w_{\text{inf}}(t)$, which is equal to w^*.

I construct the upper bound for $\zeta'(t)$ with the help of these new definitions:

$$\begin{aligned}\zeta'(t) = v(t) + \lim_{s\to\infty} \sum_{i=t+1}^{s-1} v(i) \prod_{j=t}^{i-1} w(j) &\leq \underbrace{v_{\text{sup}}(t) + \lim_{s\to\infty} \sum_{i=t+1}^{s-1} v_{\text{sup}}(t) \prod_{j=t}^{i-1} w_{\text{sup}}(t)}_{v(i) \text{ and } w(j) \text{ replaced by their suprema}} \\ &= v_{\text{sup}}(t) \sum_{i=0}^{\infty} (w_{\text{sup}}(t))^i. \end{aligned} \tag{A.5}$$

In the first step above I replaced $v(i)$ and $w(j)$ by the supremum of V_t and W_t respectively. All $v(i)$ and $w(j)$ in the definition of $\zeta'(t)$ are elements of the set V_t and W_t respectively. If I replace them by their suprema, the new expression is at least as great as $\zeta'(t)$. In the second step above I simplify the expression. Use the formula for the limit of the geometric series to obtain that

$$\zeta'(t) \leq v_{\text{sup}}(t) \sum_{i=0}^{\infty} (w_{\text{sup}}(t))^i = \frac{v_{\text{sup}}(t)}{1 - w_{\text{sup}}(t)}. \tag{A.6}$$

I use the derived upper bound to show that $\zeta'(t)$ is well defined and exists. Each $\zeta'(t) = v(t) + \lim_{s\to\infty} \sum_{i=t+1}^{s-1} v(i) \prod_{j=t}^{i-1} w(j)$ has an upper bound. The sequence $v(t) + \sum_{i=t+1}^{s-1} v(i) \prod_{j=t}^{i-1} w(j)$ is increasing in s, as all $v(i)$ and $w(j)$ are positive. The limit of this sequence for $s \to \infty$ (which is the definition of $\zeta'(t)$) does therefore exist.

In the next step I calculate the limit $\lim_{t\to\infty} \zeta'(t)$ and show that it exists. In order to do that I construct a lower bound of $\zeta'(t)$ in the same way as I constructed the upper bound above. I use the defined infima of V_t and W_t instead of the suprema:

$$\begin{aligned}\zeta'(t) = v(t) + \lim_{s\to\infty} \sum_{i=t+1}^{s-1} v(i) \prod_{j=t}^{i-1} w(j) &\geq v_{\text{inf}}(t) + \lim_{s\to\infty} \sum_{i=t+1}^{s-1} v_{\text{inf}}(t) \prod_{j=t}^{i-1} w_{\text{inf}}(t) \\ &= v_{\text{inf}}(t) \sum_{i=0}^{\infty} (w_{\text{inf}}(t))^i = \frac{v_{\text{inf}}(t)}{1 - w_{\text{inf}}(t)}. \end{aligned} \tag{A.7}$$

We already know the limit of the upper and lower bound for $t \to \infty$. It is the same for both bounds. I use the sandwich theorem to obtain the limit of $\zeta'(t)$:

$$\frac{v_{\text{inf}}(t)}{1 - w_{\text{inf}}(t)} \leq \zeta'(t) \leq \frac{v_{\text{sup}}(t)}{1 - w_{\text{sup}}(t)}. \tag{A.8}$$

Taking t to infinity on both sides yields

$$\begin{aligned}
\lim_{t\to\infty}\frac{v_{\inf}(t)}{1-w_{\inf}(t)} \leq & \lim_{t\to\infty}\zeta'(t) & \leq \lim_{t\to\infty}\frac{v_{\sup}(t)}{1-w_{\sup}(t)} \\
\frac{v^*}{1-w^*} \leq & \lim_{t\to\infty}\zeta'(t) & \leq \frac{v^*}{1-w^*} \\
\Rightarrow & \lim_{t\to\infty}\zeta'(t) & = \frac{v^*}{1-w^*} = \zeta^{*\prime}.
\end{aligned} \tag{A.9}$$

The solution guess $\zeta'(t)$ converges. But is it a solution of the difference equation (A.1)? To show that $\zeta'(t)$ is a solution of (A.1), it is sufficient to show that it fulfills Equation (A.3). Reconsider Equation (A.3) and the definition of $\zeta'(t)$ from Equation (A.4):

$$\text{Eq.(A.3) revisited:} \qquad \zeta(t) = \underbrace{v(t) + \lim_{s\to\infty}\sum_{i=t+1}^{s-1} v(i)\prod_{j=t}^{i-1} w(j)}_{=\zeta'(t)} + \underbrace{\lim_{s\to\infty}\zeta(s)\prod_{j=t}^{s-1} w(j)}_{=0 \text{ for } \zeta'(t)},$$

$$\text{Eq.(A.4) revisited:} \qquad \zeta'(t) = v(t) + \lim_{s\to\infty}\sum_{i=t+1}^{s-1} v(i)\prod_{j=t}^{i-1} w(j).$$

It is easy to see that the guess (A.4) fulfills (A.3) as the first part of (A.3) is the same as $\zeta'(t)$. The second part of (A.3) evaluated for $\zeta'(t)$ is zero. To see that use the limit of $\zeta'(t)$ in the second part:

$$\lim_{s\to\infty}\zeta'(s)\prod_{j=t}^{s-1} w(j) = \zeta^{*\prime}\underbrace{\prod_{j=t}^{\infty} w(j)}_{=0} = \zeta^{*\prime}\times 0 = 0, \tag{A.10}$$

$$\prod_{j=t}^{\infty} w(j) = 0 \text{ as } w(j) < 1\ \forall j.$$

It is true that $\zeta'(t)$ is a convergent solution of Equation (A.1). But the Theorem is even stronger. It says that $\zeta'(t)$ is the unique convergent solution. In the final step I show the uniqueness of $\zeta'(t)$ by contradiction.

Suppose there exists a convergent solution to Equation (A.1) $\hat{\zeta}(t)$, with $\lim_{t\to\infty}\hat{\zeta}(t) = \hat{\zeta}^*$ and $\hat{\zeta}(t) \neq \zeta'(t)$. As $\hat{\zeta}(t)$ is a solution of Equation (A.1) it fulfills Equation (A.3):

$$\hat{\zeta}(t) = v(t) + \lim_{s\to\infty}\sum_{i=t+1}^{s-1} v(i)\prod_{j=t}^{i-1} w(j) + \lim_{s\to\infty}\hat{\zeta}(s)\prod_{j=t}^{s-1} w(j).$$

Use the limit of $\hat{\zeta}(t)$ to derive a contradiction:

$$\hat{\zeta}(t) = v(t) + \lim_{s\to\infty}\sum_{i=t+1}^{s-1} v(i)\prod_{j=t}^{i-1} w(j) + \underbrace{\hat{\zeta}^*\prod_{j=t}^{\infty} w(j)}_{=0} = v(t) + \lim_{s\to\infty}\sum_{i=t+1}^{s-1} v(i)\prod_{j=t}^{i-1} w(j) = \zeta'(t).$$

This is a contradiction to $\hat{\zeta}(t) \neq \zeta'(t)$. $\zeta'(t)$ as defined in Equation (A.4) is the unique convergent solution of Equation (A.1).

q.e.d.

A.2 Monotonicity of Difference Equations

Theorem 2 *Consider a difference equation of the form*

$$\zeta(t) = v(t) + w(t)\zeta(t+1)$$

with $\lim_{t\to\infty} v(t) = v^*$, $\lim_{t\to\infty} w(t) = w^*$, $0 < w(t) < 1$, $v(t) \geq 0$ *and* $0 < w^* < 1$. *Use the following notation:*

$$br(t) = \frac{v(t)}{1 - w(t)}.$$

By Theorem 1 the difference equation has a unique convergent solution $\zeta(t)$. *Consider only the convergent solution.*
a.i) If $br(t) > br(t+1)$ $\forall t$ *it follows that* $\zeta(t) > \zeta(t+1) > \zeta^*$ $\forall t$.
a.ii) If $\zeta(t) > \zeta(t+1)$ *it follows that* $\zeta(t) < br(t)$.
b.i) If $br(t) < br(t+1)$ $\forall t$ *it follows that* $\zeta(t) < \zeta(t+1) < \zeta^*$ $\forall t$.
b.ii) If $\zeta(t) < \zeta(t+1)$ *it follows that* $\zeta(t) > br(t)$.

Proof:

Just consider part a). Part b) follows analogously. I begin with the proof of a.i). I divide the proof into several steps showing five intermediary results. All premises of the theorem are assumed to hold throughout the whole proof.

The following relation plays a central role in the proof:

$$\zeta(t) > \zeta(t+1) \Leftrightarrow br(t) > \zeta(t+1). \tag{A.11}$$

To see that it is true use the definition of $br(t)$ and $\zeta(t)$:

$$\begin{aligned} \zeta(t) &> \zeta(t+1) && \text{/use the definition of } \zeta(t) \\ v(t) + w(t)\zeta(t+1) &> \zeta(t+1) && \text{/move all } \zeta(t+1) \text{ to the right} \\ \frac{v(t)}{1-w(t)} &> \zeta(t+1) && \text{/use the definition of } br(t) \\ br(t) &> \zeta(t+1). \end{aligned}$$

Note that

$$\lim_{t\to\infty} br(t) = \zeta^* \quad \text{and} \quad br(t) > \zeta^* \ \forall t. \tag{A.12}$$

The first part of (A.12) follows from Theorem 1, which gives the limes of $\zeta(t)$:

$$\lim_{t\to\infty} br(t) = \lim_{t\to\infty} \frac{v(t)}{1-w(t)} = \frac{v^*}{1-w^*} = \zeta^*.$$

The second part of (A.12) is true, as $br(t)$ is decreasing in t and converges to ζ^*.

I use (A.11) and (A.12) to prove the following result:

Result A.1 $\forall \epsilon > 0$ *if* $\zeta(t+1) \in (\zeta^* - \epsilon, \zeta^* + \epsilon)$*, then* $\zeta(t) > \zeta^* - \epsilon$*.*

The result says that if $\zeta(t+1)$ is in an epsilon interval around the limit ζ^*, then the prior element $\zeta(t)$ is greater than the lower bound of that interval.

Proof Result A.1:

Consider two distinct cases. In case 1 $\zeta(t+1)$ is in the lower half of the epsilon interval, in case 2 $\zeta(t+1)$ is in the upper half.

Case 1: $\zeta^* - \epsilon < \zeta(t+1) \leq \zeta^*$

$$\begin{aligned} \zeta(t+1) &\leq \zeta^* && \text{/use } \zeta^* < br(t) \text{ from Relation (A.12)} \\ \zeta(t+1) &< br(t) && \text{/use Relation (A.11)} \\ \zeta(t+1) &< \zeta(t) && \text{/use } \zeta^* - \epsilon < \zeta(t+1) \\ \zeta^* - \epsilon &< \zeta(t) \end{aligned}$$

The result holds in case 1. Consider case 2:

Case 2: $\zeta^* < \zeta(t+1) < \zeta^* + \epsilon$
Subtract ϵ from both inequalities to obtain that

$$\zeta^* - \epsilon < \zeta(t+1) - \epsilon < \zeta^*. \tag{A.13}$$

Use Relation (A.12) to obtain that

$$\zeta(t+1) - \epsilon < \underbrace{\zeta^* < br(t)}_{\text{from (A.12)}}.$$

Use the definition of $br(t)$ and rearrange to obtain that

$$\begin{aligned} \zeta(t+1) - \epsilon &< \underbrace{\frac{v(t)}{1 - w(t)}}_{br(t)} && /(1 - w(t)) \\ \zeta(t+1) - w(t)\zeta(t+1) - \epsilon(1 - w(t)) &< v(t) && / + w(t)\zeta(t+1) \\ \zeta(t+1) - \epsilon(1 - w(t)) &< \underbrace{v(t) + w(t)\zeta(t+1)}_{\zeta(t)} \\ \zeta(t+1) - \epsilon(1 - w(t)) &< \zeta(t) \end{aligned}$$

It holds that $\zeta(t+1) - \epsilon < \zeta(t+1) - \epsilon(1 - w(t))$ as $(1 - w(t)) < 1$. Combine this with last lines above to see that $\zeta(t+1) - \epsilon < \zeta(t)$. From (A.13) it follows that $\zeta^* - \epsilon < \zeta(t+1) - \epsilon$. Therefore it holds that $\zeta^* - \epsilon < \zeta(t)$. The result holds in case 2.

Result A.2 *If* $\zeta(t+1) \geq \zeta(t+2) \Rightarrow \zeta(t) > \zeta(t+1)$.

This result is very powerful for inductions. It says that once one finds two elements of the sequence where $\zeta(t+1) \geq \zeta(t+2)$, for the prior element it holds that $\zeta(t) > \zeta(t+1)$.

Proof Result A.2:

$$
\begin{aligned}
\zeta(t+1) &\geq \zeta(t+2) && \text{/multiply by } w(t+1) \text{ and add } v(t+1)\\
v(t+1) + w(t+1)\zeta(t+1) &\geq \underbrace{v(t+1) + w(t+1)\zeta(t+2)}_{\zeta(t+1)} &&\\
v(t+1) + w(t+1)\zeta(t+1) &\geq \zeta(t+1) && \text{/move all } \zeta(t+1) \text{ to the right}\\
\underbrace{\frac{v(t+1)}{1-w(t+1)}}_{br(t+1)} &\geq \zeta(t+1) &&\\
br(t+1) &\geq \zeta(t+1) && \text{/use } br(t) > br(t+1)\\
br(t) &> \zeta(t+1) && \text{/use Relation (A.11)}\\
\zeta(t) &> \zeta(t+1). &&
\end{aligned}
$$

Result A.2 is true.

Result A.1 and A.2 will be helpful to obtain the next result.

Result A.3 $\forall \epsilon > 0 \quad \exists t_\epsilon$ *s.t.*
i) $\zeta(t_\epsilon) \in (\zeta^* - \epsilon, \zeta^* + \epsilon)$ *and* $\forall t \geq t_\epsilon,\ \zeta(t) \in (\zeta^* - \epsilon, \zeta^* + \epsilon)$,
ii) if $t_\epsilon \neq 0$ *it follows that* $\zeta(t_\epsilon - 1) \geq \zeta^* + \epsilon$ *and therefore* $\zeta(t_\epsilon - 1) > \zeta(t_\epsilon)$,
iii) $\forall t \leq t_\epsilon - 1 \quad \zeta(t-1) > \zeta(t)$ *and* $\zeta(t) \geq \zeta^* + \epsilon$.

Part i) says that for every epsilon interval around the limit ζ^* one can find an element of $\zeta(t)$ which I call $\zeta(t_\epsilon)$. This element and all following elements are in the epsilon interval around ζ^*.

Part ii) says that the prior element $\zeta(t_\epsilon - 1)$ is above the epsilon interval. It follows that $\zeta(t_\epsilon - 1) > \zeta(t_\epsilon)$. If the epsilon is too large, all elements of $\zeta(t)$ are in the epsilon interval. $\zeta(t_\epsilon - 1)$ does not exist in this case as $t_\epsilon = 0$. Here, part ii) makes no statement. However, this case does not concern us as I will use the result later for cases where $(t_\epsilon - 1)$ exists.

Part iii) makes ii) even stronger. Not only is $\zeta(t_\epsilon - 1) > \zeta(t_\epsilon)$, but all prior elements are strictly decreasing. You may have already guessed that this follows from Result A.2, given that $\zeta(t_\epsilon - 1) > \zeta(t_\epsilon)$. Of course iii) holds only if such prior elements exist. Again this will not bother us later.

Proof Result A.3:

As I am referring to the convergent solution, it is true that

$$\forall \epsilon > 0 \quad \exists t'_\epsilon \quad s.t. \quad \forall t \geq t'_\epsilon \quad \zeta(t) \in (\zeta^* - \epsilon, \zeta^* + \epsilon). \tag{A.14}$$

This is merely the definition of a convergent sequence. Let T_ϵ be the set of all these t'_ϵ. Define t_ϵ as the minimum of T_ϵ: $t_\epsilon = \min T_\epsilon$. t_ϵ exists as there exists at least one t'_ϵ and T_ϵ is bounded from below by 0. Part i) is true as t_ϵ fulfills (A.14). It holds that $\zeta(t_\epsilon - 1) \notin (\zeta^* - \epsilon, \zeta^* + \epsilon)$, otherwise t_ϵ would not be the minimum. By means of the careful definition of t_ϵ I find an element of $\zeta(t)$ such that this element $\zeta(t_\epsilon)$ is in the epsilon interval around ζ^*, but the prior element $\zeta(t_\epsilon - 1)$ is not. From Result A.1 it follows that $\zeta(t_\epsilon - 1) > \zeta^* - \epsilon$. Combining this with $\zeta(t_\epsilon - 1) \notin (\zeta^* - \epsilon, \zeta^* + \epsilon)$ yields $\zeta(t_\epsilon - 1) \geq \zeta^* + \epsilon$. As $\zeta(t_\epsilon) \in (\zeta^* - \epsilon, \zeta^* + \epsilon)$ it follows that $\zeta(t_\epsilon - 1) > \zeta(t_\epsilon)$. Part ii) is true.

Use $\zeta(t_\epsilon - 1) > \zeta(t_\epsilon)$ and Result A.2 to obtain that $\zeta(t_\epsilon - 2) > \zeta(t_\epsilon - 1)$. By induction it holds that $\zeta(t-1) > \zeta(t)$ $\forall t \leq t_\epsilon - 1$. As $\zeta(t_\epsilon - 1) \geq \zeta^* + \epsilon$ it holds that $\zeta(t) \geq \zeta^* + \epsilon$ $\forall t \leq t_\epsilon - 1$.

Result A.4 $\zeta^* \leq \zeta(t) \quad \forall t.$

Proof Result A.4:

I prove Result A.4 by contradiction. Suppose there exists a t_0 s.t. $\zeta(t_0) < \zeta^*$. Therefore there exists an $\epsilon > 0$ s.t. $\zeta(t_0) < \zeta^* - \epsilon$. By Result A.3 i) there exists a t_ϵ s.t. for all $t \geq t_\epsilon \quad \zeta(t) \in (\zeta^* - \epsilon, \zeta^* + \epsilon)$. As $\zeta(t_0)$ is smaller than $\zeta^* - \epsilon$ it follows that $t_0 < t_\epsilon$. From Result A.3 ii) or iii) it follows that $\zeta(t_0) \geq \zeta^* + \epsilon$. This is a contradiction to $\zeta(t_0) < \zeta^*$.

Result A.5 $\zeta^* < \zeta(t) \quad \forall t.$

Proof Result A.5:

I prove the result by contradiction. Suppose $\exists t_0$ s.t. $\zeta(t_0) = \zeta^*$. I have to consider two cases. In case 1 there exists a $t_1 > t_0$ s.t. $\zeta(t_1) > \zeta^*$. In case 2 $\zeta(t) = \zeta^*$ for all $t > t_0$. I do not have to consider cases where $\zeta(t) < \zeta^*$. These cases are ruled out by Result A.4.

Case 1: There exists a $t_1 > t_0$ s.t. $\zeta(t_1) > \zeta^*$. Therefore there exists an $\epsilon > 0$ s.t. $\zeta(t_1) > \zeta^* + \epsilon$. By Result A.3 i) there exists a t_ϵ s.t. for all $t \geq t_\epsilon \quad \zeta(t) \in (\zeta^* - \epsilon, \zeta^* + \epsilon)$. As $\zeta(t_1)$ is greater than $\zeta^* + \epsilon$ it follows that $t_1 < t_\epsilon$. As $t_0 < t_1$ it is true that $t_0 < t_\epsilon$. From Result A.3 ii) or iii) it follows that $\zeta(t_0) \geq \zeta^* + \epsilon$. This is a contradiction to $\zeta(t_0) = \zeta^*$.

Case 2: $\zeta(t) = \zeta^*$ for all $t > t_0$. Therefore it is true that $\zeta(t_0 + 1) = \zeta(t_0 + 2)$. From Result A.2 it follows that $\zeta(t_0) > \zeta(t_0 + 1)$. As $\zeta(t_0 + 1) = \zeta^*$ it follows that $\zeta(t_0) > \zeta^*$. This is a contradiction to $\zeta(t_0) = \zeta^*$.

I use Result A.5 to prove part a.i) of Theorem 2. Again I use a proof by contradiction. Remember, a.i) says $\zeta(t) > \zeta(t+1)$ $\forall t$.

Suppose there exists a t_0 s.t. $\zeta(t_0) \leq \zeta(t_0+1)$. By Result A.5 it is true that $\zeta(t_0+1) > \zeta^*$. Therefore there exists an $\epsilon > 0$ s.t. $\zeta(t_0+1) > \zeta^* + \epsilon$. By Result A.3 i) there exists a t_ϵ s.t. for all $t \geq t_\epsilon$ $\zeta(t) \in (\zeta^* - \epsilon, \zeta^* + \epsilon)$. As $\zeta(t_0+1)$ is greater than $\zeta^* + \epsilon$ it follows that $t_0 + 1 < t_\epsilon$ and therefore $t_0 < t_\epsilon$. From Result A.3 ii) or iii) it follows that $\zeta(t_0) > \zeta(t_0+1)$ which is a contradiction to $\zeta(t_0) \leq \zeta(t_0+1)$.

Part a.ii) which says $br(t) > \zeta(t)$ follows easily:

$$
\begin{aligned}
\zeta(t) &> \zeta(t+1) && \text{/multiply by } w(t) \text{ and add } v(t)\\
v(t) + w(t)\zeta(t) &> \underbrace{v(t) + w(t)\zeta(t+1)}_{\zeta(t)} \\
v(t) + w(t)\zeta(t) &> \zeta(t) && \text{/move all } \zeta(t) \text{ to the right}\\
\underbrace{\frac{v(t)}{1-w(t)}}_{br(t)} &> \zeta(t) \\
br(t) &> \zeta(t).
\end{aligned}
$$

q.e.d.

A.3 Propositions

Proposition 1 *Let α be in $(1/2, 1)$. If $B_0 < B_0'$ it follows that $\zeta_0(0, c, B_0) \leq \zeta_0(0, c, B_0')$.*

Proof:

Recall Equation (4.18):

$$\zeta_0(t) = \sum_{i=t}^{\infty} v_0(i) \prod_{j=t}^{i-1} w_0(j) \tag{A.15}$$

whereby

$$
\begin{aligned}
v_0(t) &= \beta(\pi_t \alpha(1+c) + b_t),\\
w_0(t) &= \beta a_t.
\end{aligned}
$$

Note that I use $\prod_j^i = 1$ if $i < j$.

Use the definitions of a_t and b_t from Equation (4.1) and (4.2) to obtain that

$$v_0(t) = v_0(t, \pi_t) = \beta(\underbrace{(2\alpha - 1 + \alpha c)}_{>0 \text{ as } \alpha > 1/2} \pi_t + 1 - \alpha), \tag{A.16}$$

$$w_0(t) = w_0(t, \pi_t) = \beta(\alpha + \underbrace{(1 - 2\alpha)}_{<0 \text{ as } \alpha > 1/2} \pi_t). \tag{A.17}$$

Recall Equation (4.6), which gives the evolution of debt:

$$B_{t+1} = P(B_t) = -(1-2\alpha)^2 B_t^2 + (1-2\alpha)^2 B_t + \alpha(1-\alpha).$$

Remember, $B_t \leq 1/2$ and as $P(B_t)$ is strictly increasing on $[0, 1/2]$,

$$B_0 < B_0' \Rightarrow B_t < B_t' \ \forall t.$$

Recall Equation (4.4):

$$\pi_t = \pi_t(B_t) = 1 - \alpha - (1-2\alpha)B_t.$$

Use Equation (4.6), (4.4), (A.16), (A.17) and (A.15) to see that $\zeta_0(t)$ is a function of B_t. In particular, $\zeta_0(0)$ is a function of B_0.

Define π_t' as $\pi_t' = \pi_t(B_t')$. Use Equation (4.4) to see that the following statement is true:

$$\text{if } B_0 < B_0' \text{ and } \alpha > 1/2 \Rightarrow B_t < B_t' \text{ and } (1-2\alpha) < 0 \Rightarrow \pi_t < \pi_t'. \tag{A.18}$$

Note that $\pi_t \leq 1/2$ as $\pi_t(B_t)$ is increasing in B_t and $B_t \leq 1/2$. Using (A.18) in Equation (A.16) and (A.17) yields

$$B_0 < B_0' \Rightarrow v_0(t) < v_0'(t) \text{ and } w_0(t) > w_0'(t),$$

where I define $v_0'(t)$ and $w_0'(t)$ as $v_0(t, \pi_t')$ and $w_0(t, \pi_t')$.

Define $f(T_1, T_2)$ as

$$f(T_1, T_2) = \sum_{i=T_1}^{T_2} v_0(i) \prod_{j=T_1}^{i-1} w_0(j).$$

Use Equation (4.6), (4.4), (A.16) and (A.17) to see that f is a function of B_{T_1}. Take an arbitrary T_2. I will show by induction that $f(T_1, T_2, B_{T_1}) < f(T_1, T_2, B_{T_1}') \ \forall T_1 < T_2$. First step: show that $f(T_2-1, T_2, B_{T_2-1}) < f(T_2-1, T_2, B_{T_2-1}')$:

$$\begin{aligned}
f(T_2-1, T_2, B_{T_2-1}) &< f(T_2-1, T_2, B_{T_2-1}') \\
&\Leftrightarrow \text{by the def. of } f \\
v_0(T_2-1) + v_0(T_2)w_0(T_2-1) &< v_0'(T_2-1) + v_0'(T_2)w_0'(T_2-1) \\
&\Leftarrow \text{ as } v_0(T_2) < v_0'(T_2) \\
v_0(T_2-1) + v_0'(T_2)w_0(T_2-1) &< v_0'(T_2-1) + v_0'(T_2)w_0'(T_2-1) \\
&\Leftrightarrow v_0'(T_2) \text{ to the left and as } w_0(T_2-1) > w_0'(T_2-1) \\
v_0'(T_2) &< \frac{v_0'(T_2-1) - v_0(T_2-1)}{w_0(T_2-1) - w_0'(T_2-1)} \\
&\Leftrightarrow \text{ by def. of } v_0 \text{ and } w_0 \\
v_0'(T_2) &< \frac{\beta(2\alpha + \alpha c - 1)(\pi_{T_2-1}' - \pi_{T_2-1})}{\beta(2\alpha - 1)(\pi_{T_2-1}' - \pi_{T_2-1})}
\end{aligned}$$

$$\Leftrightarrow$$

$$v_0'(T_2) < \frac{2\alpha + \alpha c - 1}{2\alpha - 1}.$$

$v_0'(T_2) = v_0(T_2, \pi_{T_2}')$ and is a strictly increasing in π_{T_2}'. As the maximum of π_{T_2}' is $1/2$, the maximum of $v_0'(T_2)$ is $v_0(T_2, \pi_{T_2}' = 1/2) = \frac{1}{2}\beta(1 + \alpha c)$. The last line above is true if the left hand side below is true:

$$\frac{1}{2}\beta(1 + \alpha c) < \frac{2\alpha + \alpha c - 1}{2\alpha - 1} \Rightarrow v_0'(T_2) < \frac{2\alpha + \alpha c - 1}{2\alpha - 1}.$$

It is easy to check that the inequality on the left is true as $1/2 < \alpha < 1$, $0 < \beta < 1$ and $c \geq 0$. Therefore $f(T_2 - 1, T_2, B_{T_2-1}) < f(T_2 - 1, T_2, B_{T_2-1}')$ is true.

Step two, suppose $f(T_1 + 1, T_2, B_{T_1+1}) < f(T_1 + 1, T_2, B_{T_1+1}')$ is true. Show that $f(T_1, T_2, B_{T_1}) < f(T_1, T_2, B_{T_1}')$ is true:

$$\begin{aligned}
f(T_1, T_2, B_{T_1}) &< f(T_1, T_2, B_{T_1}') \\
&\Leftrightarrow \text{ by def. of } f \\
v_0(T_1) + f(T_1 + 1, T_2, B_{T_1+1})w_0(T_1) &< v_0'(T_1) + f(T_1 + 1, T_2, B_{T_1+1}')w_0'(T_1) \\
&\Leftarrow \text{ as } f(T_1 + 1, T_2, B_{T_1+1}) < f(T_1 + 1, T_2, B_{T_1+1}') \\
v_0(T_1) + f(T_1 + 1, T_2, B_{T_1+1}')w_0(T_1) &< v_0'(T_1) + f(T_1 + 1, T_2, B_{T_1+1}')w_0'(T_1) \\
&\Leftrightarrow f \text{ to the left and as } w_0(T_1) > w_0'(T_1) \\
f(T_1 + 1, T_2, B_{T_1+1}') &< \frac{v_0'(T_1) - v_0(T_1)}{w_0(T_1) - w_0'(T_1)} \\
&\Leftrightarrow \text{ by def. of } v_0 \text{ and } w_0 \\
f(T_1 + 1, T_2, B_{T_1+1}') &< \frac{\beta(2\alpha + \alpha c - 1)(\pi_{T_1}' - \pi_{T_1})}{\beta(2\alpha - 1)(\pi_{T_1}' - \pi_{T_1})} \\
&\Leftrightarrow \\
f(T_1 + 1, T_2, B_{T_1+1}') &< \frac{2\alpha + \alpha c - 1}{2\alpha - 1}.
\end{aligned}$$

The maximum of $f(T_1+1, T_2, B_{T_1+1}')$ is $f(T_1+1, T_2, B_{T_1+1}' = 1/2)$ as $f(T_1+1, T_2, B_{T_1+1}')$ is increasing in B_{T_1+1}' and $B_{T_1+1} \leq 1/2$. By the definition of f it holds that $f(T_1+1, T_2, 1/2) < f(T_1+1, \infty, 1/2)$. Use Equation (A.15) and the definition of f to see that $f(T_1 + 1, \infty, 1/2) = \zeta_0(T_1 + 1, 1/2)$. By Result 6 ζ_0 is decreasing in t, as $1/2 > B^*$ and $\alpha > 1/2$. From Theorem 2 it follows that $\zeta_0(T_1 + 1, 1/2) < br_0(T_1 + 1, 1/2)$. Use the definition of br_0 to obtain that $br_0(T_1 + 1, 1/2) = (\beta + \alpha\beta c)/(2 - \beta)$. Combining all these inequalities yields $f(T_1 + 1, T_2, B_{T_1+1}') < (\beta + \alpha\beta c)/(2 - \beta)$. If $(\beta + \alpha\beta c)/(2 - \beta)$ fulfills the last line above, $f(T_1 + 1, T_2, B_{T_1+1}')$ also fulfills it:

$$\frac{\beta + \alpha\beta c}{2 - \beta} < \frac{2\alpha + \alpha c - 1}{2\alpha - 1} \Rightarrow f(T_1 + 1, T_2, B_{T_1+1}') < \frac{2\alpha + \alpha c - 1}{2\alpha - 1}.$$

It is easy to check that the inequality on the left hand side is true. Therefore, if $B_0 < B_0'$ it is true that $f(T_1, T_2, B_{T_1}) < f(T_1, T_2, B'_{T_1})$ for all T_2 and for all $T_1 < T_2$. Set $T_1 = t$ and take T_2 to infinity on both sides to obtain that

$$f(t, \infty, B_t) \leq f(t, \infty, B_t').$$

Use Equation (A.15) and the definition of f to see that $f(t, \infty, B_t) = \zeta_0(t, B_t)$. Therefore

$$\zeta_0(t, B_t) \leq \zeta_0(t, B_t'). \tag{A.19}$$

This holds especially for $t = 0$:

$$\zeta_0(0, B_0) \leq \zeta_0(0, B_0'). \tag{A.20}$$

q.e.d.

Proposition 2 *Let α be in $(1/2, 1)$ and $B_0 > B^*$. $\zeta_1(0, c)$ is a function of the debt in the initial period B_0. If $B_0 < B_0'$ it follows that $\zeta_1(0, c, B_0) \leq \zeta_1(0, c, B_0')$.*

Proof:

Recall Equation (4.19):

$$\zeta_1(t) = \sum_{i=t}^{\infty} v_1(i) \prod_{j=t}^{i-1} w_1(j) \tag{A.21}$$

whereby

$$v_1(t) = \beta(\pi_t(1-\alpha) + \pi_t \alpha \zeta_0(t+1)), \tag{A.22}$$

$$w_1(t) = \beta(1 - \pi_t). \tag{A.23}$$

Use Equation (4.6), (4.4), (A.24), (A.25) and (A.21) to see that $\zeta_1(t)$ is a function of B_t. In particular, $\zeta_1(0)$ is a function of B_0. It holds that $B^* < B_0 < B_0'$ and $\alpha > 1/2$. From (A.18) it follows that $\pi_t < \pi_t'$. Define $\zeta_0'(t+1)$ as $\zeta_0'(t+1) = \zeta_0(t+1, B'_{t+1})$. As $B_0 < B_0'$, it follows from (A.19) that $\zeta_0(t+1) \leq \zeta_0'(t+1)$. Using these two properties in Equation (A.24) and (A.25) yields $v_1(t) < v_1'(t)$ and $w_1(t) > w_1'(t)$ whereby

$$v_1'(t) = \beta(\pi_t'(1-\alpha) + \pi_t' \alpha \zeta_0'(t+1)), \tag{A.24}$$

$$w_1'(t) = \beta(1 - \pi_t'). \tag{A.25}$$

As $B^* < B_0'$, it follows from Result 6 that $\zeta_0'(t+1) < \zeta_0'(t)$.

Define $f(T_1, T_2)$ as

$$f(T_1, T_2) = \sum_{i=T_1}^{T_2} v_1(i) \prod_{j=T_1}^{i-1} w_1(j).$$

Use Equation (4.6), (4.4), (A.24) and (A.25) to see that f is a function of B_{T_1}. Take an arbitrary T_2. I will show by induction that $f(T_1, T_2, B_{T_1}) < f(T_1, T_2, B'_{T_1}),\ \forall T_1 < T_2$. First step, show that $f(T_2-1, T_2, B_{T_2-1}) < f(T_2-1, T_2, B'_{T_2-1})$:

$$\begin{aligned}
f(T_2-1, T_2, B_{T_2-1}) &< f(T_2-1, T_2, B'_{T_2-1}) \\
&\Leftrightarrow \\
v_1(T_2-1) + v_1(T_2)w_1(T_2-1) &< v'_1(T_2-1) + v'_1(T_2)w'_1(T_2-1) \\
&\Leftarrow \text{ as } v_1(T_2) < v'_1(T_2) \\
v_1(T_2-1) + v'_1(T_2)w_1(T_2-1) &< v'_1(T_2-1) + v'_1(T_2)w'_1(T_2-1) \\
&\Leftrightarrow v'_1(T_2) \text{ to the left and as } w_1(T_2-1) > w'_1(T_2-1) \\
v'_1(T_2) &< \frac{v'_1(T_2-1) - v_1(T_2-1)}{w_1(T_2-1) - w'_1(T_2-1)} \\
&\Leftrightarrow \text{ by def. of } v_1 \text{ and } w_1 \\
v'_1(T_2) &< \frac{\beta(1-\alpha)(\pi'_{T_2-1} - \pi_{T_2-1})}{\beta(\pi'_{T_2-1} - \pi_{T_2-1})} + \\
&\quad + \frac{\beta(\pi'_{T_2-1}\zeta'_0(T_2) - \pi_{T_2-1}\zeta_0(T_2))\alpha}{\beta(\pi'_{T_2-1} - \pi_{T_2-1})} \\
&\Leftarrow \text{ as } \zeta_0(T_2) \leq \zeta'_0(T_2) \\
v'_1(T_2) &< (1-\alpha) + \frac{(\pi'_{T_2-1}\zeta'_0(T_2) - \pi_{T_2-1}\zeta'_0(T_2))\alpha}{\pi'_{T_2-1} - \pi_{T_2-1}} \\
&\Leftrightarrow \\
v'_1(T_2) &< (1-\alpha) + \alpha\zeta'_0(T_2) \\
&\Leftrightarrow \text{ by def. of } v'_1(T_2) \\
\beta\pi'_{T_2}(1-\alpha) + \beta\alpha\pi'_{T_2}\zeta'_0(T_2+1) &< (1-\alpha) + \alpha\zeta'_0(T_2) \\
&\Leftarrow \text{ as } \beta\pi'_{T_2} < 1 \\
(1-\alpha) + \alpha\zeta'_0(T_2+1) &< (1-\alpha) + \alpha\zeta'_0(T_2) \\
&\Leftrightarrow \\
\zeta'_0(T_2+1) &< \zeta'_0(T_2), \text{ which is true.}
\end{aligned}$$

Therefore $f(T_2-1, T_2, B_{T_2-1}) < f(T_2-1, T_2, B'_{T_2-1})$ is true.

Step two, suppose $f(T_1+1, T_2, B_{T_1+1}) < f(T_1+1, T_2, B'_{T_1+1})$ is true. Show that $f(T_1, T_2, B_{T_1}) < f(T_1, T_2, B'_{T_1})$ is true:

$$\begin{aligned}
f(T_1, T_2, B_{T_1}) &< f(T_1, T_2, B'_{T_1}) \\
&\Leftrightarrow \text{ by def. of } f \\
v_1(T_1) + f(T_1+1, T_2, B_{T_1+1})w_1(T_1) &< v'_1(T_1) + f(T_1+1, T_2, B'_{T_1+1})w'_1(T_1)
\end{aligned}$$

$$\begin{aligned}
&\Leftarrow \text{ as } f(T_1+1,T_2,B_{T_1+1}) < f(T_1+1,T_2,B'_{T_1+1})\\
v_1(T_1)+f(T_1+1,T_2,B'_{T_1+1})w_1(T_1) &< v'_1(T_1)+f(T_1+1,T_2,B'_{T_1+1})w'_1(T_1)\\
&\Leftrightarrow \text{ as } w_1(T_1) > w'_1(T_1)\\
f(T_1+1,T_2,B'_{T_1+1}) &< \frac{v'_1(T_1)-v_1(T_1)}{w_1(T_1)-w'_1(T_1)}\\
&\Leftarrow \text{ by def. of } v_1 \text{ and } w_1\\
&\quad\ \text{and using } \zeta_0(T_1+1) \le \zeta'_0(T_1+1) \text{ as above}\\
f(T_1+1,T_2,B'_{T_1+1}) &< (1-\alpha)+\alpha\zeta'_0(T_1+1).
\end{aligned}$$

By the definition of f it holds that $f(T_1+1,T_2,B'_{T_1+1}) < f(T_1+1,\infty,B'_{T_1+1})$. Use Equation (A.21) and the definition of f to see that $f(T_1+1,\infty,B'_{T_1+1}) = \zeta_1(T_1+1,B'_{T_1+1})$. As $1/2 > B^*$ and $\alpha > 1/2$, by Result 6 ζ_1 is decreasing in t. By Theorem 2 it holds that $\zeta_1(T_1+1,B'_{T_1+1}) < br_1(T_1+1,B'_{T_1+1})$. Combine these inequalities to obtain that $f(T_1+1,T_2,B'_{T_1+1}) < br_1(T_1+1,B'_{T_1+1})$. Let me continue with the last line above:

$$\begin{aligned}
f(T_1+1,T_2,B'_{T_1+1}) &< (1-\alpha)+\alpha\zeta'_0(T_1+1)\\
&\Leftarrow \text{ as } f(T_1+1,T_2,B'_{T_1+1}) < br_1(T_1+1,B'_{T_1+1})\\
br_1(T_1+1,B'_{T_1+1}) &< (1-\alpha)+\alpha\zeta'_0(T_1+1)\\
&\Leftrightarrow \text{ by def. of } br_1\\
\frac{\beta(\pi'_{T_1+1}(1-\alpha)+\alpha\pi'_{T_1+1}\zeta'_0(T_1+2))}{1-\beta(1-\pi'_{T1+1})} &< (1-\alpha)+\alpha\zeta'_0(T_1+1)\\
&\Leftarrow \text{ as the left hand side is increasing in } \beta \text{ and } \beta<1\\
\frac{\pi'_{T_1+1}(1-\alpha)+\alpha\pi'_{T_1+1}\zeta'_0(T_1+2))}{1-(1-\pi'_{T1+1})} &< (1-\alpha)+\alpha\zeta'_0(T_1+1)\\
&\Leftrightarrow\\
(1-\alpha)+\alpha\zeta'_0(T_1+2) &< (1-\alpha)+\alpha\zeta'_0(T_1+1)\\
&\Leftrightarrow\\
\zeta'_0(T_1+2) &< \zeta'_0(T_1+1), \text{ which is true.}
\end{aligned}$$

Therefore, if $B_0 < B'_0$ it is true that $f(T_1,T_2,B_{T_1}) < f(T_1,T_2,B'_{T_1})$ for all T_2 and for all $T_1 < T_2$. Set $T_1 = t$ and take T_2 to infinity on both sides to obtain that

$$f(t,\infty,B_t) \le f(t,\infty,B'_t).$$

Use Equation (A.21) and the definition of f to see that $f(t,\infty,B_t) = \zeta_1(t,B_t)$. Therefore

$$\zeta_1(t,B_t) \le \zeta_1(t,B'_t). \tag{A.26}$$

This holds especially for $t = 0$:

$$\zeta_1(0,B_0) \le \zeta_1(0,B'_0). \tag{A.27}$$

q.e.d.

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Abstract (English)

In the absence of a centralized goods market and perfect credit markets, precautionary savings, driven by pessimistic expectations, can lead to a lack of aggregate demand and involuntary unemployment in steady state, whereas optimistic expectations would keep the economy in a steady state with full employment. While the steady state with full employment is efficient, the steady state with unemployment is not. Once the economy is in the inefficient steady state, under certain conditions there is no equilibrium path which leads back to the efficient steady state. This result was shown by Chamley (2014), assuming that agents face idiosyncratic preference shocks of a certain probability. I show that this result is robust to changes in the probability of preference shocks. It holds for nearly all probabilities of these shocks. In addition, I show that rare preference shocks lead to higher unemployment in the inefficient steady state than frequent shocks. This is because the higher the probability of idiosyncratic shocks, the higher the fraction of agents with a high propensity to consume. A higher fraction of agents with a high propensity to consume leads to less unemployment in the inefficient steady state.

Abstract (Deutsch)

In einer Ökonomie ohne zentralen Gütermarkt und mit unvollständigen Kreditmärkten können pessimistische Erwartungen Haushalte dazu bringen Ersparnisse als Rücklage zu bilden. Dieses Sparverhalten führt zu einem Rückgang an aggregierter Nachfrage, welcher in letzter Konsequenz Arbeitslosigkeit verursacht. Optimistische Erwartungen hingegen würden dafür sorgen, dass die Ökonomie in einem Zustand von Vollbeschäftigung bleibt. Sobald die Ökonomie in einem Steady State mit Arbeitslosigkeit ist, gibt es unter bestimmten Bedingungen keinen Gleichgewichtspfad mehr zurück zur Vollbeschäftigung. Diese Ergebnisse wurden von Chamley (2014) in einem Model gezeigt. Haushalte sind in diesem Model individuellen Schocks in ihren Präferenzen ausgesetzt. Anstatt eines zentralen Marktes nimmt Chamley an, dass Tauschpartner durch einen Zufallsmechanismus zusammen finden. Allerdings muss er für seinen Beweis eine bestimmte Wahrscheinlichkeit für Schocks in den Präferenzen annehmen. Ich erweitere in dieser Arbeit Chamleys Ergebnis und zeige, dass es für die meisten möglichen Wahrscheinlichkeiten von Schocks in den Präferenzen gilt. Zusätzlich kann ich zeigen, dass Arbeitslosigkeit im entsprechenden Steady State umso niedriger ist, je höher die Wahrscheinlichkeit von Schocks in den Präferenzen ist. Schocks müssen jedoch eine entsprechend der Wahrscheinlichkeit angemesse Intensität haben. Je wahrscheinlicher Schocks sind, desto höher ist der Anteil an Haushalten mit einer hohen Konsumneigung. Ein hoher Anteil an Haushalten mit einer hohen Konsumneigung bedingt eine geringere Arbeitslosigkeit im Steady State.

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03/2012-08/2014:	*University of Vienna, Austria* Master's programme in Economics Research orientated focus
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