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# TABLE OF CONTENT

<b>1</b>	<b>INTRODUCTION .....</b>	<b>4</b>
<b>2</b>	<b>MARKET LIQUIDITY RISK MANAGEMENT FRAMEWORK.....</b>	<b>6</b>
2.1	<i>International Regulatory Framework for Liquidity Risk Management .....</i>	6
2.2	<i>Liquidity Risk from the Perspective of Policy Makers.....</i>	9
2.3	<i>What is Market Liquidity?.....</i>	10
2.4	<i>Financial Market Liquidity versus Market Liquidity of an Asset .....</i>	10
<b>3</b>	<b>WHY IS THERE MARKET ILLIQUIDITY? .....</b>	<b>12</b>
3.1	<i>Market Structure and Trading Mechanism.....</i>	12
3.2	<i>Source of Market Illiquidity.....</i>	17
3.3	<i>Factors of Market Illiquidity.....</i>	18
<b>4</b>	<b>QUANTIFICATION OF MARKET LIQUIDITY .....</b>	<b>22</b>
4.1	<i>Spread Measures .....</i>	22
4.2	<i>Execution Quality Measures .....</i>	25
4.3	<i>Other Measures .....</i>	28
<b>5</b>	<b>QUANTIFICATION OF MARKET LIQUIDITY RISK .....</b>	<b>30</b>
5.1	<i>Market Liquidity versus Market Liquidity Risk .....</i>	30
5.2	<i>Relation between Market Liquidity Risk and Market Risk .....</i>	30
5.3	<i>Criteria for Evaluation of Models.....</i>	31
5.4	<i>STANDARD MODELS .....</i>	32
5.4.1	<i>Adverse Price Impact Measures.....</i>	32
5.4.2	<i>Spread Adjustment Models.....</i>	33
5.4.3	<i>Comparison and Evaluation of Standard Models.....</i>	35
5.5	<i>ADVANCED MODELS.....</i>	38
5.5.1	<i>Market Price Response Approach .....</i>	38
5.5.2	<i>Weighted Spread Models.....</i>	40
5.5.3	<i>Other Economic Models.....</i>	41
5.5.4	<i>Comparison and Evaluation of Advanced Models.....</i>	42
5.6	<i>MODELS FOR ILLIQUID MARKETS .....</i>	47
5.6.1	<i>Zero Rates Return.....</i>	47
5.6.2	<i>Roll's Measure .....</i>	48
5.6.3	<i>Turnover .....</i>	48
5.6.4	<i>Amihud's Illiquidity Ratio .....</i>	48
5.6.5	<i>Comparison and Evaluation of Models for Illiquid Markets.....</i>	49
<b>6</b>	<b>MARKET LIQUIDITY AND ASSET PRICES .....</b>	<b>53</b>
6.1	<i>Market Liquidity Risk Premium.....</i>	53
6.2	<i>Liquidity Adjusted CAPM.....</i>	53
6.3	<i>Liquidity CAPM.....</i>	54
<b>7</b>	<b>CHALLENGES FOR MEASUREMENT AND MANAGEMENT OF MARKET LIQUIDITY RISK .....</b>	<b>56</b>
<b>8</b>	<b>CONCLUSION .....</b>	<b>60</b>
	<b>LIST OF REFERENCES.....</b>	<b>62</b>

## LIST OF ABBREVIATIONS

BCBS	Basel Committee on Banking Supervision
BIS	Bank for International Settlements
CEBS	Committee of European Banking Supervisors
DEAR	Daily Earning at Risk
EBA	European Banking Authority ( <i>former CEBS</i> )
ECB	European Central Bank
ICAAP	Internal Capital Adequacy Assessment Process
ILAAP	Internal Liquidity Adequacy Assessment Process
IS	Implementation Shortfall
LCR	Liquidity Coverage Ratio
L-VaR	Liquidity Adjusted Value at Risk
MiFID	Markets in Financial Instruments Directive
NASDAQ	National Association of Securities Dealers Automated Quotations
NSFR	Net Stable Funding Ratio
NYSE	New York Stock Exchange
SREP	Supervisory Review Process
VaR	Value at Risk
Vwap	Volume Weighted Average Price
WS	Weighted Spread

# 1 INTRODUCTION

Liquidity risk is one of the most important risks any financial institution exposed to. Not addressing of this risk might cause significant consequences, including banking collapse, and by extension, the stability of the financial system itself. Most bank failures occur due to issues around managing liquidity risk (Villafranca & Hamdi, 2013, p. 2). Historically, ‘funding liquidity risk’ was in the main focus of regulators and policy makers attracting also the major attention of academic researches; whereas ‘market liquidity risk’ as a quantifiable concept was less researched and attracted less attention from the side of regulators and practitioners (Loebnitz, 2006, p. 94). Yet not addressing of market liquidity risk properly will lead to underestimation of exposed risk and the required risk capital as the real loss potential and P&L volatilities will not be correctly estimated. Bangia *et al.* (1998) shows that ignoring the market liquidity risk factor can produce underestimates of market risk in emerging markets by as much as 25-30% (p.1).

In particular, since 2008 financial crises showed that capital alone do not save financial institutions from a severe crisis and even collapse; the liquidity of a financial institution has become an integrated component of prudential regulation and supervision. The banks are explicitly required to have robust liquidity risk measurement and management systems in place, where they shall model and quantify the exposed risk and allocate capital against it. The properness of used models are examined by regulators via on-site audits during so-called Supervisory Review Process (SREP) and the violations by banks to the defined capital requirements are identified. As oppose to other risk types, standards in case of market liquidity risk is not set out explicitly in great detail by the regulators and the enforcement takes place substantially during on-site examinations (Malz, 2003, p. 39).

Against this background, this paper aims to describe and assess models that can be used by risk managers of credit institutes in their attempts to quantify market liquidity risk. The terms “bank”, “financial institute”, “credit institute” are used as synonyms in this paper. An extensive review of academic literature have been conducted in the process of selection and evaluation of the presented models in this work. The selected models are classified into three general categories: The first group of presented models, ‘standard models’, refer to those measures that are commonly used in the industry as a common practise. The second group, ‘advanced models’, refer to more sophisticated models with more economic appeal. Due to

lack of required data for the model set-up, aforementioned models may not be suitable in measuring of liquidity in emerging markets. Thus, a third group of models are introduced that are applicable in quantifying of market liquidity risk in structurally illiquid markets.

The central contribution of this work is that all presented models are assessed in terms of their abilities capturing the behaviour they meant to measure and the extent to which they are practically applicable. The main drawbacks and weaknesses of every model are discussed. The models are evaluated and compared with each other according to predefined evaluation criteria. Results of selected empirical research from literature are presented to illustrate and compare performance of different models in actual markets. Which model delivers better results under which condition is elaborated. The assessment of the models are conducted from a risk management perspective and within this framework, aims to serve as a supportive instrument for risk managers in evaluation of alternative models that would fit best to their internal requirements and portfolio characteristics.

The paper starts with a brief description of international regulatory environment around liquidity risk management for financial institutions. The market liquidity concept is defined in Chapter 2 and the source of market liquidity is examined. Chapter 3 further presents basic concepts of market microstructure and trading mechanism in order to understand the factors of illiquidity, which we later will try to find out to which extend these components are addressed by each of the chosen models. Chapter 4 presents selected measures used for quantification of 'market liquidity'. In what follows, Chapter 5 presents the selected models used for quantification of 'market liquidity risk'. Before presentation of models, the criteria for evaluation of the models are set and described. The detailed description of all three groups of models are followed with a comparison analysis according to the defined evaluation criteria. The drawbacks of every presented model are analysed. Chapter 6 shows if and how liquidity risk is priced in financial markets. Chapter 7 lists some challenges related to measurement and management of market liquidity risk. Consequently, concluding thoughts and a summary of findings of the conducted research is provided in the last chapter.

## **2 MARKET LIQUIDITY RISK MANAGEMENT FRAMEWORK**

### **2.1 International Regulatory Framework for Liquidity Risk Management**

Each credit institution is required to have adequate internal control systems in place that cover rules for organizational and operational structure and process for identifying, assessing, treating, monitoring and communicating risks. As a general rule, the following risk categories are to be taken into account:

- a) counterparty risks (known as credit risk),
- b) market risks,
- c) liquidity risks and
- d) operational risks

*(Federal Financial Supervisory Authority, 2012).*

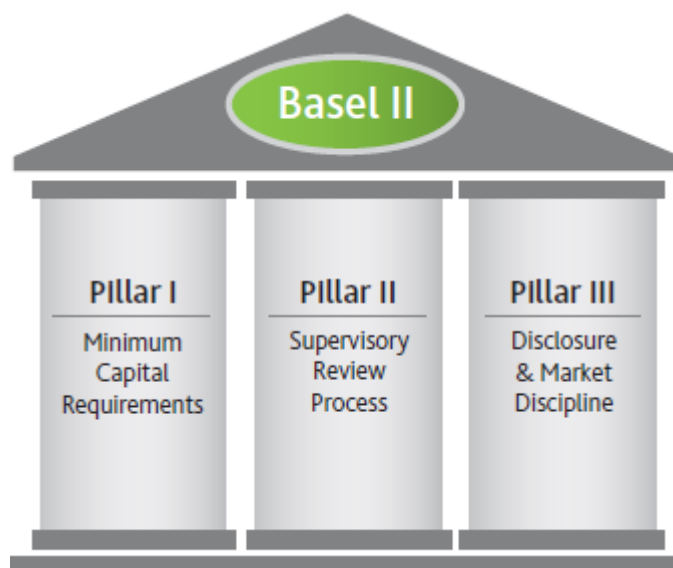
Liquidity risk management is a part of the broader overall risk management framework of a financial institution. Thus, overall ICAAP framework is very important for ensuring stability in the financial markets and therefore the compliance of financial institutes to these rules and principles are very strictly monitored by the financial supervisory authorities

#### **Basel II Regime**

With the implementation of Basel II, credit institutions have been required to quantify their exposure to credit, market and operational risks following strictly defined rules for calculation and allocating regulatory capital within so-called “Pillar I”. The principles of Basel II were integrated into local regulatory environments in implementing countries by either amending existing laws or regulations or publishing new ones. The calculation methodology for quantification of these three risk types within Pillar I was defined very precisely, leaving banks little play room for decision. The so-called “Pillar II” of Basel II, on the other hand, requires all relevant institutes to allocate internal capital (economic capital) for all material types of risks in order to ensure capital adequacy (Internal Capital Adequacy Assessment Process – ICAAP). ICAAP framework gives intentionally banks a playing room for decision since the nature of the implementation bases on the proportionality principles focusing on the

specific characteristics and the business model of each specific institute (Woschnagg, 2008, p. 103).

Below chart illustrates the famous three pillar architecture of Basel II:



*Table 1: Three Pillar Architecture of Basel II (Source:Moody'sAnalytics)*

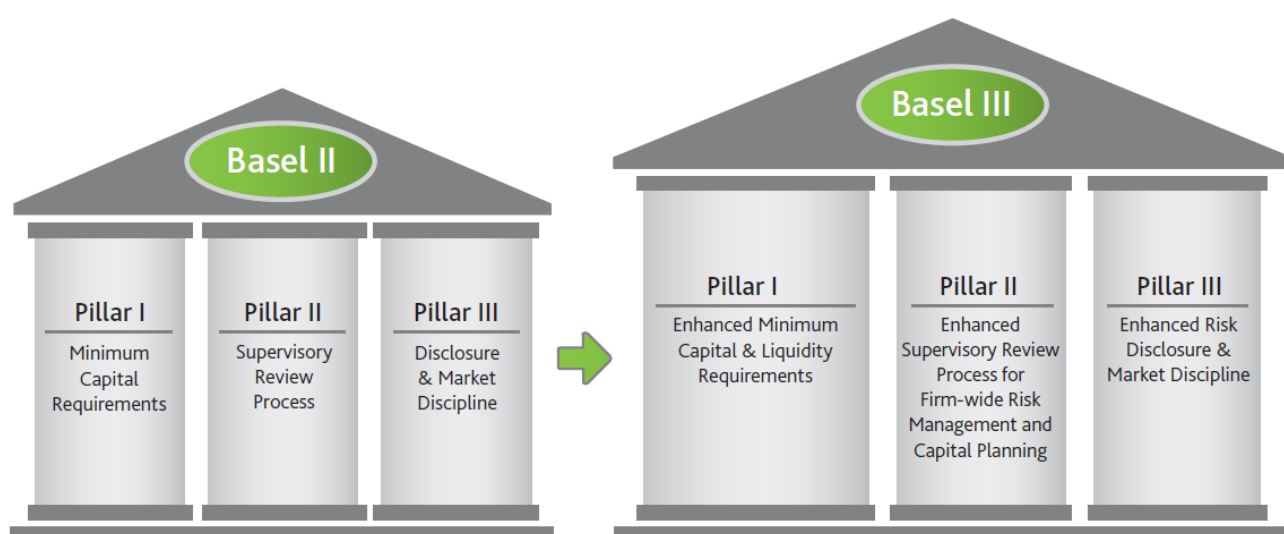
As illustrated in the above chart, Basel II framework (both Pillar I and II) puts capital into focus of risk management practices and deals with the quantification of the risk types: credit, market and operational risk. Though being one of the most material risk types, liquidity risk was not adequately addressed under Basel II regime. However, the financial and banking crisis starting from 2007 has clearly shown that allocation of capital alone do not protect financial institutions being fragile to market crisis. In order to strengthen the importance of liquidity to the functioning of financial markets and banking sector, the new Basel III framework has introduced binding liquidity requirements for institutions.

### **Basel III Regime**

Even before Lehman Brothers' collapse in September 2008, the fundamental weaknesses of Basel II framework were known. The banking sector had entered the financial crisis with too much leverage and inadequate liquidity buffers. The deficiencies in poor governance and risk management were reflected by the mispricing of credit and liquidity risk as well as excess credit growth. In response to these defects in the financial system, the Basel Committee on Banking Supervision published *Principles for Sound Liquidity Risk Management and*

*Supervision* on September 2008 (in the same month Lehman Brothers failed). In September 2010, the Group of Governors and Heads of Supervision announced higher global minimum capital standards, which followed an agreement reached in July regarding overall design of the capital and liquidity reform package, now referred as "Basel III" (Basel Committee on Banking Supervision, 2014).

Further enhancements were introduced by Basel III as a part of a broader effort to strengthen regulation and supervision of internationally active banks. Basel III framework enhances in particular Pillar I significantly by integrating liquidity and introduces new tools and methodologies for measuring and monitoring of liquidity risk for credit institutes.



*Table 2: Three Pillar Architecture of Basel II versus Basel III (Source: Moody's Analytics)*

Among those new measures, the most important is the introduction of two new liquidity risk ratios within Pillar I framework, namely Liquidity Coverage Ratio (LCR) and Net Stable Funding Ratio (NSFR). In addition to LCR and NSFR requirements, banks are required to have additional quantitative and qualitative metrics and internal models in assessing liquidity risk within Pillar II framework, where the implemented Internal Liquidity Adequacy Assessment Processes (ILAAP) by the bank is subject revision of supervisors regularly in order to ensure that banks have adequate internal strategies and processes in measuring, steering and monitoring of liquidity risk (European Banking Authority, 2014). The models presented in this paper attempts to serve as a tool for risk managers of banks in addressing market liquidity risk within their Pillar II bank-wide liquidity risk measurement and management frameworks.



## 2.2 Liquidity Risk from the Perspective of Policy Makers

In its guideline *Principles for Sound Liquidity Risk Management and Supervision*, Basel Committee on Banking Supervision (“BCBS”, 2008) defines liquidity as the “ability of a bank to fund increases in assets and meet obligations as they come due, without incurring unacceptable losses” (p.1). The fundamental role of banks in the maturity transformation of short-term deposits into long-term loans makes banks inherently vulnerable to liquidity risk, which is divided into two sub-categories by Basel Committee on Banking Supervision (BCBS) in the aforementioned guideline and described as:

### *Funding Liquidity Risk*

Funding liquidity risk is “the risk that the firm will not be able to meet efficiently both expected and unexpected current and future cash flow and collateral needs without affecting either daily operations or the financial condition of the firm” (p.1).

### *Market Liquidity Risk*

Market liquidity risk is “the risk that a firm cannot easily offset or eliminate a position at the market price because of inadequate market depth or market disruption” (p.1).

Funding liquidity risk refers to the ability of banks to fund their positions, where the market liquidity risk is the ability of trading in the markets. These two types of liquidity are strongly interrelated yet distinct, both under normal conditions and in the period of stress (BCBS, 2008, p. 9-20). Funding liquidity risk in a single bank is per se not the main concern for the regulators and policy makers; the problem arises when the funding liquidity risk is transmitted to more than one institution, when liquidity risk becomes systematic resulting in market liquidity risk (Nikolaou, 2009, p. 9).

As stated previously, of the two distinct liquidity risk types, funding liquidity risk has received the major attention from researchers and especially practitioners for its obvious significance and higher tractability for banks. This paper however focuses mainly on market liquidity risk. Though market and funding liquidity risk are intertwined on a broader conceptual level; a detailed analysis of this interrelationship is beyond the scope of this work.

## 2.3 What is Market Liquidity?

*“The word liquidity has so many facets that it is often counter-productive to use it without further and closer definition”*

*Charles Goodhart (Banque de France, 2008)*

The concept of market liquidity can be found in the literature as early as in Keynes' time. He describes an asset as more liquid, if it is more “certainly realizable at short notice without loss” (Keynes, 1930, pg. 67). 70 years after Keynes, Fernandez (1999) states (market) liquidity incorporates the key elements of *volume*, *time* and *transaction costs*. Numerous other descriptions of the term can be found in the literature by different researchers. According to Amihud, Mendelson, and Pedersen (2005) liquidity is simply the ease of trading of a security or similarly, market liquidity is “the ability to trade a security quickly at a price close to its consensus value” (Foucault, Pagano & Röell, 2013, p.8).

Even though in general it is concluded that “there is no single unambiguous, theoretically correct or universally accepted definition of liquidity” (Baker, 1996, p.1), there are certain features of liquidity that are widely accepted. Sarr & Lybek (2002) stresses five such characteristics as (i) *tightness*, which refers to low transaction costs such as low bid-ask spreads (ii) *immediacy*, which represents the speed with which the orders can be executed reflecting the efficiency of trading systems (iii) *depth*, which refers to availability of plentiful orders both above and below the actual trading price (iv) *breadth*, which represents numerous and large in volume orders with minimal impact on prices and (v) *resiliency*, which is a characteristic of a market where new orders run rapidly to correct order imbalances, without changing quoted prices (p. 5).

## 2.4 Financial Market Liquidity versus Market Liquidity of an Asset

**An asset's market liquidity** depends on the ease and speed with which large volumes can be traded without having adverse impact on its price (assumption is the absence of arrival of new information influencing the fundamental value of the asset during the trade). Hence, an asset is considered as liquid, if it is easily and timely settled, it has small transaction costs and its trade (even large volumes) has no or only very limited impact on its market price. **Financial market liquidity**, on the other hand, is the overall degree of how liquid and interchangeable

each asset is traded in a specific market. Only few researches in the literature (e.g. Chordia, Roll & Subrahmanyam, 2001) focused on measuring a market's liquidity. Most of the studies have been dedicated to measure an asset's market liquidity, which provide also an insight whether a financial market, or at least a segment of it, can be characterized as liquid (Sarr & Lybek, 2002, p. 7-8).

This paper focuses on the measures quantifying **an individual asset's market liquidity**.

### 3 WHY IS THERE MARKET ILLIQUIDITY?

In order to understand reasons leading to market illiquidity, specifics of market microstructure and trading mechanism should be investigated. In the following chapter, basic concepts of trading mechanism are examined to illustrate how specific trading environment can affect the price formation process.

#### 3.1 Market Structure and Trading Mechanism

Many of the theories in financial modelling assume an instantaneous trading at an imaginary platform with continuous market clearing prices without any friction. However, in real markets, due to market illiquidity, trading takes places with intervals, at nonmarket clearing prices and in several platforms that can be in general divided into (i) *quote driven (dealer)* markets, (ii) *order driven (auction)* markets and (iii) *hybrid* markets carrying a mix characteristics of the quote driven and order driven markets (Harris, 2003, 92-96).

##### *(i) Quote Driven Markets (Dealer Markets)*

A dealer market is a quote driven market, where customers trade only with designated liquidity suppliers called dealer or market-maker. A dealer is an intermediary who acts as counterparty for the trades of his/her customers (trader, investor) hence enhancing inventory risk (Hasbrouck, 2007, p. 14). In a quote driven market, investors deliver their orders directly to a dealer, who continuously posts bid quotes (the highest price the dealer is willing to buy a security) and ask quotes (the lowest price the dealer is willing to sell a security). Corporate bond markets in United States and Europe are good examples to dealer markets (Foucault, 2013, p.17).

In a dealer market, bid ask spreads of each dealer is not necessarily publicly available in real time (e.g. US corporate bond market). An investor should search for the best price matching his/her order by contacting different dealers and this **search for the best available price** is costly (time and effort).

Some markets allow dealers to display **indicative bid ask quotes**, where the dealer is not obligated to trade if there is a counterparty willing to trade at this specified price. Furthermore, displayed bid and ask prices are valid only for a certain quantity. It means, when a trader enters to the market, his order may not be executed at once with one price. Assuming a trader, who wants to sell a large size of a certain share: He will first approach the best price offering dealer and will sell to that dealer as much as the price is valid for. For the remaining part of his shares, the trader will approach the second best price offering dealer and so on; until he sells his desired amount of share stock. One can calculate a weighted average bid ask spread, which is **widening in trade size**. Here the investor (seller) with a large order is said to be walk down through the demand curve of aggregate dealers' bid quotes. A buyer with a large order would, on the other hand, walk up the aggregate supply curve of dealers ask prices (Foucault, Pagano & Röell, 2013, p.26). These demand and supply curves are shown in the below figure

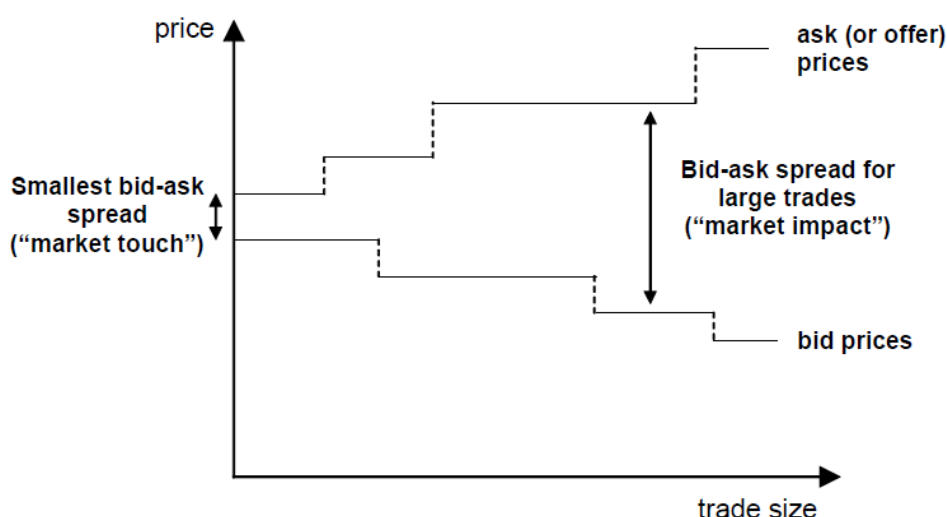


Table 3: Dealer market quotes for various trade size<sup>1</sup>

One of the debatable issues in dealer markets is the degree of **market transparency**, which represents how much information available to market participants. It is possible to negotiate a better price with a dealer. Especially institutional investors with higher volume and frequency of trading history have greater degree of **bargaining power** in executing their desired orders with more favourable prices than those publicly quoted by the market maker (Jong & Rindi,

<sup>1</sup> Source: Foucault T., Pagano M., Röell A. (2013). *Market Liquidity*. Oxford: Oxford University Press, p.26

2009, p. 12). It means, depending on the factors such as investor's customer segment (retail, institutional), order size, trading history with the dealer and frequency of trading in the market etc.; the bid-ask quotes and the resulting execution price of the same trade might differ for different traders.

*(ii) Order Driven Markets (Limit Order Markets)*

In contrary to quote driven markets, in order driven markets (or 'limit order' markets), buy and sell orders of investors are matched in a single marketplace without having any intermediary market maker in place. BATS in USA and Chi-X in Europe are examples to the limit order markets (Foucault, Pagano & Röell, 2013, p. 17). In these markets, the liquidity is granted via continuous flow of orders from market participants either through automated mechanisms or brokers. Brokers are the only intermediaries in order driven markets, who trade on behalf of the customer but do not take own position. In these markets, the orders go into a so-called 'Limit Order Book' (LOB), where the execution occur according to order precedence rules, yet not all of them use the same pricing rules (Jong & Rindi, 2009, p. 7-8).

In *continuous limit order markets*, orders of investors are executed immediately, if possible. The orders that can not be matched are placed in the LOB. In *call limit order markets* the incoming orders of market participants are entered into LOB and the matching is done with **discrete intervals** (such as once a day), where all executable orders are filled with one price. After placing all the submitted buy and sell orders in the LOB for a specific security, the number of shares which can be sold (supply) and bought (demand) for a given price will be determined. At the time of the auction, all the sell orders are listed in increasing in limit price, while buy orders are sorted increasing order of limit price. The aggregated quantity which would be sold and bought at each specific price is calculated. The transaction price of the call auction is set as the equilibrium price, where the market demand equals to supply (Foucault, Pagano & Röell, 2013, p. 18-23). Below figure illustrates the demand and supply function as well as the equilibrium pricing in call auction markets:

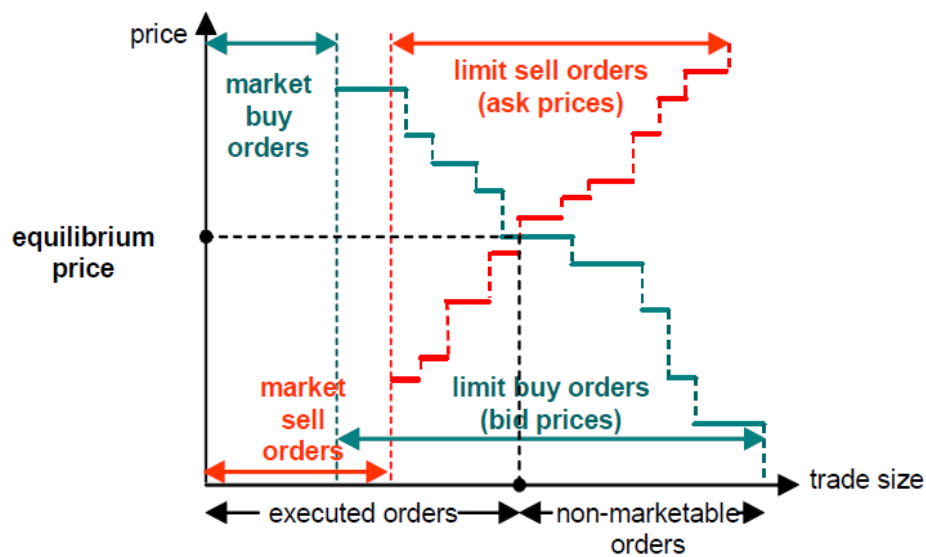


Table 4: Call auction<sup>2</sup>

With the increased usage of technology, the call auction is used limitedly. Some markets use the call auction to set the closing pricing at the end of the trading day, where in some other markets call auction is used to determine the opening price before the continuous limit order trading starts, such as NYSE-Euronext, LSE, Italian Stock Exchange. The limit orders, which were not executed in the official call auction, constructs the initial LOB of the continuous session (Foucault, Pagano & Röell, 2013, p. 21-23).

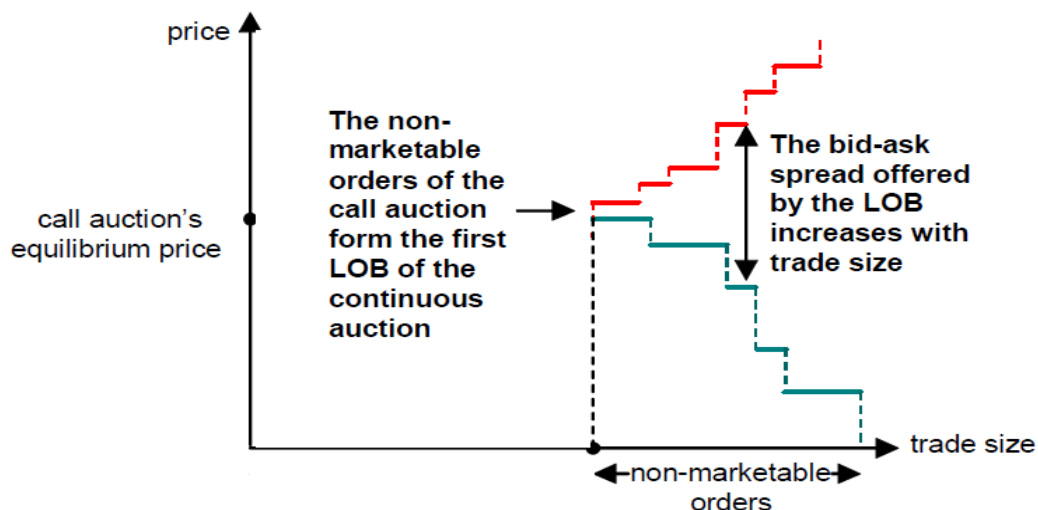


Table 5: Forming the initial limit order book of the trading day<sup>3</sup>

<sup>2</sup> Source: Foucault T., Pagano M., Röell A. (2013). *Market Liquidity*. Oxford: Oxford University Press, p.22

### *(iii) Hybrid Markets*

The market structures in the world often carry features of the quote and order driven markets. NYSE is a good example of such a hybrid set-up. NASDAQ can also be described as a hybrid nature; where it is quote driven but dealers have the option to transmit the limit orders of their customers into the electronic trading system (Jong & Rindi, 2009, p. 12).

### *Market Structure in Equity and Bond Markets*

Stocks are in general traded in organized hybrid markets, where the price is decided with an auction mechanism explained above. The so-called **price impact** of the trade, which refers to the adverse impact of that specific own trade on the quoted prices, is increasing with the trade size. Therefore, large orders are in general split up into smaller parts by the trader or intermediary in order to decrease the price impact, which increases the exposure to the so called **price risk**, which refers to the risk of adverse movements in the price of the asset due to external market factors in the time till the intended trade size is executed (Loebnitz, 2006, p. 18-19). Later in this Chapter, price risk and price impact concepts are analysed in more detail.

Bonds, on the contrary, are usually traded in the pure dealer markets (Over-the-Counter). Among those fixed income securities markets (e.g. corporate bonds, mortgage-backed securities etc), government bond market is seen as the most liquid one. Main reasons are higher volumes of trades due to limited credit risk of government bonds, being used for collateral and pricing purposes as well as higher amount of outstanding securities e.g. issuer countries take on public debt via large volumes of government bonds (Sarr & Lybek, 2002, p.29). In over-the-counter markets, price determination process is not executed by auctions; but by bilateral negotiations. The bid-ask spreads are available only for specific securities, even then they are indicative, where the dealer is not obligated to trade with these quotes. As indicated previously, traders contact dealers or other counterparties to trade in order to have the most favourable price. Different from the equity markets, where individual investors are actively involved; in bond markets the major players are institutional investors such as banks, insurance companies, pension funds and so on. As a natural consequence, the size of the orders is mostly large. Due to significant bargaining power of these institutional traders, they

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<sup>3</sup> Source: Foucault T., Pagano M., Röell A. (2013). *Market Liquidity*. Oxford: Oxford University Press, p.23



usually trade with a lower price than the pre-trade quoted price. Hence, unlike stock markets, in bonds the **price impact is decreasing in trade size** (Loebnitz, 2006, p. 42-45).

### 3.2 Source of Market Illiquidity

As indicated previously, many of the financial models assume that trading takes place in a so-called “perfect market”, in which no transaction costs exist, all traders are symmetrically informed and all potential buyers and sellers are continuously present. Everyone trades at market-clearing prices, which reflect the fundamental value of the traded assets. In such a market, above described trading mechanism would not be relevant in the process of price determination. In real markets, however, we see various frictions in the market set-up as described above. Tribe, Nixon and Sumner (2010) summarize the characteristics of actual markets versus so called perfect markets as below:

<u>Perfect Market</u>	<u>Actual Market</u> →
<ul style="list-style-type: none"> <li>➤ Large number of buyers and sellers are present; none having the power to set market prices</li> <li>➤ Free entry and exit to and from the market</li> <li>➤ No transaction costs</li> <li>➤ All traders have perfect information and foresight for all market characteristics</li> </ul>	<ul style="list-style-type: none"> <li>➤ Market power of individual buyers and sellers varies in a continuum from single-firm buyer or seller (monopoly) to large numbers of traders</li> <li>➤ Entry and exist is limited by factors such as high investment costs</li> <li>➤ The information processed varies considerably („information assymetry“)</li> </ul>

Table 6: Perfect Market vs. Actual Market<sup>4</sup>

Due to aforementioned frictions in actual markets, “the law of one price” do not hold, which is a very significant concept in modern financial theory. This law asserts every security that has the same future cash flows should have the same price as they are interchangeable. The theory implies (i) the price of the security is not influenced by any characteristics of the trade or the trader, (ii) the only expense the trader has to pay is the current market price, (iii) there is no restriction and regulation limiting the intended trade (Loebnitz, 2006, p.11). Yet, as

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<sup>4</sup> Source: Tribe, M., Nixon, F. & Sumner, A. (2010). *Economics and Development Studies*. Oxon: Routledge

stated earlier, this concept of frictionless market does not apply to most of the financial markets.

Having discussed the meaning of market friction, one can conclude that *the closer a financial market gets to a perfect market, the more liquid it is*. Therefore, a ‘liquid’ market can be characterized with low trading costs, informative prices, speed and ease in trading. It is the market structure, which decides how information is reflected in prices, how costly and fast it is to trade; and who gains and who loses from the trading activity (Zhuk, 2012).

There are several academic literature attempting comparison of different markets in terms of liquidity using different indicators as proxy. Among others, Huang and Stoll (1996) compare transaction costs in form of bid ask spreads across two rival trading platforms: They analyse two paired samples of stocks over a period of one year that are traded via dealer market NASDAQ or auction market NYSE. They show NASDAQ stock’s trading costs are twice those of same NYSE stocks (Vulkan, Roth & Neeman, 2013, p.631).

### 3.3 Factors of Market Illiquidity

In the following section, the main determinants of market illiquidity are elaborated namely, (i) *transaction costs*, (ii) *inventory risks* and (iii) *asymmetric information*.

#### 1.) Transaction Costs

Transaction costs are any costs associated with the specific trading activity. It includes all costs from the beginning of the trade (the trader delivers his/her order) till the transaction is closed (price set, transaction executed). Trading costs can occur in the form of direct or indirect costs. **Direct costs** are explicitly visible to the market participants, such as fees, commissions, taxes associated with the transaction. These are easily recognized by the trader and can simply be calculated and added to the paid market price as further expense. **Indirect costs**, on the other hand, are those implicit transaction costs, which are not as certain as direct costs to identify and quantify (Hasbrouck, 2007, p.144). The indirect costs are mainly composed of the following: (i) bid-ask spread, (ii) price risk, (iii) price impact and (iv) opportunity cost.

### (i) Bid Ask Spread

Bid-ask spread cost for a buy order is the difference between mid-price and ask price, whereas for a sell order it is the difference between mid-price and the bid price. Even though exact composition of bid ask spreads in markets are not precisely known, most researchers agree on the major components of bid ask spread, which are listed by Stoll (2001) as follows: *First*, suppliers of liquidity, such as dealers must be compensated for **order handling costs** (e.g. labor and capital needed for quote prices, order routing, execution). In a pure limit order market, where there is no professional dealer, order handling costs are expected to be lower. *Second*, dealers might have certain agreements or rules to increase spreads, where spreads reflect **non-competitive pricing**. *Third*, dealers have to be compensated for exposing to **inventory risk** in order to provide immediacy of execution. The more inventories a dealer has of a certain security, the more s/he is exposed to the risk of losses in case of a sudden fall in prices of that asset. *Fourth*, a dealer quotes bid and ask prices prior to arrival of new information. This gives to the other market participants an **option** to trade with these prices following arrival of new information. The quotes provided by dealer have to price this option. *Fifth*, dealers lose to informed trader, who has superior information compared to other market participants. Thus, quoted spreads reflect this **asymmetric information** in the market (Stoll, 2001, p.10).

### (ii) Price risk

Traders are exposed to adverse movements in the interested asset's price during order processing timeline. The longer time the execution of trade takes, the more the exposure increases to risk of losing due to adverse price movements in the market. This is especially important for transactions with larger volume, where order is split into smaller parts and execution is distributed over time (Loebnitz, 2006, p.18). Thus, when splitting an order and executing it through time horizon (in order to decrease the price impact of the transaction on the quotes), the risk of moving intended asset's prices in the unfavourable direction increases.

### (iii) Price impact

The price impact is commonly described as price response to signed order flow (order size). In a perfectly liquid market a specific trade would not have any impact on the quoted price. However, illiquidity reflects the impact of order flow on price due to adverse selection and

inventory costs. Price impact of a specific trade can be measured by taking the difference between the actual transaction price versus the midpoint quote just before the order is processed. Larger orders, which might be an indicator of informed trading to the dealer, have greater impact on prices. Since dealers do not have information on whether they are trading with an informed trader or liquidity (noise) trader, they set prices as an increasing function of the imbalances in the order flow. Hence, as the volume of trade increases, the price changes increase accordingly, which is called as 'price impact' of the trade (Amihud, Mendelson & Pedersen, 2013, p.111-112). Therefore, splitting of large orders into smaller parts and executing it over time might decrease the price impact; on the other hand, as stated previously, as the time interval increases the price risk will rise (e.g. arrival of new information to the market).

#### **(iv) Opportunity cost**

Opportunity cost of trading is the value of the best alternative forgone in order to execute the intended trade. Trading has per se a time dimension. The increase in time for order processing leads to an increase in the respective opportunity cost, since the value of accomplishments increase, which could have been achieved if the resources (time & money) have been invested elsewhere than the intended trade (Loebnitz, 2006, p.19).

#### **2.) Inventory Risk**

Increase of a dealer's inventory of a certain asset increases his exposed risk to a sudden price fall in the price of that asset. In order to decrease this risk, he can strive to find an investor or another dealer to sell the security to decrease his inventory levels (Foucault, Pagan & Röell, 2013, p. 24). Dealers, as being liquidity providers in the market, are exposed to inventory risk in absorbing order imbalances of buyers and sellers, who come to the market in different times. This inventory risk will be reflected by the dealer in the quoted prices. As dealer's inventory increases, the quoted prices will decrease. Thus, bid ask spreads would still occur even in an environment, where no transaction costs or information asymmetries exist.

#### **3.) Asymmetric Information**

The information asymmetry occurs when one party of the trade has more information than the other. An informed trader has better information than the dealer. The dealer would still be willing to trade with an informed trader, because he does not have the information if the

investor is an informed trader or not. Yet the dealer is going to adjust quotes according to order flow. An informed trader will buy, when s/he knows the current price is too low and sells when it is too high. The dealer, on the other hand, should always quote prices for both order signs. As a natural consequence, the dealer loses against informed trader. In order to stay solvent, the dealer is going to try to compensate these losses from trades he executes with uninformed traders. Hence, the bid ask spreads reflect balancing of losses to informed trading with the gains from uninformed (O'Hara, 1995, p. 53-54). Besides the factors transaction costs and inventory risk; asymmetric information is, therefore, a main driver causing market illiquidity.

## 4 QUANTIFICATION OF MARKET LIQUIDITY

In defining market liquidity in Chapter 2 we touched five characteristics of liquidity, namely *tightness*, *immediacy*, *depth*, *breadth* and *resiliency*. Even though there is no single metrics, which would capture all these characteristics fully; there are some measures, which are commonly used in quantifying il/liquidity. These measures can be mainly divided into three groups: (i) *spread measures*, (ii) *execution quality measures* and (iii) *other measures*.

### 4.1 Spread Measures

Spread measures are most commonly used for measuring transaction costs. As illustrated in the previous chapter, dealers will reflect all costs they are exposed into the bid-ask spreads they quote, including order processing costs, asymmetric information costs, inventory-carrying costs and oligopolistic market structure costs. Hence, it can be assumed that bid-ask spreads capture all of these cost components.

#### (i) Quoted Spread

The most intuitive spread measure is the **quoted spread**, which is calculated as the difference between the highest bid  $b$  and lowest ask  $a$  price:

$$S \equiv a - b$$

In addition to the above **absolute spread**, one can calculate the **relative spread**, which is obtained by normalizing the absolute spread with the mid-price  $m$ :

$$m = \frac{a + b}{2} \quad S \equiv \frac{S}{m} \equiv \frac{a - b}{m}$$

For small orders, which can be executed by the best bid or ask price, the quoted spread is the most commonly used measure for liquidity. For larger orders, where intended volume of trade exceeds the quantities stated for the best bid and ask quotes, a **weighted average bid-ask spread** is calculated from the quoted spreads as illustrated in the following:

$$s(q) \equiv \frac{\bar{a}(q) - \bar{b}(q)}{2}$$

where

$\bar{a}(q)$  = average execution price for a market buy order of the size  $q$   $\bar{a}(q)$

$\bar{b}(q)$  = average execution price for a market sell order of the size  $q$

(Foucault, Pagano & Röell, 2013, p.49-50)

(ii) Effective Spread

The quoted spreads address the liquidity of a hypothetical transaction; the **effective spread** instead looks at the trading costs of an actual transaction by taking the difference between the mid-quote  $m$  and the transaction price  $p$

The absolute effective half spread:

$$S_e \equiv d (p - m)$$

where

$p$  = average execution price

$d$  = +1 for buy orders and -1 for sell orders

Relative effective spread:

$$s_e = \frac{S_e}{m} = d \times \frac{p - m}{m}$$

(Huang & Stoll, 1996, p. 324)

(iii) Realized Spread

Both quoted and effective spreads reflect the transaction costs from the view of the trader. One can assume that this 'loss' realized by trader refers to the amount of 'profit' the dealer realizes from that one specific trade. Here this intuition is misleading. The reason is, after execution of trade, the price usually adjust in the direction of the realized trade. This decreases the profit of the dealer from the processed order. Assuming a dealer, who buys 100 shares at a price of 28, when the bid and ask prices are 28 and 29. If the dealer unwind his position immediately at the ask price, he would make a profit of 1. But if the bid-ask spreads decline to, 27.5 and 28.5 his expected profit decreases to zero - assuming he is equally likely to unwind his position at the best bid or ask price. Here both quoted and effective spreads have a tendency to overestimate the profit of the dealer. The realized spread, in contrary, compares the execution price of the trade with a mid-quote, which is right after the execution of the trade (after 5-10 minutes) in order to let the market to reflect the price impact in the

new mid-quote. The realized spread illustrates the real profit of the dealer adjusted for the price movement in the direction of the trade.

For a buy order:

$$S_r \equiv p_t - m_{t+\Delta} = \underbrace{(p_t - m_t)}_{\text{Effective Spread}} - \underbrace{(m_{t+\Delta} - m_t)}_{\text{Mid-quote Revision}}$$

(Zhuk, 2012).

In spite being most commonly used measures of ill/liquidity, spread measures have their deficiencies. First of all, the required data is not always available to market participants. As discussed in previous chapter, bid-ask spreads, even they are available in real time, do not always reflect the real picture of the current trading prices, both in the dealers and limit order markets. For instance, there are some ‘hidden’ orders recorded in the limit order book, where the information about the order is not publicly available. Similarly, in some quote driven markets dealers can quote only ‘indicative’ prices and/or quantities, where they are not obligated to execute any trade under these conditions. As a result, some outlier quotes will be eliminated from the publicly available bid-ask spread data in order not to distort the real picture. Due to aforementioned market aspects such as hidden orders, indicative quotes, execution of trade inside the quote (e.g. recalling our discussion from Chapter 3 on bargaining power of large institutionalized traders), we can conclude that the spread measures can under certain circumstances over- or underestimate liquidity (Foucault, Pagano & Röell, 2013, p.49-55). However, despite their deficiencies, spread measures are still commonly used mainly due to their intuitive nature and ease of calculation.

#### (iv) Transaction Prices vs. Return Covariance

Roll (1984) develops a model, where bid-ask spreads are estimated based on transaction prices. The model assumption is that fundamental value of an asset follows a martingale process

$$\begin{aligned} m_t &= m_{t-1} + u_t \\ E_{t-1}[u_t] &= 0 \\ \text{Var}_{t-1}[u_t] &= \sigma_u^2 \end{aligned}$$

In a competitive market, dealers set prices such that their expected profits are zero. The bid price in the market being (  $m_t - c$  ) and ask price (  $m_t + c$  ); transaction prices are



$$p_t = m_t + q_t c$$

$d_t = +1$  for buy orders and  $d_t = -1$  for sell orders

Since buy and sell orders are equally likely  $E[q_t] = 0$

Price change:

$$\Delta p_t = p_t - p_{t-1} = (q_t - q_{t-1}) \times c + u_t$$

Since  $\text{Var}[q_t] = 1$  ; covariance:

$$\gamma_0 = \text{Var}[\Delta p_t] = \text{var} [(q_t - q_{t-1}) \times c + u_t] = 2c^2 + \sigma_u^2$$

$$\gamma_1 = \text{cov}[\Delta p_t, \Delta p_{t-1}] = \text{cov} [(q_t - q_{t-1}) \times c + u_t, (q_{t-1} - q_{t-2}) \times c + u_{t-1}] = -c^2$$

This reflects Roll's estimate of the absolute value of bid-ask spread, also known as Roll's measure:

$$S_r = 2\sqrt{-\text{cov}[\Delta p_t, \Delta p_{t-1}]}$$

(Foucault, Pagano & Röell, 2013, p.59-60)

Stoll (2000) analyses transaction data for a sample of 1,706 NYSE stocks and 2,184 Nasdaq stocks in a time horizon of 3 months. He estimates the half-Roll's measure as 3.81 cent for NYSE and 11.15 cent for Nasdaq. He also reports average values for the quoted and effective spreads for his sample. On average, the half-quoted spread on the NYSE is 7.9 cents where it is 12.6 cents on Nasdaq. The half-effective spreads are 5.6 and 10.7, respectively. Thus, it can be concluded Roll's measure underestimates the quoted spread and for NYSE it also underestimates effective spread (Stoll, 2000, p. 1510).

## 4.2 Execution Quality Measures

Another way of measuring implicit trading costs is to evaluate the execution quality of the specific order. By using execution quality measures, even if the quote data is not available to calculate spread measures (e.g. lack of information on bid ask quotes at the time of trading for *quoted spread*, right before the trade for *effective spread* and right after the trade for *realized spread*) it is still possible to calculate the trading costs. In the following part, the most commonly used execution quality metrics are illustrated.

### Volume Weighted Average Price (Vwap)

If mid-quotes are not available, another benchmark price has to be chosen to compare the trade price. Day opening/closing prices are sometimes used. But more often an average of transaction prices in a time interval (usually one day) is used. Volume weighted average price (VWAP) is a popular measure, which takes total \$ volume of trading and relates it to the total number of shares traded in one security in a given time interval, hence delivers an average transaction price in that time period

$$VWAP = \frac{\$ \text{ volume of trading}}{\# \text{ of shares traded}} = \frac{\sum_{t \in T} p_t \times |x_t|}{\sum_{t \in T} |x_t|}$$

where

$p_t$  = price of the  $t$ -th trade

$x_t$  = volume of the  $t$ -th trade

(Foucault, Pagano & Röell, 2013, p.55-56)

Traders use VWAP to measure the performance of their brokers; traders compares the average price they got with VWAP to evaluate the execution quality. However, there are several obstacles using VWAP. Firstly, it depends on the order itself. For instance, if the executed own trade construct a large portion of the total trade volume of that specific day, then using VWAP as a benchmark, the trader can measure the exposed transaction cost inappropriately low. For a very illiquid security, if the own transaction of the trader is the only one in that day, the VWAP will imply a trading cost of zero. VWAP is in this context also open to be manipulated by brokers. If a broker has a large buy order from an institution, s/he will split the order into smaller pieces as possible in order to minimize the price impact of the trade. As a result, the time of execution of the overall order will increase, which will automatically cause an increase in the investor's exposed opportunity cost of delay in trading as well as the risk of not execution of the order (Foucault, Pagano & Röell, 2013, p.55-56). Implementation shortfall (IS) is therefore used as a measure to address this problem of opportunity cost for delay in order execution

### Implementation Shortfall

None of the presented measures so far considers time dimension of execution quality of the trade. One of the ways to evaluate trading efficiency is implementation shortfall, which is the

difference between the value of a hypothetical “paper portfolio” and the actual purchased portfolio. Assuming a trader desk of an institution gives 10,000 shares of buy market order for a certain security, given the last available price is \$ 30 (the cost of hypothetical paper portfolio would then be  $10,000 * 30 = \$ 300,000$ ). After execution of trade, the trading desk sees that the order was filled with the execution price of \$ 30,08. However, the total cost of the trade was realized as \$ 302,000, inducing an average price per share \$ 30,20. Here \$ 2,000 reflects the implementation shortfall of the total trade (20 cent per share). If we assume \$ 800 commission charged by the brokerage house; the rest \$ 1200 is due to other components of the implementation shortfall (Strong, 2009, p.527). According to Cheng (2003), in addition to known costs of commission, fees and taxes; implementation shortfall has the components of bid-ask spread, market trend, liquidity impact, opportunity cost and slippage (p. 26).

The value of implementation shortfall is negligible for small orders in large markets. However, it is very important to evaluate the implementation shortfall when it comes to large orders or if small markets are involved. The main reason is, in reality the paper portfolios outperform actual ones. Looking at the return on the paper portfolio would give a wrong assessment of the investment decision and implementation. Paper portfolio is a hypothetical imaginary portfolio that consists of all the assets trader intends to purchase when placing the order, where all the purchases are assumed to be executed at mid-quote prices at the time trader decides to buy these securities. Paper portfolios do not expose to any of the transaction costs that actual portfolios do. Leinweber (1995) calculates the theoretical (paper) return of the *Value Line Portfolio* (a model portfolio based on the recommendations of the Value Line newsletter) as 26.2 percent in the time between 1979 and 1991. The actual realized return in this period was on the other hand only 16.1 resulting in a significant implementation shortfall amount as high as 10.1 percent (Leinweber, 1995, p. 40).

## Real and Paper Value Line Portfolios

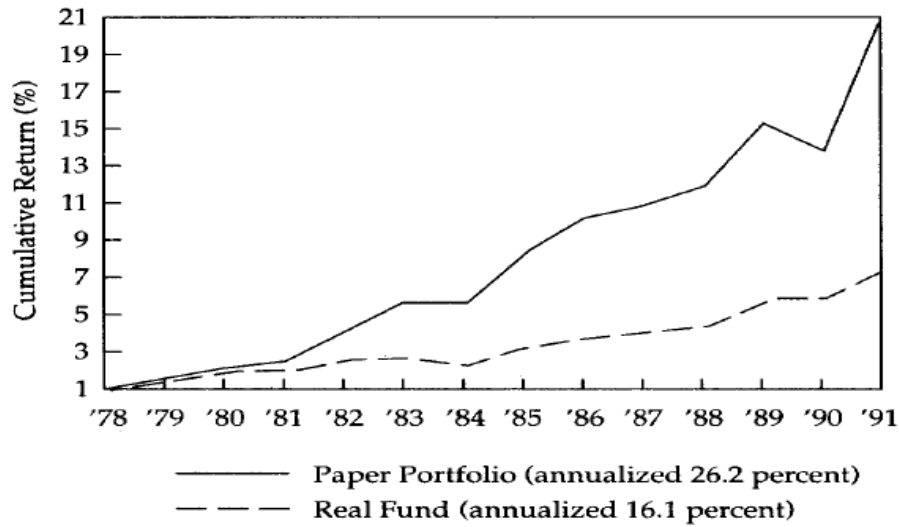


Table 7: Paper portfolio return versus actual portfolio return<sup>5</sup>

### 4.3 Other Measures

#### Volume Based Measures

Looking at the trade volume is one of the most traditionally used (and intuitive) measure of liquidity. The turnover rate is often being used, which proportionate the trading volume to the outstanding volume of the asset. It indicates the number of times the outstanding shares change hand. Sarr & Lybek (2002) illustrates the equation as below:

$$V = \sum P_i \times Q_i$$

where

$V$  = dollar volume traded

$P_i$  = price of the  $i$  trade during a specific period

$Q_i$  = quantity of the  $i$  trade during a specific period

Turnover  $T_n$  is then calculated as

$$T_n = V / (S \times P)$$

where

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<sup>5</sup> Leinweber, D.J. (2002). *Using Information from Trading in Trading and Portfolio Management: Ten Years Later*. California Institute of Technology Working Paper 1135

$S$  = outstanding amount of asset

$P$  = average price of the trade  $i$

The problem with turnover ratio or in general measures solely based on trading volume is that the times when new information arrives to the market are the times when the volatility increases followed by widening of bid-ask spread. Due to increased volume of trading, the turnover ratio will increase and falsely imply an increase of liquidity even though the trading costs are high (Sarr & Lybek, 2002, p.12). Hence, the volatility of the turnover should also be taken into account. In times of stress, for instance, volatilities usually tend to increase even though bid-ask spreads are also higher than those values under normal market conditions.

### Price Impact Measures

Using a vector auto regression analysis, Hasbrouck (1991) provides empirical evidence that the buy orders tend to raise the mid-prices, while sell orders decrease it. This movement in the transaction price from the current quote is called as trade's price impact, which increases with the degree of high illiquidity. The order's impact on the current market mid-price can be shown as

$$\Delta m_t = \lambda x_t + \varepsilon_t$$

where

$m_t$  = mid-price change between  $t-1$  and  $t$

$x_t$  = signed volume of the  $t$ -th trade

$\lambda$  = price impact coefficient

(Zhuk, 2012)

$\lambda$  can be described as a measure of reaction/sensitivity of prices to the trade. Higher  $\lambda$  implies more price reaction, hence more illiquidity associated with the asset.  $\lambda$  equal to zero would mean the prices do not change at all at any given volume of trade, implying a perfectly liquid asset. Stoll (2000) investigated a sample of NYSE / NASDAQ stocks and found that for 98% of them  $\lambda$  is positive (p.1495-1499). One obstacle about this measure is that it requires detailed information on the signed order flows and quantities, which often is not publicly available.

## 5 QUANTIFICATION OF MARKET LIQUIDITY RISK

Following chapter presents market liquidity risk as a quantifiable concept and relates it to the market liquidity concept developed in the previous part. Methods for quantification of the imposed risk exposure are illustrated and compared. The evaluation of the models are conducted mainly with respect to their ability to capture components of exposed risks to liquidity adequately; thus can serve as a tool for risk managers in their attempts to quantification of market liquidity risk.

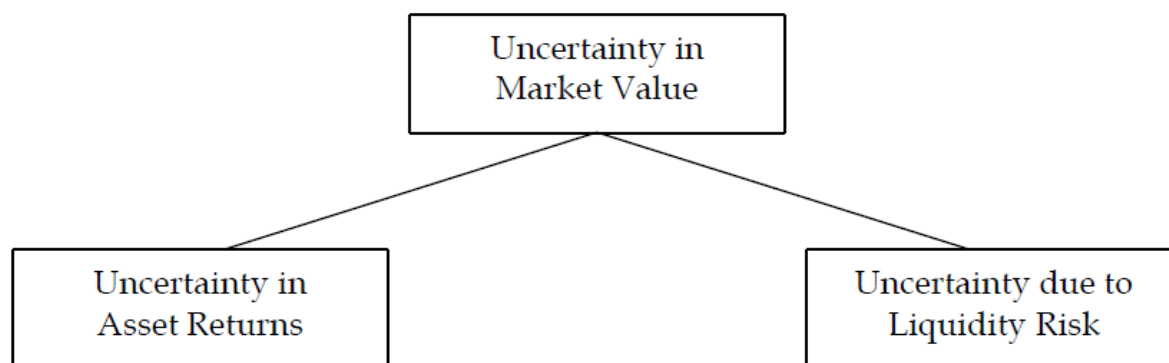
### 5.1 Market Liquidity versus Market Liquidity Risk

In previous chapters, we identified market liquidity with the degree of existence of complete markets, where the factors transaction costs, inventory risk and asymmetric information are the main determinants of the market ill/liquidity. We illustrated that in real markets, the actual transactions have a so-called ‘price impact’ on the current prices, where investors face a price concession comparing to the current market price of the asset. This price concession induces a loss for the trader. Against this background, Loebnitz (2006) describes **market liquidity** as the discounted expected price concession required for an immediate trade of an asset under a specific trading strategy. Here it would be more adequate to consider the total trade (sale/purchase of the total intended quantity) rather than looking into smaller pieces, which would give a deceiving picture. Due to the uncertainty in the amount of aforementioned price concession; the trader is exposed to a ‘risk’, which is associated with the potential loss in case the adverse price movement is worse than the expected value by the trader. This leads to a formal definition of **market liquidity risk**, which is the risk of losses due to uncertainty of deviation of the price from the expected values in immediate trading of an asset under a specific trading strategy (Loebnitz, 2006, p. 55-61). This definition enables to use statistical quantities to describe loss distribution as risk measure for a specific time horizon.

### 5.2 Relation between Market Liquidity Risk and Market Risk

Basel Committee on Banking Supervision (1996) defines **market risk** as the risk of losses arising from adverse movements in market prices, such as changes in equity prices, interest rates, foreign-exchange rates, commodity prices (p.1). Bangia (1999) conceptually split

uncertainty in market value of an asset (i.e. its overall market risk) into following two categories:



*Table 8: Comparison of Actual Returns with Cumulative Normal Returns<sup>6</sup>*

Financial institutions use a number of mathematical and statistical models to measure market risk. Among those approaches, **Value-at-Risk (VaR)** is the most commonly used measure in the industry and has become a regulatory standard in measuring market risk. However, conditional VaR models focus solely on capturing the risk due to uncertainty of asset returns; where the uncertainty component due to liquidity risk is completely ignored. As a result, traditionally used unconventional VaR models systematically underestimate the exposed risk. Indeed in times of 2007-2008 financial crisis many banks reported more than 30 days in which the losses were more than VaR, where an average of 3-5 days would be the norm (Mehta, Neukirchen, Pfetsch, 2012, p.1).

Even though market risk and market liquidity risk refer to distinct concepts, they are very strongly interconnected. Thus, measurement of the both risk types should be incorporated in an integrated framework. In the following chapters of this paper, some of the models built in this framework will be examined.

### 5.3 Criteria for Evaluation of Models

There are several models developed by academicians and practitioners attempting to quantify market liquidity risk with accompanying empirical research and back-testing work to compare performance of different models in use. In general, in evaluating of a quantification model

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<sup>6</sup> Source: Bangia, A., Diebold, F., Schuermann, T., Stroughair, J.D. (1998). *Modeling Liquidity Risk With Implications for Traditional Market Risk Measurement and Management*. University of Pennsylvania Wharton Financial Institution Center Working Papers 99-06. p. 3

from perspective of risk managers, below two criteria should be taken into account (Malz, 2003, p.45)

**1.) Degree to which the liquidity statistic actually captures the behaviour it meant to measure**

In previous chapters, we explained some basic concepts of market microstructure leading to illiquidity. We listed characteristics and factors of market illiquidity. In general attempts for modelling fail in addressing all these components of the risk. As a result, the models often either under- or overestimate the exposed actual risk.

**2.) The extent to which high quality data are likely to be available**

In deciding what model to use, a very important factor to be considered is the availability of data the model requires for the model set-up. Here another important aspect would be the reliability of the available data: it is very essential to analyse the degree of data quality prior to implementation of any model to have reliable results.

In the following, existing models for quantification of market liquidity risk will be broadly divided into two groups and examined separately, namely (i) *standard models* and (ii) *advanced models*. The former refers to those models which are commonly used in financial industry; where the latter refers to more sophisticated models.

## **5.4 STANDARD MODELS**

### **5.4.1 Adverse Price Impact Measures**

These models try to capture market liquidity risk by measuring the adverse price impact. But instead of quantifying the price impact of individual intended trade, they choose a simplified approach, in which additional market risk due to time interval between submitting an order and execution will be measured. The number of trading days for liquidation of a position will be denoted with  $T$ . If we assume the overall position is hold for  $T$  days, then the  $T$ -day VaR could be calculated with the famous square root of time rule:

$$\text{1-day position VaR} \times \sqrt{1^2 + 1^2 + \dots + 1^2} = \text{1-day asset VaR} \times \sqrt{T}$$

(Malz, 2003, p.50)



Basel Accord requires banks to report 10-day VAR instead of 1-day VAR. Applying the above formula, 1-day asset VaR would be multiplied with the square root of 10, which is 3,1623. Thus 10-day VAR would be more than 3 times of daily VaR. However, this approach is an overestimate of VaR; the VaR should indeed be between 1-day position VaR and 1-day position  $\text{VaR} \times \text{SQRT}(T)$ . Because the position size will not stay the same during holding period  $T$ , rather it will decrease and we will hold the position size of  $1, \frac{T-1}{T}, \frac{T-2}{T}, \dots, \frac{T-2}{T}$ ; instead of  $1, 1, 1, \dots, 1$ . As a result:

$$\begin{aligned} & \text{1-day position VaR} \times \sqrt{1 + \left(\frac{T-1}{T}\right)^2 + \left(\frac{T-2}{T}\right)^2 + \dots + \left(\frac{1}{T}\right)^2} \\ &= \text{1-day asset VaR} \times \sqrt{\sum_{t=1}^T \left(1 - \frac{t-1}{T}\right)^2}, \end{aligned}$$

which is equal to

$$\text{1-day position VaR} \times \sqrt{\frac{(1+T)(1+2T)}{6T}}$$

(Malz, 2003, p.50)

If we assume  $T$  as 10 trading days, then the adjustment to the overnight VaR as shown in the above formula would be 1,9621. This corresponds to an increase of 96% of the VaR, which means almost doubling of the overnight VaR of the position.

In implementing above approach, the most crucial aspect is to decide the holding period of  $T$  for a position.  $T$  is generally estimated as

$$T = \frac{\text{position size}}{\text{daily trading volume}}$$

where daily trading volume is averaged over a time horizon (i.e. month). Alternatively, traders estimate  $T$  based on their judgement on market conditions and their trading experience over time (Malz, 2003, p.51).

#### 5.4.2 Spread Adjustment Models

In addressing liquidity risk, Bangia (1998) uses transaction costs as a proxy and adjusts the conditional VaR model taking the bid-ask spread into account. He uses traditional conventional VaR models for measuring price risk and an adjustment consisting of a certain percentile of the relative spread distribution is used (Relative Spread = (Ask Price – Bid Price) / Midprice). In calculating so-called **Liquidity Adjusted Value At Risk (L-VaR)**, Bangia attempts to combine market and liquidity risk by incorporating both a 99<sup>th</sup> percentile

movement in the underlying asset as well as a 99<sup>th</sup> percentile movement in the spread, which he shows graphically as follows

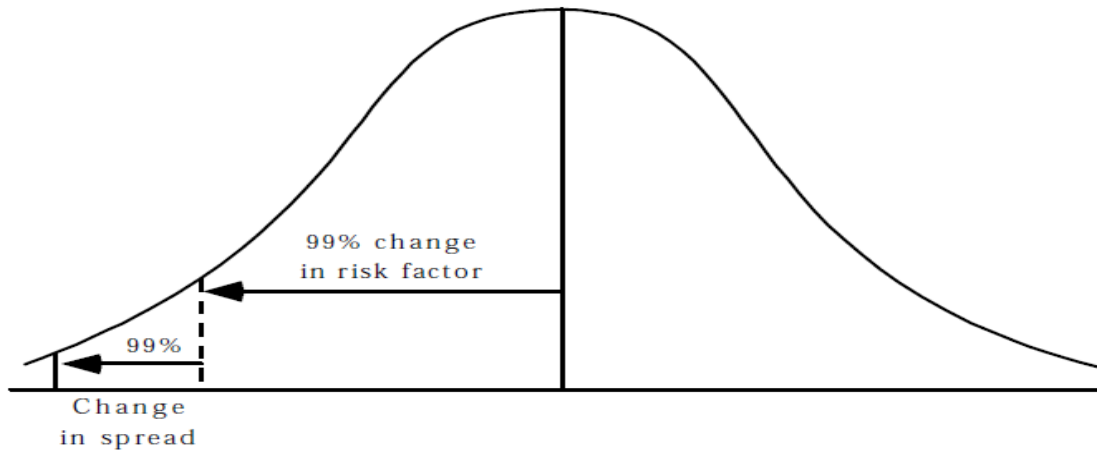


Table 9: Combining market and liquidity risk via L-VaR<sup>7</sup>

The model can be described as

$$L - VaR = 1 - \exp(z\sigma_r) + (\mu_s + \hat{z}_s\sigma_s)$$

where

$\sigma_r$  = variance of continuous mid-price return

$\sigma_s$  = variance of the bid ask spread

$\mu_s$  = mean of bid ask spread

$z$  = percentile of the normal distribution for given confidence

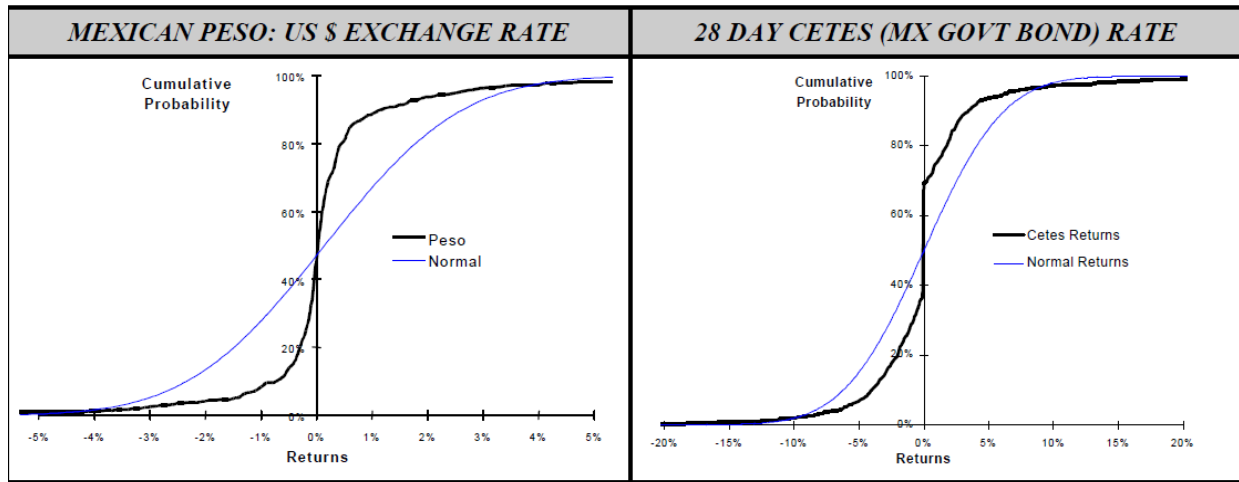
$\hat{z}_s$  = empirical percentile of the spread distribution

(Ernst, Stange & Kaserer, 2012, p.135)

One of the drawbacks of the model is that it assumes normal distribution of market returns, where in many markets the assumption of normality does not hold. Below chart illustrates an example for violation of normality assumption by comparing of actual returns with cumulative normal returns for two emerging market instruments, Mexican Peso/US Dollar exchange rate and 28-day cetes (Mexican government bond) rate (Bangia, 1998, p.9)

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<sup>7</sup> Source: Bangia, A., Diebold, F., Schuermann, T., Stroughair, J.D. (1998). Modeling Liquidity Risk With Implications for Traditional Market Risk Measurement and Management. University of Pennsylvania Wharton Financial Institution Center Working Papers 99-06. p. 8



Sample Period: Jan'94-April'96

Table 10: Comparison of Actual Returns with Cumulative Normal Returns<sup>8</sup>

Here the usage of standard deviation multiples assuming a normal distribution (e.g. 2.33 for worst 1% return) causes underestimation of the exposed risk. In order to address the issue of non-normality, Bangia (1999) designs a correction factor  $\theta$  to take into account leptokurtic or “fat-tailed” distributions: the parametric assumption asserts multiplication of  $2.33\sigma_s$  with a  $\theta$  greater than one, where  $\theta$  stands for an explicit function of the unconditional kurtosis of the return distribution (p.9).

Ernst (2012) uses a similar approach to Bangia (1999); but he assumes a non-normal distribution for future prices and spreads, where this distribution is estimated via Cornish-Fisher approximation

$$L - VaR = 1 - \exp(\mu_r + \tilde{z}_r \sigma_r) \times (1 - \frac{1}{2} (\mu_s + \tilde{z}_s \sigma_s))$$

(Ernst, Stange, Kaserer, 2012. p. 135-136)

The notations in the above formula are as noted above for Bangia (1999), where  $\tilde{z}$  is the non-normal-distribution percentile adjusted for skewness and curtosis according to Cornish-Fisher expansion.

#### 5.4.3 Comparison and Evaluation of Standard Models

In comparing and evaluating of Standard Models described above, the two criteria that we set in Chapter 5.3 are used:

<sup>8</sup> Source: Bangia, A., Diebold, F., Schuermann, T., Stroughair, J.D. (1998). Modeling Liquidity Risk With Implications for Traditional Market Risk Measurement and Management. University of Pennsylvania Wharton Financial Institution Center Working Papers 99-06. p. 9

**Criteria Nr 1.) Degree to which the liquidity statistic actually captures the behaviour it meant to measure**

The most commonly used standard model attempting to capture **adverse price impact** is VaR model that is adjusted for market liquidity risk. Despite it is so commonly used and become a standard in the industry, VaR model has very significant drawbacks. Lawrance and Robinson (1995) raise their critics on VaR stating: “*Can we be 98% confident that no more than an amount of  $L$  [the VaR estimate at  $\alpha = 0.98$ ] would be lost in liquidating the position? The answer must be ‘no!’*”. Assuming a trader would like to liquidate a position at time  $t$ , where during the next 1 day his/her standing order could not be executed in the market (due to illiquidity). After 24 hours of no trading, the position is liquidated at prices which are drawn from a (pre-specified) distribution, which is unaffected by the process of liquidation. In the act of liquidation, the price would move adversely against the trader, especially for large orders and illiquid assets, the cost of liquidation can be very significant (Loebnitz, 2006, p. 52-55).

Hisata and Yamai (2000) summarizes the main drawbacks of unconditional VaR models as (i) price impact is not considered (influence of the trader’s own trade on price) (ii) trading at mid-prices assumed, hence no consideration of influence of bid ask spreads (iii) it assumes the bank’s position can be liquidated in a short period of time (p.84). These assumptions are not easy to hold not only in times of crisis but also under normal market conditions.

The commonly used attempts to adjust the VaR to account for liquidity risk, by increasing the applied volatility or by increasing the time horizon in calculating the VaR are not solid and risk sensitive approaches in addressing the liquidity risk, as we defined in this paper. First, it is very difficult to decide on adequate number for time-to-liquidate measure  $T$ . The commonly used methods, such as dividing position size to trading volume as an estimate for  $T$  can be quite misleading and might lead to underestimation of liquidity risk especially in stressed markets. In times of crisis, the volatility increases. In general, trading volume increases in times market volatility rises. The increase in trading volume would lead to a decrease in  $T$  referring to decrease in time-to-liquidation. This is contradictory to the real behaviour in actual markets, where it is unusual for traders to liquidate a position much quicker in times of stress. Volume-based liquidity measures in general lack the essence of addressing the liquidity

risk and there is little evidence of their association with other liquidity measures. For instance, Chordia *et al.* (2001) illustrates very low even negative correlations between trading volumes and bid-ask spreads for US stocks (Malz, 2003, p.51-52).

The second group of standard models discussed in this paper is so-called **spread adjustment models**, where their biggest drawback lie on the fact that they neglect the price impact. They assume quoted spreads reflect the price impact and the trade will occur within the spread (independent of the trade volume). Especially in case of large trade volumes, this basic underlying assumption of the model will lead to significant underestimation of the liquidity risk. Hence, the model is not appropriate for measuring liquidity risk in particular for those assets where price impacts are non-decreasing in trading volume (e.g. stocks). For other instruments with non-increasing price impact functions (such as bonds) Bangia's L-VaR could be used as a proxy. One way to address this issue would be to use effective or weighted average spreads (as described in Chapter 5.5.2), however this adjustment would require transaction prices, which would then eliminate the most advantageous feature of the model, which is being most practical to implement as it requires the least data (Loebnitz, 2006, p.72-88). Another conceptual problem with Bangia (1999) approach is that it assumes normal distribution of market returns, where in many markets we know the assumption of normality does not hold. Ernst (2012) tries to address this drawback, where he assumes a non-normal distribution for future prices and spreads, where this distribution is estimated via Cornish-Fisher approximation. Even though Ernst (2012) lead to a more accurate estimate comparing to Bangia (1999), the same critic for neglecting the price impact is still valid for this model. Below table shows acceptance rate of the both spread adjustment models by order size, where acceptance rate represents the percentage of stocks with statistically significant precise risk estimation according to Kupiec (1995):

Acceptance Rate	Order size (€ thousand)															
	Min	10	25	50	75	100	150	200	500	750	1000	2000	3000	4000	5000	All
Ernst (2012)	79%	82%	77%	65%	55%	51%	35%	29%	19%	11%	13%	15%	10%	7%	4%	44%
Bangia (1999)	39%	44%	31%	18%	16%	13%	11%	8%	7%	6%	5%	0%	0%	0%	0%	16%

Table 11: Acceptance rate of Spread Adjustment Models by order size<sup>9</sup>

<sup>9</sup> Source: Source: Ernst, C., Stange, S., Kaserer, C. (2012). *Measuring Market Liquidity Risk: Which Model Works Best?* Center for Entrepreneurial and Financial Studies, Working Paper Series, No. 1, p.141

As expected, performance of both models is decreasing significantly as the order size increases, since they rely solely on bid-ask spread data and do not account for order size. Even though Ernst (2012) significantly outperforms Bangia (1999), its performance can still be evaluated as poor (comparing to more advanced models that will be discussed in the following part) considering its overall acceptance rate which is less than 50%. Nevertheless, it should be stressed that despite their drawbacks, spread adjustment models provide certainly a more prudent and realistic approach than conventional VaR models discussed previously, which totally neglect the spread component (Loebnitz, 2006, p. 88).

### **Criteria Nr 2.) Extent to which high quality data are likely to be available**

The most practical model to implement for practitioners would obviously be adjusting the unconventional VaR model, which is already extensively used by banks for their market risk quantification. These models do not require any extensive additional data and their model set-up is also quite easy to build and integrate into day-to-day risk management practises. That is also the reason why despite their significant drawbacks, they are a common practise in the industry.

Spread adjustment models, Bangia (1999) and Ernst (2012), require time series of midpoint prices and spread data. Both models are quite practical to implement due to their ease of calculation and not requiring a detailed set of data.

In general, it can be concluded that in comparison to the advanced models which will be described below, the standard models require less data and their model set-up is relatively easy to implement.

## **5.5 ADVANCED MODELS**

### ***5.5.1 Market Price Response Approach***

Berkowitz (2000) measures market liquidity risk by addressing the price impact. He describes liquidity as uncertainty in changes of price of an asset which goes beyond the reactions to the 'exogenous' factors in the market. It means, the price changes occurring in case of trading of an asset can be used as an indicator for liquidity (isolated from those changes caused by external market dynamics such as interest rate changes, arrival of new information to the market etc.). The model is described by Loebnitz (2006) as illustrated below:

The transaction price at time  $t$  can be written as

$$p_t = p_{t-1} + x_t - \theta q_t$$

where

$q_t$  = trade size

$x_t$  = exogenous factors at time  $t$

$\theta q_t$  in the equation is a measure for negative price impact (the sign is negative for sell, positive for buy orders). Berkowitz formulates the trading as an optimization problem, where trader tries to optimize the revenues from the sale of an asset. The assumptions are the reactions of market participants to the exogenous market factors are rational and there are no informed traders

$$\max_{q_t} E_t \left[ \sum_t^T p_t q_t \right]$$

subject to

$$\sum_t^T q_t = M_t$$

where

$M_t$  = total number of units that is to be sold until  $T$

Bertsimas and Lo (1998) proves that  $q_t^* = \frac{M_t}{T}$  is the optimal solution to the above problem.

With the availability of the historical data on portfolio value and net flows, one can derive the liquidity coefficient  $\theta$  with the following regression:

$$p_{t+1} - p_t = \alpha + x_{t+1} - \theta q_t^* + \varepsilon_t$$

Given the below mean of the portfolio and variance from the price impact

$$E_t(y_{t+1}) = Q_t' (p_t + Ex_{t+1} - \theta E q_t^*)$$

$$Q_t' (\text{var}[\theta q_t]) Q_t$$

where

$Q_t$  = an  $N \times 1$  vektor of asset positions ( $Q_t'$  is the transpose of this vector)

$y_{t+1}$  = portfolio value at time  $t+1$

To calculate VaR, the one-step ahead mean and variance can be forecasted or the entire forecast distribution of the portfolio value assuming tractable distributions for the factor prices and the distribution of the trades  $q_t$  can be estimated. It should be noted that the liquidity coefficient  $\theta$  addresses both spread and price impact (Loebnitz, 2006. p. 73-74).

### 5.5.2 Weighted Spread Models

For these models limit order book data is used. The liquidity cost measure weighted spread (WS) is taken as a basis, which is calculated as

$$WS_t(q) := \frac{a_t(v) - b_t(v)}{P_{mid,t}}$$

where

$a_t(v)$  = volume weighted ask price of trading  $v$  shares

$b_t(v)$  = volume weighted bid price of trading  $v$  shares

$q$  = total order size

(Ernst, Stange & Kaserer, 2012, p. 137)

Total order size  $q$  is split up and executed in smaller parts, where  $q / P_{mid} = v$

Ernst, Stange & Kaserer (2012) further presents the L-VaR for different models using weighted spread approach as summarized in the following (p.137-138)

Francois-Heude and van Wynendaele (2001):

$$LVaR(q) = 1 - \exp(-z\sigma_r) \left( 1 - \frac{\mu(q)_{ws}}{2} \right) + \frac{1}{2} (WS_t(q) - \mu(q)_{ws})$$

$z$  = normal percentile of the mid-price return distribution

$\sigma_r$  = standard deviation of the mid-price return distribution

$\mu(q)_{ws}$  = average spread for a security for order  $q$

$WS_t(q)$  = the spread at time  $t$

Giot and Grammig (2005):

They define net return as



$$r_{net_t}(q) = r_t \times (1 - \frac{WS_t(q)}{2})$$

and liquidity adjusted risk is calculated as

$$LVaR(q) = 1 - \exp(\mu_{r_{net}(q)} + Z_{ST} \sigma_{r_{net}(q)})$$

where  $Z_{ST}$  is the percentile of the t-distribution.

Stange and Kaserer (2008):

They calculate the VaR based on empirical percentiles as

$$LVaR(q) = 1 - \exp(\mu_{r_{net}(q)} + \hat{Z}_{(q)} \sigma_{r_{net}(q)})$$

where  $\hat{Z}$  is the empirical percentile of the net return distribution

Modified Bangia Model:

The Bangia (1999) model, presented in Chapter 5.4.1, can also be modified using weighted spreads and Cornish-Fisher approximation (analog to Ernst (2012))

$$LVaR(q) = 1 - \exp(\mu_r + \hat{Z}_t \sigma_t) \times (1 - \frac{1}{2} (\mu_{ws}(q) + \hat{Z}_{ws}(q) \sigma_{ws}(q)))$$

where  $\hat{Z}$  is the percentiles estimated based on Cornish-Fisher approximation.

### 5.5.3 Other Economic Models

There are several other more sophisticated models measuring liquidity risk in the market microstructure literature. Among those Cetin *et al.* (2004) suggests a stochastic supply curve to an asset's price which is changing depending on the transaction size. Hence, the direction of the trade and the intended volume of the trade decide the trade price. Jarrow and Protter proposes a linear supply curve with randomly changing slope coefficients as a first approximation referring to Cetin *et al.* (2004), Blais & Protter (2005) and Blais (2006). The supply curve function has a different slope coefficient in normal times than it is in times of crisis, where the price impact in crisis is larger than under normal circumstances. As for risk management purposes, the estimation of asset price process (e.g. geometric Brownian motion) only in extreme cases has a vital importance, hence the formulation can be reduced omitting the coefficients for the normal case. The slope coefficient is estimated for each asset via a simple regression. Having calculated the estimated slope coefficient, the portfolio value and

VaR figures can be driven given the supply curve. The model developed by Almgren and Chriss (2000) considers additionally the trading strategies. It tells a trader the optimal trading strategy prior to the trade given a desired total size of intended trade. The attempt is to minimize both transactions costs and price risk associated with the trade. The model results in an efficient frontier which illustrates the minimum expected cost for different levels of uncertainty. The optimal strategy for trade is then decided dependent on the risk aversion of the trader (Loebnitz, 2006, p.74-90).

Kyle (1985), on the other hand, takes asymmetric information component of ill/liquidity as a base for measurement. Kyle considers a market where both informed and uninformed traders submit a market order for an asset and the market maker (dealer) set a price responding to aggregate order flow, where expected value of his/her gain is equal to zero. The uninformed trader submits an order  $u$ , while informed trader submits an order  $x$  depending on his information on asset value  $v$ . Kyle illustrates the linear equilibrium in which market maker set price as

$$p_t = E(v | \mathfrak{F}_t, u_t + x_t) = p_{t-1} + \lambda_t (u_t + x_t)$$

A large demand to a certain security can be an indicator for the dealer that there might be an informed trader in the market, who knows the asset is currently undervalued. Thus the dealer raises prices closer to  $v$ . In order to prevent this, the informed trader's order size  $u$  is limited in size. In the above equation,  $\lambda$  represents a quite precise measure for market illiquidity, capturing price change per unit of net order flow, in other words, so-called market impact (Amihud, Mendelson & Pedersen, 2005, p. 297).

#### 5.5.4 Comparison and Evaluation of Advanced Models

In comparing and evaluating of Advanced Models described above, the two criteria that we set in Chapter 5.3 will be used:

##### **Criteria Nr 1.) Degree to which the liquidity statistic actually captures the behaviour it meant to measure**

The advanced models, are mainly built on the microstructure aspects of the markets, which we analysed extensively in this paper. In this respect, they address the market liquidity risk concept more accurately in comparison to the standard models we presented earlier, yet they have also certain drawbacks.

As per **market price response** models, considering its accuracy for describing market liquidity, Berkowitz (2000) approach seems as a proper measure for liquidity, where  $\theta$  captures directly the price impact and indirectly the spread components (expected bid-ask spread and volatility of bid-ask spread). However, the empirical performance measure of the model does not support this assumption. Below table shows regression estimates of the liquidity measure  $\theta$  based on the empirical analysis conducted by Ernst, Stange & Kaserer (2012)

**Liquidity Coefficient  $\theta$  in € per million shares**

	Index				
	DAX	MDAX	SDAX	TECDAX	All
<b>Mean</b>	0,03	0,30	5,24	0,37	1,84
<b>Median</b>	0,01	0,06	0,17	0,04	0,03
<b>Max</b>	0,23	12,50	1.777,00	24,40	1.777,00
<b>Min</b>	-0,12	-14,30	-53,10	-3,95	-53,10
<b>Std.Dev.</b>	0,05	1,37	91,90	2,94	52,20
<b>Signif. fraction at 95% confidence</b>	53%	36%	45%	54%	44%
<b>Signif. fraction at 99% confidence</b>	44%	27%	37%	46%	36%

Table 12: Comparison of Actual Returns with Cumulative Normal Returns<sup>10</sup>

The above illustrated regression produces positive and negative estimates for  $\theta$ , which is not inline with our assumption that liquidation of a position would always lead to a price concession. It can also be seen that for only about half of the stocks  $\theta$  significantly different than zero. These results leads to strong doubt for the accuracy of the model results (Ernst, Stange & Kaserer, 2012, 136).

The second group of models using **weighted spread** as a measure for liquidity delivers an estimate from a transaction perspective. In comparison to above measures capturing price impact (e.g. Berkowitz (2000)), weighted spread measures can be evaluated as a more precise measure for ex-ante, order size differentiated liquidity cost beyond the bid-ask spread depth (Stange & Kaserer, 2008, p.7). In this framework, they can be assed also more accurate in comparison to standard spread adjustment measures such as Bangia (1998,1999), which proxies liquidity costs of any order size with the quoted bid-ask spreads. Within the models

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<sup>10</sup> Source: Source: Ernst, C., Stange, S., Kaserer, C. (2012). *Measuring Market Liquidity Risk: Which Model Works Best?* Center for Entrepreneurial and Financial Studies, Working Paper Series, No. 1, p.137

using weighted spread as proxy, Stange & Kaserer (2008) can be chosen over other presented alternatives as they take empirical percentiles instead of some parametric method and avoid any assumption on distribution, in particular for liquidity cost, as in the case of Giot and Grammig (2005) and Francois-Heude and van Wynendaele (2001). Their approach takes the percentile of net return distribution and treats price and liquidity risk not as separate but as interconnected factors (Stange & Kaserer, 2008, p.10). Below table shows acceptance rate of presented weighted spread models by order size, where acceptance rate represents the percentage of stocks with statistically significant precise risk estimation according to Kupiec (1995):

Acceptance Rate	Order size (€ thousand)															
	Min	10	25	50	75	100	150	200	500	750	1000	2000	3000	4000	5000	All
<b>Modified Bangia model</b>	81%	80%	76%	77%	82%	79%	79%	75%	66%	63%	60%	78%	63%	58%	56%	74%
<b>Stange and Kaserer</b>	78%	73%	76%	79%	77%	77%	82%	77%	71%	71%	75%	50%	42%	41%	34%	74%
<b>Giot and Grammig</b>	62%	59%	58%	57%	63%	65%	76%	81%	85%	88%	84%	92%	86%	82%	88%	71%
<b>F.Heude &amp; v. Wynendaele</b>	3%	15%	15%	13%	16%	12%	14%	14%	18%	21%	20%	8%	9%	6%	12%	15%

*Table 13: Acceptance rate of Weighted Spread Models by order size<sup>11</sup>*

The last group of advanced models we described above, so called **other economic models**, address market liquidity risk from conceptual perspective much more accurately than others. However, they are generally not implemented by practitioners, either because of data limitations or because they are not appropriate outside of a model context. As this paper mainly aims to present tools that can be used by risk managers in financial institutions for quantification of market liquidity risk in practise, these models will be briefly discussed and not examined extensively.

Among the advance models existing in microeconomic literature, the stochastic supply curve approach from Cetin is one of the most intuitively appealing and easy to implement. It assumes a stochastic supply curve for an asset's price as a function of transaction size; thus trade price is determined by the position size and order sign (Loebnitz, 2006, p.78). The model addresses both bid-ask spread and price impact component of market liquidity risk properly. The model's biggest drawback, on the other hand, lies on its simplistic assumption of linear supply curve. The model developed by Almgren and Chrisis (2000) is, amongst all the models presented in this paper, the most complete model in terms of capturing market

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<sup>11</sup> Source: Source: Ernst, C., Stange, S., Kaserer, C. (2012). *Measuring Market Liquidity Risk: Which Model Works Best?* Center for Entrepreneurial and Financial Studies, Working Paper Series, No. 1, p.141

liquidity risk as it is defined in this paper (Loebnitz, 2006). It incorporates trading strategy and risk aversion of trader in a proper manner. The biggest strength of Almgren and Chriss (2000) model is that it is firmly grounded in market microstructure theory and empirical evidence. The model incorporates the concepts of price risk and price impact in a consistent framework. Despite its elegant approach capturing market liquidity risk, one can argue its reliability out of a modelling concept due to the following debatable aspects (1) continuous time model (not realistic – discrete trading), (2) price impact formulations (non-random, cross impacts – estimation effort), (3) trading strategy (Linear static strategy – not realistic but would be sufficient for risk management purposes), (4) asset price process (Wiener process – too thin tails; the most important drawback for using it for risk management purposes) and (5) coefficient estimations (Estimation from crisis times is problematic) (Loebnitz, 2006, p.81-114).

#### **Criteria Nr 2.) Extent to which high quality data are likely to be available**

**Market price models** are the most convenient approaches in terms of availability of required data. Even though model performance of Berkowitz (2000) is not very superior, it is still a legitimate alternative as it requires only transaction data, specifically: time series of midpoint prices, transaction prices and transaction size. In the absence of daily transaction prices, mid-prices can be used, which might lead to underestimate the exposed liquidity risk (Ernst, Stange & Kaserer, 2012, p.141).

It is more difficult to retrieve data requested by the **weighted spread models**. For these models limit order book data is used. In order to calculate weighted spread, volume weighted ask and bid prices are necessary for intended order size. This data is not available in all markets; even it is available its reliability for the purpose of model usage is questionable. In some markets, for example, traders are allowed to submit so-called ‘hidden orders’, which will not be publicly quoted. Or, in trading of large volume of government bonds especially by large institutions, the pricing is set through bilateral negotiations between the trader and market makers and the actual transaction price might deviate from those listed bid and ask prices. Thus, the extracted data by practitioners for the model set-up might be not complete and can lead to distorted model results.

In their research paper, Ernst, Stange & Kaserer (2012) compares the empirical performance of different models. In a 5.5-year stock sample they show which model provides the most accurate results by executing a comparative back-testing of daily risk forecasts for selected models, which were presented above. The below table illustrates the results of this study

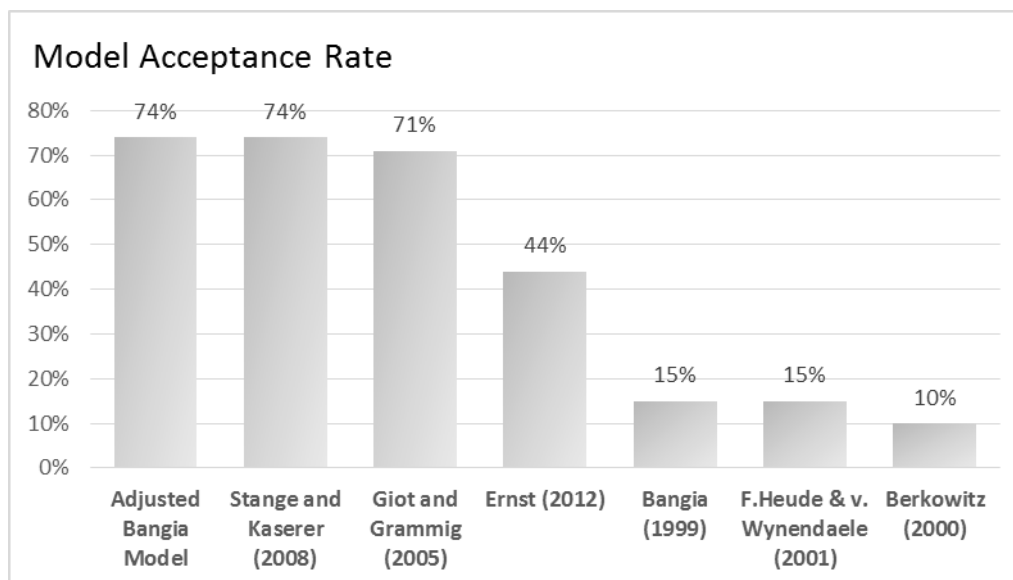


Table 14: Ranking of market liquidity risk models by overall acceptance rate<sup>12</sup>

*Not: The model called “Adjusted Bangia Model” is a modified version of the model using weighted spreads and Cornish-Fisher approximation - analog to Ernst (2012). Refer to Chapter 5.5.2*

Ernst, Stange & Kaserer (2012) shows with the above research that availability of data is the key driver for accuracy in risk forecasting. The models based on limit order data (Stange/Kaserer, Giot/Gramming and Adjusted Bangia Model) perform better than those using bid-ask spreads (Ernst and Bangia) or transaction data (Berkowitz). It is concluded that Berkowitz (2000) should only be used if nothing else than transaction data is available. If limit order book data is available, approaches based on empirical (Stange & Kaserer (2011)) or t-distributed net returns (Giot and Grammig (2005)) model show statistically satisfactory results. They also conclude that in case only bid ask spread data is available, Ernst (2012) outperforms Bangia model (p. 143).

<sup>12</sup> Source: Ernst, C., Stange, S., Kaserer, C. (2012). *Measuring Market Liquidity Risk: Which Model Works Best?* Center for Entrepreneurial and Financial Studies, Working Paper Series, No. 1, p.140

As discussed earlier, the biggest drawback of the last group of advanced models, so-called **other economic models**, is their burdensome data requirement. Most of them require historical data series on signed orders (direction) or size of the trade volumes, which are usually not readily available to market participants. Even if the data is available, most of these measures are still not appropriate to implement in real life out of a model context because of their underlying assumptions; such as Kyle's  $\lambda$ , in which the equilibrium price can be held constant while only liquidity-related quantities fluctuate (Malz, 2003, p.45). The Almgren and Chriss (2000, 2003, 2005) model that we identified as the most complete model in the previous section from a pure theoretical perspective, for example, require time-series of midpoint prices, transaction prices, transaction sizes and execution times, which are often not available in the requested time intervals (Leobnitz, 2006, p.92). For some models, even though the requested data would be somewhat available in the market, the benefits intended to be received from the model results may not pay off considering its computational burden. As a result, given the degree of complexity and extensive data requirement of these models, they are usually not preferred by practitioners.

## 5.6 MODELS FOR ILLIQUID MARKETS

The above described models are not always suitable for those emerging markets, where the market tends to deviate significantly from the hypothetical complete market concept. The biggest problem in those markets is that the lack of data set does not allow to build up a proper model capturing different aspects of ill/liquidity. There are simplified approaches developed for quantification of market illiquidity in these markets, some of which will be presented in the following part.

### 5.6.1 *Zero Rates Return*

A commonly used and intuitive method measuring of liquidity in so-called illiquid markets is looking at the frequency, in which no trades occur. Especially in very thin markets, there are days even weeks where an asset is not traded. In those cases the exchange report a so-called **stale price**, which is the realized transaction price of the last trade. It is possible to use a proxy, which measures illiquidity as a function of no-trade days, described as days with zero daily returns (Foucault, Pagano & Röell, 2013, p.59).

Lesmond, Ogden, and Trzcinka (1999) provides a zero-return measure, which relates the number of zero-return days to the number of trading days in a given month. The so-called LOT (1999) measure is noted as

$$ZR_{i,t} \equiv \frac{N_{i,t}}{T_t}$$

where  $T_t$  is the number of trading days in month  $t$  and  $N_{i,t}$  is the number of zero-return days of stock  $i$  in month  $t$  (Lee, 2006, p.13).

### 5.6.2 Roll's Measure

In illiquid markets, the historical time series for bid-ask spreads are usually not available. Roll (1984) develops a model, where bid-ask spreads are measured solely based on the transaction prices. Roll's estimate of the absolute value of the bid-ask spread, also known as Roll's measure, is defined as:

$$S_r = 2\sqrt{-\text{cov}[\Delta p_t, \Delta p_{t-1}]}$$

(Foucault, Pagano & Röell, 2013, p.59-60)

Chapter 4.1 provides a more detailed illustration of underlying assumptions and derivation of the Roll's measure.

### 5.6.3 Turnover

Volume-based measures are very intuitive and simple, thus are frequently used as a proxy for liquidity. Amongst other measures taking trading volume as a base, 'turnover' is the proportion of the trade volume in shares of stock to the total number of outstanding shares. Turnover (TO) is therefore defined as:

$$TO_{iy} = \frac{1}{N_{iy}} \sum_{t=1}^{N_{iy}} \frac{V_{iyt}}{n_{iyt}}$$

where  $V_{iyt}$  is trade volume in shares of stock  $i$  on day  $t$  in year  $y$ , and  $n_{iyt}$  is the number of shares outstanding of stock  $i$  on that day (Minovic, 2012, p. 785).

### 5.6.4 Amihud's Illiquidity Ratio

Amihud (2002) tries to estimate price impact from the returns and volumes. Amihud's Illiquidity Ratio (ILLIQ) relates absolute stock return to dollar volume of the trade. It



measures price impact by reflecting daily price response to one dollar of trading volume. Empirical research conducted shows that ILLIQ have a positive and significant impact on expected returns of a sample of NYSE stocks between 1964 and 1997 (Amihud, 2002, p.32). Amihud's illiquidity measure is calculated as follows:

$$Amihud_{i,t} = \overline{|r_t|/|Vol_t|}$$

the average is over all days with nonzero volume, where  $Vol_t$  is the dollar volume of trade at time  $t$  and  $r_t$  is the return at time  $t$  in absolute terms (Hasbrouck, 2007, p. 93).

### 5.6.5 Comparison and Evaluation of Models for Illiquid Markets

In comparing and evaluating of Measures for Illiquid Markets described above, the two criteria that we set in Chapter 5.3 will be used:

#### **Criteria Nr 1.) Degree to which the liquidity statistic actually captures the behaviour it meant to measure**

First, it should be noted that the above described measures are not really proper models capturing market liquidity risk as defined in this paper; but they should rather be treated as approximation attempts to be used as proxy for those illiquid markets, where the market structure does not allow to build up a more proper model framework.

**Zero Return:** The main problem of this measure lies on its underlying assumption of no movement in prices refers to no trade. This assumption is contradictory with the basic definition of a hypothetical perfectly liquid market, where trades occur continuously without having an impact on prices (in the absence of new information). In other terms, in a very liquid market, large volumes of trade should occur without moving the prices (Foucault, Pagano & Röell, 2013, p.59). Hence, describing no-trade with zero return can deliver a completely distorted picture about the illiquidity of a market.

**Roll's measure:** The basic limitation of the Roll's model lies on the two important assumptions it bases: (i) *assets are traded in an efficient market in terms of information* (ii) *the probability distribution of price changes is stationary*. Hence, balanced/uncorrelated order flow and perfect competition is assumed as well as order flows not affecting fundamentals (Roll, 1984, p. 1127).

**Turnover & Amihud's illiquidity measure:** The problem with measures based on trading volume is that the times when new information arrives to the market are the times when the volatility increases followed by widening of bid-ask spread. Due to increased volume of trading, the turnover ratio will increase and falsely imply an increase of liquidity even though the trading costs are high (Sarr & Lybek, 2002, p.12). Hence, the volatility of the turnover should also be taken into account. In times of stress, for instance, volatilities usually tend to increase even though bid-ask spreads are also higher than those values under normal market conditions. Therefore, volume based models can be quite misleading especially in illiquid markets.

When it comes to comparison of model performances; it is observed that many of the academic literature focuses on the analyses of the risk, return and volatility for emerging markets, but few addresses the liquidity in emerging markets. Among those researches, Lesmond (2005) estimates the liquidity measures: Lesmond, Ogden, and Trzcinka measure (LOT), turnover, Roll (1984) and Amihud (2002) for emerging markets for which daily prices are available. In the below graph, Lesmond illustrates bid-ask spread versus the other four liquidity measures for 23 emerging markets from 1993 to 2000:

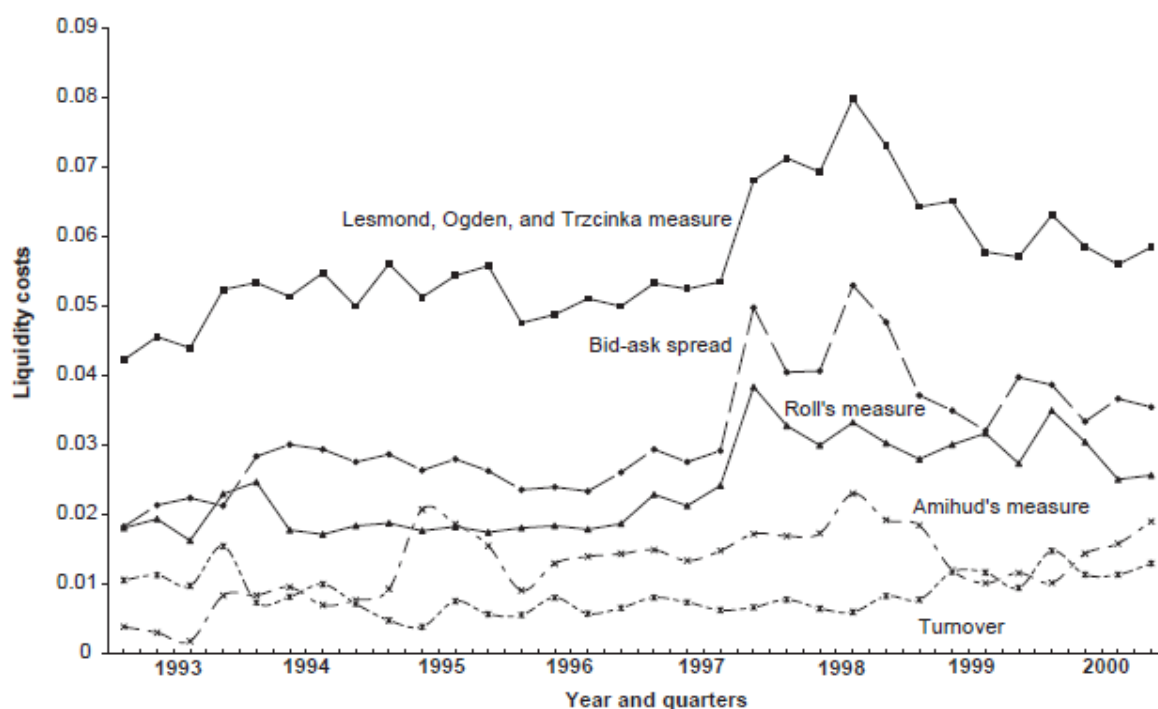


Table 15: Ranking of market liquidity risk models by overall acceptance rate<sup>13</sup>

<sup>13</sup> Source: Lesmond, D.A. (2005). *Liquidity of Emerging Markets*. *Journal of Financial Economics* 77 (2005), p.428

Comparing LOT measure with each of the other estimator measures, Lesmond (2005) concludes that in most of the emerging markets, LOT measure is superior to the other measures at explaining bid-ask spread (plus commission cost). According to the results of the above study, LOT is superior to turnover in all markets and Roll's measure is superior to LOT only in the Grecian market. Amihud's illiquidity measure is superior to LOT only in Greece and Argentina. Comparing Amihud's measure against the other two, Amihud is superior to turnover in 15 markets and to Roll's measure in 14 markets out of 23. In explaining cross-country differences with respect to liquidity, LOT and Roll delivers the best results, where LOT is over 80% and Roll's measure is over 49% correlated with bid-ask spread. The volume based models (Amihud and turnover) are downward biased in the low liquidity markets (Lesmond, 2005, p. 437-445).

Bekaert, Harvey, and Lundblad (2005) illustrate similar results to Lesmond (2005), where they show zero-return proportion has a correlation of 0.30-0.42 with other liquidity measures such as proportional bid-ask spread and Amihud (2002). They further illustrate that in the countries where spread data is available, zero-return is highly correlated (67%) with bid-ask spread (Lee, 2006, 46). In line with the findings of Lesmond (2005); Bekaert, Harvey, and Lundblad (2005) also assert that turnover is not a proper measure of liquidity in emerging markets. Bekaert *et al.* (2007) further indicates that only LOT is a sustainable applicable measure for illiquidity in emerging markets (Minovic, 2012, p.784-786).

### **Criteria Nr 2.) Extent to which high quality data are likely to be available**

Despite its limitations, **zero return** is still a quite practical measure, as it requires only the time series of the daily return and not the trade volume data; thus relatively easy to implement. However, the drawback of this model in terms of required data is that for estimation of parameters, it requires a long enough period which is not always available in illiquid markets. Another data related problem is that observation of a significant amount of zero-returns, i.e. more than 80%, makes zero-return measure invaluable (Minovic, 2012, p.783-784).

As per **turnover and Amihud's illiquidity ratio**, in most of the emerging illiquid markets, the volume data is not available on a daily basis. Second problem is, in these markets the volume data is plagued by trends and outliers. Moreover, these measures require positive

volume during the time interval the model is built, however in some illiquid markets non-trading is a very commonly observed issue (Bekaert, Harvey & Lundblad, 2005, p.6).

**Roll model** is a quite useful measure as it does not require any information on bid and ask prices; it is sufficient to have data series of market prices to run this simple model. Yet the data availability concerns raised above for illiquid markets are also valid for this model.

## 6 MARKET LIQUIDITY AND ASSET PRICES

### 6.1 Market Liquidity Risk Premium

Assume an investor, who buys asset X paying an ask price of 100. After one year, the price of the security goes up: ask price to 105 and bid price to 103 (mid-price 104). Investor liquidates his position after 1 year by selling it with this bid price (103). On the paper, the investor's return is 5% considering the ask-ask price increase. However, the realized return of the investor is only 3%. Transaction costs reduce the return of investor, hence the investor is going to expect compensation for the expected transaction costs. As a result, the transaction costs lead to lower prices and therefore higher expected returns (Jong & Rindi, 2009, p.116). This premium for market liquidity risk is considered in the asset pricing literature, where the models usually measures liquidity risk as the covariance between market returns and liquidity.

### 6.2 Liquidity Adjusted CAPM

Amihud and Mendelson (1986) suggest adjusting CAPM to account for liquidity:

$$E[R] = r_f + \beta(E[R_M] - r_f) + \mu S$$

where

$r_f$  = is the risk free rate

$E[R_M] - r_f$  = market risk premium

$\beta$  = asset's beta

$S$  = relative bid-ask spread

$\mu$  = expected trading frequency (e.g.  $\mu = 0.4$  means the investor trades 40% of his position per period)

(Jong & Rindi, 2009, p.117)

Liquidity Adjusted CAPM asserts that the expected returns on assets depend on the risk free rate of return, the risk premium depending on the beta of the stock (as suggested by the classical CAPM) and additionally the bid ask spreads and expected trading frequency, so-called **(il)liquidity premium**. It implies that the higher the liquidity premium, the higher the

market liquidity risk and the expected rate of return on a specific asset (Jong & Rindi, 2009, p.117).

Amihud and Medelson (1991) provided also empirical evidence on the fact that liquidity has an impact on the yield of an asset. They studied U.S. treasury bills and notes with matched maturities representing same cash flows and default risk. But their liquidity deviated from each other: historically, bills have lower transaction costs than notes, thus bills said to be more liquid. The average bid ask spreads in the data sample for notes was 0.0303 percent of their price (3bp) and 0.00775 percent (0.07bp) for bills. Their empirical test showed that notes usually trade with a discount to identical bills due to their higher illiquidity. They calculated the annualized return for treasury notes as 6.52% and 6.09% that for bills inducing an 43 bp average yield difference between notes and bills.

Acharya and Pedersen (2005) investigated cross-sectional regressions of the liquidity-adjusted CAPM for 25 value-weighted size portfolios using monthly data during 1964–1999. They concluded the liquidity-adjusted CAPM explains the data better than the standard CAPM

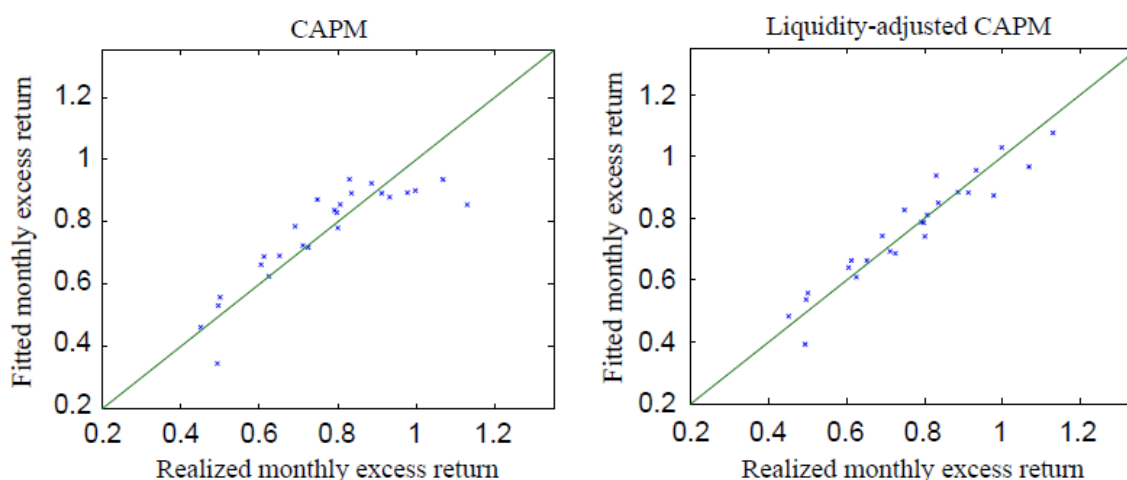


Table 16: Fitted CAPM and Liquidity-adjusted CAPM returns vs. Realized returns<sup>14</sup>

### 6.3 Liquidity CAPM

Acharya and Pedersen (2005) developed also an adjusted version of CAPM, where required returns on an asset depend on asset's expected liquidity as well as covariance between its own

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<sup>14</sup> Source: Acharya, V., Pedersen, L. H. (2005). *Asset Pricing with Liquidity Risk*. *Journal of Financial Economics* 77 (2005). p. 396

and market return and liquidity (p. 1). In their model, the expected rate of return is determined by expected transaction costs and asset's beta by using net returns (after transaction costs). Jong & Rindi (2009) illustrates the model as follows (p. 123)

The asset pricing equation is

$$E(r_i) = \alpha + \mu E(c_i) + \lambda \beta_i^{net}$$

where

$E(r_i)$  = expected excess return on asset  $i$

$E(c_i)$  = expected transaction costs

$\mu$  = implicit trading frequency of the asset

$\lambda$  = risk premium for covariance with the market return

$\beta_i^{net}$  is driven by a simple regression of net (after transaction costs) returns of asset  $i$  on the net returns on the market portfolio:

$$\beta_i^{net} = \frac{Cov(r_{it} - c_{it}, r_{mt} - c_{mt})}{Var(r_{mt} - c_{mt})}$$

where the net beta can be decomposed into below four components:

$$\beta_i^{net} = \beta_{1i} + \beta_{2i} - \beta_{3i} - \beta_{4i}$$

$$\beta_i^{net} = \frac{Cov(r_{it}, r_{mt})}{Var(r_{mt} - c_{mt})} + \frac{Cov(c_{it}, c_{mt})}{Var(r_{mt} - c_{mt})} - \frac{Cov(r_{it}, c_{mt})}{Var(r_{mt} - c_{mt})} - \frac{Cov(c_{it}, r_{mt})}{Var(r_{mt} - c_{mt})}$$

The first beta  $\beta_{1i}$  is the traditional CAPM beta and the other betas measure different component of the liquidity risk.

Acharya and Pedersen (2005) also performed empirical analysis to test their assumptions, where they estimated betas for different portfolio of stocks in NYSE and AMEX with data from 1962 and 1999. They found a 4.6% difference in the returns of the highest and lowest liquidity portfolio; where 3.5% is the compensation for expected liquidity and the remaining 1.1% is the compensation for liquidity risk (p.378).

## 7 CHALLENGES FOR MEASUREMENT AND MANAGEMENT OF MARKET LIQUIDITY RISK

### (i) Issues Related to Quantification of Market Liquidity Risk

As oppose to funding liquidity risk, quantification of market liquidity risk does not require very complex IT systems and set-ups. In measuring funding liquidity risk, a bank is required to have advanced IT structures, which enables to generate cash flow balances from all relevant asset, liability and off balance sheet items both for short and long term horizons. The necessary IT investment and allocation of staff for execution comes with its costs which is significantly higher in comparison to efforts dedicated for management of market liquidity risk.

The challenge for measurement of market liquidity, on the other hand, lies in the unavailability of the data what most of the economically advanced models would require. As discussed previously, even the basic information such as bid-ask spreads are not always available for every individual security (e.g. government bond market). Even if the time series of bid-ask spreads exist for an asset, it is not certain, whether the quotes are so-called ‘firm’ prices, where market makers are obligated to trade with these quotes or the prices are just ‘indicative’. As we have elaborated in detail, large institutional investors (e.g. banks) have certain bargaining powers on these quoted prices, where the real transaction price might deviate from those publicly available listed quotes. As a result, it is not possible for every asset to obtain reliable time series on bid-ask spreads, which is a main indicator of illiquidity.

Similarly, other data requirements such as existing order sizes, order sign, flows are not always easy to obtain. Not only in the over-the-counter markets, where trades take place based on bilateral negotiations between investor and dealers; also in organized hybrid markets traders are most of the time allowed to submit so-called ‘hidden’ orders, where the order size and direction is not visible to market participants. These deficiencies in the availability of data (in particular in emerging markets, where illiquidity is a significant component of market structure) make it difficult to run proper quantification models. Due to that reason, it is a common implementation that market liquidity will be covered with certain add-ons to the currently implemented models measuring market risks.



Another challenge is that as oppose to the some other risk types, the guidelines regulating market liquidity risk are not very explicit. There is no model developed so far (or suggested by the policy makers/regulators) that addresses all aspects of market liquidity adequately and can be conveniently used by practitioners in their day-to-day risk management practices. There is a trade-off between simple and more complicated models, some of which were presented in previous chapters of this paper. Simple models, which are convenient due to the ease of their calculation and reasonable amount of data requirements, serve as a fair proxy; but they do not take into account all the factors influencing liquidity precisely. While more sophisticated models lead to more reliable results; but they require often data, as discussed earlier, which is either not available or the time and costs which would be required to obtain data, set-up the model, overcome the computational burdens may not always pay off the benefits expected to be utilized using the model.

(ii) *Issues Related to Management of Market Liquidity Risk*

*“Of the maxims of orthodox finance none, surely, is more anti-social than the fetish of liquidity, the doctrine that it is a positive virtue on the part of investment institutions to concentrate their resources upon the holding of 'liquid' securities” (Keynes, 1936, p.155).*

Today financial institutions are required by regulators to hold adequate levels of “liquidity buffers” which can be used to offset any additional outflows that may arise under stress conditions. In its *Guidelines on Liquidity Buffers & Survival Periods*, European Banking Authority (2009) instructs institutions that their “liquidity buffer should be composed of cash and core assets that are both central bank eligible and highly liquid in private markets” (p.14). Thus, financial institutions are leaded to direct their investments towards those ‘liquid assets’, which are ‘central bank eligible’ referring to those securities that can be pledged in central banks as collateral to obtain in return cash for a specific time period. The strategy of majority of banks in case of increased liquidity needs under stress conditions is not generating liquidity through fire-sales of assets; but rather to generate liquidity by using these eligible assets in repurchase agreements with central banks through open market operations (Loebnitz, 2006, p.95). This results in a systemic risk putting a heavy burden on central banks to be considered as the main liquidity provider under stress. In order to decrease these dependency, the above mentioned EBA guideline requires that “banks will have to demonstrate adequate diversification in the total composition of the buffer so as to guarantee to supervisors that they

are not relying too heavily on access to central bank facilities as their main source of liquidity” (p.16). Institutions are required to have contingency plans (funding emergency plans), where they have to prove the regulator that operationally detailed plans exist on acquirement of additional funds in the existence of stress.

In this regulatory environment, where banks mostly have central bank eligible government bonds or other investment grade bonds in their investment portfolios, the market liquidity risk is rather an important issue for their trading books. Here the problem arises due to common usage of so called “market-sensitive risk management systems” and Basell II regulations. Today the trading desks of banks are guided by similar exposure limits, which are driven by similar models used by banks in quantifying market risk. As a result, they react similarly to the changes in the markets, which lead to herding behaviour. These models consider mostly two components, namely volatilities of the assets in the portfolio and the correlation between them. If, for example, volatilities of some assets increase significantly due to arrival of new information to the market, risk limits of these banks might be breached. In order to mitigate the risk exposure, some banks might sell the most volatile or most highly correlated securities. The selling of these assets by more institutions might result in even more adverse price changes and increase volatilities even further. This might trigger the risk limits of other banks and force them to take similar action. Thus the commonly used models of today in management of positions cause herding behaviour and an overall systemic risk in the whole financial market (Loebnitz, 2006, p.65).

Persuad (2000) analyses the phenomena of how market-sensitive risk management systems might create systematic risk, where he illustrates this herding behaviour and limit breach cycle with the following figure:

### A vicious cycle of herding and DEAR limits

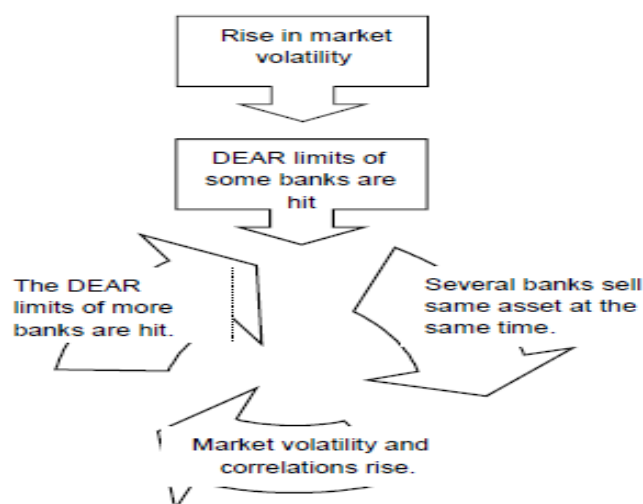


Table 17: Vicious cycle of herding and DEAR limits (Daily Earning at Risk)<sup>15</sup>

Here policy makers are responsible for consulting institutions on how to define, quantify, monitor and manage their exposure to risk adequately. The policies should illustrate in a precise way adequate methodologies for quantification of risk, which base on market microstructure factors rather than ambiguous add-ons to the used models. Within this framework, the responsibility of the regulators is not limited to instruct and monitor institutions in their risk management practices; but also to strive for taking corrective actions against market anomalies that create illiquidity in markets. One of the key measures is to increase transparency in the financial markets. As shown previously, the unavailability of information to market participants, deviations from the complete markets and asymmetric information paradigm are the main causes for illiquidity. Policy makers and regulators shall address these market imperfections. Introduction of the regulations such as Markets in Financial Instruments Directive (MiFID) in recent years is an example to those attempts mitigating opaqueness in the financial markets.

<sup>15</sup> Source: Persuad, A. D. (2000). *Sending the herd off the cliff edge: The disturbing interaction between herding and market-sensitive risk management practices*. Bank of International Settlement Papers No.2. p. 237

## 8 CONCLUSION

In this paper a large set of models quantifying market liquidity risk are presented and their performances are compared in terms of the degree to which they capture the behaviour they meant to measure and the extent to which they are applicable out of a theoretical modelling context in practice. The analysis was executed from a risk management perspective and aims to provide a supportive document for risk managers in financial institutions in deciding which model/s to implement in their liquidity risk management frameworks.

First, the conducted analysis conclude that a very significant aspect that should be considered in the selection of a model is the degree to which high quality data is available that the model would require. In the modelling context, there is a trade-off between ‘standard models’ that are quite easy to implement versus ‘advanced models’ that has more economic appeal. This research suggests wherever the data availability allows, advanced models should be preferred as they address the components of liquidity more adequately. The model proposed by Almgren and Chrisis (2000), for instance, is a very elegant model and academic researches (e.g. Loebnitz, 2006) assert the model captures the market liquidity risk aspect as it is defined in the market microstructure study.

Secondly, in cases where the availability of data doesn’t allow application of any advanced models, standard models can be implemented keeping in mind that it might lead to underestimation of the exposed risk due to their simplified nature and drawbacks related to their underlying assumptions. From the discussed standard models in this paper, Bangia (1999) is suggested due to its simple model set-up and limited data request. However, the model assumption of normal distribution of future prices and spreads, might lead to misleading results. Here the assumption of normality should be addressed by the practitioner, such as done by Ernst (2012), where he estimated the distribution parameters by using Cornish-Fisher approximation. If available, using weighted spread data would also further improve the model results - described as ‘Modified Bangia Model’ in Chapter 5.5.2. One of the most common approach of using unconditional VaR models by ‘adjusting’ it with a simple multiplier (e.g. time-to-liquidation estimation) or arbitrarily increasing the used volatilities to account for liquidity risk component is not suggested, as this approaches being very risk insensitive and not capturing the liquidity risk as we defined in this paper.

Lastly, in structurally illiquid markets where the availability of data does not allow the usage of any above mentioned methods, the introduced third group of measures can be applied. Here it should be kept in mind that these measures should only be considered as an approximation indicating 'market liquidity' rather than a proper model quantifying 'market liquidity risk'. Among the discussed approaches in this paper, the measure developed by Lesmond, Ogden, and Trzcinka (1999) based on zero rates of return (so-called LOT ratio) is suggested based on its model performance. Last but not least, it is important to note that even though being very intuitive and tempting approaches -in particular in those illiquid markets where finding data is an issue; any volume-based measure shall solely be used as an indicator for illiquidity, but they are definitely not proper tools for any risk quantification and shall not be preferred in the risk management processes.

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## **APPENDICES**

### **Appendix Nr.1: Abstract in English**

As banks being one of the main liquidity providers in the system; regulators and policy makers are very much concerned with the liquidity position of these institutions in order to ensure a robust liquidity framework in the overall financial system. This paper aims to describe and compare selected models quantifying market liquidity risk and to identify which model delivers more adequate results and thus could be preferred for implementation by practitioners. Even though all the models described in this work can be used by any market participant; the model comparison is performed mainly from a risk management perspective with a clear focus on the regulatory and internal requirements for financial institutions. The paper starts with a description of international regulatory requirements for management of liquidity risk. In what follows, the concept of market liquidity is explained and its main sources related to market microstructure and trading mechanism are explained. Based on the developed concepts, market liquidity risk is defined as a quantifiable factor and selected models for its measurement are described. After presentation of every selected model group, main weaknesses of each model are discussed and it is evaluated to what extent the presented model addresses the components of market illiquidity the way it is described in this paper. The analysis includes also elaboration of the requested data set for model set-up and comparison of the practical applicability of the models. The consequent chapter discusses challenges to be considered for measurement and management of market liquidity risk. And in conclusion a brief summary of the findings of this research are provided as well as model/s suggestions that could fit for the purpose of measurement and monitoring of market liquidity risk.

## **Appendix Nr.2: Abstract in German (Zusammenfassung in deutscher Sprache)**

Da Banken eine wesentliche Rolle bei der Bereitstellung von Liquidität im Finanzsystem darstellen, sind Regulatoren und Gesetzgeber sehr besorgt über die Liquiditätsposition dieser Institute, um ein robustes Rahmenwerk im gesamten Finanzsystem sicherzustellen. Diese Arbeit zielt darauf ab, selektierte Modelle zur Quantifizierung des Marktliquiditätsrisikos zu beschreiben und zu vergleichen, und dasjenige Modell zu identifizieren, das die adäquatesten Ergebnisse liefert und daher von den Anwendern für die Umsetzung bevorzugt werden könnte. Obwohl alle in dieser Arbeit beschriebenen Modelle von jedem Marktteilnehmer verwendet werden kann, wird der Modellvergleich hauptsächlich aus der Perspektive des Risikomanagements ausgeführt mit klarem Fokus auf regulatorische und interne Anforderungen für Finanzinstitute. Das Papier beginnt mit einer Beschreibung der internationalen regulatorischen Anforderungen zum Management des Liquiditätsrisikos. Nachfolgend wird das Konzept der Marktliquidität und die Hauptquellen in Bezug auf die Market-Mikrostruktur und Handelsmechanismus erklärt. Basierend auf die entwickelten Konzepte wird das Marktliquiditätsrisiko als quantifizierbarer Faktor definiert und selektierte Modelle zu dessen Messung beschrieben. Nach der Präsentation jedes selektierten-einzelnen Modellgruppe, werden die wesentlichen Schwächen jeden Modelles diskutiert und es wird evaluiert, in welchem Ausmaß das präsentierte Modell die Komponenten des Marktliquiditätsrisikos adressiert, so wie in diesem Papier beschrieben. Die Analyse inkludiert ebenfalls die Ausarbeitung der erforderlichen Datensets zur Modellierung und zum Vergleich der praktischen Anwendbarkeit des Modells. Das nachfolgende Kapitel diskutiert die Herausforderungen, die bei Messung und Management des Marktliquiditätsrisikos beachtet werden müssen. Abschließend werden eine kurze Zusammenfassung der Erkenntnisse dieser Studie sowie Modellvorschläge präsentiert, die zur Messung und Management des Marktliquiditätsrisikos herangezogen werden können.

## **Appendix Nr.3: Author's Curriculum Vitae**

### **PERSONAL DATA**

Name: Canan CALISKAN

Contact: canan.caliskan@outlook.at

### **EDUCATION**

10.2009 - Present      University of Vienna  
Master of Science Degree  
Specialization: Financial Markets & Controlling

09.2003 - 02.2008      Bilkent University, Ankara  
Bachelor of Science Degree  
Field of Study: Business Administration

### **WORK EXPERIENCE**

08.2015 - Present      Sberbank Europe AG, Vienna  
Advisor to Chief Risk Officer

06.2013 - 08.2015      Sberbank Europe AG, Vienna  
Market Risk Management

01.2012 - 05.2013      DenizBank AG, Vienna  
Credit Risk Management

08.2009 - 01.2012      DenizBank AG, Vienna  
Risk Management

### **SKILLS**

German, English, Turkish - Excellent command