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## Simulations of Antihydrogen Radiative Decay in ASACUSA

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#### Abstract

CPT symmetry, which is one of the most fundamental symmetries in physics, has so far resisted all attempts at falsification. Nevertheless, common extensions of the Standard Model, like string theory and quantum gravity, do not necessarily conserve CPT symmetry. Thus, searching for CPT violations at low energies may be a good method to discover physics beyond the Standard Model. A sign of this phenomena would be a deviation between the hyperfine structure of hydrogen and anti-hydrogen. The ASACUSA (Atomic Spectroscopy And Collisions Using Slow Antiprotons) collaboration realised a Rabi-like hyperfine spectroscopy experiment for this reason. There, positrons and anti-protons are mixed together in a nested Penning trap inside a special magnetic field configuration, a so-called double CUSP trap, to form bound states, before they are released into the beam-line apparatus. The quantum states of these anti-hydrogen atoms cannot be measured easily, however the 3-body recombination process, through which they are formed, is likely to produce excited Rydberg states, upon creation. As the hyperfine measurement can only be performed with ground state antihydrogen, it is important to be able to assess the antihydrogen atoms decay rate in the specific spectrometer apparatus of the ASACUSA experiment. This work contains various Monte-Carlo simulations of antihydrogen propagation through the ASACUSA beam-line, which traces the quantum states, the position and the velocity of the atoms. Eight billion antihydrogen atoms were simulated for this reason on a cluster. These are the first simulations that considered the current beam line setup, the double CUSP trap and an accurate initial n-state distribution. The magnetic field, which has an impact on the trajectory and the decay rates, is contained in the simulation. In the results, the influence of the initial principal quantum number and the sextupole magnetic field, used for spin analysis in the Rabi-type set up, are discussed in particular. Different initial velocities are compared with each other, and characteristics such as the signal to noise ratio, or the signal intensity, are specified. Furthermore, the feasibility of additionally using the electric field information is presented. The implementation of the influence of the electric field on the decay rates of Rydberg states, have been completed and tested in principle.


## Zusammenfassung

Die CPT Symmetrie, welche eine der fundamentalsten Symmetrien der Physik darstellt, konnte allen bisherigen Versuchen von Falsifizierung Stand halten. Dennoch existieren Erweiterungen des Standard Modells der Teilchenphysik, wie die String Theorie oder die Quantengravitation, die nicht notwendigerweise CPT erhaltend sind. Daher ist die Suche nach CPT Verletzungen bei niedrigen Energien möglicherweise eine gute Methode um Erkenntnisse jenseits des Standard Modells zu erlangen. Ein Anzeichen für eine solche CPT Verletzung wäre ein Unterschied zwischen der Hyperfeinstruktur von Wasserstoff und Antiwasserstoff. Die ASACUSA (Atomic Spectroscopy And Collisions Using Slow Antiprotons) Kollaboration realisierte ein Rabi ähnliches Hyperfeinstruktur Spektroskopie Experiment zu diesem Zwecke. Hierbei werden Positronen mit Antiprotonen in einer verschachtelten Penning-Falle innerhalb einer speziellen magnetischen Feldkonfiguration, einer sogenannten double CUSP Falle, zusammengeführt um Bindungszustände herzustellen, bevor sie in das Strahlrohr entlassen werden.
Die Quantenzustände dieser Antiwasserstoffatome können nicht direkt gemessen werden, jedoch ist es durch die 3-Körper Wechselwirkung durch welche sie gebildet werden, sehr wahrscheinlich, angeregte Rydberg Zustände zu erhalten. Da die Hyperfeinstruktur Messung nur mit Grundzustandsatomen durchgeführt werden kann, ist es wichtig, auf die Antiwasserstoffzerfallsraten innerhalb der Spektroskopieapparatur des ASACUSA Experiments zurück greifen zu können.
Diese Arbeit enthält verschiedene Monte-Carlo Simulationen der Antiwasserstoff Ausbreitung durch das ASACUSA Strahlrohr, welche die Quantenzustände, die Position und die Geschwindigkeit der Atome verfolgt. Acht Milliarden Antiwasserstoffatome wurden aus diesem Grund auf einem Cluster simuliert. Sie stellen die ersten Simulationen mit der aktuellen Apparatur, der doppelten CUSP Falle, und einer realistischen Zustandsverteilung dar. Das Magnetfeld, welches entscheidenden Einfluss auf die Trajektorie und die Zerfallsraten hat, ist in der Simulation berücksichtigt. In den Ergebnissen werden besonders der Einfluss des Anfangsquantenzustandes und des Magnetfeldes des Sextupole Magneten, welches für die Eigendrehimpuls Selektion für den Rabi ähnlichen Aufbau benötig wird, untersucht. Verschiedene Anfangsgeschwindigkeiten werden miteinander verglichen, und Charakteristika wie das Signal- Rauschverhältnis, oder die Signalintensität, untersucht.
Weiters wird die Einbindung der zusätzlichen Information des elektrischen Feldes, präsentiert. Die Implementation des Einflusses des elektrischen Feldes auf die Zerfallsraten der Rydberg Zustände, wurde fertig gestellt, und das Prinzip getestet.

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Chapter 1

## Introduction

### 1.1 CPT-symmetry and antiparticles

In 1928 Paul Dirac showed that Albert Einstein's theory of special relativity implies, that every particle in the universe has a corresponding antiparticle through equation 1.1 [1]. In the following equation of the special relativity, $p$ represents the particles momentum, $m$ the particles rest mass, $c$ the speed of light, and $E$ the total relativistic energy of the particle.

$$
\begin{equation*}
E= \pm \sqrt{p^{2} c^{2}+m^{2} c^{4}} \tag{1.1}
\end{equation*}
$$

In 1932, the first observation of antimatter took place. Carl D. Anderson published a historic photo of the positively charged electron, which later became known as the positron, and was shortly afterwards awarded with a Nobel Prize [2].

In 1964, Cronin and Fitch observed $2 \pi$-decays from neutral Kaon mesons at the Brookhaven Alternating Gradient Synchrotron. This was a milestone in physics and the development of the Standard Model [3]. Only the short lived $K_{1}$ should decay into two $\pi$ 's, as $K_{1}$ is an eigenstate of CP transformation with the eigenvalue 1 , whereas a $\pi$ carries a CP eigenvalue of minus one, which is a multiplicative quantum number. The long lived $K_{2}$ is an eigenstate of CP with the eigenvalue minus one and hence should decay into three $\pi$ 's. However, this experiment observed $2 \pi$ 's at the end of the beamline showing that the observed long lived state $K_{L}$ is not a pure $K_{2}$ state. Instead, it is a mixture of $K_{1}$ and $K_{2}$, which is only possible with a small violation of the CP symmetry. So, CP symmetry was shown to be also violated in weak interactions. This CP-violation meant that a physical result will change under the simultaneous application of a charge conjugation (C) and a parity transformation (P). It was a small effect, but it had severe structural consequences for the construction of the Standard Model. Even though the CP-violation is generally expected to play a role in the matter antimatter imbalance of the universe, the effect is too small to explain the observed matter dominance. The explained Kaon experiment was awarded in 1980 with the Nobel Prize for Physics.

However, since 1954 [4] the standard model is based on a more fundamental symmetry called CPT. It extends the described CP-transformation to include time reversal operation T , and predicts that all interactions are invariant under a CPT-transformation. So far, nothing contrary has ever been observed. A schematic description of the CPT-transformation can be seen in figure 1.1. The CPT-Theorem proves that CPT symmetry has to be conserved. It is based on the following assumptions (1) pointlike particles (2) local interactions (3) Lorentz invariance of a quantum field theory.

The CPT-theory states that matter and antimatter have identical physical properties (e.g. mass, lifetime) and opposite additive quantum numbers (e.g. charge, magnetic moment).

The current violation in the matter/antimatter symmetry in the Universe could be a sign of violating the CPT symmetry. This could point to new physics Beyond the Standard Model (BSMTheory).


Figure 1.1: Schematic view of CP- and CPT-transformations with its discrete transformation of $C, P$, and T. The picture is from Bertalan Juhasz.

A comparison of different experimental results on CPT symmetry tests are given in figure 1.2.


Figure 1.2: Summary of relative (length of the red bars) and absolute (left edge of the bar) precisions of different CPT symmetry testing experiments. The dotted line symbolizes the possible precision for the maser, which is not possible for antimatter. From [5].

### 1.2 Beyond the Standard Model

The Standard Model unified three of the four fundamental forces: (1) the strong, (2) the weak, (3) and the electromagnetic one. Even though it has successfully explained almost all experimental results and precisely predicted a wide variety of phenomena, it is incomplete. Dark matter, dark energy, gravity, and the imbalance of matter and antimatter are not covered. One of the models which tries to describe physics beyond the Standard Model is SME (Standard Model Extension) from Don Colladay and Alan Kostelecky [6]. It is an effective field theory which incorporates potential CPT and Lorentz violation. It combines the Standard Model and General Relativity and allows small violating effects of CPT and Lorentz-invariance. The model implies gauge invariance and energy-momentum conservation and is compatible to existing field theories. In [7], Don Collady and Alan Kostelecky provide a theoretical framework for low-energy effects of spontaneous CPT violation and partial breaking of Lorentz invariance.

### 1.3 Hyperfine splitting of antihydrogen

The ASACUSA collaboration (Atomic Spectroscopy And Collisions Using Slow Anti-protons) aims to measure the ground-state hyperfine structure of antihydrogen to compare it with its matter counterpart. The principle that a antihydrogen beam can be achieved is demonstrated in [8].

The ground state hyperfine transition of hydrogen is one of the most precisely measured values in nature with a relative precision of $10^{-12}$ [9], but so far it was never measured for antihydrogen. One reason for this precision is that frequencies can be measured and tuned very well.

The test with the highest relative precision on CPT-symmetry to date is the $K / \bar{K}$ mass measurement with a relative precision of $\triangle m / m \approx 10^{-18}$, which is equivalent to an absolute precision of $10^{5} \mathrm{~Hz}$ [10].

## Chapter 2

## Theory

The required theoretical foundation is presented in this chapter, where four major topics are covered. We will start with the hyperfine-structure of antihydrogen to understand the experiment and the simulation. The next part will explain the Zeeman- and Stark-effect to understand the influence of the external fields on the force and on the decay rates of the atoms. In the last section we will take a deeper look on the numerical package which is used to calculate the atomic decay rates. For the vector notation bold symbols are used, e.g. v.

### 2.1 Hyperfine structure

The hyperfine splitting occurs primarily due to the proton and the electron having a magnetic moment. This magnetic moment $\boldsymbol{M}_{\boldsymbol{I}}$ is caused by the proton spin $\boldsymbol{I}$ and is given by $\boldsymbol{M}_{\boldsymbol{I}}=g_{p} \mu_{n} \boldsymbol{I} / \hbar$ where $\mu_{n}$ is the nuclear magneton given by $\frac{q_{p} \hbar}{2 M_{p}}$. It can be seen why this effect is so small. The proton mass is placed in the denominator of the last equation, hence the nuclear magneton is approximately 2000 times smaller than the Bohr-magneton $\mu_{B}=e \hbar / 2 M_{e}$, where $M_{e}$ is the electron mass and $e$ is the elementary charge. The energy-shift results from the interaction between the proton magnetic dipole moment and the magnetic field generated by the electron.

In the 1 s -state (ground state) of hydrogen, there is a hyperfine-splitting, but no fine-splitting because the angular momentum is zero. The fine-splitting is the splitting that can be explained by the spin-orbit interaction, and calculated with the relativistic Dirac-equation. The electron can take the possible values $m_{S}= \pm \frac{1}{2}$ and the proton $m_{I}= \pm \frac{1}{2}$ for the magnetic spin quantum numbers. With this we are able to determine a set of quantum numbers to describe this system through the total angular momentum $\mathbf{F}=\mathbf{J}+\mathbf{I}$ with the quantum numbers $F$ and $M_{F}$. The quantum number $F$ determines the magnitude of the total angular moment of the antiproton and the positron, and $M_{F}$ the projection of this value on a specific axis, e.g. z-axis. For the ground state, this leads to a singlet state $\left(F, M_{F}\right)=(0,0)$, and a triplet state $\left(F, M_{F}\right)=(1,0),(1,-1),(1,1)$. It is summarized in figure 2.1.

Without an external field the $F=1$ states are degenerated. Hence, only two different energy levels exist for hydrogen in ground state without an external field. One energy for $F=1$, and one for $F=0$. The transition frequency between these states is the hyperfine transition for hydrogen
and one of the most accurately measured values in physics. It is presented in equation 2.1 and sometimes called the 21 cm line. In this context HFS stands for Hyper Fine Splitting.

$$
\begin{equation*}
\nu_{H F S}=1420405751.7667(9) \mathrm{Hz} \text { [11] } \tag{2.1}
\end{equation*}
$$



Figure 2.1: Singlet- and triplet-state of hydrogen in ground state. From [12].

### 2.2 The Zeeman effect

If the hydrogen atom is placed in an homogenous magnetic field $\mathbf{B}$, the free Hamiltonian $\mathcal{H}_{0}$, with the momentum operator $\mathbf{P}$ and the potential $V(r)$,

$$
\begin{equation*}
\mathcal{H}_{0}=\frac{\mathbf{P}^{2}}{2 m_{e}}+V(R) \tag{2.2}
\end{equation*}
$$

has to be extended by $\mathcal{H}_{Z}$

$$
\begin{equation*}
\mathcal{H}_{Z}=-\mathbf{B} \cdot\left(\frac{q}{2 m_{e}} \mathbf{L}+\frac{q}{m_{e}} \mathbf{S}-q \boldsymbol{M}_{\boldsymbol{I}}\right) \tag{2.3}
\end{equation*}
$$

to the total Hamiltonian

$$
\begin{equation*}
\mathcal{H}=\mathcal{H}_{0}+\mathcal{H}_{Z} . \tag{2.4}
\end{equation*}
$$

In equation 2.3 B is the magnetic field operator, $\mathbf{L}$ the angular momentum operator, $\mathbf{S}$ the spin operator and $\boldsymbol{M}_{\boldsymbol{I}}$ is the magnetic moment of the nuclei as described in section 2.1. The Zeemanterm $\mathcal{H}_{Z}$ considers (1) the magnetic moment of the electron orbital angular momentum, (2) the magnetic moment of the electron spin, and (3) the magnetic moment of the nuclus.

The energy of a dipole $\boldsymbol{\mu}$ in an external magnetic field $\mathbf{B}$ shifts according to equation 2.5.

$$
\begin{equation*}
\mathcal{E}_{\mu}=-\boldsymbol{\mu} \cdot \mathbf{B} \tag{2.5}
\end{equation*}
$$

In the case of antihydrogen the magnetic moment will align parallel to the magnetic field axis for $\left(F, M_{F}\right)=(1,1)$ and $\left(F, M_{F}\right)=(0,0)$ and anti-parallel for $\left(F, M_{F}\right)=(1,-1)$ and $\left(F, M_{F}\right)=(1,0)$. The states can also be described according to their antiproton- and positron spin, where for example the state $|\downarrow, \downarrow\rangle$ is a tensor product of the positron spin with the antiproton spin $\left|\downarrow_{e^{+}}\right\rangle \otimes\left|\downarrow_{\bar{p}}\right\rangle$.

$$
\begin{align*}
(1,-1) & =|\downarrow, \downarrow\rangle  \tag{2.6}\\
(1,0) & =\frac{1}{\sqrt{2}}(|\downarrow, \downarrow\rangle+|\uparrow, \uparrow\rangle)  \tag{2.7}\\
(1,1) & =|\uparrow, \uparrow\rangle  \tag{2.8}\\
(0,0) & =\frac{1}{\sqrt{2}}(|\downarrow, \uparrow\rangle-|\uparrow, \downarrow\rangle) \tag{2.9}
\end{align*}
$$

It is helpful to classify $H$ or $\bar{H}$ into two types, based on their behaviour in an external inhomogeneous magnetic field. The atoms which align parallel to the magnetic field tend to move to a higher field region to minimize their energy, we will call them HFS (high field seekers) from now. This should not be confused with HFS for hyper fine splitting in equation 2.1 and from now this abbreviation will be exclusively used for high field seekers. The opposite ones are moving towards lower field regions and will be called LFS (low field seekers). Due to the opposite charge signs, the LFS and HFS quantum state classification is different for hydrogen and antihydrogen, as presented in table 2.1.

Table 2.1: Overview of HFS and LFS for hydrogen and antihydrogen

|  | LFS states | HFS states |
| :---: | :---: | :---: |
| $H$ | $(1,1),(1,0)$ | $(1,-1),(0,0)$ |
| $\bar{H}$ | $(1,-1),(1,0)$ | $(1,1),(0,0)$ |

In figure 2.2 the Breit-Rabi diagram for hydrogen and antihydrogen is shown. The influence of the external magnetic field on certain states can be seen in figure 2.2. In the ASACUSA experiment the $\pi_{1}$ and the $\sigma_{1}$ transitions are induced in a spin flip cavity, which will be discussed in section 3.2.2.

If the magnetic moment is known, the force on the antihydrogen atom can be calculated. By combining equation 2.5 with equation 2.10

$$
\begin{equation*}
\mathbf{F}=-\nabla \mathcal{E} \tag{2.10}
\end{equation*}
$$

we obtain

$$
\begin{equation*}
\mathbf{F}=\nabla(\boldsymbol{\mu} \cdot \mathbf{B})=(\nabla \cdot \boldsymbol{\mu}) \mathbf{B}+\boldsymbol{\mu}(\nabla \cdot \mathbf{B}) \tag{2.11}
\end{equation*}
$$



Figure 2.2: Breit-Rabi diagram for hydrogen and antihydrogen including the hyperfine transitions $\sigma_{1}$ and $\pi_{1}$. The magnetic quantum number $m$ is flipping sign from $H$ to $\bar{H}$. From [13].
which represents the force on a magnetic dipole in an external magnetic field. Furthermore a torque $\mathbf{M}=\boldsymbol{\mu} \times \mathbf{B}$ is acting on the atom.

The main content of this work deals with the radiative decay of antihydrogen Rydberg-states into lower levels or to the ground state and their trajectory according to the force in equation 2.11. Analytical investigations into radiative decays of hydrogen atoms have been undertaken in [14]. Some of these results will now be summarised.

They revealed equations 2.12 and 2.13

$$
\begin{align*}
\mathcal{E}_{E} & =3 e a_{0} n(n-1) E / 2  \tag{2.12}\\
\mathcal{E}_{B} & =\mu_{B}(n-1) B / 2 \tag{2.13}
\end{align*}
$$

which show the maximal energy-shifts due to an external electric field $\mathbf{E}$ or magnetic field $\mathbf{B}$, where $a_{0}$ is the Bohr radius $a_{0}=4 \pi \epsilon_{0} \hbar^{2} / M_{e} e^{2}$ and $\epsilon_{0}=8.854 \times 10^{-12} \mathrm{~F} \mathrm{~m}^{-1}$ the vacuum permittivity. The following definition is used, $E=|\mathbf{E}|$ and $B=|\mathbf{B}|$. If electric and the magnetic fields are both present, the highest energy shift occurs when the fields are perpendicular to each other. This maximal energy shift is shown in equation 2.14 .

$$
\begin{equation*}
\mathcal{E}_{E B}=\sqrt{\left(3 e a_{0} n E\right)^{2}+\left(\mu_{B} B\right)^{2}(n-1) / 2} \tag{2.14}
\end{equation*}
$$

This information could be used to force a faster decay by applying externals field configurations. This is crucial for the experiment, as the antihydrogen atoms have to be in the ground state by the time they reach the spectroscopy section. This will be further explained in chapter 3. The higher the energy of a certain state, the more decay channels are reachable and hence the lifetime of this specific state is shorter. We can also determine the point, where the energy shift caused by an external field is larger than the energy gap of two neighbouring n-states. For example, for
a magnetic field strength of $B=3.0 \mathrm{~T}$ it is the case for $n \geq 21$, and for an electric field strength of $E=12000 \mathrm{Vm}^{-1}$ it is the case for $n \geq 28$. From that point the calculation of the energy states has to deal with crossed energy levels.

States with high principal quantum numbers and low angular momentum quantum number will decay fast as there exist a wide range of possible final states. In this work the following selection rules for the quantum numbers are used, where $n$ is the principal quantum number, $l$ the orbital quantum number, $m_{l}$ the magnetic quantum number, $s$ the spin projection quantum number, $j$ the total angular momentum quantum number and $m_{j}$ the projection of the total angular momentum quantum number.

$$
\begin{align*}
n & =1, \ldots, n_{\max }  \tag{2.15}\\
l & =0, \ldots, n-1  \tag{2.16}\\
m_{l} & =-l, \ldots, 0, \ldots, l  \tag{2.17}\\
s & =-1 / 2,+1 / 2  \tag{2.18}\\
j & =s, \ldots,(l+s)  \tag{2.19}\\
m_{j} & =-(l+s), \ldots, 0, \ldots,(l+s) \tag{2.20}
\end{align*}
$$

A circular state, which is defined with $l=n-1$, decays in general slower, as the only possible decay sequence is $n \Rightarrow n-1 \Rightarrow n-2 \Rightarrow \ldots$.
[14] also showed that without an applied E-field the atom goes very likely into circular states. Furthermore, states with $n \leq 10$ are decaying very likely directly into the ground state. Transitions with no change of of the total angular momentum $(\Delta j=0)$ can only decay by a two photon decay, which is a rare event. These states are called "meta-stable" states. An example is hydrogen in a $2 s$ state.

### 2.2.1 Weak-field and strong-field Zeeman effect

In the strong-field limit the spin operator $\mathbf{S}$ and angular momentum operator $\mathbf{L}$ are coupled much more to the external magnetic field than to each other, hence these quantities are decoupled. In this case, the appropriate quantum numbers are $\left|n, l, m_{l}, m_{s}\right\rangle$ where $m_{l}$ and $m_{s}$ are the magnetic quantum numbers. The projection of the magnetic moment on the magnetic field axis is presented in equation 2.21 according to [15].

$$
\begin{equation*}
\mu_{Z}=\left(g_{l} m_{l}+g_{s} m_{s}\right) \mu_{B} \tag{2.21}
\end{equation*}
$$

Where $g_{l}$ is the orbital $g$-factor [16], and $g_{s}$ is the g -factor of the electron with a value of $g_{s} \approx-2.00232$. For strong magnetic fields the Zeeman energy for ground state antihydrogen is going linearly with the external magnetic field, as shown in figure 2.3, according to [17]. The proton spin and the electron spin can be viewed as decoupled. In this case, the spin quantum numbers numbers are good quantum numbers.

In the weak-field limit the spin of the electron and the orbital angular momentum can be seen as


Figure 2.3: Zeeman diagram for the strong magnetic field limit. The $1 s$-state of hydrogen is presented. The proton spin and the electron spin can be seen as decoupled. The direction of the electron spin and the proton spin are indicated for each curve. For the anti parallel configurations of the proton spin $m_{I}$ and the electron spin $m_{S}$, the energy is shifted by the half value of the hyperfine splitting for $B=0 \mathrm{~T}$.
a coupled system, where the magnetic field is too weak to separate it. The good quantum numbers are in this case $\left|n, l, j, m_{j}\right\rangle$, where $j$ is the eigenvalue of $\mathbf{J}=\mathbf{L}+\mathbf{S}$ and $m_{j}$ the projection of it on the z -axis, which is the magnetic field axis. In the weak field limit the spin-orbit interaction dominates over the effect of the external magnetic field.

If the weak field Zeeman energy splitting exceeds the fine structure term from equation 2.22,

$$
\begin{equation*}
\mathcal{E}_{f s}=-13.6 \mathrm{eV} \frac{\alpha^{2}}{n^{2}}\left(\frac{1}{j+\frac{1}{2}}-\frac{3}{4 n}\right) \tag{2.22}
\end{equation*}
$$

where $\alpha$ stands for the fine structure constant ( $\alpha \approx 1 / 137$ ), then the weak field quantum numbers $\left(n, l, j, m_{j}\right)$ are good quantum numbers. If the strong field Zeeman energy splitting exceeds the fine structure term, then the strong field quantum numbers are good quantum numbers. A simulation software has been developed [18] which is a part of this work, and where strong field and weak field differentiation is applied.

In a weak field state $\left|n, l, j, m_{j}\right\rangle$ the projection of $\boldsymbol{\mu}$ on the magnetic field can be calculated according to equation 2.23 [19]. $g_{l}$ is again the orbital g -factor $g_{l}=1$, and $g_{s}$ the electron g -factor.

$$
\begin{align*}
\mu_{Z} & =m_{j} g_{j} \mu_{B}  \tag{2.23}\\
g_{j} & =g_{l} \frac{j(j+1)-s(s+1)+l(l+1)}{2 j(j+1)}+g_{s} \frac{j(j+1)+s(s+1)-l(l+1)}{2 j(j+1)} \tag{2.24}
\end{align*}
$$

In 2.24 the contribution from the nuclear magnetic moment is approximated to be zero. This is due to the fact that it's of the order $m_{e} / m_{p}$ smaller then the contribution of the electrons magnetic moment.

### 2.3 Stark-effect

The Stark-effect is the analogous effect to the Zeeman-effect, but for external electric fields and occurs due to the interaction energy between the electric dipole moment $\mathbf{P}$ and the field $\mathbf{E}$. The additional Hamiltonian is shown in equation 2.25.

$$
\begin{equation*}
\mathcal{H}_{S}=-\mathbf{E} \cdot \mathbf{P} \tag{2.25}
\end{equation*}
$$

Even for the strongest, laboratory created electric fields, $\mathcal{H}_{S} \ll \mathcal{H}_{0}$ is still valid. However, for a very strong field it is possible that the Stark-effect has a stronger influence compared to the fine-structure-term or the Zeeman-term.

The obvious approach would be to now calculate, using the quantum numbers, the electric dipole moment. Then with this the force via

$$
\begin{equation*}
\mathbf{F}=\nabla(\mathbf{E} \cdot \mathbf{p}) \tag{2.26}
\end{equation*}
$$

There is an existing formalism to calculate the Stark-effect of a system. By taking the nonrelativistic Schroedinger equation, in parabolic coordinates, and applying a pertubation potential to it, the energy due to the linear Stark-effect can be written as

$$
\begin{equation*}
\mathcal{E}_{s}=-\frac{Z^{2}}{2 n^{2}}+\frac{3}{2} \frac{E n}{Z}\left(n_{1}-n_{2}\right) \tag{2.27}
\end{equation*}
$$

according to $[19$, p. 228]. In equation $2.27, Z$ stands for the proton number, $n$ for the principal quantum number, $E$ for the absolute value of the electric field strength, and $n_{1}$ respectively $n_{2}$ represent the electric quantum numbers (or parabolic quantum numbers). We see that there is no dependence on the magnetic quantum number $m$. The electric quantum numbers can go from 0 to $n-1$. Hence, the highest separation between two Stark states can be obtained between $n_{1}=0, n_{2}=n-1$ and $n_{1}=n-1, n_{2}=0$. This maximum energy splitting is given by equation 2.28 .

$$
\begin{equation*}
\mathcal{E}_{s, \max }=3 E \frac{n(n-1)}{Z} \tag{2.28}
\end{equation*}
$$

If the electric quantum numbers are not available but the energy level is well known, the force on the atom could be calculated through $\mathbf{F}=-\nabla \mathcal{E}$.

### 2.4 Flexible Atomic Code

In this simulation, which will be discussed in further detail in chapter 4, an external software package called FAC (Flexible Atomic Code) [20] is invoked in order to calculate the decay rates.

We will take the chance to explain how the package is calculating the rates and the theoretical background behind it. FAC is Fortran based and developed by Ming Feng Gu. The software employs a fully relativistic approach, based on the Dirac-equation, to solve the Hamiltonian and evaluate the eigenvalues. The computation of following atomic processes are possible.

- Direct collisional excitation
- Ionization by electron impact
- Recombination (radiative and dielectronic)
- Radiative decay in external fields

This work needs only focus on the calculation of the radiative decay. For this purpose it solves the Hamiltonian $\mathcal{H}$ in equation 2.29.

$$
\begin{equation*}
\mathcal{H}=\mathcal{H}_{0}+\mathcal{H}_{B}^{(1)}+\mathcal{H}_{B}^{(2)}+\mathcal{H}_{E} \tag{2.29}
\end{equation*}
$$

$\mathcal{H}_{0}$ is again the field-free Hamiltonian

$$
\begin{equation*}
\mathcal{H}_{0}=\mathcal{H}_{D} \tag{2.30}
\end{equation*}
$$

where in our case of antihydrogen, this Hamiltonian consists of only one single electron Hamiltonian $\mathcal{H}_{\mathcal{D}} . \mathcal{H}_{B}^{(1)}$ is the linear Zeeman term

$$
\begin{equation*}
\mathcal{H}_{B}^{(1)}=\mu_{B}(2 \mathbf{S}+\mathbf{L}) \cdot \mathbf{B} \tag{2.31}
\end{equation*}
$$

$\mathcal{H}_{B}^{(2)}$ is the diamagnetic Zeeman term

$$
\begin{equation*}
\mathcal{H}_{B}^{(2)}=\frac{1}{2} \mu_{B}^{2}|\mathbf{B} \times \mathbf{r}|^{2} \tag{2.32}
\end{equation*}
$$

and $\mathcal{H}_{E}$ describes the interaction with the electric field.

$$
\begin{equation*}
\mathcal{H}_{E}=\mathbf{E} \cdot \mathbf{r} \tag{2.33}
\end{equation*}
$$

In these equations $\mathbf{S}, \mathbf{L}, \mathbf{r}$ are the spin operator, the orbital angular momentum operator and the position operator, respectively. $\mu_{B}=5.788 \times 10^{-5} \mathrm{eV} \mathrm{T}^{-1}$ is the Bohr-magneton, and $\mathbf{E}$ and $\mathbf{B}$ are again the electric and magnetic field vectors [21].

For the calculation of the transition rates between certain states, one has to find the atomic state function according to equation 2.34 , where $\psi_{\nu}$ are the basis states and $b_{\nu}$ are the eigenvalues
(mixing coefficients) of $\mathcal{H}$. FAC diagonalises $\mathcal{H}$ to get the needed eigenvalues and eigenvectors. It should be mentioned that $\psi_{\nu}$ are single-electron wavefunctions derived from a self-consistent Dirac-Fock-Method where the radial Dirac equation is solved for a known potential. This will be not shown in detail.

$$
\begin{equation*}
\Psi=\sum_{\nu} b_{\nu} \psi_{\nu} \tag{2.34}
\end{equation*}
$$

Once the Hamiltonian is diagonalized it can be represented by $\mathcal{H}_{i j}=\left\langle\psi_{i}\right| \mathcal{H}\left|\psi_{j}\right\rangle$. The Eigenvalue problem itself is solved with a standard linear algebra algorithm. After this effort the transition rates can be derived through the S-matrix elements of equation 2.35 .

$$
\begin{equation*}
\left.S_{f i}=\left|\left\langle\Psi_{f}\right|\right| O_{M}^{L}| | \Psi_{i}\right\rangle\left.\right|^{2} \tag{2.35}
\end{equation*}
$$

$O_{M}^{L}$ is algebraically a tensor operator and represents the single multipole operator. Calculating the S-matrix element leads to the following oscillator strength.

$$
\begin{equation*}
f_{f i}=[R]^{-1} \frac{\omega}{g}(\alpha \omega)^{2 R-2} S_{f i}=(2 R+1)^{-1} \frac{\omega}{g}(\alpha \omega)^{2 R-2}|M|^{2} \tag{2.36}
\end{equation*}
$$

In this formula, $\omega$ is the energy difference between the initial state and the final state, with $\omega=E_{i}-E_{f} .[R]$ is given by $[R]=2 R+1$, where $R$ is the rank of the multi-pole-operator. In our case $R=1$ for a dipole. Furthermore, $g$ is the degeneration of the state with $g=(2 J+1)$. $\alpha$ is again the fine-structure-constant. With this oscillator strength the Einstein coefficients can be calculated.

$$
\begin{equation*}
A_{f i}=2 \alpha^{3} \omega^{2} f_{f i} \tag{2.37}
\end{equation*}
$$

The Einstein coefficient describes the probability of a state changing from state $i$ to state $f$, which leads to the decay rate $\Gamma$ for a radiative decay by spontaneous emission.

$$
\begin{equation*}
\Gamma_{f i}=A_{f i} \tag{2.38}
\end{equation*}
$$

The reciprocal of the decay rate is the lifetime $\mathcal{T}_{f i}$ of this specific transition.

$$
\begin{equation*}
\mathcal{T}_{f i}=\frac{1}{\Gamma_{f i}} \tag{2.39}
\end{equation*}
$$

## Chapter 3

## Experimental Setup

### 3.1 Overview of the experiment

In figure 3.1 the ASACUSA setup is shown in detail. It is situated at the Antiproton Decelerator (AD) at CERN. The $e^{+}$, which are produced by a $\beta^{+}$decay of a ${ }^{22} \mathrm{Na}$ source, are stored in the positron accumulator. To create $\bar{p}$, a high energy proton beam from the Proton Synchrotron (PS) at CERN is directed onto an iridium target, where the process $p+p \rightarrow p+p+p+\bar{p}$ takes place. The resulting antiprotons are delivered to the AD where they are slowed down to approximately 5.4 MeV . Typically the AD delivers $2 \cdot 10^{7}$ antiprotons every 100 s . For ASACUSA the $\bar{p}$ pass through an RFQD (Radio Frequency Quadrupole Decelerator) which further decelerates them. Following this, they are caught and trapped in the MUSASHI accumulator, which is a penningtype trap. To create antihydrogen, $e^{+}$and $\bar{p}$ are injected into the double CUSP-trap.

### 3.2 Antihydrogen spectroscopy

ASACUSA aims to measure the hyperfine structure of antihydrogen with a Rabi-like experiment. The Rabi experiment is a further development of the Stern-Gerlach experiment from 1922 [23], and was first published in 1938 by I. Rabi [24]. The general idea is shown in figure 3.2.

The experimental setup of the ASACUSA collaboration is very similar to this original idea. The beam is first polarised by the double CUSP field, which focuses LFS states in the direction of the detector. After that, the beam passes through a spin-flip cavity, where a $\sigma_{1}$ transition is induced. In this transition, low field seeking $\bar{H}$ atoms are transformed into high field seeking ones. After the cavity, the beam passes through a superconductiong sextupole magnet, which acts as a state selector by deflecting any HFS off the beam axis. The remaining particles then encounter the detector. The whole spectroscopy beam line, which is also accurately emulated in the simulation software, is shown in figure 3.3.

The four main components in the beam line will now be covered. These are:

1. the double CUSP trap,


Figure 3.1: 3D drawing of the ASACUSA experiment with the sodium source for the $\beta$ decay, the positron accumulator, the MUSASHI antiproton accumulator, the double CUSP for the antihydrogen production, the micro-wave cavity, the sextupole magnet and the antihydrogen detector. From [22].


Figure 3.2: An overview of the Rabi-like experiment. An unpolarised beam enters the beam line from the left hand side. After that, the beam is polarised by a magnetic field gradient and the resulting beams split, with only one of the polarised beams being allowed further into the beam line. The polarised beam enters a resonator, which has a radio-frequency field superimposed with a static field. There the atoms can undergo a spin flip. Finally, the spin states are again separated with an inhomogeneous magnetic field, and the remaining beam that reaches the detector can be counted.


Figure 3.3: The ASACUSA spectroscopy beam line with the double cusp production trap. The upper picturer shows a rendering of the beamline CAD drawing. The bottom picture is a schematic view of the apparatus including the coordinate system. Both pictures show the main components. The double CUSP on the very left, followed by the microwave cavity, followed by the superconducting sextupole magnet, and the antihydrogen detector on the very right. $\bar{H}$ is produced in the red ellipsoid. The blue area shows the trajectories of $\bar{H}$ atoms. These pictures were produced by Doris Pristauz-Telsnigg and edited and published in [13].
2. the microwave cavity,
3. the sextupole magnet,
4. and the detector.

### 3.2.1 double CUSP

The double CUSP trap [25] consists of two pairs of anti-Helmholtz coils and a stack of multi-ring electrodes. The configuration is able to produce a radially symmetric electric and magnetic field. It is able to (1) trap, (2) cool, (3) compress, and (4) mix the positrons and the antiprotons. The magnetic field inside the double CUSP preferentially focuses LFS, and defocuses HFS.

In 2014 the CUSP trap was upgraded from a single-type to a double-type CUSP. Now, instead of one set of anti-Helmholtz coils, the trap consists of two pairs of anti-Helmholtz coils, which enables better trapping, mixing and polarisation of the beam. The beam line setup in figure 3.3 also shows the arrangement of the double CUSP. The results presented in this work are the first simulations that consider the double CUSP field-map.

The field distribution inside the trap plays an important role for the polarisation of the $\bar{H}$ beam. The magnetic field map, and the gradient of this field map for the formerly used single CUSP trap are shown in figures 3.4 and 3.5 , respectively. The magnetic field gradient is plotted as a function of the z - and the r -coordinate. As this field is radially symmetric, the radial and the axial component is sufficient to describe the vector field. The field maps for the CUSP and the double CUSP were produced by Simon van Gorp with a software tool-kit named COMSOL ${ }^{1}$. The field gradients were obtained by differentiation of those field maps.


Figure 3.4: The magnetic field strength for the single CUSP is plotted as a function of the $z$ - and the r-coordinate, for the radial and the axial vector component. The $z$-coordinate is measured from the cavity centre. The formation region of the $\bar{H}$ is at $r=0 \mathrm{~cm}$ and $z=$ -174 cm .

[^0]

Figure 3.5: The gradient of the magnetic field strength for the single CUSP is plotted as a function of the $z$ - and the r-coordinate, for the radial and the axial vector component. The $x$-coordinate is measured from the cavity centre.

In figure 3.6 the magnetic field of the double CUSP and figure 3.7 the magnetic field gradient of the double CUSP is plotted as a function of the $z-$ and the r-coordinate. It can be seen from the radial field components that for the double CUSP the Helmholtz-pairs are arranged in a back to back configuration. Single CUSP and double CUSP provide both a positive, radial field gradient, in order to focus the LFS and defocus the HFS. In figure 3.9 the crucial part of the field gradient is enlarged. With the double CUSP configuration the mixing happens at $r \approx 0 \mathrm{~cm}$ and $z=-184.0 \mathrm{~cm}$ with respect to the centre of the cavity. We see that the double CUSP is advantageous, as it provides more area with positive field gradient. In this area the single CUSP generates one spike of a positive field gradient, whereas the double CUSP delivers two peaks, with each of these being broader than the single CUSP peak. As a consequence, the $\bar{H}$ passes through a larger region of positive field gradient during its time of flight in the double CUSP. Hence, they are better focused along the beam line axis. The electric field of the double CUSP is shown in figure 3.8, where the mixing region is highlighted.


Figure 3.6: The magnetic field strength for the double CUSP is plotted as a function of the $z$ - and the r-coordinate, for the radial and the axial vector component. The $x$-coordinate is measured from the cavity centre. The formation region of the $\bar{H}$ is at $r=0 \mathrm{~cm}$ and $z=$ -184 cm .


Figure 3.7: The gradient of the magnetic field strength for the double CUSP is plotted as a function of the $z$ - and the r-coordinate, for the radial and the axial vector component. The $x$-coordinate is measured from the cavity centre.


Figure 3.8: The electric on-axis potential of the nested CUSP trap used in 2015 is shown. The antiprotons with an energy of 20 eV are injected by pulling up the upstream potential barrier. The red line shows the energy of the positron plasma when captured in the potential valley. The values on the $z$-axis are adapted to the scale used in this work, where the zero point is in the centre of the cavity. From [22].


Figure 3.9: The gradient of the magnetic field strength around the mixing region for the single CUSP is plotted as a function of the $z$ - and the r-coordinate on the left picture, and for the double CUSP on the right picture. More area of a positive field gradient is provided by the double CUSP.

### 3.2.2 Microwave cavity

The microwave cavity consists of a double strip-line resonator, which is surrounded by a pair of Helmholtz coils, according to the picture 3.10. The coils produce a static and homogeneous magnetic field and are necessary to prevent Majorana spin-flips [26]. Majorana spin flips are induced by strong fluctuations of the magnetic field. To excite a $\sigma_{1}$ transition, the external magnetic field from the Helmholtz-coils and the oscillating magnetic field from the cavity have to be parallel. To excite a $\pi_{1}$ transition, the static and the oscillating field have to be perpendicular [27]. In the current set-up only the $\sigma_{1}$ transition can be induced. Investigating the $\pi_{1}$ transition is planned for the future. The cavity is tuned around 1.4 GHz .


Figure 3.10: A photo from the spin flip cavity, where the two Helmholtz coils can be seen. From [12].


Figure 3.11: Magnetic field lines (red) of the sextupole magnet. The black parts symbolizes the magnet poles, $N$ for north-pole, $S$ for south-pole. From [28].

### 3.2.3 Sextupole magnet

The superconducting sextupole magnet is used as a spin state selector by deflecting HFS off-axis, and focusing LFS onto the detector. It can be thought of as a lens. Again, if the correct transition frequency is tuned, the LFS would be turned into HFS. Afterwards, they will be defocused by the sextupole magnetic field. A drop in count rate can then be measured on the detector.

In figure 3.11 the magnetic field lines of the sextupole in the $x-y$-plane are shown. The coordinate system is shown in figure 3.3. We see that the magnetic flux density increases if we move from the center to the outside. Furthermore, it is important to note that in the centre the magnetic field is zero. To avoid HFS-states travelling in the centre through the sextupole, a beam blocker is installed upstream of the cavity in the field ionizer chamber, which blocks the central part of the beam. This beam blocker is not part of the simulation. The equation of the magnetic field of the sextupole magnet is

$$
\begin{equation*}
B(r)=c r^{2} \text { with } c=\text { constant } . \tag{3.1}
\end{equation*}
$$

The gradient of the field is always pointing away from the center to the outside.

$$
\begin{equation*}
\nabla B(r)=2 c \mathbf{r} \tag{3.2}
\end{equation*}
$$

The sextupole field is characterised with a constant magnetic field per distance squared. In our case the value is given by the manufacturer (TESLA) with

$$
\begin{equation*}
\frac{2|B(r)|}{r^{2}}=2400 \mathrm{~T} \mathrm{~m}^{-2} \text { at } 400 \mathrm{~A} \tag{3.3}
\end{equation*}
$$

This gives a maximal magnetic field strength of 3.0 T at a distance of 50 mm from the centre through the sextupole coil.

### 3.2.4 Detector

To confirm the presence of $\bar{H}$ a CPT-detector (Charged Pion Tracker) is used [13, 29]. It consists of an in-vacuum calorimeter (BGO) made of Bismuth Germanate ( $B i_{4} G e_{3} O_{12}$ ) [30] and an outer hodoscope [31].

The BGO is a 2-D calorimeter with a diameter of 90 mm . When a $\bar{H}$ atom hits the BGO, an annihilation of $\bar{p}+p$ or $\bar{p}+n$ takes place. The annihilation is on average about three charged pions [32], which can be detected. The BGO measures the deposited energy of the annihilation. The $\vec{p}$ 's and its charged annihilation products create light in the BGO scintillator. The BGO is coupled directly to four photomultiplier tubes with each tube containing $8 \times 8$ read-out channels. With this it provides a spatial resolution of the hits in the x -y-plane (16x16).

The BGO is surrounded by a two-layer hodoscope with 32 scintillator-bars per layer (figure 3.12). When the annihilation products leave the BGO they deposit energy in the hodoscope scintillators. Each of the 64 hodoscope scintillator bars is equipped with four SiPM (Silicon Photo Multiplier), two on each end [33]. As there are two layers of the hodoscope, track reconstruction is possible. The coincidence between the BGO and the hodoscope helps to reconstruct the track and to identify $\bar{H}$. Due to the fact that there are SiPM's on each side of the hodoscope, a certain z-axis resolution can be obtained. The detection efficiency is very crucial, as the number of $\bar{H}$ atoms we expect, is low.


Figure 3.12: Schematic of the CPT detector. [30]

## Chapter 4

## Implementation and coding

### 4.1 Introduction of the software kit

Geant4 (GEometry ANd Tracking) [34] is a software toolkit developed at CERN for the simulation of the interaction between particles and matter. It is used by a a lot of experiments and research projects in the domains of high energy physics, astrophysics, space science, medical physics and radiation protection. It is also used by the ASACUSA collaboration in order to calculate the propagation of $\bar{H}$.

Geant4 is based on C++ and therefore takes advantage of polymorphism. With this mechanism core classes like the event handler, the detector construction or the step action class can be replaced by a user defined class, which contains all properties of the base class through inheritance. This is important to have maximal flexibility, and to implement functionalities like radiative decay of atoms, that were never considered in Geant4.

Over the past few years ASACUSA has built a complete simulation package based on Geant4 to provide Monte-Carlo simulations for anti-hydrogen. It covers the tracking of neutral atoms through its travel in the apparatus, from the production point in the double CUSP to the detector. It also contains the correct annihilation process of $\bar{H}$. Furthermore the beam line geometry, the electric and magnetic fields and the detector properties are built into the software model. Later a radiative decay tool for $H$ and $\bar{H}$ was added in order to have the correct quantum state information at each point. The tool fully considers the influence of the magnetic field environment on the decay rates and the force. This work was done by Chloé Malbrunot and Rikard Lundmark in 2013 [18]. In this work a further extension and new simulation results are shown.

### 4.2 Implementing the current beamline setup

To begin, the beam line geometry was to be updated. The previous software was based on the 2012 beam line setup. It was necessary to keep the old construction in the simulation available
in order to be able to reproduce results from previous papers. This was achieved by loading different macro files, which contain the different geometries. New macro files had to be created and additionally the following classes had to be extended or changed: hbarDetConst, hbarCPTConst, hbarBeamlineConst.


Figure 4.1: GDML-Export of the ASACUSA simulation geometry, which shows the previous beam line from 2013. It was printed by ROOT's GDML viewer.


Figure 4.2: GDML-Export of the ASACUSA simulation geometry, which shows the current beam line including all geometrical implementations done. It was printed by ROOT's GDML viewer.

In figure 4.1 the previous beam line and in figure 4.2 the updated beam line as it is implemented in the simulation is presented. The new implemented parts are highlighted. Due to this extension the beam line became longer. As a consequence the solid angle from the mixing area to the BGO detector is reduced. The reduction of solid angle is a disadvantage, because less $\bar{H}$ 's will arrive at the detector. On the other hand the changes brought an extension between the mixing area and the micro-wave cavity, which gives the atoms more time for the deexcitation to ground state. The exact numbers are summarised in table 4.1.

Table 4.1: The geometrical consequences of the beamline extension are shown.

|  | total distance | changes to the previous <br> beamline set up | consequences |
| :--- | :--- | :--- | :--- |
| mixing point to cavity | 1840 mm | +90 mm | more $\bar{H}$ 's cascade down <br> to GS |
| mixing point to BGO | 3695 mm | +489 mm | reduction of the solid <br> angle by $26 \%$ |

Additionally, a GDML-export (Geometry Description Markup Language) function was implemented, to export the geometry in the GDML format. This implementation was done in order to be able to lad it into the analysing software.

### 4.2.1 Sequence of the simulation

As mentioned before, in 2013, a decay tool was built on top of the ASACUSA simulation. The approach was to create a static database with a software tool that calls the FAC, which was presented in section 2.4 , in order to achieve a fast simulation of the anti-hydrogen decay.

After an $\bar{H}$ atom with initial quantum numbers, and an initial velocity and direction is created, the simulation proceeds following calculations in each simulation step.

1. The possible decay channels for the current quantum state and the current field strength are accessed from the database.
2. A new quantum state of strong field or low field quantum numbers is applied to the $\bar{H}$ particle, or the current quantum state is retained, according to the probabilities.
3. Calculate the new magnetic moment according those new quantum numbers.
4. Compute force and the trajectory of the particle according the field gradient and the magnetic moment.

### 4.3 Extension of the decay library creator

So far, the simulation only considers the B-field information and no E-field is taken into account for the calculation of the force and the decay. The motivation for implementing it is the fact that we have electric fields inside the experiment (double CUSP), and that we expect, according the previous theory chapter, to observe a faster decay due to Stark-mixing (E-field + B-field). Also, the reduction of the decay time through an additional external electric field could be studied within simulation software.

The whole software is split into two main parts. One is the tool to create the decay library and one is the simulation itself. In order to implement the E-field and the angle between the E-field and the B-field (3D grid), both tools had to be changed.


Figure 4.3: The main modules of the decay library builder.

The decay library builder itself is split into three parts as seen in figure 4.3. Bingen invokes FAC and creates, for each transition and field, a folder with three binary files. These files contain energy levels, oscillator strengths and decay rates. Rootgen takes the folders and stores the information from FAC into a ROOT-file. In a last step, this ROOT-file is taken by rootfitmake to create a minimized root database, which can be read by Geant4. rfac_decayableObject.cc on the chart is compiled by the simulation itself and can be seen as an interface class between the decay library and the ASACUSA simulation. We see that the first two modules of the decay library creator can be left untouched. For the extension into a three dimensional grid, only the rootfitmake module had to be changed, according figure 4.4.


Figure 4.4: Necessary changes to the decay database creator.

The data is stored in an abstract data type, called "interpolated objects". This class contains not only the values but also the interpolator itself. In order to evaluate the decay rates for a three-dimensional grid of a magnetic field, an electric field and an angle between these vectors, an interpolator had to developed.

### 4.3.1 Triliniar interpolator

When going from pure B-field information to B-field, E-field, and the angle between them, we had to upgrade from a linear to a trilinear interpolator. A linear interpolator was chosen in order to minimize the computation time and data size. The simulation requests the decay rates by delivering the current quantum state, the B-field, the E-field and the angle between them. The database is
organised so that for each key (current state) a 3D-map of decay rates is available. The changes, which had to be done for developing a 3D-database, are highlighted in figure 4.4.

The interpolator container, which holds the interpolator objects, stores in this case the trilinear interpolator objects. The principle of creating the 3D decay library can be seen in figure 4.5. If the full 3D information is detected this algorithm is automatically activated. The 3D interpolator needs a complete grid. It means that for each (E,B,Angle)-tupel and for each decay channel a decay rate has to be available. Otherwise the interpolator algorithm cannot be performed. But, for some transition channels on certain grid points, the database does not deliver a decay rate, if the transition is not allowed for example. In that case a transition rate of zero is applied.
Further, it was important to have a quick sorting algorithm as this has to be performed in every simulation step. This was ensured by the following principle. The actual value is divided by the grid-interval, and afterwards this value is type-cast to an integer number. Then this integer is the index of the array. For more than one dimension an offset has to be taken into account.


Figure 4.5: Saving a $3 D$ grid in the database. E are the electric field values, $B$ the magnetic field values, $A$ the angle between the electric and magnetic field and $R$ the decay rate that are stored in the database. $\# E$ is the number of different $E$-field values, $\# B$ is the number of different $B$-field values and \#A is the number of different angles available in the database. The difference from one field or angle value to the nearest neighbour give the discretizations $d E$, $d B$ and $d A$. The database entries with the largest field value or the largest angle are assigned with the largest index $n, m$ and $k$.

In equation 4.1 to 4.10 , we see the complete calculation of the trilinear interpolator. First of all, the code is evaluated according to its surrounding grid points. Then, these eight grid points are labelled from $C_{000}$ to $C_{111}$ and put into the algorithm presented in the following equations. $A_{d}$, $B_{d}$ and $E_{d}$ are the current field values and the current angle in the simulation, to which the decay rate has to be evaluated.

$$
\begin{align*}
C_{00} & =C_{000}\left(1-A_{d}\right)+C_{100} A d  \tag{4.1}\\
C_{10} & =C_{010}\left(1-A_{d}\right)+C_{110} A d  \tag{4.2}\\
C_{01} & =C_{001}\left(1-A_{d}\right)+C_{101} A d  \tag{4.3}\\
C_{11} & =C_{011}\left(1-A_{d}\right)+C_{111} A d \tag{4.4}
\end{align*}
$$

This first four lines reduce it to a plane consisting out of four grid points, $C_{00}$ to $C_{11}$. After that, these are further reduced to a line with two points by 4.5 and 4.6.

$$
\begin{align*}
& C_{0}=C_{00}\left(1-B_{d}\right)+C_{10} B_{d}  \tag{4.5}\\
& C_{1}=C_{01}\left(1-B_{d}\right)+C_{11} B_{d} \tag{4.6}
\end{align*}
$$

At the end a single linear interpolator computes the result as shown in equation 4.7.

$$
\begin{equation*}
C=C_{0}\left(1-E_{d}\right)+C_{1} E_{d} \tag{4.7}
\end{equation*}
$$

The figures $A_{d}, B_{d}, E_{d}$ are the distances between the requested point and the grid points (A for Angle, B for B-field, E for E-field).

$$
\begin{align*}
A_{d} & =\frac{A-A_{0}}{A_{1}}  \tag{4.8}\\
B_{d} & =\frac{B-B_{0}}{B_{1}-B_{0}}  \tag{4.9}\\
E_{d} & =\frac{E-E_{0}}{E_{1}-E_{0}} \tag{4.10}
\end{align*}
$$

### 4.3.2 Simulation tool

Beside the database, the simulation tool had to be extended in order to enable to process the 3D information. In the simulation tool it was necessary to require the E-field, calculate the angle between the two field vectors and put all three parameters inside the hbarStepAction. There, the decay object containing the decay information, is calculated. In the rfac_decayableObject the corresponding function was overloaded to automatically call the right interpolated object.

After the upgrade the software automatically detects if a 3D grid is used. If so, the data is stored in a new datatype called quadruple (decay rate, E-field, B-field, angle), instead of a pair (decay rate, B-field). If the simulation is accessing a data point for a certain field point, the interpolator class is called directly. The whole structure of the simulation and the decay library tool can be seen in figure 4.6. When the extension to a 3D-database was planned, it was decided to use the same structure as for the 1D one. This unfortunately proved to be too computational intensive.


Figure 4.6: Necessary changes of the software modules.

### 4.3.3 Handling large databases

The extension of the software package brought with it a variety of additional problems. The upgrade from a one dimensional database (B-field), to a three-dimensional database (B-field, E-field, angle) includes a non-linear increase of datasize and computation time. First of all the number of grid points changed from $n$ to $n^{3}$, and furthermore each data point occupies more space because the stored interpolator-objects are more complex. The existing database with the pure B-field covered 20 grid points. The newly created database includes a grid of $10 * 10 * 10=1000$ points.

Another problem occurred when trying to generate a 3D ROOT-database from the binary output of FAC with rootgen. A severe memory leak made it impossible to generate a large decay database. After 20 GB the server crashed due to an internal memory overflow. If this happens, the server has no chance to write an error message into the shell. This fact made it hard to localise the problem at first. A solution was developed by building a modified version called rootgen_serial. This tool calls rootgen and closes it after a specific data size is reached. Then, a new ROOT-file is initiated to continue. With this procedure no data points are lost, and the resulting ROOT-files can be concatenated afterwards. Thus, also the ROOT-file limitation of 100 GB is circumvented. The resulting 3D decay library is about 80 files with a total size of about 300 GB. The postprocessor rootfitmake was accordingly modified to connect the different ROOT-files to a merged tree structure. This can be made by using ROOT's TChain class, which can merge an arbitrary number of ROOT-files into one data structure.

The last problem, which has not been solved yet, is to convert the 3D rootgen database to a rootfitmake database. Looping over the TChain tree and collecting the data-points requests too much computational time. For example, a server with 40 cores would require one year for this task. In section 4.4 we will cover some potential alternatives.

However, the proof of principle for the anti hydrogen decay in an electric and magnetic field was successfully performed with a very small database ( $3 \times 3 \times 3$ grid, $n \leq 10$ ), by observing trajectories and quantum state changes for a few quantum states.

### 4.4 Organisation of iterative simulations

To compute billions of $\bar{H}$ atoms, as it was done in this work, an appropriate algorithm has to be used. This algorithm has to (1) create a macrofile for each set-up, (2) invoke the simulation hbar_gshfs with this macro file, (3) create a list of created output files, and (4) show the progress. There was a python-script edited to do this job. Multi-threading was used to have a consecutive load on each core ( 8 per server) and the processes had to be started in the background.


Figure 4.7: consecutive simulations with a python script

## Discussion

The changes to the beam line geometry increased the length of the apparatus, and therefore decreased the solid angle seen by the detector. The motivation for implementing the electric field information into the software was mentioned. Unfortunately, we faced many problems. The decay data for the magnetic and the electric field was created, however it has not yet been processed into a database that can be used by the simulation. There are at least three potential approaches to proceed.

- Leaving the last conversion step out and dealing with the 300 GB data-base. Shifting some tasks into the simulation.
- Changing the data structure to a simpler one.
- Calculating the decay rates in each step separately.

The first potential solution could be very awkward in terms of handling. It is not fully known by how much the simulation time would increase, when the algorithm has to access a 300GB data file. A SSD hard disk would be mandatory for this solution.

The second potential solution could be appropriate if it would be possible to convert the binaries in one step into a Geant4 usable data-base, filled with minimal information (no interpolators, no oscillator strength, etc). With this, it should be possible to reduce the size and especially the computation time. For this reason the interpolator could be put into the step action of the simlation tool.

The third potential solution could increase simulation time drastically, but it avoids interpolation errors as it could calculate the decay rates accurately for each point. It could be worth while to check if an alternative to FAC exists, which treats exclusively $H$ or $\bar{H}$. FAC is a tool for arbitrary atoms and in our case used for antihydrogen. For the sake of completeness we want to mention, that from [14] and [19] the energy levels of hydrogen in external fields can be calculated analytically, under certain restrictions. In [14] a simplification of degenerate perurbation theory with further approximations was used. This implies uncertainties that could add up to large errors in the case of many simulation steps. In [19] only a treatment of an external electric field or an external magnetic field are treated, but not the combination of it.

## Chapter 5

## Results

### 5.1 Simulations with a monoenergetic beam of LFS in various nstates

### 5.1.1 Velocity, energy, temperature

In atomic physics it is common to refer to the beam energy in temperature units (Kelvin). It is mostly done in the context of a beam of particles with Boltzmann distributed velocities. To get a clearer picture the results presented in this work were done with mono-energetic beams, hence we preferably use the SI-unit of velocity ( $\mathrm{m} \mathrm{s}^{-1}$ ).

We will take initial velocities derived from the mean value of Boltzman distributions, with temperatures of $50 \mathrm{~K}, 150 \mathrm{~K}, 500 \mathrm{~K}$ and 1000 K . Equation 5.1 is the mean velocity of a Boltzman distribution in three dimensions. This velocity can be translated with equation 5.2 into energy, where $\bar{v}$ is the mean velocity, $E$ is the kinetic energy, $k$ is the Boltzmann factor, $m_{\bar{H}}$ is the mass of an hydrogen atom in atomic units $u$.

$$
\begin{array}{r}
\bar{v}=\sqrt{\frac{8 k T}{m \pi}} \\
E=\frac{1}{2} m \bar{v}^{2} \\
k=8.617 \times 10^{-5} \mathrm{eV} \mathrm{~K}^{-1} \\
m_{\bar{H}}=1.007825 u \\
1 u=931.4941 \mathrm{MeV} \mathrm{c}^{-2} \tag{5.5}
\end{array}
$$

The final numbers used in this work are summarised in table 5.1.

Table 5.1: velocity-energy-temperature equivalents

| mean velocity | energy | temperature |
| :---: | :---: | :---: |
| $1000 \mathrm{~m} \mathrm{~s}^{-1}$ | 5.22 meV | 50 K |
| $1800 \mathrm{~m} \mathrm{~s}^{-1}$ | 16.9 meV | 150 K |
| $3200 \mathrm{~m} \mathrm{~s}^{-1}$ | 53.4 meV | 500 K |
| $4600 \mathrm{~m} \mathrm{~s}^{-1}$ | 110 meV | 1000 K |

### 5.1.2 Introduction

In the upcoming study, a hit-ratio is defined as the percentage of $\bar{H}$ atoms detected at the BGO with respect to the number of $\bar{H}$ atoms produced at the source, according to equation 5.6. Signal is defined as the percentage of $\bar{H}$ atoms that reach the cavity in ground state AND hit the detector with respect to the number of $\bar{H}$ atoms produced. This is summarised in equation 5.7. This distinction is made as hyperfine measurements can only be made on $\bar{H}$ atoms that are in the ground state when they reach the cavity.

$$
\begin{equation*}
\text { hit-ratio }=100 \cdot \frac{\text { number of } \bar{H} \text { atoms detected at the BGO }}{\text { number of } \bar{H} \text { atoms produced }} \tag{5.6}
\end{equation*}
$$

$$
\begin{equation*}
\text { signal }=100 \cdot \frac{\text { number of } \bar{H} \text { atoms that reach the cavity in GS and hit the detector }}{\text { number of } \bar{H} \text { atoms produced }} \tag{5.7}
\end{equation*}
$$

The simulations shown in this section use a decay library with only magnetic field information. These results show the effect of the initial principal quantum number and the velocity on the signal and noise at the detector for different sextupole magnetic fields.

In this section we used a mono-energetic beam with solely LFS in a circular state with the smallest magnetic quantum number. That means we are using a quantum set of $n, l=n-1$, $m_{l}=-l, m_{s}=-1 / 2$. This state is the LFS state with the biggest force acting on it (pointing to lower magnetic field values). These will be labelled as strong LFS (sLFS) from now. For each point in the plots, consisting of a certain quantum state, initial velocity and sextupole field value, $10000 \bar{H}$ atoms were simulated. For the study in this section, we measured the amount of $\bar{H}$ atoms and their quantum state at three significant positions along the beam-line. These positions are summarised in table 5.2 , where distance is measured relative to the cavity-center along the z -axis. The opening angle (apex angle of the cone of the source) was set to two degrees. Such a small angle makes a signal of $100 \%$ possible. The BGO detector has a diameter of 90 mm which spans a cone from the mixing point $(z=-184 \mathrm{~cm})$ with an apex angle of $4.2 \times 10^{-4}$ degrees. The current experimental beam-line set-up with the double CUSP field-map is used. All non-trivial simulation parameters are listed in table 5.3 for readers reference.

Table 5.2: The three relevant positions for the plots in section 5.1

| place | distance |
| :--- | ---: |
| source center | -1.84 m |
| cavity entrance | -0.0525 m |
| sextupole entrance | 0.680 m |
| BGO detector | 1.855 m |

The error bars are calculated according to quadratic Gaussian error propagation for a counting experiment. The error calculation for such a counting experiment is presented below. $N_{1}$ stands for the number of experimental runs, which is in our case represented by the total number of particles sent per plot point on the plot. In this section it will be always $N_{1}=10000 . N_{2}$ is the number of events. In our case, events can be either the number of $\bar{H}$ hits at the detector or the number of $\bar{H}$ hits from atoms that where at the cavity in ground state (signal).

$$
\begin{align*}
f & =\frac{N_{2}}{N_{1}}  \tag{5.8}\\
\Rightarrow \triangle f & =f \cdot \sqrt{\frac{1}{N_{1}}+\frac{1}{N_{2}}} \tag{5.9}
\end{align*}
$$

### 5.1.3 Simulation parameters

The simulation parameters used for this study are presented in the following table 5.3.

Table 5.3: Simulation parameters

| description | macro entry | value |
| :---: | :---: | :---: |
| particle type | /gun/setParticle | hbar |
| number of $\bar{H}$ 's per point | /run/beamOn | 10000 |
| beam-line setup | /control/execute | cusp2014.mac |
|  | /control/execute | spectroscopy_hbar 2014.mac |
| polarisation | not applicable (see QN's) | sLFS |
| initial principal QN n | /gun/setStateN | $1 ; \ldots ; 30$ |
| azimuthal QN l | /gun/setStateL | $n-1$ (circular) |
| 2x total angular momentum QN j | /gun/setStateTwoMJ | $-2 n+1$ |
| 2x magnetic QN ml | /gun/setStateML | $-n+1$ |
| 2 x spin QN s | /gun/setStateTwoMS | -1 |
| used decay library | /gun/setQuantumStateFile | hstates.root |
| lower hysteresis for field regime transitions | /gun/setLowerHysteresis | 0.5 |
| upper hysteresis for field regime transitions | /gun/setUpperHysteresis | 5.0 |
| randomised kinetic energy | /gun/setRandomEnergy | off |
| user defined kinetic energy | /gun/setBeamEnergy | $\begin{aligned} & 5.22 \mathrm{meV} ; 16.9 \mathrm{meV} \text {; } \\ & 53.4 \mathrm{meV} ; 110 \mathrm{meV} \end{aligned}$ |
| shoot particle into a cone | /gun/setRandomDirection |  |
| opening solid angle of that cone | $\begin{aligned} & \text { /gun/setSourceOpening } \\ & \text { Angle } \end{aligned}$ | 2.0 |
| source position is gaussian with FWHM in x-direction | /gun/setSourceFWHM_X | 0.2 cm |
| FWHM in y-direction | /gun/setSourceFWHM_X | 0.2 cm |
| FWHM in z-direction | /gun/setSourceFWHM_X | 0.0 cm |
| cut off gaussian at this value | /gun/setSourceMaximum <br> Radius | 0.1 cm |
| follow correct F and M QN | /gun/setRandomState | follow |
| particles produced per second | /gun/setProductionRate | 1 |
| initial quantum state randomized | /gun/setRandomState | off |
| position of source center (mixing) | /gun/setSourceCenter | -1.84 |
| use a two-dimensional flat beam | /gun/setSlice | off |
| maximum B-field in sextupole | /field/setMaximumValue | 0.2 T...; 3.6 T |
| cusp B-field-map used | /setup/cusp/ <br> setCuspMagAsciiFile | B_fieldmap_double CUSP_1000x1000.txt |
| cusp E-field-map used | $\begin{aligned} & \text { /setup/cusp/ } \\ & \text { setCuspElAsciiFile } \end{aligned}$ | Erz_FI_well.txt |
| process specific tracks | /tracking/processTracks | primary |

### 5.1.4 Signal and hit-ratio for different magnetic field values and principal quantum numbers

In figures 5.1 to 5.4 the hit-ratio and the signal is plotted versus the magnetic field strength for certain initial principal quantum numbers. $100 \%$ signal means that all atoms of that initial state reached the cavity in the ground-state AND hit the detector. There were a few representative initial principal quantum numbers taken. Each quantum number has its own color and the signal can be distinguished from the hit-ratio by its symbol.

The magnetic field scan can help to find an optimal magnetic field of the sextupole. Bear in mind, that in this chapter only sLFS were considered. As the sLFS are circular states, the decay is expected to be relatively slow, as we discussed in chapter 2 . On the other hand, the focusing effect for sLFS is maximal.

The particles are being emitted randomly into the opening angle. It has the unit of a solid angle and was defined for the following study with $2^{\circ}$.


Figure 5.1: Signal and hit-ratio plotted as a function of the magnetic field. Lower initial $n$-states are shown on the left picture, and higher inital n-states on the right picture. For these plots an initial velocity of $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$, and an opening angle of $2^{\circ}$ were used.


Figure 5.2: Signal and hit-ratio plotted as a function of the magnetic field. Lower initial $n$-states are shown on the left picture, and higher inital n-states on the right picture. For these plots an initial velocity of $v=1800 \mathrm{~m} \mathrm{~s}^{-1}$, and an opening angle of $2^{\circ}$ were used.


Figure 5.3: Signal and hit-ratio plotted as a function of the magnetic field. Lower initial $n$-states are shown on the left picture, and higher inital n-states on the right picture. For these plots an initial velocity of $v=3200 \mathrm{~m} \mathrm{~s}^{-1}$, and an opening angle of $2^{\circ}$ were used.


Figure 5.4: Signal and hit-ratio plotted as a function of the magnetic field. Lower initial $n$-states are shown on the left picture, and higher inital n-states on the right picture. For these plots an initial velocity of $v=4600 \mathrm{~m} \mathrm{~s}^{-1}$, and an opening angle of $2^{\circ}$ were used.

## Discussion

It can be seen from figure 5.1, that for a low velocity of $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$ the sextupole magnetic field can have a wide range of values from 0.5 T to 2.5 T in order to achieve a maximum signal for low initial n-states. For very low and very high magnetic fields, the number is decreasing due to "over-focusing" or "under-focusing". For low quantum numbers, at these velocities, the signal is equal its hit-ratio, as the time of flight is long enough to decay to ground state before reaching the cavity. For higher quantum states, the difference between signal and hit-ratio is increasing, and from $n \geq 26$ there is no signal contribution any more. Furthermore, two resonance peaks appear for the hit-ratio of $n=26$ and $n=30$. The reason for these peaks is the fact that for these initial quantum numbers and velocity, a magnetic field around 2 T is optimal and the focal point is at the BGO. Going to higher or lower field values, the strong coupling to the magnetic field due to the high quantum number is causing an over-focusing or under-focusing effect. We see that for these values there is no corresponding signal, instead only a hit-ratio, which causes only background. For such high quantum numbers in a circular state, the decay process is slow as described in section 2.2.

For $v=1800 \mathrm{~m} \mathrm{~s}^{-1}$, in figure 5.2, a higher magnetic field than 1.2 T is needed in order to have the same focal point. This is due to the particles travelling faster and spending less time in the magnetic field. For $n=18$ the signal is at $90 \%$, whereas the hit-ratio is at $100 \%$. This indicates that most of these initial states will be at ground state when they reach the sextpole and this explains the weak influence of the magnetic field on the hit-ratio and signal. Higher n-states $(n \geq 22)$ are sensitive to the magnetic field and they add almost only hits and no signal. Hence these atoms can be counted as noise and could be reduced by a higher magnetic sextupole field. For the upcoming simulations in section 5.2 , which use a complete initial n-state distribution, it would be a first estimation to consider a magnetic field lower than $B=2.5 \mathrm{~T}$ for the $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$ study, and a magnetic field higher than $B=3.0 \mathrm{~T}$ for the $v=1800 \mathrm{~m} \mathrm{~s}^{-1}$ study. Rather than trying to obtain a low background, we are interested in generating the maximal signal to noise ratio. In our case the background is the noise. As discussed in section 3.2.3, the maximal field strength that can be created in the sextupole at the experiment is currently $B=3.0 \mathrm{~T}$.

In the case of $v=3200 \mathrm{~m} \mathrm{~s}^{-1}$, in figure 5.3, there is no more signal to observe from $n=18$ upwards. To decrease the pure hit-ratio, a higher magnetic field is beneficial with the downside of decreasing the low n-state signal amplitude. We further see that for this velocity, and principal quantum numbers up to $n=14$, the signal is the same as the hit-ratio, so for these values no noise is expected. Interestingly, $n=14$ has a higher signal value than lower initial principal quantum numbers. This can be explained by the fact that the focusing of the double CUSP for this state is stronger than for lower quantum numbers. For $n=18$ almost all atoms reach the detector, but only $\approx 50 \%$ of them can be counted as signal. That means that this initial quantum number is focused well for $v=3200 \mathrm{~m} \mathrm{~s}^{-1}$ in the double CUSP and in the sextupole, but only every second atom is decaying to ground state by the time the cavity is reached. Higher quantum numbers of $n \geq 22$ do not produce any signal, but $100 \%$ hit-ratio up to a sextupole magnetic field of 1.5 T . From that point onwards an over-focusing takes place in the sextupole as the hit-ratios decrease.

The highest simulated velocity of $v=4600 \mathrm{~m} \mathrm{~s}^{-1}$, in figure 5.4 , shows a similar behaviour to the $v=3200 \mathrm{~m} \mathrm{~s}^{-1}$ study, but all signal and hit-ratio values are reduced and can barely climb over $80 \%$ for the $n=14$ case. Up to a quantum number of $n=14$ the signal has again the same amplitude as
the hit-ratio. For $n=14$ it is about $10 \%$ higher than lower quantum numbers (for $v=3200 \mathrm{~m} \mathrm{~s}^{-1}$ it was $10 \%$ ). For $n=18$ the hit-ratio does not reach the $100 \%$ mark, instead $80 \%$. From these only every fourth $\bar{H}$ is reaching the cavity in ground state to create a signal. Only the high quantum numbers ( $n \geq 22$ ) can reach $100 \%$ hit-ratio for such a high velocity. For these states, the magnetic moment is large enough to focus them onto the detector, but none of those events contribute signal.

### 5.1.5 Signal depending on the initial principal quantum number n for different magnetic fields

Figure 5.5 and figure 5.6 present the signal as a function of the principal initial quantum number for different sextupole fields. It was the same simulation data used as for the previous analysis in subsection 5.1.4. Again, all four velocities are analysed.


Figure 5.5: Signal is plotted as a function of the initial principal quantum number for different $B$-fields. An initial velocity of $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$ is shown on the left picture, and an initial velocity of $v=1800 \mathrm{~m} \mathrm{~s}^{-1}$ on the right picture. An opening angle of $2^{\circ}$ was used.


Figure 5.6: Signal is plotted as a function of the initial principal quantum number for different $B$-fields. An initial velocity of $v=3200 \mathrm{~m} \mathrm{~s}^{-1}$ is shown on the left picture, and an initial velocity of $v=4600 \mathrm{~m} \mathrm{~s}^{-1}$ on the right picture. An opening angle of $2^{\circ}$ was used.

## Discussion

In the figures 5.5 and 5.6 it can be seen that very low quantum numbers ( $n \leq 15$ ) are not necessarily ideal to achieve a high signal. It is interesting to note that there is often a peak structure to observe.

For the $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$ study, in figure 5.5 , there is a resonance peak at $n=15$ for very low and very high fields, whereas for medium magnetic fields there is no peak to observe. The mentioned quantum number is the best trade-off between being focused by the double CUSP and having time to decay down to ground state before the cavity. The fact that the peak structure exist also for a zero magnetic sextupole field confirms that the double CUSP is the reason for this peak. Lower quantum numbers show a large difference between the highest magnetic field ( $\approx 50 \%$ signal ) and the optimal magnetic field ( $\approx 100 \%$ signal). The reason for this is that lower initial quantum numbers have a worse spacial distribution at the sextupole entrance. Hence, they require an optimised field strength. For initial quantum numbers of $n \geq 16$ all signals begin to decrease, because the $\bar{H}$ cannot decay down to ground state in time. For a medium field strength between 0.6 T and 2.4 T the regression of the signal for high quantum numbers is slower than for the minimal and the maximal magnetic field. For $n \geq 25$ the signal is no longer observable.

For the $v=1800 \mathrm{~m} \mathrm{~s}^{-1}$ study, in figure 5.5, the resonance peak is slightly shifted to the right compared to the low velocity case. Another difference is that the maximal magnetic field of 3.6 T is able to reach a maximal signal up to an initial quantum number of $n=16$. This is because for higher velocities an over focusing in the sextupole does not happen for ground state atoms with 3.6 T. For $n \geq 16$ all curves drop in the same way. From that point the ability to decay to the ground state before the cavity is reached, is the dominating effect. For $n \geq 23$ the signal is no longer observable.

For the high velocity studies it is not possible to reach a maximal signal for low quantum numbers. In the $v=3200 \mathrm{~m} \mathrm{~s}^{-1}$ case, in figure 5.6 , there is again a resonance peak at $n=15$. The maximum field of 3.6 T produces the largest signal amplitudes. For low initial quantum numbers there are $\approx 95 \%$, and at the peak $\approx 100 \%$ for $n=15$. Generally, a higher field is better to produce large signals. Without a magnetic field in the sextupole, low quantum numbers produce a signal of $\approx 55 \%$ and a maximal signal of $\approx 90 \%$ at the peak. For $n>16$ all curves drop in the same way, as for these initial quantum numbers the decay to ground state is not possible on average. For $n \geq 21$ the signal is no longer observable.

The highest velocity study with $v=4600 \mathrm{~m} \mathrm{~s}^{-1}$ shows a similar behaviour to the $v=3200 \mathrm{~m} \mathrm{~s}^{-1}$ case, but all amplitudes are reduced, according to figure 5.6. There is again a resonance peak at $n=15$, but the maximal signal reaches only $\approx 80 \%$ for the maximal field of 3.6 T . The influence of the magnetic field is weaker than beforehand, as the time of the $\bar{H}$ in the field is shorter. For low quantum numbers the signal is distributed from $\approx 50 \%$ to $\approx 65 \%$ for different magnetic field values, whereas a higher magnetic field produces a higher signal. For $n \geq 20$ the signal is no longer observable.

### 5.1.6 Hit-ratio depending on the initial principal quantum number $\mathbf{n}$ for different magnetic fields

Figure 5.7 and 5.8 present only the hit-ratio, similar to the previous section. The hit-ratio is shown as a function of the initial principal quantum number for different magnetic field strengths. It is the same simulation data used as for the previous analysis in subsection 5.1.4. Again, all four velocities are analysed.


Figure 5.7: Hit-ratio is plotted as a function of the initial principal quantum number for different magnetic fields. An initial velocity of $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$ is shown on the left picture, and an initial velocity of $v=1800 \mathrm{~m} \mathrm{~s}^{-1}$ on the right picture. An opening angle of $2^{\circ}$ was used.


Figure 5.8: Hit-ratio is plotted as a function of the initial principal quantum number for different magnetic fields. An initial velocity of $v=3200 \mathrm{~m} \mathrm{~s}^{-1}$ is shown on the left picture, and an initial velocity of $v=4600 \mathrm{~m} \mathrm{~s}^{-1}$ on the right picture. An opening angle of $2^{\circ}$ was used.

## Discussion

In the figures 5.7 and 5.8 it can be seen that very low quantum numbers $(n \leq 10)$ are not necessarily ideal to achieve a high hit-ratio. We have to remember that high hit-ratios do not necessarily result in a good signal. For low quantum numbers we see, in general, similar behaviour compared to the previous study for the signal, as for low quantum numbers almost all atoms are able to reach the
cavity in ground state and in this case there is no difference between signal and hit-ratio.

For the $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$ study, in figure 5.7 , there is, as in the previous study for the signal, a resonance peak at $n=15$ for very low and very high fields, whereas for $B=0.6 \mathrm{~T}, B=1.2 \mathrm{~T}$ and $B=2.4 \mathrm{~T}$ there is not such a significant peak. For quantum numbers of $n \leq 15$ the behaviour is similar to the previous study, as for these quantum numbers almost all atoms are able to reach the cavity in ground state. For higher quantum numbers the picture is different, as a hit-ratio can be produced also for higher n-states. For quantum numbers of $n>25$ a zero magnetic field value is advantageous to facilitate a high hit-ratio. Alternatively, a field value of $B=1.8 \mathrm{~T}$ could be used for high quantum numbers due to a resonance at $n=29$. These peaks occur when, for the given velocity, a given initial principal quantum number, and a given magnetic field value, the focal point is at the detector.

The $v=1800 \mathrm{~ms}^{-1}$ study, in figure 5.7 , shows in general higher hit-ratios than the $v=$ $1000 \mathrm{~m} \mathrm{~s}^{-1}$ study. Only the low fields of $B=0.0 \mathrm{~T}$ and $B=0.6 \mathrm{~T}$ are not able to produce a hit-ratio of $100 \%$ for low quantum numbers. Again, for these two values there is a resonance peak, but for a higher quantum number of $n=18$ than for the $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$ study. Furthermore, the peak is broader and higher compared to the $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$ case. For very high quantum numbers we receive for all field values a hit-ratio bigger than $10 \%$, and with no magnetic field a maximum of almost $80 \%$ for $n=30$.

The $v=3200 \mathrm{~m} \mathrm{~s}^{-1}$ study, in figure 5.8 , reveals that for low quantum numbers the hit-ratios are nearly evenly distributed from $50 \%$ to $95 \%$ for different magnetic field values. Around $n=18$ the low field points from $B=0.0 \mathrm{~T}$ to $B=1.2 \mathrm{~T}$ climb to a hit-ratio of $100 \%$, whereas the high field curves from $B=1.8 \mathrm{~T}$ to $B=3.6 \mathrm{~T}$ start decreasing at $n \geq 18$. This drop is due to the over focusing ability at such high magnetic fields.

The $v=4600 \mathrm{~m} \mathrm{~s}^{-1}$ study, in figure 5.8 , shows for low quantum numbers again nearly evenly distributed hit-ratios from $50 \%$ to only $65 \%$. For higher quantum numbers and for low field values of $B=0.6 \mathrm{~T}$ to $B=1.8 \mathrm{~T}$ the hit-ratio climbs to almost $100 \%$, whereas for higher field values of $B=2.4 \mathrm{~T}$ the hit-ratio drops at $n=25$. For $B=3.0 \mathrm{~T}$ the hit-ratio starts decreasing at $n=20$ and for $B=3.6 \mathrm{~T}$ it drops at $n=19$. So, for higher magnetic fields the over focusing starts earlier in terms of the initial principal quantum number. For zero magnetic field the hit-ratio never reaches $100 \%$.

### 5.1.7 Amount of ground state antihydrogen at different points along the beamline

In this subsection we measured the amount of ground state atoms at the relevant positions. It is obvious that the fraction of ground state atoms at the cavity entrance and the sextupole entrance is independent of the sextupole field. For the study at the BGO, according to figures 5.10 and 5.11, the magnetic field strength plays an important role. The value of $\bar{H}$ atoms in ground state is given as a percentage of the initially produced atoms, according to equation 5.10.

$$
\begin{equation*}
\text { percentage of } \bar{H} \text { atoms in GS at } \mathrm{X}=100 \cdot \frac{\text { number of } \bar{H} \text { atoms in GS at } \mathrm{X}}{\text { number of } \bar{H} \text { atoms produced }} \tag{5.10}
\end{equation*}
$$

$\begin{array}{ll}\mathrm{v}=1000 \mathrm{~m} / \mathrm{s} \longmapsto \longmapsto & \mathrm{v}=3200 \mathrm{~m} / \mathrm{s} \\ \mathrm{v}=1800 \mathrm{~m} / \mathrm{s} \longmapsto \longleftrightarrow & \mathrm{v}=4600 \mathrm{~m} / \mathrm{s} \longmapsto \\ & \end{array}$

$\mathrm{v}=1000 \mathrm{~m} / \mathrm{s}$ $\qquad$ $\mathrm{v}=3200 \mathrm{~m} / \mathrm{s}$


Figure 5.9: The percentage of $\bar{H}$ atoms in ground state at the cavity entrance is shown on the left picture, and the percentage of $\bar{H}$ atoms in ground state at the sextupole entrance is shown on the right picture. Both diagrams are plotted as a function of the initial principal quantum number for all velocities of $v=1000 \mathrm{~m} \mathrm{~s}^{-1}, v=1800 \mathrm{~m} \mathrm{~s}^{-1}, v=3200 \mathrm{~m} \mathrm{~s}^{-1}$ and $v=4600 \mathrm{~m} \mathrm{~s}^{-1}$. An opening angle of $2^{\circ}$ was used.


Figure 5.10: The percentage of $\bar{H}$ atoms in ground state at the BGO detector is plotted as a function of the magnetic field and as a function of the initial principal quantum number. An initial velocity of $v=1000 \mathrm{~ms}^{-1}$ is shown on the left picture, and an initial velocity of $v=1800 \mathrm{~m} \mathrm{~s}^{-1}$ on the right picture. An opening angle of $2^{\circ}$ was used.


Figure 5.11: $\bar{H}$ atoms in ground state at the $B G O$ for different initial principal quantum numbers and different sextupole magnetic fields at $v=3200 \mathrm{~m} \mathrm{~s}^{-1}$ and $v=4600 \mathrm{~m} \mathrm{~s}^{-1}$. An opening angle of $2^{\circ}$ was used.

## Discussion

The decay studies shown in figure 5.9 confirm that for higher velocities the ground state contribution at various points is decreasing. For very low initial principal quantum numbers, all velocities decay to the ground-state in time.

In figure 5.9, we see that at the cavity entrance the fraction of $\bar{H}$ atoms in the ground state, with a velocity of $4600 \mathrm{~m} \mathrm{~s}^{-1}$, begin to decrease at an initial quantum of $n=15$, whereas for the lowest velocity of $1000 \mathrm{~m} \mathrm{~s}^{-1}$ this does not occur until an initial quantum of $n=20$ is reached. From that point the different velocity curves decrease in parallel. At the sextupole entrance, the situation changes slightly. There is a point, e.g. for $n=20$, where a velocity of $1800 \mathrm{~m} \mathrm{~s}^{-1}$ produces a slightly higher fraction of $\bar{H}$ atoms in the ground state than for the lower velocity of $1000 \mathrm{~m} \mathrm{~s}^{-1}$. This can be explained by the fact that for $v=1800 \mathrm{~m} \mathrm{~s}^{-1}$ the double CUSP is focusing for this specific initial principal quantum number better than for $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$.

The colour plots regarding the ground state percentage at the BGO in figure 5.10 and figure 5.11 present a clear view of the magnetic field influence. For a velocity of $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$ large magnetic fields reduce the fraction of $\bar{H}$ atoms in ground state, whereas for $v=1800 \mathrm{~m} \mathrm{~s}^{-1}$ low magnetic fields reduce the fraction of $\bar{H}$ atoms in the ground state at the BGO. For both velocities, we are able to produce an amplitude of $100 \%$ for an initial principal quantum numbers of $n \leq 20$ if the right magnetic field strength is picked. If the velocity is increasing, higher magnetic field values are advantageous in order to receive more ground-state atoms at the BGO.

For the $3200 \mathrm{~m} \mathrm{~s}^{-1}$ study only a small band of initial principal quantum numbers can reach the maximal amplitude of ground state atoms at the sextupole. For the highest velocity of $4600 \mathrm{~m} \mathrm{~s}^{-1}$ the highest amplitude we can reach is $90 \%$. This is the case for only two different quantum numbers ( $n=16$ and $n=17$ ) and with a magnetic field of 3.0 T .

### 5.2 Monoenergetic beam with a quantum state distribution

### 5.2.1 Introduction

The simulation results demonstrated in this section give more realistic results, because all possible states and polarisations are considered. The n-state distribution for the next simulations were taken from [35] and can be seen in figure 5.12. During mixing, the $\bar{p}$ cloud carries out an axial oscillation between the trap potentials. In figure 5.12 an evolution time of $10 \mu \mathrm{~s}$ was considered, which is the average time an $\bar{p}$ spends in the $e^{+}$plasma during one passage. Figure 5.12 shows the number and quantum states of $\bar{H}$ produced by $10^{6} \bar{p}$ traversing the $e^{+}$cloud. The publication revealed the expected level population for two different positron plasma temperatures, $T_{e}=50 \mathrm{~K}$ and $T_{e}=100 \mathrm{~K}$. The following studies in this work will consider both positron temperatures.
Figure 5.12 shows that the level population is rising with increasing quantum number. This is what one would expect intuitively due to the fact that a higher $n$-state gives more multiplicities of sub-states $\left(l, m_{l}\right)$. The quantum numbers from $n=1$ to $n=9$ are combined to $n=1$, because all of these states decay rapidly down. The distribution was cut at $n=40$, so higher quantum numbers were ignored. This is reasonable as the double CUSP internal field ioniser disintegrates states with $n \geq 40$. This will be discussed in detail in section 5.5 further.


Figure 5.12: Anti-hydrogen bound-state level population distribution after evolution of $10 \mu \mathrm{~s}$ for various magnetic field strengths of $0 \mathrm{~T}, 1 \mathrm{~T}, 2 \mathrm{~T}, 3 \mathrm{~T}$ using a positron cloud with $10^{6} \bar{p}$ 's. The solid lines are the thermal equilibrium level population distribution. On the left picture the figures for a positron temperature of $T_{e}=50 \mathrm{~K}$ are shown and on the right one for a positron temperature of $T_{e}=100 \mathrm{~K}$. For both pictures a positron density of $n_{e}=1 \times 10^{14} \mathrm{~m}^{-3}$ was considered. From [35].

Table 5.4: Weights according to the n-state distribution in figure 5.12 and the quantum state multiplicity for each principal quantum number.

| principal quantum | n -states distribution factor $\omega_{1}$ |  | QN-multiplicity factor $M$ |
| :--- | :---: | :---: | ---: |
| number n | $T_{e}=50 K$ | $T_{e}=100 K$ |  |
| 1 | 0.0937 | 0.0220 | 2 |
| 10 | 0.0037 | 0.0007 | 200 |
| 11 | 0.0054 | 0.0010 | 242 |
| 12 | 0.0089 | 0.0016 | 288 |
| 13 | 0.0135 | 0.0024 | 338 |
| 14 | 0.0210 | 0.0036 | 392 |
| 15 | 0.0308 | 0.0052 | 450 |
| 16 | 0.0446 | 0.0074 | 512 |
| 17 | 0.0619 | 0.0102 | 578 |
| 18 | 0.0843 | 0.0138 | 648 |
| 19 | 0.1112 | 0.0180 | 722 |
| 20 | 0.1436 | 0.0232 | 800 |
| 21 | 0.1810 | 0.0291 | 882 |
| 22 | 0.2237 | 0.0359 | 968 |
| 23 | 0.2708 | 0.0435 | 1058 |
| 24 | 0.3221 | 0.0519 | 1152 |
| 25 | 0.3764 | 0.0608 | 1250 |
| 26 | 0.4329 | 0.0704 | 1352 |
| 27 | 0.4903 | 0.0801 | 1458 |
| 28 | 0.5474 | 0.0900 | 1568 |
| 29 | 0.6030 | 0.0997 | 1682 |
| 30 | 0.6559 | 0.1089 | 1800 |
| 31 | 0.7051 | 0.1174 | 1922 |
| 32 | 0.7497 | 0.1251 | 2048 |
| 33 | 0.7891 | 0.1319 | 2178 |
| 34 | 0.8230 | 0.1379 | 2312 |
| 35 | 0.8514 | 0.1430 | 2450 |
| 36 | 0.8744 | 0.1475 | 2592 |
| 37 | 0.8925 | 0.1514 | 2738 |
| 38 | 0.9062 | 0.1550 | 2888 |
| 39 | 0.9163 | 0.1582 | 3042 |
| 40 | 0.9234 | 0.1612 | 3200 |
|  |  |  |  |

The simulations in this section considered $50,000 \bar{H}$ atoms per specific quantum state $\left(n, l, m_{l}, s\right)$. The total number of states that had to be considered is $N$ according to equation 5.11.

$$
\begin{equation*}
N=\sum_{n_{\min }}^{n_{\max }} \sum_{l=0}^{n-1} \sum_{m_{l}=-l}^{+l} \sum_{s=-\frac{1}{2}}^{s=+\frac{1}{2}} 1=\sum_{n_{\min }}^{n_{\max }} 2 n^{2} \tag{5.11}
\end{equation*}
$$

So, in total $N \times 50,000 \bar{H}$ atoms had to be simulated for this study. Each state ( $n, l, m_{l}, m_{s}$ ) had then to be weighted by a factor $\omega$.

$$
\begin{equation*}
\omega=\omega_{1} \cdot \omega_{2} \tag{5.12}
\end{equation*}
$$

It considers that a $n$-state appears with a certain probability according to figure 5.12 , which defines $\omega_{1}$. Furthermore, to each n-state there are $M$ substates $\left(l, m_{l}, m_{s}\right)$ according to equation 5.13.

$$
\begin{equation*}
M_{n}=2 \sum_{l=0}^{n-1}(2 l+1)=2 n^{2} \tag{5.13}
\end{equation*}
$$

This result in the second weight factor $\omega_{2}$, which describes the multiplicity of substates to each n -state, according equation 5.14.

$$
\begin{equation*}
\omega_{2}(n)=1 / M_{n} \tag{5.14}
\end{equation*}
$$

The probability that a specific l-state is expected is given by 5.15 .

$$
\begin{equation*}
P(l)=\frac{2 l+1}{n^{2}} \tag{5.15}
\end{equation*}
$$

Most of the results in this chapter and all the plots are normalised to one $\bar{H}$ atom initially created. For this normalisation the quantity $\Xi_{1}$ is used, which is demonstrated in equation 5.16.

$$
\begin{align*}
\Xi_{1}= & 50000 \cdot\left[M_{1} \cdot \omega(n=1)+M_{10} \cdot \omega(n=10)+\ldots+\right. \\
& \left.M_{39} \cdot \omega(n=39)+M_{40} \cdot \omega(n=40)\right]  \tag{5.16}\\
= & 50000 \cdot\left[\omega_{1}(n=1)+\omega_{1}(n=10)+\ldots+\omega_{1}(n=40)\right]
\end{align*}
$$

Some results are also normalised to $10^{6} \bar{p}$, as used in figure 5.12. For this normalisation the quantity $\Xi_{2}$ is used and demonstrated in equation 5.17, where 32 relates to 32 different principal quantum numbers ( $1,10,11, \ldots, 40$ ).

$$
\begin{equation*}
\Xi_{2}=50000 \cdot 32 \tag{5.17}
\end{equation*}
$$

For example, if there were initially $50000 \bar{H}$ atoms in a specific state ( $n, l, m_{l}, m_{s}$ ) created, and these atoms caused $50 n=1$ entries in the cavity, ten the contribution of this quantum set to the amplitude is according to equation 5.18. In the end all relevant amplitudes for all states and sub states are added up and divided by $\Xi_{1}$ or $\Xi_{2}$, depending on the normalisation. In our example $R$ is the total number of $n=1$ entries in the cavity per $\bar{H}$ atom $\left(\Xi_{1}\right)$, or per $10^{6} \bar{p}\left(\Xi_{2}\right)$.

$$
\begin{array}{r}
A_{i}=50 \cdot \omega(n=22) \\
R_{1,2}=\frac{A_{1}+A_{2}+\ldots+A_{N \max }}{\Xi_{1,2}} \tag{5.19}
\end{array}
$$

The simulations presented in this section required a CPU time of 640 days. The process can be parallelised very well on different CPU cores on different servers. In table 5.5 the numbers of simulated particles and created files can be seen.

Table 5.5: Simulation quantities

| $n_{\max }$ | 40 |
| :--- | :--- |
| $n_{\min }$ | $1 ; 10$ |
| quantum states in total | 43,710 |
| root-files produced | 43,710 |
| $\bar{H}$ atoms sent per quantum state | 50,000 |
| $\bar{H}$ atoms simulated per velocity | $2.185 \times 10^{9}$ |
| data size per velocity | $18.8 G B$ |

There were several measurement points implemented in the software to ensure a variety of possible analyses. All the figures and points which were recorded are listed in table 5.6. The mentioned distance measures are related to the middle of the spin-flip cavity.

Table 5.6: A list of all the various parameters that are recorded during a simulation run. When a $\bar{H}$ atom is created it is starting with a set of parameters at the source center. When it is reaching the external field ioniser a histogram with the n-states is filled. When reaching the cavity a TTree is filled with many different parameters. At the sextupole entrance again a histogram is filled with the n-states. And at the detector again a TTree is filled with many different parameters as presented.

| place | position | parameters | data type |
| :--- | ---: | :--- | :--- |
| source center CUSP | -1.84 m | $n, l, j, m_{j}$ | TTree $\rightarrow$ Branch, |
|  |  | $E_{k i n}, \mathbf{v}, \mathbf{x}$ | textfile |
| external field ioniser | -0.408 m | $n$ | histogram |
| cavity entrance | -0.0525 m | $n, l,\left[m_{l}, m_{s}\right]$, | TTree $\rightarrow$ Branch |
|  |  | $\left[j, m_{j}\right], F, M, \mathbf{v}$ |  |
| sextupole entrance | 0.680 m | $n$ | histogram |
| BGO detector | 1.855 m | $n, l,\left[m_{l}, m_{s}\right]$, | TTree $\rightarrow$ Branch |
|  |  | $\left[j, m_{j}\right], F, M, \mathbf{x}$ |  |

### 5.2.2 Simulation parameters

The simulation parameters for this study are identical to the figures listed in table 5.3 except from the figures shown in table 5.7.

Table 5.7: Simulation parameters

| description | macro entry | value |
| :--- | :--- | :--- |
| number of $\bar{H}$ 's per point | /run/beamOn | 50,000 |
| polarisation | not applicable (see QN's) | LFS/HFS $=1 / 1$ |
| characteristics | not applicable (see QN's) | distribution |
| initial principal QN n | /gun/setStateN | $1 ; 10 ; \ldots ; 40$ |
| azimuthal QN l | /gun/setStateL | iterated all |
| 2x total angular momentum QN j | /gun/setStateTwoMJ | iterated all |
| 2x magnetic QN ml | /gun/setStateML | iterated all |
| 2x spin QN s | /gun/setStateTwoMS | iterated all |
| used decay library | /gun/setQuantumStateFile | hstates_40.root |
| user defined kinetic energy | /gun/setBeamEnergy | 5.22 meV; 16.9 meV |
| opening solid angle of that cone | /gun/setSourceOpening | 90.0 |
|  | Angle |  |
| maximum B-field in sextupole | /field/setMaximumValue | 2.4 T for $1000 \mathrm{~m} \mathrm{~s}^{-1}$ |
|  |  | 3.2 T for 1800 m s |
|  |  | B_fieldmap_double |
| cusp B-field-map used | /setup/cusp/ | CUSP_1000x1000.txt |

In this n-state distribution study the opening angle was increased to $\pi\left(\equiv 90^{\circ}\right)$ to ensure a realistic solid angle coverage. This angle describes the opening angle (apex angle) of the cone in which the initial direction of the $\bar{H}$ is randomly distributed. To reduce the computational time, we did not simulate a complete sphere but instead restrained the study to this opening angle. This can be justified by the fact that higher angles will annihilate on the double CUSP walls ( $\varnothing=80 \mathrm{~mm}$ ) before being able to escape the double CUSP magnetic field. We have verified this assertion with the following simulation where an initial velocity $1000 \mathrm{~m} \mathrm{~s}^{-1}$ and the mentioned opening angle of $\pi$ was used. At this angle a lot of $\bar{H}$ is lost due to annihilation at the wall. For higher velocities, the focusing would be even weaker and more $\bar{H}$ would be lost. The $\bar{H}$ atoms that would have been released outside of this opening angle, would have no chance to escape out of the double CUSP in the downstream direction. For sake of completeness, we want to mention that there were very rare events seen, where an $\bar{H}$ turns around inside the CUSP. But this case is, due to its rarity, negligible and therefore can be ignored.


Figure 5.13: Investigation of the focusing ability of the CUSP with $n=40$ sLFS at a velocity of $1000 \mathrm{~m} \mathrm{~s}^{-1}$. The decay process for the $\bar{H}$ atoms was switched off for this study. Only solid angles smaller than $\pi$ can escape from the CUSP. For higher velocities the focusing would be weaker.

### 5.3 Magnetic field scan of the signal to noise ratio with representative quantum states

### 5.3.1 Introduction

It was of major interest to reveal with this study the signal to noise ratio (SNR) of the apparatus. In this sense, the SNR is the signal hits divided by all other hits (noise) at the BGO. Signal hits are $\bar{H}$ atoms which arrived as LFS in ground state at the cavity AND hit the detector. Here, low field seeking ground-state atoms include all states with $n=1, F=1, M=-1$ OR $n=1, F=1$, $M=0$. So both transitions, $\pi_{1}$ and $\sigma_{1}$, are covered. Also the number for one specific transition will be presented. The noise represents all hits at the BGO that are not signal, that is the hit-ratio minus the signal.

In this section we evaluate the influence of the magnetic field strength of the sextupole magnet on the overall signal to noise ratio (SNR), in order to determine the best magnetic field value for a certain initial velocity. For the simulation only a set of representative n-quantum states were used to guarantee a reasonable computation time, but all combinations on the remaining quantum numbers $l, m_{l}, m_{s}$ were considered. The experience of this study helped to define an optimal magnetic field for the studies in section 5.4.

### 5.3.2 Simulation parameters

The parameters for this simulation are also identical to the figures listed in table 5.3 except the parameters listed in the following table 5.8.

Table 5.8: Simulation parameters

| description | macro entry | value |
| :--- | :--- | :--- |
| number of $\bar{H}$ atoms per point | /run/beamOn | 30,000 |
| polarisation | not applicable (see QN's) | LFS/HFS $=1 / 1$ |
| characteristics | not applicable (see QN's) | distribution |
| initial principal QN n | /gun/setStateN | $1 ; 15 ; 25 ; 40$ |
| azimuthal QN l | /gun/setStateL | iterated all |
| 2x total angular momentum QN j | /gun/setStateTwoMJ | iterated all |
| 2x magnetic QN ml | /gun/setStateML | iterated all |
| 2x spin QN s | /gun/setStateTwoMS | iterated all |
| used decay library | /gun/setQuantumStateFile | hstates_40.root |
| user defined kinetic energy | /gun/setBeamEnergy | 5.22 meV; 16.9 meV |
| opening solid angle of that cone | /gun/setSourceOpening | 90.0 |
|  | Angle |  |
| maximum B-field in sextupole | /field/setMaximumValue | $0.0 \mathrm{~T}, 0.4 \mathrm{~T}, \ldots, 3.6 \mathrm{~T}$ |
| cusp B-field-map used | /setup/cusp/ | B_fieldmap_double |
|  | setCuspMagAsciiFile | CUSP_1000x1000.txt |

### 5.3.3 Magnetic field scan with an initial velocity of $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$

In the following the results and plots for the magnetic field scan of an initial velocity of $v=$ $1000 \mathrm{~m} \mathrm{~s}^{-1}$ are presented.


Figure 5.14: The noise and the signal at the $B G O$ is plotted as a function of the sextupole magnetic field for a positron temperature of 50 K on the left picture, and for a positron temperature of 100 K on the right picture. An initial $\bar{H}$ velocity of $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$ and an opening angle of $90^{\circ}$ was used.


Figure 5.15: The signal to noise ratio is plotted as a function of the sextupole magnetic field for both positron temperatures of $T_{e}=50 \mathrm{~K}$ and $T_{e}=100 \mathrm{~K}$. An initial $\bar{H}$ velocity of $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$ and an opening angle of $90^{\circ}$ was used.

### 5.3.4 Magnetic field scan with with an initial velocity of $v=1800 \mathrm{~m} \mathrm{~s}^{-1}$

In the following the results and plots for the magnetic field scan of an initial velocity of $v=$ $1800 \mathrm{~m} \mathrm{~s}^{-1}$ are presented. Again, for each magnetic field point four $n$-states ( $n=1, n=15, n=25$, $n=40$ ) with all sub-states were simulated, with an amount of 30 thousand $\bar{H}$ atoms per state.


Figure 5.16: The noise and the signal at the $B G O$ is plotted as a function of the sextupole magnetic field for a positron temperature of 50 K on the left picture, and for a positron temperature of 100 K on the right picture. An initial $\bar{H}$ velocity of $v=1800 \mathrm{~m} \mathrm{~s}^{-1}$ and an opening angle of $90^{\circ}$ was used.


Figure 5.17: The signal to noise ratio is plotted as a function of the sextupole magnetic field for both positron temperatures of $T_{e}=50 \mathrm{~K}$ and $T_{e}=100 \mathrm{~K}$. An initial $\bar{H}$ velocity of $v=1800 \mathrm{~m} \mathrm{~s}^{-1}$ and an opening angle of $90^{\circ}$ was used.

## Discussion

It was shown in this section that the sextupole magnetic field strength influences expecially the amount of noise at the BGO heavily and that the optimal magnetic field would exceed the maximal reachable flux density of 3.0 T . The noise is resulting primarily from higher quantum numbers and they are defocused strongly due to the large magnetic moment. Also the signal amplitude can be increased when optimising the magnetic field. We observed an optimal magnetic field of 2.4 T for the $1000 \mathrm{~m} \mathrm{~s}^{-1}$ study and 3.2 T for the $1800 \mathrm{~m} \mathrm{~s}^{-1}$ case to reach the maximal SNR. We used these values to set the magnetic field for the main simulation in section 5.2.2. The exact SNR and intensities are summarised in table 5.9.

Table 5.9: Summary of magnetic field scan including the maximal SNR

| velocity | magnetic field | signal to noise ratio |  | signal in ppm |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $T_{e}=50 K$ | $T_{e}=100 K$ | $T_{e}=50 K$ | $T_{e}=100 K$ |
| $1000 \mathrm{~m} \mathrm{~s}^{-1}$ | 2.4 T | 1.82 | 1.81 | 707 | 688 |
| $1800 \mathrm{~m} \mathrm{~s}^{-1}$ | 3.2 T | 0.68 | 0.70 | 246 | 246 |

### 5.4 Evaluation of the signal to noise ratio with a full quantum state distribution

### 5.4.1 Introduction

With the prior evaluated optimal magnetic field strength from section 5.3 we performed extensive simulations with a complete quantum state distribution as presented in table 5.4. The states from $n=1$ to $n=9$ are combined to the number of $n=1$ due to the fast decay of such lown-states. The signal to noise ratio, the n-state distribution, the noise and the signal distribution with respect to the initial quantum number, the radial distribution and the velocity distribution were analysed in detail. Two initial velocities were studied, $1000 \mathrm{~m} \mathrm{~s}^{-1}$ and $1800 \mathrm{~ms}^{-1}$.

The SNR is again defined as in section 5.3.

### 5.4.2 Initial anti-hydrogen velocity of $1000 \mathrm{~m} \mathrm{~s}^{-1}$

The parameters for this simulation are also identical to the figures listed in table 5.3 except the parameters listed in the following table 5.10.

Table 5.10: Simulation parameters

| description | macro entry | value |
| :--- | :--- | :--- |
| number of $\bar{H}$ per point | /run/beamOn | 50,000 |
| polarisation | not applicable (see QN's) | LFS/HFS $=1 / 1$ |
| characteristics | not applicable (see QN's) | distribution |
| initial principal QN n | /gun/setStateN | $1,10,11, \ldots, 39,40$ |
| azimuthal QN l | /gun/setStateL | iterated all |
| 2x total angular momentum QN j | /gun/setStateTwoMJ | iterated all |
| 2x magnetic QN ml | /gun/setStateML | iterated all |
| 2x spin QN s | /gun/setStateTwoMS | iterated all |
| used decay library | /gun/setQuantumStateFile | hstates_40.root |
| user defined kinetic energy | /gun/setBeamEnergy | 5.22 meV (1000 m s ${ }^{-1}$ ) |
| opening solid angle of that cone | /gun/setSourceOpening | 90.0 |
|  | Angle |  |
| maximum B-field in sextupole | /field/setMaximumValue | 2.4 T |
| cusp B-field-map used | /setup/cusp/ | B_fieldmap_double |
|  | setCuspMagAsciiFile | CUSP_1000x1000.txt |

The optimal sextupole magnetic field value to receive the maximal SNR, was determined in section 5.3.3 to be $B=2.4 \mathrm{~T}$ for this study. The resulting SNR for an initial $\bar{H}$ velocity of $1000 \mathrm{~m} \mathrm{~s}^{-1}$ is given in equation 5.20 and 5.21 . These numbers differ slightly from the numbers presented in section 5.3, as we used here a larger sample and a different $n$-state distribution.

$$
\begin{align*}
S N R\left(v_{\bar{H}}=1000 \mathrm{~m} \mathrm{~s}^{-1}, T_{e}=50 K\right) & =1.89  \tag{5.20}\\
S N R\left(v_{\bar{H}}=1000 \mathrm{~m} \mathrm{~s}^{-1}, T_{e}=100 K\right) & =1.87 \tag{5.21}
\end{align*}
$$

In figure 5.18 the noise and signal for both positron temperature are shown, where both possible transitions, $\sigma_{1}$ and $\pi_{1}$, were considered as signal. The results are normalised to one produce $\bar{H}$ atom. The exact numbers are 813 ppm for $T_{e}=50 \mathrm{~K}$ and 804 ppm for $T_{e}=100 \mathrm{~K}$.


Figure 5.18: The total noise and the total signal per produced $\bar{H}$ at the $B G O$ is shown in this picture. The unit is parts per million. Both positron temperatures are presented for an initial $\bar{H}$ velocity of $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$, an opening angle of $90^{\circ}$ and a sextupole B-field of 2.4 T .

When normalising the results on $10^{6}$ injected $\bar{p}$ 's, we count 335 signal $\bar{H}$ atoms for the $T_{e}=50 \mathrm{~K}$ case, and 55.5 signal $\bar{H}$ atoms for the $T_{e}=100 \mathrm{~K}$ case. The numbers for both normalisations are summarised in 5.11.

Table 5.11: Signal and noise for $v_{\bar{H}}=1000 \mathrm{~m} \mathrm{~s}^{-1}$

|  | $T_{e}=50 \mathrm{~K}$ |  | $T_{e}=100 \mathrm{~K}$ |  |
| :--- | :--- | :---: | :---: | :---: |
|  | per $\bar{H}$ | per $10^{6} \bar{p}$ 's | per $\bar{H}$ | per $10^{6} \bar{p}^{\prime}$ s |
| Signal | 813 ppm | 335 | 804 ppm | 55.5 |
| Noise | 430 ppm | 177 | 830 ppm | 29.6 |
| SNR | 1.89 |  | 1.87 |  |

The simulation revealed that at the cavity and at the BGO, a ratio of $1: 1$ between low field seeking ground state $\bar{H}$ in $F=1, M=-1$ states for a potential $\pi_{1}$ transition, and $F=1, M=0$ states for a potential $\sigma_{1}$ transition, is present. Hence, we can conclude that for a pure $\sigma_{1}$ transition scan the $\bar{H}$ atoms that only allow a $\pi_{1}$ transition can be counted as noise since the frequency at
$B \neq 0$ differs substantially. As a result, the signal amplitudes reduce to $406 \mathrm{ppm}\left(T_{e}=50 K\right)$ and $402 \mathrm{ppm}\left(T_{e}=100 K\right)$ and the noise increases accordingly. The valid SNR for one specific transition is shown in equation 5.22 and 5.23.

$$
\begin{align*}
S N R_{\sigma_{1}}\left(v_{\bar{H}}=1000 \mathrm{~m} \mathrm{~s}^{-1}, T_{e}=50 K\right) & =0.49  \tag{5.22}\\
S N R_{\sigma_{1}}\left(v_{\bar{H}}=1000 \mathrm{~m} \mathrm{~s}^{-1}, T_{e}=100 K\right) & =0.48 \tag{5.23}
\end{align*}
$$

In figure 5.19 the principal quantum number distributions at the detector and at the cavity entrance are presented. Again, the amplitude is given in parts per million related to all $\bar{H}$ atoms produced.


Figure 5.19: The n-state distribution at the cavity entrance is shown on the left picture, and the n-state distribution at the $B G O$ on the right picture. Each n-state is presented for both positron temperature, and for an initial $\bar{H}$ velocity of $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$, an opening angle of $90^{\circ}$ and a sextupole $B$-field of 2.4 T . The numbers are normalised to one produced $\bar{H}$ atom.

Figure 5.20 compares the percentage of $\bar{H}$ produced in the CUSP (blue bars), with the percentage of noise at the BGO (red bars). For initial principal quantum numbers from $n=1$ to $n=40$, and uses a logarithmic scale for the percentage. The blue bars are normalised to the total number of produced $\bar{H}$, and the red bars are normalised to the total noise. So, it is an indicator of which initial principal quantum number decreases the SNR most.


Figure 5.20: The noise at the BGO is compared with the quantum state distribution of initially created $\bar{H}$ atoms. The values are given in percent of the total noise, and in percent of the total amount of $\bar{H}$ atoms produced. Both values are plotted as a function of the initial principal quantum number for a positron temperature of 50 K on the left picture, and for a positron temperature of 100 K on the right picture. It was an initial $\bar{H}$ velocity of $v=1000 \mathrm{~ms}^{-1}$, an opening angle of $90^{\circ}$ and a sextupole B-field of 2.4 T used for both pictures.

Figure 5.21 compares the percentage of $\bar{H}$ produced in the CUSP (blue bars), with the percentage of signal at the BGO (green bars). For initial principal quantum numbers from $n=1$ to $n=40$, and uses a logarithmic scale for the percentage. The blue bars are normalised to the total number of produced $\bar{H}$, and the red bars are normalised to the total signal. So, it is an indicator of which initial principal quantum number increases the SNR most.


Figure 5.21: The signal at the $B G O$ is compared with the quantum state distribution of initially created $\bar{H}$ atoms. The values are given in percent of the total signal, and in percent of the total amount of $\bar{H}$ atoms produced. Both values are plotted as a function of the initial principal quantum number for a positron temperature of 50 K on the left picture, and for a positron temperature of 100 K on the right picture. It was an initial $\bar{H}$ velocity of $v=$ $1000 \mathrm{~m} \mathrm{~s}^{-1}$, an opening angle of $90^{\circ}$ and a sextupole B-field of 2.4 T used for both pictures.

Figure 5.22 shows the radial distribution of the signal and the noise at the BGO. For this purpose the BGO was segmented into rings with 5.0 mm thickness. As mentioned in the experimental setup, the BGO has a diameter of 90 mm . The amplitude is given in ppm per $\mathrm{mm}^{2}$.


Figure 5.22: The radial signal and noise distribution at the $B G O$ is shown for $T_{e}=50 \mathrm{~K}$ on the left picture, and for $T_{e}=100 \mathrm{~K}$ on the right picture. The signal and the noise is plotted as a function of the BGO segment, from a radius of 0 mm to 45 mm . It was an initial $\bar{H}$ velocity of $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$, an opening angle of $90^{\circ}$ and a sextupole $B$-field of 2.4 T for both pictures used. The numbers are normalised to one produced $\bar{H}$ atom.

The velocity distribution at the cavity entrance for both positron temperatures are presented in figure 5.23. A $\log$ scale is used to make the tail of the distribution visible.


Figure 5.23: The velocity distribution at the cavity entrance is shown for $T_{e}=50 \mathrm{~K}$ on the left picture, and for $T_{e}=100 \mathrm{~K}$ on the right picture. The entries are plotted in logarithmic scale as a function of the velocity. It was an initial $\bar{H}$ velocity of $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$, an opening angle of $90^{\circ}$ and a sextupole B-field of 2.4 T for both pictures used. The numbers are normalised to one produced $\bar{H}$ atom.

## Discussion

We received a $S N R \approx 0.5$ for one transition. The lower the SNR , the more difficult a hyperfine scan will be. A high background (noise) makes it more difficult to identify the drop in the count rate. But it should be considered that a signal pattern can be identified also for a $S N R<1$, if the expected signal is known, which is the case for this experiment.

With different positron temperatures of $T_{e}=50 \mathrm{~K}$ and $T_{e}=100 \mathrm{~K}$ only very little differences
in the results were observed, when the results are normalised to one produced $\bar{H}$ atom. The reason for this is the fact that the relative differences between the two $n$-state distributions in figure 5.12 are small. Whereas, when normalising it to $10^{6} \bar{p}$ 's, the signal amplitudes between $T_{e}=50 \mathrm{~K}$ and $T_{e}=100 \mathrm{~K}$ show a large difference. We count 335 usable $\bar{H}$ atoms per $10^{6}$ injected $\vec{p}$ 's for the $T_{e}=50 \mathrm{~K}$ case, but only 55.5 usable $\bar{H}$ atoms per $10^{6} \bar{p}$ 's for the $T_{e}=100 \mathrm{~K}$ case. The SNR is not affected by the kind of normalisation.

We also observed that the amount of usable $\bar{H}$ that reaches the BGO is approximately one out of one thousand produced $\bar{H}$ atoms for an opening angle of $90^{\circ}$.

In the quantum state distribution in figure 5.19, we see that the ground state is the most probable quantum number at the cavity and at the BGO. At the BGO, higher quantum states are strongly suppressed relative to the cavity distribution. This can be explained by the defocusing effect of the strong sextupole field and by the longer decay time. This is an important fact to reduce the noise. The quantum states of $n=3$ and $n=4$ cannot be seen on the plots as the numbers are very small. We have seen in section 2.2 that such low quantum numbers are very likely to decay directly into ground state. The relatively high fraction of $n=2$ states is due to metastable $\bar{H}$ atoms. These are $2 s$ states $(n=2, l=0)$, that have very low decay rates.

Figure 5.20 shows the quantum state distribution of the initially created $\bar{H}$ atoms and the noise with respect to the initial principal quantum number. It revealed that the noise is produced by high initial $n$-states of $n \geq 20$. This is interesting, because it shows that by avoiding such high quantum numbers, the noise can be reduced drastically at that given velocity. Figure 5.21 shows the quantum state distribution of the initially created $\bar{H}$ atoms and the signal. It revealed that most of the signal is produced by quantum numbers around $n=25$. Also interesting is the fact, that the initial ground state atoms only contribute $\approx 0.6 \%$ to the total signal.

The radial distribution of the signal and noise in figure 5.22 shows that the SNR is better for a BGO area with a radius between $5 \mathrm{~mm}-35 \mathrm{~mm}$. With the electronic read-out and the position resolution of the BGO detector, this information can be used to improve the hyperfine scans by increasing the SNR as a trade-off with lower statistics.

The velocity distribution was presented in figure 5.23. It revealed that it is not normal distributed. By summing over the entries at the cavity from $v=550 \mathrm{~m} \mathrm{~s}^{-1}$ to $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$, we obtain 2097 ppm for $T_{e}=50 \mathrm{~K}$ and 2191 ppm for $T_{e}=100 \mathrm{~K}$. On the other hand, we got, from $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$ to $v=1400 \mathrm{~m} \mathrm{~s}^{-1}$, a sum of 3140 ppm for $T_{e}=50 \mathrm{~K}$ and a sum of 3141 ppm for $T_{e}=100 \mathrm{~K}$. Considering an initial velocity of $1000 \mathrm{~m} \mathrm{~s}^{-1}$, this result implies a net acceleration of the $\bar{H}$. The reason for this is the magnetic field gradient in z-direction inside the double CUSP, which was shown prior in section 3.2.1. In the axial positive gradient regime the LFS states are accelerated in the z -direction towards the cavity.

### 5.4.3 Initial antihydrogen velocity of $1800 \mathrm{~m} \mathrm{~s}^{-1}$

This chapter shows an identical analysis to section 5.4 .2, but with a higher initial $\bar{H}$ velocity of $1800 \mathrm{~m} \mathrm{~s}^{-1}$. The simulation parameters are identical to the ones in the section 5.4.2, except a defined kinetic energy of 16.9 meV and a sextupole magnetic field value of $B=3.2 \mathrm{~T}$, as determined in section 5.3.4. The resulting SNR for an initial $\bar{H}$ velocity of $1800 \mathrm{~m} \mathrm{~s}^{-1}$ is given in equation, 5.24 and 5.25 . These number differ again slightly from the numbers presented in section 5.3 for the same reasons as beforehand.

$$
\begin{align*}
S N R\left(v_{\bar{H}}=1800 \mathrm{~m} \mathrm{~s}^{-1}, T_{e}=50 K\right) & =0.72  \tag{5.24}\\
S N R\left(v_{\bar{H}}=1800 \mathrm{~m} \mathrm{~s}^{-1}, T_{e}=100 K\right) & =0.71 \tag{5.25}
\end{align*}
$$

The total signal and the total noise is presented in figure 5.24. These numbers are normalised to one produce $\bar{H}$ atom. In this case the signal is 267 ppm for $T_{e}=50 \mathrm{~K}$ and 263 ppm for $T_{e}=100 \mathrm{~K}$. So, if $10^{6} \bar{H}$ atoms are being produced according to the distribution, we can expect about 270 usable $\bar{H}$ and total hits of about 650 at the detector.


Figure 5.24: The total noise and the total signal per produced $\bar{H}$ at the $B G O$ is shown in this picture. The unit is parts per million. Both positron temperatures are presented for an initial $\bar{H}$ velocity of $v=1800 \mathrm{~ms}^{-1}$, an opening angle of $90^{\circ}$ and a sextupole $B$-field of 3.2 T .

When normalising the results to $10^{6}$ injected $\bar{p}$ 's, we count 109 signal $\bar{H}$ atoms for the $T_{e}=50 \mathrm{~K}$ case, and 18.2 signal $\bar{H}$ atoms for the $T_{e}=100 \mathrm{~K}$ case. The numbers for both normalisations are summarised in 5.12.

Table 5.12: Signal and noise for $v_{\bar{H}}=1800 \mathrm{~m} \mathrm{~s}^{-1}$

|  | $T_{e}=50 \mathrm{~K}$ |  | $T_{e}=100 \mathrm{~K}$ |  |
| :--- | :---: | :---: | :---: | :---: |
|  | per $\bar{H}$ | per $10^{6} \bar{p}{ }^{\prime} \mathrm{s}$ | per $\bar{H}$ | per $10^{6} \bar{p}{ }^{\prime} \mathrm{s}$ |
| Signal | 267 ppm | 109 | 263 ppm | 18.2 |
| Noise | 372 ppm | 153 | 370 ppm | 25.5 |
| SNR | 0.72 |  | 0.71 |  |

We measured again, that at the cavity and at the BGO a ratio of $1: 1$ between low field seeking ground state $\bar{H}$ atoms in $F=1, M=-1$ states ( $\pi_{1}$ transition), and $F=1, M=0$ states ( $\sigma_{1}$ transition), is present. The signal for a $\sigma_{1}$ transition is reduced to $133 \mathrm{ppm}\left(T_{e}=50 \mathrm{~K}\right)$ and 131 $\operatorname{ppm}\left(T_{e}=100 K\right)$. The valid SNR for one specific transition is shown in equation 5.26 and 5.27.

$$
\begin{align*}
S N R_{\sigma_{1}}\left(v_{\bar{H}}=1800 \mathrm{~m} \mathrm{~s}^{-1}, T_{e}=50 K\right) & =0.26  \tag{5.26}\\
S N R_{\sigma_{1}}\left(v_{\bar{H}}=1800 \mathrm{~m} \mathrm{~s}^{-1}, T_{e}=100 K\right) & =0.26 \tag{5.27}
\end{align*}
$$

The following plots and studies are equivalent to the study beforehand, but with an initial velocity of $1800 \mathrm{~m} \mathrm{~s}^{-1}$.


Figure 5.25: The n-state distribution at the cavity entrance is shown on the left picture, and the $n$-state distribution at the $B G O$ on the right picture. Each n-state is presented for both positron temperature, and for an initial $\bar{H}$ velocity of $v=1800 \mathrm{~m} \mathrm{~s}^{-1}$, an opening angle of $90^{\circ}$ and a sextupole B-field of 3.2 T . The numbers are normalised to one produced $\bar{H}$ atom.


Figure 5.26: The noise at the BGO is compared with the number of initially created $\bar{H}$ atoms. The values are given in percent of the total noise, and in percent of the total amount of $\bar{H}$ atoms produced. Both values are plotted as a function of the initial principal quantum number for a positron temperature of 50 K on the left picture, and for a positron temperature of 100 K on the right picture. The $y$-axis is using a logarithmic scale. It was an initial $\bar{H}$ velocity of $v=1800 \mathrm{~m} \mathrm{~s}^{-1}$, an opening angle of $90^{\circ}$ and a sextupole $B$-field of 3.2 T for both pictures used.


Figure 5.27: The signal at the $B G O$ is compared with the quantum state distribution of initially created $\bar{H}$ atoms. The values are given in percent of the total signal, and in percent of the total amount of $\bar{H}$ atoms produced. Both values are plotted as a function of the initial principal quantum number for a positron temperature of 50 K on the left picture, and for a positron temperature of 100 K on the right picture. The $y$-axis is using a logarithmic scale. It was an initial $\bar{H}$ velocity of $v=1800 \mathrm{~m} \mathrm{~s}^{-1}$, an opening angle of $90^{\circ}$ and a sextupole $B$-field of 3.2 T used for both pictures.


Figure 5.28: The radial signal and noise distribution at the $B G O$ is shown for $T_{e}=50 \mathrm{~K}$ on the left picture, and for $T_{e}=100 \mathrm{~K}$ on the right picture. A logarithmic scale is used. The signal and the noise is plotted as a function of the $B G O$ segment, from a radius of 0 mm to 45 mm . It was an initial $\bar{H}$ velocity of $v=1800 \mathrm{~m} \mathrm{~s}^{-1}$, an opening angle of $90^{\circ}$ and a sextupole $B$-field of 3.2 T for both pictures used. The numbers are normalised to one produced $\bar{H}$ atom.


Figure 5.29: The velocity distribution at the cavity entrance is shown for $T_{e}=50 \mathrm{~K}$ on the left picture, and for $T_{e}=100 \mathrm{~K}$ on the right picture. The entries are plotted in logarithmic scale as a function of the velocity. It was an initial $\bar{H}$ velocity of $v=1800 \mathrm{~m} \mathrm{~s}^{-1}$, an opening angle of $90^{\circ}$ and a sextupole B-field of 3.2 T for both pictures used. The numbers are normalised to one produced $\bar{H}$ atom.

## Discussion

With a higher velocity than in the previous study, we realise that the SNR is dropping to $\approx 0.26$ for a specific transition. This means to receive one usable hit for every three background hits. To reach this value, we had to raise the maximal sextupole magnetic field to 3.2 T , which is basically not possible in the current experimental setup.

The fraction of usable $\bar{H}$ that reaches the BGO decreased to approximately one out of four thousand produced atoms, which is a quarter of the previous section 5.4.2.

Again, with different positron temperatures of $T_{e}=50 \mathrm{~K}$ and $T_{e}=100 \mathrm{~K}$, only very little differences in the results were observed when normalising to one produced $\bar{H}$. When normalising the results to $10^{6} \bar{p}$ 's, the signal amplitudes between $T_{e}=50 \mathrm{~K}$ and $T_{e}=100 \mathrm{~K}$ show a large difference. We count 109 usable $\bar{H}$ atoms per $10^{6}$ injected $\bar{p}$ 's for the $T_{e}=50 \mathrm{~K}$ case, but only 18.2 usable $\bar{H}$ atoms per $10^{6} \bar{p}$ s for the $T_{e}=100 \mathrm{~K}$ case.

The quantum state distribution in figure 5.25 shows a similar behaviour but with lower amplitudes. For example, the ground state fraction in the cavity is decreased from 1100 (ppm), for the $1000 \mathrm{~m} \mathrm{~s}^{-1}$, to $900(\mathrm{ppm})$ for the $1800 \mathrm{~m} \mathrm{~s}^{-1}$ study. That is due to the less double CUSP focusing effect of higher velocities. Of the $n=1$ states that reached the BGO, there is a fraction of $20 \%$ noise. These $\bar{H}$ atoms were either HFS, or not in ground state when reaching the cavity.

In this study initial $n$-states of $n \geq 19$ produce the majority of the noise at the detector, as presented in figure 5.26. In the previous study the noise began with initial n-states of $n \geq 20$. The distribution of the signal, according to figure 5.27, shows a significant maximum at $n=24$. For higher initial $n$-states the signal contribution is decreasing, even though the production is increasing. The contribution of initial ground state atoms to the signal is higher compared to the previous $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$ study, with $0.9 \%$ instead of $0.6 \%$.

The radial distribution of the signal and noise in figure 5.28 shows, for this velocity, an even more drastic picture. The BGO area from a radius of 25 mm to 45 mm contribute almost nothing to the signal, instead only noise. Here, a great improvement to the SNR can be achieved by excluding this region in the data analysis.

In this study we also observed a net acceleration of the $\bar{H}$ atoms with an initial velocity of $1800 \mathrm{~m} \mathrm{~s}^{-1}$, according to figure 5.29 . By integrating the entries at the cavity from $v=1600 \mathrm{~m} \mathrm{~s}^{-1}$ to $v=1800 \mathrm{~m} \mathrm{~s}^{-1}$, we receive 1878 ppm for $T_{e}=50 \mathrm{~K}$ and 1882 ppm for $T_{e}=100 \mathrm{~K}$. On the other hand we get from $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$ to $v=1400 \mathrm{~m} \mathrm{~s}^{-1}$, a sum of 2388 ppm for $T_{e}=50 \mathrm{~K}$ and a sum of 2399 ppm for $T_{e}=100 \mathrm{~K}$.

### 5.4.4 Polarisation at the cavity

A publication in 2014 [36] stated that in a certain mixing region inside the CUSP, a favourable polarisation can be reached for ground state $\bar{H}$ atoms at the exit of the double CUSP. Favorable polarisation means in this context that the fraction of LFS is higher than the fraction of HFS. The subsection 5.4.4 is dedicated to the question of polarisation.

In this section we also analyse the polarisation of the $\bar{H}$ at the cavity entrance. For this analysis the same data were used as in previous sections 5.4.2 and 5.4.3, so all initial parameters can be taken from section 5.2.2. Again, for the $1000 \mathrm{~m} \mathrm{~s}^{-1}$ study a maximal sextupole field of 2.4 T and for the $1800 \mathrm{~m} \mathrm{~s}^{-1}$ a field of 3.2 T was taken. Former studies on the polarisation of the double CUSP can be read in [36]. We use the same definition regarding the polarisation $\mathcal{P}$ of the beam. The definition is given in equation 5.28. The value $f$ is the number of $\bar{H}$ entering the cavity in a LFS or HFS state.

$$
\begin{equation*}
\mathcal{P}=\frac{f_{L F S}-f_{H F S}}{f_{L F S}+f_{H F S}} \tag{5.28}
\end{equation*}
$$

The polarisation compares the fraction of LFS with the fraction of HFS. To classify ground state particles, table 2.1 has to be used. Hence, the case with $F=1, M=0$ and $F=1, M=-1$ are LFS-states, and $F=1, M=1$ and $F=0, M=0$ are HFS-states. For higher quantum numbers $(n>1)$ we classify the state with the force on the atom via the known force equation 2.11 in chapter $2, \mu_{z}<0$ stands for LFS and $\mu_{z}>0$ for HFS. The resulting conditions for specifying a LFS-state are shown in equation 5.29 for the weak field limit,

$$
\begin{equation*}
\mu_{z}=m_{j} g_{j} \mu_{B} \quad \Rightarrow \quad m_{j} g_{j}<0 \tag{5.29}
\end{equation*}
$$

and in equation 5.30 for the strong field limit, where $g_{l}=1$ and $g_{s} \approx 2$.

$$
\begin{equation*}
\mu_{z}=\left(g_{l} m_{l}+g_{s} m_{s}\right) \mu_{B} \quad \Rightarrow \quad g_{l} m_{l}+g_{s} m_{s}<0 \tag{5.30}
\end{equation*}
$$

The polarisations are summarised for $v_{\bar{H}}=1000 \mathrm{~m} \mathrm{~s}^{-1}$ in table 5.13, and for $v_{\bar{H}}=1800 \mathrm{~m} \mathrm{~s}^{-1}$ in table 5.14. We distinguished the analysis for three different cases. First, $\bar{H}$ that started in the ground-state, second $\bar{H}$ that started with $n \geq 1$, and finally the polarisation for both conditions.

Table 5.13: Summary of the polarisation at the cavity entrance for $v_{\bar{H}}=1000 \mathrm{~m} \mathrm{~s}^{-1}$

| velocity of $\bar{H}$ and positron temperature |  | $n=1$ |  |
| :--- | :---: | :---: | :---: |
|  | P | LFS $(\%)$ | HFS (\%) |
| $v_{\bar{H}}=1000 \mathrm{~m} \mathrm{~s}^{-1}, T_{e}=50 K$ | $\mathcal{P}=0.33$ | $66.5 \%$ | $33.5 \%$ |
| $v_{\bar{H}}=1000 \mathrm{~m} \mathrm{~s}^{-1}, T_{e}=100 K$ | $\mathcal{P}=0.33$ | $66.5 \%$ | $33.5 \%$ |
|  |  | $n>1$ |  |
| $v_{\bar{H}}=1000 \mathrm{~m} \mathrm{~s}^{-1}, T_{e}=50 K$ | P | $\mathrm{LFS}(\%)$ | HFS (\%) |
| $v_{\bar{H}}=1000 \mathrm{~m} \mathrm{~s}^{-1}, T_{e}=100 K$ | $\mathcal{P}=0.76$ | $87.9 \%$ | $12.1 \%$ |
|  | $\mathcal{P}=0.76$ | $88.0 \%$ | $12.0 \%$ |
| $v_{\bar{H}}=1000 \mathrm{~m} \mathrm{~s}^{-1}, T_{e}=50 K$ |  | all $n$ |  |
| $v_{\bar{H}}=1000 \mathrm{~m} \mathrm{~s}^{-1}, T_{e}=100 K$ | P | LFS $(\%)$ | HFS $(\%)$ |

Table 5.14: Summary of the polarisation at the cavity entrance for $v_{\bar{H}}=1800 \mathrm{~m} \mathrm{~s}^{-1}$

| velocity of $\bar{H}$ and positron temperature |  | $n=1$ |  |
| :--- | :---: | :---: | :---: |
|  | P | LFS (\%) | HFS (\%) |
| $v_{\bar{H}}=1800 \mathrm{~m} \mathrm{~s}^{-1}, T_{e}=50 K$ | $\mathcal{P}=0.21$ | $60.7 \%$ | $39.3 \%$ |
| $v_{\bar{H}}=1800 \mathrm{~m} \mathrm{~s}^{-1}, T_{e}=100 K$ | $\mathcal{P}=0.21$ | $60.6 \%$ | $39.4 \%$ |
|  |  | $n>1$ |  |
| $v_{\bar{H}}=1800 \mathrm{~m} \mathrm{~s}^{-1}, T_{e}=50 K$ | P | LFS (\%) | HFS (\%) |
| $v_{\bar{H}}=1800 \mathrm{~m} \mathrm{~s}^{-1}, T_{e}=100 K$ | $\mathcal{P}=0.85$ | $92.3 \%$ | $7.7 \%$ |
|  | $\mathcal{P}=0.85$ | $92.4 \%$ | $7.6 \%$ |
|  |  | all $n$ |  |
| $v_{\bar{H}}=1800 \mathrm{~m} \mathrm{~s}^{-1}, T_{e}=50 K$ | P | LFS (\%) | HFS (\%) |
| $v_{\bar{H}}=1800 \mathrm{~m} \mathrm{~s}^{-1}, T_{e}=100 K$ | $\mathcal{P}=0.75$ | $87.7 \%$ | $12.3 \%$ |

## Discussion

As expected we observed a better polarisation for ground-states at a lower velocity of $1000 \mathrm{~m} \mathrm{~s}^{-1}$. For higher states it is interesting that the picture is reversed. The higher velocity study of $1800 \mathrm{~m} \mathrm{~s}^{-1}$ shows a polarisation of $\mathcal{P}=0.85$ for $n>1$ states, where the lower velocity brought $\mathcal{P}=0.76$. The higher $n>1$ states polarisation for $1800 \mathrm{~m} \mathrm{~s}^{-1}$ results in a higher total polarisation compared to the $1000 \mathrm{~m} \mathrm{~s}^{-1}$ case. The reason for this is that the focal point of the double CUSP depends on the state and the velocity. For higher n-states (larger magnetic moment) the focal point at $v=1800 \mathrm{~m} \mathrm{~s}^{-1}$ is closer to the cavity than for the $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$ study. In the later case, the focal point is placed more upstream of the cavity (over focusing). For ground state atoms the focal point
of the $v=1800 \mathrm{~m} \mathrm{~s}^{-1}$ case is further downstream of the cavity than for the $v=1800 \mathrm{~m} \mathrm{~s}^{-1}$ case. Hence, in this case the polarisation is better for a velocity of $1800 \mathrm{~m} \mathrm{~s}^{-1}$.

### 5.5 Study of high n-states at the CUSP internal field ioniser

### 5.5.1 Introduction

In this section we study the focusing ability of the double CUSP and investigate how many atoms are able to reach the double CUSP internal field ioniser by starting with an initial principal quantum number of $n=40$ and all possible sub-states $\left(l, m_{l}, m_{s}\right)$. Again for this study we used a mono-energetic beam with an initial velocity of $1000 \mathrm{~m} \mathrm{~s}^{-1}$.

The double CUSP internal field ioniser should not be confused with the external field ioniser which was presented in table 5.5. The field ioniser in the double CUSP consists of a cylindrical electrode with a voltage applied on it. The device is circular with a diameter of 80 mm . It is placed 5 cm upstream of the CUSP center which is equivalent to -164 cm from the cavity centre, and 20 cm downstream from the $\bar{H}$ mixing point. This ionisation well provides a monitoring of the $\bar{H}$ synthesis. The ionisation energy of an atom falls with a raising principal quantum number. An $\bar{H}$ is field-ionised if the condition $n \geq(3.2 / E)^{1 / 4} \times 10^{2}$ is satisfied, where $E\left(\mathrm{~V} \mathrm{~cm}^{-1}\right)$ is the electric field strength. The average field strength in the field ioniser is $139 \mathrm{~V} \mathrm{~cm}^{-1}$, which means that $\bar{H}$ atoms with $n \gtrsim 39$ can be field ionised [8]. After the ionisation, the $\bar{p}$ 's are trapped in the the field-ionisation well [37]. When this well is opened the $\bar{p}$ 's annihilate and are being counted with a pion-tracker.

In this simulation we measured the total amount of $\bar{H}$ that reaches the double CUSP internal field ioniser, the fraction of $n=40 \bar{H}$ at this position, and also the particle which reached the detector. The sextupole magnetic field was switched off for this investigation. The $\bar{H}$ was released into the whole half sphere of $2 \pi$ to be sure all $\bar{H}$ that could contribute are included. The solid angle of the CUSP internal field ioniser from the mixing point is $1.27 \%$ of the whole sphere. All simulation settings for this study can be seen in the next section 5.5.2.

### 5.5.2 Simulation parameters

The parameters for this simulation are identical to the numbers listed in table 5.3 except the figures which are presented in the following table 5.15.

Table 5.15: Simulation parameters

| description | macro entry | value |
| :--- | :--- | :--- |
| number of $\bar{H}$ per point | /run/beamOn | 50,000 |
| polarisation | not applicable (see QN's) | LFS/HFS $=1 / 1$ |
| characteristics | not applicable (see QN's) | distribution |
| initial principal QN n | /gun/setStateN | 40 |
| azimuthal QN l | /gun/setStateL | iterated all |
| 2x total angular momentum QN j | /gun/setStateTwoMJ | iterated all |
| 2x magnetic QN ml | /gun/setStateML | iterated all |
| 2x spin QN s | /gun/setStateTwoMS | iterated all |
| used decay library | /gun/setQuantumStateFile | hstates_40.root |
| user defined kinetic energy | /gun/setBeamEnergy | 5.22 meV (1000 m s ${ }^{-1}$ ) |
| opening solid angle of that cone | /gun/setSourceOpening | 180.0 |
|  | Angle |  |
| maximum B-field in sextupole | /field/setMaximumValue | 0.0 T |
| cusp B-field-map used | /setup/cusp/ | B_fieldmap_double |
|  | setCuspMagAsciiFile | CUSP_1000x1000.txt |

### 5.5.3 Results

The main results are summarised in the table 5.16, where the percentage are normalised to the total number of atoms produced. In table 5.17 the quantum number distribution at the field ioniser is shown, where the small fractions for $n=2$ to $n=35$ are combined to one value.

Table 5.16: Summary of the CUSP field ioniser entries

| number of $\bar{H}$ sent | $8 \times 10^{7}$ |
| :--- | :---: |
| fraction of $\bar{H}$ reaching the field ioniser | $2.18 \%$ |
| fraction of $\bar{H}$ reaching the BGO-detector | $5 \times 10^{-4}$ |

Table 5.17: Summary of the CUSP field ioniser entries

| n at the field ioniser | fraction |
| :--- | :---: |
| $n=40$ | $85.70 \%$ |
| $n=39$ | $2.20 \%$ |
| $n=38$ | $1.22 \%$ |
| $n=37$ | $0.82 \%$ |
| $n=36$ | $0.63 \%$ |
| $n=2, \ldots, n=35$ | $4.35 \%$ |
| $n=1$ | $5.08 \%$ |

## Discussion

Through this analysis we saw that the effective solid angle of the internal field ioniser for $n=40$ states is $2.18 \%$, whereas the geometrical solid angle is $1.27 \%$. Furthermore, we observed that the
majority of particles stayed in the initial quantum state of $n=40$ and hence would be ionised by the CUSP internal field ioniser. Only a very small portion of 490 ppm arrived at the detector and the fraction of ground state atoms at the field ioniser is relatively high, at $5 \%$. This is because we started with all possible sub-states. This includes low orbital angular momentum $(l=0)$, which can decay down quickly. For such a low velocity of $1000 \mathrm{~m} \mathrm{~s}^{-1}$ the distance of 20 cm , is enough to let a fraction decay down.

## Chapter 6

## Discussion and Outlook

The geometry of the beamline and the detector was updated to the 2014 set up, which is currently used by the experiment. The additional length brought a loss in solid angle from the source to the detector, but also a slightly larger distance between the source and the cavity. This is beneficial as the $\bar{H}$ atoms have more time to de-excite to the ground state. The theory chapter revealed that it is very attractive to apply an additional E-field as this would instigate a faster decay. Perpendicular field vectors of the electric and the magnetic field can increase the decay rates. This is important as it would increase the signal amplitude that can be used for a hyperfine scan. The implementation of the electric field, and the angle between the electric and the magnetic field was done in the simulation software and the decay library creator. With the computation of a suitable 3D data base we faced severe problems due to the combinatorial increase of the data size. Alternative solutions were evaluated in order to enable the creation of a decay library. In particular, the idea of simplifying the data structure should be addressed in a first approach.

Several simulations with strong low field seekers and an opening angle of $2.0 \%$ were addressed. This opening angle was chosen in order to disentangle the magnetic field of the double CUSP with all other effects, like sextupole magnetic field, initial quantum number and velocity, but also to obtain high amplitudes for the hit-ratio or the signal at the detector. These were the first simulations, which considered the new beam-line set up and the double CUSP field map. We learned in the first section that the optimal magnetic field of the sextupole is not necessarily the largest for such an opening angle. For low initial quantum numbers, the signal and the hit-ratio were equal. For higher velocities, the signal is reduced, as the time for the de-excitation is smaller. The magnetic field of the sextupole is crucial for the hit-ratio and the signal. We observed, for certain sextupole fields, resonance peaks in the hit-ratio, for example at $n=26$ and $n=30$ given a velocity of $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$. For higher velocities, the sensitivity to the magnetic field is higher than for low velocities. When studying the signal and the hit-ratio as a function of the initial principal quantum number, for a low velocity, and for very low or very high magnetic fields a resonance peak exist around $n=15$. This quantum number is focused stronger than lower initial quantum numbers due to the double CUSP magnetic field gradient. As a consequence they have a better spacial resolution at the sextupole entrance. Additionally, they decay down to the ground state quickly and therefore reach the cavity very likely in ground state. When going to higher velocities an optimal magnetic field plays a more important role, while for the maximal velocity a full signal is no longer reachable at all. Only for higher initial quantum numbers the hit-ratios differ from the signal study, as the hit-ratio only depends weakly on the decay to the ground state. Higher initial
quantum numbers produce a strongly suppressed signal, whereas when going to higher n-states the hit-ratios even increase in some cases. For the highest velocity of $v=4600 \mathrm{~m} \mathrm{~s}^{-1}$, the magnetic field has to be increased in order to get high hit-ratios and signals. Furthermore it was shown that high n-states will not contribute any usable signal at that velocity, but instead cause hits at the detector (noise). It was also shown that higher quantum numbers are very sensitive to over focusing due to the sextupole magnetic fields. Interestingly, for every velocity and large initial quantum numbers having zero magnetic sextupole field was a good compromise in order to receive a high hit-ratio.

To carry out realistic simulations that can reproduce results from the experiment, we considered the expected n -state distribution. Also, the opening angle was increased to $\pi\left(\equiv 90^{\circ}\right)$ to guarantee that all atoms that could contribute to the result were covered. This opening angle was analysed beforehand by a simulation run on strong LFS states. After that, a large magnetic field scan was performed to evaluate the optimal magnetic field at the sextupole to perform the n-state distribution simulation. The impression from the first simulation section was confirmed, that the maximal magnetic sextupole field is not necessarily the best. It showed that there is a significant maximal SNR that can be reached with a specific sextupole magnetic field value. In particular, the noise could be reduced heavily when setting the optimal magnetic field value. With this parameters, the presented signal to noise ratios were reached for the final simulations. For the $1000 \mathrm{~m} \mathrm{~s}^{-1}$ study, we observed a $S N R \approx 1.9$ with a sextupole magnetic field of 2.4 T . This SNR considered that both transitions contribute to the signal. In the real experiment only one transition frequency can be scanned. A more valid SNR, for one specific transition, of $S N R_{\sigma_{1}} \approx 0.5$ was obtained. When the beam velocity increases the SNR tends to get lower, due to the fact that the signal amplitude is reduced strongly. For $1800 \mathrm{~m} \mathrm{~s}^{-1}$ the SNR for one transition reduces to $S N R_{\sigma_{1}} \approx 0.26$. To recognise a known signal structure with such low SNR is possible by using matched filtering, for example. For the lower velocity of $1000 \mathrm{~m} \mathrm{~s}^{-1}$, one out of 1000 atoms could be counted as useable signal, whereas for the higher velocity of $1800 \mathrm{~m} \mathrm{~s}^{-1}$ one out of 4000 count as a signal. This is a promising result for the further progress of the experiment. When normalising the signal amplitudes to one produced $\bar{H}$ atom we only observed a slight difference between the positron temperatures of $T_{e}=50 \mathrm{~K}$ and $T_{e}=100 \mathrm{~K}$, due to the fact that the relative differences of the used n -state distributions are small. We received a signal of 813 ppm for $T_{e}=50 \mathrm{~K}$ and 804 ppm for $T_{e}=100 \mathrm{~K}$ per produced $\bar{H}$ in the $1000 \mathrm{~m} \mathrm{~s}^{-1}$ case. In the $1800 \mathrm{~m} \mathrm{~s}^{-1}$ case we received a signal of 267 ppm for $T_{e}=50 \mathrm{~K}$ and 263 ppm for $T_{e}=100 \mathrm{~K}$ per produced $\bar{H}$. When normalising it to $10^{6}$ injected $\bar{p}$ 's there is a large difference in the signal amplitudes. For the $1000 \mathrm{~m} \mathrm{~s}^{-1}$ case we received 335 signal $\bar{H}$ atoms for $T_{e}=50 \mathrm{~K}$, and only 55.5 signal $\bar{H}$ atoms for $T_{e}=100 \mathrm{~K}$. For the $1800 \mathrm{~m} \mathrm{~s}^{-1}$ there appear 109 signal $\bar{H}$ atoms for $T_{e}=50 \mathrm{~K}$, and only 18.2 signal $\bar{H}$ atoms for $T_{e}=100 \mathrm{~K}$. Furthermore, the analysed n -state distribution at the cavity has shown a strong contribution of ground state atoms, a very low contribution of low n-states, but an increasing distribution to higher n-states. After the sextupole magnet, the high states are reduced by more than a factor of ten, but the ground state atoms decreased by only a factor of two. The reduction of the high n -states is done predominantly by the defocusing of the sextupole magnet, and the slight decrease of the $n=1$ states is due to the solid angle losses. When the signal and noise distribution was compared to the initial quantum number distribution, we observed that the noise is strongly increasing with the initial quantum number, whereas the signal contribution is decreasing from $n>25$ for the $1000 \mathrm{~m} \mathrm{~s}^{-1}$, and from $n>24$ for the $1800 \mathrm{~m} \mathrm{~s}^{-1}$ case. We also found a high fraction of $n=2$ which were identified as 2 s states ( metastable). The analysis of the velocity distribution showed a net acceleration of $\bar{H}$ for both initial velocities. The $1000 \mathrm{~m} \mathrm{~s}^{-1}$ study revealed that $60 \%$ of $\bar{H}$ at the cavity travelled with $v>1000 \mathrm{~m} \mathrm{~s}^{-1}$ and $40 \%$ with $v<1000 \mathrm{~m} \mathrm{~s}^{-1}$. The acceleration in the $1800 \mathrm{~m} \mathrm{~s}^{-1}$ study was a little less, with $56 \%$ of the $\bar{H}$ travelling with $v>1800 \mathrm{~m} \mathrm{~s}^{-1}$ and $44 \%$ of the $\bar{H}$ with $v<1800 \mathrm{~m} \mathrm{~s}^{-1}$.

This acceleration can be explained through the axial magnetic field gradient in the double CUSP which acts on the dipole moment of the $\bar{H}$. It is to emphasise, that the used opening angle of $90^{\circ}$ relates to a coverage of the whole sphere of $15 \%$. The solid angle discussion showed, that the $\bar{H}$ atoms which propagates into the other $85 \%$ of the sphere, can be seen as lost.

In this work there were some possible optimisations revealed to increase the SNR. If the outer area of the BGO detector is ignored in the analysis, a lot of noise can be omitted, hence the SNR increases. Furthermore, when plotting the noise per quantum numbers we saw that the majority of noise is produced by initial principal quantum numbers greater than 20 . This information can be used to reduce the noise again. Along the beamline there is a field ioniser installed at the entrance of the spin-flip cavity. This device ionises atoms with a quantum number of $n>12$, and could be used to exclude noise.

When analysing the polarisation of the collected data, we observed for $n=1$ states a better polarisation at the lower velocity of $1000 \mathrm{~m} \mathrm{~s}^{-1}$ than for the $1800 \mathrm{~m} \mathrm{~s}^{-1}$ study, as we expected. For higher $n$-states it is interesting that the picture is reversed. The higher velocity study of $1800 \mathrm{~m} \mathrm{~s}^{-1}$ shows a polarisation of $\mathcal{P}=0.85$ for $n>1$ states, where the lower velocity brought $\mathcal{P}=0.76$. The higher $n>1$ states polarisation for $1800 \mathrm{~m} \mathrm{~s}^{-1}$ results in a higher total polarisation compared to the $1000 \mathrm{~m} \mathrm{~s}^{-1}$ case. The reason for this is that the focal point of the double CUSP depends on the state and the velocity. For higher n-states (larger magnetic moment) the focal point of $v=1800 \mathrm{~m} \mathrm{~s}^{-1}$ is closer to the cavity than for the $v=1000 \mathrm{~m} \mathrm{~s}^{-1}$ study. In the later case the focal point is placed more upstream of the cavity (over focusing).

In a further study we analysed the effective solid angle of the apparatus into the double CUSP internal field ioniser, which is placed 20 cm downstream of the mixing point. When starting with $n=40$ and all possible sub-states $\left(n, l, m_{l}, m_{s}\right)$, and $v=1000 \mathrm{~m} \mathrm{~s}^{-1}, 86 \%$ stay in the $n=40$ state. Approximately $5 \%$ are even able to decay to ground state. In total $2.18 \%$ reach the field ioniser, which is almost twice as high as the geometrical solid angle.

For the future progress there is a lot of left to research. More initial velocities could be studied, in order to get the whole Boltzmann-distribution. This study can help to get a realistic picture of the experiment, as the $\bar{H}$-beam velocity will in fact be distributed, and the mean value of the distribution is not known yet. It can also help to measure this unknown velocity distribution, once enough data from the experiment is collected. How the external field ioniser at the cavity entrance can improve the SNR could also be investigated, as we stated before that especially high initial states are responsible for the noise at the BGO. Furthermore, a proper decay library with a good resolution including the electric field information could be elaborated in order to test different E-field contributions for a faster decay to ground state. Also, other potential improvements like restricting the BGO area could be reviewed in detail.

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[^0]:    ${ }^{1}$ A proprietary software tool to solve differential equations, by COMSOL Inc. (www.comsol.com)

