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„A construction heuristic for the bus line planning and scheduling problem“

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Abstract

In rural areas public transportation plays a huge role because many people depend on it. There are just two types of public transportation in rural areas: trains and buses. This work provides a solution procedure to develop a bus network system in Steyr Land. The objective is to maximize demand satisfaction, taking into account a restriction on the number of buses and the travel time of passengers.

A heuristic method is used for this work and it is divided into five steps. In the first step, the number of stations needed for the bus network system is determined. In the second step, the number of routes and total covered demand for the bus network system are found. In the third step, the number of buses and frequency per hour on each line, and in addition, the operator and user costs are defined. In the fourth step, the centers are found out, which can be used for selling tickets, information etc. In the last step, the bus network system is checked if there might be improvements, which further reduce minimize the operator and user costs.

Abstract

In ländlichen Gebieten spielt öffentlicher Verkehr eine große Rolle, da ein großer Teil der Bevölkerung von ihm abhängig ist. Es gibt zwei Arten öffentlicher Verkehrmittel: Züge und Busse. Diese Arbeit präsentiert ein Lösungverfahren für die Erstellung von Buslinien für das Gebiet Steyr Land. Das Ziel dieser Arbeit ist es einen Buslinienplan zu erstellen welcher soviel Nachfrage wie möglich abdeckt und eine maximale Anzahl von Bussen nicht überschreitet.

Das Lösungverfahren ist heuristisch und besteht aus fünf Schritten. Im ersten Schritt werden die Stationen für den Buslinienplan erstellt. Im zweiten Schritt werden die Routen für den Buslinienplan erstellt. Im dritten Schritt bestimmt man die Anzahl der Busse und Frequenz von ihnen sowie auch die Kosten des Betreibers und der Kunden. Im vierten Schritt findet man Zentren, die dann für Tickets, Information und so weiter benutzt werden können. Im letzten Schritt checkt man, ob der Buslinienplan verbessert und die Kosten des Betreibers und der Kunden reduziert werden können.

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1. Introduction

Nowadays, public transportation is quite important, because there is a vast number of people who do not have other options. For instance, people who cannot afford a car, do not have a driving license, are underage or who have a physical disability. As a consequence, public transportation is an alternative to cars for this group of people. On the other hand, those who have a vehicle and could drive may decide otherwise, either because they want to save money, avoid traffic or even as an environmental consciousness on their behalf. In addition, many people use public transportation as well to reach a destination faster and comfortably, while they are working instead of driving.

In addition, public transportation should be efficient in all dimensions, such as, the frequency of transportation vehicles, time table, the number of vehicles, the locations of the stations, and also the attainment of a destination without the need of changing lines. Efficiency in terms of frequency implies the optimal number of transportation vehicles a line of transportation can achieve per hour such that commuters should not wait for long intervals of time. From the point of view of commuters, they would reach a high level of satisfaction in cases when they do not have to wait long for a vehicle to pick them up. However the operators should consider the number of vehicles per line and station in order for them to optimize the frequency of vehicles per hour in every line. If the number of commuters in a specific route is big, the operators should increase the frequency of vehicles per hour in that line. In case of low number of commuters on the route, they should be able to cut the frequency of vehicles per hour in order to reduce costs and be more efficient since these vehicles can also be used in other more busy routes. Being efficient in terms of time schedule implies that the operators should take into consideration punctuality, delays, fixing the delays, peaks, low demand on a route, providing information about possible delays and alternative solutions in case of delays such that commuters attain their desired destination and gain maximum utility from the public transportation system.

Furthermore, public transportation for most rural areas in Austria is based purely on a bus system and trains.

The public transportation in rural areas differs significantly from that of urban areas; foremost, the vehicle frequency in rural areas is lower compared to that of urban areas as a consequence of the lower number of commuters per line. Additionally, as already mentioned above, there exist just two types of public transportation such as trains and buses; lastly, it is worth noting that stops/stations are scattered in longer distances between each other in comparison to the relatively close distance between the stops in urban areas which implies that stops are less accessible to people in rural areas.

For this reason, it plays a huge role connecting those areas with each other as well as with cities nearby, because not every person can afford a car as it was mentioned at the beginning. There are four fundamental elements of a rural public transportation system: the time schedule, frequency, promptness, and stations' locations.

The importance of these elements is derived from the fact that most of the inhabitants of rural places commute back and forth from the rural areas to the metropolitan or urban areas for work, business or other secondary or tertiary reasons.

Passengers value promptness and being on time for their errands in case of delay in the system. This essentially implies that alternative solutions should be available to them such as backup vehicles, or alternative adjacent lines that can be easily reached a foot so that they do not have to wait for long intervals of times. Ultimately the network should be efficient in terms of promptness and frequency so that delays, crowds and overlaps of more than one bus at the same time are avoided.

Public transport/Year	1990	1995	2000	2005	2010	2011	2012
Buses	8.0	8.7	9.2	9.3	9.6	9.5	9.5
Tram and metro	2.8	3.3	3.6	3.8	4.1	4.1	4.1
Railways	8.9	10.1	8.7	9.5	10.7	10.9	11.3

Table 1. PKM for different modes of Public Transportation (all number in billions)

Table 1 shows the passenger kilometre (pkm) of different modes of public transportation in Austria in the years from 1990 to 2012 (Statistical pocketbook, 2014). As can be seen, the railways are the most used mode of transportation. Buses were used more frequently or at the same level as railways in some years. While, tram and metro were used half less frequently than public buses.

It can be concluded, that public bus plays a huge role in the daily life of Austrian citizens.

For this reason, the study is concentrated on the design of efficient bus network system.

The process of developing a bus network system consists of: network design, frequency setting, timetable development, bus scheduling and driver scheduling (Ceder and Wilson, 1986).

The procedure of forming or building a bus network is quite complex. There are several things that should be accounted for, such as: Optimal frequency, when accounted for the number of buses per line, timetable suitability such that it will closely coincide with the commuters' needs at particular times during the day or night. If there is a problem of deciding how many buses are needed, the number of commuters should be considered and optimized accordingly. On the other hand, if there is an issue of determining the number of bus stops, it should be optimized, considering the distance between each bus stops on the route and other constraints.

Study Objectives

The goal of this study is to develop a computer-based design for a bus network in Steyr Land, Austria. In order to reach this goal the following aspects should be fulfilled:

1) Identify the number of stations the bus network needs
2) Identify the number of routes to satisfy as much as possible the demand in the region Steyr
3) Identify the frequency and the number of buses in the network system

To reach these goals the work of Baaj and Mahmassani (1991) will be used. They offer a solution approach that includes the following major steps:

1) Route generation approach
2) Network analysis procedure
3) Transit center selection
4) Network improvement procedure

Overview

In chapter 2 the literature review is presented in order to show, how the previous studies were trying to find the efficient bus network system.

In chapter 3 the problem will be described to its objective function, constraints and decision variable.

Chapter 4 will provide a brief solution methodology. Also, all algorithms will be described.

In chapter 5 the results of this study will be presented.

2. Literature Review

The following approaches optimized the bus network system: Lampkin and Saalmans (1967), Rea (1971), Silman, et al. (1974), Mandl (1979), Dubois, et al. (1979), Hasselstrom (1981), Ceder and Wilson (1986) , Van Nes et, al. (1988), Baaj (1990), and Israeli and Ceder (1991), Pattnaik et al. (1998), Fusco et al. (2002) , Tom and Mohan (2003), Zhao (2006), Afandizadeh et al. (2013), Renato Arbex and Claudio Barbieri da Cunha.(2014).

Most those approaches were trying to minimize the generalized cost (user cost and/or operator cost). Hasselstrom (1981) proposed maximizing consumer surplus to handle with variable demand. In the Hasselstrom's model (1981), a direct model is used to estimate a demand matrix. The constraints of this model were total operator cost, fleet size and service frequency. As a commuter behaviour was used multiple path assignment model. It means that the number of acceptable paths is generated and then each commuter is assigned to the best path. The model of Hasselstrom (1981) used a complex two-level optimization procedure which first reduces the network by eliminating links that are not used by commuters or used seldom. Then, the routes of this network are selected by assigning frequencies using a linear programming model which maximizes the number of transfers from a link network, which means transfers are possible at every node to a public transit network, in other words transfers only at the intersections. The decision variables of this model are route and frequency.

Another approach was offered by Van Nes et al. (1988) to maximize the number of direct trips. As a demand, the model of Van Nes et al. (1988) used a direct demand model based on the simultaneous distribution-modal split model. The constraints of this model were fleet size and service frequency. As a commuter behaviour was used multiple path assignment model. The solution techniques of this model of Van Nes et al. (1988) assigned the frequencies to a pre-selected set of possible routes and increased the frequency on the route with the highest efficiency ratio, the ratio is defined as the number of extra commuters.

Baaj (1990) found out that the multi-objective function nature of the network design problem is important. In the model of Baaj (1990), the total demand satisfied and its components are checked versus total time and its components as well as fleet size of the network. In his work, the demand is assumed fixed. The constraints on Baaj's model (1990) were the following: circuity factor, load standard, route ridership volume. The same commuter behaviour was used multiple path assignment model. The solution technique generates routes from initial skeletons. Additional nodes can be added to skeletons, used the expansion algorithm. Decision variables as in the previous approaches were used route and frequency.

Israeli and Ceder (1991) considered the minimization of generalized cost and fleet size in two objective functions. Demand was assumed fixed and independent of service quality. The model's constraints were total operator cost, fleet size, and service frequency. A commuter behaviour was used multiple path assignment model as well as in the last previous approaches. It means that the number of acceptable paths is generated and then each commuter is assigned to the best path. Israeli and Ceder (1991) enumerated all possible routes and applied a route length constraint to erase routes with travel time, exceeding the least-time route by a specific threshold.

Pattnaik et al. (1998) proposed a two step algorithm for the bus network design problem. In the first step the set of routes was generated. In the second step the genetic algorithm is applied to find the best set of routes in order to minimize the total system cost. The algorithm was applied in Madras, India, with 25 nodes and 39 links.

Fusco et al. (2002) identified a transit network configuration, which consists of a set of routes and associated frequencies in order to minimize the system cost. Their approach is based on genetic algorithm and combines transit network design methods, which were proposed by Baaj and Mahmassani (1991).

Tom and Mohan (2003) followed the approach of Pattnaik et al. (1998), which proposed to minimize the total system cost, including bus operating cost as well as commuters' total travel time.

Zhao (2006) proposed simulated annealing global search scheme to minimize user cost as well as the number of transfers.

Afandizadeh et al. (2013) proposed genetic algorithm to solve the bus network design problem which consists of frequency determination, assignment procedure, and network evaluation procedure. The objective function takes also into consideration depot assignment, penalty for empty seats to optimize fleet capacity, the penalty for unsatisfied demand.

Renato Arbex and Claudio Barbieri da Cunha. (2014).proposed a solution for the bus network design problem which takes into consideration demand satisfaction with minimum total travel time, including a transfer penalty. Then assign frequency to each route in order to find out the waiting time as well as the fleet size of the system. The objective function is multi-objective due to the fact that the author wants to minimize the total travel time as well as the fleet size.

3. Problem Overview

The aim of this work is to satisfy as much as possible the total covered demand by generating stations, routes and to provide transportation that minimizes the total travel time of each commuter as well as total number of buses because every bus costs much investment from the operator. The demand for this study is assumed fixed. It means that demand is independent of service quality, seasonality (people ride to work, children should ride to the school, most people want to ride to another cities, shopping at the weekends).

In this study the constraints are the following: the distance between a station and its next closest stations should be at least 1,000 meters. 1,000 meters were chosen because many runs with different constraints such as 500, 1,250, and 1,500 were completed and the 1,000 meters distance gave the best results for the operator as well as the commuter. The distance between a commuter and his/her closest station should be equal or less than 350 meters. The reason the 350m distance was the same as for the 1,000 meters case. On one hand, the number of routes 30, 35, 40 were taken to check the level of demand satisfaction, which can be covered. The number 40 was chosen due the fact that it covers the highest level of demand satisfaction. On another hand, the fixed covered demand with the following levels 80, 85, 90 were chosen to determine the number of routes needed. The minimum frequency of each route should not be less than 1, which means that the number of buses per hour in two ways should be more than 1. Route round time should not be higher than 120 minutes according to National Cooperative Highway Research Program (NCHRP) Number 69 (1980).

It is assumed that there are a lot of different commuters using the public transportation daily and most of them want to reach their destinations without transfers, but for a better service, the study should also consider one transfer and two transfers.

In this study the decision variables are frequency, the number of buses on each line/route, the routes, and the number of stations.

4. Solution Methodology

The first step is to create the network of bus stations. This means, arranging all necessary bus stations which need to be in the system and interlink them with each other in a specific way. For this, the “Station generation approach” will be used. This approach uses geometric calculation to find the minimum distance between a point (commuter) and a line.

The next step is to determine the bus lines/routes. This was done by the “Route generation procedure”. The advantage of this procedure is that routes/lines can be generated by using the shortest path algorithm thereby they can be expanded, as well as they are built in such a way as to increase the direct demand satisfaction as well as demand satisfaction via 1-2 transfers. Lines/Routes will keep on being created until constraints are fulfilled.

The third step is to find the frequency and the number of buses needed for each line/route. It takes into consideration if the commuters can reach the final destinations, how they can reach, what are the possible routes/lines, on which nodes/station they can transfer.

The next step consists of finding out the centres for the bus network. This part plays also a role in this study because it allows to identify where the centres should be located making the bus network more efficient. To clarify, the “centres” help to determine which stations are needed the most. Once the above is defined and understood, it is taken into consideration for providing opportunities for joint development.

The last approach on the matter in hand is the “Network improvement procedure”. It helps to avoid routes where there are few commuters, in other words, where their frequency is too low, after several improvements iteration procedures they will be erased from the bus network.

4.1. Station Generation Approach

At the beginning of this study, the list composed of 85 station’s coordinates with the connectivity list were given. In addition, 22903 commuter coordinates were provided, Whole this information was presented in order to generate more stations and increase the number of commuters, who can reach a closest bus station and use the public transportation frequently. To determine the bus stations and their locations the following constraints should be fulfilled: Distance between a commuter and his/her closest potential station should be within 350 meters. The second constraint is that the distance between any stations should be longer or equal to 1000 meters.

In order to generate the number of stations, the minimum distance between commuter point and line will be used.

The idea is the following. First, coordinates of a commuter and the coordinates of two end nodes that can be represented as stations on the same line are taken. In the first step, the idea is to find out whether a potential station can be placed on the same line with both end nodes/stations. If it can, then in the second step, the distance between a commuter and the station is found, if the distance is within the first constraint, provided above, it is concluded that the first test has been passed. The next step is to find the distance between the station, which was found in the first step and its next closest stations. Also, if the distance does not violate the second constraint, then the second test is passed. Once, the number of potential stations for the considered commuter is found, the station with the minimum distance for the considered commuter is put into use. At the end, the commuters who can reach also the same station are checked and if that happens to be the case, they should be assigned to this station.

```
for all commuters in the list
    for all edges in the list
      find a potential station and its coordinates
      find distance b/w commuter and the pot. station
      if distance ≥ 350
         take the next edge
      else
         for all stations in the list
           if distance b/w each station in the list and the pot. station ≤ 1000
             take the next edge
           else
             save the pot. station
         End for
    End for
  find the station with min. distance to current commuter
  delete current commuter from the list
  for all commuters in the list
     if a commuter can reach the same station, delete him/her from the list
  End for
```

Algorithm 1. Station Generation Approach

3.2. Route Generation Procedure (RGP)

This chapter will explain the “Route generation procedure”. There are three steps that need to be taken into consideration: the initial skeletons, skeletons expansion to complete a route and check if the desired covered demand is satisfied.

Skeleton means a route that is not completed yet. Route means a route that is already expanded and completed.

At the beginning of this procedure, the idea is to set the initial skeletons for the bus network or in other words, the number of completed routes, which should be generated for this bus network system.

The RGP starts generating skeletons. The number of skeletons (M) is chosen 40, which means that this bus network system should have not less 40 completed routes. RGP does not terminate until all completed routes are generated and then the level of demand satisfaction is met. As was mentioned, other constraints are to see the number of routes needed for the network in order to satisfy the certain level of demand. For this procedure, the number of skeleton (M) is chosen to be 1. The algorithm will start generating the routes until the specific level of demand is satisfied.

Node-pairs for generating initial skeletons are taken from a sorted demand matrix. The first step is to sort the demand matrix in decreasing order. Once the demand matrix is sorted RGP chooses the first node-pair with the highest demand. After M node-pair is selected, the RGP selects nodes along the shortest path to form M skeletons. In order to find the shortest path the Dijkstra algorithm has employed.

After a skeleton is found, it should be checked if expansion is possible. The expansion is an important part of this procedure due to the fact that it helps to increase the length of the skeleton, and more stations in the skeleton which lead into increasing the total covered demand. The procedure has 4 steps, the first step analyses the nodes which can be connected to end nodes of analysed skeleton. Then, the second checks if those nodes are already in some other routes. The third provides information whether the node makes a circuity, which means if the distance between Beginning-Node and End-Node in this skeleton is not longer than the distance by using Dijkstra for the same two nodes. While the fourth selects the node with the highest demand. If the skeleton cannot be expanded, it leads into generation of a completed route without expansion. When the skeleton is already expanded, all node-pairs in the completed route will be checked and not considered for further routes.

The last step of RGP is to check if it can already terminate. In order for that to be possible it can be said that the RGP should satisfy certain direct demand or demand via one transfer. If demand cannot be satisfied within RGP routes, the procedure should be repeated. The purpose of this algorithm is to maximize total demand within constraints.

```
sort the demand matrix
create a list where all potential stations for expansion will be saved
for all node-pair in demand matrix
    find the skeleton for current node-pair
    find the total length of this skeleton
      while total length of current skeleton <= 120
          expand the skeleton
      End while
    if total number of skeletons is reached
      find total demand satisfied via 0 transfer
      find total demand satisfied via one transfer
      If total demand satisfied is within the constraint
        stop
```

Algorithm 2. Route Generation Procedure

INPUT Information

The RGP requires the following information at the beginning:

1. Network: The number of bus nodes (stations), the connectivity list, the number of initial skeletons (at least 1 should be set).
2. Demand: level of demand satisfied directly or via one transfer should be chosen.
3. Node Insertion rule: there are four node selection and insertion heuristics which can be selected: Maximum demand insertion (MD), Maximum demand per minimum time insertion (MDMT), Maximum demand per minimum route length increase insertion (MDMI) and Maximum demand per minimum cost insertion(MOMe).

Expansion of Skeletons to Routes

In this section, the expansion procedure is discussed to complete the route. This procedure is trying to find the candidate for insertion into the route. Each candidate has to satisfy a demand increase and length constraints after expansion. The procedure terminates if no candidate has been found.

Before explaining the procedure in more details, each candidate has to follow the following criteria:

1. The resulting route does not form a loop after insertion
2. The resulting route does not become circuitous
3. The node still shows a low percentage of its total originating demand which is satisfied directly. This means if the node has already been inserted into other routes, and it satisfied much of its total originating demand directly and it will not contribute much demand satisfaction for others. In other words, the routes should avoid having the same nodes because the demand satisfaction will not be increased.

The Selection and the Insertion of the Nodes

The procedure follows 4 steps, each node has to be checked out and if it cannot satisfy any of the necessary step, it then has to be erased from the list of nodes for expansion.

Route-Looping Test

The idea of this test is to find the nodes which are connected to the end nodes of the skeleton. If the skeleton has already several nodes, say 1-2-3-4-5, this test checks which nodes are neighbours to 1 and 5.

Node-Sharing Test

The list of nodes is checked to remove nodes that have a high percentage of their originating demand satisfied in other routes. The sharing factor for this study is 75%. If a node has higher demand satisfied, it will be removed from the list of nodes for expansion.

These criteria is based on **<u>the following data</u>**:

1) If a node is inserted into many routes with much of the originating demand already satisfied, it will not contribute an increase in demand satisfaction directly.

2) In a transit network, commuter trips can be completed either directly or by transfers. Therefore, demand originating at a specific node that cannot be reached directly could be reached via transfers. If the node is inserted with much of the originating demand already satisfied directly will not be profitable.
3) From the computational point of view, it would be useless to use the node which can be reached just via 0-transfer. If such as node is taken into consideration for expansion, then the opportunity to maximize the total covered demand via 1-2 transfers will be lost. (Mao-Chang Shih and Hani S. Mahmassani 1994).

Route Circuity Test

Each node that has already passed the previous tests, should be checked on circuity. Each node is taken and the algorithm checks the shortest path between that node and the end node and the trip time or distance between that node and the end node within the route. If a factor exceeds 1.5, the node has to be removed from the list of nodes for expansion. The **factor** can be found the following formula:

Distance within the route = a
Distance within the shortest path = b

Factor $= \frac{a}{b}$ <=1.5

where,
Distance within route is a distance between the node that is considered for insertion and the last node in the skeleton, which should be expanded.
Distance within the shortest path is the distance between the node that is considered for insertion into the route and the last in the skeleton, which should be expanded, using the Dijkstra algorithm.

Sorting-Property Test

There are four ways of choosing a node that needs to be inserted into a route:

1) Direct demand can be satisfied by inserting the candidate node into the route: It will lead to higher demand satisfaction, but this test does not consider any operator costs.

2) Direct demand satisfaction can be increased by inserting the candidate node into the route, which will result into an increase in total in-vehicle time. It will reflect the user benefits versus user costs. It means if the node is inserted, a commuter has to spend more time in the bus due to the fact that a route will become longer. It means, if the route has 8 stations and the passenger wants to ride from station 10 to the station 30 then he/she needs to ride through 1 station 15, before the node was inserted. If the node is inserted between stations 15 and 30, then the commuter has to ride already 2 stations, what leads to the result of total in-vehicle time increase.
3) Direct demand satisfaction can be increased by inserting the candidate node into the route, as a result the total round time for the current route will increase. It will lead to higher demand satisfaction considering the operator costs. In other words, there is a route with 8 stations and total round time 30 minutes. If the considered node is inserted into this route, then the route would be increased by 1 station. This procedure will increase the total round time of this route.
4) Direct demand satisfaction can be increased by inserting the candidate node into the route but increase in sum of total in-vehicle time and round time should be considered. I will lead to higher demand satisfaction with regards to the users' costs and the operators' costs. In this situation the tests "2" and "3" are examined at the same time.

In the current study, a third strategy (demand satisfaction will be increased by inserting the candidate node into the route per increase in round trip time) will be used because the main goal is to maximize the total demand satisfaction as well as to account for the operator's costs.

If there is still a node with a higher demand increase, it should be checked by the constraint. If a route with a new node exceeds the total round time, which was set to be 120, then that node cannot be considered as a candidate, but if that node passed, then the procedure should be repeated to find a new node with existing nodes under expansion.

The following is an example for this whole algorithm: suppose that the node-pair with the highest covered demand satisfied is 10-25, which should build a skeleton. Assuming the skeleton became 10-30-40-25 and the total length of this skeleton is 50 minutes. It should be checked whether the skeleton could be expanded to build a longer completed route. In the expansion procedure, the first step is to check whether the stations 10 and 25 have stations, which are connected to them. Once again, assuming the following stations for expansion could be added: 15, 23, and 11. In the next step would be to check the originating demand for each station under expansion. First, the originating demand (1) from the station under expansion and each station in the skeleton that is connected to the current station under expansion is found. Second, the originating demand (2) from the current station under expansion and each station in the bus network system is found. At the end, the ratio will be found and if the ratio is within 75%, then the current station under expansion can be considered for the further steps. The formula for ratio is $\frac{originating\ demand(1)}{originating\ demand(2)}$. In the third step, the circuity should be checked. Suppose, there are just two stations left in the list of stations for expansion such as 15 and 11. There should be one more assumption, that is, that station 15 is connected with the station 10 in the skeleton and the station 11 is connected with the station 25. First, the distance between station 15 and station 25 should be found. This means, the distance between stations 15-10, 10-30, 30-40, and 40-25 is to be found. Then, the shortest path algorithm should be introduced in order to find the shortest distance between the station 15 and the station 25. Once, these distances are found, the distance ratio should be applied, which has to be within 1.5. In other words, the distance can exceed the distance by using shortest path algorithm by no more than 50%. Supposing both stations passed the circuity test. In the next step would be that the station will be taken, which gives the highest increase in total covered demand for the bus network system. It is considered that the station 15 gives the highest increase in total covered demand. In the last step, total length of the skeleton should be checked and if the total length is within 120 minutes, the station 15 should be added into the skeleton. After this procedure, it can be concluded that the skeleton became longer with 15-10-30-40-25. The procedure should be repeated till the total length is exceeded or there are no stations anymore for expansion. The skeleton will be renamed into the completed route.

Summary of RGP

RGP can generate the efficient routes/lines for a bus network system by providing the following important factors (Mao-Chang Shih and Hani S. Mahmassani 1994):

1. RGP selects the highest node-pairs from the demand-matrix to form the skeletons. It will be lead to a high direct demand satisfaction.
2. It builds the routes, using the shortest path.
3. The RGP generates sets of routes which correspond to different node selection and insertion strategies, directness levels, where user and/or operator costs are taken into consideration.

4. It includes important service planning factors such as length, loop avoidance and route structure.
5. The RGP allows for the improvement of efficiency: the selection of minimum number of routes or fixing number of routes, different constraints for expansion as well as different strategies for node selection and insertion can be implemented.

3.3. Transit Route Configurations (TRUST)

This chapter will consider the next approach for the bus network system “Transit Route Configurations” or “TRUST”.

TRUST is a program that evaluates a given set of bus routes and associated frequency to find service quality, the operator cost, and the user cost. The procedure finds the percentage of total covered demand, which can be reached via 0-transfer, 1-trasnfer, 2-transfer, or cannot be satisfied at all. Furthermore, it finds the link-flow for each route or in other words, the number of commuters, which ride on each link of each route. In addition, the procedure also finds the best frequency and the optimal number of buses for each route.

Overview of TRUST

Trust is a program that finds the following information:

1. The total travel time for the whole network(waiting time, in-vehicle time, penalty time (time for a commuter to walk to the next station if he/she should use 1-2 transfer))
2. The demand satisfied directly , via 1 transfer, via 2 transfers and also unsatisfied demand
3. The link flow on each route
4. The frequency and number of buses for each route as well as total number of buses for the whole network

TRUST requires the following data as input data

1. Node-list (round time list) – it is a list of each route with its round time(time measured in minutes, which is needed for a bus to complete a route in two ways)
2. Connectivity-list- is the list of all nodes which are connected with each other.

3. Demand-matrix- is a matrix with all nodes (stations) and demand assigned to every single one of them.
4. Route-list- is the list of all routes in the network.
5. Frequency-list- in this study is taken to be 10, it is used on al routes. The number 10 signifies the number of buses per hour.
6. Round-trip-times- is a list of all round-time on every route.
7. Bus-seating-capacity- in this study is taken to be 55.
8. Transfer-penalty- for this work 3 minutes are used on each route.
9. Max-Load-Factor- for this study is taken to be 1.2 factor. This means if the normal load capacity of the bus is 55 passengers and crush load capacity is 66. The maximum load factor is crush load capacity/normal load capacity.

The objective function of trust is to minimize total travel time for each commuter and number of buses for the operator within the constraints. So the objective function can be also explained as a tradeoff between user cost (total travel time) and operator cost (the number of buses).

The Assignment Model and Computation of Transit Network Descriptions

The assignment process is used to find out whether a specific node-pair with its demand can be reached via 0, 1, 2 transfers. If the node-pair exists in any routes, the procedure checks how the node-pair can be reached. Once the node-pair is reached, the number of routes which can be used for commuters, in-vehicle time, waiting time and penalty time as well as link-flow are found. Link-flow shows the covered demand by each link on each route.
After all node-pairs were checked, the highest link-flow from each route is taken and the number of buses as well as the frequency for each route will be found.

The Assignment for 0-Transfer

After checking whether the node-pair already exists in any route, the procedure finds out if the node pair can be reached, using the direct transfer/0 transfer. The route with minimum in-vehicle time will be chosen for those commuters, routes with a threshold higher than 50 percent of the minimum in-vehicle time are rejected in the process.

If there are two routes, **the following formula** should be used to find the demand for each route:

(Frequency of first route/ (frequency of first route + frequency of second route)* demand between node pair).
The **average waiting time** for them can be calculated:

60/ (2*(frequency of first route + frequency of second route)).

The link-flow for then are found:
(Frequency of first route / Total frequency of both) * covered demand within the node-pair.
The same is done for the second route.

It means that all routes have the same waiting time.
Transfer penalty is not used for assignment for 0-transfer process.

The Assignment for 1-Transfer

The assignment for 1-transfer has the following steps.
First, the TRUST checks all the combinations for that node pair and intersection node. If both routes have the same intersection node and the first route has the first node of the node-pair and the second has the second or vice versa, it means that the node-pair can be reached via 1-transfer.
After the number of routes for assignment has been found, the procedure finds the route with the minimum total travel time, and then rejects those routes, which exceed the minimum total travel time by 10%.
The **formula below** is used to find the total travel time for each route if two routes were selected:

Total travel time = in-vehicle time between first node and intersection node + in-vehicle time between the intersection node and the second node + [60/ (2*f1)] + [60/ (2*f2)] + transfer penalty.

The explanation of the formula is as follows: after the node-pair was checked and the procedure found out that it can be reached via one-transfer, then it should find the total travel time for commuter on that route. Each commuter should first wait (f1) to get into the bus, ride till the intersection node (where a transfer is made), that time is calculated as in-vehicle time between the first node and intersection node. Then, a commuter should walk to another station (it is just assumed) and wait, this should be considered as penalty time and waiting time (f2). After the commuter gets into the bus again, he/she should ride till the last node/station, this should be measured as in-vehicle time between the intersection node and second node.

To clarify the calculations of f1 and f2 the **following example** is given:

f1 is the total frequency of all starting routes.
f2 is the total frequency of all routes, where commuters transfer.
The node-pair is 10-25.
The first route is 10-20-30-50. The frequency of it is 15.
The second route is 5-20-30-25. The frequency of it is 20.
The starting route is the first route, because a commuter wants to reach a destination from the node/station 10. The average waiting time for f1 is 60/ (2*15) =2. The same formula can be applied for transfer node. The average waiting time for f2 is 60/ (2*20) = 1.5.
Once, total travel time and the routes which passed the test with minimum total travel time are found. **The link-flow** for each route should be determined:

((Frequency of first route/ (Total frequency of routes with starting node) * demand satisfaction between two nodes)/ the number of routes with starting node.

For example, the same two routes which were used in the first example are used also here too. There are 2 commutes, who want to reach the station/node 25, going from the station/node 10.
The first commuter goes to the node/station 20 and at this station/node transfers.
For the first path the link-flow for the first and second route is the following:
(15/15)*2/2 = 1, this means, that the path from 10-20 for the first route and from 20-40 for the second route has 1 covered demand.
The same should be found for the second path. The second commuter wants to go from 10 to 25 and at the node/station 30, he/she transfers.
((15/15)*2)/2 =1, this means, that the path from 10-30 for the first route and from 30-25 for the second route has 1 covered demand. The total link for both are the following:
10-20 = 2; 20-30 = 1 – for the first route
20-30 = 1; 30-25 = 2 – for the second route.

If a commuter cannot also reach his/her destination with 1-transfer, then the procedure checks the last possibility for the commuter, using 2-transfer assignment process.

The Assignment for 2-Transfer

The procedure for 2-transfer starts with searching for all the routes with exactly two transfers between given node pair. First, TRUST takes the first node, checks whether that node can be connected with another route, having the same intersection node (1). After TRUST takes that route with the same intersection node (1), it finds whether there are routes with the same intersection node (2) between the second route and the third route, if there is a node, those routes can be connected, creating 2-transfer trip. At the end, the last route should be checked whether it has the node that left in the node-pair. Supposed, there are i and j nodes, if i has been already found, then j should be checked in the last route and if there is that node, the procedure can complete the assignment.

After the procedure finds the number of routes, which can be used for the assignment, the next step is to reject all routes which exceed the minimum total travel time by 10%.

The formula to find total travel time for 2-transfer:

Total travel time = in-vehicle time between first node in node pair and first intersection node + in-vehicle time between first intersection node and second intersection node + in-vehicle time between second intersection node and the second node in node pair + [60/ (2*f1)] + [60/(2*f2)] +[60/(2*f3)]+(2*transfer penalty).

The same procedure as in the first transfer can be applied here, but just with 3 routes.

If at the end, the commuter could not find any transfer with which he/she can reach his/her final destination, then the covered demand in that node pair should be considered as an unsatisfied demand and should be placed into the unsatisfied list.
After the procedure finished, there should be lists with 0-transfer demand, 1-transfer demand, and 2-transfer demand. Total travel time and its components (in-vehicle time, network-waiting time, and penalty time) are computed within TRUST.

Computation of Route Frequencies and Number of Buses

Once all elements are computed and the final list-of-link flows is assigned, the next process can be started.

First, the highest link of flow for each route should be chosen.

The formula below finds the number of buses on each route:

$$\text{Number of buses on route (k)} = \frac{Maximum\ link\ flow\ of\ the\ route * Total\ round\ time\ of\ that\ route}{60 * maximum\ load\ factor * bus\ capacity}$$

where,

Maximum load factor 1.2 as well as bus capacity are given.

The following formula computes the frequency for each route:

$$\text{Frequency on route (k)} = \frac{Number\ of\ buses\ on\ that\ route * 60}{Total\ round\ time\ of\ that\ route}$$

The total fleet size can be computed:

Total number of buses = $\sum$number of buses on each route

TRUST is an effective tool for the network design which has different configurations: route, frequency, bus seating capacity, max-load factor, transfer penalty, thresholds which can be easily modified in order to find the best bus network system.

To delineate the process of TRUST more precisely, below a flow chart and a pseudocode have been provided.

for all node-pairs in the demand matrix
  if node-pair can be found in the in the routes
    if node-pair can be reached via 0-transfer in the current route, save route
      find routes that do not exceed the minimum in-vehicle time
      update Waiting time
      update In-vehicle time
      update Total travel time

update link-flow
update satisfied demand via 0-transfer
if-else node-pair can be reached via one transfer in routes, save routes
find the routes that do not exceed the minimum total travel time
update Waiting time
update Penalty time
update In-vehicle time
update Total travel time
update link-flow
update satisfied demand via one-transfer
if-else node-pair can be reached via two transfer in routes, save routes
find the routes that do not exceed the minimum total travel time
update Waiting time
update Penalty time
update In-vehicle time
update Total travel time
update link-flow
update satisfied demand via two-transfer
End for
find the maximum link-flow for each route
for each link-flow
find the number of buses needed for each route
find the frequency needed for each route
End for
find the total number of buses for the network system

Algorithm 3. TRUST

3.4. Transit Center Selection

Transit center selection can be also considered as an important part of an efficient bus network system. For instance, if the information is provided with stations, which stations have the highest demand, those stations can be provided with additional service. For example, there can be ticket’s cashiers, cafes, taxi etc.

The process of identifying the proper centers is the following:

1. Create an empty list of stations, which should be considered for centers.
2. Find stations, which are directly connected to two other station, if a station is connected to less than two stations, that station cannot be considered for any ensuing steps.
3. Here are the stations should be defined for potential transit center within service area. For example, there is a station, which should be considered as the station for transit center. Each stations, which are in the bus network system, should be checked if they are located within the range equal or longer than 15 minutes. If that is a case, then the covered demand should be calculated. For instance, the station which is considered for center is A. There are 2 more stations: B, C, which are located within time equal or higher than 15 minutes. Then, the covered demand should be calculated as follows: A-B, A-C, B-A, B-C, C,A, C-B. If the covered demand is not satisfied by 150 commuters, the node/station A should be erased from the list of potential centers. 150 commuters were chosen because in the paper Baaj and Mahmassani that was the case.
4. The next step is the most difficult one because for each considered station should be taken into account the following parameters: “transferring demand”, “terminating demand” and “originating demand”. The first type, the transferring demand is the demand which goes through the considered station or in another words the number of people who uses this considered station for transfer. While, the terminating demand is a demand which ends the journey at the considered station. Finally, the originating demand is a demand which starts at that station.
5. Find the station with the highest demand.
6. Check all potential stations and if there are stations which are located in the distance more than 15 minutes, these should be then considered as transit centers.

```
create a list of stations, which should be considered for centers
for each station in the routes
  find the number of stations connected to each of them
  if the number of stations connected < 2
    delete the station for further consideration
End for
for each station for transit center
  for each station in the routes
    find the stations which are within 15 minutes to pot. transit center
    find originating demand for each of them
  End for
  if total originating demand satisfied > 150
    save the station for further consideration
End for
for each station for transit center
  find the originating, transferring, terminating total covered demand
End for
find the station with highest total covered demand
for each station for transit center
  if station > 15 minutes to the selected centers
    add into the list for transit centers
End for
```

Algorithm 4. Transit Center Selection

3.5 Network improvement Procedure

The network improvement procedure allows checking, which routes can be improved. For example, if there is a route and it has just one bus per hour. This route can be checked whether it can improved, joined with other routes or that route should be just erased if it cannot be joined with any.

There are two improvement methods used in this study. The first one is discontinuation of service on low ridership routes and the second is route joining.

Discontinuation of service on low ridership routes - is a procedure which finds the route with the frequency less than 1 and eliminates it from the bus network. It leads in most situations to lower total demand satisfaction, reduce fleet size, the total number of miles the bus should drive.

Route joining - is a procedure which looks for a route with low ridership, then finds whether that route can be connected to another one which needs additional buses. After, the connection that route should be removed from the list of routes. For instance, there are 40 routes, there is 1 route with the frequency 1 that route can be connected to another route, after the connection appeared that route should be deleted from the list. The new list will have 39 routes, but there is a new route, which became longer due to joining algorithm. As a result, the total waiting time and transfer time should be reduced. The fleet size can be increased and also the direct demand will be increased, but not in all situations.

In order to show the procedure, the flow chart and pseudocode for joining improvement procedure are given below.

1: **for** all routes
2:   **if** frequency of route ≤ 1
3:     **for** all routes
3:         **if** route ≠ route in the line two **AND** the number of buses ≤ 10 **AND** it can be joined
4:           join route with frequency ≤ 1 with the route that needs more buses
5:           **break for**
6: **End for**
7: **for** all routes
8:   **if** frequency of route ≤ 1
9:     Delete the route from the network
10: **End for**

Algorithm 5. Joining improvement procedure

4. Computation Results

In this chapter, data will be provided from the beginning to the end. Also, additional information regarding each algorithm will be mentioned in order to show how those algorithms can be used in most bus network problems.

At the beginning of this study the following information was given:

The number of commuters who live in that area is 22903

The number of stations is 85 stations.

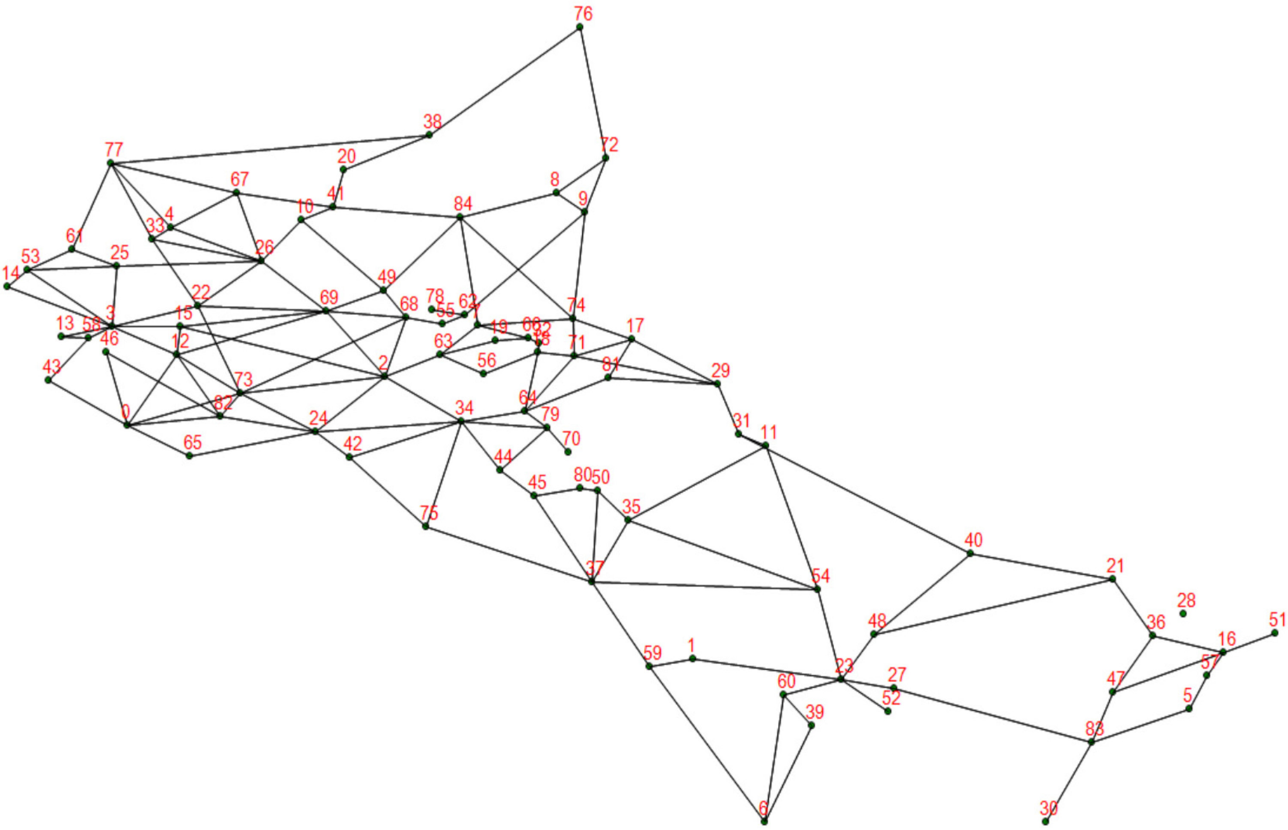

Figure 1. Map of Stations before the "SGA"

The Figure 1 shows stations for bus network system. The number of stations is not enough due to many reasons. A large number of commuters cannot reach any of the stations, which leads to lower demand satisfaction.

As a result, the station generation approach has been used in this study to create more stations and check whether most people can reach their closest station in the bus network system. There are some commuters who cannot reach any of the stations because they might live or work too far from any of them. For this reason, it is still considered if the commuter walks more than 350 meters, use a taxi to his/her closest station.

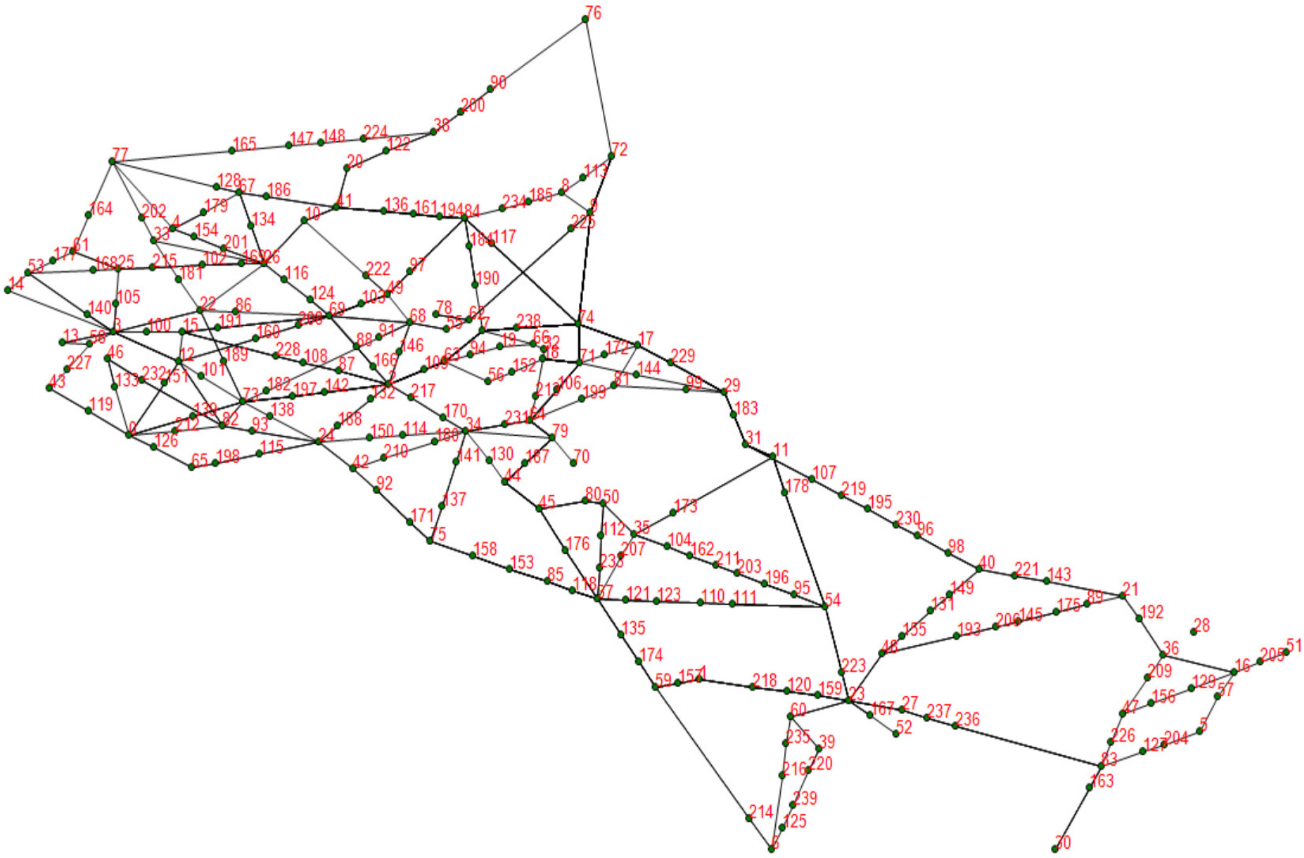

Figure 2. Map of Stations after the "SGA"

After station generation approach was run, the number of stations required were 240 which is almost 3 times more as when the approach is not used. This approach gives a good solution because the network has more stations that can lead to higher total demand satisfaction due to the fact that most people should just walk to the closest station and not anymore use another vehicle to reach work, home, school, etc.

When all stations are generated, the RGA algorithm should be run, which will create skeletons from the nodes below, expand them to create a complete route. The results of this approach is represented in the **Figure 3.**

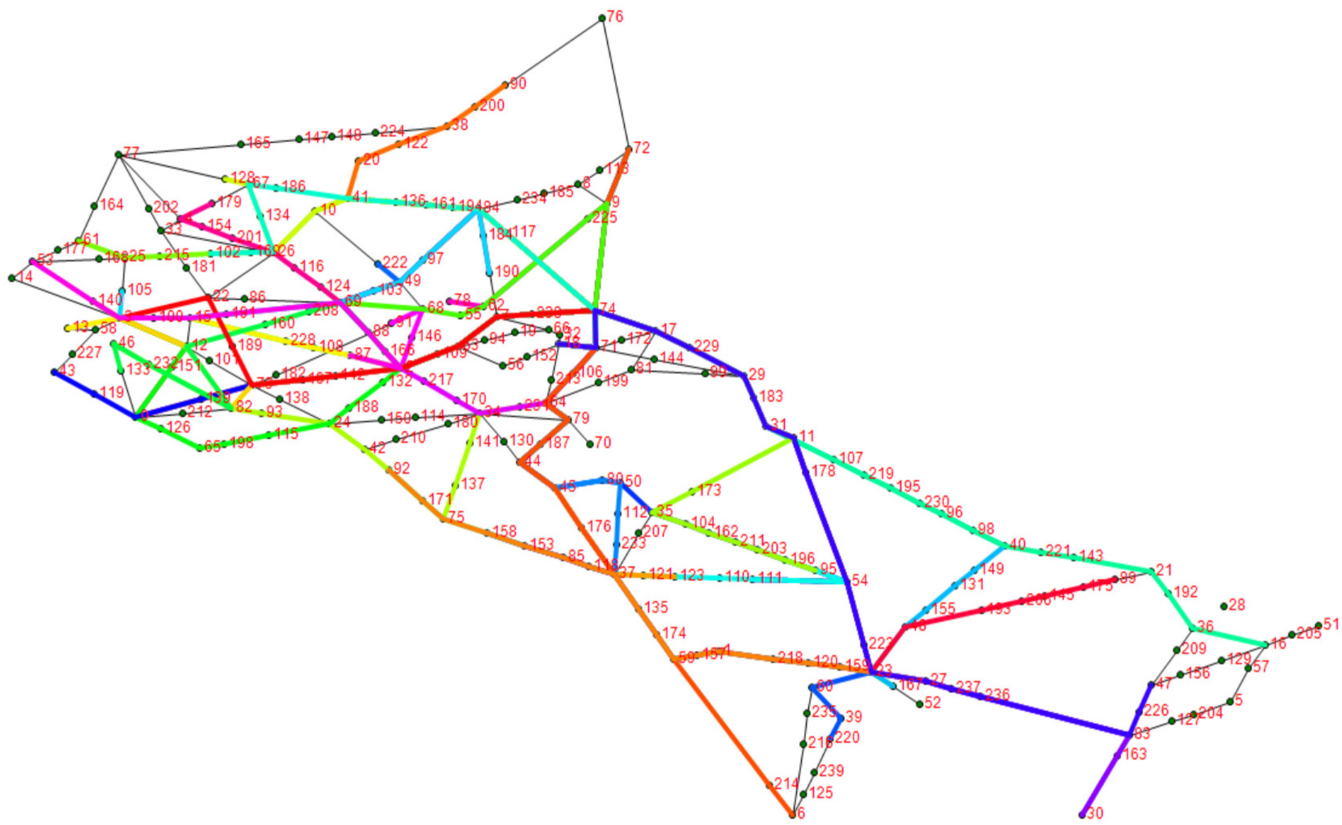

Figure 3. Map of Colored 40 routes after the “RGA”

The most important conclusion after analyzing the **Figure 3** with these routes is that the most important stations are presented in the bus network system. Most journeys start or end at the main station Steyr that is number “74” on the map where most commuters work or live.

In addition, as was shown that the covered demand is not satisfied fully and due to this reason, not all stations can be covered within the bus network system.

In addition, as mentioned at the beginning the idea is to find out the number of routes needed for different fixed levels of demand satisfaction. For this purpose, the algorithm “RGA” uses different constraints for total covered demand. The first constraint is 80% total demand satisfaction and the number of routes needed for this level.

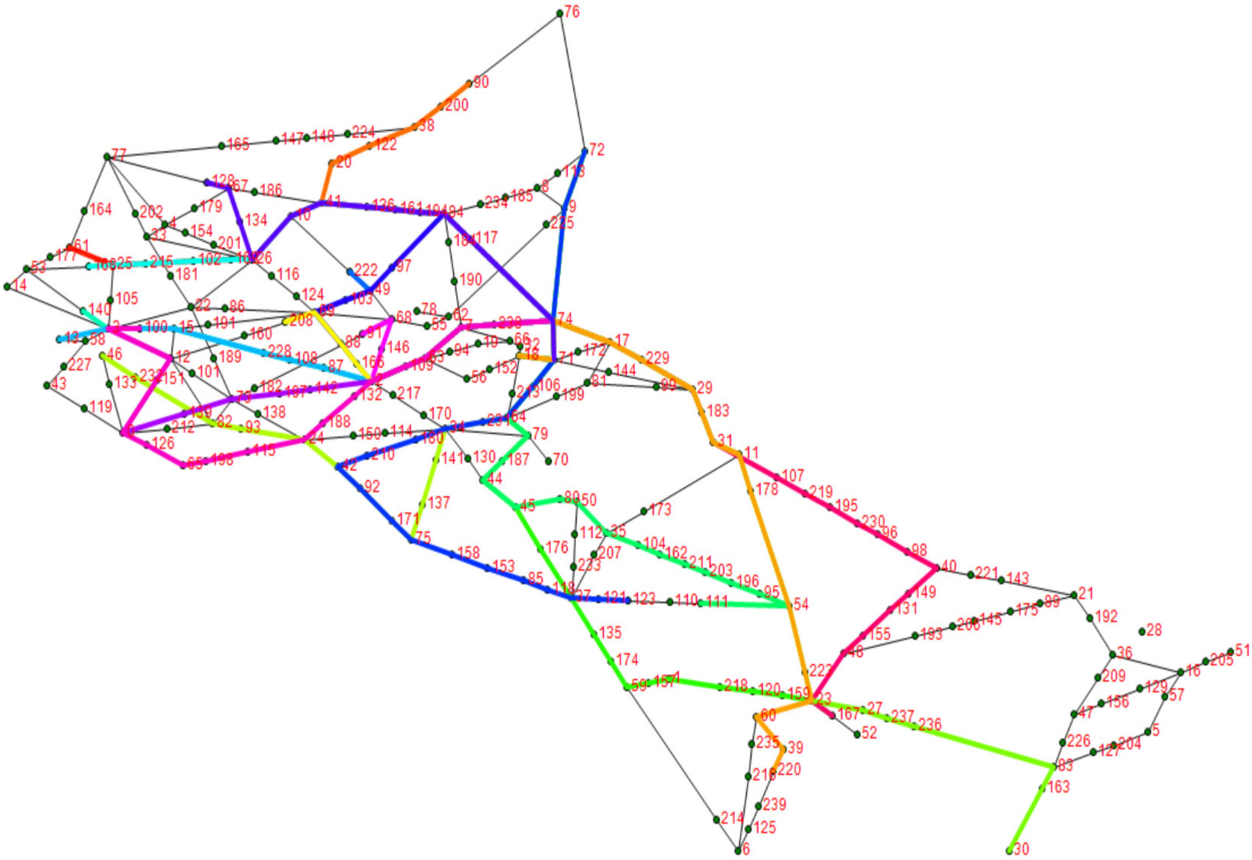

Figure 4. Map of Colored 19 routes for covered demand higher or equal to 80% after the "RGA"

After the algorithm was run, the results obtained is represented in the **Figure 4**. It is clear to see the difference between the fixed number of routes and when the purpose is just to satisfy the certain level of demand satisfaction. For example, in the **Figure 4** the stations from 40 to 16 as well as from 48 to 99 are not covered at all. There is also difference in the regions where total covered demand is much higher. For example, the buses do not ride to the following stations: 154, 179, 201, 4, 184, 190, 119, 43, 83, etc. There is just one identical fact in compared to the **Figure 3** is that the station "74" Steyr is used in almost each route, what can be again concluded as efficient bus network.

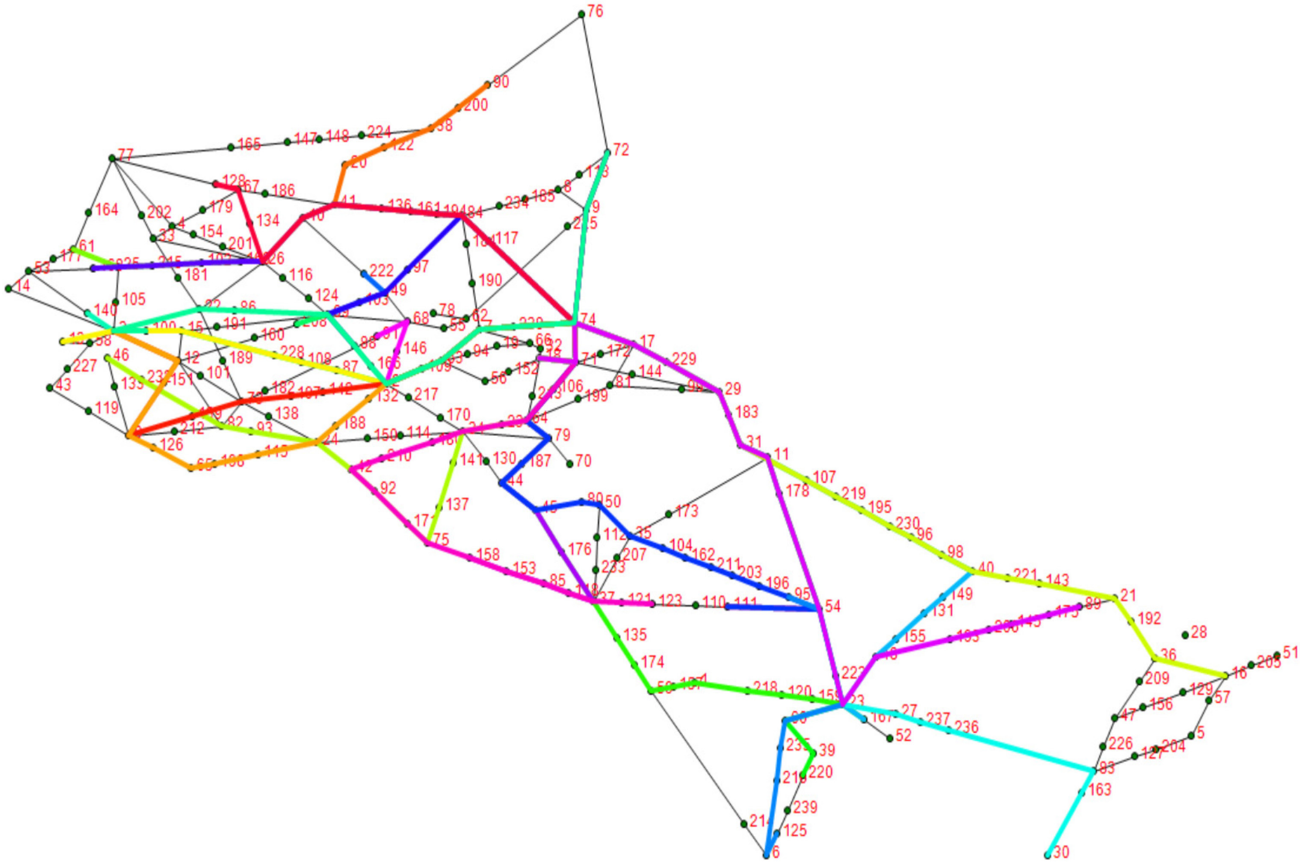

Figure 5. Map of Colored 25 routes for covered demand higher or equal to 85% after the “RGA”

Figure 5 gives a good overview of the bus network, when the purpose is to satisfy total covered demand within 85%. There is interesting fact due to the fact that skeletons were determined differently. In the **Figure 3**, the buses do not bring people to the stations 235, 219, 125. Of course there are much more stations which are not covered when the purpose to satisfy the 85% of total covered demand.

The network looks much widely used because all stations with the highest demand is already satisfied.

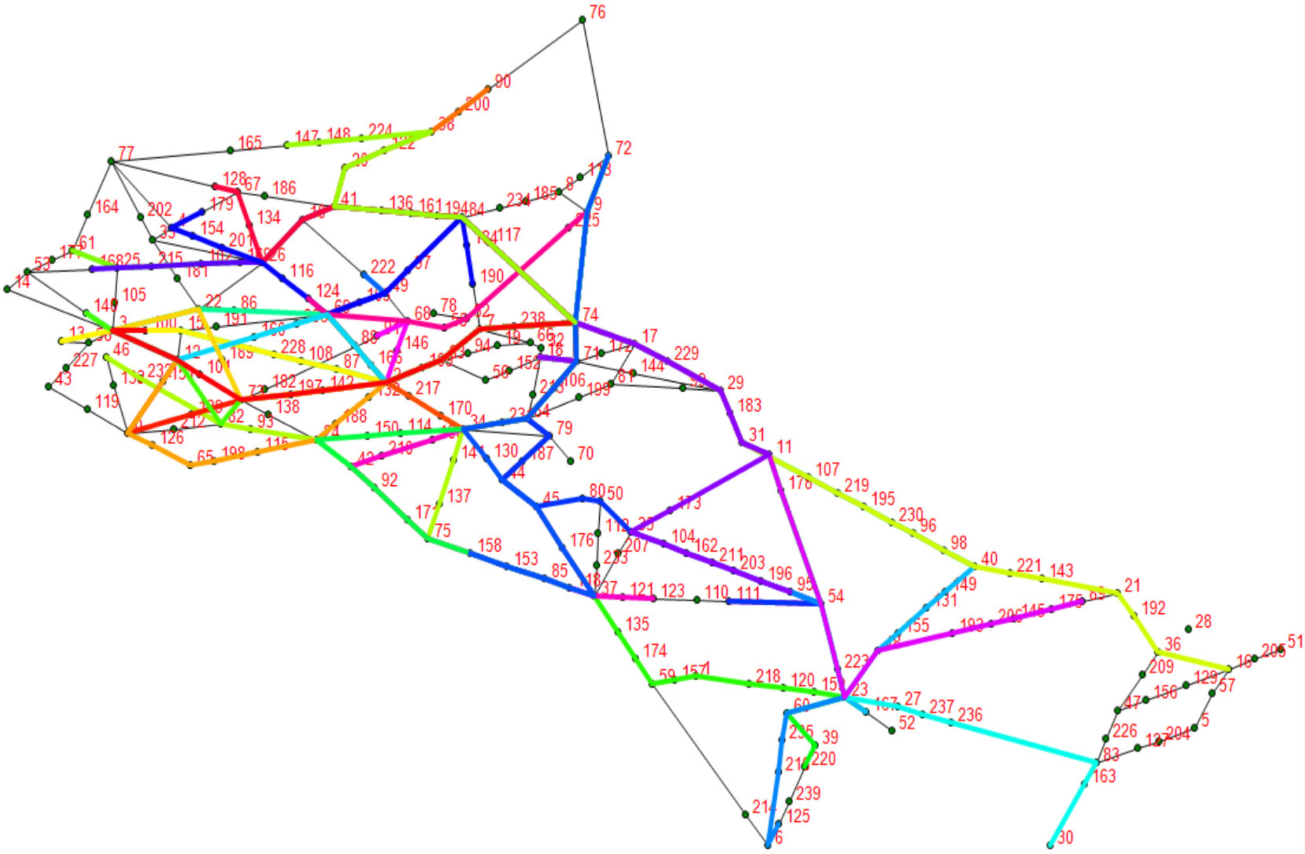

Figure 6. Map of Colored 36 routes for covered demand higher or equal to 90% after the "RGA"

Figure 6 represents the bus network, when the aim is to satisfy total covered demand within 90%. The network has already 36 routes and covers almost as many stations as the network with fixed 40 routes. There is again difference, because in the network with fixed routes, there are stations which are not covered indeed they are covered in the network which is shown in the **Figure 6**. For example, the stations 235, 219,125 as well as in the **Figure 5** which are covered, they are not covered in the **Figure 1**. Still, there are some station in the **Figure 6**, which are not covered, when in the figure 10, they are completely covered.

The next algorithm is **TRUST**, which can be considered as the most important algorithm of this study because its results play a big role not just for the operator but also for commuters.

Constraint	***90% covered demand***		***85% covered demand***		***80% covered demand***		***40 routes***	
Waiting Time(min.)	13.58%	8,389.04	14.22%	7,599.96	14.83%	6,348.55	13.38%	8,473.39
In-Vehicle Time(min.)	64.30%	39,691	67.63%	36,140	71.98%	30,809	66.99%	42,405
Penalty Time(min.)	22.12%	13,656	18.15%	9,702	13.19%	5,649	19.63%	12,429
Total Travel Time(min.)	61,736.04		53,441.96		42,806.55		63,307.39	
Covered demand via 0-transfer	75.74%	17,348	72.19%	16,532	69.36%	15,885	78.93%	18,076
Covered demand via 1-transfer	14.53%	3,300	13.12%	3,006	12.11%	2,773	12.46%	2,852
Covered demand via 2-transfer	0.40%	93	0.12%	28	0.00%	0	0.37%	87
Unsatisfied demand	9.43%	2,162	14.57%	3,337	18.53%	4,245	8.24%	1,888
Total km/meters of all routes	996.24 2	996,242	708	707,883	533.091	533,091	1,086.18	1,086,178
Number of buses	230.54		220		183.6		228.76	
Number of routes	36		25		19		40	

Table 2. List of important data for each different run

Table 2 describes the most important findings in this study. Firstly, it is important to say that 40 routes cover the highest covered demand via 0-transfer. It can be achieved because many routes with different stations and if a commuter wants to go to work or home, he/she does not need to transfer at other station, which will take additional time. The second important finding is waiting time. Again, with 40 routes, the waiting time is lowest what can be again achieved due to many routes, stations, and not necessity to transfer. The total covered demand, which can be achieved within 40 routes, is 91.76%. Another interesting finding is that if the operator wants to maximize the fixed covered demand with a 90% level it can be found out that there are more buses needed because there are already more passengers who should transfer and spend more time waiting for the bus as well as going to another stations what can be considered as penalty time.

If the operator decided to find the number of routes needed for just 80% of covered demand. It should be a bus network with just 19 routes and 183 buses. In such network, passengers spend most time in the bus and not waiting and going to transfer. Also, network is twice smaller according to the total km of all routes if it is compared to the 40 routes network.

As an example from the real world bus network is Vienna bus lines. Currently there are approximately 500 buses with 43 routes along 360 kilometers. The number of passengers annually is about 120 million. Another example is OEBB Postbus that is the largest bus company is Austria. It has 900 lines with 2,200 buses. The number of commuters is 228 million annually. The length of all lines/routes is 148 million kilometers. The number of stations is 22,000. If the city Steyr is examined. There are currently 160 stations with the length of 87 kilometers. The number of passengers is 4.2 million annually. The number of buses is 26, which operate on 9 lines/routes. All buses start and end their journey at the station “Bahnhof” what is the same finding, which determined in this study (almost all journeys start and end at the station “74” which can be name as “Bahnhof”. The difference between the city of Steyr and the study is that the study considers not just the city but also all rural areas around the city.

In this section, the transit center will be described and the table with the best stations, which should be used as centers, will be provided.

The purpose for transit centers is very important due to the fact that, the operator could provide information wisely to create additional service at these centers. Once this is achieved, it could be perfect environment for retail stores where they could offer their service such as shops, restaurants, cafes, taxis and among others. This phenomenon is mostly seen at train stations with several connectivity, such as trains, buses, and even subways. So due to the vast number of people, it turned out to be the perfect spot for Malls to be located.

Node/station	Total Demand covered
74	16544
3	8115
23	6021
54	5949
83	2512
40	505
25	384
0	344

Table 3. Centers for the constraint 40 routes

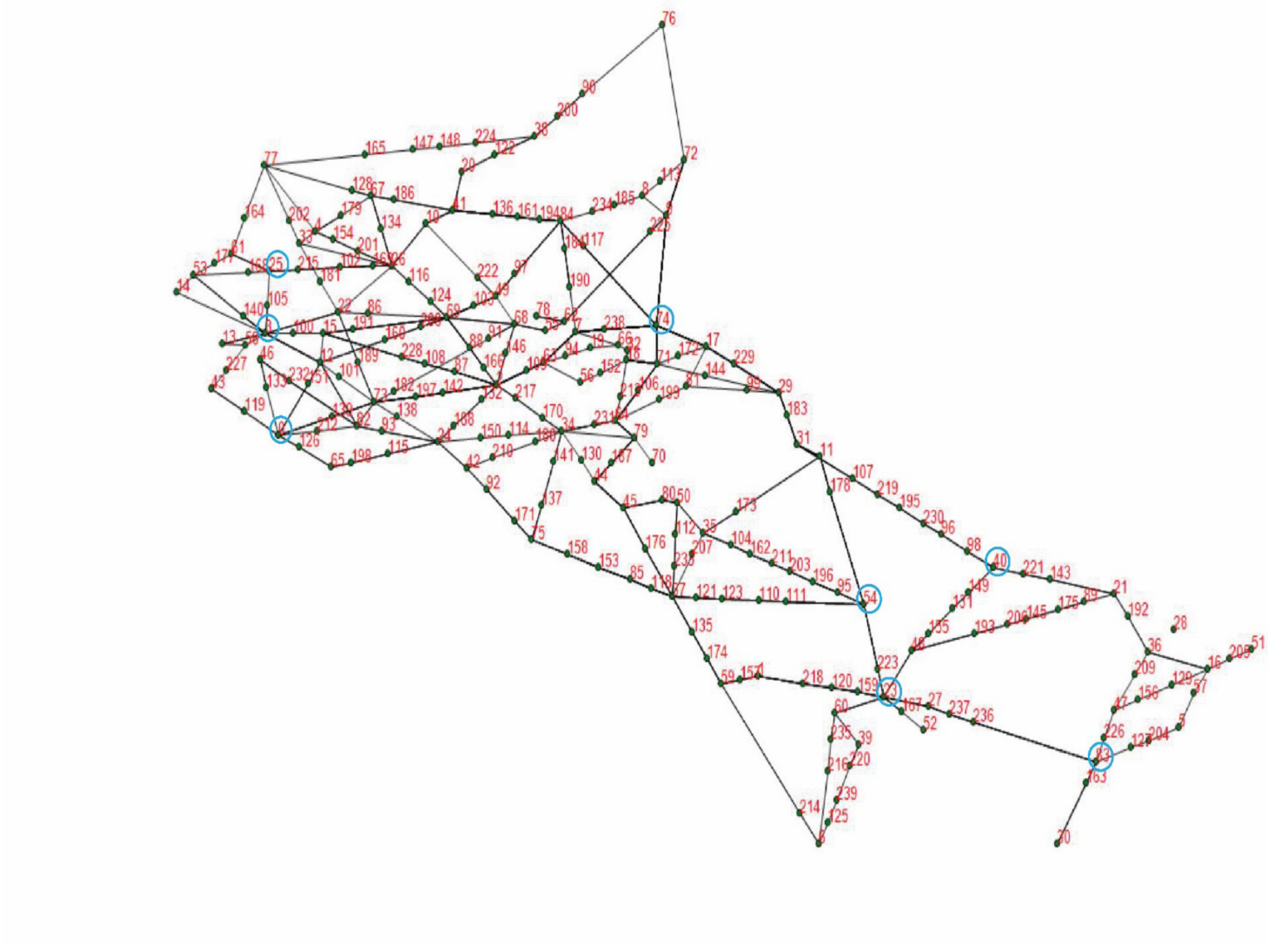

Figure 7. Map of Centers for the constraint 40 routes

The centers are listed in decreasing order according to their demand. So, as the routes in **table 3** can be analyzed and found out that the main center should be actually the **station 74** because almost all buses drive to that station or drive out of that station. The second station is **station 3** with high demand, but it has much lower demand than the station 74. Obviously, the operator could just consider one center at the station 74, but since algorithm found out otherwise, it would be more efficient to have those 8 stations as centers.

5. Conclusion

After all runs were made, it is concluded that the bus networks, which were found in this study are efficient in theory because do not have any route with low frequency. Furthermore, as was mentioned the region, which is examined is Styer, which has the station number 74, almost all buses go from or to that station. In addition, the demand is covered with different levels, which means that the operator can choice the bus network according to his aims. In addition, the transit centers are located within the rules, which mentioned in the transit center's explanation. This was possible thanks to the data provided in this study, which indeed helped to understand how it would be better to build the bus network system not just from the point of view of the operator, but also for the commuters. As a results, the solution obtained here could also be used for further studies when the purpose is to minimize waiting time, frequency, fuel costs. Different constraints can be applied for this study as well as in other studies such as: the total round time for each route can be increased, decreased, the circuity can be ignored when the expansion is applied, different walking distance for commuter to the closest station as well as distance between each station also can be considered.

As in the real world can be seen that most bus networks are built and implemented it does not guarantee the efficiency of the bus network. In the world, there are a lot of bus network systems, which transport a vast number of people. Those networks are changed and re-checked all the time in order to improve the comfort, minimize cost of operators as well as the cost of commuters.

The same procedure should be implemented for this study, just after it is checked in the real world, the efficiency can be measured.

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