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Dott. Flavio Del Santo

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Abstract

Quantum information theory investigates the capabilities of quantum systems for information processing. The counterintuitive features of quantum mechanics, such as superposition principle or quantum entanglement, paved the way to new possibilities which do not have classical counterparts in information theory. In modern times, these genuine quantum effects have been exploited to achieve novel outstanding results, such as quantum computing, quantum cryptography or quantum communication. The field of quantum communication explores the principles of quantum theory in order to accomplish certain tasks and protocols that are impossible by using classical resources. In a general sense, communication is the process of transmitting a message from a sender to a receiver. Such a process usually requires to encode the information in some information carriers (e.g. electromagnetic waves, voltage signals, mechanical waves, etc.), which obey the laws of physics. From that perspective, quantum physics with its counterintuitive laws and principles that do not have corresponding classical counterparts, provides a novel and powerful framework for communication and information processing in general.

The main objective of the present thesis is to investigate a model of communication that is restricted to *limited resources* (as measured by the amount of information carries), and the *finite speed of propagation*. We show that communication fulfilling the mentioned restrictions, is fundamentally limited for classical systems. On the contrary, when quantum effects are allowed, one can surpass the classical limitations. In fact, quantum communication bounded to the exchange of a single particle (in spatial superposition) with finite speed can result in "two-way signalling", which is impossible by using classical resources. In the language of multipartite communication games, we show that the probability of success is always less than the unity, and it monotonically decreases as the number of players grows, for all classical models of communication. On the other hand, communication via a quantum particle that is prepared in superposition of different spatial locations, allows to win the game with certainty, independently of the number of players. The game has been explicitly characterized for two, three and five parties, and generalized to an arbitrary number of players. Moreover, we propose a possible experimental implementation of the three-party game by

using single-photons.

Zusammenfassung

Quanteninformationstheorie untersucht die Möglichkeiten mit Quantensystemen Information zu verarbeiten. Die ungewöhnlichen Eigenschaften der Quantenmechanik, wie beispielsweise Superposition und Verschränkung, erlauben neue Wege der Informationsverarbeitung, die in klassischen Theorien nicht denkbar wären. In den letzten Jahren wurden diese besonderen Quanteneffekte dazu genutzt, um außergewöhnliche Resultate, wie zum Beispiel Quantencomputer, Quantenkryptographie und Quantenkommunikation, zu erreichen. Das Feld der Quantenkommunikation erforscht die Prinzipien der Quantenmechanik, um bestimmte Aufgaben und Protokolle zu realisieren, die mit klassischen Ressourcen unmöglich wären. Im Allgemeinen ist Kommunikation der Prozess bei dem eine Nachricht von einem Sender zu einem Empfänger übermittelt wird. Solch ein Vorgang benötigt die Kodierung der zu sendenden Information auf einem Signalträger (elektromagnetische Wellen, Spannung, mechanische Wellen, etc.), der den Gesetzen der Physik gehorcht. Aus dieser Perspektive ermöglicht die Quantenmechanik mit ihren der Intuition widersprechenden Prinzipien einen völlig neuen Rahmen für Kommunikationstheorie und Informationsverarbeitung. Das Ziel der vorliegenden Arbeit ist es, ein Modell der Kommunikation mit beschränkten Ressourcen (gemessen an der Anzahl an Informationsträgern) und endlicher Ausbreitungsgeschwindigkeit zu untersuchen. Wir zeigen, dass Kommunikation, welche die obigen Restriktionen erfüllt, für klassische Systeme grundlegenden Schranken gehorcht. Im Gegensatz dazu können diese klassischen Barrieren durch Quanteneffekte überwunden werden. Tatsächlich kann Quantenkommunikation mit nur einem Teilchen in räumlicher Superposition und mit endlicher Ausbreitungsgeschwindigkeit zu „Zweiwegkommunikation“ führen, was mit rein klassischen Ressourcen unmöglich wäre. Im Kontext von Mehrspieler Kommunikationsspielen zeigen wir einerseits, dass für klassische Theorien die Gewinnwahrscheinlichkeit immer unter Eins liegt und mit der Anzahl der Spieler monoton sinkt. Andererseits erlaubt Kommunikation mittels eines Quantenteilchens in Superposition verschiedener Aufenthaltsorte das Spiel unabhängig von der Anzahl der Spieler immer mit Sicherheit zu gewinnen. Dieses Protokoll wird explizit für zwei, drei und fünf Parteien charakterisiert und danach für eine beliebige Anzahl an Spielern verallgemeinert. Des Weiteren schlagen wir eine mögliche experimentelle

Implementation mit einzelnen Photonen für ein Spiel mit drei Parteien vor.

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Introduction

Since its systematic formulation in the 1920s, it was clear that quantum mechanics (QM) requires a radically new way of looking at the nature. After a first stage of mere phenomenological explanations of experimental anomalies, since the mid 1920s QM acquired a unique consistent formalism (thanks to the pivotal contributions of physicists the likes of Heisenberg, Born, Jordan and Schrödinger). Nevertheless the interpretation of the formalism remained an open problem and have raised many controversies in the scientific community. Indeed, it took a short time before scientists realized that they were dealing not only with a new theory, but with some of the most involved philosophical problems that physics ever had to face. Crudely speaking, although QM is at present the most corroborated theory ever (in terms of experimental predictions), it breaks some of the *a priori* concepts that have been the foundations of natural science for centuries. In fact, QM seems to be fundamentally incompatible with a deterministic description of Nature, which is assumed in Newtonian and in relativistic mechanics. As a matter of fact, Bell's theorem [1] has ruled out the possibility of completing QM with underlying local deterministic models.

Quantum formalism, allows new peculiar effects - such as the superposition principle or quantum entanglement - which do not have any classical counterpart. Historically, these genuine quantum effects have provided the ground for the formulation of paradoxical examples, which were aimed at finding flaws in quantum mechanics (or in its interpretations). In this regard, the year 1935 marked a turning point for the critics of QM, since two of the most illustrious among them, Erwin Schrödinger and Albert Einstein, proposed two pivotal *gedankenexperimenten* which have changed the conception of foundation of QM thereafter. The former proposed a thought experiment which has gone down in history as the *Schrödinger's cat paradox* [2]. Exploiting one of the quintessential quantum effects, the *quantum superposition principle*, Schrödinger set up a particular hypothetical arrangement, leading to the conclusion that quantum mechanics is paradoxical, since it allows the possibility of preparing states of macroscopic objects, e.g. a cat both dead and alive at the same time, in what is called a coherent quantum superposition. In the same year Einstein, Podolsky and Rosen, put forward a distinguished *gedankenexperiment*, know as the *EPR* *paradox* [3], which aimed at demonstrating the incompleteness of QM, thanks to another genuine quantum effect know as *quantum entanglement*.

An epochal turning point surely is the line of research inaugurated by John Bell in the mid-1960s. Extending the results of EPR, he managed to put forward a theorem [1] capable of experimentally discriminating between (local) 'classical' theories and quantum mechanics. Historically, the research stimulated by Bell's theorem helped much to oppose the widespread pragmatic approach in physics in the postwar era, and to bring back some attentions to the fundamental studies in QM. Foundation of quantum mechanics have effectively contributed to gain a more profound understanding of quantum theory, which eventually allowed to also develop a plethora of new practical applications. In fact, through a long and winding road, the mentioned peculiar quantum effects led to the development of quantum information theory and its practical applications known as quantum technologies [4]. It has been indeed explicitly pointed out that "if the name of a field indicated its parentage, then the 'Quantum' in 'Quantum Information' would refer to Quantum Foundations" [5]. Quantum information theory investigates the capabilities of quantum systems for information processing with a number of innovative possibilities which have no classical counterpart, such as quantum cryptography, quantum communication and quantum computation (see e.g. [6, 7, 8, 9]).

The present work is specifically focused on a theoretical analysis of quantum communication tasks. Quantum communication, mainly explores quantum correlations shared by distant parties (e.g. quantum entanglement) in order to accomplish certain tasks and protocols that are fundamentally forbidden by the mere use of classical resources, such as the violation of Bell's inequality [1] or quantum teleportation [10]. The present work of thesis aims at providing a contribution towards a deeper comprehension of the fundamental and irreconcilable differences between QM and classical theories, by means of communication tasks. In particular we characterize novel communication protocols that are fundamentally forbidden in classical physics.

In chapter 1 we review Bell's theorem and its modern phrasing in terms of quantum games [7, 16]. The main result of this work is exposed in chapter 2, wherein we develop a novel task for quantum communication by exploring quantum superpositions. In fact, a single quantum particle in spatial superposition enables "two-way" signaling, which is essentially forbidden in classical physics. To quantify the discrepancy between the classical and the quantum cases, we formulate the problem as a quantum game played by distant parties, firstly for two, three and five players, and then providing a generalization for N-parties. Chapter 3 is devoted to a detailed description of a possible experimental implementation of the proposed protocol, adapted for single-photon quantum optics experiments. In chapter 4 we discuss briefly the philosophical consequences and relations of the model presented here with the notions of 'local realism' and 'macro-realism'.

Chapter 1

Quantum Vs classical theories: Bell's inequalities

1.1 Local realism and Bell's inequalities

As it was recalled in the introduction, in a distinguished paper entitled ”on the Einstein-Podolsky-Rosen paradox” [1], John Bell derived a theorem, that can be formulated in terms of inequalities, which states that no 'local hidden variables theory' can reproduce the same predictions of QM. Although a proper definition of 'local hidden variables' is imperative, and it will be largely discussed in what follows, on an intuitive level, this can be though as strongly related to the assumptions underlying classical physical theories, such as locality, realism, determinism, etc. Remarkably, such rather philosophical concepts can be put to the experimental test thanks to Bell's inequalities.

In order to derive the pivotal result of Bell's theorem, we follow here a typical operational formulation, like the one recently provided by Brunner *et al.* in [7]. Therein, the authors consider a scenario in which two observers, traditionally called Alice and Bob, are located at two distant (even space-like separated) positions. Alice and Bob are interested in characterizing the correlations of some experiments carried out at their separate local positions. Correlations are usually encoded into physical systems that can be some kind of material objects or particles. Hereinafter we refer to these objects also as *information carriers*. Let us assume that two information carriers are sent one to Alice and the other to Bob, and that they have interacted in past, for instance having been produced by a common source (which might have established correlations between them). Alice and Bob can carry out some measurements on their respective object, in order to measure possible correlations in their results. To this end, Alice chooses to perform a measurement with setting labeled by x, and she finds a certain outcome a. Bob carries out a measurement with setting y, and finds the outcome b. The two observers then repeat their measurements many times, and even when they chose the same initial settings x and y, the outcomes a and b may vary from one run to another (i.e. a deterministic behavior is not assumed in general). After a sufficient number of experimental runs, they can find a good estimation of the probability distribution $p(a,b|x,y)$ which governs the dependence of the outcomes on measurement settings.

Now, since Alice and Bob are very far away and they are freely choosing their local measurement settings, the common sense would suggest that the joint probability of finding a and b given x and y is independent (i.e. factorizable). Nevertheless, one finds in general

$$p(a,b|x,y) \neq p(a|x)\ p(b|y). \tag{1.1}$$

Therefore, it may look like that the local measurement settings somehow statistically influence outcomes of distant experiments. Although this result might seem at first sight astonishing, Brunner and co-authors clarify that, in principle, "the existence of such correlations is nothing mysterious. In particular, it does not necessarily imply some kind of direct influence of one system on the other, for these correlations may simply reveal some dependence relation between the two systems which was established when they interacted in the past" [7]. The absence of direct influences is the operational evidence of the so-called "no-communication" (no-signaling) theorem, which states that (both classical and quantum) correlations cannot be used to communicate information between distant parties.

The observed correlations can in fact be thought as the exterior result of some 'common memory' of the previous interaction, that the two objects have carried along. Indeed, one can restore the local behavior assuming that some *hidden variables* λ take into account all correlations. In such a way, the probabilities in (1.1) must become independent and thus factorize as:

$$p(a,b|x,y,\lambda) = p(a|x,\lambda)p(b|y,\lambda). \tag{1.2}$$

If one is concerned with the nature of the hidden variables λ, John Bell himself stated in his original paper [1] that "it is a matter of indifference in the following whether λ denotes a single variable or a set, or even a set of functions, and whether the variables are discrete or continuous". In any case λ is conventionally treated as a continuous single variable, that might assume different values from one run to another, governed by a probability distribution $q(\lambda)$. Therefore, (1.2) takes the general form

$$p(a,b|x,y) = \int_\Lambda d\lambda q(\lambda)p(a|x,\lambda)p(b|y,\lambda), \tag{1.3}$$

where Λ is the domain of λ.

The expression (1.3) is referred to as condition of *local realism* (LR). Since the existence of such a decomposition was derived under the mere assumption of having local hidden variables that factorize the joint probability distribution. In chapter 4 we will discuss in greater detail the assumption of LR and the possible misunderstanding of the interpretation of this condition, which may lead to tremendous consequences on foundation of physics.

Provided with the condition of LR, the famous result of Bell's theorem, is now a matter of an easy mathematical derivation. As a matter of simplicity we discuss the case where Alice and Bob deal with binary inputs and outputs, i.e. x, $y \in \{0,1\}$and a, $b \in \{-1,+1\}$. The expectation value of ab (correlation) is given by

$$\langle a_x b_y \rangle = \sum_{a,b} ab\ p(a,b|x,y). \tag{1.4}$$

Out of these, one can construct the following quantity

$$S := \langle a_0 b_0 \rangle + \langle a_0 b_1 \rangle + \langle a_1 b_0 \rangle - \langle a_1 b_1 \rangle. \tag{1.5}$$

Now, we insert the condition of LR (1.3) into (1.4), and obtain

$$\langle a_x b_y \rangle = \int_\Lambda d\lambda q(\lambda) ab \sum_a p(a|x,\lambda) \sum_b p(b|y,\lambda) = \int_\Lambda d\lambda q(\lambda) \langle a_x \rangle_\lambda \langle b_y \rangle_\lambda, \tag{1.6}$$

where $\langle a_x \rangle_\lambda = \sum_a a\ p(a|x,\lambda)$ and $\langle b_y \rangle_\lambda = \sum_b b\ p(b|y,\lambda)$ are the local expectation values. This yields to

$$S = \int_\Lambda d\lambda q(\lambda) \Big(\langle a_0 \rangle_\lambda \langle b_0 \rangle_\lambda + \langle a_0 \rangle_\lambda \langle b_1 \rangle_\lambda + \langle a_1 \rangle_\lambda \langle b_0 \rangle_\lambda - \langle a_1 \rangle_\lambda \langle b_1 \rangle_\lambda \Big). \tag{1.7}$$

Since $\langle a_0 \rangle_\lambda$,$\langle a_1 \rangle_\lambda$,$\langle b_0 \rangle_\lambda$,$\langle b_1 \rangle_\lambda \in [-1,1]$, one can easily prove that the quantity S is bounded by

$$S_{(LR)} = \langle a_0 b_0 \rangle + \langle a_0 b_1 \rangle + \langle a_1 b_0 \rangle - \langle a_1 b_1 \rangle \leq 2. \tag{1.8}$$

The subscript (LR) denotes that the condition of local realism has been enforced to derive this inequality.

The results (1.8) is known as the Clauser-Horne-Shimony-Holt inequality (CHSH) [11], named after the physicists who firstly derived it in 1969, and it constitutes the easiest non-trivial Bell's inequality. Such inequalities have a pivotal importance because they provide a quantitative and experimentally testable bound for any local-realistic theory (in the sense of (1.3)).

We turn now to quantum mechanics. This theory allows the preparation of states which are called *entangled* or *non-separable*.[1] Let us consider that the same distant observers Alice and Bob now share a maximally entangled (bipartite)

[1]Quantum mechanics postulates that a joint state of two systems A and B lives in the tensor product of the two systems, i.e. $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$. A pure state $|\psi\rangle_{AB} \in \mathcal{H}_{AB}$ is then defined to be separable if $|\psi\rangle_{AB} = |\psi\rangle_A \otimes |\psi\rangle_B$. A non-separable state is referred to as entangled.

state, say the following *singlet* state

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B\right), \tag{1.9}$$

where $|0\rangle$ and $|1\rangle$ are the eigenstates of the standard Pauli z-matrix σ_z associated to the eigenvalues +1 and -1, respectively. As before, we assume binary outcomes $a, b \in \{-1, 1\}$, meaning that A and B measure dichotomic oservables. In this case, we associate to the measurement settings x, y the three dimensional vectors $\vec{x}$, $\vec{y}$ (this is called the Bloch representation for two-level systems, see e.g. [12]), which denote the measurement direction. In such representation the measured observables are represented by $\vec{x} \cdot \vec{\sigma}$ for Alice and $\vec{y} \cdot \vec{\sigma}$ for Bob. It is possible to show that correlation, given the choices x and y, reads [1]

$$\langle a_x b_y \rangle = -\vec{x} \cdot \vec{y}. \tag{1.10}$$

We choose $x = 0, 1$ to label the measurements in two orthogonal directions $\hat{e}_1$, $\hat{e}_2$, whereas $y = 0, 1$ labels the measurement directions $-(\hat{e}_1 + \hat{e}_2)/\sqrt{2}$ and $(-\hat{e}_1 + \hat{e}_2)/\sqrt{2}$, respectively. For this particular choice of measurement settings, we get the following correlations: $\langle a_0 b_0 \rangle = \langle a_0 b_1 \rangle = \langle a_1 b_0 \rangle = 1/\sqrt{2}$ and $\langle a_1 b_1 \rangle = -1/\sqrt{2}$. Plugging these values into the definition of S (1.5), one finds

$$S_{(Q)} = 2\sqrt{2} > 2 = S_{(LR)}, \tag{1.11}$$

where the subscript (Q) refers to quantum theory.

This is the second crucial result of Bell's inequalities: the quantum formalism violates (for certain combinations of states and measurements) the predictions of local realism (1.3). This result is sometimes referred to as Bell's theorem.

1.2 'Non-local' games

In modern quantum information theory, Bell's inequalities are often rephrased in the more intuitive terms of 'non-local' games [7, 14, 15, 16, 17]. In the framework of 'non-local' games, the usual 'observers' are referred to as 'players'. An outside party, called the *referee*, provides each player with an input and challenges the players to cooperate in order to answer some questions related to the assigned inputs. Thus, each player, fully aware of the rules, returns an answer. The referee then checks whether the answers are correct, according to some expression called 'predicate' [7], specified by the rules of the game. The players win the game only if their answers fulfill the predicate. Moreover, players can previously agree on any possible strategy, in order to optimize their results. However, once the game starts, they cannot longer communicate (this ensures to test the strength of correlations between distant parties).

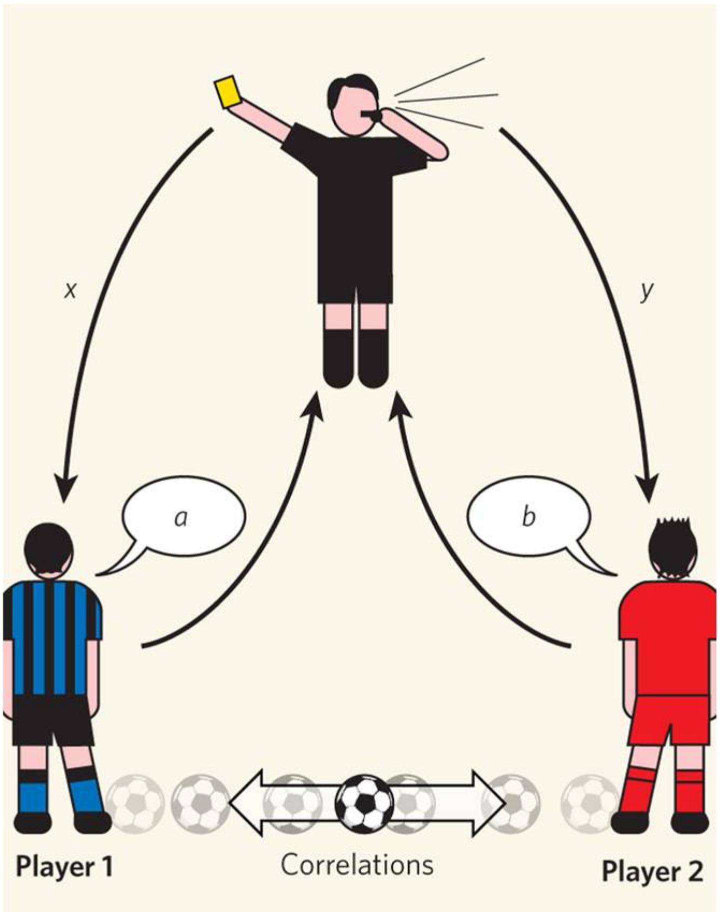

Figure 1.1: Bell's inequalities can be phrased in terms of quantum games. This figure is reproduced from [16].

The easiest (non-trivial) version of a 'non-local' game (Fig. 1.1) involves two players, Alice and Bob, who are given binary inputs, $x, y \in \{0, 1\}$ and they shall return binary answers to the referee, i.e. $a, b \in \{0, 1\}$. Let us consider the following predicate that A and B are supposed to fulfill

$$x \cdot y = a \oplus b, \tag{1.12}$$

where the symbol $\oplus$ denotes the sum modulo-2. This is called the CHSH game, the easiest and most famous example of a class of games called XOR games [7], because the predicate (1.12) can be phrased in terms of logic operators as XOR($answers$) = AND($inputs$). This class of games has been fully analyzed also for an arbitrary number of players [14, 15].

In the CHSH game, one finds the following values for each possible pair of inputs and answers:

inputs			answers		
x	y	$x \cdot y$	a	b	$a \oplus b$
0	0	0	0	0	0
0	1	0	0	1	1
1	0	0	1	0	1
1	1	1	1	1	0

Out of 16 combinations of inputs and answers, the game is won only for combinations which satisfy the predicate (1.12). If the settings are uniformly and randomly distributed, we get the following expression for the win probability

$$\begin{aligned} P_{win} &= \frac{1}{4}\, p(a \oplus b = 0|x = 0, y = 0) + \frac{1}{4}\, p(a \oplus b = 0|x = 0, y = 1) + \\ &+ \frac{1}{4}\, p(a \oplus b = 0|x = 1, y = 0) + \frac{1}{4}\, p(a \oplus b = 1|x = 1, y = 1). \end{aligned} \tag{1.13}$$

Writing explicitly the probabilities appearing in (1.13), one finds

$$p(a \oplus b = 0|x = 0, y = 0) = p(a = 0, b = 0|x = 0, y = 0) + p(a = 1, b = 1|x = 0, y = 0), \tag{1.14}$$

$$p(a \oplus b = 0|x = 0, y = 1) = p(a = 0, b = 0|x = 0, y = 0) + p(a = 1, b = 1|x = 0, y = 0), \tag{1.15}$$

$$p(a \oplus b = 0|x = 1, y = 0) = p(a = 0, b = 0|x = 0, y = 0) + p(a = 1, b = 1|x = 0, y = 0), \tag{1.16}$$

$$p(a \oplus b = 1|x = 1, y = 1) = p(a = 0, b = 1|x = 0, y = 0) + p(a = 1, b = 0|x = 0, y = 0). \tag{1.17}$$

We now define the correlations C_{xy} between inputs x and y

$$\begin{aligned} C_{xy} &:= p(a = 0, b = 0|x, y) - p(a = 0, b = 1|x, y) + \\ &- p(a = 1, b = 0|x, y) + p(a = 1, b = 1|x, y). \end{aligned} \tag{1.18}$$

Moreover the probabilities have to be normalized, i.e.

$$\sum_{a,b=0}^{1} p(a,b|x,y) = 1 \tag{1.19}$$

Inserting the normalization condition together with the expressions for C_{xy}, the four terms of (1.13) then read

$$p(a \oplus b = 0|x = 0, y = 0) = \frac{1}{2}(1 + C_{00}), \tag{1.20}$$

$$p(a \oplus b = 0|x = 0, y = 1) = \frac{1}{2}(1 + C_{01}), \tag{1.21}$$

$$p(a \oplus b = 0|x = 1, y = 0) = \frac{1}{2}(1 + C_{10}), \tag{1.22}$$

$$p(a \oplus b = 1|x = 1, y = 1) = \frac{1}{2}(1 - C_{11}). \tag{1.23}$$

Thus, the probability of winning becomes

$$P_{win} = \frac{1}{2} + \frac{1}{8}\left(C_{00} + C_{01} + C_{10} - C_{11}\right). \tag{1.24}$$

The quantity in parenthesis in (1.24) is nothing else than S (as defined in (1.5). We already know from the last section that, if the condition of local realism is enforced, $S_{(LR)} = 2$ at most. Hence, the probability of winning the game is upper-bounded by

$$P_{win}^{(LR)} = \frac{3}{4}. \tag{1.25}$$

In contrast, in a quantum scenario, S can violate this bound and take the values up to $S_{(Q)} = 2\sqrt{2}$. Consequently, the game can be won with a probability as high as

$$P_{win}^{(Q)} = \frac{1}{2}(1 + \frac{1}{\sqrt{2}}) \approx 85\%. \tag{1.26}$$

Chapter 2

Quantum communication with limited resources

Generally speaking, communication is the process of transmitting a message (information) from a sender to a receiver [18]. We usually think of sending a physical information, i.e. a message embodied in a (physical) information carrier, and send it as a signal, such as voltage signals, speech, video or radar. In the classical world, physical systems that carry the information obey the laws of classical physics. For example, the electromagnetic signal propagates in space according to Maxwell's equations, thus the speed of information transfer is fundamentally limited to that of light. Similarly, in radio communications, we know that information flows from a radio emitter to the radio receiver but not vice-versa, as it follows from the causality principle. In other words, the capabilities and limitations of communication, and information processing in general, are governed by the laws of physics. Quantum theory allows for novel possibilities that can surpass some of the limitations imposed by classical physics.

In the present chapter, we investigate a model of communication that is restricted to the following fundamental assumptions:

A1 - ***Limited resources.*** The amount of information carriers is limited.

A2 - ***Finite speed of propagation.*** Information carriers travel with finite velocity (upper-bounded by that of light).

Our goal is here to show that the model suffers from fundamental limitations when restricted to classical systems. On the other hand, quantum mechanics allows for a novel possibility, i.e. to put the particles in superposition of spatially distinct locations, and we shall prove here that this is a more powerful resource for communication (as compared to its classical counterpart). In particular, we show that communication restricted to the exchange of a single quantum particle that is coherently distributed at different spatial locations can result in a two-way signaling, which is essentially impossible by using classical resources. Based on the model, we introduce a simple game played by distant players that are supposed to accomplish certain tasks by exchanging (limited) communication. Our aim is to investigate the fundamental difference between quantum and classical scenario, showing that, for certain tasks, the win probability is bounded and strictly lower than 1 for all classical strategies. In contrast, quantum information carriers in (spatial) superpositions enable the players to accomplish the task with certainty. Unlike many quantum information protocols based on entanglement and quantum correlations between different parties, our task involves only a single quantum particle and it is based solely on the superposition principle. In this respect our findings are similar to the recent proposals exploring quantum superpositions for information processing purposes, such as quantum processes without a defined causal order [19], superposition of orders [20] and directions [21], quantum combs [22], quantum switch [23] and quantum causal models [24]. Some of these novel phenomena have been demonstrated in recent experiments [25, 26].

In the present chapter, we define and analyze in detail a bipartite game, and extend the result to an increasing number of players. Finally, we provide a generalization of it for an arbitrarily large number of players (also in the limit of very many of them).

2.1 "Two-way signaling" with a single quantum particle

Consider a classical model of communication where two agents, say Alice and Berta,[1] are located at the distance d from each other and they are allowed to communicate via single information carrier. Here as carrier, we think of a particle or object that can travel with the finite speed bounded by c. For example, if Alice holds the particle, she can imprint the message in it and send it to Berta. The message needs at least d/c time to arrive at Berta's side. We assume that the communication channel is open for a certain time window of $d/c \leq \tau < 2d/c + \epsilon$, where $\epsilon \geq 0$ is a small constant, such that the particle has enough time to arrive at Berta's side, but not to come back to Alice. This is what we mean when we refer to limited communication, i.e. within the time window τ, A and B can exchange only "one-way" communication. Before they are assigned the inputs, Alice and Berta, fully aware of the rules, are granted an *initialization phase*, in which they are allowed to communicate without restrictions and agree on any strategy to be adopted in the subsequent game .Their task is to answer a certain question imposed by referee as good as possible, i.e. with the maximal probability

[1] The name of the second player has here been changed from the traditional Bob to Berta, in honor of Berta Karlik, the physicist who has been the first woman to become full professor at the University of Vienna (and in the whole Austria), as late as 1956 [28].

of success. Therefore such a task is to be thought as probabilistic (the imposed questions and answers may vary from run to run).

At the time $t = 0$, A and B are respectively given the input variables x and y by the referee and they are asked to return the output variables a and b at the later time $t = \tau$. If we represent the communication in the space-time diagram (see Fig. 2.1.a), it is clear that there are only two possible options, i.e. if the particle was in possession of Alice at $t = 0$, she can encode her input in it and send it to Berta, but she gets no information on Berta's input, or vice-versa. In the formalism of causal diagrams [24], there are two possible causal relations between the variables x, y, a, b (see Fig. 2.1.b), i.e. either x influences a and b, whereas y influences b only or y influences a and b, whereas x influences a only. Therefore, the probability distribution $p(ab|xy)$ is a classical mixture of one-way signaling distributions, i.e.

$$p(ab|xy) = \lambda p_A(a|x) p_{A \prec B}(b|xy) + (1-\lambda) p_B(b|y) p_{B \prec A}(a|xy), \tag{2.1}$$

where symbol $\prec$ denotes the direction of signaling, e.g. $A \prec B$ denotes the case of A sending her particle to B. The probability distribution (2.1) is completely characterized by a the "so-called" classical polytope [7], and its facets are represented by the Bell's-like inequalities which impose the limits on classical model. For the case of binary inputs $x, y = 0, 1$ and binary outputs $a, b = 0, 1$, there are only two inequivalent inequalities [27]

$$p(a = y, b = x) \leq \frac{1}{2}, \tag{2.2}$$

$$p(x(a \oplus y) = y(b \oplus x) = 0) \leq \frac{3}{4}, \tag{2.3}$$

known as two variants of "guess your neighbor's input" (GYNI) game [17], i.e. the players are supposed to guess the inputs of their partners. Here the inputs x and y are uniformly distributed, i.e. $p(x, y) = 1/4$. We will focus on (2.2) which when translated into the language of communication game results in the requirement of computing the neighbor's input, i.e. for given inputs x and y, A and B are asked to reveal the input of their partners. Formally speaking, they win the game when their outcomes satisfy $a = y$ and $b = x$. Clearly, in classical scenario, the probability of success is bounded by (2.2).

In contrast to the Bell scenarios (see Chapter 1), the parties are allowed to exchange communication during the game, which however is restricted by the abovementioned assumptions A1-A2. Moreover, the game can be played either in a classical or quantum scenario, depending on whether the information carrier is granted the possibility to exploit quantum effects (superposition principle, specifically). In either case, the players shall adopt the optimal strategy to answer referee's question about the value assigned to the other player's input. The

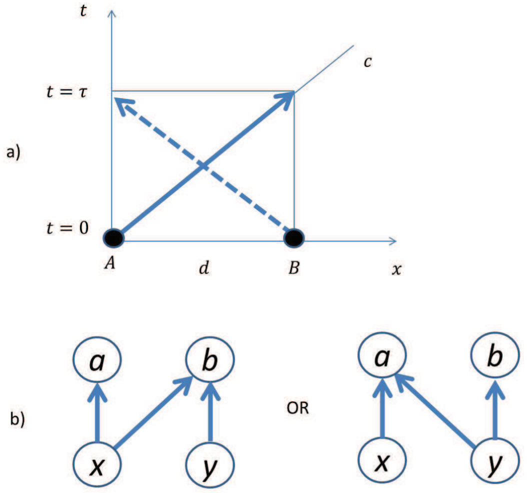

Figure 2.1: **a) Space-time diagram.** Within the time window τ either A signals to B or B to A. **b) Causal diagram.** The variables x and y are in common past of a and b. There are two possible causal relations between the variables, i.e. either x influences a and b, whereas y influences b only (left) or y influences a and b, whereas x influences a only (right).

question then arises: is it possible to perform such a task in a more efficient way by means of the use of spatial quantum superpositions?

In the quantum scenario, the single particle used in the process of communication is not spatially localized within either of the two parties, but it is prepared in a equally weighted quantum superposition between A and B, i.e. $\frac{1}{\sqrt{2}}(|A\rangle + |B\rangle)$. For the sake of simplicity, we will use the second quantization formalism (see section 2.5) and write the initial state as

$$|\psi\rangle_{in} = \frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B) = \frac{1}{\sqrt{2}}(\hat{a}^\dagger + \hat{b}^\dagger)|0\rangle_A|0\rangle_B, \tag{2.4}$$

where, for example, $|1\rangle_A|0\rangle_B$ indicates that particle is localized with Alice, whereas Bob has zero (vacuum) particles in possession. The operators $\hat{a}^\dagger$ and $\hat{b}^\dagger$ are the standard ladder operators that create a particle on A and B side, respectively. Note that ladder operators are used here just for convenience reasons, and as long as we are dealing with a single particle, our results are completely independent of the distinguishability property or type of particle used.

Now let us imagine to interpose a unitary device right in the middle between Alice's and Berta's respective positions (at the distance $d/2$). The device acts such that if the particle is sent from A to B it is "half-reflected" and "half-transmitted" in a coherent way (see in Fig. 2.2). A similar situation is found if the particle is sent from B to A. The unitary device serves as a communication channel, and it can be realized in practice by putting a simple potential barrier for material particles or a 50:50 beam splitter for the case of single-photons. Generally, we shall refer to such a device simply as beam splitter (BS). A beam

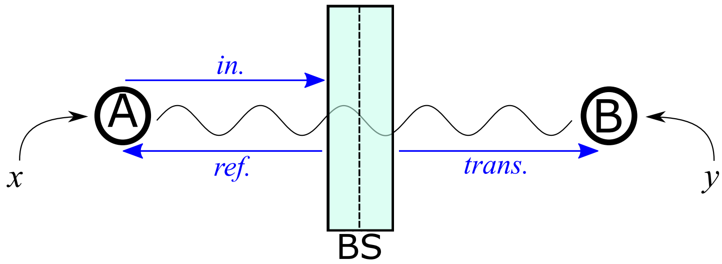

Figure 2.2: Two players Alice (A) and Berta (B) both receive a binary input x and y. A beam-splitter (BS) is interposed halfway between the parties. If the particle was initially localized, say with A (*in.*), it gets "half-reflected" (*ref.*) and "half-transmitted" (*trans.*), represented by blue arrows. However, if when the particle is superposed between both the locations A and B (wavy line), the particle will end up deterministically localized in either positions A or B.

splitter is mathematically represented by the 2×2 unitary operator:

$$BS = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \tag{2.5}$$

In quantum information theory this operator represents the *Hadamard gate* [9], which plays an important role in quantum computing. Formally speaking, such an operator represents the two-dimensional discrete *Fourier matrix*, $\mathscr{F}_2$ (for an insight on Fourier matrices see e.g.[57]). Therefore we can completely identify BS with $\mathscr{F}_2$.

The operator $\mathscr{F}_2$, when applied to single-particle states, provides the following transformations:

$$|1\rangle_A|0\rangle_B \xrightarrow{\mathscr{F}_2} \frac{1}{\sqrt{2}}(|1\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B), \tag{2.6}$$

$$|0\rangle_A|1\rangle_B \xrightarrow{\mathscr{F}_2} \frac{1}{\sqrt{2}}(|1\rangle_A|0\rangle_B - |0\rangle_A|1\rangle_B). \tag{2.7}$$

It is particularly convenient to explicitly write transformation rules for ladder operators under the action of $\mathscr{F}_2$:

$$\hat{a}^\dagger \xrightarrow{\mathscr{F}_2} \frac{1}{\sqrt{2}}(\hat{a}^\dagger + \hat{b}^\dagger), \tag{2.8}$$

$$\hat{b}^\dagger \xrightarrow{\mathscr{F}_2} \frac{1}{\sqrt{2}}(\hat{a}^\dagger - \hat{b}^\dagger). \tag{2.9}$$

We define the operator T as the inverse of $\mathscr{F}_2$,

$$T := \mathscr{F}_2{}^{-1},$$

with the corresponding transformation for ladder operators

$$\frac{1}{\sqrt{2}}(\hat{a}^\dagger + \hat{b}^\dagger) \xrightarrow{T} \hat{a}^\dagger, \tag{2.10}$$

$$\frac{1}{\sqrt{2}}(\hat{a}^\dagger - \hat{b}^\dagger) \xrightarrow{T} \hat{b}^\dagger. \tag{2.11}$$

It ought to be remarked that the two-dimensional Fourier matrix is involutory, i.e. it is equal to its own inverse, $\mathscr{F}_2 = \mathscr{F}_2^{-1}$.

Let us now turn to the operations that Alice and Berta are locally carrying out after they have received their respective inputs. In fact, assume that they have previously agreed to encode their inputs by adding a local phase to the state, in the following way:

$$\hat{a}^\dagger \to (-1)^x \hat{a}^\dagger, \tag{2.12}$$

$$\hat{b}^\dagger \to (-1)^y \hat{b}^\dagger. \tag{2.13}$$

Since binary inputs have been assigned to both players, four different cases can occur:

x	y	encoding inputs	state of the system with inputs encoded
0	0	$\xrightarrow{(x,y)}$	$\lvert\psi\rangle^{00} = \frac{1}{\sqrt{2}}(\hat{a}^\dagger + \hat{b}^\dagger)\lvert 0\rangle_A \lvert 0\rangle_B$
0	1	$\xrightarrow{(x,y}$	$\lvert\psi\rangle^{01} = \frac{1}{\sqrt{2}}(\hat{a}^\dagger - \hat{b}^\dagger)\lvert 0\rangle_A \lvert 0\rangle_B$
1	0	$\xrightarrow{(x,y)}$	$\lvert\psi\rangle^{10} = \frac{1}{\sqrt{2}}(-\hat{a}^\dagger + \hat{b}^\dagger)\lvert 0\rangle_A \lvert 0\rangle_B$
1	1	$\xrightarrow{(x,y)}$	$\lvert\psi\rangle^{11} = \frac{1}{\sqrt{2}}(-\hat{a}^\dagger - \hat{b}^\dagger)\lvert 0\rangle_A \lvert 0\rangle_B$

Here $|\psi\rangle^{xy}$, refers to the state of the system after encoding the inputs x and y by means of (2.12), (2.13).

Now the players exchange communication, by sending their respective 'part of particle' to the partner. The constrain of limited communication A1 is satisfied, since at any instant during the game only one particle is present. The referee can even make a random inspection, by measuring the number of particles and their position. Clearly, he shall always find a single particle only.

Let us now focus on the action of BS. During the communication exchange the state of the system $|\psi\rangle^{xy}$, is transformed under the action of beam splitter: $BS\,[|\psi\rangle^{xy}] = T\,[|\psi\rangle^{xy}]$. There are four different states $|\psi\rangle^{xy}$ and they transform in the following way

$$T\left[|\psi\rangle^{00}\right] = T\left[\frac{1}{\sqrt{2}}(\hat{a}^\dagger + \hat{b}^\dagger)|0\rangle_A|0\rangle_B\right] = \hat{a}^\dagger|0\rangle_A|0\rangle_B = |1\rangle_A|0\rangle_B, \tag{2.14}$$

$$T\left[|\psi\rangle^{01}\right] = T\left[\frac{1}{\sqrt{2}}(\hat{a}^\dagger - \hat{b}^\dagger)|0\rangle_A|0\rangle_B\right] = \hat{b}^\dagger|0\rangle_A|0\rangle_B = |0\rangle_A|1\rangle_B, \tag{2.15}$$

$$T\left[|\psi\rangle^{10}\right] = T\left[\frac{1}{\sqrt{2}}(-\hat{a}^\dagger + \hat{b}^\dagger)|0\rangle_A|0\rangle_B\right] = -\hat{b}^\dagger|0\rangle_A|0\rangle_B = -|0\rangle_A|1\rangle_B, \tag{2.16}$$

$$T\left[|\psi\rangle^{11}\right] = T\left[\frac{1}{\sqrt{2}}(-\hat{a}^\dagger - \hat{b}^\dagger)|0\rangle_A|0\rangle_B\right] = -\hat{a}^\dagger|0\rangle_A|0\rangle_B = -|1\rangle_A|0\rangle_B. \tag{2.17}$$

At the final stage, players can locally detect the presence of the particle, and in any case they find two possible outcomes: either Alice or Berta has the particle. In fact, (2.14) is equal to (2.17) up to a global phase and the same holds for (2.15) and (2.16). Alice and Berta can now extract the parity of the inputs, $s := x \oplus y$, where the symbol $\oplus$ denotes the sum modulo-2. More precisely, if the particle is to be found with Alice, the initial inputs could be either $x = y = 0$ or $x = y = 1$, corresponding to parity $s = 0$. On the contrary, if the particle is eventually localized with Berta, the initial inputs could only be either $x = 0, y = 1$ or $x = 1, y = 0$, with the corresponding $s = 1$. Therefore, the players can deterministically extract the value of parity, and from there, they can easily extract the value of the neighbor's input, i.e. $a = p \oplus x = y$ for A, and $b = p \oplus y = x$ for B. Thus the players are able to answer the request of the referee with certainty, i.e. with probability $P_Q = 1$. The protocol is summarized in Table 2.1.

inputs		parity	final state	measurement
x	y	$s(x,y)$	$\|\psi\rangle_f^{xy}$	
0	0	0	$\|1\rangle_A\|0\rangle_B$	particle with A
1	1		$-\|1\rangle_A\|0\rangle_B$	
0	1	1	$\|0\rangle_A\|1\rangle_B$	particle with B
1	0		$-\|0\rangle_A\|1\rangle_B$	

Table 2.1: **Summary of the two-party quantum game**. A referee assigns a binary input, x (y) to player A (B). After encoding their inputs via local operations and exchanging a single particle, the players are able to guess the parity of the inputs with certainty, only by means of local measurements.

2.2 Three-party game

In the present section we extend the result found in previous section to the case of three parties. The same set of assumptions A1,A2, as defined in section 2, are in force. As before, the players are granted to exchange an unlimited amount of communication and adopt an optimal strategy during the *initialization phase*, and then restrict to limited communication via a single particle. Nonetheless, the scope of the game needs to be slightly revised in order to show the quantum advantage for communication.

Quantum scenario

The game involves now three players, A, B and C, who are spatially separated and disposed on the vertices of an equilateral triangle as displayed in Fig. 2.3. The distance between a vertex and the geometrical center of the triangle is set to $d/2$. As before, the speed of particle is bounded by c, and time window within which the exchange of communication is allowed is set to $d/c \leq \tau \leq d/c + \epsilon$. So, let us assume a single particle prepared in an equally weighted superposition

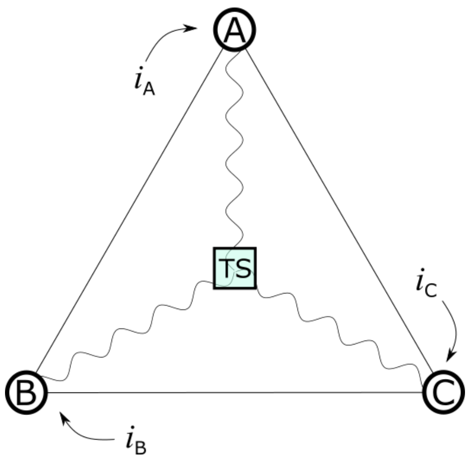

Figure 2.3: Three players, arranged on the vertices of a equilateral triangle, receive an initial *input string* (i_A, i_B, i_C), and share a single quantum particle prepared in spatial superposition (wavy line). A unitary device (*tri-splitter*, TS) is placed in the geometrical center of the triangle.

between A, B and C. Such a superposition is graphically represented in Fig. 2.3

as a wavy line. This initial state reads:

$$\begin{aligned}|\psi\rangle_{in} &= \frac{1}{\sqrt{3}}(|0\rangle_A|0\rangle_B|1\rangle_C + |0\rangle_A|1\rangle_B|0\rangle_C + |1\rangle_A|0\rangle_B|0\rangle_C) \\ &= \frac{1}{\sqrt{3}}(a^\dagger + b^\dagger + c^\dagger)|0\rangle_A|0\rangle_B|0\rangle_C, \end{aligned} \tag{2.18}$$

where $a^\dagger$, $b^\dagger$ and $c^\dagger$ are the standard creation operators acting on modes associated to the spatial locations of A, B, and C, respectively. As a matter of convenience, hereinafter we identify a single-particle state with just the linear combination of ladder operators:

$$|\psi\rangle_{in} = \frac{1}{\sqrt{3}}(a^\dagger + b^\dagger + c^\dagger)|0\rangle_A|0\rangle_B|0\rangle_C \equiv \frac{1}{\sqrt{3}}(a^\dagger + b^\dagger + c^\dagger). \tag{2.19}$$

This is just a formal identification and the real state can be recovered by applying the corresponding combination of ladder operators to the vacuum state. We can now define a unitary operator which is going to play the analogous role of BS (defined in (2.5)) for the two-party game. In the case of three parties, we shall use the three-dimensional Fourier matrix:

$$\mathscr{F}_3 = \frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega\end{bmatrix}. \tag{2.20}$$

with $\omega = e^{2\pi i/3}$. The corresponding transformation rules for ladder operators read

$$a^\dagger \xrightarrow{\mathscr{F}_3} \frac{1}{\sqrt{3}}(a^\dagger + b^\dagger + c^\dagger), \tag{2.21}$$

$$b^\dagger \xrightarrow{\mathscr{F}_3} \frac{1}{\sqrt{3}}(a^\dagger + \omega b^\dagger + \omega^2 c^\dagger), \tag{2.22}$$

$$c^\dagger \xrightarrow{\mathscr{F}_3} \frac{1}{\sqrt{3}}(a^\dagger + \omega^2 b^\dagger + \omega c^\dagger). \tag{2.23}$$

Similarly to the two-party game, we define TS as the inverse of $\mathscr{F}_3$,

$$TS := \mathscr{F}_3{}^{-1}. \tag{2.24}$$

It is straightforward to verify that:

$$\frac{1}{\sqrt{3}}(a^\dagger + b^\dagger + c^\dagger) \xrightarrow{TS} a^\dagger, \tag{2.25}$$

$$\frac{1}{\sqrt{3}}(a^\dagger + \omega b^\dagger + \omega^2 c^\dagger) \xrightarrow{TS} b^\dagger, \tag{2.26}$$

$$\frac{1}{\sqrt{3}}(a^\dagger + \omega^2 b^\dagger + \omega c^\dagger) \xrightarrow{TS} c^\dagger. \tag{2.27}$$

The operator TS, is called *tri-splitter* (or *tritter* [40]), because it splits a beam in three equiprobable paths. Chapter 3 is devoted to a detailed analysis of the possible experimental realization of TS, inclusive of the required geometrical arrangement. The tritter TS is placed in the geometrical center of the triangle with vertices A, B, C (see Fig. 2.3).

Similarly to the bipartite case, the main task for players is to guess the correct answer to some referee's requests, that shall be directly related to the other players' inputs (we will introduce a generalization of GYNI game defined in previous section). Nonetheless, we postpone the explicit exposition of game rules to the end of the section, since it is worthwhile to discuss some features of the present arrangement firstly.

The game starts with the referee assigning a random input to each of the players. The inputs, $i_A, i_B, i_C \in \{0, 1, 2\}$, are labeled with the corresponding letter of the player to whom they are assigned. We call *input string* the ordered triplet $X = (i_A, i_B, i_C)$. Clearly, the total number of possible input strings is $3^3 = 27$. Assume now that the players encode their inputs by applying the following local phase to the input state $|\psi\rangle_{in}$, i.e.

$$a^\dagger \to \omega^{i_A} a^\dagger, \tag{2.28}$$

$$b^\dagger \to \omega^{i_B} b^\dagger, \tag{2.29}$$

$$c^\dagger \to \omega^{i_C} c^\dagger. \tag{2.30}$$

There are in total 27 possible states $|\psi\rangle^{i_A i_B i_C}$ after the encoding transformation. However, only three states can be perfectly distinguished in a single-shot experiment (in general, for a d-level quantum system, only d orthogonal states are perfectly distinguishable). Therefore, we cannot expect to distinguish all 27 states after encoding, hence an additional restriction is needed:

Constrain on the inputs (CI) for three-party game. The referee assigns an input string $X = (i_A, i_B, i_C)$, by randomly choosing (with equal probability) among the following nine possibilities:
$\{(0, 0, 0), (1, 1, 1), (2, 2, 2), (0, 1, 2), (1, 2, 0), (2, 0, 1), (0, 2, 1), (1, 0, 2), (2, 1, 0)\}$.

The introduction of such a restriction is necessary to enable the players to distinguish the final states with certainty. Indeed, if the CI is enforced, there are nine possible states after encoding and we group then into three different classes:

i_A	i_B	i_C	encoding inputs	state of the system with inputs encoded $\|\psi\rangle^{i_A i_B i_C}$
0	0	0		$\frac{1}{\sqrt{3}}(a^\dagger + b^\dagger + c^\dagger)$
1	1	1		$\frac{\omega}{\sqrt{3}}(a^\dagger + b^\dagger + c^\dagger)$
2	2	2		$\frac{\omega^2}{\sqrt{3}}(a^\dagger + b^\dagger + c^\dagger)$
0	1	2		$\frac{1}{\sqrt{3}}(a^\dagger + \omega b^\dagger + \omega^2 c^\dagger)$
1	2	0	$\xrightarrow{(i_A,i_B,i_C)}$	$\frac{\omega}{\sqrt{3}}(a^\dagger + \omega b^\dagger + \omega^2 c^\dagger)$
2	0	1		$\frac{\omega^2}{\sqrt{3}}(a^\dagger + \omega b^\dagger + \omega^2 c^\dagger)$
0	2	1		$\frac{1}{\sqrt{3}}(a^\dagger + \omega^2 b^\dagger + \omega c^\dagger)$
1	0	2		$\frac{\omega}{\sqrt{3}}(a^\dagger + \omega^2 b^\dagger + \omega c^\dagger)$
2	1	0		$\frac{\omega^2}{\sqrt{3}}(a^\dagger + \omega^2 b^\dagger + \omega c^\dagger)$

The first state in each class (i.e. $|\psi\rangle^{000}$, $|\psi\rangle^{012}$ and $|\psi\rangle^{021}$) corresponds exactly to the ones on the left-hand side of the transformation relations (2.25)-(2.27), and they are perfectly distinguishable (orthogonal states). The states in the second and third row are instead equal to the one in first row up-to a global phase (ω and ω^2 for the second and third states, respectively). This means that, only the states from different classes can be distinguished perfectly (the classes are mutually orthogonal). After encoding, the parties then send their 'parts of particle' to TS. And the application of TS on $|\psi\rangle^{i_A i_B i_C}$ is straightforward, since it is directly given by equations (2.25)-(2.27). Therefore, after the transformation, there are nine possible final states:

$$TS\left[|\psi\rangle^{000}\right] = TS\left[\frac{1}{\sqrt{3}}(a^\dagger + b^\dagger + c^\dagger)\right] = a^\dagger, \tag{2.31}$$

$$TS\left[|\psi\rangle^{111}\right] = \omega \cdot TS\left[|\psi\rangle^{000}\right] = \omega a^\dagger, \tag{2.32}$$

$$TS\left[|\psi\rangle^{222}\right] = \omega^2 \cdot TS\left[|\psi\rangle^{000}\right] = \omega^2 a^\dagger, \tag{2.33}$$

$$TS\left[|\psi\rangle^{012}\right] = TS\left[\frac{1}{\sqrt{3}}(a^\dagger + \omega b^\dagger + \omega^2 c^\dagger)\right] = b^\dagger, \tag{2.34}$$

$$TS\left[|\psi\rangle^{120}\right] = \omega \cdot TS\left[|\psi\rangle^{012}\right] = \omega b^\dagger, \tag{2.35}$$

$$TS\left[|\psi\rangle^{201}\right] = \omega^2 \cdot TS\left[|\psi\rangle^{012}\right] = \omega^2 b^\dagger, \tag{2.36}$$

$$TS\left[|\psi\rangle^{021}\right] = TS\left[\frac{1}{\sqrt{3}}(a^\dagger + \omega^2 b^\dagger + \omega c^\dagger)\right] = c^\dagger, \tag{2.37}$$

$$TS\left[|\psi\rangle^{102}\right] = \omega \cdot TS\left[|\psi\rangle^{021}\right] = \omega c^\dagger, \tag{2.38}$$

$$TS\left[|\psi\rangle^{210}\right] = \omega^2 \cdot TS\left[|\psi\rangle^{021}\right] = \omega^2 c^\dagger. \tag{2.39}$$

From equations (2.31)-(2.39) it is evident that the particle is to be found with

A if the input string X was chosen from the first class. The same relation holds for the location of particle at B and C, and the second and third class of inputs, respectively. Therefore, it is natural to divide inputs into three corresponding categories, i.e.

Set A			Set B			Set C		
0	0	0	0	1	2	0	2	1
1	1	1	1	2	0	1	0	2
2	2	2	2	0	1	2	1	0

Associated to these categories, we assign the following binary questions (of which the answers are labeled as 'YES' or 'NO') asked by the referee:

- Question to player A (QA): is the given input string from set A?
- Question to player B (QB): is the given input string from set B?
- Question to player C (QC): is the given input string from set C?

In analogy to the two-party game, at the final stage of the game, each player performs a measurement to detect the location of the particle. If the particle is to be found localized in A, this necessarily means that the initial input string belongs to Set A. Therefore, when A is asked, she knows the right answer to QA with certainty. In particular, A wins the game if she answers 'YES' every time she finds the particle in her possession, and 'NO' otherwise. Similarly, there is a one-to-one correspondence between the detection of the particle in B or in C and corresponding classes (Set B and Set C, respectively). Therefore, B and C can answer as well with certainty the question asked by the referee. We conclude that players win the game with probability $P_Q = 1$. The main steps of this section are summarized in Table 2.2.

Classical scenario

Let us turn now to the classical case. In contrast to the quantum case, only classical communication can be used and, as such, no quantum superpositions of the particle are allowed. Consider at first the case in which the particle must follow the same geometrical constrain of passing thought the center of the triangle. This can be imagined as a communication channel, e.g. an actual physical constrain (tunnels or optical fibers, etc.), which forces the particle to travel only along fixed and well defined paths. Since the channel stays open only for the period τ, the information carrier can travel between any two players (via the center of the triangle), but can never reach the third one, resulting again in "one-way" communication between a pair of parties. In this case, one of the most efficient strategies for players is to agree in advance that one of them, say B, will always send her input to another player, say A. Notice that all the nine input strings are univocally identified by only two ordered input values, out of three. This means that A can acquire the full information on $X = (i_A, i_B, i_C)$ after getting B's input. The other two players, on the other hand, only know their own respective inputs, and therefore they do not have enough information to determine the exact answers. So, one of the best strategy is that B and C agree to answer with 'NO' always. Since A knows the entire input string X, she can answer with certainty always. Now, if the referee randomly asks only one player to answer his request for each run of the game, the probability of guessing is $P'_{class} = \frac{1}{3} \cdot 1 + \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} = \frac{2}{3}$. On the contrary, if all the players are asked to answer at every run, the classical probability of success is bounded by $P_{class} = \frac{1}{3}$.

As a more general instance, one can release the constrain of passing via the center, and consider a classical particle free to take a 'shortcut', traveling along the edges of the triangle. To this extent, one should compute how far can travel a particle within the time window τ. In the considered case (see Fig. 2.4) the maximal distance is d (without loss of generality we have here set $\epsilon = 0$). On the other hand, twice the edge of triangle is longer than the allowed maximal distance. In particular, the particle traveling along the edges (each of which are ℓ long) can only cover $d = \frac{2}{\sqrt{3}}\ell < 2\ell$. Therefore only a single round of "one-way" communication (between two players) can be exchanged within τ. Therefore, the analysis is completely equivalent to the one given in the preceding paragraph.

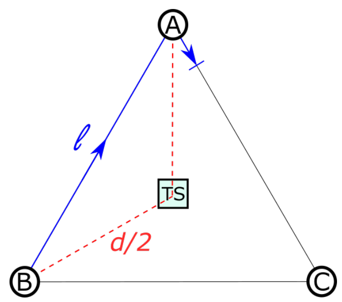

Figure 2.4: Path traveled by the classical particle within τ (in blue). Only one round of "one-way" communication can be exchanged between the players.

input strings			final state	measurement	answers		
i_A	i_B	i_C	$\|\psi\rangle_f^{i_A i_B i_C}$		QA	QB	QC
0	0	0	$a^\dagger$	particle with A	YES	NO	NO
1	1	1	$\omega a^\dagger$				
2	2	2	$\omega^2 a^\dagger$				
0	1	2	$b^\dagger$	particle with B	NO	YES	NO
1	2	0	$\omega b^\dagger$				
2	0	1	$\omega^2 b^\dagger$				
0	2	1	$c^\dagger$	particle with C	NO	NO	YES
1	0	2	$\omega c^\dagger$				
2	1	0	$\omega^2 c^\dagger$				

Table 2.2: **Summary of the three-party quantum game**. A referee assigns an input (i_A, i_B, i_C) to each of the players, A, B and C. The set of inputs is categorized into three different classes. After encoding their inputs via local operations and exchanging communication, the players are able to distinguish whether the input string belongs to a certain class, and thus they in the game with certainty.

2.3 Five-party game

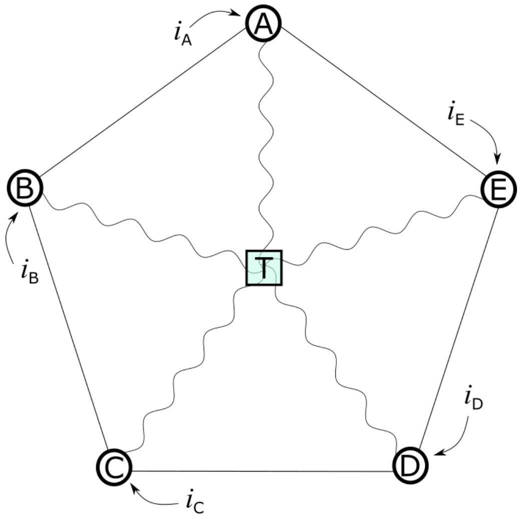

Figure 2.5: Five players, disposed on the vertices of a regular pentagon, receive an initial input string (i_A, i_B, i_C, i_D, i_E). They share a single-particle state in spatial quantum superposition (wavy line). A linear device T is placed in the geometrical center of the pentagon.

Once we have fully characterized a three-party game, it is rather natural to generalize the game to a grater number of parties, simply by following the same set of assumptions A1-A2. Although a proper generalization to an arbitrary number of parties is presented in the next section, we deem it useful to shortly analyze another particular instance, too. So, let us consider now a five-party game. With reference to Fig. 2.5, the players, A, B, C, D and E are disposed on the vertices of a regular pentagon (the distance between a vertex and the center is set to $d/2$). As in the previous case, they are challenged by a referee to guess the correct answer to some requests. At the beginning of every run, the referee assigns an input string $X = (i_A, i_B, i_C, i_D, i_E)$, where $i_k = 0, 1, 2, 3, 4$.

We start by considering the quantum scenario, namely an initial state $|\psi\rangle_{in}$, in spatial superposition between all the five parties:

$$|\psi\rangle_{in} = \frac{1}{\sqrt{5}}(a^\dagger + b^\dagger + c^\dagger + d^\dagger + e^\dagger)|0\rangle_A|0\rangle_B|0\rangle_C|0\rangle_D|0\rangle_E, \tag{2.40}$$

where $a^\dagger, b^\dagger, c^\dagger, d^\dagger, e^\dagger$ are the creation operators acting on the the modes associated to locations A, B, C, D and E, respectively. We then recall the explicit form of the five-dimensional Fourier matrix:

$$\mathscr{F}_5 = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^2 & \omega^3 & \omega^4 \\ 1 & \omega^2 & \omega^4 & \omega & \omega^3 \\ 1 & \omega^3 & \omega & \omega^4 & \omega^2 \\ 1 & \omega^4 & \omega^3 & \omega^2 & \omega \end{bmatrix}, \tag{2.41}$$

where we redefine $\omega = e^{2\pi i/5}$. The matrix $\mathscr{F}_5$ transform the ladder operators in the following way

$$a^\dagger \xrightarrow{\mathscr{F}_5} \frac{1}{\sqrt{5}}(a^\dagger + b^\dagger + c^\dagger + d^\dagger + e^\dagger), \tag{2.42}$$

$$b^\dagger \xrightarrow{\mathscr{F}_5} \frac{1}{\sqrt{5}}(a^\dagger + \omega b^\dagger + \omega^2 c^\dagger + \omega^3 d^\dagger + \omega^4 e^\dagger), \tag{2.43}$$

$$c^\dagger \xrightarrow{\mathscr{F}_5} \frac{1}{\sqrt{5}}(a^\dagger + \omega^2 b^\dagger + \omega^4 c^\dagger + \omega d^\dagger + \omega^3 e^\dagger), \tag{2.44}$$

$$d^\dagger \xrightarrow{\mathscr{F}_5} \frac{1}{\sqrt{5}}(a^\dagger + \omega^3 b^\dagger + \omega c^\dagger + \omega^4 d^\dagger + \omega^2 e^\dagger), \tag{2.45}$$

$$e^\dagger \xrightarrow{\mathscr{F}_5} \frac{1}{\sqrt{5}}(a^\dagger + \omega^4 b^\dagger + \omega^3 c^\dagger + \omega^2 d^\dagger + \omega e^\dagger). \tag{2.46}$$

Clearly, $T := \mathscr{F}_5{}^{-1}$ inverts the order of transformations in the equations (2.42) - (2.46). The players have previously agreed to encode their inputs locally by means of local phases, i.e.

$$\begin{aligned} a^\dagger &\to \omega^{i_A} a^\dagger, \\ b^\dagger &\to \omega^{i_B} b^\dagger, \\ c^\dagger &\to \omega^{i_C} c^\dagger, \\ d^\dagger &\to \omega^{i_D} d^\dagger, \\ e^\dagger &\to \omega^{i_E} e^\dagger. \end{aligned} \tag{2.47}$$

As in previous example of three-party game, an additional constraint to inputs is needed:

Constrain on the inputs (CI) for five-party game. The referee assigns an input string $X = (i_A, i_B, i_C, i_D, i_E)$, by randomly choosing (with equal probability) among the 25 possibilities displayed on the left-hand side of the following table:

i_A	i_B	i_C	i_D	i_E	encoding inputs	state with inputs encoded
0	0	0	0	0		$\|\psi\rangle_1 := \frac{1}{\sqrt{5}}(a^\dagger + b^\dagger + c^\dagger + d^\dagger + e^\dagger)$
1	1	1	1	1		$\omega\|\psi\rangle_1$
2	2	2	2	2		$\omega^2\|\psi\rangle_1$
3	3	3	3	3		$\omega^3\|\psi\rangle_1$
4	4	4	4	4		$\omega^4\|\psi\rangle_1$
0	1	2	3	4		$\|\psi\rangle_2 := \frac{1}{\sqrt{5}}(a^\dagger + \omega b^\dagger + \omega^3 c^\dagger + \omega^4 d^\dagger + \omega^5 e^\dagger)$
1	2	3	4	0		$\omega\|\psi\rangle_2$
2	3	4	0	1		$\omega^2\|\psi\rangle_2$
3	4	0	1	2		$\omega^3\|\psi\rangle_2$
4	0	1	2	3		$\omega^4\|\psi\rangle_2$
0	2	4	1	3		$\|\psi\rangle_3 := \frac{1}{\sqrt{5}}(a^\dagger + \omega^2 b^\dagger + \omega^4 c^\dagger + \omega d^\dagger + \omega^3 e^\dagger)$
1	3	0	2	4		$\omega\|\psi\rangle_3$
2	4	1	3	0	$\xrightarrow{(i_A,i_B,i_C,i_D,i_E)}$	$\omega^2\|\psi\rangle_3$
3	0	2	4	1		$\omega^3\|\psi\rangle_3$
4	1	3	0	2		$\omega^4\|\psi\rangle_3$
0	3	1	4	2		$\|\psi\rangle_4 := \frac{1}{\sqrt{5}}(a^\dagger + \omega^3 b^\dagger + \omega c^\dagger + \omega^4 d^\dagger + \omega^2 e^\dagger)$
1	4	2	0	3		$\omega\|\psi\rangle_4$
2	0	3	1	4		$\omega^2\|\psi\rangle_4$
3	1	4	2	0		$\omega^3\|\psi\rangle_4$
4	2	0	3	1		$\omega^4\|\psi\rangle_4$
0	4	3	2	1		$\|\psi\rangle_5 := \frac{1}{\sqrt{5}}(a^\dagger + \omega^4 b^\dagger + \omega^3 c^\dagger + \omega^2 d^\dagger + \omega e^\dagger)$
1	0	4	3	2		$\omega\|\psi\rangle_5$
2	1	0	4	3		$\omega^2\|\psi\rangle_5$
3	2	1	0	4		$\omega^3\|\psi\rangle_5$
4	3	2	1	0		$\omega^4\|\psi\rangle_5$

In complete analogy to the three-partite case, one can see that the states in rows 2-4 are up-to a global phase equal to the ones displayed in the first row. Having in mind the action of the inverse transform $\mathscr{F}_5^{-1}$, it is clear that particle ends up with A if the input was from the first block. An analogous situation is to be found for all the other players. Therefore, we shall divide the inputs into the following five sets:

Set A	Set B	Set C	Set D	Set E
0 0 0 0 0	0 1 2 3 4	0 2 4 1 3	0 3 1 4 2	0 4 3 2 1
1 1 1 1 1	1 2 3 4 0	1 3 0 2 4	1 4 2 0 3	1 0 4 3 2
2 2 2 2 2	2 3 4 0 1	2 4 1 3 0	2 0 3 1 4	2 1 0 4 3
3 3 3 3 3	3 4 0 1 2	3 0 2 4 1	3 1 4 2 0	3 2 1 0 4
4 4 4 4 4	4 0 1 2 3	4 1 3 0 2	4 2 0 3 1	4 3 2 1 0

The referee then asks a different question to each of the players, to which they can only return binary answer, labeled as 'YES' or 'NO':

- Question to player A: is the given input string from set A?
- Question to player B : is the given input string from set B?
- Question to player C: is the given input string from set C?
- Question to player D: is the given input string from set D?
- Question to player E: is the given input string from set E?

After encoding, the parties send their 'parts of particle' to the center of the pentagon, where T is implemented. The states get transformed under the action of T, according to the reversed order of (2.42)-(2.46). At the final stage, each player performs the measurement and only one of them finds the particle at her position. In that case, the winning answer to the question addressed to corresponding player is 'YES', otherwise she shall reply with 'NO'. It is clear that the players win the game with certainty.

For what concerns the best classical strategy, we can start by restricting the communication to occur along the same channels of the quantum one, thus passing through the center of the pentagon. In such a case, only one round of "one-way" communication between two players can be exchanged within τ. Even though this constrain is released, and the particle can travel from one player to another along the edges of the pentagon, still only one classical communication is allowed within τ. This is because twice the length of the edge of a regular pentagon is longer than d. So, even in the best classical scenario, only one player would know the exact answer with certainty, namely the one who receives the input from another player via the communication process. Thus, the optimal strategy is that all the other players agree to always answer with 'NO' since only one 'YES' per run wins the game.

If the referee asks the question only one player per run, choosing her randomly (each player has in average 1/5 of probability of being asked), the best classical probability to guess the right answer is $P'_{Class} = \frac{1}{5}\cdot 1 + \frac{1}{5}\cdot\frac{1}{5} + \frac{1}{5}\cdot\frac{1}{5} + \frac{1}{5}\cdot\frac{1}{5} + \frac{1}{5}\cdot\frac{1}{5} = \frac{9}{25}$. On the other hand, if the referee asks all the player to answer, the probability of guessing the right result is $P_{Class} = \frac{1}{5}$. In both cases, the corresponding quantum scenario reveals $P_Q = 1$.

2.4 N-party generalization

In this section we address the following question: Does quantum superposition principle provides a stronger resource for communication when very many parties are involved? In order to answer this question we need to find the general rules for arbitrary number of parties.

We start by considering N players ($N \geq 2$) disposed on the vertices of a convex regular polygon with N edges, with the distance between a vertex and the geometrical center $d/2$. We label the players with j such that $0 \leq j \leq N-1$. The number of player can be arbitrarily large, but we add the constrain that N must be a prime number, for a reason that will become clear in what follows.

Similarly to the analysis provided in previous sections, the conjunction of the assumptions A1-A2 is still in force, and we impose to have only one particle for communication. The communication is restricted to occur within the same time window $d/c \leq \tau \leq d/c + \epsilon$. Moreover, the players are allowed to exchange unrestricted information in an initialization phase, during which they can agree on any legal strategy.

The actual game starts (at $t = 0$) when the usual referee assigns to the jth player an input x_j, where $x_j \in [0, N-1] \subset \mathbb{Z}$. Here we label an N-element input string as $X = (x_0, x_2, \cdots, x_{N-1})$. In analogy with the three-party and five-party games, a constraint on the inputs needs to be added to the game in order to experience the advantage provided by quantum superpositions (compared with the corresponding classical resource). In fact, it has been shown in sections 2.2 and 2.3, that only certain input strings allow the players to accomplish their tasks with certainty. In particular for the three-party and the five-party cases, the acceptable input strings were restricted to $3^2 = 9$ over $3^3 = 27$ and to $5^2 = 25$ over $5^5 = 3125$ possibilities respectively, as described by the corresponding CI (see sections 2.2 and 2.3). It is well known from combinatorial analysis that the total number of possible input strings is given by the number of symbols (i.e. possible values that the single inputs can assume) to the power of the length of the string. Hence, in this case the total number of input strings is N^N, whereas we have clear sign that only N^2 of them can be taken. We can generalize CI to the N-party game as

Constrain on the inputs (CI). At the beginning of every run of the game, the referee assigns an input string $(x_1, x_2, \cdots, x_{N-1})$, defined by two integers (n, m), such that

$$x_k = nk + m \pmod N, \tag{2.48}$$

with (n, m) picked randomly from the set $n, m = 0, \ldots, N-1$ (with probability $1/N^2$). In such a way, we limit the accepted input strings to N^2 over a total number of N^N.

The referee then divides the allowed input strings into N sets corresponding to those which share the same value of k. The referee addresses the following question to each of the N players, who can only answer 'YES' or 'NO':

- Question to kth player (Q_k) : *Does the given input string X satisfies $n = k$?*

In order to win the game, only one player has to answer with "YES" ($n = k$ is true for one k only), while the rest shall answer with "NO". Let us consider separately the best solution to the problem posed by the referee in both quantum and classical scenario.

Quantum scenario

In a quantum scenario, the N players are granted the possibility of using quantum superposition principle to prepare their initial state. A single particle is prepared in equally weighted superposition of Ndifferent locations. The initial state, in second quantized formalism reads:

$$\begin{aligned} |\psi\rangle_{in} &= \frac{1}{\sqrt{N}}(\sum_{j=0}^{N-1} a_j^\dagger)\bigotimes_{k=0}^{N-1}|0\rangle_k \\ &\equiv \frac{1}{\sqrt{N}}(\sum_{j=0}^{N-1} a_j^\dagger), \end{aligned} \tag{2.49}$$

where $a_j^\dagger$ is the creation operator only acting on the mode associated to the spatial location of the jth player, i.e. $a_j^\dagger|0\rangle_j = |1\rangle_j$, whereas $\otimes$ denotes the standard tensor product. The last member of (2.49) is a shorter notation, already encountered in (2.19), that neglects the explicit writing of the vacuum state on which the ladder operators acts, but describes the state merely as the corresponding linear combination of ladder operators. As a matter of simplicity this notation is to be adopted in what follows.

We start by introducing the N-dimensional Fourier matrix $\mathscr{F}_N$:

$$\{\mathscr{F}_N\}_{k,j} = \frac{1}{\sqrt{N}}\omega^{k\cdot j}, \tag{2.50}$$

where $\omega = e^{2\pi i/N}$. The matrix $\mathscr{F}_N$ transforms the ladder operators in the following way:

$$a_k^\dagger \xrightarrow{\mathscr{F}_N} \frac{1}{\sqrt{N}}\sum_{j=0}^{N-1}\omega^{k\cdot j}a_j^\dagger. \tag{2.51}$$

As before, we set $T_N := \mathscr{F}_N^{-1}$, therefore we have

$$\frac{1}{\sqrt{N}}\sum_{j=0}^{N-1}\omega^{k\cdot j}a_j^\dagger \xrightarrow{T_N} a_k^\dagger. \tag{2.52}$$

Similarly to our previous analysis, a unitary device implementing the action of T_N is placed in the geometrical center of the regular polygon. All the N players have previously agreed to encode their own inputs into the initial state $|\psi\rangle_{in}$, using the following local operations (adding of phases):

$$a_j^\dagger \to \omega^{x_j} a_j^\dagger. \tag{2.53}$$

Referring to the new state which encodes all the inputs as $|\psi\rangle_{x_j}$, it is now possible to compute its evolution under the application of T_N. In fact, thanks to CI, only the input strings of the form $x_k = nk + m \pmod N$ are allowed, therefore the state after encoding the inputs reads:

$$|\psi\rangle_{x_k} = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} \omega^{k\cdot j+\varphi} a_j^\dagger = \frac{\omega^\varphi}{\sqrt{N}} \sum_{j=0}^{N-1} \omega^{k\cdot j} a_j^\dagger. \tag{2.54}$$

In the next step the players exchange their communication by sending their 'part of particle' towards the other players (via the polygon center), letting the state evolve through the unitary device T_N. The process occurs, once again, within the time window τ which would correspond to a particle traveling, with a finite velocity, from one player to another via the center of the polygon (in the corresponding classical scenario).

After the communication process, the final state $|\psi\rangle_f$ is obtained. Recalling the action of T_N from (2.52) and (2.54), the final state reads:

$$|\psi\rangle_f = T_N|\psi\rangle_{x_k} = T_N \left[\frac{\omega^\varphi}{\sqrt{N}} \sum_{j=0}^{N-1} \omega^{k\cdot j} a_j^\dagger \right] = \omega^\varphi a_k^\dagger. \tag{2.55}$$

Since ω^φ is a global phase, when the players perform a measurement (detection of particles) on the final state, they will find N possible distinct states, one for each k. Therefore if and only if the given input string was from the set k, the particle is to be found localized with the kth player. This player will thus answer 'YES' to the referee's request, whereas all the other players, who do not detect the particle, will answer to the respective question with 'NO'. Thus, they win the game with certainty. In conclusion, all the players are able, using a strategy based on quantum superposition principle, to achieve the win probability $P_Q = 1$, independently of the number of players.

Classical scenario

Here we analyze a classical scenario, in which players exchange only one information carrier and, without loss of generality, we assume that it is initially localized with one of them. As it happened in the other variants of this game analyzed in the previous sections, the players agree on a strategy during the initialization phase. Firstly, we consider that the particle is bounded to travel through the center of the regular polygon on which vertices the players are located. Therefore, within the time window τ, only a single round of "one-way" communication is possible, independently of the number of players. Also in this case the best strategy seems to be that a player, say the kth one, sends her input to another player, say the lth one. In such a way, player l knows the given input string and she is able to answer with certainty the question that the referee posed to her. In fact, since l has in possession $x_k = nk + m$ and $x_l = nl + m$, she can simply find the difference $x_k - x_l = n(k - l)$ and from there she can extract the value of $n = (x_k - x_l)(k - l)^{-1}$ (N is chosen to be a prime number, thus the division modulo N is well defined). Therefore, l can verify $n = l$ with certainty. Nevertheless, the rest of $N - 1$ players do not have any information on inputs of the other players. As it was argued in previous examples, they should answer 'NO' to their respective question, since only one 'YES' per run is allowed. In this way, the players achieve a win probability of $1/N$. Another optimal strategy is to fix one player who will always reveal 'YES' and other ones shall answer with 'NO'. Again they achieve the same value of $1/N$, yet curiously enough, in this particular case they do not have to exchange any communication.

As a second and slightly more general instance of game, we release the restriction on the possible paths that can be traveled by the classical particle. The players can now send communications along the edges of the polygon. Without loss of generality, consider the communication to occur in series from say the jth player, to the next one, and so forth until the time window τ closes. For a convex regular polygon, it is easy to find the maximal number of classical communications, n_{cc}, since it corresponds to the maximal number of edges contained in d, i.e. $n_{cc} = \lfloor \frac{d}{\ell} \rfloor$, where ℓ is the length of the polygon edge and $\lfloor . \rfloor$ denotes the integer part function. Referring to figure 2.6, it is trivial to infer the following

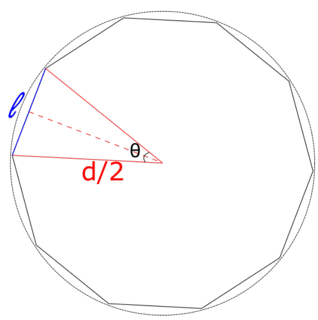

Figure 2.6: A generic convex regular polygon with edge ℓ, inscribed in a circle with radius $r = d/2$. For regular polygons the ration between $2r$ and ℓ only depends on the number of edges N.

relations from elementary trigonometric relations:

$$\frac{d}{2} = \frac{\ell}{2sin(\frac{\theta}{2})}, \tag{2.56}$$

where θ is the central angle subtended by the edge ℓ. For any regular polygon $\theta = \frac{2\pi}{N}$ and

$$\frac{d}{2} = \frac{\ell}{2sin(\frac{\pi}{N})}. \tag{2.57}$$

Therefore, the number of classical communication allowed within the chosen time window, as a function of the number of parties N, is given by

$$n_{cc} = \left\lfloor \frac{1}{sin(\frac{\pi}{N})} \right\rfloor. \tag{2.58}$$

The behavior of (2.58) is displayed in figure 2.7, where the number of classical communication is plotted versus the number of parties (values of N up to 30 are shown).

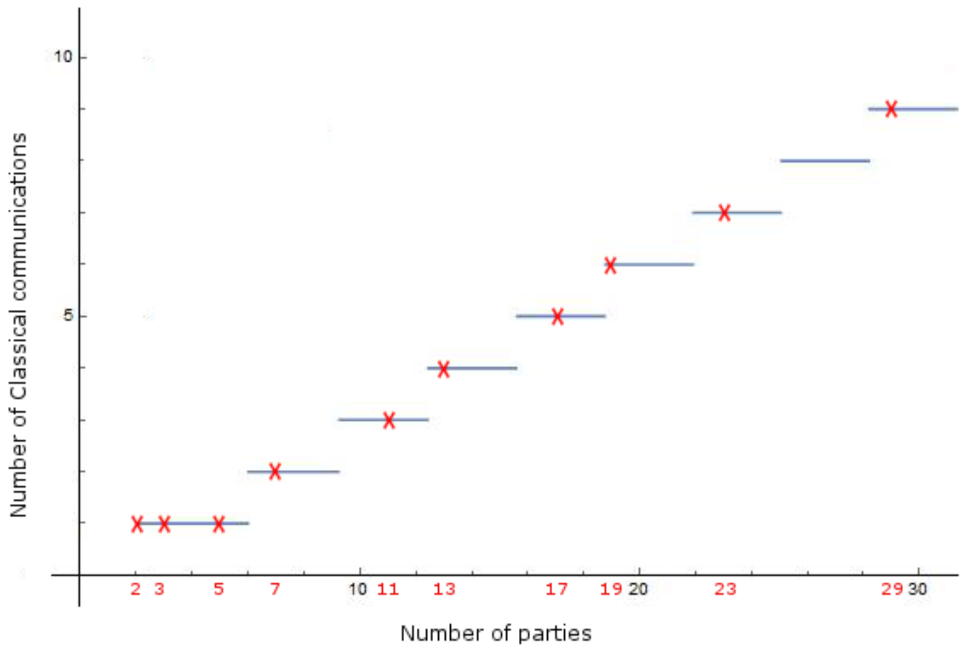

Figure 2.7: Number of classical communication (y-axis) allowed within the time window τ, as a function of the number of parties N (x-axis). The prime numbers are highlighted in red.

So, coming back to the best classical strategy, the players agree to send their input in a series of n_{cc} consecutive rounds of "one-way" communication. In such a way up to n_{cc} players will know the right answer to the question of the referee, whereas all the remaining N-n_{cc} players, who do not receive any information about the other players' inputs, shall agree to always answer 'NO', since only one question over N has 'YES' as correct answer. Again in comparison with the special cases presented in the previous sections, a first possibility is that the referee only asks one player per run to answer the respective question. If this choice is randomly made according to a uniform distribution, each player is asked to answer in average 1/N times. As we have shown, only n_{cc} players would be able to answer with certainty the right answer, whereas the remaining ones have the chance of $1/N$ to guess the correct answer. Therefore the probability of success is $P_{class}(N)' = n_{cc}\frac{1}{N} + (N - n_{cc})\frac{1}{N^2}$. On the other hand, if all the players are supposed to answer referee's questions, the best classical probability of success is

$$P_{class}(N) = \frac{n_{cc}(N)}{N} = \left\lfloor \frac{1}{sin(\frac{\pi}{N})} \right\rfloor \frac{1}{N}. \tag{2.59}$$

It is remarkable that such classical probabilities are monotonically decreasing functions of N, and always assume values smaller than the unity (for every $N > 2$), whereas the corresponding quantum probability is always equal to 1, independently of N. As a further comparison, it is also of some interest to analyze how the classical probability behaves when very many parties ($N \to \infty$) are involved:

$$\lim_{N\to\infty} P'_{class}(N) = \lim_{N\to\infty} P_{class}(N) \simeq \lim_{N\to\infty} \frac{1}{N sin(\frac{\pi}{N})} = \lim_{N\to\infty} \frac{1}{\frac{sin(\pi/N)}{\pi/N}\pi} = \frac{1}{\pi}. \tag{2.60}$$

The same result can be found with geometrical considerations. In fact, while the number of parties increases, the polygon on which the players are located approaches a circle. In this case, the edge length can be well approximated by an arc length, i.e. $\ell = 2\pi/N$, therefore, the number of classical communications is $n_{cc} = \frac{2}{2\pi/N} = \frac{N}{\pi}$. Consequently, the corresponding probability of success reads $P_{class} = \frac{1}{\pi}$, which is consistent with the solution provided by (2.60).

2.5 Superposition or entanglement?

So far, we have shown that quantum system can violate the classical bounds imposed by assumptions A1-A2. One may wonder: what is the resource that enables quantum advantage? In Bell-like scenarios it is clear that quantum entanglement between distant parties allows for stronger-than-classical correlations. In the case analyzed here, one may argue that the quantum superposition principle allows for a fundamental difference between quantum mechanics and classical physics. On the other hand, the state of a single particle in spatial superposition can be seen as an entangled state when described in the second quantisatized notation (Fock's formalism). Indeed, the problem of determining whether the state of a particle in superposition between two spatial position is actually entangled has long been matter of debate.

As early as the 1990s Tan *et al.* investigated the possible non-local properties of single-photons, and proposed the first experiment to violate Bell's inequalities using single-particle entanglement [29]. At present, a number of works have strengthened the conviction that states of the type (2.4) are actually entangled. This has been shown using theoretical arguments for single-photon states [30, 31] and recently also for single-electron states [32]. Moreover, entanglement and violations of Bell's inequalities with single-photons have been experimentally demonstrated using heterodyne measurements [33] and homodyne tomography [34, 35]. Single-photon entangled states find also application in quantum information and quantum communication protocols [36].

Superpositions of single-particle states are the main object of investigation in this work. The preparation of such a state can be realized by sending a single particle located at the position A through a beam splitter (2.5), which creates a superposition of the particle between the two positions A and B. This correspond to the transformation

$$|A\rangle \to \frac{1}{\sqrt{2}}(|A\rangle + |B\rangle). \tag{2.61}$$

As already mentioned, an equivalent description of the same scenario, is given by the second quantization formalism., i.e. we write $|A\rangle = |1\rangle_A|0\rangle_B$ and $|B\rangle = |0\rangle_A|1\rangle_B$, meaning one particle in mode A and zero particles (vacuum) in mode B, and vice-versa. Therefore, the transformation of beam splitter in second quantization reads

$$|1\rangle_A|0\rangle_B \to \frac{1}{\sqrt{2}}(|1\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B). \tag{2.62}$$

This clearly shows creation of entanglement in Fock space (mode entanglement) thanks to the action of a beam splitter. In other words, the beam splitter acts as an entangling quantum gate.

It ought to be stressed that the first quantization formalism describes a state living in a two-dimensional Hilbert space $\mathcal{H}$ spanned by $|A\rangle, |B\rangle$. Whereas, the latter formalism describes the state as a four-dimensional vector in the Hilbert space, constructed as the composition of the subsystems A and B, $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$, and spanned by the basis vectors $|0\rangle_A|0\rangle_B, |0\rangle_A|1\rangle_B, |1\rangle_A|0\rangle_B, |1\rangle_A|1\rangle_B$.

Nonetheless, the physical constrain of having a well defined number of particles reduces the space to a two-dimensional subspace generated by $|0\rangle_A|1\rangle_B, |1\rangle_A|0\rangle_B$. This allowed us, in section 2.1, to define BS as a 2×2 matrix, despite the use of the second quantisatized notation. In general, a state of a single particle in superposition between N different positions, lives in a N-dimensional Hilbert space, whereas the same state described in terms of second quantization (2.49) lives in a 2^N-dimensional space. Yet, the physical constrain of allowing only a single particle plays the role of a superselection rule which restricts the space to a N-dimensional subspace of single-particle excitations.

Chapter 3

Experimental implementation with single photons

In this chapter we investigate the possible experimental realization of the quantum game proposed in the previous sections. We consider here the case of a quantum optics experiment and single-photon in superposition between different spatial modes (path degrees of freedom). In particular, we analyze in detail the three-partite case. The generalization to an arbitrary number of parties is straightforward.

Following the derivation of the game, the players can be thought, from an experimental perspective, as spatially separated stations, arranged to fulfill the required geometrical configuration (i.e. they are disposed on the vertices of a regular triangle). Single-photon sources, commonly used for quantum optics experiments, seems to be here the natural choice for generating the carrier of communication. The source emits a photon which is split into three beams (paths) coherently (via some unitary device that implements $\mathscr{F}_3$ in order to prepare an initial state given by the eq. (2.19)). Subsequently, each path is guided towards the observers A, B and C, and the photon arrives at their position at the same time, say $t = 0$. Each observer is provided with a local tunable phase shifter and a photodetector. Just after receiving their inputs ($t \simeq 0$) the players encode them using the corresponding phase shifter (see eq. (2.28)-(2.30)). The observers then guide their modes to the center of the polygon, which implements the action of TS (according to the transformation defined in (2.25)-(2.27)). After TS, the photon goes back towards players and at the final stage the detectors allow the final read out.

The experimental realization of the operators involved in the game, i.e. the corresponding Fourier matrix, requires a more careful analysis. Since Fourier matrices are unitary, they can be experimentally implemented using elementary optical devices [39]. It is indeed well known that any element of the group $U(2)$

can be experimentally realized by a lossless beam splitter[1] with an additional phase shift between the splitting modes [37, 38, 39].

The matrix describing such a device transforms input modes (k_1, k_2) into the output modes (k'_1, k'_2), as follows:

$$\begin{bmatrix} k'_1 \\ k'_2 \end{bmatrix} = \begin{bmatrix} e^{i\phi} sin\theta & e^{i\phi} cos\theta \\ cos\theta & -sin\theta \end{bmatrix} \cdot \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}. \tag{3.1}$$

This matrix is fully characterized by the two parameters θ and ϕ. The latter parameter, is here referred to as the *phase parameter*, since it accounts for the relative phase of $e^{i\phi}$ between the modes. Such a phase can be experimentally implemented by inserting a phase shifter (usually a birefringent crystal) on photon's path. The other parameter, θ, is here refereed to as the *angular parameter* and it characterizes the coefficients of reflectivity (R) and transmittancy (T):

$$T = cos^2\theta,$$

$$R = sin^2\theta.$$

Here we will assume that an experimentalist is occupied by elementary building-blocks, such as standard beam splitters and phase shifters only. In such a case, it is very useful to notice that a simple Mach-Zehnder interferometer, composed of two mirrors, two standard beam splitters and two phase shifters (as displayed in Fig. 3.1), can substitute any variable beam splitter of the form (3.1).

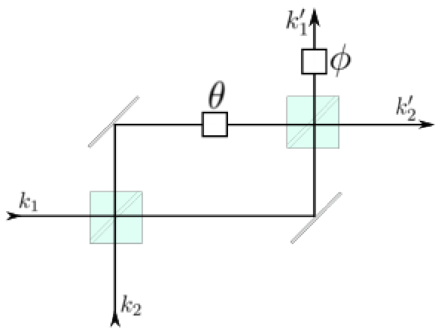

Figure 3.1: A Mach-Zehnder interferometer can be used to implement any non-standard beam splitter, by the modulation of two phase shifters $e^{\theta i}$ and $e^{\phi i}$ (labelled with θ and ϕ, respectively).

Using this construction as a basic building block, Reck *et al.* [39] have shown that every element of $U(N)$ group (the set of $N \times N$ unitary matrices) can be decomposed into the product of matrices acting as $U(2)$ transformations on two-

[1]With beam splitter we refer here to a device capable of dividing a beam into two ones, with an arbitrary ratio between the intensity of the outcomes, depending on some internal parameters. We refer here to the half-transmitting and half-reflecting beam splitter as "standard" or 50:50 beam splitter.

dimensional subspaces of the whole N-dimensional Hilbert space. The product of these matrices corresponds, in an experimental setup, to sequential application of tunable beam splitters, as defined in (3.1). We label as T_{ij} the N-dimensional matrix that acts non-trivially in the subspace ij, with the transformation defined in (3.1). If a general $N \times N$ unitary matrix U_N, is multiplied from the right with the sequence of $N-1$ nearest-neighbor T_{ij} matrices, i.e.

$$U_N \cdot T_{N,N-1} \cdot T_{N,N-2} \cdots T_{N,1} = \begin{bmatrix} U_{N-1} & 0 \\ 0 & e^{i\alpha} \end{bmatrix}, \tag{3.2}$$

the dimension is effectively reduced for one, i.e. from N to $N-1$. This defines a recursive algorithm that can be applied the necessary number of times in order to transform the original U_N into a diagonal matrix with phase factors on diagonal. We define D to be the inverse of such diagonal matrix, i.e

$$U_N \cdot T_{N,N-1} \cdot T_{N,N-2} \cdots T_{2,1} \cdot D = \mathbb{1}. \tag{3.3}$$

It is not difficult too see, that the number of required beam splitters of the type (3.1) is $\frac{N(N-1)}{2}$. Additional N phase shifters (non-zero elements of the matrix D) are needed for the full decomposition.

We turn now to the possible experimental realization and we refer to the setup schematically described in Fig. 2.3. The goal here is to provide a suitable geometrical arrangement of elementary devices (beam splitters and phase shifters) in order to implement operator TS defined in (2.24). We start by considering the application of the present algorithm, to decompose the 3×3 Fourier matrix $\mathscr{F}_3$ into a sequence of operators T_{32}, T_{31}, T_{21}. The problem of the experimental realization of a *tritter* has been discussed already in [40]. According to (3.3), the decomposition of the matrix $\mathscr{F}_3$ reads:

$$\mathscr{F}_3 \cdot T_{32} \cdot T_{31} \cdot T_{21} \cdot D = \mathbb{1} \tag{3.4}$$

$$\Longrightarrow \mathscr{F}_3 = (T_{32} \cdot T_{31} \cdot T_{21} \cdot D)^{-1}.$$

Yet, one has to recall that TS, is actually the inverse of $\mathscr{F}_3$, thus:

$$TS := \mathscr{F}_3^{-1} = T_{32} \cdot T_{31} \cdot T_{21} \cdot D.$$

It is easy to verify that the matrices fulfilling this relation are the following ones:

$$T_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} e^{\frac{\pi}{3} i} & \frac{1}{\sqrt{2}} e^{\frac{\pi}{3} i} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}, \tag{3.5}$$

$$T_{31} = \begin{bmatrix} \sqrt{\frac{2}{3}}e^{\frac{2}{3}\pi i} & 0 & \sqrt{\frac{1}{3}}e^{\frac{2}{3}\pi i} \\ 0 & 1 & 0 \\ \sqrt{\frac{1}{3}} & 0 & -\sqrt{\frac{2}{3}} \end{bmatrix}, \tag{3.6}$$

$$T_{21} = \begin{bmatrix} \frac{i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{3.7}$$

$$D = \begin{bmatrix} e^{\frac{5}{6}\pi i} & 0 & 0 \\ 0 & e^{\frac{5}{6}\pi i} & 0 \\ 0 & 0 & e^{-\frac{2}{3}\pi i} \end{bmatrix} := \begin{bmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & e^{i\alpha_3} \end{bmatrix}. \tag{3.8}$$

Thus, the matrix TS can be implemented by the ordered application of three phase shifters (one for each diagonal element of D) and three beam splitters with ratios T/R presented in Table 3.1 (each with an additional output phase as in (3.1)). This result, together with the explicit values of the angular and phase parameters is reported in Table 3.1.

operator	angular parameter (θ)	phase parameter (ϕ)	beam splitter	
			T	R
T_{32}	$\frac{\pi}{4}$	$\frac{\pi}{3}$	1/2	1/2
T_{31}	$\frac{1}{2}(\pi - arctan(2\sqrt{2}))$	$\frac{2}{3}\pi$	1/3	2/3
T_{21}	$-\frac{3}{4}\pi$	$-\frac{\pi}{2}$	1/2	1/2

Table 3.1: A *tri-splitter* (operator TS) can be experimentally implemented by three beam splitters characterized by the values of the parameters ϕ and θ.

The experimental setup also requires to fulfill the geometrical arrangement of Fig. 2.3, namely TS must be disposed on the geometrical center of an equilateral triangle, of which the vertices are the players. A possible geometrical solution is proposed in Fig. 3.2.

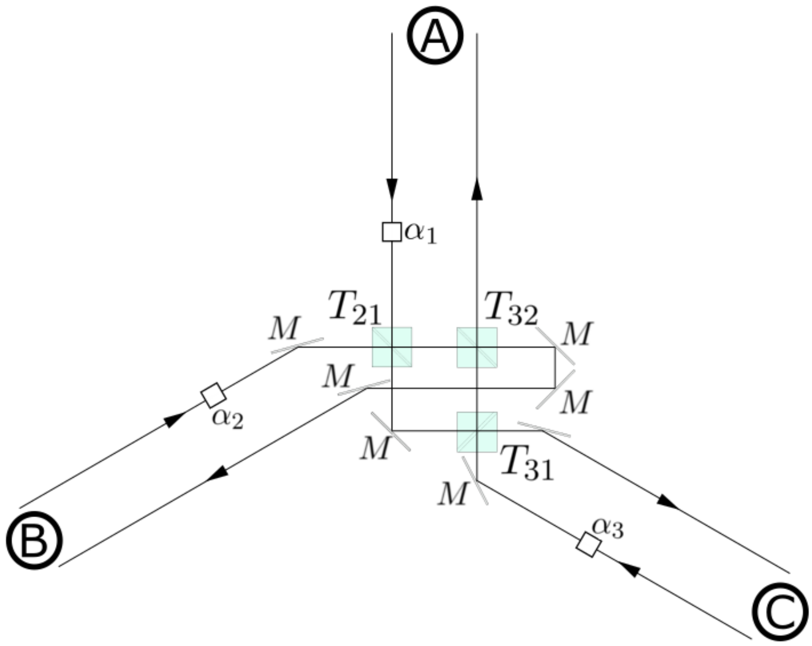

Figure 3.2: A possible experimental arrangement of the thee-party game. The operator TS has been decomposed into elementary optical devices, i.e. beam splitters (T_{32}, T_{31} and T_{21}) and phase shifters (α_1, α_2 and α_3). The apparatus is supplemented by mirrors (Ms) that serve to guide the photons from A, B, C towards the center (where TS is implemented) and back to players.

Chapter 4

Philosophical issues

4.1 Assumptions underlying Bell's theorem: local realism

Although Bell's theorem has celebrated its first half a century, there is no global consensus on its interpretation. This rather simple mathematical derivation is leading not only to a contradiction with the prediction of QM on a theoretical basis. Actual experimental violations of the bounds imposed by the theorem have been demonstrated in a plethora of more and more ambitious experiments (recently, loophole-free experiments have been realized, i.e. [41] [42]). Now, an observed violation of Bell's theorem necessary implies, according to classical logic (*modus tollens*), that the conjunction of the assumptions used to derive it is untenable. But which are exactly these assumptions?

A tacit underlying assumption in the original proposal of Bell, is the 'freedom of choice' (or 'free will') of the parties, who are completely free to locally choose their settings. The relaxation of this assumption would in any case bring to a form of 'super-determinism', but, in Bell's words: "I do not expect to see a serious theory of this kind"[43], and we will not consider this possibility in what follows.

On the mathematical level, we have shown in chapter 1 that the main assumption of Bell's inequalities is the existence of a set of hidden variables λ, governed by a probability distribution $q(\lambda)$ which allows to write the conditioned probability distribution as in equation (1.3). The meaning of these variables, and thus their implications in physics, are open to interpretations, and has been referred to with a number of expressions: "The λ [...] can pop up under many guises such as, e.g. 'the physical state of the systems as described by any possible future theory' , 'local beables', 'the real state of affairs', 'complete description of the state', etc. Since λ do not appear in quantum mechanics, thus they are (good old) hidden variables"[44]. Related to the role played by λ there are contrasting schools of thought which attribute different interpretations to the assumption leading to the condition (1.3), that we have here called *local realism*. In a recent paper [44], Zukowski and Brukner have refuted two quite accredited views on the assumptions underlying Bell's inequalities (besides 'free will'):

View 1. The unique fundamental assumption leading to Bell's inequalities is locality.

View 2. There are additional premises besides locality (e.g. 'realism', 'determinism'), but they can all be inferred from locality.

Therefore the implication that both of these views attribute to Bell's inequalities is that of ruling out once and for all locality from all the possible theories violating them, including quantum mechanics. We also agree that such positions are quite untenable, and that locality alone cannot be thought as the fundamental concept leading to Bell's inequality. Another quite popular view, which can be find e.g. in [45], should be also critically revised:

View 3. Both locality and realism are two independent fundamental assumptions to derive Bell's theorem.

Realism is here associated to the existence of the hidden variables λ and its distribution $q(\lambda)$, whereas locality to the separability condition which leads from equation (1.1) to (1.2). But in fact, the definition of λ is empty by itself in this context, and its introduction becomes meaningful only when it allows to write the expression (1.2), i.e. when it decouples the local probabilities.

In conclusion, Bell's inequalities rely on a "compound condition"[44] as a whole, which we have called local realism (sometimes referred to 'local causality'). Indeed, such a condition is intuitively deeply related to the concept of realism and locality, and yet it is not possible to formally enforce either of the two conditions one by one. A violation of Bell's inequalities merely means that local realism (in conjunction with that of 'free will') is untenable, and nothing more.

4.2 Leggett-Garg inequality and macrorealism

Reminiscent of Bell's inequalities, in 1985, Leggett and Garg proposed a new family of inequalities [46], with the scope of testing a possible break down of quantum mechanics at the 'macroscopic' scale. This is done by the investigation of Scrödinger's cat-like states, namely the study of the possibility of preparing quantum superpositions of macroscopic objects (defined according to some reasonable criteria).

The formalization of this concept is completely derived by adopting some of fundamental assumptions, which all together go under the name of *macrorealism* [47]:

Macrorealism per se. A macroscopic object which has available to it two or more macroscopically distinct states is at any given time in a definite one of those states.

Non-invasive measurability. It is possible in principle to determine which of these states the system is in without any effect on the state itself or on the subsequent system dynamics.

Induction. The properties of ensembles are determined exclusively by initial conditions (and in particular not by final conditions)."

Out of these simple assumptions, it is possible to derive a series of inequalities which play analogous role that Bell's inequalities play for local realism. Following the formalism provided in [48], one can consider a macroscopic object described by a set of 'macroscopically distinct' [47] variables $\{Q, Q', ...\}$. Let the object be prepared, in a series of runs, always in the same initial state, and let the time always reset to $t = 0$ at the beginning of each run. If variable $a \in \{Q, Q', ...\}$ is measured at time $t_a > 0$ and the variable $b \in \{Q, Q', ...\}$ is measured at $t_b > t_a$, macrorealism allows to write the joint probabilities of these events as

$$p(a_{t_a}, b_{t_b}) = \sum q(\lambda)p(a_{t_a}|\lambda)p(b_{t_b}|\lambda). \tag{4.1}$$

Such convex decomposition is completely analogous to the (discrete) condition of local realism (1.3), and we call it condition of *macrorealism*. It is easy to verify that in the simplest case, i.e. when the variables can only assume binary values $Q \in \{-1, 1\}$, macrorealism (4.1) leads to the *Leggett-Garg inequalities* of the CHSH-type

$$C_{t_1t_2} + C_{t_2t_3} + C_{t_3t_4} - C_{t_1t_4} \leq 2, \tag{4.2}$$

were $C_{t_at_b} := \langle Q_{t_a}Q_{t_b}\rangle$ are the temporal correlations, and $t_1 < t_2 < t_3 < t_4$.

Violation of Leggett-Garg inequalities are a pervading phenomenon at the microscopic quantum level [48, 49, 50], but only in recent years technological progress approached a regime in which experimental tests on macroscopic objects seems to be feasible. Superpositions of magnetic-flux states in superconducting quantum interference devices (SQUID) [47, 51, 52] are good candidates, as well as nanometer-sized massive objects [53], nano-sphere [54] or large molecules [55, 56].

4.3 Quantum communication with limited resources: meaning of the assumptions

The new game discussed in chapter 2, shows that, under certain conditions, communication tasks are more efficient when genuine quantum effects are exploited. This helps to shed light on the intrinsic fundamental difference between classical and quantum theories. In the present chapter it was recalled that both Bell's theorem and the Leggett-Garg inequalities test quantum mechanics against classical alternatives, on the basis of the fundamental assumptions of local realism and macrorealism, respectively. In this spirit, in chapter 2, we have proposed a new model based on the assumptions of *limited resources* and *finite speed of propagation*. When referring to classical scenarios throughout our derivation, we have always enforced the additional underlying assumption of *definiteness of the position* (we denote it as Assumption 3, A3), namely that the spatial location of physical objects is well defined at any time. This resulted in the fundamental restriction to "one-way" communication if one uses only a single particle. On the contrary, it appears evident from the proposed model that quantum formalism (specifically the possibility of using coherent quantum superpositions) violates the classical restrictions, and thus implies that the conjunction of assumptions A1-A3 is untenable. We deem it of great interest to discuss the relation that these assumptions A1-A3 have, if any, with local realism and macrorealism.

Indeed, assumption A1 and A3, are strongly related to the idea of realism, since they assume that the two physical properties 'number of information carriers' and 'position' are well defined at any moment in time. These assumptions are in fact fulfilling the *macrorealism per se* leading to Leggett-Garg inequalities. Notice however that the second assumption of macrorealism, i.e. the *non-invasive measurability*, is not required for the model proposed here, since the latter only requires a single final measurement to be performed by the parties, whereas all the other processes - including the encoding of the inputs - are carried out by means of unitary operators.

On the other hand the last assumption A2, is directly related to the request of compatibility with relativity, and thus in direct connection with the no-signaling condition [7] and to locality in a broad sense.[1].

[1]In his original paper for instance, Bell defined locality as the property demanding that "if two measurements are made at places remote from one another the [setting of one measurement device] does not influence the result obtained with the other" [48]

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Flavio Del Santo

Curriculum Vitae et Studiorum

Staudgasse 33/7
1180 Vienna, Austria
✉ *delsantoflavio@gmail.com*

Affiliations

Quantum Optics, Quantum Nanophysics and Quantum Information (Prof. Brukner), Faculty of Physics, University of Vienna.
Member of the "**Wiener Kreis Society** - For The Advancement Of Scientific World Conceptions", Vienna.
Member of the international association "**Basic Research Community for Physics**'', BRCP (http://basic-research.org/).
Founding member of "**THINK Association** - Verein zur Vernetzung kritischer WissenschafterInnen" (Austria, ZVR-Zahl: 577491579).

Education

2015-present **Master studies in Physics**, *University of Vienna*, Austria.

2009-2014 **Bachelor in Physics and Astrophysics**, *Universita' degli studi di Firenze*, Florence, Italy.

2009 **Matura**, *Liceo Scientifico "Leonardo da Vinci"* , Florence, Italy.

Publications

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- **"Think^3 - an interdisciplinary international conference on the foundations of science"**, organised by students of the University of Vienna for students - to be held in Tata, Hungary on 13-17.07.2017 (https://thinkconference.net)
- **Symposium "Shut Up and Contemplate!"**, held at the University of Vienna on 03.03.2017 (https://shutupandcontemplatesymposium.wordpress.com)
- First Symposium **"WIP - Women in Physics"**, held at the University of Vienna on 22.06.2015
- Study day **"History and Philosophy of Science - Evolution of Scientific Thinking"**, held at the Faculty of physics of Florence on 05.11.2012
- Promoter and co-organiser of the initiatives of the student representatives at the University of Florence: "Bio-Impact: an Analysis on the Difficult Relation between Human and Nature" 09/05/2013, "The Last Fermat Theorem" 12/03/2013, "Genius, Revolution, Mathematic - Second Centenary from the Birth of Evariste Galois" 25/10/2011, "What is Nuclear? Introductory Lectures on the Processes Exploited in the Nuclear Power Plant" 29/03/2011, "Evolution of Observation: Once upon a time...Astronomy" 13/12/2010.