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„dS Vacua and Starobinsky Inflation in 4d N=1 Supergravity“

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Christoph Oliver Roupec, B.Sc.

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Contents

Abstract

In 1980 A. A. Starobinsky presented in [1] a model of inflation that uses a Planck-suppressed order R^2 term in the Einstein-Hilbert action to describe the exponential expansion of the universe. Such an expansion was first postulated by Guth in [2] to solve the *horizon* and *flatness* problem of cosmology.

The Starobinsky model of inflation, in the context of supergravity, has been discussed before, for example in [3], [4] and [5]. In this work however we want to construct generalized Starobinsky type inflation models that arise from a supergravity that is the low energy limit of string theory. We hope that on the one hand this restriction gives us particles in form of the moduli arising from compactifying the string theory and on the other hand that our work here gives the possibility to use cosmological data to check some aspects of string theory and therefore its validity.

The first chapter is dedicated to the classical concepts of cosmology that we require for our work. In the last section we will look at the classical Starobinsky inflation in order to motivate our later work. The basics of supergravity and how we can get a supergravity from string theory will be outlined in chapter 2. There the STU-model of supergravity will be presented, which shall serve as the main ingredient for the work of chapter 3. In the penultimate chapter we will start our work on simple STU-models with polynomial superpotentials and check their relevance for inflation. To conclude the chapter we investigate possibilities to generalize the model via adding additional fields or loosening our restrictions on the form of the potential.

1 Classical Inflation

1.1 Motivation

The period of inflation is caused by an unknown mechanism that causes the universe to expand exponentially happening somewhere in the time frame of $10^{-35}s$ to $10^{-14}s$ [6] after the big bang. There are a number of reasons to *postdict* inflation as a part in the history of our universe, the most popular cited ones are the uniform distribution of the cosmic microwave background (CMB) [7] and the so called *flatness problem* [8].

1.1.1 The cosmic microwave background (CMB)

The CMB [9] is the radiation from the *surface of last scattering* [7], the time the universe cooled down enough from its hot and dense initial state for light to propagate freely, in other words, the universe became transparent. This radiation is the oldest observable we can measure directly about our universe. It corresponds to a thermal radiation of $2.725K$ and fits the curve of a black body almost exactly [10], as can be seen from figure 1. It is also almost perfectly homogeneous in every direction [11]. In figure 2 the CMB temperature is plotted for all directions. The maximal difference of two temperatures is merely $600\mu K$, meaning the CMB is in thermal equilibrium, which we call the *horizon problem.* Without inflation not all points on the surface of last scattering (see figure 3) were in causal contact (i.e.: were not able to interact) in the past since the universe did not exist long enough for thermal equilibrium to settle in. Thus, there is no reason that the CMB should be uniform in every direction. In fact there should be around 10^4 patches of the CMB corresponding to angles of $2°$. [13]
If the universe underwent a period of exponential expansion one such patch (in thermal equilibrium) could be stretched to the size of the surface of last scattering, resolving the horizon problem.

1.1.2 The flatness problem

The apparent lack of substantial curvature in the observable universe means that an incredible amount of fine-tuning of the initial conditions of our universe would be necessary if there is no mechanism that causes this state. We call this the *flatness problem.*
In our universe the contribution to the energy density from curvature is almost equal to zero [14]. Conversely that sets the normalized energy density to $\Omega_T \simeq 1$, meaning that the energy density is due to matter, radiation

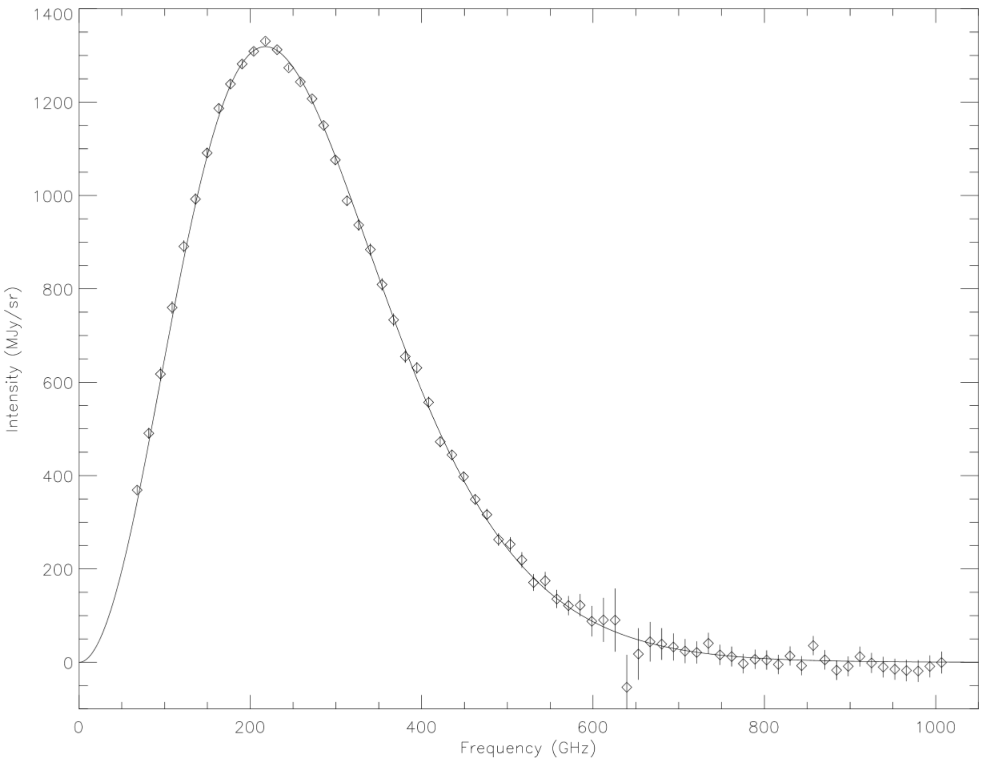

Figure 1: The black-body-spectrum of the CMB. The line is an ideal black-body while the diamonds give the measurement results. (Graph taken from [10])

and dark energy alone [1]. This is related to the curvature (in a Friedmann universe) by [7]

$$\Omega_T - 1 = \frac{K}{\dot{a}^2(t)} = \frac{3K}{8\pi G \rho_c(t) a^2(t)}. \tag{1.1}$$

Todays experimental bounds, according to [14], are

$$\frac{K}{\dot{a}^2(t)} < 0.005,$$

meaning that our universe is almost flat (Minkowski). Today the energy density is dominated by the cosmological constant Λ and hence the critical density ρ_c is constant, but in past times our universe underwent a matter and a radiation dominated era [7] where $\rho_c(t)$ was proportional to $a(t)^{-3}$

[1] These quantities and the statements below will be discussed in detail in chapter 1.2

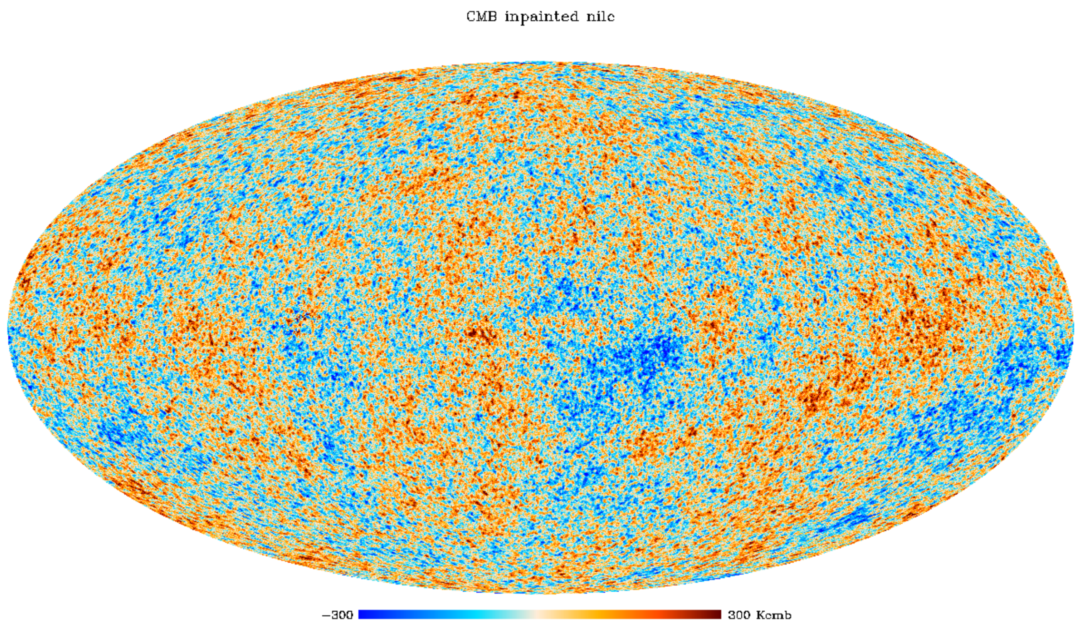

Figure 2: Heat-map of the fluctuation of the CMB. The scale is in μK. (Taken from [11])

and $a(t)^{-4}$ respectively, where $a(t)$ is the scale parameter in the Friedman-Lemaître-Robertson-Walker (FLRW) metric [2]. This means that a small deviation from $\Omega_T = 1$ will grow over time, meaning that at the beginning of the universe Ω_T had to be very close to 1 to achieve today's flat universe. Going back in time to when the temperature of the universe was around the Planck scale [3], we find that:

$$\Omega_T - 1 \simeq 10^{-62}.$$

This seems unlikely, even in a perfectly flat universe ($K = 0$) we would expect some local variation of K (the curvature) which would be grow in time. During inflation $\dot{a}(t)$ is very large and supresses the $K/\dot{a}(t)$ term in (1.1) which can explain our flat universe.

[2] They each modelled a homogenous, isotropic universe on their own, for an explanation of the metric see [7].

[3] We will see how certain quantities can be devolved back in time in chapter 1.3.2

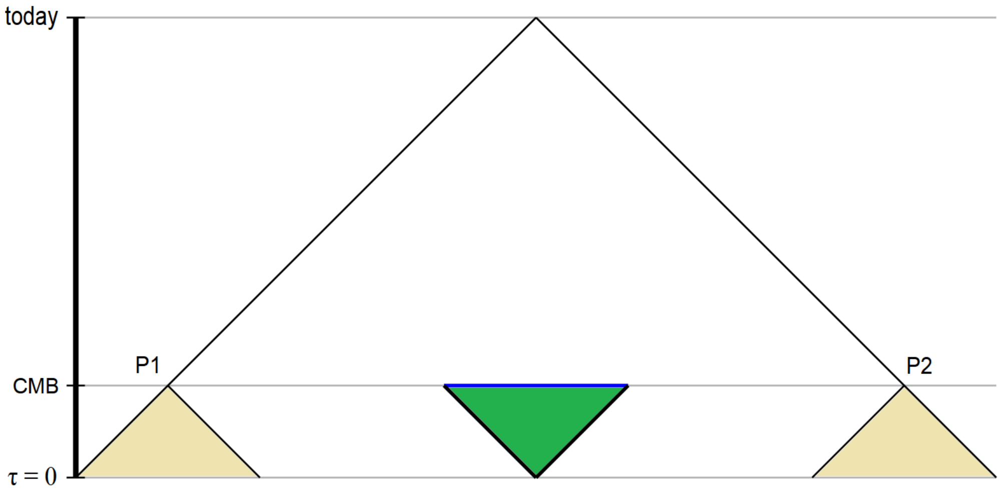

Figure 3: Schematic picture of the CMB photons propagating from the surface of last scattering. Without inflation points, P1 and P2 cannot be in thermal equilibrium. Only areas of the size of the blue line can have a causal connection. (Figure not to scale)

1.2 The Standard Model Of Cosmology

To study inflation we need the basic notions of the standard model of cosmology called the ΛCDM [4] model. At very large scales ($\gtrsim 100Mpc$ [15]) the observable universe is *isotropic* and *homogeneous* [7], meaning it looks the same in every direction and the distribution of matter (and other forms of energy) is uniform.
Such a universe can be described by the Friedman-Lemaître-Robertson-Walker (FLRW) metric [7]:

$$ds^2 = -dt^2 + a(t)^2 \left(1 + K \frac{(x_i)^2}{1 - K x_i^2} \right) dx_i^2 \tag{1.2}$$

($i = 1, 2, 3,\ (x_i)^2 = \delta^{ij} x_i x_j$) where $a(t)$ is the scale factor in units of length. K can have values 0 and ± 1 which correspond to a flat universe ($K = 0$), a negative curvature ($K = -1$) or a positive curvature ($K = 1$).
In spherical coordinates this reads

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right). \tag{1.3}$$

In a universe with this metric (for example our universe for large distances) an object at co-moving coordinate r has the distance

[4] Λ is the cosmological constant and CDM stands for cold dark matter

$$d(r,t) = a(t)\int_0^r \frac{dr'}{\sqrt{1-Kr'^2}} = a(t)\times\begin{cases} arcsin(r) & K=+1 \\ arsinh(r) & K=-1 \\ r & K=0 \end{cases} \tag{1.4}$$

and thus moves away from us for fixed r if $a(t)$ grows with time. Concretely an object fixed at r moves away from us with velocity

$$v(r,t) = \frac{\dot{a}(t)}{a(t)}d(r,t) = H(t)d(r,t)$$

which amounts to Hubble's observation [16] of an expanding universe, visualized in figure 4.

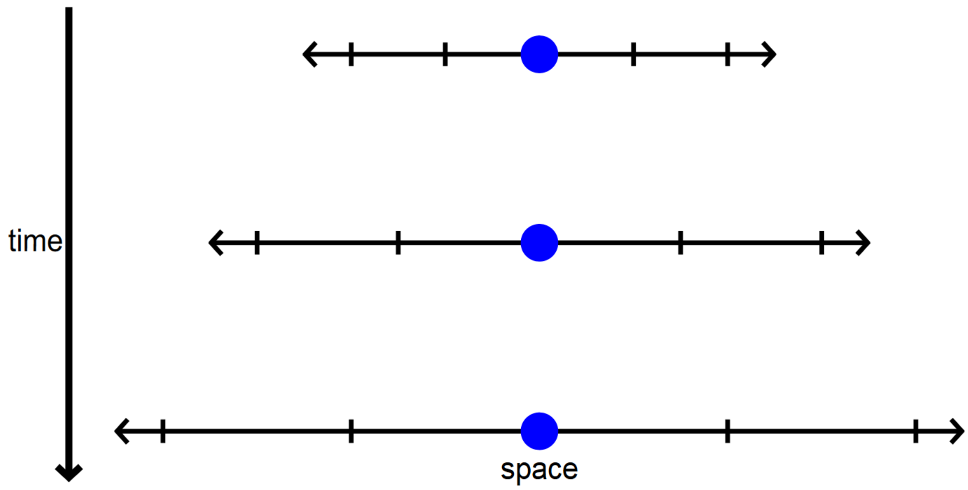

Figure 4: In an expanding universe all points move away from each other. Note that the center point is not special in any way, taking any other point as a reference paints the same picture.

1.2.1 Friedmann equations

We have seen that a time dependent scale factor $a(t)$ leads to an expanding (or contracting) universe, and in fact there is good experimental reason to assume time dependence of $a(t)$, namely the evident expansion of the universe first observed by Hubble [16]. Using the Einstein equations of general relativity (GR) one can see that the time dependence of the scale factor is governed by the energy content of the universe. The Einstein equations with a cosmological constant [17] are

$$R_{\mu\nu} + \left(-\frac{1}{2}R + \Lambda\right)g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{1.5}$$

and in a universe with an FLRW metric (1.2) they reduce to

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 + \frac{K}{a(t)^2} - \frac{\Lambda}{3} = \frac{8\pi G}{3}\rho(t) \tag{1.6}$$

$$\frac{\ddot{a}(t)}{a(t)} - \frac{\Lambda}{3} = -\frac{4\pi G}{3}\left(\rho(t) + 3p(t)\right) \tag{1.7}$$

where, due to isotropy and homogeneity [17], we set $T_{ij} = p(t)g_{ij}$ as well as $T_{00} = \rho(t)$ (due to isotropy $T_{0i} = 0$). These are the *first and second Friedmann equations* which govern the time evolution of the scale parameter. Note that by introducing the energy density $\rho(t)$ and the pressure $p(t)$ we have essentially approximated the universe as a fluid and thus omitted the "graining" due to the concentration of energy in galaxies and other structures. Noting that the cosmological constant Λ appears in the same way as $\rho(t)$ and $p(t)$ in (1.6) and (1.7) we can perform shifts

$$\begin{aligned} \rho &\to \rho - \frac{\Lambda}{8\pi G} \\ p &\to p + \frac{\Lambda}{8\pi G} \end{aligned} \tag{1.8}$$

to get rid of the cosmological constant and find the Friedmann equations as

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 + \frac{K}{a(t)^2} = \frac{8\pi G}{3}\rho(t) \tag{1.9}$$

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{3}\left(\rho(t) + 3p(t)\right). \tag{1.10}$$

The Friedmann equations describe the large scale evolution of our universe as long as we know the complete energy content of the universe.

1.2.2 Critical density

Since we know that our universe is almost exactly flat on large scales it is useful to define the energy density of a flat universe. If we set $K = 0$ in (1.9) we find the *critical density*

$$\rho_c(t) = \frac{3H(t)^2}{8\pi G}. \tag{1.11}$$

The required critical density for our observable universe today is according to [14]:

$$\rho_c(t_0) \simeq 10^{-26}\frac{kg}{m^3}.$$

It proofs useful to also define the *normalized energy density* for all types of energy in the universe as

$$\Omega_i(t) = \frac{\rho_i(t)}{\rho_c(t)} \tag{1.12}$$

where i runs over all types of energy i.e.: matter (m), radiation (r) and the cosmological constant (Λ). With that definition, equation (1.9) reads

$$\Omega_T(t) = \sum_i \Omega_i(t) = 1 + \frac{K}{\dot{a}(t)^2}. \tag{1.13}$$

This equation correlates the total normalized energy density with the curvature of the universe. For $\Omega_T = 1$ the universe is flat while $\Omega_T > 1$ describes a closed universe and $\Omega_T < 1$ an open one. The current upper bound for the curvature of our universe is [14]

$$\left|\frac{K}{\dot{a}_0^2}\right| < 0.005.$$

1.3 Energy content and expansion of the universe

We approximate our universe as filled by "dust" of different kind: radiation, "ordinary" matter and the cosmological constant. All these satisfy the equation of state [7]

$$p(t) = w\rho(t) \tag{1.14}$$

and the continuity equation

$$\dot{\rho}(t) + 3H(t)\left[\rho(t) + p(t)\right] = 0,$$

which we can derive from (1.9) and (1.10). After a series of manipulations we arrive at

$$\rho(t) \propto a(t)^{-3(1+w)} \tag{1.15}$$

where w is a constant that depends on the type of matter in question.

- Non-relativistic matter [5] can be approximated to have only energy given by its mass (we neglect the kinetic energy). We start with a cube of length $a(t_{in})l$ filled with some amount of matter, possibly stars, nebulas or even galaxies. The energy density in this box is $\rho_m = E/(a(t_{in})l)^3 = M/(a(t_{in})l)^3$ where we set the speed of light $c = 1$ as we will do from

[5] Generally all matter that cannot be approximated as moving at the speed of light. Solid matter is considered to be non-relativistic almost always.

here on almost always. During the expansion of the universe the volume of the box will change and thus

$$\rho_m(t) \propto a(t)^{-3} \tag{1.16}$$

which corresponds to $w_m = 0$ and from the equation of state (1.14) we see that the pressure for ordinary matter vanishes ($p_m(t) = 0$).
Currently to the best of our knowledge the energy density of non-relativistic matter is given to be [14]

$$\Omega_{m,0} = 0.308 \pm 0.012 \tag{1.17}$$

which includes all visible matter, neutrinos and even dark matter which is predicted from gravitational effects.

- For radiation we consider an amount of photons of wavelength $a(t_{in})\lambda$ in a cube of initial volume $[a(t_{in})l]^3$. When we let the cube expand, the energy $E = [2\pi]/[a(t_{in})\lambda]$ [6] of the radiation will be diluted proportionally to $a(t)^{-3}$. Furthermore, the energy decreases because of the $a(t)^{-1}$ factor and we arrive at a behaviour for radiation given as

$$\rho_r(t) \propto a(t)^{-4} \tag{1.18}$$

and thus $w_r = 1/3$ and therefore $p_r(t) = 1/3\ \rho_r(t)$ for radiation.
Relativistic matter plays only a minor role in today's energy density. The energy density is given to be [14]

$$\Omega_{r,0} \approx 10^{-4}. \tag{1.19}$$

Due to the time evolution of the energy density (1.18) radiation actually dominated the expansion behaviour of the universe at early times.

- For the cosmological constant Λ we see from (1.8) that

$$\rho_\Lambda(t) = -p_\Lambda(t) = \frac{\Lambda}{8\pi G} \tag{1.20}$$

and thus $w_\Lambda = -1$. This means that the energy density of a constant does not change in time or dissipate. During an expansion of the universe more vacuum with the usual energy density is created. This will also exclude a large cosmological constant as the source of inflation as

[6] We will generally set $\hbar = 1$.

we will discuss below.
The energy density of the cosmological constant [14]

$$\Omega_{\Lambda,0} = 0.692 \pm 0.012 \tag{1.21}$$

dominates today. This energy density is commonly called *dark energy* and its origin is not understood. It is only clear that it is responsible for the accelerated expansion of the universe [18].

1.3.1 Time evolution of the universe

As can be guessed from above, due to the different time behaviour of different kinds of energy the universe's expansion was mostly determined by one of the above types during different times. Using the Friedmann equation (1.9) (neglecting curvature) and the general evolution of the energy density according to (1.15) with an arbitrary starting point:

$$\rho(t) = \rho_0 \left(\frac{a(t)}{a_0} \right)^{-3(1+w)} \tag{1.22}$$

we find after some algebraic manipulations

$$a(t)^{(1+3w)/2} \dot{a}(t) = \sqrt{\frac{8\pi G}{3} \rho_0 a_0^{3(1+w)}}$$

which can be separated and integrated to give

$$\frac{2}{3(1+w)} a^{(3(1+w))/2} = \sqrt{\frac{8\pi G}{3} \rho_0 a_0^{3(1+w)}} + const$$

after choosing $a(t=0) = 0$ and demanding $a(t_0) = a_0$ we get

$$a(t) = a_0 \left(\frac{t}{t_0} \right)^{2/(3(1+w))} \quad , \quad w \neq -1. \tag{1.23}$$

Evidently this does not hold for the cosmological constant but for that case the Friedmann equation (1.9) (again neglecting K) with $\rho(t) = const$ gives immediately

$$a(t) = a_0 e^{H_0(t-t_0)}. \tag{1.24}$$

1.3.2 Redshift

A photon gets stretched during the expansion of the universe according to

$$\lambda_0 = \frac{a(t_0)}{a(t_1)}\lambda_1.$$

For expansion ($a(t_0) > a(t_1)$), the photon's wavelength will move more towards the red - it becomes redshifted. It is useful to introduce the *redshift parameter*

$$z = \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{a(t_0)}{a(t_1)} - 1 \tag{1.25}$$

as a measure for time in the universe. For example the time the CMB was released was around $z \approx 1000$ [7], also meaning that the universe was 1000 times smaller at that point in time than it is now.

1.4 Inflation

With the tools presented above we now want to solve the problems of sections 1.1.1 and 1.1.2. In particular we want to see how a rapidly expanding universe solves these problems and we will even get an estimate of the required expansion to solve them.

1.4.1 The horizon problem revisited

The problem of the uniform distribution of the CMB can be solved in the context of inflation by stretching a small patch of the CMB that is in thermal equilibrium to a size that is in agreement with our observable universe. This is schematically pictured in figure 5.
To tackle this problem we introduce the conformal time via

$$dt^2 = a(\tau)^2 d\tau^2. \tag{1.26}$$

With this transformation the FLRW metric reads

$$ds^2 = a(\tau)^2 \left(-d\tau^2 + \frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right), \tag{1.27}$$

which gives a the flat Minkowski metric multiplied by an overall factor for $K = 0$.
Using conformal time we can calculate the time from the beginning of the

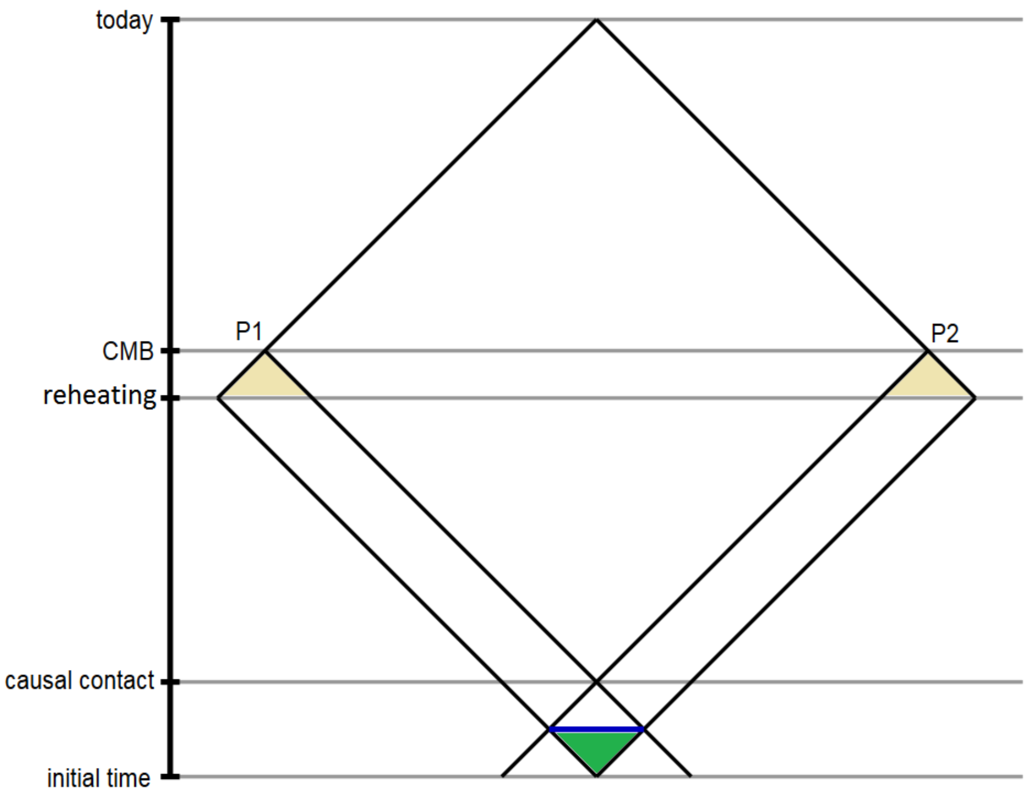

Figure 5: The expansion during inflation stretches a patch in causal contact to the size of today's observable universe.

thermal process until the end of the expansion process where the universe *reheats* as

$$\tau_{reh} - \tau_i = \int_{\tau_i}^{\tau_{reh}} d\tau' = \int_{t_i}^{t_{reh}} \frac{dt'}{a(t')} = \int_{a_i}^{a_{reh}} \frac{da}{a\dot{a}} = \int_{a_i}^{a_{reh}} \frac{da}{a^2 H_{inf}}$$
$$\approx \frac{1}{a_i H_{inf}} - \frac{1}{a_{reh} H_{inf}} \approx \frac{1}{a_i H_{inf}},$$

where we assumed the Hubble parameter to be constant during the duration of inflation and $a_i \ll a_{reh}$ due to the exponential expansion. We need to compare this to the time evolution after the end of inflation which we can approximate by

$$\tau_0 - \tau_{reh} = \int_{\tau_{reh}}^{\tau_0} d\tau' = \int_{a_{reh}}^{a_0} \frac{da}{a^2 H} \approx \frac{1}{a_{reh}^2 H_{inf}} (a_0 - a_{reh}) \approx \frac{a_0}{a_{reh}^2 H_{inf}},$$

where it was assumed that the universe after inflation was *radiation dominated* such that $a(t) = a_0\sqrt{t/t_0}$ and $a^2H = const \approx a_{reh}^2 H_{inf}$.
To solve the horizon problem we need

$$\tau_{reh} - \tau_i \gtrsim \tau_0 - \tau_{reh}$$

which gives, using our calculations from above,

$$\frac{a_{reh}}{a_i} \gtrsim \frac{a_0}{a_{reh}} \approx \frac{T_{reh}}{T_0}{}^7. \tag{1.28}$$

If we assume the temperature around the time of reheating to be slightly below the GUT scale: $T_{reh} \approx 10^{14}GeV$ [19] we find ($T_{cmb,0} \approx 2.725K \approx 10^{-4}eV$):

$$\frac{a_{reh}}{a_i} \gtrsim 10^{27} \approx e^{60}.$$

This factor defines the number of *e-folds* $N_e = log(e^{60}) = 60$ required. It is a convenient measure for the amount the universe has to expand during inflation.

1.4.2 The flatness problem revisited

In chapter 1.1.2 we discussed that the apparent lack of curvature needs an incredible amount of fine-tuning in the initial conditions of the universe and thus is problematic. If we assume a period of inflation we have $a(t) \propto \dot{a}(t)$ during that period which deals with the $K/\dot{a}(t)^2$ term in (1.1). Assuming again $T_{cmb} \approx 10^{14}GeV$ and

$$\frac{K}{\dot{a}_i} \approx 1,$$

we can use (1.1) as well as the critical density from (1.11) and transport these quantities back in time with our knowledge from 1.3 to find [8]

$$\Omega_T(t_{cmb}) - 1 = \frac{K}{\dot{a}(t_{cmb})^2} \approx 10^{-53}$$

$$\Rightarrow \frac{\dot{a}(t_{cmb})}{\dot{a}(t_i)} \approx \frac{a(t_{reh})}{a(t_i)} \gtrsim 10^{53/2} \approx e^{60},$$

which is similar to our above estimate.

[7] Assuming the CMB to be black body radiation it will follow Wien's law: $\lambda_{max}T = const$ and thus from the behaviour of λ under expansion we find $T(t_1) = a_0/a(t_1)T_0$.

[8] At matter-radiation equality $z = 3400$ and at the electro-weak phase transition $z = 10^5$ [20].

1.5 Scalar Particle Inflation

The easiest way to describe a rapidly expanding universe is by a large cosmological constant, however the energy density of a constant does not dissipate during the expansion and thus the expansion does not stop, meaning that inflation via a large cosmological constant is not possible.
We are looking for a way to have a finite inflationary period. This is possible for a scalar particle in a potential. At first the particle sits in a false vacuum state (a local minimum of the potential, see figure 6) or on a very flat slope

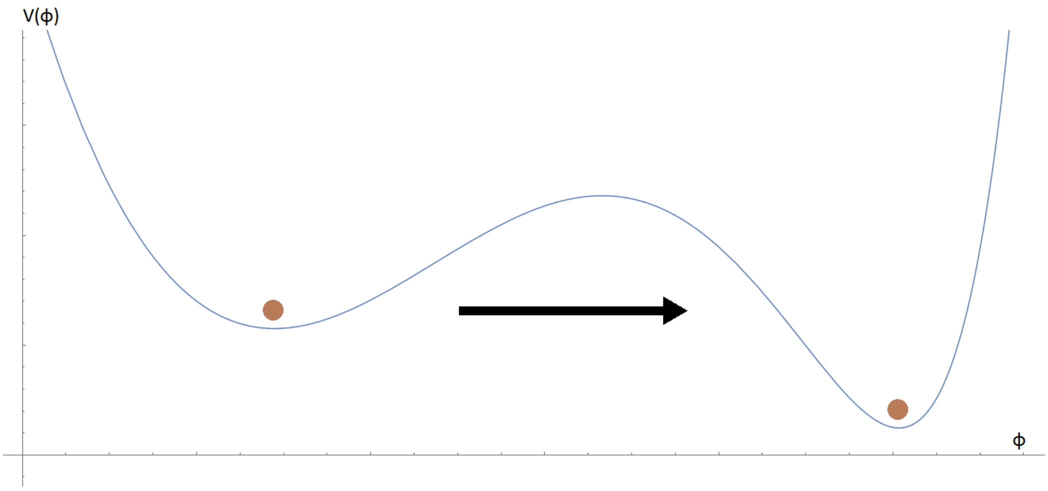

Figure 6: The particle rests in a false vacuum of its potential, representing a large cosmological constant, until it tunnels to the true vacuum of the theory.

(as pictured in figure 7) and then tunnels or rolls off to the true vacuum. The scalar interacts with other particles and loses its energy/reheats the universe. The true minimum of the scalar potential then corresponds to the observed (small) cosmological constant.
The first model for inflation proposed (by Guth [2]) was a false vacuum potential where the particle sits at a local minimum (see figure 6) and then tunnels to the true vacuum. This can happen at different space-time locations independently and results in bubbles forming with lower energy inside. These bubbles expand outwards and our universe could be inside such a bubble.
In practice, however, the problem is that the energy from this transition gathers on the wall of the bubble, leaving the inside essentially empty. To resolve this the bubbles need to collide and form our homogeneous universe. The problem with this is that space continues to inflate and thus the bub-

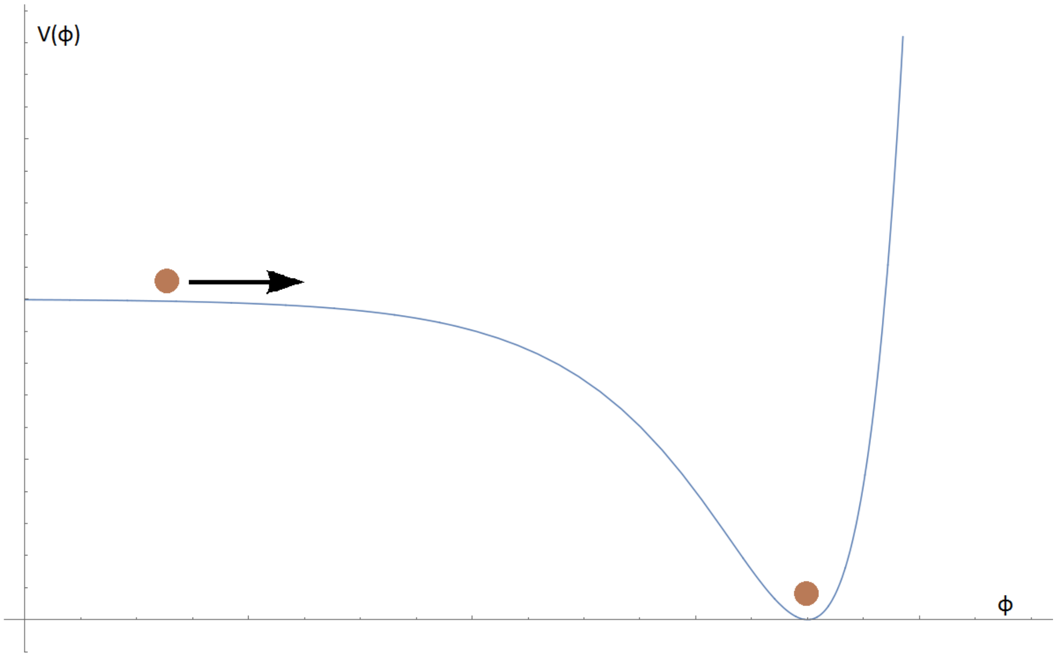

Figure 7: The particle moves along a very flat part of the potential for a long time before it reaches the vacuum of the theory.

bles move away from each other faster than they expand meaning that no collisions can ever take place [7].
What remains possible is a particle on a slope that starts to roll towards a stable vacuum [21]. The particle rolls slowly along the flat part of the potential and behaves like a large cosmological constant until it reaches the steep part of the potential where the kinetic energy becomes important and thus no longer emulates a cosmological potential and inflation ends. Around its true minimum the particle can interact with standard model (SM) particles and the remaining potential energy gives today's (true) cosmological constant.

1.5.1 Scalar Particle in a FLRW Universe

The action of a scalar particle in a general space time is given as [7]

$$S_\phi = \int d^4x\sqrt{-g}\left[-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right] \tag{1.29}$$

where $g = det(g_{\mu\nu})$. In a FLRW universe the line element is given by (1.2). Varying (1.29) for ϕ gives its equation of motion as

$$\ddot{\phi} + 3H\dot{\phi} - \frac{(\vec{\nabla}^2\phi)}{a^2(t)} + V'(\phi) = 0 \tag{1.30}$$

($\vec{\nabla}^2 = \partial_i \partial^i$) where $H = \frac{\dot{a}(t)}{a(t)}$ is the Hubble parameter and $V' = \partial V / \partial \phi$ is the partial derivative of the potential with respect to the field ϕ.
During inflation the scale parameter $a(t)$ grows exponentially and thus the equation of motion (1.30) reduces to

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \tag{1.31}$$

1.6 Slow-Roll inflation

As mentioned above, the most successful way to describe inflation is via a scalar particle in a potential with a very flat part. During *slow-roll* inflation, the kinetic energy ($\propto \dot{\phi}^2$) is negligible compared to the potential energy ($V(\phi)$). Furthermore, because of the expansion of space-time during inflation it is possible to also neglect other sources to the energy density (matter, radiation). To check possible models for inflation it is useful to introduce two dimensionless slow-roll parameters [7]. The Friedmann equation (1.9) with our scalar field as source takes the form [9]

$$H^2 = \frac{1}{3M_P^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right). \tag{1.32}$$

For $\dot{\phi}^2 \ll V(\phi)$ (meaning that ϕ changes very slowly with time) the scalar potential $V(\phi)$ is nearly constant and thus, so is H^2. To parametrize the change in the Hubble parameter we introduce

$$\epsilon = -\frac{\dot{H}}{H^2}. \tag{1.33}$$

Conversely, for inflation to happen it is necessary that $\epsilon \ll 1$. Moreover, since we require inflation to last for a sufficient long time, ϵ cannot change too fast either. This motivates us to introduce

$$\eta = \frac{\dot{\epsilon}}{H\epsilon} \tag{1.34}$$

as a second slow roll parameter. It gives the change of ϵ per Hubble time and we require $\eta \ll 1$ for inflation.
During inflation it is useful to measure time in terms of e-folds which we introduced in chapter 1.4. This is done by defining

$$dN := dln(a) = Hdt. \tag{1.35}$$

[9]Remeber that $T^\mu_\nu = (\partial_\nu \phi)\ [\partial \mathcal{L}]/[\partial(\partial_\mu \phi)] - \delta^\mu_\nu \mathcal{L}$ and that T^{00} is energy density while T^{ii} is pressure. For a homogeneous field the pressure is the same in all three space directions, hence $\rho + 3p$ (see below).

With this, the number of e-folds is given by

$$N_e = \int_{a_i}^{a_f} dln(a) = ln\left(\frac{a_f}{a_i}\right) = \int_{t_i}^{t_f} Hdt \approx H_{inf}(t_f - t_i),$$

where we approximated the Hubble parameter as constant during inflation, as we did before. The second Friedmann equation (1.10) with a homogeneous scalar field reads

$$\frac{\ddot{a}(t)}{a(t)} = \dot{H} + H^2 = -\frac{1}{3M_P^2}\left(\dot{\phi}^2 - V(\phi)\right) \tag{1.36}$$

and using (1.32) we find for the Hubble parameter:

$$\dot{H} = -\frac{\dot{\phi}}{2M_P^2}. \tag{1.37}$$

Putting this into (1.33) we find ϵ as

$$\epsilon = \frac{\dot{\phi}^2}{2M_P^2 H^2} \tag{1.38}$$

and for η:

$$\eta = \frac{\dot{\epsilon}}{H\epsilon} = 2\left(\frac{\ddot{\phi}}{\dot{\phi}H} + \epsilon\right). \tag{1.39}$$

1.6.1 The slow-roll equations

Up until now our considerations were mostly general however during inflation we need to have η and ϵ to be rather small, as we discussed above. This allows us to use approximations to leading order to describe slow-roll inflation. For $\epsilon \ll 1$ we find from (1.32) and (1.38) that

$$H^2 \approx \frac{V}{2M_P^2} \tag{1.40}$$

during slow-roll inflation which means that the Hubble constant is given by the scalar potential and since the scalar field does not change a lot during inflation, neither does H. Additionally imposing $|\eta| \ll 1$, we can use (1.39) as well as the equation of motion for ϕ (1.31) to find

$$3H\dot{\phi} \approx -V'(\phi). \tag{1.41}$$

With the time derivative of this equation,

$$3\dot{H}\dot{\phi} \approx -\dot{\phi}V''(\phi),$$

and the other two equations (1.40) and (1.41) from before, we can write approximate expressions during slow-roll inflation for ϵ (1.33) and η (1.34):

$$\epsilon \approx \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2, \tag{1.42}$$

$$\eta \approx 2M_P^2\left(-\frac{V''}{V}+\left(\frac{V'}{V}\right)^2\right). \tag{1.43}$$

For model building, it is fortunate that the slow-roll parameters depend only on the scalar potential and its derivatives. For convenience, we introduce another set of slow roll parameters based on the expressions above:

$$\epsilon_V := \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \approx \epsilon, \tag{1.44}$$

$$\eta_V := M_P^2\frac{V''}{V} \approx -\frac{1}{2}\eta+\epsilon. \tag{1.45}$$

With these equations, it is easy to see that our conditions on the slow-roll parameters translate to conditions on the derivatives of the scalar potential. In particular the smallness of ϵ_V, corresponding to a constant Hubble parameter, translates to a small first derivative of the potential. Our requirement for the duration of inflation makes it necessary for the second derivative of the potential to be small, too.
Finally we can relate the scalar potential to the number of e-folds needed for inflation via (1.42) and (1.44). We now write

$$N_e = \int_{t_i}^{t_f} H dt \approx \int_{\phi_i}^{\phi_f} \frac{1}{M_P\sqrt{2\epsilon_V}}|d\phi| = \frac{1}{M_P^2}\left|\int_{\phi_i}^{\phi_f} d\phi\frac{V(\phi)}{V'(\phi)}\right|, \tag{1.46}$$

which relates the number of required e-folds directly to the scalar potential.

1.7 Experimental bounds on inflation models

Currently, collecting data from measurements of the cosmic microwave background is our only way to get restrictions on possible inflation models. The best data available is from the ESA's[10] Planck collaboration which measures the black body spectrum of the CMB at different frequencies and also looks at the polarizations of the CMB photons. This data allows us to look back in

[10]European Space Agency

time to about 50 to 60 e-folds before inflation ended. Their results exclude models in terms of the spectral tilt [11] [12]

$$n_s \approx 1 - 6\epsilon_V + 2\eta_V \tag{1.47}$$

and the tensor-to-scalar ratio

$$r \approx 16\epsilon_V. \tag{1.48}$$

In figure 8 we see a recent plot of the data with some types of models filled into the plot. Theoretically the value of n_s can be larger than 1, but data

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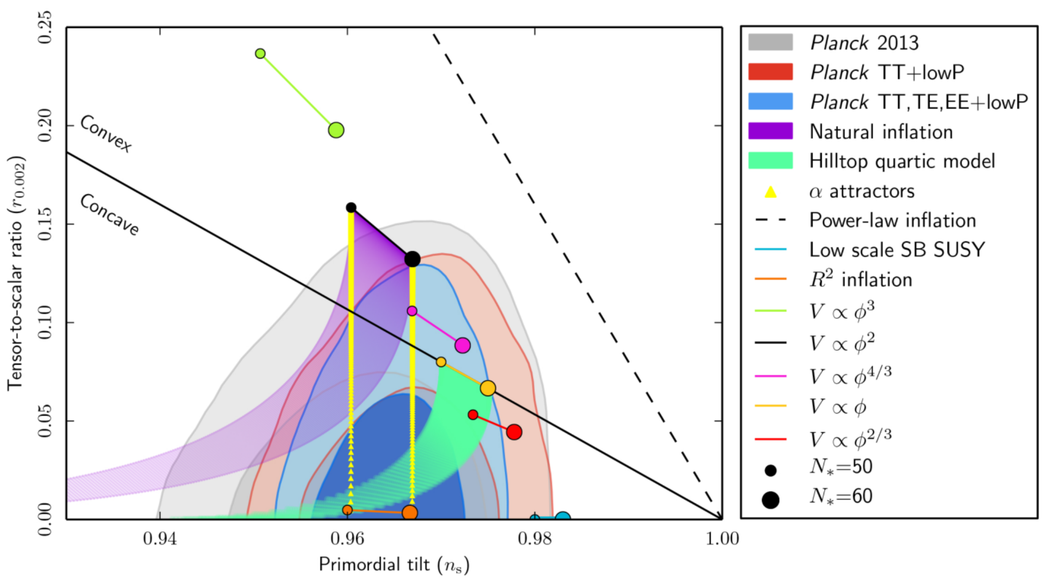

Figure 8: Data from ESA's Planck mission with predictions from different inflation models drawn over it. Notably, natural inflation and certain polynomial potential are unlikely. The Starobinsky-type inflation is denoted as R^2-inflation. Figure taken from [22].

shows that $n_s < 1$ is required. The tensor-to-scalar ratio is experimentally constrained to be smaller than 0.07 with a confidence level of 95%.
As can be seen by the data, some potential models are already excluded by the data and a lot of further models are unlikely. Predictions from past improvements also indicate that the r ratio will go down even further.

1.8 Starobinsky Inflation

One of the earliest models of inflation was proposed by A. A. Starobinsky in [1]. He considered an additional term in the Einstein-Hilbert action [23] proportional to the squared curvature scalar R^2:

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left[R + \frac{\kappa}{2M_P^2} R^2 \right]. \tag{1.49}$$

Here κ is some real parameter and has to be rather large in order for the Planck-suppressed R^2 term to matter.
To see that this can lead to a suitable inflation potential one performs a conformal transformation on the metric:

$$g_{\mu\nu} \to \Omega^2 g_{\mu\nu} = \bar{g}_{\mu\nu}, \tag{1.50}$$

with $\Omega^2 = 1 + \kappa\, R/M_P^2 = e^{-\omega}$. Under this transformation the curvature scalar transforms like

$$\begin{aligned}\bar{R} &= e^{-2\omega} \left[R - (2d-1)\partial^\mu \partial_\mu \omega - (d-1)(d-2)(\partial^\mu \omega)(\partial_\mu \omega) \right] \\ &= \Omega^{-2} \left[R - (2d-1)\partial^\mu \partial_\mu log(\Omega) - (d-1)(d-2)(\partial^\mu log(\Omega))(\partial_\mu log(\Omega)) \right]\end{aligned}$$

with the space-time dimension d. The $\partial_\mu \partial^\mu \omega$ term is a total derivative and therefore can be omitted. Defining a scalar field ϕ as

$$\phi = M_P \sqrt{\frac{3}{2}} log \left(1 + \kappa \frac{R}{M_P^2} \right) \tag{1.51}$$

we find in 4 space-time dimensions (omitting $\partial_\mu \partial^\mu \omega$):

$$\bar{R} = e^{-\sqrt{2/3}\,\phi/M_P} \left[\frac{M_P^2}{\kappa} \left(e^{\sqrt{2/3}\,\phi/M_P} - 1 \right) - \frac{1}{M_P^2} (\partial_\mu \phi)(\partial^\mu \phi) \right].$$

Squaring this but keeping only terms up to second order in the fields (no interactions) yields:

$$\bar{R}^2 = \frac{M_P^4}{\kappa^2} \left(1 - e^{-\sqrt{2/3}\,\phi/M_P} \right)^2 - \frac{2}{\kappa} \underbrace{\Omega^{-2} g^{\mu\nu}}_{=:\bar{g}^{\mu\nu}} (\partial_\mu \phi)(\partial_\nu \phi) + \cdots .$$

Inserting this in the action (1.49) and writing everything in terms of barred quantities one finds

$$S = \int d^4x \sqrt{-\bar{g}} \left[\frac{M_P^2}{2} \bar{R} - \frac{1}{2} (\bar{\partial}_\mu \phi)(\bar{\partial}^\mu \phi) + \underbrace{\frac{M_P^4}{4\kappa} \left(1 - e^{-\sqrt{2/3}\,\phi/M_P} \right)^2}_{=:-V(\phi)} \right]. \tag{1.52}$$

The form of this potential, which is the same as in figure 7, allows, for sufficiently large κ, for inflation with the slow roll parameters (for $\phi \gg M_P$ during inflation)

$$\eta_V = -\frac{4}{3} e^{-\sqrt{2/3}\,\phi/M_P} \tag{1.53}$$

$$\epsilon_V = \frac{4}{3}\left(e^{-\sqrt{2/3}\,\phi/M_P}\right)^2 = \frac{3}{4}\eta_V^2. \tag{1.54}$$

This model has a spectral tilt of

$$n_S \approx 0.96 \tag{1.55}$$

and a tensor to scalar ratio of

$$r \approx 3 \cdot 10^{-3} \tag{1.56}$$

where we have estimated $\phi/M_P \approx 5$ by using equation (1.46) with the goal to get at least 60 e-folds of inflation. These values lie well within current data [22] and can be seen in figure 8.

2 dS Vacua and Inflation in Supergravity

Supergravity is the field theory that arises when one imposes local supersymmetry as an additional gauge symmetry in combination with the Poincaré group. A gauged version of supersymmetry was first mentioned by P. Nath and R. Arnowitt [24] and later, a complete model of supergravity was popularized by Daniel Z. Freedman, Peter van Nieuwenhuizen and Sergio Ferrara [25]. In order to get the required tools for our work below we will review a few basic concepts of supersymmetry and supergravity. A thorough and complete introduction to these topics can be found for example in [26] and [27].

Later in this chapter we review how supergravity theories are related to low-energy limit compactifications of string theories and motivate our decision to restrict ourselves to such a theory.

2.1 Supersymmetry

Supersymmetry (SUSY) is a symmetry that relates fermions with bosons and vice versa. Under this symmetry, particles form a doublet or multiplet where the particles are related via supersymmetry transformations. SUSY is a unique maximal extension to the Poincaré algebra of space-time, which would be motivation enough to study this theory. The algebra relations are:

$$\begin{aligned}
\{Q^A_\alpha, \bar{Q}_{\dot{\beta}B}\} &= 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu \delta^A_B, \\
\{Q^A_\alpha, Q^B_\beta\} &= \{\bar{Q}_{\dot{\alpha}A}, \bar{Q}_{\dot{\beta}B}\} = 0, \\
[P_\mu, Q^A_\alpha] &= [P_\mu, \bar{Q}_{\dot{\alpha}A}] = 0, \\
[M_{\mu\nu}, Q^A_\alpha] &= -\frac{1}{2}(\sigma_{\mu\nu})^\beta_\alpha Q^A_\beta, \\
[M_{\mu\nu}, \bar{Q}^A_{\dot{\alpha}}] &= +\frac{1}{2}(\bar{\sigma}_{\mu\nu})^{\dot{\beta}}_{\dot{\alpha}} Q^A_{\dot{\beta}}, \\
[P_\mu, P_\nu] &= 0, \\
[M_{\mu\nu}, P_\rho] &= \eta_{\nu\rho} P_\mu - \eta_{\mu\rho} P_\nu, \\
[M_{\mu\nu}, M_{\rho\sigma}] &= \eta_{\nu\rho} M_{\mu\sigma} - \eta_{\mu\rho} M_{\nu\sigma} - \eta_{\nu\sigma} M_{\mu\rho} + \eta_{\mu\sigma} M_{\nu\rho}.
\end{aligned} \tag{2.1}$$

Here, Q and $\bar{Q}$ are the newly introduced supersymmetry generators with $\alpha, \dot{\alpha}, \beta, \dot{\beta} = 1, 2$ while the P and M generate the Poincaré transformations. We use the usual conventions for (anti-)commutators and $\sigma^\mu = (\mathbb{1}, \sigma^i)$ the Pauli matrices σ^i. $\sigma^{\mu\nu}$ is the anti-symmetric product of two Pauli matrices: $\sigma_{\mu\nu} = 1/2(\sigma_\mu\sigma_\nu - \sigma_\nu\sigma_\mu)$. Note also that the supersymmetry generators carry spin indices - in fact the supersymmetry is a fermionic symmetry. The above version of the symmetry includes labels A and B which count different SUSY generators. From here on we will only consider $\mathcal{N} = 1$ supersymmetry, meaning that we have only one set of such generators. Another important remark is that neither the Poincaré nor the SUSY algebra requires 4 space-time dimensions. In fact, supergravity is commonly considered in 10 dimensions, as is (super-)string theory.

So far supersymmetry has not been observed in nature or any experiments we are able to set up. This means that SUSY has to be broken below energies of at least $14TeV$ [11]. On the other hand SUSY could solve a lot of problems we face in modern physical theories. For example SUSY can lead to a unification of the forces of the standard model of particle physics (SM) and it gives rise to possible dark matter candidates. Furthermore, since SUSY is the maximal symmetric extension of the Poincaré group, it is only natural to investigate such a beautiful theory.

[11] The energies reached by CERN's large hadron collider (LHC).

2.1.1 A representation of SUSY

As already mentioned, we will restrict ourself to the case $\mathcal{N} = 1$ SUSY and for now we will only consider massless particles, this means that we can always move to a frame where the particle has momentum $P_\mu = (-E, 0, 0, E)$ $(P_\mu P^\mu = 0)$. With this assumption, the anti-commutator of two SUSY generators from (2.1) becomes

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2 \begin{pmatrix} 2E & 0 \\ 0 & 0 \end{pmatrix}. \tag{2.2}$$

Ignoring the other generators for a moment, we can introduce creation and annihilation operators just like for a harmonic oscillator in regular quantum mechanics as

$$\begin{aligned} a &= \frac{1}{2\sqrt{E}} Q_1, \\ a^\dagger &= \frac{1}{2\sqrt{E}} \bar{Q}_{\dot{1}}, \end{aligned} \tag{2.3}$$

where the normalization was chosen for convenience. In terms of the creation and annihilation operators the algebra relation from (2.2) reads:

$$\begin{aligned} \{a, a^\dagger\} &= 1 \\ \{a, a\} &= \{a^\dagger, a^\dagger\} = 0. \end{aligned} \tag{2.4}$$

Note that according to (2.2) Q_2 and $\bar{Q}_2$ anti-commute and thus must be represented by 0.
Now to demonstrate the action of the SUSY generators, we assume a lowest helicity state such that $a\Omega = 0$. Acting with the creation operator $a^\dagger$ on Ω we generate a state with higher helicity. For example, take a scalar ϕ as the lowest helicity state. After acting with $a^\dagger$, we get a spin-1/2 state $\chi = a^\dagger \phi$. Additional action with the creation operator on χ gives $a^\dagger \chi = 0$. We see that we have a chiral doublet (ϕ, χ) under SUSY transformations. If we start from a spin-1/2 particle, say λ, and act on it with $a^\dagger$ we get a vector A_μ belonging in the vector doublet (λ, A_μ).
One of the reasons that we restrict ourself to the $\mathcal{N} = 1$ case is, besides simplicity, that for $\mathcal{N} = 2$ every matter multiplet (i.e.: containing no gauge fields) would have to start with a right-handed Weyl fermion λ_R that gets mapped into two scalars which in turn get mapped to a single left-handed fermion. This is inconsistent with the standard model of particle physics since the electron can either be left- or right-handed and according to this sits in the SU(2) singlet (right-handed) or doublet (left-handed). Under SUSY these two particles would need to be in a $\mathcal{N} = 2$ multiplet and thus have the same

quantum numbers. Due to this observation one usually studies $4d$ $\mathcal{N}=1$ models in order to relate the SM to a supersymmetric theory.
One question that arises is whether the particles in the multiplets have the same number of possible states, which would be expected. In order to show that this is the case we introduce the fermion number operator $(-1)^F$ with eigenvalue $+1$ for bosons and -1 for fermions. Due to the anti-commuting (fermionic) nature of the SUSY generators it follows that $(-1)^F Q_\alpha = -Q_\alpha(-1)^F$. Using the cyclic property of the trace [12], the following holds for any representation of the SUSY algebra (2.1):

$$Tr\left((-1)^F\{Q_\alpha,\bar{Q}_{\dot{\beta}}\}\right) = Tr\left(-Q_\alpha(-1)^F\bar{Q}_{\dot{\beta}} + Q_\alpha(-1)^F\bar{Q}_{\dot{\beta}}\right) = 0.$$

On the other hand we can also use the algebra relation directly to find

$$Tr\left((-1)^F\{Q_\alpha,\bar{Q}_{\dot{\beta}}\}\right) = 2\sigma^\mu_{\alpha\dot{\beta}}\cdot Tr\left((-1)^F P_\mu\right) = 0,$$

which for any non-zero momentum P_μ means that

$$Tr\left((-1)^F\right) = 0. \tag{2.5}$$

We see that there are an equal number of fermionic and bosonic states in any given finite dimensional representation.
This in fact gives a restriction to the possible actions we can write down in supergravity and thus also on our work to build a model for inflation: Since the chiral multiplet contains a fermion χ with four off-shell degrees of freedom and it satisfies $\partial\chi = 0$, the scalar in the doublet necessarily has to be a complex one (without any condition on the first derivatives).

2.2 Superspace

It is sometimes convenient to formulate a supersymmetric theory in an abstract space-time called superspace in which the SUSY transformations can be interpreted as *translations in fermionic space directions*. These additional directions are labelled θ and $\bar{\theta}$ and are Grassmann-valued[13]. Via the exponential map the non-trivial anti-commutator (for $\mathcal{N}=1$) of two SUSY generators from (2.1) gives the group element

$$g(x,\theta,\bar{\theta}) = e^{-xP-\theta Q-\bar{\theta}\bar{Q}}, \tag{2.6}$$

[12] $Tr(abc) = Tr(bca) = Tr(cab)$

[13] Grassman variables are anti-commuting numbers $ab=-ba$ and from this it also follows that $a^2=0$.

which has the same form as a usual translation but with the additional directions θ and $\bar{\theta}$. The group elements of the SUSY group, constructed in this manner, act on functions $f\left(g(x,\theta,\bar{\theta})\right) = f(x,\theta,\bar{\theta})$ on this abstract space-time.
The infinitesimal action of the SUSY generators on an arbitrary function in superspace is determined by

$$(\xi Q)f(x,\theta,\bar{\theta}) = \left[f\left(g(0,\xi,0)\cdot g(x,\theta,\bar{\theta})\right) - f\left(g(x,\theta,\bar{\theta})\right)\right]|_{\mathcal{O}(\epsilon)}.$$

Using the Baker-Campbell-Hausdorff formula [29] as well as the algebra relations (2.1) we find

$$(\xi Q f) = \left[f(x + i\xi\sigma^\mu\bar{\theta}, \xi + \theta, \bar{\theta}) - f(x,\theta,\bar{\theta})\right]|_{\mathcal{O}(\epsilon)}. \tag{2.7}$$

From this it is evident that the action of the SUSY generator Q also affects the x-coordinate. With (2.7) we can represent the generators as differential operators:

$$\begin{aligned} Q_\alpha &= \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu \\ \bar{Q}_{\dot{\alpha}} &= \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\bar{\sigma}^\mu_{\dot{\alpha}\alpha}\theta^\alpha\partial_\mu \\ P_\mu &= \frac{\partial}{\partial x^\mu}, \end{aligned} \tag{2.8}$$

which satisfy the algebra relations of (2.1).
Note that we chose to represent the SUSY generators by a *left translation*. In a completely analogous way we can represent Q and $\bar{Q}$ by a *right translation* and find:

$$\begin{aligned} D_\alpha &= \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu \\ \bar{D}_{\dot{\alpha}} &= \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\theta^\alpha\partial_\mu. \end{aligned} \tag{2.9}$$

Notably, they commute with the left translations.

2.2.1 Superfields

Our goal is to formulate a Lagrangian field theory on superspace. To that end we have to consider fields on superspace which we can later identify with fields in our ordinary space-time. Due to the Grassmann nature of the coordinates θ and $\bar{\theta}$, every field in superspace has an exact expansion that terminates at order $\theta^2\bar{\theta}^2$ [14]:

$$\Phi(x,\theta,\bar{\theta}) = \phi(x) + \theta\chi(x) + \bar{\theta}\bar{\zeta}(x) + \cdots + \theta^2\bar{\theta}^2 E(x). \tag{2.10}$$

[14] Note that $\theta^2 = \theta^\alpha\theta_\alpha$.

We now want to use such fields to formulate a Lagrangian theory where the action is of the form

$$S = \int d^4x \int d^2\theta \int d^2\bar{\theta}\mathcal{L}(x, \theta, \bar{\theta}).$$

Note that we have to integrate over the complete superspace, in particular including the coordinates θ and $\bar{\theta}$. Another important thing to notice is that by the rules of Grassmann integration only terms in $\mathcal{L}$ that are proportional to $\theta^2\bar{\theta}^2$ will contribute to the physical theory. All others yield 0 after the integration over the coordinates θ and $\bar{\theta}$.

A chiral superfield satisfies $\bar{D}_{\dot{\alpha}}\Phi = 0$ and due to this restriction the superspace expansion of a chiral superfield is

$$S(x, \theta, \bar{\theta}) = s(x) + \sqrt{2}\theta\psi(x) + \theta^2 F_s(x). \tag{2.11}$$

where on the right hand side we use coordinates $y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$. Recently it was shown that by using nilpotent or otherwise constrained chiral superfields it is possible to construct dS vacua in supergravity, see for example [33].

2.3 Supergravity

So far the SUSY transformations we looked at were global transformations with a fixed parameter ϵ for Q or correspondingly $\bar{\epsilon}$ for $\bar{Q}$. By allowing these parameters to depend on the space-time coordinates we get *local* supersymmetry. As we will see in a moment this already implies that the theory becomes a theory of gravity called *supergravity* [27]. Most of the work done here and in the next chapter is based on [28].
We can write the algebra (2.1) as commutators using the space-time dependent parameters. The part relevant for us reads:

$$\begin{aligned}
[\epsilon(x)Q, \bar{\epsilon}(x)\bar{Q}] &= 2\epsilon(x)\sigma^\mu\bar{\epsilon}(x)P_\mu, \\
[\epsilon(x)Q, \epsilon(x)Q] &= [\bar{\epsilon}(x)\bar{Q}, \bar{\epsilon}(x)\bar{Q}] = 0, \\
[P_m u, \epsilon Q] &= [P_\mu, \bar{\epsilon}\bar{Q}] = 0, \\
[P_\mu, P_\nu] &= 0,
\end{aligned} \tag{2.12}$$

from which we see that the commutator of 2 SUSY transformations amounts to a translation with time-dependent parameter $2\epsilon(x)\sigma^\mu\bar{\epsilon}(x)$. This means that our theory has to be invariant under diffeomorphisms and requires the metric $g_{\mu\nu}(x) = e^a_\mu(x)e^b_\nu(x)\eta_{ab}$ and the vielbein $e^a_\mu(x)$ to be dynamical fields, which is equivalent to having a theory of gravity. To write down a supersymmetric theory of gravity we need, in accordance with the spinor nature of the SUSY parameter, a gauge field that carries also spinor indices. This gauge field is called the *gravitino* and transforms like

$$\Psi_{\mu\alpha} \to \Psi_{\mu\alpha} + \partial\epsilon_\alpha(x). \tag{2.13}$$

The gravitino together with the vielbein $e^a_\mu(x)$ forms the multiplet (e^a_μ, Ψ_μ) under $\mathcal{N} = 1$ SUSY. The pure supergravity action contains only terms formed by this multiplet but is still quite convoluted, to leading order:

$$S = \frac{M_P^2}{2} \int d^4x\sqrt{-g}\left[R - \bar{\Psi}_\mu \gamma^{\mu\nu\rho}\left(\partial_\nu + \frac{1}{4}\omega_{\nu ab}\gamma^{ab}\right)\Psi_\rho + \mathcal{O}(\Psi^4)\right], \tag{2.14}$$

with the spin connection $\omega_{\mu ab}$ and the anti-symmetrised gamma matrices $\gamma^{\mu\nu\rho}$.

2.3.1 Matter in supergravity and the bosonic action

Supergravity does not yield any other particles besides the gravitino and the vielbein field, but we can add any number of chiral or vector multiplets that we wish to. These particles are not restricted in any way except for the Poincaré- and (local) supersymmetry of the theory. Note that in addition to these symmetries there are internal symmetries allowed, like the gauge groups of the standard model of particle physics. The complete possible action is written down for example in chapter 18 of [27] and the action is discussed in detail in that book. We will only focus on the parts relevant to our work here.

An important feature of a supergravity action is that the SUSY relates bosons to fermions, which allows us to focus on the bosonic part. The fermionic physics follows from the action of the SUSY transformations. For $4d$, $\mathcal{N} = 1$ supergravity, this means that the complete field content is given by the gravity doublet (e^a_μ, Ψ_μ) with N_c chiral matter multiplets (ϕ^I, χ^I) and N_v vector multiplets (λ^A, A^A_μ) corresponding to some gauge groups $G = G_1 \times G_2 \times \cdots \times G_{N_v}$. With this we can write down the *bosonic supergravity action*

$$\begin{aligned} S = \int d^4x\sqrt{-g}\Bigg[&\frac{M_P^2}{2}R - K_{I\bar{J}}\hat{\partial}_\mu\phi^I\hat{\partial}^\mu\bar{\phi}^{\bar{J}} - V_F - V_D \\ &- \frac{Re(f_{AB})}{4}F^A_{\mu\nu}F^{\mu\nu\,B} + i\frac{Im(f_{AB})}{4}F^A_{\mu\nu}\tilde{F}^{\mu\nu\,B}\Bigg], \end{aligned} \tag{2.15}$$

where the first term, proportional to the curvature scalar R, is also present in the Einstein-Hilber action [17]. The kinetic term of the scalar field is determined by the Kähler potential which is real valued and depends only on the scalar fields of the theory. It gives us the *Kähler metric*

$$K_{I\bar{J}} = \partial_I \partial_{\bar{J}} K \tag{2.16}$$

with the definition of the partial derivative

$$\partial_I = \frac{\partial}{\partial \phi^I}.$$

The gauge covariant derivatives

$$\hat{\partial}\phi^I = \partial_\mu \phi^I + i A_\mu^A K^{I\bar{J}} \partial_{\bar{J}} \mathcal{D}_A \tag{2.17}$$

with the real valued D-term $\mathcal{D}_A$ correspond to the gauge group G_A. Like the Kähler potential the D-term only depends on the scalar fields. Under a gauge transformation of G_A the scalar fields transform as

$$\phi^I \to \phi^I - i\theta^A K^{I\bar{J}} \partial_{\bar{J}} \mathcal{D}_A. \tag{2.18}$$

Next up are the two scalar potentials V_F and V_D where the first one is

$$V_F = e^{\frac{K}{M_P^2}} \left(K^{I\bar{J}} D_I W \overline{D_J W} - 3\frac{|W|^2}{M_P^2} \right), \tag{2.19}$$

which is completely determined by the Kähler potential $K(\phi^I, \bar{\phi}^{\bar{J}})$, and the superpotential $W(\phi^I)$, which is a holomorphic function of the scalar fields. The *Kähler covariant derivative* $D_I W$ is given as

$$D_I W(\phi) = \partial_I W + \frac{1}{M_P^2} W \partial_I K. \tag{2.20}$$

The second scalar potential is called the D-term potential and is determined by the holomorphic gauge-kinetic function $f_{AB}(\phi^I) = f_{BA}(\phi^I)$ and reads

$$V_D = \frac{1}{2} \left(Re(f) \right)^{-1\,AB} \mathcal{D}_A \mathcal{D}_B. \tag{2.21}$$

An important remark is that, since the gauge-kinetic function f_{AB} determines the kinetic terms of the vector fields (as we will see shortly), they have to be positive definite and thus $V_D \geq 0$. In the second line we have the kinetic terms of the gauge fields determined by the field strength tensor and its dual:

$$\begin{aligned} F_{\mu\nu}^A &= \partial_\mu A_\nu^A - \partial_\nu A_\mu^A \\ \tilde{F}^{\mu\nu\,A} &= \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}^A. \end{aligned} \tag{2.22}$$

In conclusion, $4d\ \mathcal{N}=1$ supergravity action is determined by two real valued functions $K(\phi^I, \phi^{\bar{J}})$ and $\mathcal{D}_A(\phi^I, \bar{\phi}^{\bar{J}}$ as well as a pair of holomorphic functions $W(\phi^I)$ and $f_{AB}(\phi^I)$.
As we will discuss later, we will restrict our theory further by requiring it to be a type IIB compactification of a string theory. In particular, in the specific way we choose to do our compactification it will not give rise to any gauge fields[15] and thus the action we will be discussing reads

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - K_{I\bar{J}}\partial_\mu\phi^I\partial^\mu\bar{\phi}^{\bar{J}} - V_F(\phi^I, \bar{\phi}^{\bar{J}})\right]. \tag{2.23}$$

This action will be invariant under SUSY if it is combined with the corresponding fermionic action.

2.3.2 Supersymmetry breaking

Just like in the Higgs-mechanism, SUSY can be preserved or broken at a minimum of the scalar potential V_F of (2.23). In the case of a pure F-term potential SUSY remains unbroken if

$$D_I W = \partial_I W + \frac{1}{M_P^2}W\partial_I K = 0 \tag{2.24}$$

at the minimum of the scalar potential. With this restriction the value of the potential at the minimum is

$$V_F|_{D_I W=0} = -3e^{\frac{K}{M_P^2}}\frac{|W|^2}{M_P^2} \leq 0. \tag{2.25}$$

Remembering our discussion of chapter 1.5 we note that the resulting cosmological constant of such a potential is negative or zero, which contradicts our observations (and would correspond to an AdS or Minkowski space-time respectively). We conclude that SUSY has to be broken to explain today's cosmological constant. It is only necessary that supersymmetry is broken for energies below a certain threshold $E_{break} > E_{LHC} \approx 10^4 GeV$ which implies $exp[K/M_P^2]K^{I\bar{J}}D_I W\overline{D_J W} > (10^4 GeV)^4$ and to explain our cosmological constant we find

$$V_{today} \approx 10^{-120}M_P^4 \approx (2.4\cdot 10^{-12}GeV)^4 \approx e^{\frac{K}{M_P^2}}\left(K^{I\bar{J}}D_I W\overline{D_J W} - 3\frac{|W|^2}{M_P^2}\right),$$

[15] This is not general for type IIB theories and depends on specific choices during the process of compactification.

which means that the terms must cancel very precisely.
From our condition for unbroken SUSY (2.24) we can derive a more convenient condition to find the minimum of the scalar potential than calculating $\partial_I V_F$. Differentiating the scalar potential V_F (2.19) we find:

$$\partial_I V_F = e^{\frac{K}{M_P^2}} \left[(\partial_I K) \left(K^{I\bar{J}} D_I W \overline{D_J W} - 3\frac{|W|^2}{M_P^2} \right) + \partial_I \left(K^{I\bar{J}} D_I W \overline{D_J W} \right) - 3\partial_I \frac{|W|^2}{M_P^2} \right].$$

Remembering our condition for a SUSY minimum (2.24)

$$D_I W = \partial_I W + \frac{1}{M_P^2} W \partial_I K = 0$$

and applying this to the above we find:

$$\begin{aligned} \partial_I V_F &= e^{\frac{K}{M_P^2}} \left(-3\frac{|W|^2}{M_P^4} \partial_I K - 3\partial_I \frac{|W|^2}{M_P^2} \right) \\ &= -\frac{3}{M_P^2} e^{\frac{K}{M_P^2}} \underbrace{\left(\frac{|W|^2}{M_P^2} \partial_I K + \partial_I |W|^2 \right)}_{\propto D_I W} = 0. \end{aligned}$$

So it follows that for SUSY to remain unbroken we require[16]:

$$D_I W = 0 \;\Rightarrow\; \partial_I V_F = 0. \tag{2.26}$$

In conclusion we only need to solve $D_I W = 0$ to find the minimum of the scalar potential. Furthermore, remembering our previous remark that the scalar potential is positive semi-definite we see that the minimum is stable for $V|_{min} = 0$. If the minimum of the scalar potential is < 0, we have AdS geometry and the masses are allowed to be negative as long as they are above the Breitenlohner-Freedman bound, which is the case as long as $D_I W = 0$.

2.3.3 Planck suppressed operators and the eta-problem

In chapter 1.8 we looked at Starobinsky-type inflation, an interesting model that we want to consider further later on. In that model we introduced a term proportional to R^2 into the action that was Planck suppressed, with no justification. In supergravity the Kähler potential receives corrections of the form

$$c_n \frac{(\phi\bar{\phi})^n}{M_P^{2n-2}}$$

[16] Note that the inverse is not true: $\partial_I V_F \not\Rightarrow D_I W = 0$.

(with some constant c_n) and since the scalar potential (2.19) has a pre factor $exp[K/M_P^2]$ the corrections in K lead to a change in V_F of form:

$$V \to V + \sum_n \delta V_n \quad \text{with} \quad \delta V_n \propto c_n \frac{(\phi\bar{\phi})^n}{M_P^{2n-2}} V. \tag{2.27}$$

This means that a correction of order $n = 2$ leads to a correction of order c_2 to the slow roll parameter η_V as defined in (1.45). From this we see that in supergravity Planck suppressed operators cannot automatically be dismissed.

2.4 Supergravity from string theory

Why string theory? As mentioned above supergravity does not contain any physical restrictions on the particles we consider, besides the symmetries. Moreover it does not give us any particles to work with. In supergravity the way to get inflation is to assume a suitable scalar particle in an equally suitable potential and write down the solution. By restricting ourselves to a theory of supergravity that arises as the low energy limit of a string theory we get particles as candidates for our inflaton[17] and also restrict the theory by the symmetries and properties of string theory. Furthermore string theory is UV complete and thus we can, in principle, calculate corrections from the Planck suppressed operators that we discussed above. Such operators might either allow inflation by their additional dynamics or spoil it with their presence.
Discussing inflation in the context of such a theory may also allow us, with improving data from cosmological experiments, to actually check some properties of string theories and see if it is a good candidate for a high energy theory. For example, as discussed in [30], we expect the value of the tensor to scalar ratio r to be restricted to much lower values in the near future. The prognosis for these advances is shown in figure 9 and we expect to either confirm non-zero r or get at least a greatly improved bound, further restricting possible models for inflation.

2.4.1 Energy scales and compactification

Since it is unfortunately too hard to completely solve string theories without restriction, we have a hard time building a model that can give predictions for cosmology. Luckily in the low energy limit of a string theory, below the string scale $E \ll M_S = 1/\sqrt{\alpha_s}$, where $\sqrt{\alpha_s}$ is the string length, string theories reduce to $10d$ supergravities with particles instead of strings.

[17] The particle whose dynamics are responsible for inflation.

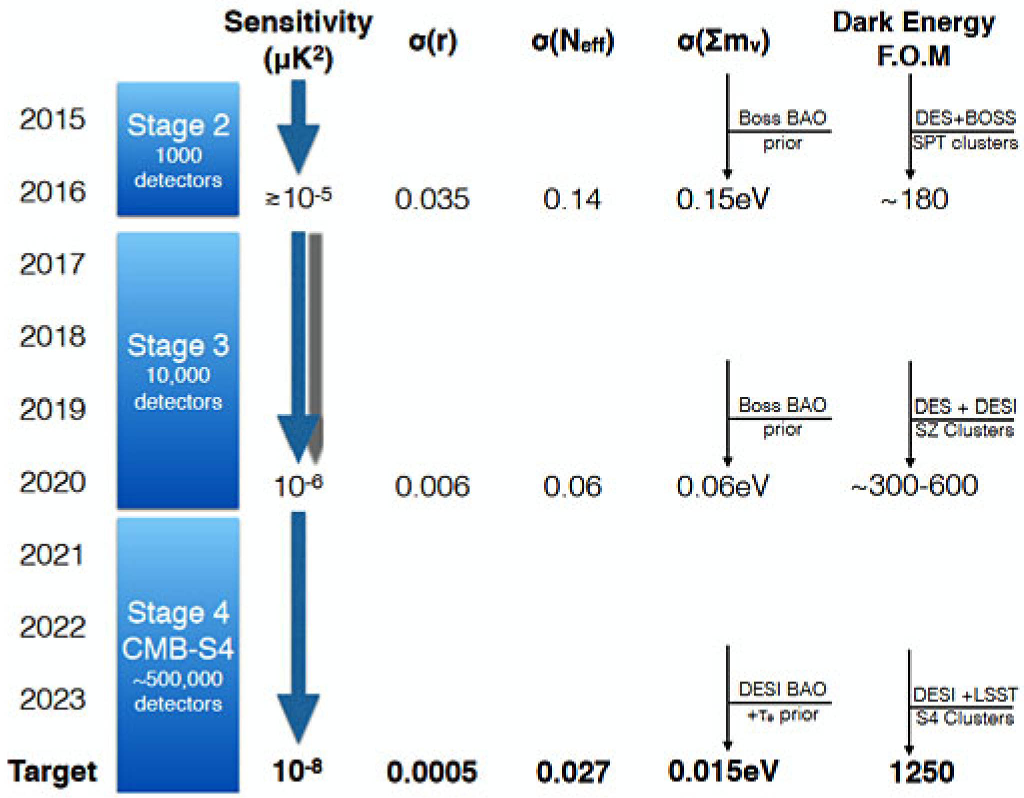

Figure 9: The prognosis for the improvement of cosmological data taken from [30]. Most interesting for analysing inflation models is the column labelled $\sigma(r)$, which gives the one sigma confidence for the value of the tensor to scalar ratio r.

Another important scale is the Kaluza-Klein (KK) scale that arises from the compactification of a $10d$ supergravity to a $4d$ one. For each dimension we compactify, we get a scale given by the radius of the rolled-up dimension: $1/R$. This is relevant because due to the compactification we will get new particles. For example consider a $5d$ space-time $\mathbb{R}^{3,1} \times S^1$ where the circle S^1 has radius R. The space-time can be split up into x^μ and the additional coordinated $y = y + 2\pi R$. A real scalar field on this $5d$ space-time has a Fourier expansion

$$\phi(x^\mu, y) = \sum_{k\in\mathbb{Z}} \phi_k(x^\mu) e^{ik\frac{y}{R}},$$

where the reality condition of the field ϕ also implies $\phi_{-k} = \bar{\phi}_k$ for the coefficients. The action of the $5d$ scalar gets reduced to a $4d$ action by integrating over the additional coordinate:

$$\begin{aligned}
S &= -\int d^4x \int dy \partial_M \phi \partial^M \phi \\
&= -\int d^4x \int dy \left(\partial_\mu \phi \partial^\mu \phi + \partial_y \phi \partial^y \phi\right) \\
&= -\int d^4x \int dy \sum_{k,l} \left(\partial_\mu \phi \partial^\mu \phi - \frac{kl}{R^2} \phi_k \phi_l\right) e^{iy\frac{k+l}{R}} \\
&= -\int d^4x (2\pi R) \sum_k \left(\partial_\mu \phi_k \partial^\mu \phi_{-k} + \frac{k^2}{R^2} \phi_k \phi_{-k}\right).
\end{aligned}$$

This final $4d$ action corresponds to the action of a massless $4d$ scalar and an infinite number of massive scalar particles, called the Kaluza-Klein tower, with masses set by the inverse radius of compactification $M_{KK} = 1/R$. These particles are usually neglected by restricting our theory to energies below the KK-scale of $M_{KK} = 1/R$.

The familiar Planck scale in $4d$ is in this case not independent but is related to the compactification volume $vol_6 = \mathcal{V}(\alpha_s)^3$ such that $M_P = g_S^{-1/4} \mathcal{V}^{1/2} M_s$, with the string coupling $g_s = e^\phi$, in which the real scalar field ϕ is called the *dilaton*. In particular our $10d$ supergravities are derived in the limit of small string couplings $g_s \ll 1$ and large internal volumes $\mathcal{V} \gg 1$.

In conclusion, to trust our theory, we require

$$E \ll M_{KK} \ll M_s \ll M_P \approx 2.4 \cdot 10^{18} GeV.$$

These restrictions are valid in general for supergravity theories that stem from a string compactification and in particular need to hold during inflation. Because of this we are allowed to neglect very heavy particles with masses way above the Hubble scale $H \approx V_{inf}^2 / M_P$ like the KK-modes. Other particles can be around that scale but still contribute. Our model below will use scalar particles from a compactification and we will try to tune our model such that one of them is lighter than the rest and call it the inflaton that is responsible for inflation.

2.4.2 The moduli of type IIB compactification

For the rest of this work we will deal with models from a specific type of string theory namely type IIB [31]. The low energy limit of this theory is a $10d$ $\mathcal{N} = 2$ supergravity which in turn gets compactified to a $4d$ $\mathcal{N} = 1$ theory. This theory contains 2 10d scalars, the dilaton e^ϕ and the axion C_0. It is possible to combine them into a complex scalar

$$S = C_0 + ie^{-\phi}, \tag{2.28}$$

called the axio-dilaton. Two other objects that appear in this theory are the two-forms B_{MN} and C_{MN}, however we will compactify in such a way that they do not give rise to any scalar particles. The final object in the theory is the 4-form C_{MNOP} which will give rise to scalar fields in $4d$. Those scalars can be combined with the geometric moduli[18] from the metric to give complex scalars for our theory.
Other than the aforementioned fields the only remaining object we have is the $10d$ metric g_{MN} that will give us additional scalar fields. How this plays out exactly is beyond the scope of this work but we shall show in the toy example of the compactification of a torus what types of fields arise. A torus $T^2 = S^1 \times S^1$ can be viewed as a parallelogram as shown in figure 10, with

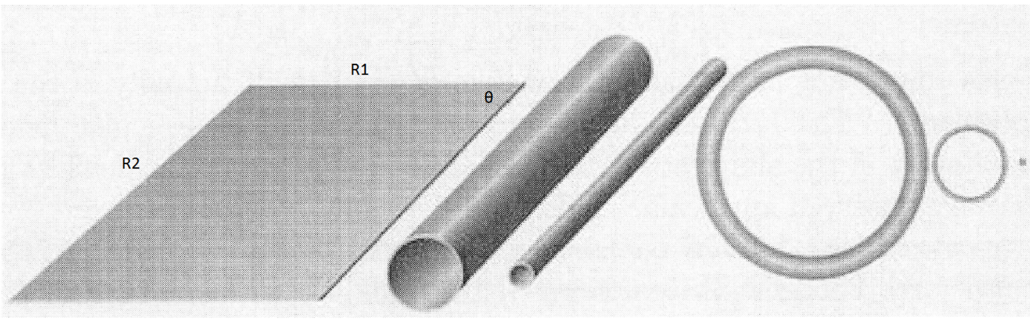

Figure 10: We view a torus as a parallelogram by identifying two opposing sides, practically rolling it up to a cylinder at first and then by identifying the boundary circles to a torus. Making both radii small we eventually compactify this (internal) volume away. Image taken from [32]
.

the sides identified. With this picture in mind it is easy to see that we can describe a torus with only 3 real parameters: The length of the sides R_1 and R_2 as well as the angle θ between two sides. Another way to categorize a torus with these parameters is by noting that the volume is given by $R_1 \cdot R_2$ and the shape is given by R_1/R_2 and θ. Compactifying a dynamical metric $g_{MN}(x^\mu, y^I)$ on a torus to $4d$, we will find that the size and shape parameters belong to the internal metric $g_{y^I y^J}$ and will manifest themselves as scalar fields.
In more sophisticated compactifications, one usually uses Calabi-Yau manifolds [34]. However one can construct singular limits of CY-manifolds from orbifolds of three T^2s. For the case of an $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold one relates the tori via a $\mathbb{Z}_3$ symmetry which leaves us again with the above 3 parameters.
As hinted above, the volume $R_1 \cdot R_2$ combines with a scalar from the 4-form

[18]Scalar fields that arise due to a compactification are called *moduli*.

C_{MNOP} to give the complex modulus T, called a *Kähler-* or *volume-modulus*. The two shape parameters on the other hand combine to give the *complex structure* modulus U. In general there can be multiple moduli T^i and U^i, depending on the compactification. This simple compactification scheme is called the STU-model, after the names of the appearing moduli. In terms of supergravity, this theory is given by the Kähler and superpotentials:

$$\begin{aligned} &K = -ln\left(-i(S-\bar{S})\right) - 3ln\left(-i(T-\bar{T})\right) - 3ln\left(-i(U-\bar{U})\right), \\ &W = 0. \end{aligned} \tag{2.29}$$

A note on the regime of our theory: The previous conditions of $g_S \ll 1$ and $\mathcal{V} \gg 1$ translate to our moduli to require $Im(S) \gg 1$ and $Im(T) \gg 1$ respectively.

2.4.3 The Gukov-Vafa-Witten flux superpotential

The above discussion leads us to a very simple model of supergravity from the compactification of string theory. In fact, due to $W = 0 \Rightarrow V = 0$, and thus, inflation is impossible. An important idea, discussed in [35], is to introduce *fluxes*, similar to the fluxes in electro-magnetism derived from the field strength tensor $F_{\mu\nu}$. If we consider a simple torus compactification we have

$$\begin{aligned} &-\frac{1}{4}\int d^4x \int d^2y\sqrt{-g}F_{MN}F^{MN} \\ &-\frac{1}{4}\int d^4x \int d^2y\sqrt{-g}\left(F_{\mu\nu}F^{\mu\nu} + F_{y^1y^2}F^{y^1y^2}\right). \end{aligned}$$

As long as $F_{y^1y^2}$ is non-vanishing, the second term will give a contribution to the scalar potential. This contribution is connected to the moduli from the metric since the metric is involved in the contractions: $F^{y^1y^2} = g^{y^1y^i}g^{y^2y^j}F_{y^iy^j}$.

In string theory, this method was first done by Gukov, Vafa and Witten in [36]. In our compactification scheme we have the 2 two-forms B_{MN} and C_{MN} that gave no scalar particles due to our choice of orientifold projection. However we still can use them to get fluxes because

$$G_{MNO}(S) = \partial_{[M}B_{NO]} - S\partial_{[M}C_{NO]} \tag{2.30}$$

can be non-vanishing.

We can now use these non-zero fluxes along the internal directions to get contributions to the scalar potential. With the field strength from (2.30) and the holomorphic 3-form $\Omega_{J_1J_2J_3}(U^i)$, containing the structure moduli U^i, integrated over the internal CY-manifold, we get from a proper reduction the supergravity given in terms of the Kähler- and superpotential as

$$\begin{aligned} K &= -ln\left(-i(S-\bar{S})\right) - 3ln\left(-i(T-\bar{T})\right) - 3ln\left(F(U^i,\bar{U^{\bar{\imath}}})\right) \\ W &= \int_{CY} d^6y\sqrt{g_{CY}}G_{I_1I_2I_3}(S)g_{CY}^{I_1J_1}g_{CY}^{I_2J_2}g_{CY}^{I_3J_3}\Omega_{J_1J_2J_3}(U^i) =: W_{GVW}(S,U^i), \end{aligned} \tag{2.31}$$

where, in case of multiple complex structure moduli U^i,

$$F(U^i,\bar{U^{\bar{\imath}}}) = \int_{CY} d^6y\sqrt{g_{CY}}\Omega_{I_1I_2I_3}(U^i)g_{CY}^{I_1J_1}g_{CY}^{I_2J_2}g_{CY}^{I_3J_3}\bar{\Omega}_{J_1J_2J_3}(\bar{U^{\bar{\imath}}}). \tag{2.32}$$

To illustrate an important feature, let us take a look at the situation for one Kähler modulus T. We find the Kähler-covariant derivative and the $T\bar{T}$ component of the Kähler metric to be [19]

$$\begin{aligned} D_TW &= \partial_TW + W\partial_TK = -\frac{3W}{T-\bar{T}} \\ K_{T\bar{T}} &= \partial_T\partial_{\bar{T}}K = -\frac{3}{(T-\bar{T})^2}, \end{aligned}$$

which leads us to

$$K^{T\bar{T}}D_TW\overline{D_TW} = -\frac{(T-\bar{T})^2}{3}\left(-\frac{3W}{(T-\bar{T})}\right)\left(\frac{3\bar{W}}{(T-\bar{T})}\right) = 3|W|^2$$

and using this result we find that $K^{T\bar{T}}D_TW\overline{D_TW}$ cancels the $-3|W|^2$ in equation (2.19) to yield

$$V = e^K\left(K^{S\bar{S}}D_SW\overline{D_SW} + K^{U^i\bar{U}^{\bar{\imath}}}D_{U^i}W\overline{D_{U^i}W}\right). \tag{2.33}$$

The feature of T to cancel the $-3|W|^2$ term of the scalar potential is called *no-scale property*. From the above we also see that the Kähler metric governs the kinetic terms of the moduli and thus has to be positive definite, making V the sum of two positive definite parts. Finally the Kähler modulus T enters the scalar potential V only via the pre-factor $exp[K]$ and thus is $\propto (i(T-\bar{T}))^{-3}$, meaning that the volume modulus T will be minimized at $Im(T) \to \infty$, which decompactifies our internal dimensions and thus the whole theory. For the case that the other moduli have an extremum $D_SW = D_{U^i}W = 0$, the scalar potential has a minimum $(\partial_T)^nV = 0$ and the T modulus remains a flat direction and massless. Furthermore, since $D_SW = D_{U^i}W = 0$ provides the same number of constraints as the complex fields have degrees of freedom, we generically find solutions where all fields take a fixed value.

[19]From here on we will set $M_P = 1$ whenever possible.

SUSY breaking In this model supersymmetry remains unbroken as long as $D_T W = -3W/(T - \bar{T}) = 0$, meaning that $W = 0$ as well. If this holds true, all derivatives are zero and we have a stable supersymmetric solution. However for $W \neq 0$ SUSY is broken but all masses are still ≥ 0 because we already saw that $V \geq 0$, so any critical point with $V = 0$ is a global minimum and thus the masses likewise have to be ≥ 0.

2.4.4 The KKLT construction of dS vacua

To match current data we need to achieve $V_{min} > 0$, corresponding to a dS space-time. This was first done by Kachru, Kallosh, Linde and Trivedi who introduced an *uplift term* to the superpotential in [37].
We will only outline how this works by continuing our simple example with one T modulus from above. As mentioned, the T modulus remains a flat direction in the minimum. However this is not allowed by string theory because no continuous global symmetries are possible in its framework [31]. The Kähler- and superpotential of equation (2.31) are invariant under a shift symmetry $Re(T) \to Re(T) + c,\ c \in \mathbb{R}$. Symmetries of this type are broken to discrete symmetries via non-perturbative effects and the potentials receive corrections:

$$\begin{aligned} K &= -ln\left(-i(S - \bar{S})\right) - 3ln\left(-i(T - \bar{T})\right) - 3ln\left(F(U^i, \bar{U}^{\bar{i}})\right), \\ W &= W_{GVW}(S, U^i) + A(S, U^i)e^{iaT}, \end{aligned} \tag{2.34}$$

where $a \in \mathbb{R}$. Since we require $Im(T) \gg 1$ and thus $exp[iaT] \ll 1$, we can set U^i and S to their minimum values, essentially integrating those fields out of the theory. With this we have

$$\begin{aligned} A(U^i, S) &= A(U^i_{min}, S_{min}) = const \\ W_0 &:= W_{GVW}(S_{min}, U^i_{min}) = const \end{aligned}$$

and are left with a single field model given in terms of Kähler- and superpotential:

$$\begin{aligned} K &= -3ln\left(-i(T - \bar{T})\right) \\ W &= W_0 + Ae^{iaT}, \end{aligned} \tag{2.35}$$

where we choose $W_0,\ A \in \mathbb{R}$ as a simplification. As usual we look at the Kähler-covariant derivative which reads

$$D_T W = iaAe^{iaT} - 3\frac{W_0 + Ae^{iaT}}{T - \bar{T}} \stackrel{!}{=} 0.$$

Defining $T := b + i\rho$ we look at the real and imaginary part separately:

$$
\begin{aligned}
0 &= Re(D_T W) = -aAe^{-a\rho}Im(e^{iab}) - 3\frac{Ae^{-a\rho}Im(e^{iab})}{2\rho} \\
0 &= Im(D_T W) = aAe^{-a\rho} + 3\frac{W_0 + Ae^{-a\rho}}{2\rho}.
\end{aligned}
$$

The real part is solved by $b = 0$ since $Im(exp[i \cdot 0]) = Im(exp[0]) = Im(1) = 0$, while the imaginary part has an implicit solution given by

$$
W_0 = -Ae^{-a\rho_{min}}\left(1 + \frac{2}{3}a\rho_{min}\right) \neq 0.
$$

This means that, since $\rho_0 \gg 1$, we require $|W_0| \ll 1$ in order for our theory to be trustworthy. These are the conditions for a supersymmetric minimum in our theory and we need to look at the value of the scalar potential (2.19) at the minimum:

$$
\begin{aligned}
V_{min} &= e^K\left(K^{T\bar{T}}D_T W\overline{D_T W} - 3|W|^2\right) \\
&= -3e^K|W|^2 = -3\frac{1}{8\rho_{min}^3}\left|W_0 + Ae^{-a\rho_{min}}\right|^2 \\
&= -\frac{3}{8\rho_{min}}\left|\frac{2}{3}\rho_{min}aAe^{-a\rho_{min}}\right|^2 \\
&= -\frac{a^2A^2e^{-2a\rho_{min}}}{6\rho_{min}} < 0.
\end{aligned} \tag{2.36}
$$

We need to check that this is in fact the only minimum because otherwise, another positive minimum would be possible. This can be achieved by plotting the first line in (2.36) as shown in figure 11.
Therfore it is non-trivial to get a dS vacuum and we need to modify our model somewhat to arrive at a theory that can describe our observations of a small, positive cosmological constant. We will follow [37] and add an uplift term to our superpotential. This term can arise in string theory from anti-D3-branes ($\overline{D3}$). In terms of our supergravity language, this can be achieved by adding a chiral multiplet N, where the scalar part of N is actually given by the fermion bilinear $\bar{\chi}\chi$. The potentials for this models are:

$$
\begin{aligned}
K &= -3ln\left(-i(T - \bar{T})\right) + N\bar{N}, \\
W &= W_0 + Ae^{iaT} + \mu N = W_{KKLT} + \mu N,
\end{aligned} \tag{2.37}
$$

with some number μ. To get a bosonic answer we need to set $N = 0$ at the end of our considerations. With this in mind we find:

$$
\begin{aligned}
D_N W &= \mu + W\bar{N} = \mu, \\
D_T W &= D_T W_{KKLT},
\end{aligned}
$$

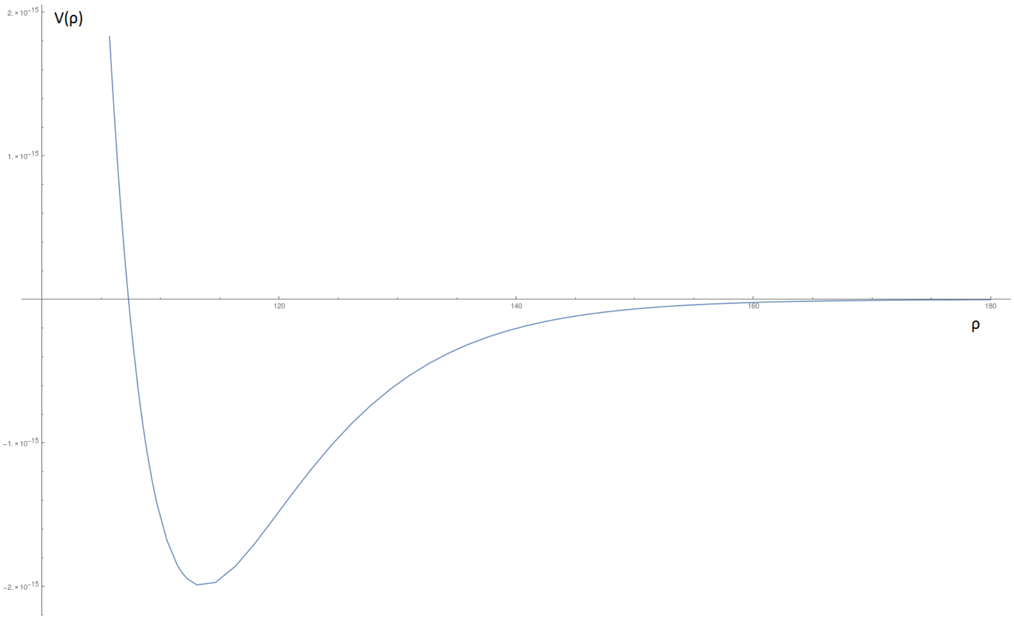

Figure 11: The scalar potential of (2.36) plotted for $a = 0.1$, $A = 1$ and $W_0 = -10^{-4}$. We see that no other minimum exists and that the minimum we found is, in fact, negative.

which immediately tells us that there are no supersymmetric solutions for $\mu \neq 0$. Our discussion about $D_T W$ remains unchanged because we have to set $N = 0$. The scalar potential (2.19) becomes

$$\begin{aligned} V &= \frac{1}{8\rho^3}\left(K^{T\bar{T}} D_T W_{KKLT} \overline{D_T W_{KKLT}} + |\mu|^2 - 3|W_{KKLT}|^2\right) \\ &= V_{KKLT} + \frac{|\mu|^2}{8\rho^3}, \end{aligned} \tag{2.38}$$

in the presence of an uplift. Looking at this we see that our efforts have worked out. The added positive part to the otherwise negative minimum of the scalar potential can *lift* our potential above zero as shown in figure 12, thereby yielding a positive cosmological constant. Note that the SUSY-breaking is set by μ and the cosmological constant is given by $V_{min} + \frac{|\mu|^2}{8\rho^3}$ with V_{min} from equation (2.36). We can fine-tune our parameters to fix both the value of the cosmological constant as well as the scale at which supersymmetry breaks.

On the topic of the "uplifting field": We argued before that the superfield N does not give rise to a bosonic part which might contribute. This

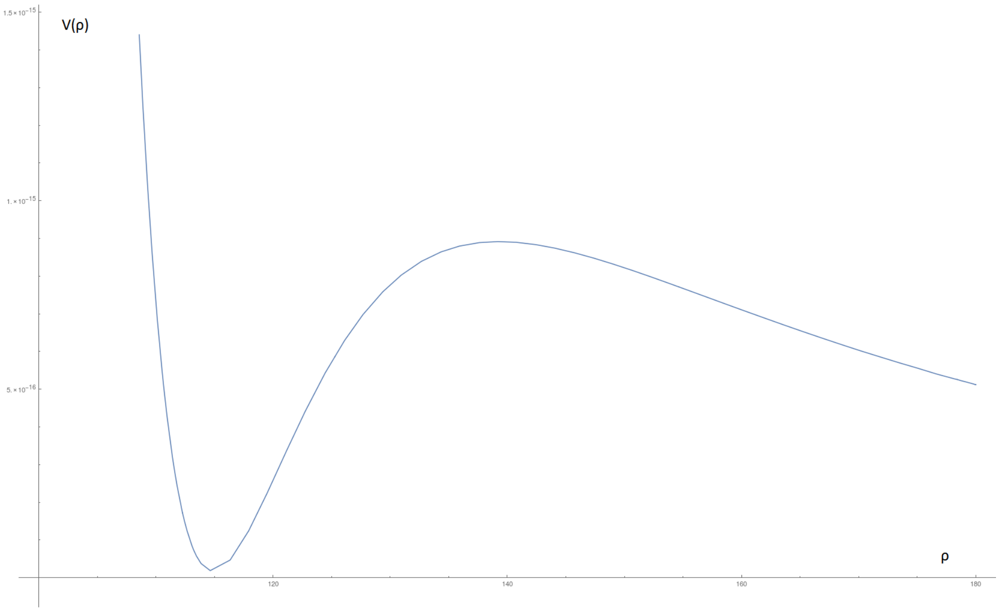

Figure 12: The scalar potential of (2.38) with the uplift term for $|\mu|^2 = 2.4 \cdot 10^{-8}$. It is evident that it is possible to get a positive cosmological constant with this method.

allowed us to set $N = 0$ after all of our calculations. One way to achieve this is by using *constrained* superfields. One particular way was shown in [33] where nil-potent superfields, for which $N^2 = 0$ holds, were used. Consider for example the chiral field

$$N = n + \sqrt{2}\theta\psi + \theta^2 F_n, \quad N^2 = 0. \tag{2.39}$$

The condition in terms of the component fields reads

$$N^2 = n^2 + 2\sqrt{2}\theta\psi + \theta^2 \left(2nF_n - \psi^2\right) = 0$$

and we see from the highest component term that we can write the scalar component as

$$n = \frac{\psi^2}{2F_n}, \tag{2.40}$$

which actually solves all three equations. This is what we meant by stating that there is no bosonic degree of freedom in N. However, it also implies that supersymmetry is non-lineary realized. The corresponding action was first discussed by Vuolkov and Akulov in [38] and [39], while the connection between the VA-action and the fermion in the nil-potent superfield was established in [40].

3 Models for Starobinsky-type inflation in Supergravity

In this work we use the *saxions* $Im(S)$ and $Im(U)$ to build a model for inflation. Usually one uses one of the *axions*, particles that do not appear in the Kähler potential as the inflaton since they do not receive potentially problematic Planck suppressed corrections and retain their shift symmetry. We hope that in an explicit compactification of string theory the saxions can be used as inflatons and produce the required scalar potential.
Armed with the tools of the previous chapters our goal is now to investigate a model using primarily $Im(U)$ as inflaton and try to find inflationary dynamics that resemble the Starobinsky type of inflation of chapter 1.8. We are not looking to reproduce the potential (1.52) exactly but only its general form is important, constants and the like can be adjusted to fit our needs. We want to have a positive minimum, corresponding to a positive constant term in the expansion of the scalar potential and an infinite (or at least long enough) flat part of the potential for inflation to happen.
We begin by stating the form of the Kähler- and superpotential we want to consider. They arise from compactifications of type IIB string theory on CY_3 orientifolds. The general form of the potentials was first derived in [41] and for our specific model we have, with a single Kähler modulus T:

$$\begin{aligned} K =& -ln\left(-i(S-\bar{S})\right) - 3ln\left(-i(T-\bar{T})\right) \\ & - ln\left((-i)^3\kappa_{ijk}(U^i-\bar{U}^i)(U^j-\bar{U}^j)(U^k-\bar{U}^k)\right) \\ W =& \left[\begin{pmatrix} f_0 \\ f_{1i} \\ f_2^i \\ f_3 \end{pmatrix} - S\begin{pmatrix} h_0 \\ h_{1i} \\ h_2^i \\ h_3 \end{pmatrix}\right] \cdot \begin{pmatrix} 1 \\ U^i \\ \kappa_{ijk}U^jU^k + g_i \\ \kappa_{ijk}U^iU^jU^k + g_iU^i + iq \end{pmatrix} \end{aligned} \tag{3.1}$$

where all parameters are real and $\kappa_{ijk} \in \mathbb{R}$ are the intersection numbers that encode the relation of the Us. For one light U we find for the potentials [20]

$$\begin{aligned} K &= -ln\left(-i(S-\bar{S})\right) - 3ln\left(-i(U-\bar{U})\right) - 3ln\left(-i(T-\bar{T})\right), \\ W &= \sum_{\alpha=0}^{3} a_\alpha U^\alpha + S\sum_{\alpha=0}^{3} b_\alpha U^\alpha, \end{aligned} \tag{3.2}$$

where the parameters were combined for each order and renamed for convenience. Note that for the simple STU-model all terms need to be real and thus $q = 0$.

[20] Here α is not numbering the fields but gives the power of the field.

In (3.1) we can assume that some of the fields U^i are heavy compared to the rest and they do not produce any dynamics during inflation. In that case we can integrate them out, effectively setting them to a constant value, with this we can find a general potential of form

$$\begin{aligned} K &= -ln\left(-i(S-\bar{S})\right) - p \cdot ln\left(-i(U-\bar{U})\right) - 3ln\left(-i(T-\bar{T})\right), \\ W &= \sum_{\alpha=0}^{n} a_\alpha U^\alpha + S\sum_{\alpha=0}^{n} b_\alpha U^\alpha, \end{aligned} \tag{3.3}$$

where $p = n = 1, 2, 3$, for the light modulus U. In our work here we will use the imaginary part of this light modulus as the inflaton and investigate its scalar potential for possible inflationary dynamics.
We proceed by taking different values and minimizing the directions by setting $D_I W = 0$. This method gives us conditions on parameters or fields which reduce the amount of free parameters. After using up all equations we are left with an expansion of the scalar potential in orders of U which we then review for compatibility with inflation.

3.1 Remarks about the model

Kinetic terms of the moduli It is important to note that the kinetic terms of the moduli, as seen in (2.23), are canonical when setting the moduli to $exp[\pm\phi]$. For example, consider a single complex structure modulus $Im(U) \to exp[\pm\phi]$. From (2.23) we find by using the Kähler potential of (3.3) we find

$$\begin{aligned} K_{U\bar{U}}\partial_\mu U\partial^\mu U &= \frac{-1}{((U-\bar{U}))^2}\partial_\mu U\partial^\mu U \approx -e^{2\phi}\left(e^{-\phi}\partial_\mu\phi\right)\left(e^{-\phi}\partial^\mu\phi\right) + \cdots \\ \Rightarrow K_{U\bar{U}}\partial_\mu U\partial^\mu U &\approx -\partial_\mu\phi\partial^\mu\phi + \cdots, \end{aligned}$$

where the $\cdots$ are contributions from the real part of the modulus U. This together with the fact that the superpotential is a polynomial makes it natural to try to relate this model with classical Starobinsky inflation.

Relation to the Starobinsky potential In the classical model of Starobinsky type inflation of chapter 1.8 we found a potential with the form given by

$$1 + e^{-2\sqrt{\frac{2}{3}}\phi} - 2e^{-\sqrt{\frac{2}{3}}\phi}.$$

In order to identify our modulus with the scalar particle appearing here we substitute

$$Im(U) \to e^{\phi}.$$

Other, higher negative-order terms that might appear in the expansion have no corresponding term in the Starobinsky model of 1.8 but do not spoil inflation. Since we do not seek to reproduce the Starobinsky model exactly but only the general shape of its potential those terms only have to behave and we will not require them to vanish.
Positive order terms however need to vanish because they diverge and spoil the form of the potential, preventing a long flat part for slow-roll inflation.

General restriction on the potential Looking at the scalar potential (2.19) we see that the pre-factor is $\propto U^{-p}$ and thus, since we do not want the potential to grow at infinity, we see that the rest of the potential can at most be of power U^p. This gives a restriction on the maximal power n in the superpotential W, which has to satisfy $n \leq p/2$. As a sanity check we still will consider these models in our work below but we expect them to fail. In a final attempt to build a model for inflation we will look at models where the above condition is not satisfied in order to have a positive power in the series expansion of the scalar potential. There we will try to have a long, flat part of the potential, suitable for inflation, but with monotonically growing potential for $\phi \to \pm\infty$.

The imaginary term in the potential The observant reader might have noticed that there is a term proportional to iq, with $q \in \mathbb{R}$ appearing in (3.1) that we then chose to neglect. One does not expect this term to change the results for the simple model and in fact it is often not considered for STU-models. We expect that the parameter q cannot change our conclusions in a meaningful way due to the reality condition on our parameters.
Nevertheless the impact of the iq term was checked for models with $p = 1, 2, 3$ and $n \leq p/2$ and indeed no changes in the results were found.

The axio-dilaton The potential possibility for the axio-dilaton S to be the inflaton is already addressed in this model as for the $n = p = 1$ model the potentials are invariant under the exchange $S \leftrightarrow U$.

The T modulus An important remark is that in all of this we do not need to consider the T modulus as of now. If we find a working model we will have to consider this modulus again and try to stabilize it. Reintroducing the T modulus may spoil our model, however stabilizing the T modulus to begin with will not improve the behaviour (i.e. a model that does not work without T will also not work with T) and thus, to make things simple, we do not consider T for now.

Axions As mentioned before the particles that do not appear in the Kähler potential, specifically the real parts of U and S, are called axions. We will often set them to zero without loss of generality because it is at value zero that they are at the minimum of their respective potentials. This is especially important for higher power models where the calculations, even with the use of a computer program, become too involved to solve on normal hardware. We employed this for models with $n, p \geq 2$.

A note on the absence of Ae^{iaT} and the uplift term As we mentioned in chapter 2.4.4 the shift symmetry present in (3.3) is broken and we need to introduce a term Ae^{iaT} into the superpotential corresponding to that fact. Furthermore in order to get a positive minimum value of the scalar potential we need to introduce an uplift term. Neither of these terms is present in (3.3). We omitted them because they are not needed for general considerations. As soon as we find our desired potential we can introduce both these terms and build the dS vacuum we are after.

3.2 Simple models for n and p and methodology

First off we want to look at the models given by the Kähler- and superpotential (3.3) and we will try to construct a potential with the general behaviour as seen in 7: A minimum with a steep potential to one side and an infinitely long flat part for the inflaton to roll down on.
Even though the formulas seem to be rather tame and manageable, due to the amount of terms involved the help of a computer algebra program, namely Mathematica, was employed to arrive at results in a timely manner. The code is based on the works of [42].

3.2.1 Outline of employed code and procedure

Our code is supplied with the potentials of (3.3) with the values of p and n that we want to consider. In the first step the Kähler-covariant derivatives, Kähler-metric and scalar potential are calculated. For convenience these objects are converted to real- and imaginary-part and split up if necessary. We then use the equations

$$\begin{aligned} Re(D_U W) &= 0, \\ Re(D_S W) &= 0, \\ Im(D_S W) &= 0, \end{aligned} \tag{3.4}$$

to fix, for example, the values of the S field ($Re(S)$ and $Im(S)$) and one of the parameters that appear in the superpotential.

Armed with these conditions we expand the scalar potential (2.19) in orders of our inflaton $Im(U)$ for $Im(U) \to \infty$[21] and check the form of this potential. In particular we require there to be no positive powers of $Im(U)$, a constant term corresponding to the energy scale of inflation and real coefficients. The last condition is of importance as soon as roots appear in the series and requires us to tune parameters to retain reality. Positive powers that appear in the initial expansion of the scalar potential can be set to zero by introducing further restrictions on the parameters.

3.2.2 Results for models up to $n = 3$, $p = 3$

The results for all models with $n = 1, 2, 3$ and $p = 1, 2, 3$ are shown in table 3.2.2. All models terminate due to a number of different reasons:

- *NS:* The potential is not usable in the first place because it has too few terms in the expansion or the constant term is missing, thus giving no cosmological constant.
- $Im(S) = 0$: The dilaton as given by $Im(S)$ has to be zero and we can no longer trust our supergravity as discussed in chapter 2.4.2.
- *RED:* In the process of eliminating positive order of $Im(U)$ we either reduce the model to one of the lower order ones or the expansion of the potential with the conditions on the parameters is not suitable for inflation.

	$n = 1$	$n = 2$	$n = 3$
$p = 1$	NS	$Im(S) = 0$	RED
$p = 2$	NS	$Im(S) = 0$	NS
$p = 3$	NS	NS	RED

Table 1: The summary of the results of the simple models for $n = 1, 2, 3$ and $p = 1, 2, 3$. All models fail according to one of the problems mentioned above.

Discussion of all models

- *n=1, p=1:* After solving $D_S W = 0$ the scalar potential vanishes identically.

[21] Inflation for large fields.

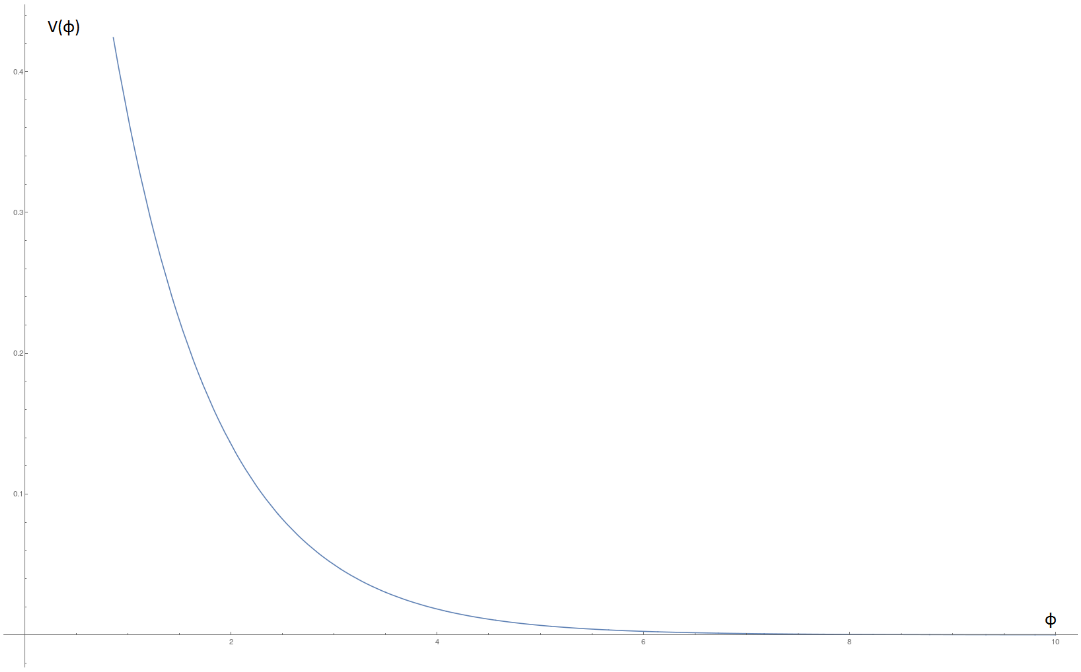

Figure 13: The general behaviour of a potential that is only given by a term with a single negative power. The inflaton cannot roll towards a stable minimum.

- *n=1, p=2:* Here we find a single term potential with only one negative power $\propto Im(U)^{-1}$. A potential like this is not suitable for inflation starting at $\phi \approx \infty$ as is evident from figure 13.

- *n=1, p=3:* For this case the conditions from $D_S W = 0$ are again sufficient to arrive at a single term potential in this case $\propto Im(U)^{-2}$.

- *n=2, p=1:* Solving $D_S W = 0$ requires $S_i = 0$ which is not compatible with our requirements on the moduli from chapter 2.4.2.

- *n=2, p=2:* The same problem arises as in the previous model, solving the constraints requires $Im(S) = 0$ which is not acceptable.

- *n=2, p=3:* This is the first model for which we use the simplification to set the axions $Re(U) = Re(S) = 0$. With this we find our first multi-term potential, however, one of them is a positive power. The potential has the general form $AIm(U) + B + CIm(U)^{-1}$ which cannot be used as seen in figure 14. Setting it to zero by a condition on the parameters renders the potential unusable.

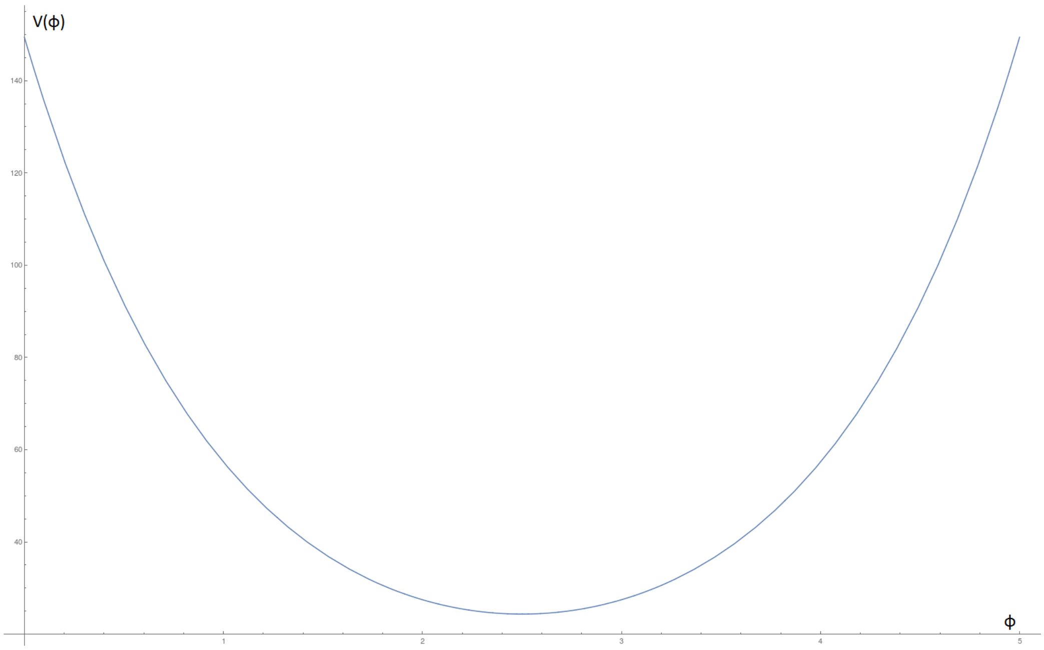

Figure 14: A potential with single positive and negative power gives generically no flat part on which the particle can roll during inflation. We see that we need to get rid of the positive order and require a higher negative one.

- *n=3, p=1:* The series for the potential of this model has enough terms to require a more thorough investigation than the previous models. It starts with a positive power of $Im(U)^4$ and importantly has a constant term. The term $Im(U)^4$ proved resilient and in the end we found that the model either becomes not usable or reduces to the model for n=2, p=1.

- *n=3, p=2:* Here we again find a promising potential expansion with positive powers. It is possible to reduce it to an interesting expansion without the positive powers but sadly also without a constant term which we would need for inflation.

- *n=3, p=3:* Again we find a series for V that might lead to a usable potential but we need to get rid of some positive powers. Due to the amount of parameters this is a somewhat lengthy process and leads us to find that the model reduces to a previous one with all the problems this implies.

Unfortunately we find that each attempt fails for one reason or another. Below we review our attempts to generalize the model in order to get a working model for inflation.

3.3 Adding another field

One method that might introduce enough additional dynamics to allow us to build a suitable potential for inflation is to introduce another field to the model. We can allow for two different complex structure moduli in (3.1) to become light and we call the additional one F [22] to distinguish the two. Looking at (3.1) we see that we can get a number of different superpotentials that are polynomial in U and F. We will consider three different models that can arise for by setting some of the $U^i \to U$ or $\to F$.

3.3.1 Maximal model with U^2

First if we take $\kappa_{ijk}U^iU^jU^k = U^2F$ we find potentials that can be written down as:

$$\begin{aligned} K &= -ln\left(-i(S-\bar{S})\right) - 3ln\left(-i(T-\bar{T})\right) \\ &\quad - 2ln\left(-i(U-\bar{U})\right) - ln\left(-i(F-\bar{F})\right), \\ W &= a_0 + a_1U + a_2U^2 + c_1F + c_2UF + a_3U^2F \\ &\quad + S\left(b_0 + b_1U + b_2U^2 + d_1F + d_2UF + b3_3U^2F\right) + iq\left(a_3 + b_3S\right). \end{aligned} \tag{3.5}$$

One general fact to note is that not every term in the superpotential has a unique parameter due to the general form given in (3.1). This is of course a restriction on the model, limiting the number of parameters we can tune in order to arrive at our required potential form. Note also that for this model the imaginary coefficient iq does not necessarily vanish. As for all fields we will require $D_FW = 0$. With the conditions on the parameters that arise by solving this equation as well as $D_SW = 0$ we try to find our scalar potential expansion. Note that our reality condition on the scalar potential does not change and we have to impose conditions on the parameters accordingly.
For this particular case we find a potential with a positive term $\propto Im(U)$ which we have to set to zero. Unfortunately due to the roots appearing in the coefficients of the series it is not possible to achieve a real potential. No matter how one tunes the parameters there will always either be an imaginary term or the potential becomes unsuitable for inflation.

[22] Not to be confused with the F-term.

3.3.2 Model with F^2

Another possibility we can explore is to choose the κ_{ijk} such that $h_2^i \kappa_{ijk} U^j U^k = 0$ and $\kappa_{ijk} U^i U^j U^k = UF^2$. For this case the potentials read:

$$\begin{aligned} K &= -ln\left(-i(S-\bar{S})\right) - 3ln\left(-i(T-\bar{T})\right) \\ &\quad - ln\left(-i(U-\bar{U})\right) - 2ln\left(-i(F-\bar{F})\right), \\ W &= a_0 + a_1 U + c_1 UF^2 + fF^2 \\ &\quad + S\left(b_0 + b_1 U + c_2 F^2 + d_2 UF^2\right) + iq\left(c_1 + d_2 S\right). \end{aligned} \tag{3.6}$$

Important to note is that in this model, other than the term iqS, the fields S and U appear on equal footing.
With these choices it is possible to use either $Im(U)$ or $Im(F)$ as the inflaton and to minimize other directions. Choosing for example U we find after minimizing F and S a potential expansion with a constant term and even powers of $Im(U)$, starting at the positive power $Im(U)^2$. We can easily eliminate this positive power by a condition on a parameter and find that it only eliminates the term we aimed for. Substituting $Im(U) \to e^{\phi}$ we have for the first three terms in the series expansion of the scalar potential a potential of form

$$V \approx \alpha - \beta e^{-2\phi} + \gamma e^{-4\phi},$$

where we have chosen one parameter such that the second term is negative and assumed T to be stable for now. With such a potential we can achieve a scalar potential as depicted in figure 15. While this is not quite the same as a the Starobinsky type potential from figure 7 it might actually work as the characteristics for $\phi \to \infty$ allow for inflation to happen.
However there is one thing we still need to check: In our attempt to have a potential with only negative powers of $Im(U)$ we imposed another condition on one of the parameters. This condition as well as the condition we use to get a negative sign in front of the term $\propto Im(U)^{-2}$ must be compatible with our solution of the equations $D_S W = D_F W = 0$ that restrict our fields $Im(S)$ and $Im(F)$. As a matter of fact we find that substituting all these conditions into the the solution for the fields leads to a vanishing field $Im(F)$ for large field values of $Im(U)$ and a complicated condition on parameters to have $Im(S) \in \mathbb{R}$. Since we require $Im(U^i) > 1$ this contradicts our stability requirements that we described in chapter 2.4.2. Again this is the end of the model.

3.3.3 Superpotential without squared terms

Finally we can consider a model where none of the fields appear with higher power, via $\kappa_{ijk} U^j U^k = UF$ and $\kappa_{ijk} U^i U^j U^k = 0$ we find for the Kähler- and

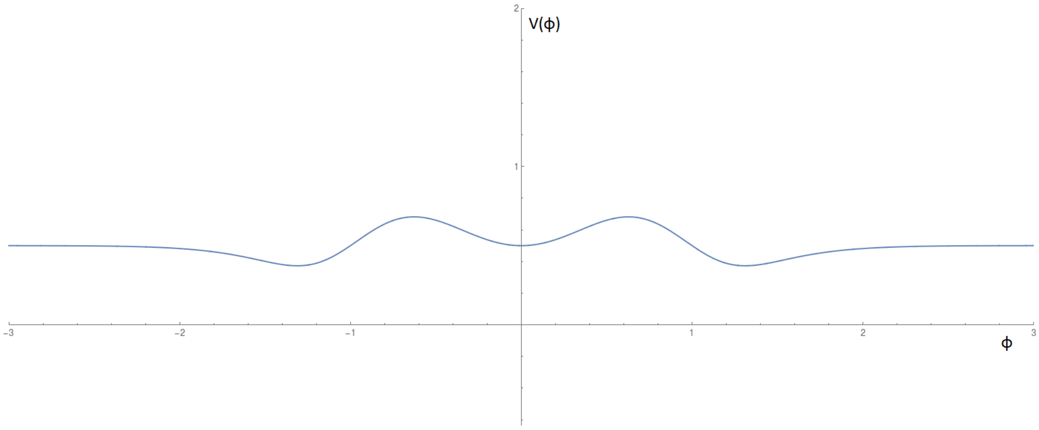

Figure 15: The first two terms of the potential found by using the potentials (3.6) give a symmetric potential with a fake vacuum for $\phi = 0$ and allow inflationary dynamics when ϕ starts at large values. By choice of parameters it seems to be possible to achieve the correct behaviour in correspondence with our current observations.

superpotential:

$$\begin{aligned} K &= -ln\left(-i(S-\bar{S})\right) - 3ln\left(-i(T-\bar{T})\right) \\ &\quad - ln\left(-i(U-\bar{U})\right) - ln\left(-i(F-\bar{F})\right), \\ W &= a_0 + a_1 U + f_1 UF + f_2 SUF \\ &\quad + b_0 S + iq\left(c + dS\right). \end{aligned} \tag{3.7}$$

For this model there is no way to minimize the $Im(F)$ and $Im(S)$ direction simultaneously and thus inflation using $Im(U)$ is not possible.

Conclusion By considering the three models of this chapter we have exhausted the possibilities of (3.1) because in the limit of large U only the highest order in the superpotential will be relevant. For the case that $\kappa_{ijk}U^iU^jU^k \propto U^3$ the superpotential is of form $W = const \cdot U^3 + \cdots$ and for the behaviour of the potential the remaining terms are irrelevant. If the dominant power appearing is $const \cdot U^2 \cdot (other\ \ fields) + \cdots$, we can call the other fields F and the relevant part for $U \to \infty$ is given by the U^2 term. Similarly for $const \cdot U \cdot (other\ fields) + \cdots$ we can call the other fields F or F^2 and the behaviour is yet again determined by the U term and the rest is not important. So we see that we have dealt with the most general case of (3.1).

3.4 Plateau model for inflation

In the previous attempts we always tried to make the positive powers of $Im(U)$ vanish, corresponding to an infinitely long flat part of the potential. Strictly speaking this is not required. We can imagine a potential as symbolically pictured in figure 16, with a long finite flat part, that grows continuously after that. It is only important that the flat part is long enough to allow for our target amount of e-folds and that in the limits for $\phi \to \pm\infty$ the potential is monotonically growing, meaning that there are no other (local) minima. This can be achieved by allowing a positive power of $Im(U)$ appearing in the

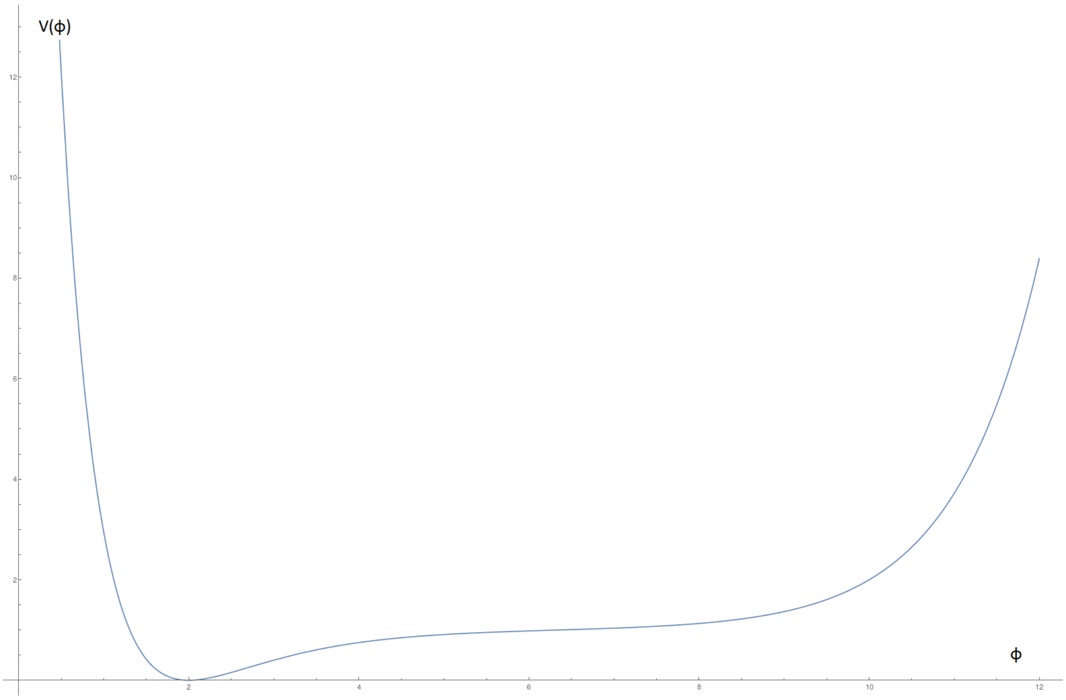

Figure 16: A possible potential for inflation that grows for both limits $\phi \to \pm\infty$. The long flat part in the middle allows for slow-roll inflation.

series expansion of the scalar potential to be present, but with a very small parameter such that the flat part of the potential is sufficiently long.
We pursue that goal by proceeding as normal, but instead of trying to set the parameters in front of the positive terms to zero, we set it to a parameter $\epsilon > 0$ that we can use to set the length of the flat part of the potential.
We used the model (3.5) with the additional field present because it was the most promising one we investigated so far. However we found no improvement, there are still either roots appearing that cannot be made real without either spoiling the form of the potential or the aforementioned parameter

ϵ appearing in other terms of the expansion, like the constant term, that should not be small.
For example, after setting the parameter in the term $\propto Im(U)$ to ϵ, we found three different roots of the parameters in the constant term. The general form of this term is

$$\frac{r\sqrt{X} + s\sqrt{X}\sqrt{Y}}{t\sqrt{Z}}$$

For this to give a real solution we need either all roots to be real or both $\sqrt{X}$ and $\sqrt{Z}$ be imaginary and $\sqrt{Y}$ real. For one particular case we found

$$\begin{aligned} X &= a_2 a_3 c_2 \\ Y &= \frac{a_2 b_3 (a_3 b_1 - a_1 b_3 + a_2 d_2)}{a_3 c_2} \\ Z &= -b_3 (a_3 b_1 - a_1 b_3 + a_2 d_2) \end{aligned}$$

and we immediately note $Y = -a_2/(a_3 c_2) Z$. Now if $sign(a_2 a_3 c_2) = -1$ we see that $Y = |a_2/(a_3 c_2)|\, Z$ and thus if Z is imaginary so is Y. For $sign(a_2 a_3 c_2) = +1$ we have $Y = -\,|a_2/(a_3 c_2)|\, Z$ and thus if Z is real Y will be imaginary.
These kind of cases appear in all situations and the only remaining option is to begin to set parameters to zero which in turn sets whole terms in the series of the scalar potential to zero. This reduces the amount of terms in our series expansion of the scalar potential and always leads to an unusable potential form.

4 Conclusion and outlook

In this work we investigated the possibility to construct a potential that reproduces the main features that we require from inflation, namely around 60 e-folds of inflation and the slow-roll parameters (1.44) and (1.45), compatible with the best current bounds from experiments. To that end we looked at type IIB string theory compactifications on Calabi-Yau orientifolds in the limit of large complex structure moduli. We investigated whether a complex structure modulus can be used as an inflaton in this limit in order to produce a scalar potential similar in form to the Starobinsky model. We started with the STU-model with only one such complex structure modulus.
We found that the simplest models for a polynomial superpotential did not allow for inflationary dynamics to arise. The model seems to be too restrictive to accommodate all our additional requirements of reality while staying in the region of the model ($Im(S) \gg 1$) where it is still reliable.

Additional attempts to expand our model showed some signs of improvement but ultimately neither adding an additional field nor allowing for only a finite flat part of the potential did yield the desired results.
Moving forward we hope that it will possible to construct a model of inflation in a similar framework or to at least find a way to proof generally that inflation is not possible in such a model.

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A Englisch Abstract

Abstract

In 1980 A. A. Starobinsky presented a model of inflation that uses a Planck-suppressed order R^2 term in the Einstein-Hilbert action to describe the exponential expansion of the universe. Such an expansion was first postulated by Guth to solve the *horizon* and *flatness* problem of cosmology.

The Starobinsky model of inflation, in the context of supergravity, has been discussed before. In this work however we want to construct generalized Starobinsky type inflation models that arise from a supergravity that is the low energy limit of string theory. We hope that on the one hand this restriction gives us particles in form of the moduli arising from compactifying the string theory and on the other hand that our work here gives the possibility to use cosmological data to check some aspects of string theory and therefore its validity.

The first chapter is dedicated to the classical concepts of cosmology that we require for our work. In the last section we will look at the classical Starobinsky inflation in order to motivate our later work. The basics of supergravity and how we can get a supergravity from string theory will be outlined in chapter 2. There the STU-model of supergravity will be presented, which shall serve as the main ingredient for the work of chapter 3. In the penultimate chapter we will start our work on simple STU-models with polynomial superpotentials and check their relevance for inflation. To conclude the chapter we investigate possibilities to generalize the model via adding additional fields or loosening our restrictions on the form of the potential.

B German Abstract - Zusammenfassung

Zusammenfassung

Bereits 1980 hat A. A. Starobinsky ein Inflationsmodell vorgestellt, in dem er einen Planck-unterdrückten Term, proportional zu R^2, in der Einstein-Hilbert Wirkung betrachtet. Damit gelang es ihm eine exponentielle Expansion unseres Universums zu beschreiben. Die Notwendigkeit solch einer Expansion wurde erstmals von Guth beschrieben, um das *Horizont*- und das *Flachheitsproblem* der Kosmologie zu lösen.

Im Kontext von Supergravitation wurde das Starobinsky-Modell für Inflation bereits betrachtet. In dieser Arbeit wollen wir versuchen, Starobinsky-ähnliche Inflationsmodelle im Rahmen von Supergravitationsmodellen, die der Grenzwert niedriger Energie von Typ IIB Stringtheorie sind, zu konstruieren. Wir hoffen, dass wir auf diesem Weg einerseits Teilchen vom Kompaktifizierungsprozess erhalten und andererseits, dass es möglich ist kosmologische Messdaten zu verwenden, um Vorhersagen der Stringtheorie zu überprüfen.

Im ersten Kapitel besprechen wir die klassischen Konzepte der Kosmologie, die wir für unsere Arbeit benötigen. Im letzten Unterkapitel betrachten wir das Modell von Starobinsky in diesem Kontext. Die Grundlagen der Supergravitation werden in Kapitel zwei betrachtet, wo wir auch motivieren wollen, warum wir eine spezielle Kompaktifizierung von Typ IIB Stringtheorie betrachten. Außerdem wird das STU-Modell besprochen, unser wichtigstes Werkzeug für die weitere Arbeit. Im vorletzten Kapitel schließlich werden wir diese Grundlagen verwenden, um Inflationsmodelle zu konstruieren und deren Konsistenz zu überprüfen. Zum Abschluss dieses Kapitels und der Arbeit betrachten wir Verallgemeinerungen unserer Modelle.