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## „Energy loss of anti-protons in a positron plasma"

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#### Abstract

Faculty for Physics Stefan Meyer Institute for subatomic physics Master of Science Energy loss of anti-protons in a positron plasma

by Michael Bartel

This Masterthesis aims to verify if the radial drift of anti-protons to the outside of the positron plasma is a possible source of the drop in anti-hydrogen production rate in the CUSP trap of the ASACUSA experiment at CERN. The radial drift is caused by energy loss through collisions of the anti-protons with positrons and we use simulations to get a qualitative timescale of this drift to compare to the experimental results. In our approximate approach we investigate isolated collisions of the two particles in a strong magnetic field and use a Classical Trajectory Monte Carlo scheme to arrive at an averaged energy loss rate and furthermore a radial drift time for the anti-proton. The simulation results of the $\bar{p}$ drift times have the same order of magnitude, tens of seconds, as the observed anti-hydrogen yield drop, but more extensive studies are necessary to


 come to a conclusion.Universität Wien

## Zusammenfassung

Fakultät der Physik<br>Stefan Meyer Institut für subatomare Physik<br>Master of Science<br>Energieverlust von Anti-Protonen in einem Positronenplasma

von Michael Bartel

Das Ziel dieser Masterarbeit ist es den möglichen Zusammenhang zwischen dem Abfall der Anti-Wasserstoff-Erzeugung mit der radialen Expansion der Anti-Protonen in der CUSP Falle des ASACUSA Experimentes (CERN) zu untersuchen. Diese radiale Bewegung wird durch Kollisionen der Anti-Protonen mit dem Positronenplasma verursacht und dadurch verlassen erstere das Plasma und stehen nicht mehr zur Anti-WasserstoffProduktion zur Verfügung. Eine qualitative Ermittlung der Zeitskala auf welcher die Expansion stattfindet, wurde unter Anwendung von Monte Carlo Simulationen der Kollisionen der beiden Teilchen in einem starken Magnetfeld erreicht. Dabei kamen die in der CUSP Falle vorherrschenden physikalischen Parameter zur Anwendung um einen Vergleich mit den Messungen der Anti-Wasserstoff-Produktion zu ermöglichen. Das Ergebnis dieser Untersuchung ist, dass die von uns in erster Näherung ermittelte Expansionszeitskala die selbe Größenordnung hat, wie die zugrundeliegende Messung der Produktionsraten. Jedoch sind weitergehende Simulationen von Nöten um ein aussagekräftiges Ergebnis zu erhalten.

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## Contents

Abstract ..... iii
Zusammenfassung ..... iv
Acknowledgements ..... V
Contents ..... vii
1 The ASACUSA Experiment ..... 1
1.1 Theory ..... 1
1.1.1 Basic symmetries in particles physics ..... 1
1.1.1.1 P Symmetry ..... 1
1.1.1.2 CP Symmetry ..... 1
1.1.1.3 CPT Symmetry ..... 2
1.1.2 CPT violation in anti-hydrogen experiments ..... 4
1.2 The ASACUSA Experiment ..... 7
1.2.1 Overview ..... 8
1.2.2 Preparation of the anti-proton plasma ..... 8
1.2.3 Preparation of positrons ..... 9
1.2.4 CUSP trap ..... 10
1.2.5 Spin flip cavity, Sextopole Magnet and Detector ..... 12
1.3 Motivation and aim of the Thesis ..... 13
2 Theory ..... 17
2.1 Basics of plasma physics ..... 17
2.1.1 Definition of a plasma ..... 17
2.1.1.1 Debye length ..... 17
2.1.1.2 Plasma oscillation ..... 19
2.1.2 Motion of charged particles in electromagnetic fields ..... 20
2.1.2.1 Cyclotron motion ..... 20
2.1.2.2 Magnetron motion ..... 21
2.2 Centrifugal separation ..... 23
2.3 Rutherford scattering ..... 25
3 Simulation Program ..... 27
3.1 Motivation ..... 27
3.2 CTMC - Classical trajectory Monte Carlo method ..... 28
3.2.1 Application of the Monte Carlo method ..... 29
3.3 MCSIM ..... 30
3.3.1 Initial simulation parameters ..... 31
3.3.2 Impact disc ..... 32
3.3.3 Simulation convergence ..... 33
3.3.4 MCSIM output ..... 36
3.4 SIMBUCA ..... 37
3.4.1 Changes to SIMBUCA ..... 37
4 Simulation results ..... 41
4.1 Rutherford scattering ..... 42
4.1.1 Influence of simulation box size ..... 45
4.1.2 Comparison of different frames of reference ..... 46
4.2 Scattering in a magnetic field without radial positron energy ..... 48
4.2.1 Lorentz invariance ..... 50
4.2.2 Scaling laws for the magnetic field strength for an axial velocity of $V=20000 \mathrm{~m} / \mathrm{s}$ ..... 52
4.2.3 Scaling laws for the magnetic field strength for an axial velocity of $V=66000 \mathrm{~m} / \mathrm{s}$ ..... 56
4.3 Scattering in a magnetic field with radial positron energy ..... 61
4.3.1 Lorentz invariance ..... 61
4.3.2 Scaling with radial Velocity ..... 63
4.3.3 Scaling with radial temperature ..... 64
4.4 Robicheaux simulations ..... 66
4.5 Adding anti-proton radial energy ..... 70
4.5.1 $\overline{\mathrm{p}}$ outward drift ..... 72
4.5.2 Influence of different magnetic fields ..... 73
4.5.3 Comparison with $\mathrm{B}=0$ ..... 77
4.5.4 Analysis of different positron temperatures ..... 81
4.5.5 Analysis of different anti-proton energies ..... 83
4.5.6 CUSP Parameters ..... 86
4.5.6.1 Conclusions ..... 91
5 Resume ..... 93
Bibliography ..... 95
List of Figures ..... 95
List of Tables ..... 99

## Chapter 1

## The ASACUSA Experiment

The ASACUSA collaboration (Atomic Spectroscopy And Collisions Using Slow Antiprotons) aims to produce and measure anti-hydrogen to find a possible CPT violation or boundaries for the model predicted by an extension of the standard model briefly described in section 1.1. The experimental setup in section 1.2 was able to produce anti-hydrogen in 2010 [1], a beam of cold $\overline{\mathrm{H}}$ in 2012 [2] and had a first run of the full experiment in 2014 [3]. The results of 2010 and 2012 showed an unexpected decrease of anti-hydrogen production concentrated on a time scale of about 20s. The aim of this master thesis is to identify possible processes to account for this drop and is further outlined in section 1.3.

### 1.1 Theory

### 1.1.1 Basic symmetries in particles physics

### 1.1.1.1 P Symmetry

In the last century two of three main symmetries of particle physics have been found to be broken under certain conditions. After a suggestion of T.D. Lee and C.N. Yang in 1956 [4] an experiment performed by C.S. Wu et al. in 1957 [5] showed that the parity symmetry ( P , spatial mirroring transforming between left and right handed chirality), even though conserved in electromagnetic, gravitational and strong interaction, is found to be broken in beta decay and therefore in weak interaction in general.

### 1.1.1.2 CP Symmetry

Just a few years later, in 1964, James Cronin and Val Fitch found a violation of CPsymmetry in the decay of Kaons [6]. The charge conjugation symmetry (C) transforms particles into anti-particles and vice-versa and is broken separately as well as in the
combination CP that induces the more realistic transition from particle to anti-particle, as there are only left-handed particles and right-handed anti-particles. In the experiment they measured the decay of Kaons and anti-Kaons in two different channels connected to the eigenstates of the CP-Operator

$$
\begin{align*}
\left|K_{S}\right\rangle & =\frac{1}{\sqrt{2}}\left(\left|K^{0}\right\rangle+\left|\bar{K}^{0}\right\rangle\right), \\
\left|K_{L}\right\rangle & =\frac{1}{\sqrt{2}}\left(\left|K^{0}\right\rangle-\left|\bar{K}^{0}\right\rangle\right), \tag{1.1}
\end{align*}
$$

with $C P\left|K_{S}\right\rangle=+1\left|K_{S}\right\rangle$ and $C P\left|K_{L}\right\rangle=-1\left|K_{L}\right\rangle$. The eigenstates are also called shortlived $K_{S}$ and long-lived $K_{L}$ with regards to their respective lifetime of $\Gamma_{S}^{-1}=1 \times 10^{-10} \mathrm{~s}$ and $\Gamma_{L}^{-1}=5 \times 10^{-8} \mathrm{~s}[7]$. Assuming that CP holds, the $K_{S}$ state can now decay into two pions, $\left(\pi^{0} \pi^{0}\right)$ or $\left(\pi^{+} \pi^{-}\right)$, or for higher angular momentum with odd quantum number into $\left(\pi^{+} \pi^{-} \pi^{0}\right)$, as all of those states have the same +1 eigenvalue of the CP transformation. $K_{L}$ on the other hand can not decay into a two pion state, but only into states with CP eigenvalue ( -1 ). That is $\left(\pi^{+} \pi^{-} \pi^{0}\right.$ ) for quantum number 1 even or $\left(\pi^{0} \pi^{0} \pi^{0}\right)[7]$.

The experiment of J. Cronin and V. Fitch measured the decay of $K_{L}$ and found signatures of forbidden decay into two pions with a probability of $\epsilon \sim 2.0 \times 10^{-3}$, also called the CP violation parameter [7].

### 1.1.1.3 CPT Symmetry

The last bastion of basic symmetries in particle physics, the CPT symmetry that adds time inversion invariance T, still holds up to this moment. It was proofed, using Lorentz invariance, by Wolgang Pauli [8] and Gerdart Lüders [9].

If CPT symmetry is not broken then one would assume that our baryonic universe must consist of $50 \%$ matter and $50 \%$ anti-matter, as its origin lies in the pair-creation process that would force them in the same way to annihilate on contact. Obviously we live in a stable universe such that there is no anti-matter in our close vicinity, so where did those $50 \%$ go? One theory is that there are whole galaxies made of anti-matter scatted throughout the universe. In this case their interaction with the intergalactic medium (IGM) as well as statistically speaking half of the merging galaxies should show annihilation signatures that have not been observed to this moment. On the other hand if a very small CPT violation is allowed, the disparity of matter and anti-matter can be explained.

One possible implementation by Don Colladay and V. Alan Kosteleckỳ provides a theoretical framework introducing CPT breaking terms into the standard model at the price
of partially broken Lorentz invariance [10]. This reference presents a generalized form of a CPT violating Lagrangian,

$$
\begin{align*}
\mathcal{L} & =\mathcal{L}_{0}-\mathcal{L}^{\prime} \\
\mathcal{L}^{\prime} & =\frac{\lambda}{M^{k}}\langle T\rangle \bar{\psi} \Gamma(i \delta)^{k} \psi+\text { h.c. } \tag{1.2}
\end{align*}
$$

where $\psi$ is the particle field, $k \leq 2, \lambda$ a dimensionless coupling constant, $M$ denotes a mass scale to ensure the Lagrangian has mass/energy dimension $4, \Gamma=G_{\alpha} \gamma^{\alpha}$ is a general spinor matrix element spanned by the Dirac matrices $\gamma^{\mu}$ and $T$ is a general tensor, with expectation values that break CPT and Lorentz symmetry. The lowest order terms for $k=0$ have then the general form

$$
\begin{equation*}
\mathcal{L}_{a}^{\prime}=a_{\mu} \bar{\psi} \gamma^{\mu} \psi \quad \quad \mathcal{L}_{b}^{\prime}=b_{\mu} \bar{\psi} \gamma_{5} \gamma^{\mu} \psi \tag{1.3}
\end{equation*}
$$

where $a_{\mu}$ and $b_{\mu}$ are CPT invariant coefficients that transform as external constant fields under Lorentz transformation. Introducing them next to the free Lagrangian we arrive at a most basic form of a particle with mass $m$ that violates CPT:

$$
\begin{equation*}
\mathcal{L}=\frac{i}{2} \bar{\psi} \gamma^{\mu} \stackrel{\leftrightarrow}{\partial}_{\mu} \psi-a_{\mu} \bar{\psi} \gamma^{\mu} \psi-b_{\mu} \bar{\psi} \gamma_{5} \gamma^{\mu} \psi-m \bar{\psi} \psi \tag{1.4}
\end{equation*}
$$

where $\bar{\psi} \gamma^{\mu} \stackrel{\leftrightarrow}{\partial}_{\mu} \psi=\bar{\psi} \gamma^{\mu}\left(\partial_{\mu} \psi\right)-\left(\partial_{\mu} \bar{\psi} \gamma^{\mu}\right) \psi$. Similar CPT violating terms can arise as an extension to the Standard Model, which are chosen to be compliant with the $\mathrm{SU}(3) \mathrm{xSU}(2) \mathrm{xU}(1)$ gauge symmetry of the (minimal) Standard Model as well as energymomentum conservation, micro-causality and renormalization [10]. The actual form of the terms will not be given here, as they are out of the scope of this work, but if interested they can be found in [10, p24-29]. More importantly this extension gives rise to additional factors in observables, as shown for example in equation 1.5, enabling us to look for experimental proof of CPT violation.

The current experimental precisions are given in figure 1.1. The full bars represent past experimental results, while the open boxes state experiments currently conducted (status in 2014). None of these experimental setups found a possible CPT violation to this point.


Figure 1.1: Current experimental precision of different experiments for detection of a possible CPT violation. The right edge of the bars gives the energy (frequency) equivalent of the observable measured, while the left edge shows the absolute precision of the measurement. Status of 2014 [11].

### 1.1.2 CPT violation in anti-hydrogen experiments

The ASACUSA collaboration aims to create and do spectroscopy on anti-hydrogen. The CPT breaking terms arise for example as a correction in the $1 S \rightarrow 2 S$ transition as well as in the hyperfine splitting of the ground state. In the minimal Standard Model Extension of Kosteleckỳ and Bluhm, the former process has the disadvantage, that the leading order corrections, due to CPT violation, cancel out in the energy of the 1 S and 2 S state, such that the connected violation parameter $b_{3}$ shows up only in first order terms and is suppressed by $\alpha^{2} b_{3} / 8 \pi$ [12]. The leading order shift induced by the CPT violating terms and parameters $a, b, c, d, H$ has the form,

$$
\begin{align*}
\Delta E^{H}\left(m_{J}, m_{I}\right) & =a_{0}^{e}+a_{0}^{p}-c_{00}^{p} m_{e}-c_{00}^{p} m_{p} \\
& +\left(-b_{3}^{e}+d_{30}^{e} m_{e}+H_{12}^{e}\right) m_{J} /\left|m_{J}\right| \\
& +\left(-b_{3}^{p}+d_{30}^{p} m_{p}+H_{12}^{p}\right) m_{I} /\left|m_{I}\right|, \tag{1.5}
\end{align*}
$$

where $m_{e}$ and $m_{p}$ are the electron and proton mass, while $m_{J}$ and $m_{I}$ are their respective spin components. This energy shift is the same for the 1 S and 2 S state in hydrogen as well as in anti-hydrogen $\Delta E^{\overline{\mathrm{H}}}$, which can be calculated from $\Delta E^{H}$ by $(a, d, H) \rightarrow$ $(-a,-d,-H)$ and consequently cancel out. On the other hand this is not the case for the energy shifts of the different levels $(F, M)$ of hyperfine splitting, where $F=|\vec{F}|=$ $\left|\vec{S}_{p}+\vec{S}_{e}\right|=0,1$ is the total spin and $M=-1,0,1$ the projection onto the magnetic field [13].

$$
\begin{align*}
(F, M)=(1,1) \rightarrow \Delta E_{1}^{H} & =-b_{3}^{e}-b_{3}^{p}+d_{30}^{e} m_{e}+d_{30}^{p} m_{p}+H_{12}^{e}+H_{12}^{p} \\
(F, M)=(1,0) \rightarrow \Delta E_{2}^{H} & =-\cos (2 \theta)\left[b_{3}^{e}-b_{3}^{p}-d_{30}^{e} m_{e}+d_{30}^{p} m_{p}-H_{12}^{e}+H_{12}^{p}\right] \\
(F, M)=(1,-1) \rightarrow \Delta E_{3}^{H} & =-\Delta E_{1}^{H} \\
(F, M)=(0,0) \rightarrow \Delta E_{4}^{H} & =-\Delta E_{2}^{H} \tag{1.6}
\end{align*}
$$

The parameter $\theta$ denotes the mixing angle between the $(1,0)$ and $(0,0)$ states and is defined by $\tan (2 \theta) \sim(51 \mathrm{mT}) / n^{3} B$. The Breit-Rabi formula for the Zeeman splitting, defining the energy levels in dependence of an external magnetic field, is given in equation 1.7. From the equations one can see that for $B=0$ the anomalous energy shifts cancels the degeneracy of the triplet state.

$$
\left.\begin{array}{rl}
(F, M)=(1,1) \rightarrow E_{1}^{H} & =\frac{1}{4} E_{0}+\frac{1}{2}\left(g_{J}+g_{I}\right) \mu_{B} B+\Delta E_{1}^{H} \\
(F, M)=(1,0) \rightarrow E_{2}^{H} & =-\frac{1}{4} E_{0}+\frac{1}{2} E_{0} \sqrt{1+\left(\frac{B}{B_{0}}\right)^{2}}+\Delta E_{2}^{H} \\
(F, M)=(1,-1) \rightarrow E_{3}^{H} & =\frac{1}{4} E_{0}-\frac{1}{2}\left(g_{J}+g_{I}\right) \mu_{B} B-\Delta E_{1}^{H} \\
(F, M) & =(0,0) \rightarrow E_{4}^{H} \tag{1.7}
\end{array}\right)=-\frac{1}{4} E_{0}-\frac{1}{2} E_{0} \sqrt{1+\left(\frac{B}{B_{0}}\right)^{2}}-\Delta E_{2}^{H} .
$$

In figure 1.2 we can see the Breit-Rabi diagram with the associated spin states on the right hand side and on the left hand side the energy split for the triplet state at $B=0$. This split can artificially be attributed to a constant pseudo-magnetic field of free space [13]

$$
\begin{equation*}
\Delta B_{\mathrm{LIV}}=\frac{2 \Delta E_{1}^{H}}{g_{J} \mu_{B}} \tag{1.8}
\end{equation*}
$$

The direction of this magnetic field locally breaks Lorentz symmetry as it defines a preferred direction in space. This would additionally induce a difference between measurements conducted on different times of the day, as the angle of the vector observables changes with the rotation of earth.

From the energy of the singlet and triplet states one can derive the influence of the CPT violation on the observable frequency of state transitions. In the experiment the low field seeking states $(1,1)$ and $(1,0)$ are transformed to high field seeking states $(1,-1)$ and $(0,0)$ in a $B \sim 0$ environment, therefore only the $\pi_{1}:(1,1) \rightarrow(0,0)$ and $\sigma_{1}:(1,0) \rightarrow(0,0)$ transitions can be measured experimentally [13]. The transition frequencies can be computed from equation 1.7,


Figure 1.2: Zeeman splitting of the groud state hyperfine levels of hydrogen, adapted from [13].

$$
\begin{align*}
\sigma_{1} & : \quad(F, M)=(1,0) \rightarrow(0,0): \quad \nu_{\sigma_{1}}=\nu_{0} \sqrt{1+\left(\frac{B}{B_{0}}\right)^{2}}+\frac{2 \Delta E_{2}^{H}}{h} \\
\pi_{1} & : \quad(F, M)=(1,1) \rightarrow(0,0): \\
\nu_{\pi_{1}} & =\frac{1}{2} \nu_{0}+\frac{1}{4 \pi}\left(g_{J}+g_{I}\right) \mu_{B} B+\frac{1}{2} \nu_{0} \sqrt{1+\left(\frac{B}{B_{0}}\right)^{2}}+\frac{\Delta E_{1}^{H}-\Delta E_{4}^{H}}{h} \tag{1.9}
\end{align*}
$$

In the limit $B \rightarrow 0$ the $\sigma_{1}$ line does not have a first order CPT correction, as $\Delta E_{2}^{H}$ vanishes, but the $\pi_{1}$ line does:

$$
\begin{align*}
\nu_{\sigma_{1}} & =\nu_{0} \\
\nu_{\pi_{1}} & =\nu_{0}+\frac{1}{4 \pi}\left(g_{J}+g_{I}\right) \mu_{B} B+\frac{\Delta E_{1}^{H}}{h} \tag{1.10}
\end{align*}
$$

The energy shift of this transition is different for anti-hydrogen compared to ordinary hydrogen though. As mentioned in the paragraph below equation 1.5 some CPT violation parameters change sign for anti particles and therefore the magnitude of $\Delta E_{1}^{\overline{\mathrm{H}}}$ and $\Delta E_{1}^{H}$ is different and so is the transition energy. From the difference in the frequency of the $\pi_{1}$ and $\sigma_{1}$ lines, a direct connection to a CPT violation parameter can be established as [12]

$$
\begin{equation*}
\Delta \nu_{1-2}=\Delta\left(\nu_{\pi_{1}}-\nu_{\sigma_{1}}\right)=\left(\nu_{\pi_{1}}^{H}-\nu_{\sigma_{1}}^{H}\right)-\left(\nu_{\pi_{1}}^{\overline{\mathrm{H}}}-\nu_{\sigma_{1}}^{\overline{\mathrm{H}}}\right) \sim \frac{-2 b_{3}^{p}}{\pi} \tag{1.11}
\end{equation*}
$$

This equation only holds in the case where $B$ is about $0.3 \mathrm{~T}[12]$, such that $\hat{\kappa}=\cos (2 \theta) \sim$ 1 in equation 1.5 . Therefore an extrapolation of the measured frequency differences to higher magnetic field values is necessary to calculate the CPT violation factor $b_{3}^{p}$.

Measurement and comparison of the $\pi_{1}$ and $\sigma_{1}$ lines in hydrogen and anti-hydrogen is therefore a possible proof of CPT violation in the model of Kosteleckỳ! One can find similar possible observables that give direct access to the parameters in question, but using hyperfine splitting has several advantages.

The hyperfine transition frequency $\nu_{H F}$ in hydrogen is measured to very high precision in maser experiments [14, 15]

$$
\begin{equation*}
\nu_{\mathrm{HF}}=1420405751.767 \pm 0.002 \mathrm{~Hz} \tag{1.12}
\end{equation*}
$$

Furthermore the hyperfine frequency can be computed by means of Fermi contact interaction [16] such that in first order [13]

$$
\begin{equation*}
\nu_{\mathrm{HF}}=\frac{16}{3}\left(\frac{m_{p}}{m_{p}+m_{e}}\right)^{3} \frac{m_{e}}{m_{p}} \frac{\mu_{p}}{\mu_{N}} \alpha^{2} c R y \tag{1.13}
\end{equation*}
$$

Here $m_{p}\left(m_{e}\right)$ denote the proton (electron) mass, $\alpha=e^{2} / 2 c \epsilon_{0} h \sim 1 / 137.036$ the fine structure constant, $R y=m_{e} e^{4} / 8 c \epsilon_{0}^{2} h^{3} \sim 10.973 \times 10^{6} \mathrm{~m}^{-1}$ the Rydberg constant, $c$ speed of light and $\mu_{p}$ and $\mu_{N}$ the magnetic moment of proton respectively the nuclear magnetron. This gives rise to the possibility to calculate the (anomalous) anti-proton magnetic moment from the measurement of the absolute value of the transition frequency in anti-hydrogen with high relative precision. The best measurement currently available, conducted by the BASE experiment in 2017, has a relative precision of $2.6 \mathrm{ppb}(95 \%$ CL) [17], before the BASE experiment it was only around $0.3 \%$ [18].

### 1.2 The ASACUSA Experiment

The section before showed, that to find a CPT violation in the Kosteleckỳ model, measurement of the hyperfine splitting frequency $\pi_{1}$ is needed to a very high precision. This calls for some conditions to be fulfilled in the experiment:

- the anti-hydrogen needs to be in the ground state,
- at the point of the $\pi_{1}$ transition the magnetic field must be known precisely,
- high statistics are necessary and therefore high number of suitable anti-hydrogen atoms are required.


### 1.2.1 Overview

The schematic view in figure 1.3 shows the primary components of the experimental setup. Anti-protons are prepared in the MUSASHI trap and positrons in the $e^{+}$accumulator before both are transferred to the CUSP trap where anti-hydrogen is formed.

Important note: At the time of release of this master thesis the CUSP trap was already updated to a double CUSP trap. All calculations were done on the old CUSP setup though and therefore the following descriptions will still reference the setup used at that time.

The magnetic field of the CUSP is created by an anti-Helmholtz coil and at the downstream part the radial field gradient will focus the low field seeker states LFS $(F, M)=$ $(1,1)$ and $(1,0)$ in the $\overline{\mathrm{H}}$ beam and defocus the high field seeker states HFS $(1,-1)$ and $(0,0)$ that will annihilate on the tube surface.

The frequency of the electromagnetic field of the RF-cavity after the CUSP will be fine tuned around the expected $\nu_{\mathrm{RF}}$, which depends on the magnetic field present, such that it will hit the transition of $\overline{\mathrm{H}}$ inducing the $\pi_{1}$ spin flip (equation 1.9) on resonance. The sextupole magnet just before the detector will only focus low field seeker states, such that, if the frequency of the cavity does not induce a spin flip the anti-hydrogen will still be in a low field seeker state and pass the second sextupole magnet yielding a signal in the detector. On the other hand, if a transition from a HFS to LFS state occurs the $\overline{\mathrm{H}}$ atom will be defocused and will not reach the detector. Correlating the time-frequency sweep of the cavity with the counts on the detector yields a graph of signal versus frequency of the cavity which will reveal the $\pi_{1}$ and $\sigma_{1}$ frequencies.

### 1.2.2 Preparation of the anti-proton plasma

Bunches of about $4 \times 10^{7} \overline{\mathrm{p}}$ are created by collision of $10^{13}$ protons, which are coming from the proton synchrotron (PS) with a momentum of about $26 \mathrm{GeV} / \mathrm{c}$ and collide with a metal target. The created $\overline{\mathrm{p}}$ are injected and stored in the AD ring system, where cavities, electron and stochastic cooling provide an efficient cool-down to about 5 MeV before transported to the individual experiments [19]. The ASACUSA experiment includes a unique device, the RFQD (radio frequency quadrupole decelerator), to further reduce the energy to $10-100 \mathrm{keV}$ with only moderate loss of anti-protons [20]. The flyby spectroscopy used in this experiment necessitates to produce a beam of anti-hydrogen, so the number of available anti-protons is an exceedingly important factor. The use of the RFQD gives a huge advantage over other experiments that use degrader foils to slow down the $\overline{\mathrm{p}}$ but lose a lot of particles in the process, yielding $\overline{\mathrm{p}}$ capture efficiencies about two magnitudes smaller [21].


Figure 1.3: Schematic overview of the ASACUSA experiment [2].

After the RFQD (depicted by the green arrow in figure 1.3) the anti-protons are passing a PET foil before entering the MUSASHI trap. The anti-protons are cooled by interaction with preloaded electrons in the trap then radially compressed by a so called rotating wall electric field and stored for further use in the experiment. Typically three to five stacks of anti-protons from the AD ring are gathered inside the MUSASHI trap prior to transferring them into the CUSP trap for mixing.[22]

### 1.2.3 Preparation of positrons

The positrons needed to form anti-hydrogen, are produced by the radioactive $\beta^{+}$emitter ${ }^{22} \mathrm{Na}$ with $t_{1 / 2}=2.6$ years. The sodium carbonate crystal $\left(\mathrm{Na}_{2} \mathrm{CO}_{3}\right)$ used has an activity of a few GBq (f.e. April $2014 \sim 3.6 \mathrm{GBq}$ ) [23] and is contained in a radiation shield. The positrons are entering the $e^{+}$-accumulator, shown in figure 1.4, through a moderator made from Ne ice of a few $\mu \mathrm{m}$ thickness, that transforms a small fraction of the fast $e^{+}$entering on one side to a nearly mono energetic beam at thermal energy of the moderator exiting at the other [24]. On the exit of the accumulator a reflection moderator is mounted, emitting positrons from the same surface they entered, while thermalizing them.

In the tube in between, a strong axial magnetic field confines the positrons radially so they don't annihilate on the surface. There are two parts inside the accumulator tube. First, directly after the entry point moderator, a gas cell is situated that is filled with nitrogen gas that cools the positrons through collision. Afterwards a MRE (multi ring
electrode) provides an axial electric potential well in which the cooled positrons are captured. Strong differential pumping keeps the amount of residual nitrogen gas in the MRE part low, so that on the one hand a considerable amount of $N_{2}$ can be used in the gas cell for effective cooling while on the other hand there are only few molecules in the MRE section providing long confinement times as positrons are less likely to annihilate with the gas.


Figure 1.4: Schematic view of the positron accumulator [23].

### 1.2.4 CUSP trap

Magnetic coils on the connection tubes between the MUSASHI trap, the $e^{+}$-accumulator, and the CUSP trap guide the anti-protons and positrons to the CUSP with minimal losses. Especially the positron transport shows a high efficiency of more than $50 \%$ [25], even though there are two 90 degree bending elements on their path.

The CUSP trap itself is where anti-hydrogen atoms are forming and therefore it needs to confine both oppositely charged particles, $\overline{\mathrm{p}}$ and $e^{+}$, at the same time, so that they are able to interact with each other. This is realized in the form of a nested well trap, see figure 1.5, for axial confinement and a strong axial magnetic field for radial confinement. The latter induces a cyclotron motion in the charged anti-proton and positron (see section 2.1.2.1) so radial drift is only possible by collision processes and this drift happens over very large time scales compared to the dynamics of the plasma.

In the actual experiment, the positrons are first loaded into the CUSP trap. This is done by using the red potential $\phi_{1}$ in figure 1.5 , where the upstream part is lowered to the level marked by the dotted line. After the $e^{+}$are inside the well the potential is quickly reverted to the solid red line state $\phi_{2}$ confining the particles. Now the whole potential is slowly elevated to state $\phi_{3}$ depicted by the solid blue line, while preserving the central well shape. The positrons heat up in the transition of potentials, but they quickly radiate excess energy away as synchrotron radiation so confinement is sustained. Anti-protons can be loaded into the trap now by rapidly alternating between configuration $\phi_{3}$ and $\phi_{4}$. There are two different procedures to inject anti-protons into the


Figure 1.5: (a) Multi ring electrode (MRE) of the CUSP trap, (b) its axial magnetic field strength and (c) potential configurations of positron loading $\phi_{1}, e^{+}$cooling $\phi_{2}$, antihydrogen forming $\phi_{3}, \overline{\mathrm{p}}$ loading $\phi_{4}$ and FIT empty/regular operation mode $\phi_{5}$ [1].
positron plasma sitting in the central well. In the direct injection scheme the $\bar{p}$ are loaded into the trap with energies high enough to overcome the central potential barrier (so more than about $15-20 \mathrm{eV}$ injection energy, this was improved 2016 to about 1.5 eV ) and therefore they can traverse the plasma colliding with positrons to create $\overline{\mathrm{H}}$ atoms.

In the auto-resonance scheme the anti-protons are first cooled down to lower energies and therefore do not have enough velocity to overcome the potential to reach the plasma. They will loose energy by sympathetic cooling and settle in the inverted well. Afterwards a harmonic signal with frequency close to the harmonic potential frequency will be applied to electrode U7. This will excite the anti-proton to higher and higher axial energy with each harmonic oscillation (while radial energy does not change significantly) up until the positron plasma can be reached [26]. This has the advantage, that the energy of the $\bar{p}$ is only slightly above the electrostatic potential, so that the anti-proton has a very low axial energy while passing the positron plasma. The principle quantum number of the Rydberg state in which the anti-hydrogen is produced is very sensitive to excess energy, so to produce the anti-hydrogen in a state as low as possible, a very low initial energy is preferred [27]. The auto resonance scheme will eventually be implemented in the experiment, but for the moment direct injection is used.

In any case, while the anti-proton passes the positron cloud it has the chance to recombine to anti-hydrogen. This now neutral particle can pass the potential barrier and reach
the downstream part of the CUSP. The FIT (field ionization trap) is used to count ionized anti-hydrogen atoms that were formed in a quantum state with $n \geq 100 \times(3.2 / \epsilon)^{\frac{1}{4}}$, with $\epsilon$ the electric field strength, and collects the anti-protons in the deep harmonic potential [28]. When released in configuration $\phi_{5}$ they are transferred to the downstream exit of the CUSP trap where a detector counts the annihilation events, that directly translated to the number of anti-hydrogen produced in the appropriate forward solid angle (those would form the $\overline{\mathrm{H}}$ beam available for further analysis).

The magnetic configuration in the downstream part of the CUSP acts like an sextupole field for the generated anti-hydrogen atoms. The high field seeking spin states are defocused so that the remaining $\overline{\mathrm{H}}$ beam is composed of the states $(F, M)=(1,1)$ and $(1,0)$. If the anti-hydrogen is still in a higher Rydberg state and was not able to cascade to the ground state in time, then those particles are lost as well.

### 1.2.5 Spin flip cavity, Sextopole Magnet and Detector



Figure 1.6: Schematic view of spin selection, transition and detection elements, adapted from [23].

The polarized beam reaching the RF cavity after the CUSP trap, is exposed to the RF field with tunable frequency. The cavity sweeps over its tune range and if it is equal to one of the actual transitions $\pi_{1}$ or $\sigma_{1}$, it induces a spin flip in the anti-hydrogen beam to the HFS state. After the cavity another sextupole field removes the HFS states from the beam before the detector [29]. Mapping the frequency sweep to the time stamped data of the detector the transition lines $\pi_{1}$ and $\sigma_{1}$ can be measured. In figure 1.7 the result from a similar experiment using hydrogen is shown [3].

The detector at the end of the beam line is made from $\mathrm{Bi}_{4} \mathrm{Ge}_{3} \mathrm{O}_{12}$ crystal of 10 cm diameter and 5 mm thickness, therefore called BGO detector. It is a scintillating material of high density that is compatible with the ultra-high vacuum conditions present in the set up. The anti-protons from incident $\overline{\mathrm{H}}$ annihilate on the crystal producing photons


Figure 1.7: Measurement of $\sigma_{1}$ and $\pi_{1}$ spin flips in a hydrogen beam [3].
and, on average, three pions. The latter are detected by multiple scintillator bars that are installed around the BGO, forming a hodoscope, and covering about $50 \%$ of the total solid angle [2]. The scintillation light is picked up by the two SiPMs attached to each bar and their signal is compared to the BGO detection signal. This is done to separate an actual signal of a $\bar{H}$, that produces a coinciding signal in the BGO and in up to three scintillators, from background radiation that comes from pions created by annihilation in the CUSP or cosmic rays, producing a signal in only two bars and the BGO.

Additional scintillation plates for detecting pions are situated next to the CUSP trap, to spatially resolve annihilation of anti-protons and $\overline{\mathrm{H}}$ inside the trap with an axial precision of a few cm . Therefore the axial position of $\overline{\mathrm{p}}$ can be measured to get an idea of the distribution inside the CUSP.

### 1.3 Motivation and aim of the Thesis

In 2010 the ASACUSA experiment produced anti-hydrogen in the CUSP trap for the first time [23]. The $\overline{\mathrm{H}}$ left the nested trap part of the potential and were ionized by the FIT (see section 1.2.4). The anti-protons then accumulated in the FIT well for 5 s before the downstream potential is lowered and the $\bar{p}$ are transported to the detector. The corresponding signal is shown in figure 1.8. In the first 20 s the signal is significantly lower, because the injected anti-protons need some time to cool to energies low enough to form anti-hydrogen. The production peaks at 25 s , so that a maximal number of anti-protons interact with the positrons at sufficiently low energy, but for an unknown reason the $\overline{\mathrm{H}}$ yield steadily drops from there.

Considering that only a small number of anti-protons form $\overline{\mathrm{H}}$ and no significant increase in annihilation can be detected by the 3D scintillation plates, then there must be a process steadily impairing the conditions for anti-hydrogen formation on a time-scale of $20-50 \mathrm{~s}$. A possibility would be a mechanism partially heating positrons or anti-protons to energies too high to form significant amounts of $\overline{\mathrm{H}}$. It seems unlikely that heating acts on this time scale and consistently starts after about 20 s .


Figure 1.8: $\overline{\mathrm{H}}$ formation rate in beamtime 2010 [1].

Another possibility would be axial separation caused by axial energy loss of the antiprotons so that they can not overcome the central potential barrier in the nested trap anymore and accumulate in the bottom of one of the side wells of the harmonic potential. Even though they are cooling down, what would be favorable for $\overline{\mathrm{H}}$ formation, they do not touch the positron plasma anymore. This effect was already investigated and a RF-excitation mechanism was introduced to counter the axial separation effect. This is done by applying an RF signal to electrode U7 with frequency close to the frequency of the harmonic well the anti-proton accumulated in. This electrical field axially excites the $\bar{p}$ so that they gain enough energy again to pass the positron plasma again [2]. The most recent results from 2012 are shown in figure 1.9 where RF-excitation is applied (red dots). There is a significant increase and shift of the maximum of $\overline{\mathrm{H}}$ detected, but even then a drop acting on approximately the same time scale as before (black dots). Therefore axial separation alone does not explain the steady drop in formation efficiency.

If it is not axial separation removing anti-protons from the set of particles available for $\overline{\mathrm{H}}$ formation, then maybe they are leaving the positron plasma radially.

The physical process that is possibly responsible for a radial outward drift of the antiproton is the so called centrifugal separation. Amoretti et al. [30] present a model that directly links the collisions of anti-protons and positrons to a slow radial outward drift of the form

$$
\begin{equation*}
r_{c}(t) \sim r(0) e^{\gamma t} \tag{1.14}
\end{equation*}
$$

where $r_{c}$ is the magnetron radius of the anti-proton (see section 2.1.2.2). This model is described in more detail in section 2.2. In short, the collisions of anti-protons with positrons results in an radial energy loss of the former collision partner that couples to the magnetron motion slowly increasing its radius. In figure 1.10 we can see a similar


Figure 1.9: H formation rate in beamtime 2012, with direct injection of anti-protons(black squares) and auto resonance (red dots) taken from [2].
simulation result from an ion that cools radially by collisions with neutral buffer gas atoms.


Figure 1.10: Ion interaction with a neutral buffer gas leads to a loss of cyclotron energy that translates to an increase of magnetron radius [31]

This steady drift is the averaged product of many collisions of anti-protons with positrons so the parameter we need to determine in this thesis to calculate the outward drift is
the average radial energy loss of an anti-proton in an positron plasma.

In section 3.1 we will outline methods to calculate the radial energy loss and the simulation approach chosen in the end.

Finally in chapter 4 the results of the simulations will be presented and section 4.5.6 will discuss if the radial drift is a possible reason for the drop in $\overline{\mathrm{H}}$ yield overtime.

## Chapter 2

## Theory

### 2.1 Basics of plasma physics

The cloud of positrons in the CUSP trap (see section 1.2.4) is a non-neutral plasma therefore an introduction into the basic properties of such a plasma is necessary. The next sections are loosely following the (unpublished) lecture scripts for the theoretical plasma physics course of Prof. Dr. Kamelander of the Technical University (TU) Vienna.

### 2.1.1 Definition of a plasma

Plasma defines the fourth state of matter and generally consist of charged as well as neutral particles. Not all such particle mixtures qualify as plasma though, as it has to possess the following properties,

- the Debye length of the plasma must be much smaller than its size,
- the number of particles in a sphere with Debye radius must be much bigger than 1 ,
- and the scattering frequency of charged with neutral particles must be smaller than the respective plasma frequencies.


### 2.1.1.1 Debye length

The Debye length, also called screening length, is a specific distance scale that arises if charged particles are present. Imagine for simplicity a homogeneous gas of negatively charged ions or electrons with some finite temperature $T$. If we introduce a positive charge into the gas it will attract the negative particles, which will repulse each other in addition to their thermal motion. In the stable final configuration the negative particle density will shift towards the positive charge such that it is effectively covered by them as we can see in figure 2.1.


Figure 2.1: Schematic view of the charge shift in a negatively charged plasma introducing
a single positive charge.

The higher density of negative particles around the positive charge screens its potential, such that the gas further away is not influenced by it anymore. The thermal motion of the gas prevents the negative charges to build up to an densely packed particle layer around the positive charge, so that the screening is continuous radially outwards. For a generally applicable solution to this problem we consider a thermal plasma consisting of positive, negative and neutral particles with their respective densities,

$$
\begin{align*}
& n_{\mathrm{neg}}(\vec{r})=n_{0} \cdot e^{\frac{e \phi(\vec{r})}{k_{B} T}} \\
& n_{\mathrm{pos}}(\vec{r})=n_{0} \cdot e^{-\frac{e \phi(\vec{r})}{k_{B} T}} \tag{2.1}
\end{align*}
$$

Here $n_{0}$ is the initial equilibrium density without introducing the external charge, furthermore $e$ is the unit charge and the plasma is considered to consist of particles with positive or negative charge ( $1 e$ ). The plasma temperature $T$ causes the density to follow a Boltzmann distribution. Then the total charge density together with the external positive charge $(+Q)$ is

$$
\begin{equation*}
\rho(\vec{r})=e\left(n_{\mathrm{pos}}-n_{\mathrm{neg}}\right)+Q \delta(0) \tag{2.2}
\end{equation*}
$$

With the charge density in hand we have to solve the Poisson equation to find the solution for the total electric potential

$$
\begin{equation*}
\Delta \phi(\vec{r})=-\frac{\rho(\vec{r})}{\epsilon_{0}} . \tag{2.3}
\end{equation*}
$$

Using equation 2.1 and 2.2, applying a first order series expansion to the exponential terms and using the fact that the system is spherical symmetric around the external charge one can solve equation 2.3 and find the electric potential to be

$$
\begin{equation*}
\phi(\vec{r})=\frac{Q}{4 \pi \epsilon_{0} r} \cdot e^{-\frac{r \sqrt{2}}{\lambda D}} \quad \lambda_{D}=\sqrt{\frac{\epsilon_{0} k_{B} T}{n_{0} e^{2}}} \tag{2.4}
\end{equation*}
$$

with $\lambda_{D}$ the Debye length of the plasma. The potential consists of the part caused by the bare charge that is damped by an exponential function with length scale $\lambda_{D}$. That means for a lower temperature the Debye length will go to zero such that the negative charges are able to pile up around the central charge and shield it in a very thin layer around it, while for higher temperature the thermal motion prevents them from getting close widening the layer. The first plasma criteria now dictates that the plasma size itself must be much bigger than $\lambda_{D}$ so that the effect described can occur naturally over the whole plasma.

The second criteria in the beginning of the section is directly connected to the derivation given above. In Equation 2.1 we used the Boltzmann distribution, a statistical concept, that is only valid if there are enough particles to interact with each other and eventually settle in the average static solution used.

### 2.1.1.2 Plasma oscillation

The third criteria needs us to introduce the concept of plasma frequency. Consider a stationary neutral plasma state and now displace some negative charges in a small area by a small amount, so we are now effectively having an area of surplus negative particles next to an positively charged area. After stopping the external influence that separates the charges the two areas will attract each other to get back to the original neutral state. The accelerated particles can not immediately assume their old state, because their inertia will make them overshoot and will force them to oscillate around their former positions. Neglecting interactions and energy loss this collective oscillation has the frequency

$$
\begin{equation*}
\omega_{p}=\sqrt{\frac{n q^{2}}{\epsilon_{0} m}} \tag{2.5}
\end{equation*}
$$

This is the plasma frequency for particles with charge $q$, $n$ their local density and mass m . These oscillations are essential for the collective motions inside of a plasma, and therefore they should not be strongly dampened by interaction of the charged with
neutral particles. This scattering with neutrals can be expressed in terms of a collision frequency $\omega_{\mathrm{ne} / \mathrm{ni}}$ for, in this case, electrons and positive ions. The third plasma criteria can now simply be written as

$$
\begin{equation*}
\omega_{\mathrm{pe}, \mathrm{pi}} \gg \omega_{\mathrm{ne}, \mathrm{ni}} \tag{2.6}
\end{equation*}
$$

If this condition is not fulfilled the collisions of charged particles with neutrals dominate over the interaction between the charges and the plasma acts as a simple gas.

### 2.1.2 Motion of charged particles in electromagnetic fields

In Chapter 1 we saw that the processes investigated in this thesis, the interaction of anti-protons and positrons, takes place in an CUSP trap. The basic motion of a charged particle due to electric and magnetic fields is therefore an integral part in describing the physics involved.

The electric and magnetic fields inside the trap do not change much over the positron plasma volume, so it is a good approximation to consider the fields to be homogeneous. Then the equation of motion for a single charged particle subject to the Lorentz force is then

$$
\begin{equation*}
\frac{d \vec{v}}{d t}=\frac{q}{m}(\vec{E}+\vec{v} \times \vec{B}) \tag{2.7}
\end{equation*}
$$

The vector product in equation 2.7 naturally decomposes the motion of the particle in the direction along the magnetic field and perpendicular to it. The equation of motion along the magnetic field is described by a simple constant acceleration by the electric field, as the magnetic field does not have a component acting along it

$$
\begin{equation*}
\frac{d \vec{v}_{\|}}{d t}=\frac{q}{m} \vec{E}_{\|} \quad \vec{r}_{\|}(t)=\frac{q}{2 m} \vec{E}_{\|} t^{2}+\vec{v}(0) t+\vec{r}(0) \tag{2.8}
\end{equation*}
$$

### 2.1.2.1 Cyclotron motion

For the perpendicular direction, assume for simplicity that there is no electric but only the magnetic field present for the moment. In this case the equation of motion reduce to

$$
\begin{align*}
\frac{d \vec{v}_{\perp}}{d t}=\frac{q}{m} \vec{v}_{\perp} \times \vec{B} & \Rightarrow \frac{d \vec{v}_{\perp}}{d t}=\vec{\omega}_{c} \times \vec{B}  \tag{2.9}\\
\vec{\omega}_{c} & =-\frac{q}{m} \vec{B} \tag{2.10}
\end{align*}
$$

The simple integration finds the perpendicular motion in a magnetic field

$$
\begin{array}{ll}
v_{x}=v_{\perp} \sin \left(\omega_{c} t+\phi_{0}\right) & x(t)=-\frac{v_{\perp}}{\omega_{c}} \cos \left(\omega_{c} t+\phi_{0}\right)+x(0) \\
v_{y}=v_{\perp} \cos \left(\omega_{c} t+\phi_{0}\right) & y(t)=\frac{v_{\perp}}{\omega_{c}} \sin \left(\omega_{c} t+\phi_{0}\right)+x(0) \tag{2.12}
\end{array}
$$

This parameterizes a circular motion in the plane perpendicular to the magnetic field with angular frequency $\omega_{c}$, the cyclotron frequency, and cyclotron radius

$$
\begin{equation*}
r_{c}=\frac{v_{\perp}}{\omega_{c}} \tag{2.13}
\end{equation*}
$$

This is an extremely important property for this work, as it shows, that the motion perpendicular to the magnetic field is "bound" to it. Consider the case of an electron with room temperature $T \sim 300 \mathrm{~K}$ in an magnetic field of $B=2.8 \mathrm{~T}$. The cyclotron radius is then about 270 nm . That means that a charged particle can not move freely in this plane. Furthermore any (small) change in energy (velocity), for example by a collision, does only translate into an respective change of cyclotron radius but the average position of the particle stays on the central field line. Combined with the linear acceleration by the axial electric field this yields then a helical motion along the magnetic field as in figure 2.2.

### 2.1.2.2 Magnetron motion

In the last part we ignored, for simplicity, an electric field acting perpendicular to the magnetic field. This addition leads to the equation of motion

$$
\begin{equation*}
\frac{d \vec{v}_{\perp}}{d t}=\frac{q}{m}\left(\vec{E}_{\perp}+\vec{v}_{\perp} \times \vec{B}\right) \tag{2.14}
\end{equation*}
$$

This can be solved with a simple substitution with a constant velocity $\vec{v}_{E}$

$$
\begin{equation*}
\vec{v}_{\perp}=\overrightarrow{v^{\prime}} \perp+\vec{v}_{E} \quad \vec{v}_{E}=\frac{\vec{E} \times \vec{B}}{B^{2}} \tag{2.15}
\end{equation*}
$$

for which equation 2.14 transforms into

$$
\begin{equation*}
\frac{d \overrightarrow{v^{\prime}} \perp}{d t}=\frac{q}{m} \overrightarrow{v^{\prime}} \perp \times \vec{B} \tag{2.16}
\end{equation*}
$$

which has the same form as Equation 2.9, leading to a cyclotron motion that is superposed with the velocity $\vec{v}_{E}$, called the ExB-drift velocity. It is important to note, that this drift velocity is independent of charge, such that both anti-protons and positrons


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Figure 2.2: Helical motion of a charged particle in an electromagnetic field, taken from [32].
drift in the same direction. This result comes into play if we now consider a stationary plasma of positrons sitting in the central well of the CUSP trap (section 1.2.4). We note that the cumulative charge builds up a radial electric field (perpendicular to the magnetic field) and induce a ExB drift. In figure 2.3 we can see the radial electric field rising linearly until the plasma boundary at $r=1 \mathrm{~mm}$ and then falling off with $r^{-2}$ as expected from electrostatic theory. On top of the field generated by the positron plasma we have a contribution from the MRE electrodes that is negligible inside of the plasma, as shown by the red line.

It is important to note, that inside the plasma the radial electric field is increasing linearly, such that the ExB drift velocity is

$$
\begin{equation*}
\vec{v}_{E}(r)=\frac{\vec{E}(r) \times \vec{B}}{B^{2}}=\frac{d \vec{E} / d r \times \vec{B}}{B^{2}} \cdot r \tag{2.17}
\end{equation*}
$$

rising linearly with $r$ as well. Furthermore the radial electric field is radially symmetric, that means, that for a particle at some distance $r$ the ExB drift velocity is always tangential to the circle with radius $r$ and the particle will move along it with velocity $\vec{v}_{E}$. This movement is called magnetron motion and the radius on which the particle moves magnetron radius.


Figure 2.3: Radial electric field at the center of the positron plasma in the CUSP trap as a superposition of the field of the plasma and the outer electrodes. The red lines shows the electrode field if no plasma is present.

The angular frequency of this magnetron motion, the magnetron frequency, around the center of the positron plasma is

$$
\begin{equation*}
\omega_{0}=\frac{\vec{v}_{E}}{r}=\frac{d \vec{E} / d r \times \vec{B}}{B^{2}} \tag{2.18}
\end{equation*}
$$

We see that this frequency is constant for all particles inside of the cloud as $E \propto r$ there, which means that the whole cloud of positrons and the anti-protons within turn with the same angular velocity in the same direction and acts as a, so called, rigid rotor.

### 2.2 Centrifugal separation

In [30] Amoretti et al. present a simple model for a charged particle in an infinite plasma column and the centrifugal separation timescale. In 2.19 the equation of motion is given for an ion inside the plasma column, where $\Omega_{i}$ is the cyclotron frequency of the ion and $\omega_{0}$ is the rigid rotor (magnetron) frequency. The collision frequency $\nu$ is acting on the cyclotron motion adding or removing energy from the radial motion.

$$
\begin{align*}
& \frac{d^{2} x}{d t^{2}}=\Omega_{i}\left(\omega_{0} x+\frac{d y}{d t}\right)-\nu\left(\frac{d x}{d t}-\omega_{0} y\right) \\
& \frac{d^{2} y}{d t^{2}}=\Omega_{i}\left(\omega_{0} y-\frac{d x}{d t}\right)-\nu\left(\frac{d y}{d t}+\omega_{0} x\right) \tag{2.19}
\end{align*}
$$

Here $x$ and $y$ are the coordinates in the plane perpendicular to the magnetic field inducing the cyclotron motion. Using the complex Ansatz $u(t)=x(t)+i y(t)$ one can solve the equation of motion and finds that the magnetron radius will increase with time as

$$
\begin{align*}
r_{c}(t) & \sim r(0) e^{\gamma t}  \tag{2.20}\\
\gamma & \sim \frac{\omega_{0}^{2} \nu}{\Omega_{i}^{2}+\nu^{2}} \tag{2.21}
\end{align*}
$$

The magnetron and cyclotron frequency are given by

$$
\begin{align*}
\omega_{0} & =\frac{\omega_{p}^{2}}{2 \Omega_{e}}=\frac{n_{e} e}{2 \epsilon_{0} B}  \tag{2.22}\\
\Omega_{i} & =\frac{e B}{m_{i}} \tag{2.23}
\end{align*}
$$

The calculation of the collision frequency needs more consideration. The energy transfer we get from simulations is the change of radial energy while moving axially. Note that the collision frequency in equation 2.19 acts along the cyclotron motion of the antiproton in the radial plane. Therefore we must map the energy change to the distance covered radially by calculating the path in the radial plane, while the anti-proton moves axially.

$$
s_{\text {radial }}=s_{\text {axial }} \frac{v_{\text {radial }}}{v_{\text {axial }}}
$$

From Equation 2.19 follows

$$
\begin{align*}
\left.\frac{d E}{d x}\right|_{\text {radial }}=\left.\frac{d E}{d x}\right|_{\text {axial }} \frac{s_{\text {axial }}}{s_{\text {radial }}} & =\left.\frac{d E}{d x}\right|_{\text {axial }} \frac{v_{\text {axial }}}{v_{\text {radial }}}=F_{c}=-m_{\overline{\mathrm{p}}} v_{\text {radial }} \nu \\
\nu & =-\left.\frac{v_{a x}}{m_{\overline{\mathrm{p}}} v_{\text {radial }}^{2}} \frac{d E}{d x}\right|_{\text {axial }} \tag{2.24}
\end{align*}
$$

Generally the energy transfer will change over time, as the particle is losing energy. Therefore the collision frequency $\nu$ and the outward transport rate $\gamma$ will change as well. For a time dependent $\gamma(t)$ we can generalize equation 2.20 to

$$
\begin{equation*}
r_{c}(t) \sim r(0) e^{\int_{0}^{t} \gamma\left(t^{\prime}\right) d t^{\prime}} \tag{2.25}
\end{equation*}
$$

Therefore it is important to see that the calculations of the drift times in the sections below, that are using a constant energy transfer for a first approach, only provide a
guiding value for the time the anti-proton needs to leave the plasma!

### 2.3 Rutherford scattering

Rutherford investigated the results of the experiments of Marsden and Geiger [33] that show deflection of alpha and beta particles when passing matter, more precisely a $40 \mu \mathrm{~m}$ gold foil. He developed a theory to calculate the deflection angle for a charged particle passing an heavy atom [34].

Originally he started to look at an atom with positive core charge $N \cdot e$ and electrons with the same total charge homogeneously distributed over a sphere with Radius R. The geometrical description of the particle interaction derived from this setup can be applied the same way to the positron anti-proton collisions this thesis investigates. Figure 2.4 shows the situation of a charged particle scattered by the charged nucleus. The collision is considered elastic, so the initial and final momentum have the same magnitude but different direction. This originally stems from the fact that the nucleus was considered much heavier than the incident particle so that the recoil could be ignored.


Solving the geometrical equations leads to the famous Rutherford scattering formula for the cross section

$$
\begin{align*}
\frac{d \sigma}{d \Omega} & =\left(\frac{d_{c}}{4}\right)^{2} \frac{1}{\sin ^{4}(\theta / 2)}  \tag{2.26}\\
d_{c} & =\frac{Q_{1} Q_{2}}{4 \pi \epsilon_{0}} \frac{1}{E_{\mathrm{kin}}} \tag{2.27}
\end{align*}
$$

The parameter $d_{c}$ denotes the distance of closest approach in a head on collision and is used to simplify equation 2.26 . The results of the simulations conducted in this thesis come foremost in the form energy transfer over impact parameter $b$. Therefore we can find a corresponding equation in $[36, \mathrm{p} 626]$ that gives the exact relation for the energy transfer that is needed

$$
\begin{align*}
T(b) & =\left(\frac{Q_{1} Q_{2}}{4 \pi \epsilon_{0}}\right)^{2} \cdot \frac{2}{m_{e} v^{2}} \cdot \frac{1}{b^{2}+b_{\min }^{2}}  \tag{2.28}\\
b_{\min } & =\frac{Q_{1} Q_{2}}{4 \pi \epsilon_{0}} \frac{1}{m_{e} v^{2}} \tag{2.29}
\end{align*}
$$

for the anti-proton positron collision investigation. The parameter $b_{\text {min }}$ regularizes the energy transfer for impact parameter zero such that $T(0)=2 m_{e} v^{2}$.

## Chapter 3

## Simulation Program

This chapter deals with the set-up of the simulations and introduces the programs used in the process. The basis of the computation is the program SIMBUCA [37] written for the WITCH project [38]. It was used for the computation of the particle trajectories and interaction. The setup and configuration of SIMBUCA was done by a dedicated self-written program - MCSIM - (see section 3.3) that utilized the Monte Carlo method (see section 3.2) to produce statistical results for the observables of interest.

### 3.1 Motivation

The decision to use a Monte Carlo method stemmed from the fact that there are only a few theoretical approximate approaches available, which are complex (see [39]) and difficult to apply to our parameters. Several theories were simplified by ignoring the physics in the plane perpendicular to the magnetic field, which is needed to solve our problem (see section 1.3). It turns out, that for our parameters of interest in the CUSP, the problem is strongly chaotic and theories have a hard time to predict the outcomes anyway. Without a generally applicable comprehensible theory in hand, we turned to numerical simulations of the problem.

The first approach was to simulate the full particle dynamics in the CUSP trap with the program SIMBUCA. We were dealing with about $10^{7}$ positrons and $10^{6}$ anti-protons and as the Coulomb interaction computation scales with $N^{2}$ such a calculation is too expensive in terms of computational power needed. The fastest approximate methods available for the calculation are of order $N \cdot \log N$ where, on a reasonably powerful single computer, the computation of one time step takes about a second. The effect investigated by this thesis (see section 1.3) is of the order of tens of seconds, while the cyclotron motion of the positron allows for integration time steps not bigger than about $1 \times 10^{-13} \mathrm{~s}$, otherwise energy is not conserved in the trajectory integration. The resulting simulation time of $1 \times 10^{14} \mathrm{~s} \sim 3 \times 10^{7}$ years, is obviously not applicable as well.

If we reduce our focus to a single anti-proton, the Debye shielding, explained in section 2.1.1.1, can reduce the number of positrons that need to be simulated, as the influence of $e^{+}$further away than a few times the Debye length can be neglected. Regrettably the number of particles and the corresponding simulation time were still too high for our purposes. On the other hand, the Debye length for our parameters was of the order of $100 \mu \mathrm{~m}$ and therefore much larger than the average positron-positron distance of about $10 \mu \mathrm{~m}$, such that the interaction of the anti-proton with a single positron is barely influenced by the rest of the cloud. This means we can handle the interaction of an anti-proton with the positron cloud as the superposition of many single encounters, enabling us to use a statistical procedure like the Monte Carlo method.

The advantage is striking, as instead of simulating the interaction of millions of particles with each other, the integration simplifies to millions of calculations of a two particle interaction effectively scaling with order $N$. Furthermore for a first approach an averaged action of the positron cloud on the anti-particle is enough, so depending on the aspired precision the number of trajectories computed can be lowered even more.

The downside of this approach is, that it is only another approximation, neglecting positron-positron interaction. That means f.e. the interaction of the anti-proton with positron plasma waves would add another contribution that has to be verified separately.

### 3.2 CTMC - Classical trajectory Monte Carlo method

The Monte Carlo method is a statistical procedure to describe a process by generating an output distribution from randomly chosen parameter from the input distribution. If this is done many times, depending on the error limits and the process itself, the method gives a suitable representation of the process in question.

A very simple example is the integration of a function $f(x)$ such that

$$
\begin{equation*}
F(x)=\int_{a}^{b} f(x) d x \tag{3.1}
\end{equation*}
$$

If $f(x)$ is a very complex function, that is not integrable by analytic means, the Monte Carlo method can easily give a result, as the integral is simply the area bounded by the x -axis and the function $f(x)$ itself. In figure 3.1 we can see such a function and we want to know the integral value between $a=1$ and $b=3$. Our input distribution is now all possible points in the rectangular target area that is bounded by the x -axis and the two limits $a$ and $b$. The top boundary can be defined freely (even by any arbitrary curve, as long as the total area is known) but must fully enclose the function $f(x)$.


Figure 3.1: Integration by the Monte Carlo method.

The Monte Carlo algorithm now randomly selects points ( $x, y$ ) from the initial homogeneous distribution and simply counts if the point is under the function curve by checking if $y<f(x)$. We know the area of the rectangle and by computing the fraction of points under the function over the total number we find

$$
\begin{equation*}
\frac{N_{f(x)}}{N_{\text {total }}} \sim \frac{A_{f(x)}}{A_{\text {rect }}} \Rightarrow F(x)=\int_{a}^{b} f(x) d x \sim \frac{N_{f(x)}}{N_{\text {total }}} \cdot A_{\text {rect }} \tag{3.2}
\end{equation*}
$$

The error on this computation depends on the number of points and for $N_{\text {total }} \rightarrow \infty$ the relation becomes exact.

### 3.2.1 Application of the Monte Carlo method

Here we want to utilize the Monte Carlo method to probe the two particle scattering in a magnetic field. First we must define the input distribution to choose from. Those are then randomly picked by the algorithm and the collision is computed taking them as input. These are:

- radial position of the $e^{+}$( $\rightarrow$ impact parameter),
- axial and radial positron velocity (fixed in some simulations),
- and cyclotron phase angle of the $\overline{\mathrm{p}}$ and $e^{+}$(if the particles have radial velocity).

The radial position is picked from a homogeneously distributed disc in the $x, y$ plane, described in more detail in section 3.3.2. The positron velocities will be fixed in some of the simulations conducted in this thesis or randomly diced from an Maxwell-Boltzmann thermal distribution, but with fixed temperature. Some parameters will be kept fixed in our Monte Carlo simulations:

- magnetic field strength $B$,
- anti-proton axial and radial velocity,
- positron temperature,
- and initial axial particle position ( $\overline{\mathrm{p}}, e^{+}$).

After all parameters are set, either by having a fixed value or being randomly selected from the initial distributions, the trajectory is integrated and the particle values extracted at the end. This is repeated many times keeping the fixed values untouched and dicing new parameters from the initial distributions every time. Careful consideration is needed to decide if enough trajectories are generated to fulfill our precision goal. This is discussed in section 3.3.3.

### 3.3 MCSIM

The program MCSIM was written to implement the Monte Carlo method and provide the tools to extract and process the output data from the computed particle trajectories. The integration of the trajectories is done by the program SIMBUCA, for details see section 3.4.

The input configuration file for SIMBUCA is written by MCSIM, describing the set up and delivering the initial parameters. A sample file can be seen in figure 3.2.

## CREATEPARTICLES 2

PARTICLES antiproton $1.17109 \mathrm{e}-061.63634 \mathrm{e}-050.0-0 \quad 0 \quad-13800$ e+ $-3.54251 \mathrm{e}-06 \quad-2.36863 \mathrm{e}-05 \quad-5 \mathrm{e}-05 \quad 37104.2 \quad 39384.2 \quad-7205.64$
BUFFER 0 le-4
ODE $511 \mathrm{le}-14$
COULOMB 11
OUTPUT ./sakel_Thesis_CUSP_B2_8T_Tpos100K_Vax13800_Vrad4400_MC/thermalPlasmaT100/Impact201234 3.0327e-09 1
IDEALTRAP 0.040 .0000012 .8
NE $1.51645 \mathrm{e}-06$
Figure 3.2: Sample simulation file for SIMBUCA.

The first line simply gives the number of particles in the simulation, in our case two: the anti-proton and positron. The next line sets the initial positions and velocities for the particles determined by MCSIM and we will discuss this in section 3.3.1. The BUFFER line is not necessary and can be ignored, but the next line sets up the integration routine
of SIMBUCA.

The first number determines the integration algorithm, in this case a Dormand-Prince 5 th order scheme, and the second number activates(1) or deactivates(0) the adaptive step size function, which takes the third number in line, the initial step size, and increases and decreases it over the course of integration as necessary. This has the advantage that for the part of integration, where the two particles do not interact strongly with each other, the time step can be kept at the maximum value, determined by the fastest motion in the system, the positron cyclotron movement, which sets the upper limit to about $1 \times 10^{-13} \mathrm{~s}$. Around the point of closest distance between the particles the interaction can become so strong, that the time step needs to be reduced by the algorithm, about $1 \times 10^{-21} \mathrm{~s}$ was observed, to precisely integrate at that point. With a fixed time step the whole integration would need to run with $1 \times 10^{-21} \mathrm{~s}$, even though it is only needed for a very small part of the trajectory, slowing down overall integration by many orders of magnitude.

The COULOMB tag and the (1) after it activates the Coulomb interaction between the particles with strength $(k=1)$, which would multiplicatively modify the Coulomb force equation $F=k * F_{\text {Coulomb }}$. The next line defines the folder where the output of the trajectory will be written to by SIMBUCA and the number next to it the time step when it will be written. This number has no influence though, as only the first and last line of the output file will be used for further calculations. The IDEALTRAP line sets up a harmonic potential generated by an electrode with inner radius set by the first and field strength implicitly defined by the second number. In our simulations we don't apply any external electrical fields (only the magnetic field of the trap), as the influence of the radial field is already contained in the magnetron motion and the axial field is shorted out by the positron plasma. Therefore the field strength is set to effectively zero (SIMBUCA needs a non-zero value passed to it though). The third number gives the magnetic field strength along the axial direction. The last line simply sets the final time for the integration, that is actually not used for our computations. For details see section 3.4.1.

### 3.3.1 Initial simulation parameters

The Monte Carlo method sets the initial values for the anti-proton and positron position and velocities. It is important to note, that the coordinate system used in the simulations is set up around the collision of the anti-proton and positron (see figure 4.1) and does not correspond to the experimental setup. First of all the $\bar{p}$ starts always at axial position $z=0$. Its radial position values $(x, y)$ are determined by the constant radial velocity set in the program, such that the anti-proton is set on the respective cyclotron radius (see equation 2.13), with the center of the circular motion, also called guiding center, located on axis $(0,0)$. This implements the redefinition of impact parameter for
two particles in a magnetic field with non-zero radial velocity, that is, the radial distance between the center of cyclotron motion of the respective particles. Therefore the positron guiding center coordinates directly give the impact parameter of the simulation.

For the exact placement of the anti-proton on its cyclotron radius the phase is randomized (see section 3.2.1). Additionally there is the possibility to either use a fixed radial velocity value, or dice the number from a Maxwell-Boltzmann distribution with a fixed temperature. The axial velocity is a fixed value set in MCSIM.

For the positron velocities there is the possibility to either set the radial and axial velocity to a fixed value or dice it from a thermal distribution with fixed temperature. This can be done for both directions separately. The axial placement of the positron now depends on both values of axial velocity of $e^{+}$and $\overline{\mathrm{p}}$. In the case that $v_{z, e^{+}}<v_{z, \overline{\mathrm{p}}}$ means that the positron is set to $z=-$ boxsize for $v_{z, \overline{\mathrm{p}}}<0$ and to $z=+$ boxsize for $v_{z, \overline{\mathrm{p}}}>0$. On the other hand, if $v_{z, e^{+}}>v_{z, \overline{\mathrm{p}}}$ the positron distance to the anti-proton, that can not keep up, will increase over time and no collision occurs. In a cloud of positrons there can be a positron coming from behind with that same velocity instead and give the anti-proton a kick from behind. This is realized by inverting the relation from before to $z=-$ boxsize for $v_{z, \overline{\mathrm{p}}}>0$ and to $z=+$ boxsize for $v_{z, \overline{\mathrm{p}}}<0$. The only parameter left is the radial placement of the positron, that has several important aspects tied to it and deserves its own section.

### 3.3.2 Impact disc

Generally the radial position has two major contributions. On the one hand the cyclotron motion of the $e^{+}$defines a radius depending on its radial velocity and with the help of a randomized phase it can be placed on the respective cyclotron circle.

On the other hand, the distance from the center of this cyclotron circle to the guiding center of the anti-proton defines the generalized impact parameter for scattering in a magnetic field (see section 3.3.1). This parameter is the only distance indicator of the scattering process, because the particles will axially pass each other, and therefore significantly defines the result of the collision simulation. It is of utmost importance and in Chapter 4 we will use it to display several output values with respect to the impact parameter.

Imagine now the anti-proton axially passing through the positron cloud and randomly encountering positrons in different radial distances. To reflect this in the Monte Carlo simulation we now need to randomly place the positrons guiding center in the radial plane. It would make no sense to use the whole $\mathcal{R}^{2}$, as the Coulomb interaction strength declines with $r^{-2}$, additionally damped by Debye shielding (see section 2.1.1.1), so the
contributions of particles far out can be neglected. The question is, how to decide the maximum distance used for the impact parameter in the simulation?

With no theory at hand to give some limit, we need to implement a procedure that checks, if particles outside of a set initial radial distance have an impact on our results, and if they do, increases the range of possible impact parameter to take them into account. This procedure will be discussed in the next section 3.3.3.

In summary the positron radial position is made up by the random placement of its guiding center inside a disc of a specific radius, from here on out called the impact disc, and the added offset from placement on the cyclotron circle with random phase.

### 3.3.3 Simulation convergence

In section 3.2 we briefly discussed that we need to repeat the process subject to the Monte Carlo analysis over and over until the error of our output is below some threshold. Applied to our problem, we need to repeat the collision simulation until we have enough trajectories to satisfy some error limit.

First we need to choose an output parameter of our simulations to define an error for. If we go back to section 1.3 we see that in the end we want to calculate an energy transfer from the simulation, that is proportional to the energy transfer averaged over all trajectories calculated. Therefore our parameter of choice to base our error assessment on is the average total energy transfer between the particles.

The next step is to define the error computation itself and for that we use a statistical property of average values. The central limit theorem of mathematics states that in the limit for infinitely many independent identically distributed random numbers (in our case the total energy transfer of a collision) their average follows a normal distribution [40]. For a finite but large number N the resulting distribution can still be approximated by a normal distribution with the width

$$
\begin{equation*}
\sigma_{\bar{E}}^{2}=\frac{\sigma_{E}^{2}}{N}=\frac{\sum_{i=0}^{N}\left(E_{i}-\bar{E}\right)^{2}}{N(N-1)} \tag{3.3}
\end{equation*}
$$

If we apply this theory here, our average total energy transfer will be normally distributed and the width will give us a measure of how probable the calculated average is. Normalizing to a relative value

$$
\begin{equation*}
\bar{\sigma}=\frac{\sigma_{\bar{E}}}{\bar{E}} \tag{3.4}
\end{equation*}
$$

we have found our error variable. Increasing the number of computed trajectories, this $\bar{\sigma}$ will eventually go to zero, so we define our simulation to be convergent if $\bar{\sigma}<1 \%$.

Now we know when our simulation has enough trajectories to be considered finished, but we still have the problem from section 3.3.2 of the upper limit of the radius of the impact disc. This can be solved by first giving an educated guess for an initial size of the impact disc and creating enough collision trajectories for the simulation to converge. Then increase the disc radius and again run the simulation until it converges with the increased radius. Finally compare the average total energy transfer of the two computations and if the result did not change considerably then the impact disc size converged as well.

To be precise the value converging is the product of average energy transfer times area of the impact disc. Figure 3.3 shows a simple case to proof this statement. Assume a disc with radius $R_{1}$, with all trajectories inside yielding the energy transfer of 1.0, so the average will be 1.0 as well. Next increase it to the radius $R_{2}$, but all energy transfers outside of $R_{1}$ yield 0 , so they can in principle be neglected. But the average transfer of the disc with radius $R_{2}$ is less now, because several trajectories with transfer 0 were added to the average connected to $R_{1}$. If we look on the product of area and average we can see that this compensated for the aforementioned drop and therefore this product will not change if we increase the area to include neglectable trajectories, making it the parameter of choice for the radial convergence.


Figure 3.3: A simple example to show, that the number - area • average - converges to a constant if the radius of the impact disc is increased, including trajectories with a very small energy ( $\sim 0$ ) transfer for higher impact parameter.

Of course trajectories will in general not yield zero energy transfer, but if it is small enough to change the product $\bar{E} \cdot A$ only by an amount smaller than some threshold,
it might still be neglected. We define the simulation is convergent radially if, for an increase $R_{1} \rightarrow R_{2}$, follows

$$
\begin{equation*}
\frac{\bar{E}_{R_{2}} \cdot R_{2}^{2}-\bar{E}_{R_{1}} \cdot R_{1}^{2}}{\bar{E}_{R_{1}} \cdot R_{1}^{2}}<1 \% \tag{3.5}
\end{equation*}
$$

You have to be careful with this definition though. Consider Rutherford scattering where the energy transfer scales with $r^{-2}$ (see 2.28), so we would at one point satisfy equation 3.5. On the other hand we know that the integral over the energy transfer diverges for

$$
\begin{equation*}
r \rightarrow \infty \quad \Rightarrow \quad \int_{0}^{\infty} T(b) 2 \pi b d b \sim \ln \left(\frac{\infty}{b_{\min }}\right) \rightarrow \infty \tag{3.6}
\end{equation*}
$$

If we are in a plasma Debye shielding saves the day, as we have an additional exponential decay that forces the integral to converge quickly preventing the aforementioned case.

Theoretically it would be necessary for both radii $R_{1}$ and $R_{2}$ to converge separately in number of trajectories and then compare the results to see if they converge radially. This is a bad usage of computational power though, as the area spanned by $R_{1}$ would be probed twice without any real impact on the result. In figure 3.4 we can see an example on how to increase the impact disc size without wasting computation time.

We start with 2000 red trajectories and assume they do not yet converge in number so $10 \%$ more trajectories are computed in the smaller area depicted in green. Now the simulation converges in trajectory number and the algorithm increases the size of the impact disc by $10 \%$. Instead of computing trajectories in the whole new area, we limit them to be in the ring that was added, shown as the blue crosses. The number of new computations must yield the same trajectory density in the ring as it was present in the old disc. If we now add all of the trajectories we can again check for number convergence, this time in the bigger area. If it converged, we can do the check in equation 3.5, if not we need to add additional $10 \%$ more trajectories, but now randomized over the full area defined by $R_{2}$.

Eventually the algorithm will converge in number and radius, thus finishing the simulation.
randomly diced impact parameter


Figure 3.4: Example trajectories for the impact disc radius convergence mechanism. Shown is the initial position of the positron in the radial plane, while the anti-proton is considered to be at $(0,0)$.

### 3.3.4 MCSIM output

During the simulation the MCSIM program will continuously read in the trajectory files created by SIMBUCA and extract the first line, the initial parameters passed to SIMBUCA by MCSIM in the first place, and the last line yielding the parameters in the final state at the end of the trajectory. This data is processed and the transfered total, axial and radial energy calculated to be checked by the convergence algorithm described in section 3.3.3. The processed data is written to the simulation directory for the anti-proton and positron separately.

### 3.4 SIMBUCA

SIMBUCA stands for Simulation of Ion Motion in a Penning trap with realistic BUffergas and Coulomb interaction using $\mathbf{A}$ graphics card and as the name implies is a program that specialized in the computation of ion motion in trap configurations. The program itself and a lot of information can be found at http://sourceforge.net/projects/simbuca/, so I will only give a brief overview here.
The advantages of SIMBUCA for our purposes are

- the direct implementation of the trap configurations,
- the possibility to read in custom electrical and magnetic fieldmaps,
- fast and reliable integration routines with adaptive step size control,
- and very fast GPU accelerated N -body Coulomb interaction calculations.


### 3.4.1 Changes to SIMBUCA

SIMBUCA is used only for the computation of single trajectories, while the initial parameters are generated by MCSIM as well as the data analysis and preparation. Therefore only a few small alterations to SIMBUCA were necessary in the end.

In a scattering experiment, the physical values in the initial and final state are taken at infinite distance to each other, so that the interaction potential has initially no influence on them. This is of course only a theoretical construct and not applicable in an actual experiment or simulation. Therefore one tries to maximize those distances in order to minimize the systematic error introduced. This can only be done within the boundary given by the experiment or, in the case of simulations, by aspects of efficiency as discussed in section 4.1.1.

Originally SIMBUCA does not consider distances in order to trigger the end of the simulation, but takes a simulation time parameter in the provided .sim-file to tell the program the final time to integrate to. Even though, for classical Rutherford scattering ( $B=0$, see section 2.3), the integration time can be estimated from the initial parameters of the simulation, one runs into several problems with this approach.

It is possible to compute an approximate simulation time from the initial distance and velocity of the particles and use twice the corresponding time-until-contact for the SIMBUCA computation. In the case of very small deflection angles of the positron, the absolute value of the velocity changes only slightly and the two particles have nearly the same initial and final distance. If that potential energy difference of initial and final state is much smaller than the energy exchanged, the results are suitable, but are carrying an additional (small) systematic error. This error is strongly pronounced for
scattering events with a big change in the positron energy though. In that case the particle distance can differ strongly between initial and final state and all related values differ greatly from the ideal case of infinite distance.

A possible solution for this would be to extract the initial and final distances and manually correct for the difference in potential energy by adding correction terms to all parameters. This is possible, because of the simple symmetric nature of the bare Coulomb interaction involved in this case, but becomes far more difficult for the magnetized scattering problem investigated in this thesis. Consider the extreme case of an interaction process that transfers most axial energy along the field to the radial motion, such that the positron is now axially close to the anti-proton at final simulation time. On the other hand, even though the corresponding cyclotron radius increases due to the higher radial energy (see equation 2.13), the positron is bound radially and the final distance is fundamentally smaller than the initial one. This error can not easily be corrected, so the described use of the final simulation time as parameter is not suitable for our simulations.

The usual solution for tackling the outlined problem, or generally a scattering simulation, is to implement a different condition to stop the integration of the trajectory. In theory the initial and final state is one of infinite distance between the particles, distance being the parameter of interest. Imagine a box of infinite size around one particle, such that the initial state is the other particle entering this box and the final state has it leaving. In the realistic case this box is shrunk to a ball of the initial distance between the particles, so the final state is reached once the particles have the same relative distance again. The finite box size again introduces a systematic error, so the size must be considered carefully.

For the implementation in SIMBUCA itself, a new condition was introduced, that checks the relative distance of the particles after each time step and stops integration when the initial separation is reached again. Depending on the values of the velocities it might happen, that the particle triggers this condition after a few time steps because of cyclotron movement. This is an artifact of the finite sized box and those cases were filtered out by setting an additional check of simulation time, such that the stop condition only occurs after the particles came into contact. This was approximated by the time, the particles need to axially meet derived from the initial axial separation and velocity.

This finite box method naturally solves the problems mentioned in the beginning of this section, especially the field free case of Rutherford scattering does not need any correction terms. The systematic error comes only from the finite box size, which can be easily estimated and optimized. The case with a magnetic field still shows a subtle, but important, error additionally introduced by the finite box size. Figure 3.5 shows the axial position of a positron in a collision. The radial movement is bound to the magnetic field, so that the particle can not leave the box by radial motion. Therefore mainly the axial position triggers the stop condition for the integration. The solid and
dashed line represent two different box sizes and it can be seen, that for the larger initial axial separation the positron is "bound" for a longer time and interacts multiple times with the anti-proton leading to a completely different final state. This effect needs to be considered and the initial separation chosen carefully, such that as little as possible bound states are cut prematurely.


Figure 3.5: Sample trajectory for chaotic scattering. The axial position of the positron (green) and anti-proton (red) that is initially at rest in $z=0$. The black horizontal lines show the impact of two different box sizes. In the case of the solid line (boxsize $40 \mu \mathrm{~m}$ ) multiple scattering events take place, that are completely suppressed for the case of the dashed line (boxsize $10 \mu \mathrm{~m}$ ).

Furthermore there might be a set of initial condition leading to a bound state over a very long time. In that case the integration will be stuck for that trajectory because of the small integration timestep and the whole simulation needs to wait for this one to finish. To counter this and because SIMBUCA needs to be given an integration time to work properly, 100 times time-to-contact (time until the particles have the same axial position) is given to SIMBUCA as endpoint for the integration. That way strongly bound trajectories will not over utilize the computing resources, but another source of systematic error is introduced that needs consideration.

## Chapter 4

## Simulation results

In this chapter we will present the results of the simulations conducted with MCSIM and derive a qualitative description of the processes involved. In the end we will give the results to our initial problem presented in section 1.3.

The first step to convince ourselves that the programs introduced in Chapter 3 are working correctly is to validate the results for a well known setup. Coulomb collisions without a magnetic field, Rutherford scattering, was used for that purpose. It is the most basic form of two particle interaction and has an exact analytical solution, that can easily be cross checked. Rutherford scattering will form a basis from which we will slowly increase our parameter space one by one to the end up at the conditions present in the CUSP setup in section 4.5.6. This is to disentangle the impact of a parameter on the results as much as possible. Therefore the general outline of this chapter is:

- Rutherford scattering of $\overline{\mathrm{p}}$ and incident $e^{+}$with constant axial velocity,
- add magnetic field,
- add radial velocity for $e^{+}$,
- $e^{+}$velocity diced from a Maxwell-Boltzmann distribution,
- add fixed axial and radial velocity for the anti-proton,
- and investigate CUSP conditions.


### 4.1 Rutherford scattering

The basic concepts of Rutherford scattering were described in section 2.3 already. The goal of this section is to show, that the analytical Rutherford scattering equations are reproduced by the results of the corresponding simulations.

The anti-proton is set to the center of the coordinate system while the positron has a fixed axial distance (axial is defined to be the z -axis, while the radial plane is spanned by the x - y -axes) and varied radial position. The whole system is symmetric in respect to rotations around the axial direction and therefore the radial distance from the z -axis is the well defined impact parameter of the collision setup. The simulation program homogeneously distributes the initial positron position in a disc in the radial plane and the x -y coordinates can be mapped directly to the impact parameter by $b=r=\sqrt{x^{2}+y^{2}}$. This configuration covers all possible interaction processes, because they can always transformed to the frame of reference described here. In figure 4.1 a generalized setup is shown.


Figure 4.1: This setup for the Rutherford simulations shows the initial placement of the anti-proton in the origin of the coordinate system and a possible random starting position of the positron inside the dashed impact disc. The axial separation here is chosen to be $50 \mu \mathrm{~m}$, the number most simulations will use.

Several simulations were conducted to point out different issues of validation and their parameters are given in table 4.1. Simulation 1 and 2 are used to show agreement with the analytical results of equation 2.28 for two different velocities. The velocities appearing here and in the following sections correspond to the thermal velocity of the positron gas in the CUSP with a temperature of $100 \mathrm{~K}\left(v_{e^{+}} \sim 66000 \mathrm{~m} / \mathrm{s}\right)$, an anti-proton with (injection) energy of $1 \mathrm{eV}\left(v_{\bar{p}} \sim 13800 \mathrm{~m} / \mathrm{s}\right)$ and their combined velocity.

|  | axial separation $[\mu \mathrm{m}]$ | axial velocity $\overline{\mathrm{p}}[\mathrm{m} / \mathrm{s}]$ | axial velocity $e^{+}[\mathrm{m} / \mathrm{s}]$ |
| :---: | :---: | :---: | :---: |
| Sim 1 | 50 | -66000 | 0 |
| $\operatorname{Sim} 2$ | 50 | -79800 | 0 |
| Sim 3 | 100 | -66000 | 0 |
| Sim 4 | 1000 | -66000 | 0 |
| Sim 5 | 50 | 0 | 66000 |
| Sim 6 | 50 | -13800 | 52200 |

TABLE 4.1: Parameter values for simulations conducted to validate Rutherford scattering results.

Comparing simulations 1,3 and 4 tackle the issue of possible errors introduced by the finite box size approach, while simulation 1,5 and 6 are performed to verify the agreement with Lorenz boosting along the axial direction.

Equation 2.28 shows the loss of energy of the incident particle over the impact parameter in the frame where the heavy particle, that concentrates the center of mass in itself, hits the stationary light particle. The ability to directly apply equation 2.28 is the reason why, in this section, we will temporarily stray from the initial assumption of an anti-proton at rest. The sections below, will return to the $\overline{\mathrm{p}}$ at rest assumption. For parameters of simulation 1 in table 4.1 the analytical solution for energy change over impact parameter is,

$$
\begin{aligned}
v & =66000 \mathrm{~m} / \mathrm{s} \\
T(b) & =\frac{e^{4}}{\left(4 \pi \epsilon_{0}\right)^{2}} \cdot \frac{2}{m_{e} v^{2}} \cdot \frac{1}{b^{2}+b_{\min }^{2}} \\
& \sim 1.68 \times 10^{-16} \mathrm{eV} \cdot \frac{1 \mathrm{~m}^{2}}{b^{2}+\left(5.8 \times 10^{-8} \mathrm{~m}\right)^{2}} \\
T(0) & \sim 0.0495 \mathrm{eV}
\end{aligned}
$$

For simulation 2 the energy transfer is given by

$$
\begin{aligned}
v & =79800 \mathrm{~m} / \mathrm{s} \\
T(b) & =\frac{e^{4}}{\left(4 \pi \epsilon_{0}\right)^{2}} \cdot \frac{2}{m_{e} v^{2}} \cdot \frac{1}{b^{2}+b_{\min }^{2}} \\
& \sim 1.15 \times 10^{-16} \mathrm{eV} \cdot \frac{1 \mathrm{~m}^{2}}{b^{2}+\left(3.98 \times 10^{-8} \mathrm{~m}\right)^{2}} \\
T(0) & \sim 0.0725 \mathrm{eV}
\end{aligned}
$$

With this we can compare the simulation results to the analytical solution in figure 4.2 and find perfect agreement for both investigated relative velocities.


Figure 4.2: Simulation results and analytical solution for Rutherford scattering with the positron at rest and the anti-proton velocity of $V_{\overline{\mathrm{p}}}=66000 \mathrm{~m} / \mathrm{s}$ (top panel) respectively $V_{\overline{\mathrm{p}}}=79800 \mathrm{~m} / \mathrm{s}$ (bottom panel).

### 4.1.1 Influence of simulation box size

The next step is to make sure, that the finite box size of the simulation (see section 3.4.1) does not influence the results of the simulation. The simulation is again set up with the positron at rest and the anti-proton with an axial velocity of $V_{\overline{\mathrm{p}}}=66000 \mathrm{~m} / \mathrm{s}$. The initial axial separation of the particles defines the box size (it is actually a ball around one of the particles), such that the simulation stops when the particles have again the same spatial distance to each other after the collision. Three simulations with initial distance of $50 \mu \mathrm{~m}, 100 \mu \mathrm{~m}$ and 100 mm were conducted and the resulting trajectories subtracted from the analytical solution. The results in figure 4.3 show the difference of the simulation to the analytical solution.


Figure 4.3: Rutherford scattering with the positron at rest and the anti-proton velocity of $V_{\overline{\mathrm{p}}}=66000 \mathrm{~m} / \mathrm{s}$ showing the differences of the simulation to the analytical solution for different box sizes of $50 \mu \mathrm{~m}, 100 \mu \mathrm{~m}$ and 1 mm .

The biggest error at small $b$ is about $5 \times 10^{-5} \mathrm{eV}$ for an energy transfer of about $5 \times 10^{-2} \mathrm{eV}$ yielding a relative error of about $10^{-3}$. This error partially stems from the fact, that the analytic equation is only approximate as one of the particles is assumed to be infinitely massive, therefore the shape of the error resembles the energy transfer graph in figure 4.2. The approximation in the equation is of order of the mass fraction between anti-proton and positron of $\sim 1.8 \times 10^{-3}$, so the error above seems to fit this quite well. More interesting is the spread between results for different box sizes that comes down to a maximum of about $3 \times 10^{-5} \mathrm{eV}$ yielding a reasonable relative error of $\sim 3 \times 10^{-4}$. Therefore increasing the box size does influence the results only marginally, while linearly increasing simulation times. We decided that a box size of $50 \mu \mathrm{~m}$ gives acceptably accurate results, while keeping simulation times low.

### 4.1.2 Comparison of different frames of reference

As mentioned above, some of the simulations did not have the anti-proton at rest as described at the beginning of the chapter, but the positron while the anti-proton carried the initial velocity. In the unmagnetized case there is no preferential spatial direction so that the collision description can always be transformed to a frame where the antiproton is at rest and the positron is the incident particle closing in along the so-defined axial direction. The velocities involved are very slow and therefore the boost from the frame where the anti-proton is at rest, is described simply by a one dimensional Galilean transformation

$$
\begin{equation*}
v_{z \bar{p}}=0, v_{z e^{+}}=V \quad \xrightarrow{-V} \quad v_{z \bar{p}}=-V, v_{z e^{+}}=0 \tag{4.1}
\end{equation*}
$$

The simulation results must be invariant under this linear boost between inertial frames. The next simulation set up compared three simulations with the same relative velocity of $V=66000 \mathrm{~m} / \mathrm{s}$ but split up differently on the two particles. In the top panel in figure 4.4 we can see the results of the three simulations that show a different magnitude in transfered energy (the blue dots are in the positive part of the plane) that comes from their unlike energy in the laboratory frame. Although if the velocities involved are boosted by a Galilean transformation to the frame ( $V_{\overline{\mathrm{p}}}=66000 \mathrm{~m} / \mathrm{s}, V_{e^{+}}=0$ ) as seen in the bottom panel of figure 4.2 they coincide as expected.

With this we were able to successfully reproduce the physical behavior of Rutherford scattering, so we can be confident that the simulation program works as expected.


Figure 4.4: Top panel: simulation results for three different frames of reference with the same relative velocity $V=66000 \mathrm{~m} / \mathrm{s}$, bottom panel: Galilean transformation of the results in the top panel to the frame ( $\left.V_{\overline{\mathrm{p}}}=66000 \mathrm{~m} / \mathrm{s}, V_{e^{+}}=0\right)$.

### 4.2 Scattering in a magnetic field without radial positron energy

The last section confirmed the validity of integration, so what will happen if we were to introduce a homogeneous magnetic field. The field splits up the three dimensional space into the direction along the field $\vec{e}_{z}$, from here on out called axial direction, and the plane perpendicular to it, the radial plane. The equations of motion (Eq. 2.7) point out that the axial direction integrates the same way as in the field free case, while the radial plane is strongly influenced by the magnetic field showing cyclotron motion that binds the particle radially to the field line. The last aspect indicates that a collision between two charged particles will not show the hyperbolic trajectories of the field free case, as they can not move freely in the radial plane. Henceforth the arguably simple geometric deduction that lead to Rutherford's equation 2.28 is not applicable here. In figure 4.5 the total energy transfer to the anti-proton is shown, for a magnetic field of $B=1 \mathrm{~T}$ and initially resting anti-proton, while the positron has an axial velocity of $20000 \mathrm{~m} / \mathrm{s}$ and no radial velocity.


Figure 4.5: Total energy transfer in a collision between a resting anti-proton and an incident positron with $V=20000 \mathrm{~m} / \mathrm{s}$ in a magnetic field with $B=1 \mathrm{~T}$. The red line shows the analytic result for $B=0$.

The most outstanding feature of the energy transfer is the chaotic behavior that can be seen for an impact parameter of about $0.05 \mu \mathrm{~m} \leq b \leq 0.31 \mu \mathrm{~m}$. Chaotic behavior, since for a very small change of impact parameter the result can vary very strongly. Also inside of the chaotic region zones of regularity can be found at $b \sim 0.13 \mu \mathrm{~m}$ and $b \sim 0.17 \mu \mathrm{~m}$. The reason for the chaos can be seen in figure 4.6, it shows the axial position of the two particles over time for a single trajectory in the chaotic region. At $t=0$ they are at their initial axial distance and the positron approaches the anti-proton, then has a collision from which it emerges with considerably less energy than at the start. The positron is bound to the magnetic field line and therefore can only slightly change its distance to the
anti-proton by an increase of cyclotron radius, but for the sake of a simple description we determine the radial distance does not change by a collision. The positron energy is now too low to overcome the Coulomb attraction of the $\overline{\mathrm{p}}$ and is accelerated back and has another collision and energy exchange. This repeats over 200 times until the positron gets enough energy to increase its distance to the initial separation and the integration is considered complete. For most of the time the positron stays very close to the anti-proton axially as well as radially, in a "weakly-bound-state", exchanging energy all the time until it is ejected after $6 \times 10^{-8} \mathrm{~s}$. If the initial conditions of this trajectory are changed by a very small amount, the energy transfer of the first interaction changes and already the second collision starts with a completely different initial state yielding an output that is depending highly non-linear on the small initial change of parameters. Integrated over the whole time the positron is "bound" and exchanging energy, it is clear that the state at the end of integration can change almost independently of the initial parameter shift resulting in chaotic behavior.


Figure 4.6: Axial position of the positron (green) and anti-proton (red) that is initially at rest in $z=0$. The positron is in a weakly bound state around the anti-proton, as it can not escape radially because of the magnetic field, and does more than 200 collisions.

Another important aspect of the simulation can be seen if we look at the time after the first collision where the positron has almost enough energy to escape the electromagnetic attraction. Consider that the initial axial separation of this trajectory was not $50 \mu \mathrm{~m}$ but only $10 \mu \mathrm{~m}$. The first interaction would be the identical, as initial energies are the same, but the simulation program only checks the distance of the particles to determine the end of integration and would stop already after the first collision, such that the "bound" state is never reached. This directly shows the dependence of the result from the finite box size approach used in these simulations, thus introducing a systematic error into the analysis (see section 3.4.1). If the size is too small, the topology and density of the parameter space in the chaotic region in the end state is arguably altered
by the simulation setup!

Adding the magnetic field to the analytically known Rutherford set up, it became difficult to cross check the results with theories as many of them rely on rather crude approximations, like one dimensional collisions [41]. Nersisyan et al. [39, 42] presented some possible theoretical approaches to the problem using perturbation theory. The drawback is that the perturbed equations are only applicable, if the influence of the Coulomb attraction on the cyclotron movement is small and the circular motion is only slightly perturbed. The chaotic regions are in stark contrast to this approach and the theory is not applicable.

We therefore decided to qualitatively describe the dynamics of scattering in strong magnetic fields, by trying to establish basic scaling laws for the parameters involved. The parameters of interest were the impact parameter of start and end of the chaotic zone as well as the seemingly physical boundary for maximal energy transfer.

### 4.2.1 Lorentz invariance

Despite the complications introduced by the magnetic field it does leave the physics in axial direction untouched. Therefore the basic property of invariance regarding a Galilean transformation along the axial direction, as it was done in section 4.1.2, must be conserved. This was checked by conducting three simulations with the same relative velocity $V=79800 \mathrm{~m} / \mathrm{s}$ and a magnetic field of $B=2.8 \mathrm{~T}$. The axial velocities

- $V_{\overline{\mathrm{p}}}=0, V_{e^{+}}=79800 \mathrm{~m} / \mathrm{s}$
- $V_{\overline{\mathrm{p}}}=13800 \mathrm{~m} / \mathrm{s}, V_{e^{+}}=66000 \mathrm{~m} / \mathrm{s}$
- $V_{\overline{\mathrm{p}}}=79800 \mathrm{~m} / \mathrm{s}, V_{e^{+}}=0$
and their respective energy transfer can be found in the top panel in figure 4.7. Then the trajectories were boosted, using a Galilean transformation (equation 4.1), to the frame with $V_{\overline{\mathrm{p}}}=0, V_{e^{+}}=79800 \mathrm{~m} / \mathrm{s}$ and again compared. The bottom panel of figure 4.7 shows almost perfect coverage of the points. This comes from the fact, that all simulations use random number generators with the same initial seed, so that they should map to the same point after the boost.

There are a few single energy transfers that have no corresponding boosted points though. One reason is that the three simulations do not have the exact same number of points, so that there are some trajectories with no corresponding point in the other simulations. The other is the effect of the finite box size of the simulation as explained in the section above. Some trajectories can show strongly bound behavior and the cut off from the finite box size is different in the respective frames yielding results


Figure 4.7: Top panel: simulation results for three different frames of reference with the same relative velocity $V=66000 \mathrm{~m} / \mathrm{s}$, bottom panel: Galilean transformation of the results in the top panel to the frame ( $V_{\overline{\mathrm{p}}}=66000 \mathrm{~m} / \mathrm{s}, V_{e^{+}}=0$ ).
that can differ greatly. For some of the single points the integration time is very long compared to the rest, encouraging the argument. Most of the unpaired points come from the fact, that the simulations are converging for different number of trajectories and if the radius of the impact disc is increased in one simulation earlier than for the others, all random points diced afterwards will not map on the same boosted point anymore. In the end only a very small number of trajectories doesn't have corresponding boosted points, so that the effect on the overall result can be neglected.

### 4.2.2 Scaling laws for the magnetic field strength for an axial velocity of $V=\mathbf{2 0 0 0} \mathbf{~ m} / \mathrm{s}$

We already saw the dependence of the Rutherford scattering with the axial velocity of $\propto v^{-2}$ and now we want to find a scaling law for the magnetic field. The simulations are set up with the same initial distances of $50 \mu \mathrm{~m}$ and positron velocity of $V=20000 \mathrm{~m} / \mathrm{s}$, while the magnetic field is changed. Figure 4.8 shows the energy transfer for some values of the magnetic field, where one can see that the chaotic zone shifts and widens with lower field.


Figure 4.8: Total energy transfer dependence on magnetic field strength with axial relative velocity of $V=20000 \mathrm{~m} / \mathrm{s}$ and no initial radial movement

The regular structures inside the chaotic region appear for all values of the magnetic field and seem to scale with the zone itself. Therefore the general properties of the chaos are described to a good extend by parameterizing the impact parameter of start and end of the chaotic zone as well as the function that acts as envelope for the maximal energy transfer. The values were visually read from the data files and can be found in table 4.2. The error was estimated from reading the same specific point several times over the course of a few days and conservatively looking at the spread of the values. For the impact parameter values an error of about $\Delta b \sim 1 \mathrm{~nm}$ and for the energy values $\Delta E \sim 0.5 \mu \mathrm{eV}$ seems appropriate.

| $B[\mathrm{~T}]$ | start of chaotic <br> zone $[\mu \mathrm{m}]$ | end of chaotic <br> zone $[\mu \mathrm{m}]$ | position of <br> maximum of <br> energy transfer <br> $[\mu \mathrm{m}]$ | position of <br> maximal energy <br> transfer $[\mu \mathrm{VV}]$ | maximum of <br> maximum of av. <br> energy transfer <br> $[\mu \mathrm{m}]$ | maverage energy <br> transfer $[\mu \mathrm{eV}]$ <br> 0.001 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.01 | 0.571 | 0.699 | - | - | - | - |
| 0.05 | 0.362 | 0.796 | - | - | - | - |
| 0.1 | 0.230 | 0.830 | 0.782 | -2.730 | - | - |
| 0.25 | 0.137 | 0.754 | 0.733 | -3.50 | - | - |
| 0.5 | 0.0850 | 0.572 | 0.537 | -5.621 | 0.531 | -3.79 |
| 0.75 | 0.0602 | 0.355 | 0.403 | -9.59 | 0.393 | -6.0 |
| 1 | 0.0490 | 0.305 | 0.328 | -12.80 | 0.320 | -8.13 |
| 1.5 | 0.0350 | 0.251 | 0.227 | -16.02 | 0.278 | -10.1 |
| 2 | 0.0273 | 0.215 | 0.196 | -21.20 | 0.215 | -13.2 |
| 2.5 | 0.0225 | 0.192 | 0.173 | -26.90 | 0.192 | -16.8 |
| 4 | 0.0143 | 0.149 | 0.134 | -33.95 | 0.172 | -20.9 |
| 5 | 0.0186 | 0.132 | 0.120 | -61.19 | 0.132 | -28.6 |

TABLE 4.2: Scaling parameter extracted from the simulation results with axial velocity of $V=20000 \mathrm{~m} / \mathrm{s}$ and no radial velocity. The dash signalizes, that the corresponding point could not be determined.

Several different fit functions were tried to find the scaling laws of the magnetic field. Figure 4.9 shows the best fits found with the result, that the start of the chaotic zone and the position of the maximum of the averaged energy transfer scale with the root of the field strength while the end of the chaotic zone and the position of the maximum of the energy transfer scale with the fourth root. As we can see in the next section 4.2.3, the powers of about 0.5 and 0.25 might be a coincidence for the chosen axial velocity and only represent a good fit here.


Figure 4.9: Fit functions of the magnetic field scaling.
While the start of the chaotic zone monotonously shifts to bigger impact parameters for $B \rightarrow 0$, the end shows a deviation from the $\propto B^{1 / 4}$ behavior starting around $B \sim 0.05 \mathrm{~T}$ from which on its gets smaller for $B \rightarrow 0$. Therefore for $B$ approaching 0 the chaotic zones asymptotically converge to an impact parameter $b \sim 0.63 \mu \mathrm{~m}$ as it can be seen in figure 4.10 .

If this argument is inverted, it would mean that the origin of the chaotic behavior lies at this specific impact parameter, which would be an interesting topic to pick up theoretically in future works.


Figure 4.10: Total energy transfer dependence for low magnetic field strength with axial relative velocity of $V=20000 \mathrm{~m} / \mathrm{s}$ and no initial radial movement. It can be seen, that the boarders of the chaotic zone converge to a impact parameter of $\sim 0.63 \mu \mathrm{~m}$ from both sides. As for $B=0.0001 \mathrm{~T}$ the zone is so thin, that no trajectory falls into it, and it almost follows the $B=0$ solution.

### 4.2.3 Scaling laws for the magnetic field strength for an axial velocity of $V=\mathbf{6 6 0 0 0} \mathbf{m} / \mathrm{s}$

The other free parameter of this simulation that was fixed in the analysis above is the axial relative velocity. Applying the same procedure as above to a different axial velocity will improve the insight on scaling and further explore the parameter space.


Figure 4.11: Total energy transfer dependence on magnetic field strength with axial relative velocity of $V=66000 \mathrm{~m} / \mathrm{s}$ and no initial radial movement.

From the results in figure 4.11 and 4.12 the appropriate values can be extracted the same way as before and found in table 4.3. The regular zones inside the chaotic region are easy to recognize for magnetic fields higher than 1 T . Their shape is similar to a half wave of a sinus function, so for these simulation the position and energy transfer of the


Figure 4.12: Total energy transfer dependence on magnetic field strength with axial relative velocity of $V=66000 \mathrm{~m} / \mathrm{s}$ and no initial radial movement.
maximum of the regular arc present in the data were measured too and can be found in 4.4 .

| $B$ [T] | start of chaotic zone [nm] | end of chaotic zone [nm] | energy transfer at start $[\mu \mathrm{V} \mathrm{V}]$ | energy transfer at end $[\mu \mathrm{eV}$ ] | position of maximum of energy transfer [ nm ] | maximal energy transfer [ HeV ] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.05 | 51.6 | 64.6 | 13.5 | 13.5 | - | - |
| 0.1 | 49.1 | 67.3 | 14.0 | 13.4 | - | - |
| 0.25 | 44.8 | 71.5 | 13.7 | 13.4 | - | - |
| 0.5 | 40.1 | 74.9 | 13.6 | 13.6 | - | - |
| 0.75 | 36.8 | 76.6 | 14.1 | 14.0 | - | - |
| 1 | 34.0 | 77.2 | 14.7 | 14.5 | - | - |
| 1.5 | 30.2 | 77.0 | 14.8 | 15.4 | - | - |
| 2 | 27.2 | 75.7 | 15.2 | 16.0 | 71.9 | 30.3 |
| 2.5 | 24.8 | 74.0 | 16.0 | 16.6 | 69.3 | 32.0 |
| 3 | 22.7 | 71.9 | 16.9 | 17.5 | 66.1 | 34.0 |
| 4 | 19.7 | 67.6 | 17.6 | 19.3 | 63.8 | 38.2 |
| 5 | 17.8 | 63.9 | 17.8 | 20.5 | 59.6 | 43.0 |
| 8 | 13.5 | 54.9 | 18.6 | 25.0 | 50.7 | 56.6 |
| 10 | 11.6 | 50.5 | 19.5 | 27.2 | 47.1 | 66.1 |

TABLE 4.3: Scaling parameter extracted from the simulation results with axial velocity of $\mathrm{V}=66000 \mathrm{~m} / \mathrm{s}$ and no radial velocity. The dash signalizes, that the corresponding point could not be determined.

| $B$ [ T$]$ | position of $\operatorname{arc1}[\mathrm{nm}]$ | energy transfer of arc1 [nm] | position of arc2 [nm] | energy transfer of arc2 [nm] | position of arc3 [nm] | energy transfer of $\operatorname{arc} 3$ [nm] | position of arc4 [nm] | energy transfer of arc4 [nm] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.5 | 50.1 | 28.1 | 59.1 | 28.2 | - | - | - | - |
| 2 | 48.0 | 28.7 | 57.5 | 29.4 | - | - | - | - |
| 2.5 | 46.3 | 29.5 | 56.7 | 30.5 | - | - | - | - |
| 3 | 44.7 | 30.3 | 53.8 | 31.8 | - | - | - | - |
| 4 | 41.7 | 32.2 | 50.5 | 34.7 | 54.6 | 35.9 | 56.8 | 36.4 |
| 5 | 39.3 | 34.1 | 47.4 | 37.3 | 51.2 | 39.3 | 53.4 | 40.6 |
| 8 | 33.6 | 40.6 | 40.5 | 46.8 | 43.6 | 50.1 | 45.2 | 52.1 |
| 10 | 30.8 | 45.0 | 37.1 | 53.1 | 39.8 | 57.1 | 41.5 | 59.5 |

TABLE 4.4: The arc entries are the maximum energy transfer point of the arcs of regularity inside the chaotic zone.

In figure 4.13 the values for the impact parameter of the begin of the chaotic zone in the upper panel and the end of the chaotic zone in the bottom panel are plotted against the magnetic field strength. Unfortunately the powers of the fit are quite different from the analysis of the same variables in figure 4.9. Therefore it seems that the scaling can not be easily expressed in terms of a power law of the magnetic field and further analysis of the scaling for different axial velocities is needed.


Figure 4.13: Fit functions of the magnetic field scaling for the beginning and the end of the chaotic zones, that scale with $B^{0.18}$ and $B^{0.40}$ respectively.

### 4.3 Scattering in a magnetic field with radial positron energy

Having established two parameters in the sections above, axial velocity and magnetic field, the next step is to give the positron a radial velocity. This induces a cyclotron motion in the $e^{+}$that necessitates a redefinition of the impact parameter. This comes from the fact, that the phase of the cyclotron movement is randomized in the simulation to average over this parameter. The magnitude of the randomized initial radial velocity and its direction define a specific cyclotron trajectory and all initial conditions that are randomized to be situated on this same circle yield the same result from the simulation effectively creating an ambiguity. That means that even though the radial distance (=definition of impact parameter up to now) on this cyclotron radius is different they all contribute to the same average result.

The redefinition comes naturally from the before mentioned fact, that on average all initial conditions belonging to the same cyclotron circle are equal, making that circle or even better its center a representative distance for the collision. Therefore the new definition is simply

## Definition of impact parameter:

Radial distance of the center of cyclotron motion for the respective radial velocity

### 4.3.1 Lorentz invariance

At this point we again turn to check if the Galilean boost along the axial direction leaves our result invariant. This time we have the additional problem of an extended movement in the radial plane, that might be able to spoil our simulation. Identical parameters for axial velocities and magnetic field of $B=1 \mathrm{~T}$ are used for the simulations

- $V_{\overline{\mathrm{p}}}=0, V_{e^{+}}=79800 \mathrm{~m} / \mathrm{s}$,
- $V_{\overline{\mathrm{p}}}=13800 \mathrm{~m} / \mathrm{s}, V_{e^{+}}=66000 \mathrm{~m} / \mathrm{s}$,
- $V_{\overline{\mathrm{p}}}=79800 \mathrm{~m} / \mathrm{s}, V_{e^{+}}=0$.

We can see the total energy transfer in the top panel in figure 4.7. Then the trajectories were boosted, using a Galilean transformation (equation 4.1), to the frame with $V_{\overline{\mathrm{p}}}=0$, $V_{e^{+}}=79800 \mathrm{~m} / \mathrm{s}$ and again compared. The bottom panel of figure 4.7 shows almost perfect coverage of the points.


Figure 4.14: Top panel: simulation results for three different frames of reference with the same relative velocity $V=66000 \mathrm{~m} / \mathrm{s}$, bottom panel: Galilean transformation of the results in the top panel to the frame ( $\left.V_{\overline{\mathrm{p}}}=66000 \mathrm{~m} / \mathrm{s}, V_{e^{+}}=0\right)$.

We can see a few single and double points, but most of the trajectories end in a triple point. This again comes from the fact, that on one hand the simulations converge for a different amount of trajectories and therefore the distribution of initial parameters is not the same, leaving some points without a respective one in the other simulations. On top of that the finite box size can introduce an error here.

The great correspondence here validates the correctness of simulation and furthermore shows, that the chosen initial box size is sufficiently big.

### 4.3.2 Scaling with radial Velocity

With this new impact parameter definition we can do another survey keeping axial velocity constant at $V=20000 \mathrm{~m} / \mathrm{s}$ and the magnetic field at $B=1 \mathrm{~T}$ for the simulations presented in figure 4.15. We can see again a hard boundary on the energy transfer that is only violated by a few single trajectories, otherwise seems like a physical limit though. We can see, that it does not depend on radial velocity at all, but in section 4.2 .2 and 4.2.3 we see, that it scales with magnetic field and axial positron velocity.


Figure 4.15: Total energy transfer dependence on the radial positron velocity with axial relative velocity of $V=20000 \mathrm{~m} / \mathrm{s}$ and $B=1 \mathrm{~T}$.

The axial momentum transfer in figure 4.16 does show very interesting behavior in the form of the lower hard boundaries, that is difficult to parameterize though.


Figure 4.16: Axial momentum transfer dependence on the radial positron velocity with axial relative velocity of $V=20000 \mathrm{~m} / \mathrm{s}$ and $B=1 \mathrm{~T}$.

### 4.3.3 Scaling with radial temperature

The next logic step in our treatment is to dice the initial radial velocities from a MaxwellBoltzmann distribution of a specific temperature. In figure 4.17 we can see the transfered energy and that it shows the same hard boundary as in the case of a single velocity, as expected.


Figure 4.17: Total energy transfer dependence on the radial positron velocity with axial relative velocity of $V=20000 \mathrm{~m} / \mathrm{s}$ and $B=1 \mathrm{~T}$.

The same can be said about the axial momentum transfer in figure 4.18. The hard boundaries are not radial velocity dependent, but only scale with magnetic field strength and axial velocity of the positron.


Figure 4.18: Axial momentum transfer dependence on the radial positron velocity with axial relative velocity of $V=20000 \mathrm{~m} / \mathrm{s}$ and $B=1 \mathrm{~T}$.

### 4.4 Robicheaux simulations

In section 4.2 we saw that the addition of the magnetic field introduced chaotic behavior not described by the theories. Having added even another parameter, the radial positron energy, we can compare results to a paper from F. Robicheaux [43], which builds confidence in our implementation and results at this point. Thus recalculating the results in the paper will ensure correctness of the output our simulation program generates. The parameters used in the paper are

- magnetic field $B=1 \mathrm{~T}$,
- anti-proton at rest,
- positron axial velocity between $10000 \mathrm{~m} / \mathrm{s}$ and $60000 \mathrm{~m} / \mathrm{s}$,
- positron radial temperature 4 K .

The paper focuses on the effects of scattering in a magnetic field on the axial motion, so the two main physical parameters extracted from the simulation results are the axial momentum transfer to the anti-proton and the axial energy transfer. Radial behavior is mostly neglected, as it is assumed that any radial motion of the $\overline{\mathrm{p}}$ is exchanging energy very fast and equilibrates very quickly compared to the axial motion. In the end radial energy exchange is considered instantaneous and only the axial part is calculated. We will strive to quantify and proof this statement here and will compute the radial energy transfer as well.

First we will crosscheck our simulation results with figure 3 in the paper of Robicheaux [43], showing axial momentum transfer between the particles. In figure 4.19 we see our trajectories in red, the average over bins of 10 nm width in blue and the paper results in black. Considering the lack of precise information on the details of the simulations we get acceptable results from our simulation.


Figure 4.19: Comparison of our simulation results with the figure published in the paper of F.Robicheaux et al. [43]. The red dots are the energy transfers of our trajectories while the blue line is their binned average (bin-width 10 nm ). The black dots and lines are the results from the paper.

We can see a notable difference for $b$ close to 0 , but as in a homogeneous radial distribution the number of trajectories gets very small close to zero, this may be accounted to low statistics, but on the other hand it will have only a small influence on the total average for the same reasons. The next result to check is the variation of the momentum transfer rate $F$ with the axial velocity.

$$
\begin{equation*}
\mathcal{F}=v_{\text {axial }} A\left\langle\Delta p_{\text {axial }}\right\rangle \tag{4.2}
\end{equation*}
$$

The value $\left\langle\Delta p_{\text {axial }}\right\rangle$ is the total average of the momentum transfer over all trajectories. The algorithm is increasing the size of the impact disc as described in section 3.3.2, so that it has the area $A$ when converged. This rate has the dimension of $\left[\mathrm{Jm}^{2}\right]$ so if it is multiplied by the local positron density we directly yield the axial energy transfer $d E / d x$. Figure 4.20 shows the paper results in black and our simulations in red.


Figure 4.20: Comparison of our simulation results with the figure publicized in the paper of F.Robicheaux et al. [43]. The red dots show our result for a selection of five different axial velocities that match the black paper results acceptably well. The solid line here shows the momentum transfer rate for $T=4 \mathrm{~K}$, the dotted line for $T=8 \mathrm{~K}$ and the dashed line for $T=16 \mathrm{~K}$.

Having established confidence in our simulations with the agreeable results above, we now aim to extend the parameters space presented in Robicheaux's paper. First we can survey the temperature further and see how the curves in 4.20 look for higher thermal energies. Furthermore we are interested into the energy transfer as well, so in figure 4.21 we see the extended transfer rates for some interesting temperatures.


Figure 4.21: Top panel: axial momentum transfer rate $F$ for different values of the temperature $T$, bottom panel: total energy transfer rate $G$.

To get a step closer to CUSP conditions again, it is interesting to investigate how the magnetic field influences the results in figure 4.22 .


Figure 4.22: Top panel: axial momentum transfer rate $F$ for different values of the magnetic field $B$, bottom panel: total energy transfer rate $G$.

### 4.5 Adding anti-proton radial energy

We have now the full set of parameters, present in the experiment, at our disposal. The finite radial anti-proton velocity induces a $\overline{\mathrm{p}}$ cyclotron motion in the magnetic field, which necessitates another generalization of the impact parameter definition. This is done in accord to section 4.2, where we generalized the impact parameter for positrons with cyclotron movement.

## Definition of impact parameter:

Radial distance between the center of cyclotron motion of the respective particles.

Taking the example of parameter values present in the CUSP trap as described in 4.5.6 the cyclotron radius of the $\overline{\mathrm{p}}$ is much bigger than that of the positron. For $B=2.8 \mathrm{~T}$, $E_{\text {rad, } \overline{\mathrm{p}}}=0.1 \mathrm{eV}, \bar{v}_{\text {rad }, e^{+}, 100 \mathrm{~K}} \sim 55000 \mathrm{~m} / \mathrm{s}$ follows that $r_{c, \overline{\mathrm{p}}} \sim 16 \mu \mathrm{~m} \gg r_{c, \mathrm{e}^{+}} \sim 0.1 \mu \mathrm{~m}$. In this case a small impact parameter $(b \sim 0)$ does not lead to a hard collision, only for values of $b \sim r_{c, \overline{\mathrm{p}}}$ we will be able to efficiently transfer energy. In figure 4.23 we can see the corresponding energy transfer of the anti-proton due to a collision with positrons, that have their velocity diced from a Maxwell-Boltzmann distribution with $T=100 \mathrm{~K}$ and their radial position randomized in a disc of $R=30 \mu \mathrm{~m}$. As expected the energy transfer has a maximum around $16 \mu \mathrm{~m}$, exactly the radius of the revolving anti-proton.


Figure 4.23: Total energy transfer of a 1 eV anti-proton in a $T=100 \mathrm{~K}$ positron plasma.

Another important aspect in figure 4.23 is, that the phase of the anti-proton is randomized in the simulations to provide an average over this parameter. Henceforth the simulation is not symmetric in the radial plane and there is no direct relation between impact parameter and initial radial distance. The positron position is randomly diced within the impact disc (see section 3.3.2), but the corresponding impact parameter can have different values of radial distance to the $\bar{p}$ depending on the random anti-proton cyclotron phase. This results in different transferred energies, even though the velocities involved are the same. For a simple example consider the center of cyclotron motion of a positron to be set on the cyclotron radius of the anti-proton at $(x, y)=(16 \mu \mathrm{~m}, 0)$ and therefore having an impact parameter of $b=16 \mu \mathrm{~m}$. The phase of the $\overline{\mathrm{p}}$ is randomized to $180^{\circ}$ so it starts at $(-16 \mu \mathrm{~m}, 0)$. In the course of integration the anti-proton does advance along its cyclotron radius finishing approximately one full revolution at the time the two particles meet axially, therefore again sits at ( $-16 \mu \mathrm{~m}, 0$ ). Even though the value of the impact parameter might suggest a hard collision at the anti-proton radius,
the two particles are actually separated by $32 \mu \mathrm{~m}$ and only interacting very weakly.

This aspect combined with the randomization of positron velocities lead to a very low chance of making a hard collision seen as only a few points have a high energy transfer even though figure 4.23 contains almost $10^{6}$ trajectories.

### 4.5.1 $\overline{\mathrm{p}}$ outward drift

Having established the full parameter space in this section, we now get results that enable us to compute the outward drift rate of the anti-proton. Because of time constrains the simulations below were stopped prematurely though, so that the convergence algorithm was not able to increase the disc size (see section 3.3.3). Looking at the average energy transfers, f.e. figure 4.31 or any of the others, it falls off to zero quickly for the impact parameter close to the upper limit of $30 \mu \mathrm{~m}$. Therefore trajectories outside the simulated disc do not strongly impact the average which limits the error introduced.

Consider now a cylinder with the base area equal the impact disc and a length $d z$ in axial direction. As stated above positrons radially outside of the disc are assumed to not contribute to the energy change. The anti-proton now travels along the axial direction interacting with positrons inside of the cylinder contributing $\bar{E}$ each to the overall energy transfer. In a homogeneously dense plasma the energy transfer $d E / d z$ can therefore be expressed as

$$
\begin{equation*}
\frac{d E}{d z}=n_{e+} r^{2} \pi \bar{E} \tag{4.3}
\end{equation*}
$$

The anti-proton is moving axially through the nested trap and we assume that it does not interact while in the harmonic potential part of the trap and only transfers energy while passing through the positron cloud. We can calculate the time spent outside the plasma from the harmonic frequency of the well $T_{\text {harmonic }}=1 / f_{\text {harmonic }} \sim 2.5 \times 10^{-6} \mathrm{~s}$. The time inside the plasma can be calculated from the length of the positron cloud of about 6.6 cm (beam time 2014) and the axial velocity of the anti-proton, that depends on the injection energy (in this work an axial energy of 1 eV is adopted most of the time, so that $\left.v_{z} \sim 13800 \mathrm{~m} / \mathrm{s}\right)$. With those numbers we can define the relations in equation 4.4 that give us the macroscopic energy transfer per pass through the positron cloud and the average energy transfer per second. The latter needs careful attention, because it assumes that the energy transfer stays constant with each pass, but as the anti-proton is exchanging energy the transfer rate will change too.

$$
\begin{align*}
\frac{\Delta E}{\text { pass }} & =\frac{d E}{d z} \cdot l_{\text {plasma }} \\
\frac{\Delta E}{\Delta t} & =\frac{\Delta E_{\text {pass }}}{t_{\text {plasma }}+T_{\text {harmonic }}}=\frac{\Delta E_{\text {pass }}}{\frac{l_{\text {plasma }}}{v_{\text {axial }}}+\frac{1}{f_{\text {harmonic }}}} \tag{4.4}
\end{align*}
$$

In the next sections we will present a basic parameter survey of the important parameters of magnetic field strength $B$, positron temperature $T$ and a first approach to the impact of different initial anti-proton energies. The energy transfer rates as well as the outward transport rate $\gamma$ will be computed, as well as a representative drift time scale that, as mentioned above, gives only a guiding value for the drift effect.

Note that simulation times up to two or three weeks are necessary for a statistical sufficient amount of trajectories. Even then the results did not yet converge to the limits set in section 3.3.3, but were stopped prematurely with an error of around $10 \%$. For more reliable results a longer time to conduct simulations or more resources are needed. The results shown hereafter will concentrate on a qualitative description and yield a first result for radial outward drift at the end of this chapter.

### 4.5.2 Influence of different magnetic fields

In this section the influence of the magnetic field on the collision results is investigated for positron temperatures of 100 K and 30 K . Simulations with different magnetic field strength require a change to the initial size of impact disc, because the anti-proton cyclotron radius changes as well (see equation 2.13). Therefore the averages can not be directly compared as they are computed over all trajectories in the disc area, yielding lower averages for bigger disc sizes. The energy transfer in equation 4.3 is a more appropriate number to compare different simulations.

The tables below, show the energy transfer calculated from the simulation results for different magnetic field strength as well as the outward transport rate (see equation 2.21)

| $B[\mathrm{~T}]$ | averaged total <br> energy transfer <br> $[\mathrm{eV} / \mathrm{m}]$ | averaged axial <br> energy transfer <br> $[\mathrm{eV} / \mathrm{m}]$ | averaged radial <br> energy transfer <br> $[\mathrm{eV} / \mathrm{m}]$ | outward transport <br> rate $\gamma\left[\mathrm{s}^{-1}\right]$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | -1.123 | +0.703 | -1.826 | 25.054 |
| 2.8 | -1.217 | +0.039 | -1.256 | 0.280 |
| 5 | -0.773 | +0.068 | -0.841 | 0.018 |

TABLE 4.5: transferred energy of an anti-proton with an axial energy of 1 eV , radial energy of 0.1 eV and positron temperature of $T=30 \mathrm{~K}$ and density $n=1.5 \times 10^{14} \mathrm{~m}^{-3}$, the impact disc size is $30 \mu \mathrm{~m}$ for $B=5 \mathrm{~T}, 30 \mu \mathrm{~m}$ for $B=2.8 \mathrm{~T}$ and $90 \mu \mathrm{~m}$ for $B=1 \mathrm{~T}$

| $B[\mathrm{~T}]$ | averaged total <br> energy transfer <br> $[\mathrm{eV} / \mathrm{m}]$ | averaged axial <br> energy transfer <br> $[\mathrm{eV} / \mathrm{m}]$ | averaged radial <br> energy transfer <br> $[\mathrm{eV} / \mathrm{m}]$ | outward transport <br> rate $\gamma\left[\mathrm{s}^{-1}\right]$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | -0.670 | +0.087 | -0.756 | 10.37 |
| 2.8 | -0.447 | +0.104 | -0.551 | 0.123 |
| 5 | -0.423 | +0.023 | -0.445 | 0.010 |

TABLE 4.6: transferred energy of an anti-proton with an axial energy of 1 eV ,radial energy of 0.1 eV and positron temperature of $T=100 \mathrm{~K}$ and density $n=1.5 \times 10^{14} \mathrm{~m}^{-3}$, the impact disc size is $30 \mu \mathrm{~m}$ for $B=5 \mathrm{~T}, 30 \mu \mathrm{~m}$ for $B=2.8 \mathrm{~T}$ and $60 \mu \mathrm{~m}$ for $B=1 \mathrm{~T}$

For both temperatures we can observe that the energy transfer decreases with increasing magnetic field in both components. The same effect can be observed in figure 4.22.

From the outward transport rate we can calculate the drift time for an anti-proton starting at half the plasma radius to reach the outer boundary. We do this for two scenarios, on the one hand the realistic case where the anti-proton traverses the plasma and harmonic side wells, where no energy transfer is present, and therefore yields longer drift times. For this calculation we use equation 4.4. Section 4.5.6 features a more detailed description of this calculation. For an anti-proton to drift from radius $r$ to the edge of the positron cloud at R in a infinitely long plasma cylinder, it needs the time

$$
\begin{equation*}
t_{\infty}=\frac{\ln \left(\frac{R}{r}\right)}{\gamma} \tag{4.5}
\end{equation*}
$$

In the CUSP setup the anti-proton traverses the positron plasma with length $l_{\text {plasma }}$ and the harmonic side wells with $T_{\text {harm }}=\frac{1}{f_{\text {harm }}} \sim 2.3 \times 10^{-6} \mathrm{~s}$, such that the drift time is given by

$$
\begin{equation*}
t_{\mathrm{CUSP}}=t_{\infty} \times \frac{T_{\text {harm }}+t_{\text {plasma }}}{t_{\text {plasma }}} \quad t_{\text {plasma }}=\frac{l_{\text {plasma }}}{v_{\text {axial }}} \tag{4.6}
\end{equation*}
$$

| [\mathrm{T}]{} | $r=0.5 R_{\text {plasma }}$ <br> drift time 30 K <br> $t_{\infty}$ | $r=0.5 R_{\text {plasma }}$ <br> drift time 100 K <br> $t_{\infty}$ | $r=0.5 R_{\text {plasma }}$ <br> drift time 30 K <br> $t_{\text {CUSP }}$ | $r=0.5 R_{\text {plasma }}$ <br> drift time 100 K <br> $t_{\text {CUSP }}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | 0.03 s | 0.07 s | 0.04 s | 0.1 s |
|  | 2.47 s | 5.64 s | 3.66 s | 8.34 s |
| 5 | 37.55 s | 71.0 s | 55.53 s | 105.0 s |

TABLE 4.7: Drift times of an anti-proton with an axial energy of 1 eV , radial energy of 0.1 eV and positron temperature of $T=100 \mathrm{~K}$ and density $n=1.5 \times 10^{14} \mathrm{~m}^{-3}$, the impact disc size is $30 \mu \mathrm{~m}$ for $B=5 \mathrm{~T}, 30 \mu \mathrm{~m}$ for $B=2.8 \mathrm{~T}$ and $60 \mu \mathrm{~m}$ for $B=1 \mathrm{~T}$. The two columns on the right feature the realistic case in the CUSP, while the two columns on the left are calculated for a infinitely long plasma cylinder.




Figure 4.24: Magnetic field dependence of energy transfer for an anti-proton of $E_{\text {axial }}=$ $1 \mathrm{eV}, E_{\text {radial }}=0.1 \mathrm{eV}$ and $T=30 \mathrm{~K}$. Total energy (top panel), axial component (middle



Figure 4.25: Magnetic field dependence of energy transfer for an anti-proton of $E_{\text {axial }}=$ $1 \mathrm{eV}, E_{\text {radial }}=0.1 \mathrm{eV}$ and $T=100 \mathrm{~K}$. Total energy (top panel), axial component (middle

### 4.5.3 Comparison with $B=0$

We began the simulation analysis by cross checking our simulation with the analytically known results of Rutherford scattering. Once again we turn back to the non-magnetic case here to compare our results to. With the lack of a magnetic field there is no cyclotron motion of the anti-proton so it proofs difficult comparing to simulations where the $\overline{\mathrm{p}}$ is found on a certain radial distance. Therefore the simulation program is modified to randomly distribute the anti-protons on a fixed cyclotron radius corresponding to $B=2.8 \mathrm{~T}$ that we want to compare the $B=0$ results to.

An important aspect when comparing the following simulations, is that for the $\mathrm{B}=0$ case the same axial velocity distribution was used as for $B=2.8 \mathrm{~T}$, but without any radial energy. That means that on average the total energy of positrons in the magnetized case is three times higher.

The following figures 4.26 and 4.27 show the comparison between total energy transfer to the anti-proton for $B=0$ and $B=2.8 \mathrm{~T}$ with positron temperatures of $300 \mathrm{~K}, 100 \mathrm{~K}$, 30 K and 10 K .


Figure 4.26: Total energy transfer for an anti-proton of $E=1 \mathrm{eV}$ with positrons of axial temperature $T=300 \mathrm{~K}$ (top panel) and $T=100 \mathrm{~K}$ (bottom panel).


Figure 4.27: Total energy transfer for an anti-proton of $E=1 \mathrm{eV}$ with positrons of axial temperature $T=30 \mathrm{~K}$ (top panel) and $T=10 \mathrm{~K}$ (bottom panel).

The difference relating to $b$ approaching 0 is not yet clear. One thing that comes to mind would be the broken symmetry of the interaction, because of the magnetic field. If the positron is close to the center $b=0$, the anti-proton describes a circular orbit around the positron over the integration time. Depending on the axial velocities involved, it can be one or more full revolutions for a small relative axial velocity difference or only an arc for high velocities, because for this case the integration time is smaller than the $\overline{\mathrm{p}}$ cyclotron period. This kind of influence does not exist for the $B=0$ case and the effect of this movement might present itself differently for an impact parameter $>20 \mu \mathrm{~m}$ in the magnetized situation, where we can see that the two cases are very similar for all temperatures.

Another interesting aspect is that there are more collisions with a very high energy transfer in the collision within a magnetic field, while the effective range of the interaction, the width of the green compared to the purple peak at $b=16 \mu \mathrm{~m}$, seems smaller, independent of the temperatures.

In table 4.8 the averaged energy transfer of the anti-proton is calculated for all trajectories. The results show, that only for a 10 K plasma, the anti-proton is actually cooling. For the other temperatures positrons are transferring energy to the anti-proton.

| $T$ [K] | impact disc <br> radius [m] | averaged total <br> energy transfer [eV] | averaged total <br> energy transfer $[\mathrm{eV} / \mathrm{m}]$ |
| :---: | :---: | :---: | :---: |
| 300 | $3 \times 10^{-5}$ | $+3.178 \times 10^{-7}$ | +0.135 |
| 100 | $3 \times 10^{-5}$ | $+1.058 \times 10^{-6}$ | +0.448 |
| 30 | $3 \times 10^{-5}$ | $+9.176 \times 10^{-7}$ | +0.389 |
| 10 | $3 \times 10^{-5}$ | $-1.485 \times 10^{-5}$ | -6.300 |

TABLE 4.8: Transferred energy of an anti-proton with an axial energy of 1 eV in a $B=0$ set up.

### 4.5.4 Analysis of different positron temperatures

Figure 4.28 shows the dependence of the energy components from the positron temperature for a magnetic field of $B=2.8 \mathrm{~T}$. The dominating radial component scales directly with the temperature, as the energy transfer gets higher the colder the plasma is. The axial component shows less of this relation, for temperatures higher than 30 K there is an actual axial energy gain. Cooling becomes effective only for $T=10 \mathrm{~K}$. The averages of all trajectories are found in table 4.9.

| $T[\mathrm{~K}]$ | averaged total <br> energy transfer <br> $[\mathrm{eV} / \mathrm{m}]$ | averaged axial <br> energy transfer <br> $[\mathrm{eV} / \mathrm{m}]$ | averaged radial <br> energy transfer <br> $[\mathrm{eV} / \mathrm{m}]$ | outward transport <br> rate $\left[\mathrm{s}^{-1}\right]$ |
| :---: | :---: | :---: | :---: | :---: |
| 300 | -0.330 | -0.028 | -0.301 | 0.067 |
| 100 | -0.447 | +0.104 | -0.55 | 0.123 |
| 30 | -1.217 | +0.039 | -1.256 | 0.280 |
| 10 | -2.972 | -0.811 | -2.161 | 0.482 |

TABLE 4.9: Transferred energy of an anti-proton with an axial energy of 1 eV and radial energy of 0.1 eV in a trap with $B=2.8 \mathrm{~T}$, the impact disc size is $30 \mu \mathrm{~m}$.

The axial energy transfer, in these and almost all other simulations, does not show a clear correlation and is very "noisy". Axial cooling for 10 K that was already seen in the non-magnetized case (table 4.8) shows up here again even though it is an order of magnitude smaller. As expected radial cooling becomes more effective for lower temperatures showing a monotonous decrease.

| $T[\mathrm{~K}]$ | $r=0.5 R_{\text {plasma }}$ <br> leave time <br> $t_{\infty}$ | $r=0.5 R_{\text {plasma }}$ <br> leave time <br> $t_{\text {CUSP }}$ |
| :---: | :---: | :---: |
| 300 | 10.3 s | 15.3 s |
| 100 | 5.7 s | 8.4 s |
| 30 | 2.5 s | 3.7 s |
| 10 | 1.4 s | 2.1 s |

TABLE 4.10: Drift times of an anti-proton with an axial energy of 1 eV and radial energy of 0.1 eV in a trap with $B=2.8 \mathrm{~T}$, the impact disc size is $30 \mu \mathrm{~m}$. The second column features the realistic case in the CUSP, while the third column is calculated for a infinitely long plasma cylinder.


Figure 4.28: Temperature dependence of the energy transfer for an anti-proton of $E_{\text {axial }}=1 \mathrm{eV}, E_{\text {radial }}=0.1 \mathrm{eV}$ and $B=2.8 \mathrm{~T}$. Total energy (top panel), axial (mid-

### 4.5.5 Analysis of different anti-proton energies

One main concern of a realistic computation of the outward drift of the anti-proton is that information on the energy progression of the anti-proton energy is needed.

| axial E <br> $[\mathrm{eV}]$ | radial E <br> $[\mathrm{eV}]$ | averaged total <br> energy transfer <br> $[\mathrm{eV} / \mathrm{m}]$ | averaged axial <br> energy transfer <br> $[\mathrm{eV} / \mathrm{m}]$ | averaged radial <br> energy transfer <br> $[\mathrm{eV} / \mathrm{m}]$ | outward transport <br> rate $\left[\mathrm{s}^{-1}\right]$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.1 | -2.972 | -0.811 | -2.161 | 0.482 |
| 5 | 0.1 | -2.210 | -1.963 | -0.247 | 0.123 |
| 10 | 0.1 | -0.396 | -0.394 | -0.0026 | 0.0019 |
| 10 | 1 | -1.047 | -1.650 | +0.603 | -0.043 |

TABLE 4.11: Transferred energy of an anti-proton in a positron plasma with temperature of $T=10 \mathrm{~K}, B=2.8 \mathrm{~T}$ and impact disc size of $40 \mu \mathrm{~m}$ for $E_{\text {radial }}=0.1 \mathrm{eV}$ and $80 \mu \mathrm{~m}$ for

$$
E_{\text {radial }}=1 \mathrm{eV}
$$

Smaller energy transfer with increasing axial velocity is the expected result of the lower interaction time between the two collision bodies. Interesting is, that for increasing axial energy the up to now somewhat undefined axial energy transfer shows a distinctive peak at the anti-proton position. Even more astounding is that for 5 eV the axial transfer is maximal (in this simulation set) and gets smaller again for 10 eV . Generally one can see that for higher axial energy the share of radial energy transfer on the total gets considerable smaller. For a $1 \mathrm{eV} \overline{\mathrm{p}}$ radial energy transfer was about one magnitude bigger in absolute value, for 10 eV they are about the same.

We have no explanation yet for the behavior of the radial transfer component for the $(10 \mathrm{eV}, 1 \mathrm{eV})$ simulation. More simulations with different radial anti-proton energies are needed here.

| axial E <br> $[\mathrm{eV}]$ | radial E <br> $[\mathrm{eV}]$ | $r=0.5 R_{\text {plasma }}$ <br> leave time <br> $t_{\infty}$ | $r=0.5 R_{\text {plasma }}$ <br> leave time <br> $t_{\text {CUSP }}$ |
| :---: | :---: | :---: | :---: |
| 1 | 0.1 | 1.4 s | 2.1 s |
| 5 | 0.1 | 5.6 s | 11.7 s |
| 10 | 0.1 | 378 s | 950 s |
| 10 | 1 | - | - |

TABLE 4.12: Drift time of an anti-proton in a positron plasma with temperature of $T=$ $10 \mathrm{~K}, B=2.8 \mathrm{~T}$ and impact disc size of $40 \mu \mathrm{~m}$ for $E_{\text {radial }}=0.1 \mathrm{eV}$ and $80 \mu \mathrm{~m}$ for $E_{\text {radial }}=$ 1 eV . The third column features the realistic case in the CUSP, while the fourth column is calculated for a infinitely long plasma cylinder.


Figure 4.29: Axial velocity dependence of energy transfer for an anti-proton in a magnetic field of $B=2.8 \mathrm{~T}$ and a positron plasma with $T=10 \mathrm{~K}$. Total energy (top panel), axial


radial energy comparison: $\mathrm{T}=10 \mathrm{~K}, \mathrm{~B}=2.8 \mathrm{~T}$


Figure 4.30: Radial velocity dependence of energy transfer for an anti-proton in a magnetic field of $B=2.8 \mathrm{~T}$ and a positron plasma with $T=10 \mathrm{~K}$. Total energy (top panel),

### 4.5.6 CUSP Parameters

This section covers the important results for parameters present in the experiment in more detail. In the interaction region of the CUSP trap:

- homogeneous magnetic field of $B=2.8 \mathrm{~T}$,
- positron plasma with $T \sim 100 \mathrm{~K}$,
- fixed axial and radial velocity of the anti-proton $E_{\text {axial }}=1 \mathrm{eV}, E_{\text {radial }}=0.1 \mathrm{eV}$.

Those numbers come from theoretical assessment and measurements of the experimental conditions. The magnetic field value is extracted from field-maps simulated with the program COMSOL [44], showing a maximum of about 2.8 T at the position of the positron plasma. The value changes to about $90 \%$ at the axial plasma edges, but for qualitative results it is considered to be homogeneous.

The temperature of the positron plasma is more difficult to determine precisely. After injecting the positrons into the nested well (see figure 1.5) they will develop to a radiative equilibrium with the electrodes of the trap, such that in the ideal case the temperature of electrodes and plasma are the same. Measurements are done by slowly lowering the potential of the downstream electrode and measuring the number of escaping positrons [45]. The results of this extraction show possible positron plasma temperatures between $T \sim 100 \mathrm{~K}$ and $T \sim 300 \mathrm{~K}$ [23]. In general a lower temperature positron plasma greatly enhances the $\overline{\mathrm{H}}$ production [27], therefore temperatures as low as $T \sim 10 \mathrm{~K}$ are studied as well $[26,46]$.

There are two different procedures to inject anti-protons into the positron plasma described in section 1.2.4. For the simulations the direct-injection-procedures energies were used. In the case of adiabatic transport from the MUSASHI trap, they can be as low as 1 eV axially and 0.1 eV radially [22].

In figure 4.31 the results of the simulation are shown once more, as well as the $0.15 \mu \mathrm{~m}$ bin-averaged energy transfer components.


Figure 4.31: Top panel: total energy transfer of a 1 eV anti-proton in a 100 K positron plasma, bottom panel: average energy transfers.

Here we will take a more in-depth look at the influence of different parameters on the energy transfer. The axial positron velocity dependence can be seen in figure 4.32. The vertical line represents the axial velocity of the anti-proton $v_{\text {axial }, \overline{\mathrm{p}}}=-13800 \mathrm{~m} / \mathrm{s}$, therefore positrons on the right side of the line will have a frontal collision, those on the left will hit the $\overline{\mathrm{p}}$ from behind.


Figure 4.32: Energy transfer of the $e^{+}$depending on its velocity that is diced from a Boltzmann-distribution of $T=100 \mathrm{~K}$. Top: axial velocity, bottom: radial velocity.

Comparing the magnetized (figure 4.32, top) and unmagnetized case (figure 4.33) we see a clear asymmetry when a magnetic field comes into play.


Figure 4.33: Energy transfer of the e+ depending on its axial velocity for $T=100 \mathrm{~K}$ in the case $B=0$.

For a non-magnetic collision, the $e^{+}$to the left of the line with $v_{\text {axial }, e^{+}}=-13800 \mathrm{~m} / \mathrm{s}$ will always loose energy to the anti-proton in a collision, whereas the positrons to the right will collide from the front and gain energy the $\overline{\mathrm{p}}$ loses. This can be seen in figure 4.33.

The asymmetry in the magnetized case is not known and might partly stem from the fact, that because of the magnetic field the positron can not escape radially but stays close to the anti-proton for longer times. This is especially true for velocities a bit smaller than $-13800 \mathrm{~m} / \mathrm{s}$ as the positrons will move into the same direction as the $\overline{\mathrm{p}}$ trailing them with only slightly higher velocity while staying close axially as well as radially. So when they are loosing energy to the anti-proton by kicking them from behind the velocity difference will become even smaller enabling them to fly side-by-side, further increasing the interaction effect.

From figure 4.31, bottom panel, we can see that the loss of total energy stems primarily from radial energy loss. The axial component seems to even gain energy from the positron plasma. Averaging over all trajectories we can quantify these observations and find that axial energy is indeed increasing while the radial energy loss tips the balance to a resulting overall energy loss as seen in table 4.13. If we compare with the results for non-magnetic positrons in table 4.8 we see that for $B=0$ we yield an axial energy gain too, that is by a factor of five bigger though.

For CUSP conditions the average density is $n_{e+}=1.5 \times 10^{14} \mathrm{~m}^{-3}$ so we yield the energy transfer in table 4.13.

| impact disc <br> radius $[\mu \mathrm{m}]$ | averaged total <br> energy transfer $[\mathrm{eV} / \mathrm{m}]$ | averaged axial <br> energy transfer $[\mathrm{eV} / \mathrm{m}]$ | averaged radial <br> energy transfer $[\mathrm{eV} / \mathrm{m}]$ |
| :---: | :---: | :---: | :---: |
| 30 | -0.447 | +0.104 | -0.550 |

TABLE 4.13: Transferred energy of an anti-proton with an axial energy of 1 eV and radial energy of 0.1 eV in a $B=2.8 \mathrm{~T}$ trap with a $T=100 \mathrm{~K}$ positron plasma.

Using equation 4.4 the energy transfer per pass through the positron cloud and per second is calculated, taking the time the $\overline{\mathrm{p}}$ is outside of the plasma into account.

|  | $[\mathrm{eV} / \mathrm{m}]$ | $[\mathrm{eV} /$ pass $]$ | $[\mathrm{eV} / \mathrm{s}]$ |
| :---: | :---: | :---: | :---: |
| total energy | -0.447 | -0.029 | 4085 |
| axial energy | +0.104 | 0.0069 | 972 |
| radial energy | -0.55 | -0.036 | 5070 |

TABLE 4.14: Energy transfer rates of an anti-proton with an axial energy of 1 eV and radial energy of 0.1 eV in a $B=2.8 \mathrm{~T}$ trap with a $T=100 \mathrm{~K}$ positron plasma of density $n_{e+}=1.5 \times 10^{14} \mathrm{~m}^{-3}$ and length 6.6 cm , passing it with a frequency of $f=4.4 \times 10^{5} \mathrm{~s}^{-1}$.

For the CUSP parameters we can calculate the magnetron, cyclotron and dampening frequency

$$
\begin{align*}
\omega_{0} & =\frac{\omega_{p}^{2}}{2 \Omega_{e}}=\frac{n_{e} e}{2 \epsilon_{0} B}=4.85 \times 10^{5} \mathrm{~s}^{-1}  \tag{4.7}\\
\Omega_{i} & =\frac{e B}{m_{i}}=2.69 \times 10^{8} \mathrm{~s}^{-1}  \tag{4.8}\\
\nu & =-\left.\frac{1}{m_{\overline{\mathrm{p}}} v_{\text {radial }}} \frac{v_{\text {axial }}}{v_{\text {radial }}} \frac{d E}{d x}\right|_{\text {axial }}=3.75 \times 10^{4} \mathrm{~s}^{-1} \tag{4.9}
\end{align*}
$$

With this we find the rate of outward transport to be $\gamma=\mathbf{0 . 1 2 3} \mathbf{s}^{\mathbf{- 1}}$. Assume an anti-proton at a radial distance of 0.5 mm to the center of the plasma with a radius of 1 mm . From equation 2.20 we find the time for the anti-proton to leave the plasma to be $t=\ln 2 / \gamma=5.65 \mathrm{~s}$. This calculation assumes the anti-proton is moving through an infinitely long plasma tube. The actual situation is that the plasma is traversed only for a short time $t \sim 2.3 \times 10^{-6} \mathrm{~s}$ (for the $\overline{\mathrm{p}}$ axial velocity of $13800 \mathrm{~m} / \mathrm{s}$ ) then the anti-proton leaves the plasma axially and moves through the nested trap potential before coming back into the plasma. The anti-proton will therefore leave the plasma after $1.2 \times 10^{6}$ passes through the center of the trap. The axial frequency of the $\bar{p}$ in the nested trap is about $f_{\text {passage }} \sim f_{\text {harmonic }} \sim 4.4 \times 10^{5} \mathrm{~s}^{-1}$ yielding a total time of 8.35 s for the antiproton to leave the plasma.

### 4.5.6.1 Conclusions

An anti-proton with axial energy of 1 eV and radial energy of 0.1 eV in a positron plasma of $T=100 \mathrm{~K}$ and $B=2.8 \mathrm{~T}$ loses radial energy with the rate $d E / d x \sim-0.55 \mathrm{eV} / \mathrm{m}$, which translates to a rate of outward transport of $\gamma=0.135 \mathrm{~s}^{-1}$. Therefore a particle at radius $r$ will leave the plasma (with $R=1 \mathrm{~mm}$ ) in

| $r$ | 0.01 mm | 0.1 mm | 0.2 mm | 0.5 mm | 0.8 mm | 0.9 mm |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $t_{\text {leave }, \infty}$ | 37.5 s | 18.8 s | 13.1 s | 5.64 s | 1.81 s | 0.85 s |
| $t_{\text {leave, CUSP }}$ | 55.5 s | 27.8 s | 19.4 s | 8.35 s | 2.69 s | 1.26 s |

TABLE 4.15: Radial escape times of anti-protons with $E_{\text {radial }}=0.1 \mathrm{eV}$ and $T=100 \mathrm{~K}$ positron plasma.

The results of the simulations are giving numbers for the energy transfer and furthermore for the outward drift rate of the anti-proton. The time scales of outward drift found for CUSP parameters are around 10 s which is close to the time scale observed in the experiment.

The problem is, that the number computed from the simulation acts as guiding value for the drift time, because the energy transfer is assumed to be constant over this macroscopic time. Therefore the anti-proton would have lost its energy a million times over, see table 4.14, making this result unrealistic. Assuming that the energy transfer will vary over time, the integrated outward drift rate would probably yield time scales considerably different from the 10 s found here. So on a first glance the centrifugal separation alone might not cause anti-protons to leave the positron plasma on an experimentally significant timescale.

For convincing results more simulation have to be done, especially for different axial and radial energies to get an energy and therefore time dependent outward drift rate, that can be integrated for a realistic result of the time scale of the outward drift.

## Chapter 5

## Resume

In this thesis we have done the first step to understand the energy transfer of anti-protons in a positron plasma. To that end simulations of the two particle interaction in a strong magnetic field were conducted and analyzed on a general level with the anti-proton at rest and for different magnetic fields (section 4.2) and positron velocities/temperatures (section 4.3). The chaotic nature of the interaction (figure 4.6) left us without a suitable theory, so we tried to find some basic scaling laws for the chaotic region (section 4.2.2 and 4.2.3).

The full CUSP parameter set was simulated and roughly analyzed in section 4.5 where a brief survey for different magnetic fields and positron temperatures as well as a first approach to different anti-proton energies was conducted. The energy loss for the CUSP parameters are summarized in section 4.5.6.

This energy loss causes a radial centrifugal separation drift, that could be a suitable physical process to stop anti-hydrogen forming when anti-protons drift radially out of the positron cloud. Measurements of the $\overline{\mathrm{H}}$ yield in figure 5.1 show that the timescale in question is of the order of tens of seconds.


Figure 5.1: $\overline{\mathrm{H}}$ formation rate in beamtime 2012, with direct injection of anti-protons(black squares) and auto resonance (red dots) taken from [2].

The simulation results for the $\bar{p}$ outward drift times in table 5.1 are of the same order of magnitude as we would need, but to get to these times a constant energy loss rate was used. That means, that the anti-proton needs to loose the same $-0.55 \mathrm{eV} / \mathrm{m}$ over the whole tens of seconds time frame to achieve those drift times. Having only 0.1 eV radial energy to begin with this deduction is not realistic, but defines a guiding value for the drift times at hand. More simulations for different anti-proton energies are needed to calculate a realistic integrated drift time.

| $r$ | 0.01 mm | 0.1 mm | 0.2 mm | 0.5 mm | 0.8 mm | 0.9 mm |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $t_{\text {leave }, \infty}$ | 37.5 s | 18.8 s | 13.1 s | 5.64 s | 1.81 s | 0.85 s |
| $t_{\text {leave }, \text { CUSP }}$ | 55.5 s | 27.8 s | 19.4 s | 8.35 s | 2.69 s | 1.26 s |

Table 5.1: Radial escape times of anti-protons with $E_{\text {radial }}=0.1 \mathrm{eV}$ and $T=100 \mathrm{~K}$ positron plasma.

The conclusion for me here is, that there is not enough data to determine if centrifugal separation causes the anti-hydrogen yield drop or to completely exclude this process as possibility. The numbers tell us that the drift times themselves are in the right order of magnitude, so combining it with some other processes like heating of the anti-protons outside of the plasma or high energy loss rates even for low energies can make centrifugal separation an important factor in the yield drop and worth of further investigations.

## List of Figures

1.1 Current experimental precision of different experiments for detection of a possible CPT violation. The right edge of the bars gives the energy (frequency) equivalent of the observable measured, while the left edge shows the absolute precision of the measurement. Status of 2014 [11]. ..... 4
1.2 Zeeman splitting of the groud state hyperfine levels of hydrogen, adapted from [13]. ..... 6
1.3 Schematic overview of the ASACUSA experiment [2]. ..... 9
1.4 Schematic view of the positron accumulator [23]. ..... 10
1.5 (a) Multi ring electrode (MRE) of the CUSP trap, (b) its axial magnetic field strength and (c) potential configurations of positron loading $\phi_{1}, e^{+}$ cooling $\phi_{2}$, anti-hydrogen forming $\phi_{3}, \overline{\mathrm{p}}$ loading $\phi_{4}$ and FIT empty/regular operation mode $\phi_{5}$ [1] ..... 11
1.6 Schematic view of spin selection, transition and detection elements, adapted from [23]. ..... 12
1.7 Measurement of $\sigma_{1}$ and $\pi_{1}$ spin flips in a hydrogen beam [3]. ..... 13
$1.8 \overline{\mathrm{H}}$ formation rate in beamtime 2010 [1] ..... 14
$1.9 \overline{\mathrm{H}}$ formation rate in beamtime 2012, with direct injection of anti-protons(black squares) and auto resonance (red dots) taken from [2] ..... 15
1.10 Ion interaction with a neutral buffer gas leads to a loss of cyclotron energy that translates to an increase of magnetron radius [31] ..... 15
2.1 Schematic view of the charge shift in a negatively charged plasma intro- ducing a single positive charge. ..... 18
2.2 Helical motion of a charged particle in an electromagnetic field, taken from [32]. ..... 22
2.3 Radial electric field at the center of the positron plasma in the CUSP trap as a superposition of the field of the plasma and the outer electrodes. The red lines shows the electrode field if no plasma is present. ..... 23
2.4 Geometrical set up of Rutherford scattering. The alpha particle is repelled by the stationary nucleus and follows a hyperbolic trajectory. The dashed asymptotes span well defined angles from which the geometrical equations of Rutherford scattering can be deduced [35]. ..... 25
3.1 Integration by the Monte Carlo method. ..... 29
3.2 Sample simulation file for SIMBUCA. ..... 30
3.3 A simple example to show, that the number - area $\cdot$ average - converges toa constant if the radius of the impact disc is increased, including trajec-tories with a very small energy $(\sim 0)$ transfer for higher impact parameter. 34

$$
\begin{aligned}
& \text { 3.4 Example trajectories for the impact disc radius convergence mechanism. } \\
& \text { Shown is the initial position of the positron in the radial plane, while the } \\
& \text { anti-proton is considered to be at }(0,0) \text {. . . . . . . . . . . . . . . . . . . . } 36
\end{aligned}
$$

3.5 Sample trajectory for chaotic scattering. The axial position of the positron (green) and anti-proton (red) that is initially at rest in $z=0$. The black horizontal lines show the impact of two different box sizes. In the case of the solid line (boxsize $40 \mu \mathrm{~m}$ ) multiple scattering events take place, that are completely suppressed for the case of the dashed line (boxsize $10 \mu \mathrm{~m}$ ). ..... 39

4.1 This setup for the Rutherford simulations shows the initial placement of
the anti-proton in the origin of the coordinate system and a possible ran
dom starting position of the positron inside the dashed impact disc. The
axial separation here is chosen to be $50 \mu \mathrm{~m}$, the number most simulations
will use. ..... 42
4.2 Simulation results and analytical solution for Rutherford scattering with the positron at rest and the anti-proton velocity of $V_{\overline{\mathrm{p}}}=66000 \mathrm{~m} / \mathrm{s}$ (top panel) respectively $V_{\overline{\mathrm{p}}}=79800 \mathrm{~m} / \mathrm{s}$ (bottom panel). ..... 44
4.3 Rutherford scattering with the positron at rest and the anti-proton ve- locity of $V_{\overline{\mathrm{p}}}=66000 \mathrm{~m} / \mathrm{s}$ showing the differences of the simulation to the analytical solution for different box sizes of $50 \mu \mathrm{~m}, 100 \mu \mathrm{~m}$ and 1 mm . ..... 45
4.4 Top panel: simulation results for three different frames of reference with the same relative velocity $V=66000 \mathrm{~m} / \mathrm{s}$, bottom panel: Galilean trans- formation of the results in the top panel to the frame ( $V_{\overline{\mathrm{p}}}=66000 \mathrm{~m} / \mathrm{s}$, $V_{e^{+}}=0$ ). ..... 47
4.5 Total energy transfer in a collision between a resting anti-proton and an incident positron with $V=20000 \mathrm{~m} / \mathrm{s}$ in a magnetic field with $B=1 \mathrm{~T}$. The red line shows the analytic result for $B=0$. ..... 48
4.6 Axial position of the positron (green) and anti-proton (red) that is initially at rest in $z=0$. The positron is in a weakly bound state around the anti- proton, as it can not escape radially because of the magnetic field, and does more than 200 collisions. ..... 49
4.7 Top panel: simulation results for three different frames of reference with the same relative velocity $V=66000 \mathrm{~m} / \mathrm{s}$, bottom panel: Galilean trans- formation of the results in the top panel to the frame ( $V_{\overline{\mathrm{p}}}=66000 \mathrm{~m} / \mathrm{s}$, $V_{e^{+}}=0$ ). ..... 51
4.8 Total energy transfer dependence on magnetic field strength with axial relative velocity of $V=20000 \mathrm{~m} / \mathrm{s}$ and no initial radial movement ..... 52
4.9 Fit functions of the magnetic field scaling. ..... 54
4.10 Total energy transfer dependence for low magnetic field strength with axial relative velocity of $V=20000 \mathrm{~m} / \mathrm{s}$ and no initial radial movement. It can be seen, that the boarders of the chaotic zone converge to a impact parameter of $\sim 0.63 \mu \mathrm{~m}$ from both sides. As for $B=0.0001 \mathrm{~T}$ the zone is so thin, that no trajectory falls into it, and it almost follows the $B=0$ solution. ..... 55
4.11 Total energy transfer dependence on magnetic field strength with axial relative velocity of $V=66000 \mathrm{~m} / \mathrm{s}$ and no initial radial movement. ..... 56
4.12 Total energy transfer dependence on magnetic field strength with axial relative velocity of $V=66000 \mathrm{~m} / \mathrm{s}$ and no initial radial movement. ..... 57
4.13 Fit functions of the magnetic field scaling for the beginning and the end of the chaotic zones, that scale with $B^{0.18}$ and $B^{0.40}$ respectively. ..... 60
4.14 Top panel: simulation results for three different frames of reference with the same relative velocity $V=66000 \mathrm{~m} / \mathrm{s}$, bottom panel: Galilean trans- formation of the results in the top panel to the frame ( $V_{\overline{\mathrm{p}}}=66000 \mathrm{~m} / \mathrm{s}$, $\left.V_{e^{+}}=0\right)$. ..... 62
4.15 Total energy transfer dependence on the radial positron velocity with axial relative velocity of $V=20000 \mathrm{~m} / \mathrm{s}$ and $B=1 \mathrm{~T}$. ..... 63
4.16 Axial momentum transfer dependence on the radial positron velocity with axial relative velocity of $V=20000 \mathrm{~m} / \mathrm{s}$ and $B=1 \mathrm{~T}$. ..... 64
4.17 Total energy transfer dependence on the radial positron velocity with axial relative velocity of $V=20000 \mathrm{~m} / \mathrm{s}$ and $B=1 \mathrm{~T}$. ..... 64
4.18 Axial momentum transfer dependence on the radial positron velocity with axial relative velocity of $V=20000 \mathrm{~m} / \mathrm{s}$ and $B=1 \mathrm{~T}$. ..... 65
4.19 Comparison of our simulation results with the figure published in the paper of F.Robicheaux et al. [43]. The red dots are the energy transfers of our trajectories while the blue line is their binned average (bin-width $10 \mathrm{~nm})$. The black dots and lines are the results from the paper. ..... 67
4.20 Comparison of our simulation results with the figure publicized in the paper of F.Robicheaux et al. [43]. The red dots show our result for a selection of five different axial velocities that match the black paper results acceptably well. The solid line here shows the momentum transfer rate for $T=4 \mathrm{~K}$, the dotted line for $T=8 \mathrm{~K}$ and the dashed line for $T=16 \mathrm{~K}$. ..... 68
4.21 Top panel: axial momentum transfer rate $F$ for different values of the temperature $T$, bottom panel: total energy transfer rate $G$. ..... 69
4.22 Top panel: axial momentum transfer rate $F$ for different values of the magnetic field $B$, bottom panel: total energy transfer rate $G$. ..... 70
4.23 Total energy transfer of a 1 eV anti-proton in a $T=100 \mathrm{~K}$ positron plasma. 74.24 Magnetic field dependence of energy transfer for an anti-proton of $E_{\text {axial }}=$$1 \mathrm{eV}, E_{\text {radial }}=0.1 \mathrm{eV}$ and $T=\mathbf{3 0} \mathbf{K}$. Total energy (top panel), axialcomponent (middle panel) and radial component(bottom panel).75
4.25 Magnetic field dependence of energy transfer for an anti-proton of $E_{\text {axial }}=$ $1 \mathrm{eV}, E_{\text {radial }}=0.1 \mathrm{eV}$ and $T=\mathbf{1 0 0} \mathbf{K}$. Total energy (top panel), axial component (middle panel) and radial component(bottom panel). ..... 76
4.26 Total energy transfer for an anti-proton of $E=1 \mathrm{eV}$ with positrons of axial temperature $T=300 \mathrm{~K}$ (top panel) and $T=100 \mathrm{~K}$ (bottom panel). ..... 78
4.27 Total energy transfer for an anti-proton of $E=1 \mathrm{eV}$ with positrons of axial temperature $T=30 \mathrm{~K}$ (top panel) and $T=10 \mathrm{~K}$ (bottom panel) ..... 79
4.28 Temperature dependence of the energy transfer for an anti-proton of $E_{\text {axial }}=1 \mathrm{eV}, E_{\text {radial }}=0.1 \mathrm{eV}$ and $B=2.8 \mathrm{~T}$. Total energy (top panel), axial (middle panel) and radial component(bottom panel) ..... 82
4.29 Axial velocity dependence of energy transfer for an anti-proton in a mag-netic field of $B=2.8 \mathrm{~T}$ and a positron plasma with $T=10 \mathrm{~K}$. Totalenergy (top panel), axial (middle panel) and radial component(bottompanel).84
4.30 Radial velocity dependence of energy transfer for an anti-proton in a magnetic field of $B=2.8 \mathrm{~T}$ and a positron plasma with $T=10 \mathrm{~K}$. Total energy (top panel), axial (middle panel) and radial component(bottom panel) ..... 85
4.31 Top panel: total energy transfer of a 1 eV anti-proton in a 100 K positron plasma, bottom panel: average energy transfers ..... 87
4.32 Energy transfer of the $e^{+}$depending on its velocity that is diced from a Boltzmann-distribution of $T=100 \mathrm{~K}$. Top: axial velocity, bottom: radial velocity ..... 88
4.33 Energy transfer of the e+ depending on its axial velocity for $T=100 \mathrm{~K}$ in the case $B=0$ ..... 89
5.1 $\overline{\mathrm{H}}$ formation rate in beamtime 2012, with direct injection of anti-protons(black squares) and auto resonance (red dots) taken from [2]. ..... 93

## List of Tables

4.1 Parameter values for simulations conducted to validate Rutherford scat- tering results. ..... 43
4.2 Scaling parameter extracted from the simulation results with axial veloc- ity of $V=20000 \mathrm{~m} / \mathrm{s}$ and no radial velocity. The dash signalizes, that the corresponding point could not be determined ..... 53
4.3 Scaling parameter extracted from the simulation results with axial veloc- ity of $\mathrm{V}=66000 \mathrm{~m} / \mathrm{s}$ and no radial velocity. The dash signalizes, that the corresponding point could not be determined. ..... 58
4.4 The arc entries are the maximum energy transfer point of the arcs of regularity inside the chaotic zone. ..... 59
4.5 transferred energy of an anti-proton with an axial energy of 1 eV , radial energy of 0.1 eV and positron temperature of $T=\mathbf{3 0} \mathbf{K}$ and density $n=1.5 \times 10^{14} \mathrm{~m}^{-3}$, the impact disc size is $30 \mu \mathrm{~m}$ for $B=5 \mathrm{~T}, 30 \mu \mathrm{~m}$ for $B=2.8 \mathrm{~T}$ and $90 \mu \mathrm{~m}$ for $B=1 \mathrm{~T}$ ..... 73
4.6 transferred energy of an anti-proton with an axial energy of 1 eV ,radial energy of 0.1 eV and positron temperature of $T=\mathbf{1 0 0} \mathrm{K}$ and density $n=1.5 \times 10^{14} \mathrm{~m}^{-3}$, the impact disc size is $30 \mu \mathrm{~m}$ for $B=5 \mathrm{~T}, 30 \mu \mathrm{~m}$ for $B=2.8 \mathrm{~T}$ and $60 \mu \mathrm{~m}$ for $B=1 \mathrm{~T}$ ..... 74
4.7 Drift times of an anti-proton with an axial energy of 1 eV , radial en- ergy of 0.1 eV and positron temperature of $T=100 \mathrm{~K}$ and density $n=1.5 \times 10^{14} \mathrm{~m}^{-3}$, the impact disc size is $30 \mu \mathrm{~m}$ for $B=5 \mathrm{~T}, 30 \mu \mathrm{~m}$ for $B=2.8 \mathrm{~T}$ and $60 \mu \mathrm{~m}$ for $B=1 \mathrm{~T}$. The two columns on the right feature the realistic case in the CUSP, while the two columns on the left are calculated for a infinitely long plasma cylinder. ..... 74
4.8 Transferred energy of an anti-proton with an axial energy of 1 eV in a $B=0$ set up. ..... 80
4.9 Transferred energy of an anti-proton with an axial energy of 1 eV and radial energy of 0.1 eV in a trap with $B=2.8 \mathrm{~T}$, the impact disc size is $30 \mu \mathrm{~m}$. ..... 81
4.10 Drift times of an anti-proton with an axial energy of 1 eV and radial energy of 0.1 eV in a trap with $B=2.8 \mathrm{~T}$, the impact disc size is $30 \mu \mathrm{~m}$. The second column features the realistic case in the CUSP, while the third column is calculated for a infinitely long plasma cylinder. ..... 81
4.11 Transferred energy of an anti-proton in a positron plasma with tem-perature of $T=10 \mathrm{~K}, B=2.8 \mathrm{~T}$ and impact disc size of $40 \mu \mathrm{~m}$ for$E_{\text {radial }}=0.1 \mathrm{eV}$ and $80 \mu \mathrm{~m}$ for $E_{\text {radial }}=1 \mathrm{eV}$.83
4.12 Drift time of an anti-proton in a positron plasma with temperature of$T=10 \mathrm{~K}, B=2.8 \mathrm{~T}$ and impact disc size of $40 \mu \mathrm{~m}$ for $E_{\text {radial }}=0.1 \mathrm{eV}$and $80 \mu \mathrm{~m}$ for $E_{\text {radial }}=1 \mathrm{eV}$. The third column features the realistic casein the CUSP, while the fourth column is calculated for a infinitely longplasma cylinder.83
4.13 Transferred energy of an anti-proton with an axial energy of 1 eV and radial energy of 0.1 eV in a $B=2.8 \mathrm{~T}$ trap with a $T=100 \mathrm{~K}$ positron plasma. ..... 90
4.14 Energy transfer rates of an anti-proton with an axial energy of 1 eV and radial energy of 0.1 eV in a $B=2.8 \mathrm{~T}$ trap with a $T=100 \mathrm{~K}$ positron plasma of density $n_{e+}=1.5 \times 10^{14} \mathrm{~m}^{-3}$ and length 6.6 cm , passing it with a frequency of $f=4.4 \times 10^{5} \mathrm{~s}^{-1}$. ..... 90
4.15 Radial escape times of anti-protons with $E_{\text {radial }}=0.1 \mathrm{eV}$ and $T=100 \mathrm{~K}$ positron plasma. ..... 91
5.1 Radial escape times of anti-protons with $E_{\text {radial }}=0.1 \mathrm{eV}$ and $T=100 \mathrm{~K}$ positron plasma. ..... 94

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