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Contents

1 Introduction

One of the core issues of industrial organization is how firms deal with the issue of price-quality competition in markets. This is the central focus in each of the three chapters of my dissertation, where I address this issue in three different markets. In each chapter, I set up a model where each firm strategically chooses the quality of its product and the respective price such that it maximizes its profit, taking into consideration the optimal behavior of consumers. I use game-theoretic tools to solve each of the models.

In Chapter 2, I consider the markets of Credence goods. A credence good has certain quality attributes which are not observable. A firm usually knows the actual level of such quality attributes of the good it is producing, but its rival firms and consumers do not. This creates an information asymmetry among various agents in such markets. Examples of such unobservable quality attributes are - the amount of pesticides used during a good's production process, amount of pollution caused, involvement of any child labor, etc. This asymmetry has led to the emergence of a particular type of market institution called a certification intermediary. These act as middlemen who send some information regarding quality attributes of privately informed agents (firms) to uninformed parties (consumers). These intermediaries can be for-profit firms or non-profit firms. In this chapter, I study how an exogenous degree of horizontal product differentiation and the objective function of a certifier (for-profit or non-profit) affect the equilibrium of the market. More precisely, I take the horizontal differentiation between firms to be fixed and given, and then ask the following two questions: how does the horizontal differentiation between firms influence a certifier's (for-profit or non-profit) certification policy and therefore influences firms' decisions regarding their quality choices and decisions to opt for certification? Secondly, would a social planner choose a for-profit certifier or a non-profit certifier for a given market?

I show that under a non-profit certifier it is always the case that both firms produce the highest quality and opt for certification. This is also the case under a for-profit certifier, but only when the degree of horizontal differentiation is sufficiently high. When horizontal differentiation is low, the for-profit certifier, by charging a very high certification fee, creates maximum vertical differentiation between firms. As a result, only one firm produces the highest quality and opts for certification whereas the other firm produces the lowest quality and does not opt for certification. This asymmetry under a for-profit certifier makes the market inefficient, which provides one possible explanation for the existence of non-profit certifiers in such markets.

In Chapter 3, I examine the phenomena of selling and offering subscriptions - both of which are seen across various markets. Sometimes firms sell their products and sometimes they provide a subscription for their products. The question is : when will a firm choose which sales mode? This is the question which I seek to answer in this chapter under a durable good monopoly framework. I consider a two-period model where the qualities that are available in each period are exogenously given with the quality level that is available in the future period is higher than the quality level in the current period. Under such a framework, I ask the following question: for a given level of quality improvement for the future product, will a monopolist sell or offer subscription?

I find that in markets where innovation is slow and the product quality does not improve significantly over time, the firm will offer a subscription. However, when the quality of the product that is offered in the future period is significantly higher than in the current period, and the cost of producing it is also high, it is better for the monopolist to sell its products rather than to offer a subscription.

In Chapter 4, I address the issue of price-quality competition in the presence of search frictions in markets. From the consumer search literature, we know that search frictions lead to price dispersion in markets. However, there has been little attention given to how search frictions influence firms' decisions regarding their quality choices. In this chapter, I consider a duopoly framework in which each firm chooses the quality it wants to produce and the respective price.

The results show that the existence of homogeneous products in a market depends on choices made by firms. If search costs are small or the proportion of consumers who search costlessly (shoppers) is large, firms have incentives to differentiate themselves vertically. On the other hand, if search costs are large and the proportion of shoppers is small, this incentive does not exist. Therefore, with large search costs and a small proportion of shoppers, firms produce homogeneous products.

2 Certification Under Oligopolistic Competition

A version of this chapter is published in the journal 'Environmental and Resource Economics' [1].

2.1 Introduction

Quality attributes such as the environmental friendliness of a product are considered as credence attributes. The key feature of such attributes is that a firm usually knows the actual level of such quality attributes of the good it is producing, but its rival firms and consumers do not. This creates an information asymmetry among various agents in such markets. Examples of such unobservable quality attributes are - the amount of pesticides used during a good's production process, amount of pollution caused, involvement of any child labor, etc. This asymmetry has led to the emergence of a particular type of market institution called a certification intermediary. These act as middlemen who send some information regarding quality attributes of privately informed agents (firms) to uninformed parties (consumers). These intermediaries can be for-profit firms or non-profit firms. Curiously enough, while financial markets are dominated by for-profit certifiers, the markets in consideration are highly populated with non-profit certifiers (government agencies or NGOs). For example, the European Eco-Management and Audit Scheme (EMAS) certification is given either by the state or by the organizations and professionals authorized by the state to the firms who voluntarily commit to

[1] Environmental and Resource Economics, volume 65, pages 251 – 271 (2016) (`https://link.springer.com/article/10.1007/s10640-015-9903-3`).

perform according to EMAS regulations on environmental commitments[2], and the Forest Stewardship Council (FSC) is an international non-profit organization responsible for standard setting, independent certification and labeling of forest products, which offers customers around the world the ability to choose products from socially and environmentally responsible forestry, etc. Therefore, it seems quite natural to discover what role for-profit certifiers could play in providing certification for these environmentally friendly products.

In the literature, these unobservable quality attributes, such as, environmental friendliness of a product, or certain social attributes of a product, are considered as vertical attributes, as evidence suggests that consumers are willing to pay more for products with higher levels of these attributes[3]. In these markets, firms compete with each other by choosing their own quality attributes along vertical dimensions, deciding whether to get their products certified or not and setting prices. But along with this vertical dimension, there also exists a horizontal differentiation between firms, such as firms' locations or consumers' individual taste preferences for different firms.

In this chapter, I study how an exogenous degree of horizontal product differentiation and the objective function of a certifier (for-profit or non-profit) affect the equilibrium of the market. More precisely, I take the horizontal differentiation between firms to be fixed and given, and then ask the following two questions : how does the horizontal differentiation between firms influence a certifier's (for-profit or non-profit) certification policy and therefore influences firms' decisions regarding their quality choices and decisions to opt for certification? Secondly, would a social planner choose a for-profit certifier or a non-profit certifier for a given market?

[2]The EMAS is a voluntary environmental management instrument, which was developed by the European Commission. It enables organizations to assess, manage and continuously improve their environmental performance. The scheme is globally applicable and open to all types of private and public organizations. In order to register with EMAS, organizations must meet the requirements of the EU EMAS-Regulation.

[3]Consumers are willing to pay more for products with "Dolphin-safe" labels (Teisl et al. (2002), organic and fair trade coffee in the UK (Galarraga and Markandya (2004)), and sportswear made of organic cotton that involves lower use of pesticides and fertilizers (Casadesus-Masanell et al. (2009)).

To answer the questions I consider a market where two firms are horizontally differentiated and the degree of this differentiation is fixed and known. There is a certifier (for-profit or non-profit) in the market who announces its certification policy. Each firm chooses the quality it will produce which is its own private information, decides whether to opt for certification or not and sets a price. Consumers are willing to pay more for a higher quality product which is more costly to produce but generates higher surplus.

I find that under a non-profit certifier which certifies the highest quality products only, it is always the case that both firms produce the highest quality and opt for certification. This is also the case under a for-profit certifier but only when the degree of horizontal differentiation is sufficiently high. When horizontal differentiation is low, the for-profit certifier, by charging a very high certification fee, creates maximum vertical differentiation between firms. As a result, only one firm produces the highest quality and opts for certification whereas the other firm produces the lowest quality and does not opt for certification. This asymmetry makes the market inefficient under a for-profit certifier. Therefore, a social planner who wants to maximize social welfare would weakly prefer to have a non-profit certifier rather than a for-profit certifier to operate in such a market.

The intuition behind this is the following : a for-profit certifier cares only about its profit. In order to do so, when the horizontal differentiation between firms is low, by charging a very high fee the certifier creates maximum vertical differentiation between firms and captures a large amount of surplus from the market. When this horizontal differentiation becomes high, both firms capture sufficient monopoly power over their respective market shares which forces the certifier to reduce its fee. As a result, both firms produce the highest quality and opt for certification. Therefore, the degree of horizontal differentiation between firms plays a crucial role when a for-profit certifier chooses its certification policy and which in turn determines the nature of market equilibrium - whether it will be symmetric or asymmetric. The asymmetric equilibrium is inefficient due to the fact that firms share the market unequally and the lowest quality products are being traded in the market.

In industrial organization theory, this chapter relates to the vast literature discussing endogenous price-quality competition[4]. However, this deals with either search goods or experience goods and does not address the issue of certification. I discuss the impact of having a certifier in a market where firms are involved in price-quality competition[5]. Lizzeri (1999) studies the profit maximizing policy of certifier(s) when a firm's quality is exogenous. He finds that a monopolist certifier can extract a large amount of surplus without revealing any information, whereas if there are more than two certification agencies, there is a set of equilibria in which at least two certifiers reveal all the information and charge zero fees, making zero profits. Based on a similar framework, Albano and Lizzeri (2001) endogenize the monopolist seller's quality choice. They find different ways to implement the certifier's optimal policy, such as full disclosure with a nonlinear price schedule or a fixed fee with noisy disclosure. They also find that the certifier just needs to manipulate one dimension to obtain the optimal policy, either the price schedule or the disclosure policy. I introduce competition between the production firms. I find the optimal certification policy for a for-profit certifier. I study how competition between the production firms affects its optimal policy, and characterize the market equilibrium and present a comparative analysis between a for-profit certifier and a non-profit certifier. Interestingly, I find that under a for-profit certifier depending on the extent of horizontal differentiation between firms, both the symmetric and the asymmetric equilibrium can arise, whereas under a non-profit certifier only the symmetric equilibrium arises. The asymmetric equilibrium under a for-profit certifier makes the market inefficient.

This chapter also relates to the literature on 'Eco-labeling' in environmental economics. Mason (2006) analyzes the welfare implications of third party eco-

[4]Rosen (1974), Shapiro (1982), Chan and Leland (1982), Wolinsky (1983), Dubovik and Janssen (2012), etc.

[5]My model fits to the markets for credence goods. But as I consider a single shot game, theoretically it can also be applied to the markets for experience goods.

labeling as an imperfect and costly signal of quality. Baksi and Bose (2007) analyze the optimality of different labeling policies (self labeling and third party labeling) for credence goods when firms can cheat with respect to the labels they affix on their products. Bonroy and Constantatos (2008) study firms' behavior under perfect labeling and imperfect labeling and compare them from an efficiency point of view. Baron (2011) studies the choice of a credence standard by the firms forming a credence organization and explains how social pressure affects the standard they choose. He finds that the credence standard is lower the larger the organization, and social pressure results in a higher standard. I examine the role of a horizontal differentiation dimension between firms on a for-profit certifier's optimal certification policy in such a market and compare the efficiency of the respective market equilibrium to that with the market equilibrium under a non-profit certifier. I find that under certain reasonable assumptions, it is socially optimal to have a non-profit certifier in such a market and it is weakly preferable than having a for-profit certifier.

The chapter is organized as follows: in the next section I describe the model, in section 3 I present a basic result and discuss the beliefs of different agents about firms' quality choices. I characterize the market equilibrium under a non-profit certifier in section 4. I find the optimal certification policy for a for-profit certifier and characterize the market equilibrium under such a policy in section 5, followed by a welfare comparison in section 6. The final section concludes. The proofs of the propositions are described in the appendix along with the solution of the for-profit certifier's maximization problem.

2.2 Model

Consider a market that is comprised of a certifier, two identical firms - 1 and 2, and a unit mass of consumers. The certifier can be a for-profit firm or a non-profit firm. Competition between firms has two dimensions - horizontal and vertical. The horizontal dimension is fixed and known to everybody, and I model it by the

Hotelling line. Consumers are located uniformly on a line of unit length, and firms 1 and 2 are located on the same line at the two extreme points 0 and 1 respectively. In order to buy, a consumer has to incur a quadratic transportation cost of $t \cdot d^2$, $t > 0$, where d is the distance between that consumer and the firm from which he buys the product. The vertical dimension is modeled through firms' quality choices. Firm i can produce any quality $\theta_i \in [0, \bar{\theta}]$ at a unit cost of $c(\theta_i)$ for $i = 1, 2$, with $c(\theta)$ being increasing in θ, for $\theta \in [0, \bar{\theta}]$, and $c(0) > 0$. This choice of quality by a firm is its own private information, that is, whatever quality a firm produces is only observed by that particular firm. It is neither observed by its rival firm nor by any of the consumers[6]. Consumers have unit demand. If consumers know the quality of a product, they are willing to pay $V(\theta)$ for a product of quality θ, with $V(\theta)$ being increasing in θ. Let $S(\theta) = V(\theta) - c(\theta)$ for $\theta \in [0, \bar{\theta}]$ and assume that $S(\theta)$ is increasing in θ, with $S(0) > 0$. I use the following notations: $w(\theta) = S(\theta) - S(0)$, the incremental surplus for quality θ, for $\theta \in [0, \bar{\theta}]$, and $K = S(\bar{\theta}) - S(0)$, the maximum incremental surplus. Clearly $w(\theta)$ is also increasing in θ.

The certifier announces a certification policy which consists of a disclosure rule (D) and a non-negative certification fee (F). If a firm applies for certification, it has to pay the certification fee F to the certifier. I assume that if a firm applies for certification, the certifier can observe that particular firm's quality choice perfectly and it certifies the firm according to D. In order to maintain credibility in the market, I assume that the certifier cannot lie or cheat[7]. Formally, a fee $F : [0, \bar{\theta}] \rightarrow \mathbb{R}_+ \bigcup \{0\}$ and a disclosure rule $D : [0, \bar{\theta}] \rightarrow \mathbf{Q}$, where $\mathbf{Q}$ is the set of probability distributions on real numbers (Lizzeri (1999)). For example, a policy

[6] The reasons that I take the horizontal differentiation between firms to be given and allow each firm to choose its quality are the following. First of all, a firm's location choice is a long term decision and is difficult to change. Secondly, the environmental friendliness of a product has drawn significant attention only in recent times. Before that, firms were already selling their products without any reference to such quality attributes and over time, consumers have developed their own taste preferences for individual firms which cannot be changed suddenly. In most of the cases, environmental friendliness of a product is a voluntary commitment of a firm. Therefore, given these existing individual preferences of consumers for firms, I allow each firm to decide whether to produce an environmentally friendly product or not.

[7] There are some works which address the issue of credibility of these certification intermediaries. See Strausz (2005), Mathis et al. (2009), Peyrache and Quesada (2011).

in which only the highest quality is certified can be represented by a function that maps quality $\bar{\theta}$ to a probabilty distribution degenerate at $\bar{\theta}$ and it maps any other quality level (say θ) to a probabilty distribution degenerate at r independent of θ.

In different markets, we often find that a non-profit certifier certifies products of the highest quality only and charges a very small fee. For example, it certifies a product only if it finds that there is no child labor involved during its production process, or it certifies a product as a 'Green product' or 'Eco-friendly product' only if it finds that no pesticide is used or it satisfies certain environmental emission standards during its production process. Basically a non-profit certifier cares so much about the negative externalities caused by any quality level which is lower than the highest that it certifies products of the highest quality only. For this reason, I consider that the objective of a non-profit certifier is to maximize the number of firms producing the highest quality. Therefore, in my model a non-profit certifier certifies products of quality $\bar{\theta}$ only. On the other hand, a for-profit certifier cares only about its profit and it chooses the certification policy (F, D) in such a way that its profit is maximized. To compare welfare between the cases under a for-profit and a non-profit certifier, I introduce a social planner who wants to maximize social welfare which I measure in terms of total surplus.

The timing of the game is as follows:

1. At the first stage the certifier announces its certification policy (F, D).

2. Observing this certification policy, firms simultaneously choose qualities (θ_1, θ_2) that they will produce (this quality choice is each individual firm's private information) and decide whether to apply for certification or not. If a firm decides to apply for certification, it pays the fee F to the certifier. The certifier inspects the quality of that particular firm and announces its inspection result (R) for that firm publicly according to D.

3. After observing the certification results R (if any), firms choose prices (P_1, P_2) simultaneously.

4. At the final stage of the game, consumers observe the certification policy, certification results R (if any) and the prices charged by the firms. They consider their transportation costs and decide whether to buy and if so, which firm to buy from.

As each firm's quality choice is its private information, at stage 3 while setting prices, each firm forms an expectation regarding its rival firm's quality choice based on the information available (certification policy and certification result). Let $\theta^e_{i,r}$ denote the expected quality of firm i as perceived by its rival firm, where $\theta^e_{i,r} = \mathbb{E}[\theta_i \mid (F,\, D),\, R]$, for $i = 1,\, 2$. Similarly, at stage 4 while making buying decisions, consumers also form expectations about each firm's quality choice based on the information available (certification policy, certification results and prices) to them. Let θ^e_i denote the expected quality of firm i as perceived by the consumers where $\theta^e_i = \mathbb{E}[\theta_i \mid (F,\, D),\, R,\, P_1,\, P_2]$, for $i = 1,\, 2$.

The price chosen by firm i is denoted by P_i. If firm i chooses to produce quality θ_i, without loss of generality I can restrict its pricing strategy set to the interval $[c(\theta_i),\, V(\bar{\theta})]$, for $i = 1,\, 2$.

The payoff of each firm is its net profit. The payoff of a for-profit certifier is the total fee(s) it gets from the firm(s) who applies for certification.

The payoff of a consumer who buys is his net surplus. If he buys the product from firm i, then his payoff is the valuation of the product with quality θ^e_i minus the price charged by firm i minus the transportation cost that he incurs to reach firm i. If he knows through certification that the true quality of the product firm i is θ (e.g., in case of a non-profit certifier if a consumer sees that firm i's product is certified, he knows that its quality must be equal to $\bar{\theta}$), then $\theta^e_i = \theta$, and therefore, from buying the product from firm i his payoff is equal to the valuation of the product with quality θ minus the price charged by that firm minus the transportation cost he incurs to reach the firm. If he does not buy, his payoff is zero.

The equilibrium notion that I use is perfect Bayesian equilibrium.

2.3 Preliminary Result and Beliefs

Before moving to the main analysis, it would be helpful to understand how a firm will behave when there is no certifier in the market. This will help to study a firm's behavior when it does not apply for certification. Lemma 1 describes firms' behavior in the absence of any certifier, which along with the out of equilibrium beliefs of firms and consumers, discussed after lemma 1, significantly simplifies the analysis of the whole problem.

Lemma 1. *When there is no certifier in the market, there is no equilibrium where either of the firms produce strictly positive quality.*

Proof. Suppose on the contrary that in equilibrium one of the firms, say firm 1, produces a positive quality $\theta_1^*(> 0)$ and sets a price P_1^*. This quality choice is neither observed by firm 2, nor by consumers. If firm 1 deviates by choosing some other quality, this deviation will also not be observed by firm 2 and consumers. So whatever profit firm 1 makes by choosing quality θ_1^* and charging price P_1^*, it can always make more profit by choosing quality $\theta_1^* - \varepsilon$, where $\varepsilon > 0$, and charging the same price P_1^*, as $c(\theta_1^* - \varepsilon) < c(\theta_1^*)$. This is true for any positive quality θ_1^*. Therefore, whenever firm 1 chooses some positive quality, deviation to lower quality is always profitable to firm 1, which contradicts our assumption. Therefore in equilibrium, firm 1 will not produce any positive quality. By symmetry, the same is also true for firm 2, which completes the proof. □

Suppose a certifier announces it will certify a certain quality θ(say) only. If in equilibrium a firm (say firm i) does not apply for certification, its rival firm and consumers cannot observe its quality. In such a case, in equilibrium it can never happen that firm i produces positive quality. Because if it does so and charges a price P_i^*, it can always make more profit by lowering its quality and charging the same price P_i^*. Following a similar argument to that mentioned in lemma 1, irrespective of the price that firm i charges, its optimal quality choice is to produce quality 0.

For a firm i to apply for certification can only be a part of its equilibrium strategy if it has no incentive to deviate. If it deviates by not applying for certification, then its quality will not be observed. Therefore, irrespective of whatever out of equilibrium beliefs that its rival firm and consumers may have about its quality choice, its most profitable quality choice would be always to produce quality 0. However, these out of equilibrium beliefs play a role in a firm's pricing decision. For any out of equilibrium beliefs which assign any positive probabilty on firm i producing any positive quality will imply that the firm could charge higher price. But it does not make much sense to assign such beliefs, as a firm's cost function is common knowledge and therefore, its rival firm and consumers can draw inferences about its most profitable quality choice decision. Therefore, to make the deviation for a certified firm to not apply for certification not profitable, I assume that its rival firm and consumers will have the most pessimistic out of equilibrium beliefs about the deviating firm's quality choice, that is, they will all believe the deviating firm's quality to be 0, *i.e.*, $\theta^e_{i,r} = 0$ and $\theta^e_i = 0$ respectively, which is also the most reasonable one.

2.4 Non-profit Certifier

***Certification Policy**:* As a non-profit certifier wants to maximize the number of firms producing the highest quality, consider the following certification policy (F, D) announced by the non-profit certifier: the certification fee $F = 0$ and the disclosure rule D states that it certifies products of quality $\bar{\theta}$ only.[8] Given this certification policy, I find firms' optimal behavior.

***Firms' Behavior**:* Suppose given this certification policy, in equilibrium both firms produce quality $\bar{\theta}$ and apply for certification. Therefore through certification results, at the price setting stage firms know each other's quality choice and before buying consumers also know both firms' quality choices. Due to price competition, each of the two firms will set prices equal to $P_1^* = P_2^* = c(\bar{\theta}) + t$, capture half of

[8] For a small positive certification fee, the results do not change.

the market, and make a profit of $\frac{t}{2}$.

***Deviations**:* Now given the quality choices made by each of the firms, no firm has an incentive to deviate by charging a different price. At the quality choice stage, a firm could deviate by not applying for certification. If it does so, its quality will not be observed by others. Therefore, its most profitable quality choice would be to produce quality 0.

Suppose one firm (say firm 1) deviates by producing quality 0 and not applying for certification. In such a case, its rival firm and consumers will believe $\theta^e_{1,r} = 0$ and $\theta^e_1 = 0$. Considering these most pessimistic out of equilibrium beliefs, firm 1 will set a different price and will face a demand $\frac{(3t-K)}{6t}$, and its deviational profit will be $\frac{(3t-K)^2}{18t}$ (for details see Appendix). Clearly, the deviating firm will have positive demand only if $t > \frac{K}{3}$. The deviation will not be profitable for either of the firms, if the equilibrium profit is greater than or equal to the deviational profit, that is, if $\frac{t}{2} - \frac{(3t-K)^2}{18t} \geq 0 \iff t \geq \frac{K}{6}$. If $t < \frac{K}{6}$, this deviation could be profitable, but the deviating firm will have a positive demand only if $t > \frac{K}{3}$, which contradicts the condition $t < \frac{K}{6}$. For $t \leq \frac{K}{3}$, the deviating firm will face no demand and will make zero profit. Hence, this deviation is never profitable for either firm.[9]

Note that, the situation where both firms produce quality 0 and do not opt for certification can never be part of an equilibrium. As $S(\bar{\theta}) > S(0)$ and given one firm produces quality 0 and does not opt for certification, the other firm's best response would be to produce quality $\bar{\theta}$ and apply for certification. The minimum price that the non certified firm can charge is $c(0)$. The certified firm can capture the entire

[9] In the case where the non-profit certifier charges a very small fee ϵ for certification, if both firms opt for certification by producing the highest quality, then each firm will make a profit of $\frac{t}{2} - \epsilon$. Again, if a firm deviates by not applying for certification, it will produce quality 0. This deviating firm will have positive demand only if $t > \frac{K}{3}$. For $t > \frac{K}{3}$, the deviation will not be profitable if $\frac{t}{2} - \epsilon \geq \frac{(3t-K)^2}{18t} \iff \epsilon \leq \frac{K(6t-K)}{18t}$. The RHS of the inequality is non-negative for $t \geq \frac{K}{6}$. Hence for $t \geq \frac{K}{6}$, the certifier can charge a small certification fee ϵ such that $\epsilon \leq \frac{K(6t-K)}{18t}$ and both firms will produce the highest quality and opt for certification. For $t \leq \frac{K}{6}$, the deviating firm will make zero profit because it will face zero demand. Hence for $t \leq \frac{K}{6}$, the certifier can charge a small certification fee ϵ such that $\epsilon \leq \frac{t}{2}$ and both firms will produce the highest quality and opt for certification. As long as the non-profit certifier charges a small certification fee satisfying certain conditions to certify the highest quality only, it is always the case that both firms will produce the highest quality and will opt for certification.

market by charging a price marginally lower than $V(\bar{\theta}) - [V(0) - c(0)] - t$. In such a situation the certified firm could earn a surplus of $K - t$ and the non-certified firm earns nothing. Therefore, the strategies where both firms produce quality 0 and do not apply for certification can never be part of an equilibrium. Hence, we have the following proposition.

Proposition 1. *If a non-profit certifier certifies products of quality $\bar{\theta}$ only and charges no certification fee, there is an equilibrium which is unique and symmetric where both firms produce quality $\bar{\theta}$, opt for certification, set prices equal to $c(\bar{\theta}) + t$ and each firm captures half of the market. Each firm makes a profit of $\frac{t}{2}$.*

Under a non-profit certifier, I find that in equilibrium both firms always produce the highest quality and opt for certification. Intuitively this is clear, because as the highest quality generates maximum surplus and there is no (or only a very small) certification fee, both firms want to extract maximum surplus by producing the highest quality and revealing its true quality to consumers through certification. As horizontal differentiation increases, firms' profits also increases. But under a for-profit certifier, this may not be the case.

Remark. In a situation where horizontal differentiation vanishes (i.e., $t = 0$), the above strategies of firms mentioned in proposition 1 will still remain equilibrium strategies, as long as the certifier does not charge a certification fee. Due to perfect competition, both firms will set prices equal to the marginal cost ($= c(\bar{\theta})$), will make zero profits and the entire surplus will go to consumers. In this perfect competition case, along with this symmetric equilibrium where both firms produce quality $\bar{\theta}$ and opt for certification, there also exists an asymmetric equilibrium where one firm produces quality $\bar{\theta}$ and opts for certification, whereas the other firm produces quality 0 and does not opt for certification. As $S(\bar{\theta}) > S(0)$, the certified firm has a competitive advantage over the non-certified firm. By charging a price marginally lower than $V(\bar{\theta}) - [V(0) - c(0)]$ the certified firm can capture the entire market. Therefore, in equilibrium, the certified firm charges a price equal to $V(\bar{\theta}) - [V(0) - c(0)]$ and sells to all consumers. The non-certified firm makes zero profit whereas the certified firm earns a surplus of K. For any positive certification fee, if both firms opt for certification, due to perfect competition prices will still be equal to the marginal cost ($= c(\bar{\theta})$) and both firms will make losses. Hence, when $t = 0$, for any positive certification fee (for-profit or non-profit), only the asymmetric equilibrium will exist.

2.5 For-profit Certifier

In this section I derive the optimal certification policy for a for-profit certifier. A for-profit certifier wants to extract maximum surplus from the market, and it does so by charging a suitable positive fee F for certifying a certain quality. First I will consider the certification policy announced by a for-profit certifier to be exogenously given. That is, given that the certifier charges a fee F to certify a certain quality θ(say), $\theta \in [0, \bar{\theta}]$, I find conditions for an equilibrium in which both firms produce quality θ and opt for certification (symmetric case), and conditions for an equilibrium in which only one firm produces quality θ and opts for certification whereas the other firm does not opt for certification (asymmetric case). Later I endogenize the certification policy and find the optimal certification policy for a for-profit certifier and characterize the market equilibrium.

2.5.1 *Firms' Behavior*

Symmetric case

Proposition 2. *For a given certification policy (F_s, D), where D states that the certifier will certify products of quality θ only, $\theta \in [0, \bar{\theta}]$, there is an equilibrium which is unique and symmetric where both firms produce quality θ and apply for certification, provided $F_s \leq \frac{w(\theta)(6t - w(\theta))}{18t}$.*

Given the certification policy mentioned in proposition 2, if both firms produce quality θ and apply for certification by paying the fee F_s to the certifier, in equilibrium firms charge prices $P_1^* = P_2^* = c(\theta) + t$. Each of the firms faces a demand equal to $\frac{1}{2}$ and earns a profit of $\frac{t}{2} - F_s$ each.

Now given the certification policy, strategies taken by one firm and those taken by the other in stage 2, the optimal price for the other firm is $c(\theta) + t$. Deviation to a different price is never profitable.

Each of the firms can deviate only in stage 2. Clearly neither firm will deviate by producing quality that is higher than the certified quality and applying for certification, because by producing higher quality it will incur higher cost. But through certification its quality will be revealed as θ. So it will make less profit than equilibrium profit due to the higher cost of production. Hence this deviation is not profitable for either firm. If a firm deviates by producing quality less than θ, the certifier will not certify its quality. In that case, the deviated firm will not apply for certification. Now if one of the firms deviates by not applying for certification, the other firm and the consumers will not know its quality. In that case the best possible quality choice for the deviating firm will be to produce quality 0. To make this deviation not profitable, I assume that its rival firm and consumers will have the most pessimistic out of equilibrium beliefs about the deviated firm's quality choice, that is, they will believe its quality to be 0. If either of the firms (say firm 1) deviates by producing quality 0 and not applying for certification, then incorporating the beliefs of its rival firm ($i.e.$, $\theta^e_{1,r} = 0$) and the consumers ($i.e.$, $\theta^e_1 = 0$) we can find its optimal deviating price. This deviation will not be profitable if the fee F_s charged by the certifier is not high (see Appendix).

The certifier wants to maximize $\frac{w(\theta)(6t - w(\theta))}{18t}$ with respect to θ which is equivalent to maximizing $\frac{w(\theta)(6t - w(\theta))}{18t}$ with respect to $w(\theta)$, as $w(\theta)$ is monotonically increasing in θ. The solution of the maximization problem $\hat{\theta}$ is given by $\hat{\theta} = w^{-1}(\widehat{w(\theta)})$, where

$$\widehat{w(\theta)} = \begin{cases} w(\bar{\theta}), & if\, w(\bar{\theta}) \leq 3t \\ 3t, & if\, w(\bar{\theta}) > 3t \end{cases}.$$

Therefore the certifier could certify products of quality $\hat{\theta}$ only and charges a certification fee $\hat{F}_s$, where $\hat{F}_s = \frac{w(\hat{\theta})(6t-w(\hat{\theta}))}{18t}$. Given this certification policy, both firms produce quality $\hat{\theta}$ and opt for certification.

Asymmetric case

Proposition 3. *For a given certification policy (F_a, D), where D states that the certifier will certify products of quality θ only, $\theta \in [0, \bar{\theta}]$, there is an equilibrium which is unique and asymmetric where one firm produces quality θ and applies for certification whereas the other firm produces quality 0 and does not apply for certification, provided* $\frac{w(\theta)(6t-w(\theta))}{18t} \leq F_a \leq \frac{w(\theta)(6t+w(\theta))}{18t}$.

Given the certification policy in proposition 3, suppose in equilibrium one firm (say firm 1) produces quality 0 and does not apply for certification, whereas the other firm (firm 2) produces quality θ and applies for certification. Firm 2 has several deviation possibilities, but following a similar argument to that in proposition 2, the most profitable deviation for firm 2 would be to produce quality 0 and not apply for certification. To have this deviation not profitable for firm 2, F_a should not be too high. Among all possible deviations for firm 1, the best possible deviation is to produce quality θ and apply for certification. This deviation is not profitable for firm 1 if F_a is sufficiently high (see Appendix).

The certifier wants to maximize F_a. Note that F_a is increasing in θ and attains its maximum at $\theta = \bar{\theta}$. Therefore the certifier can certify products of quality $\bar{\theta}$ only and can charge a fee $\hat{F}_a$, where $\hat{F}_a = \frac{K(6t+K)}{18t}$.

From propositions 2 and 3, we see that depending on the certification fee F, the symmetric or the asymmetric equilibrium can arise.

2.5.2 *Optimal Certification Policy*

In principle, the set of possible disclosure rules for the certifier can be large, e.g., it certifies products of quality θ (say) only, where $\theta \in [0, \bar{\theta}]$, it certifies all products whose qualities are higher or equal to θ and discloses their exact qualities, it certifies all products whose qualities are higher or equal to θ and discloses their qualities as qualities $\geq \theta$, it sets some noisy disclosure rule, etc. Following lemma 2, I can restrict my analysis to only one kind of disclosure rule.

Lemma 2. *To find the optimal certification policy for a for-profit certifier, it is sufficient to consider only the case where the certifier charges fee F_1 to certify quality θ_1 and a fee F_2 to certify quality θ_2, with $F_1 \leq F_2$ and $0 \leq \theta_1 \leq \theta_2 \leq \bar{\theta}$.*

Proof. Suppose the certifier certifies all products whose qualities are higher or equal to θ and discloses their qualities as qualities higher or equal to θ. Then none of the firms which opts for certification will produce qualities strictly higher than θ. This is due to the fact that, if a firm applies for certification and gets certification, through the certification result its quality is revealed as higher or equal to θ. Any quality that is strictly higher than θ is neither observed by its rival firm nor by consumers. For any positive ε as $c(\theta) < c(\theta + \varepsilon)$, the optimal quality choice for the firm seeking certification would be to produce quality θ. Thus the certifier can certify products of quality θ only. If the certifier certifies all products whose qualities are higher or equal to θ and discloses their exact qualities, then either one firm or both firms will opt for certification. If one firm opts for certification, it will produce products of quality $\bar{\theta}$, as producing quality $\bar{\theta}$ generates maximum surplus. So the certifier can certify products of quality $\bar{\theta}$ only. If both firms opt for certification, they can produce the same or different qualities. If they produce the same quality, they will choose $\bar{\theta}$, and so the certifier can just certify quality $\bar{\theta}$. If the firms produce different qualities, the certifier can certify only those two qualities for which the certifier can make maximum profit. In general, whatever disclosure rule the certifier announces, firms will choose at most two qualities. Therefore, among all possible disclosure rules, the for-profit certifier can certify two qualities for which the certifier can make maximum profit. □

Profit Certifier's Maximization Problem

Suppose the certifier announces a certification policy which states that it will certify qualities θ_1 and θ_2 with $0 \leq \theta_1 \leq \theta_2 \leq \bar{\theta}$, and to apply for certification, a firm has to pay fees F_1(for quality θ_1) and F_2 (for quality θ_2), with $F_1 \leq F_2$.

Given this certification policy, suppose that in equilibrium one firm (say firm 1) produces quality θ_1 and applies for certification by paying the fee F_1, whereas the other firm (firm 2) produces quality θ_2 and applies for certification by paying the fee F_2. Certification result R will reveal firm 1's quality to be θ_1 and that of firm 2 to be θ_2. Therefore, $\theta^e_{1,2} = \theta^e_1 = \theta_1$ and $\theta^e_{2,1} = \theta^e_2 = \theta_2$. Firm 1 and firm 2 set prices P_1^* and P_2^* respectively and make profits π_1^* and π_2^* respectively (see Appendix).

Deviations for firm 1: Given the strategies taken by firm 2 and the strategies taken by firm 1 in stage 2, P_1^* is the optimal price that firm 1 can charge. Therefore, firm 1 has no incentive to deviate at the price setting stage.

Among all possible deviations for firm 1 in stage 2, the most profitable deviation is either to produce quality θ_2 and apply for certification or to produce quality 0 and not apply for certification. If firm 1 deviates by producing quality θ_2 and applies for certification by paying the fee F_2, through certification its quality will be revealed as θ_2. Firm 1 will set a price equal to $c(\theta_2) + t$, capture half of the market and make a profit of $\frac{t}{2} - F_2$. This deviation will not be profitable for firm 1 if $\pi_1^* \geq \pi_1^D$, *i.e.* if

$$\frac{(3t + S(\theta_1) - S(\theta_2))^2}{18t} - F_1 \geq \frac{t}{2} - F_2. \tag{2.1}$$

If firm 1 deviates by producing quality 0 and not applying for certification, its quality is not observed. To make this deviation not profitable for firm 1, I assume that in such a situation firm 2 and all consumers will have the most pessimistic out of equilibrium beliefs about firm 1's quality, that is, they will all believe that firm 1's quality is 0 which implies $\theta^e_{1,2} = \theta^e_1 = 0$. Given this out of equilibrium belief, this deviation will not be profitable for firm 1 if π_1^* is greater than or equal to its deviational profit (see Appendix), that is, if

$$F_1 \leq \frac{(S(\theta_1) - S(0))(6t + S(\theta_1) + S(0) - 2S(\theta_2))}{18t} = f_1(\theta_1, \theta_2). \tag{2.2}$$

Deviations for firm 2: Similarly, it is not profitable for firm 2 to deviate only at the price setting stage. The most profitable deviations for firm 2 would be either to produce quality θ_1 and apply for certification or to produce quality 0 and not apply for certification. The first deviation will not be profitable if $\pi_2^* \geq \pi_2^D$, *i.e.* if

$$\frac{(3t + S(\theta_2) - S(\theta_1))^2}{18t} - F_2 \geq \frac{t}{2} - F_1, \tag{2.3}$$

and imposing similar out of equilibrium beliefs ($\theta_{2,1}^e = \theta_2^e = 0$) the second deviation will not be profitable if

$$F_2 \leq \frac{(S(\theta_2) - S(0))(6t + S(\theta_2) + S(0) - 2S(\theta_1))}{18t} = f_2(\theta_1, \theta_2). \tag{2.4}$$

Given the certification policy, if conditions (2.1)-(2.4) are satisfied, one firm would opt for certification of quality θ_1 and the other would opt for certification of quality θ_2. In that case, the certifier earns a surplus of $F_1 + F_2$, and it wants to maximize $F_1 + F_2$ with respect to $\theta_1$and θ_2, where $\theta_1, \theta_2 \in [0, \bar{\theta}]$ and $\theta_1 \leq \theta_2$, such that conditions $(2.1) - (2.4)$ are satisfied. This is equivalent to maximizing $f_1(\theta_1, \theta_2) + f_2(\theta_1, \theta_2)$ with respect to θ_1 and θ_2, where $\theta_1, \theta_2 \in [0, \bar{\theta}]$ and $\theta_1 \leq \theta_2$, such that conditions $(1) - (4)$ are satisfied.

Using Kuhn Tucker conditions, I obtain the following solution for the certifier's maximization problem (see Appendix):

1. $\hat{\theta}_1 = 0$, $\hat{F}_1 = 0$, and $\hat{\theta}_2 = \bar{\theta}$, $\hat{F}_2 = \frac{K(6t+K)}{18t}$, for $t \leq \frac{K}{2}$,

2. $\hat{\theta}_1 = \hat{\theta}_2 = \bar{\theta}$, $\hat{F}_1 = \hat{F}_2 = \frac{K(6t-K)}{18t}$, for $t > \frac{K}{2}$.

In the first case where $\hat{\theta}_1 = 0, \hat{F}_1 = 0$ is equivalent to no certification. In this case, the certifier earns a surplus $\frac{K(6t+K)}{18t}$ and in the second case a surplus $\frac{K(6t-K)}{9t}$.

Note that for $t \leq \frac{K}{2}$, the firm which produces quality 0 will have a demand of $\frac{(3t-K)}{6t}$, which is positive for $t > \frac{K}{3}$. If $t \leq \frac{K}{3}$, the firm which produces quality $\bar{\theta}$ and opts for certification captures the entire market. In this case the certified firm can charge a price equal to $V(\bar{\theta}) - [V(0) - c(0)] - t$ and can capture the entire market. Hence the certifier can charge a different fee. If the certified firm deviates by not opting for certification, it will produce quality 0 and therefore will share the market equally with the other firm and will make a profit of $\frac{t}{2}$. The fee (F) that the certifier can charge to make this deviation not profitable for the certified firm satisfies the following:

$$K - t - F \geq \frac{t}{2} \Longleftrightarrow F \leq K - \frac{3t}{2}.$$

In this case the maximum fee $(\hat{F})$ the certifier can charge is equal to $K - \frac{3t}{2}$. Combining all these points, I obtain the following proposition.

Proposition 4. *In all equilibria the optimal certification policy for a for-profit certifier is to certify products of quality $\bar{\theta}$ only. The optimal certification fee $\hat{F}$ and market equilibrium is characterized by the following: there exists a unique pair of threshold values $(t^* = \frac{K}{3}, t^{**} = \frac{K}{2})$ of t such that,*

1) for $0 \leq t \leq t^*$, there exists an equilibrium which is unique and asymmetric where $\hat{F} = K - \frac{3t}{2}$, one firm produces quality $\bar{\theta}$ and opts for certification, whereas the other produces quality 0 and does not opt for certification, and the certified firm captures the entire market,

2) for $t^* < t \leq t^{**}$, there exists an equilibrium which is unique and asymmetric where $\hat{F} = \frac{K(6t+K)}{18t}$, one firm produces quality $\bar{\theta}$ and opts for certification, whereas the other produces quality 0 and does not opt for certification, and both firms make positive profits,

3) for $t > t^{**}$, there exists an equilibrium which is unique and symmetric where $\hat{F} = \frac{K(6t-K)}{18t}$, both firms produce quality $\bar{\theta}$ and opt for certification, and both firms make equal profits.

From Proposition 4, I find that in all equilibria a for-profit certifier always certifies products of the highest quality only, but depending on the degree of horizontal differentiation, it charges different certification fees which lead to the asymmetric equilibrium or the symmetric equilibrium. When this horizontal differentiation is very low (when t is smaller than t^*), that is the competition between firms is very high, it charges such a high fee that only one firm produces the highest quality, opts for certification and captures the entire market, whereas the other firm produces the lowest quality and sells nothing. As the horizontal differentiation becomes larger than t^* but smaller or equal to t^{**}, the certifier charges a different fee but still high enough so that only one firm produces the highest quality and opts for certification. In this case, as competition between firms reduces, the non-certified firm which produces the lowest quality also gains some market power and sells to some consumers. This asymmetric equilibrium vanishes, as the horizontal differentiation becomes larger than t^{**}. Here as competition between firms becomes sufficiently low, both firms gain sufficient market power, therefore the certifier charges a much lower fee so that both firms produce the highest quality and opt for certification.

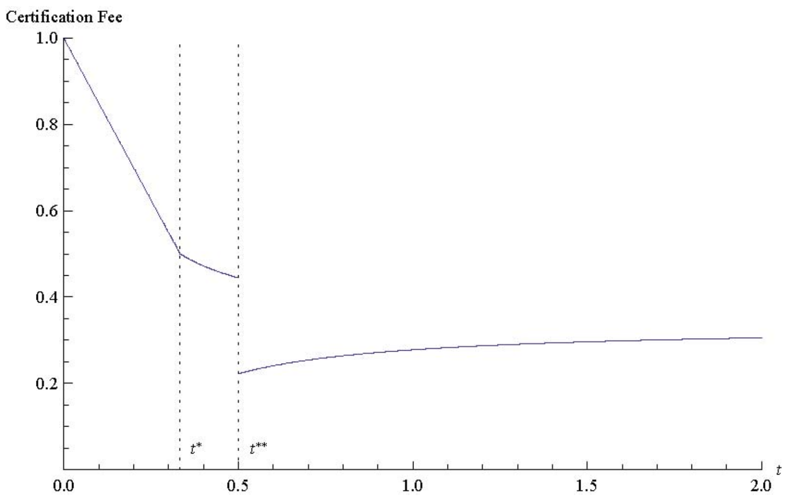

Figure 2.1: Graph of certification fee function vs t for $K = 1$. The vertical dotted lines in the left and the right are drawn at the points $t = t^*$ and $t = t^{**}$ respectively.

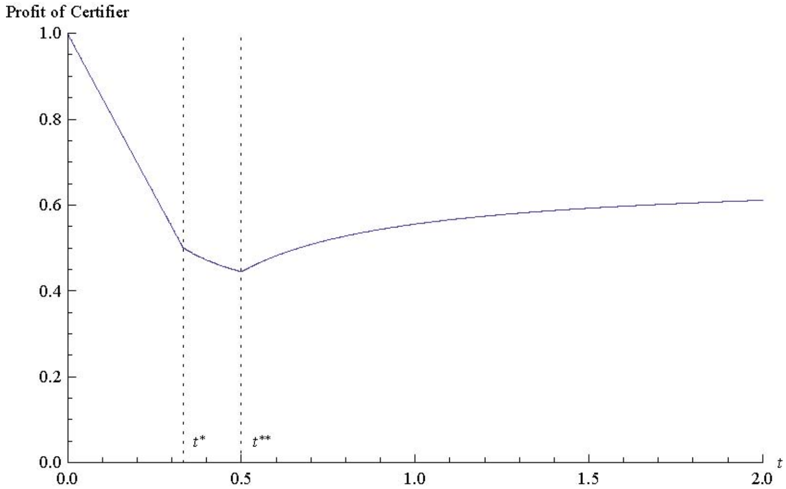

Figure 2.2: Graph of the certifier's profit function vs t for $K = 1$. The vertical dotted lines in the left and right are drawn at the points $t = t^*$ and $t = t^{**}$ respectively.

Figures 2.1 and 2.2 show respectively how the certification fee charged by a for-profit certifier and its corresponding profit varies as the degree of horizontal differentiation between firms increases. The certification fee is discontinuous due to the change in equilibrium structure and is non-monotonic in t. As a consequence, the profit of the certifier is also non-monotone in t. One might think that as competition between firms becomes low, the certifier will make more profit which is not true here. The certifier makes more profit when the competition between firms is very intense and it does so by creating maximum vertical differentiation between the two firms.

2.6 Welfare Analysis

A social planner wants to maximize social welfare. I measure social welfare by calculating the total surplus generated in each of the equilibria under a non-profit certifier and a for-profit certifier. As a non-profit certifier does not charge a certification fee, the total surplus in this case is just the sum of firms' total profits and consumer surplus. Under a for-profit certifier, the total surplus is the sum of industry profit (i.e., certifier's profit plus firms' total profits) and consumer surplus.

Under a non-profit certifier, each firm captures half of the market, sets the same price $c(\bar{\theta}) + t$ and makes a profit of $\frac{t}{2}$. Total consumer surplus will be $2\int_0^{\frac{1}{2}}[V(\bar{\theta}) - c(\bar{\theta}) - t - tx^2]dx = S(\bar{\theta}) - \frac{13t}{12}$.

Therefore under a non-profit certifier the total surplus is always $t + S(\bar{\theta}) - \frac{13t}{12} = S(\bar{\theta}) - \frac{t}{12}$, which is the maximum surplus that can be generated from the market.

t	Industry Profit	Consumer Surplus	Total Surplus
$0 < t \leq t^*$	$K - t$	$S(0) + \frac{2t}{3}$	$S(\bar{\theta}) - \frac{t}{3}$
$t^* < t \leq t^{**}$	$t + \frac{K^2}{9t}$	$\frac{K^2}{18t} + \frac{S(\bar{\theta})+S(0)}{2} - \frac{13t}{12}$	$\frac{K^2}{6t} + \frac{S(\bar{\theta})+S(0)}{2} - \frac{t}{12}$
$t > t^{**}$	t	$S(\bar{\theta}) - \frac{13t}{12}$	$S(\bar{\theta}) - \frac{t}{12}$

Table 1: The total surplus under a for-profit certifier

Table 1 gives the total surplus under a for-profit certifier for different values of t. Under a non-profit certifier, as each firm captures half of the market, the total transportation cost that is incurred by consumers is minimal. Both firms sell products of the highest quality which generates maximum surplus. Moreover, it can be checked easily that for $0 < t \leq t^{**}$, the total surplus under a non-profit certifier is strictly higher than that under a for-profit certifier. For $0 < t \leq t^*$, under a for-profit certifier as the certified firm captures the entire market, the total transportation cost incurred by the consumers is higher, which leads to the inefficiency. For $t^* < t \leq t^{**}$, the inefficiency arises for two reasons. Firstly, firms have unequal market shares and hence the total transportation cost incurred by consumers is higher. Secondly, as some consumers buy the lowest quality products, this yields lower surplus. For $t > \frac{K}{2}$, under both types of certifiers, only the symmetric equilibrium exists where both firms produce the highest quality and opt for certification. In this case, the total surplus remains the same under both types of certifier because under a for-profit certifier some surplus gets transferred from the firms to the certifier. Therefore, a social planner would weakly prefer to have a non-profit certifier rather than a for-profit certifier to operate in such markets.

2.7 Conclusion

In this chapter I have analyzed a symmetric duopolistic market where each firm's choice concerning certain quality attributes such as environmental friendliness of its product is its own private information and have found that the degree of horizontal differentiation among firms plays a crucial role in a for-profit certifier's optimal certification policy. Under a for-profit certifier, there exists a threshold value of the degree of horizontal differentiation below which the certifier charges such a high fee that only one of the firms produces the highest quality and opts for certification, whereas the other firm produces the lowest quality and does not opt for certification. Moreover, when the horizontal differentiation is sufficiently small, the certified firm captures the entire market. Basically when competition between the firms is very high, in order to make maximum profit, a for-profit certifier, by charging a very high, fee creates maximum vertical differentiation between firms. Above this threshold value, that is, when the competition between firms becomes low, both firms gain sufficient monopoly power over their respective market shares. This compels the certifier to reduce its fee significantly and both firms produce the highest quality and opt for certification. Under a non-profit certifier, this asymmetric equilibrium vanishes. In this case, only the symmetric equilibrium exists where both firms produce the highest quality and opt for certification, which is also the most efficient. The asymmetric equilibrium arising under a for-profit certifier makes the market inefficient, and therefore, it is socially optimal to have a non-profit certifier rather than a for-profit certifier in such a market. This could be viewed as one of the possible reasons that we find mostly non-profit certifiers operating in such markets.

The for-profit certifier always certifies the highest quality because of the assumption that surplus generated by quality is strictly increasing with quality. If the surplus function is non-monotonic, it will certify that quality level which generates maximum surplus. In line with the literature, I assume that the surplus function is strictly increasing in quality, and under this assumption the for-profit certifier always certifies the highest quality only, which makes it easier to compare with a non-profit certifier case. Under both types of certifier, only the highest quality products are certified. It is only through the certification fee that the for-profit certifier can create an asymmetry in the market.

Future research could introduce competition between for-profit certifiers. It would be interesting to see if different certifiers certify different quality levels, but it seems very likely that all the for-profit certifiers will certify the highest quality only, but due to Bertrand competition the certification fee will be zero, which is similar to one of the results in Lizzeri (1999). Another possible future development of this work could be to introduce heterogeneity among consumers in terms of their valuations for qualities, and study its effects.

Appendix

Proof of Proposition 1: Both firms produce $\bar{\theta}$ and opt for certification. Let a consumer located at point $\hat{d}$ be indifferent between buying from firm 1 and firm 2. Then,

$$V(\bar{\theta}) - P_1 - t\hat{d}^2 = V(\bar{\theta}) - P_2 - t(1-\hat{d})^2 \Longleftrightarrow \hat{d} = \frac{P_2 - P_1 + t}{2t}.$$

The demand faced by firm 1 is given by $D_1 = \frac{P_2 - P_1 + t}{2t}$ and that of firm 2 is $D_2 = 1 - \hat{d} = \frac{P_1 - P_2 + t}{2t}$. Profit functions of firms 1 and 2 are $\pi_1 = [P_1 - c(\bar{\theta})][\frac{P_2 - P_1 + t}{2t}]$ and $\pi_2 = [P_2 - c(\bar{\theta})][\frac{P_1 - P_2 + t}{2t}]$ respectively. Therefore, firm 1 will maximize π_1 with respect to P_1 and firm 2 will maximize π_2 with respect to P_2. The two first order conditions and the symmetry yield $P_1^* = P_2^* = c(\bar{\theta}) + t$. Each of the firms faces a demand equal to $\frac{1}{2}$ and earns a profit of $\frac{t}{2}$ each.

Most profitable deviation: Suppose either of the firms (say firm 1) deviates by producing quality 0 and not applying for certification. Imposing $\theta^e_{1,r} = 0$ and $\theta^e_1 = 0$, the indifferent consumer is given by $\hat{d} = \frac{P_2 - P_1^D + t - V(\bar{\theta}) + V(0)}{2t}$. Therefore, the demand faced by firm 1 is $D_1 = \frac{P_2 - P_1^D + t - V(\bar{\theta}) + V(0)}{2t}$ and that of firm 2 is $D_2 = 1 - \hat{d} = \frac{P_1^D - P_2 + t + V(\bar{\theta}) - V(0)}{2t}$. The profit functions of firms 1 and 2 will be $\pi_1^D = [P_1^D - c(0)][\frac{P_2 - P_1^D + t - V(\bar{\theta}) + V(0)}{2t}]$ and $\pi_2 = [P_2 - c(\bar{\theta})][\frac{P_1^D - P_2 + t + V(\bar{\theta}) - V(0)}{2t}]$ respectively. Solving for P_1^D and P_2 from the two first order conditions I obtain $P_1^D = \frac{2c(0) + c(\bar{\theta}) + 3t + V(0) - V(\bar{\theta})}{3}$ and $P_2 = \frac{2c(\bar{\theta}) + c(0) + 3t - V(0) + V(\bar{\theta})}{3}$. Substituting P_1^D and P_2 in D_1 and D_2, I obtain $D_1 = \frac{(3t-K)}{6t}$ and $D_2 = \frac{(3t+K)}{6t}$. The deviational profit for firm 1 will be $\pi_1^D = \frac{(3t-K)^2}{18t}$. Rest of the proof is given in section 4.

Proof of Proposition 2: Both firms produce θ and opt for certification. Let a consumer located at point $\hat{d}$ be indifferent between buying from firm 1 and firm 2. Then,

$$V(\theta) - P_1 - t\hat{d}^2 = V(\theta) - P_2 - t(1-\hat{d})^2 \iff \hat{d} = \frac{P_2 - P_1 + t}{2t}.$$

The demand faced by firm 1 is given by $D_1 = \frac{P_2 - P_1 + t}{2t}$ and that of firm 2 is $D_2 = 1 - \hat{d} = \frac{P_1 - P_2 + t}{2t}$. The profit functions of firms 1 and 2 are $\pi_1 = [P_1 - c(\theta)][\frac{P_2 - P_1 + t}{2t}] - F_s$ and $\pi_2 = [P_2 - c(\theta)][\frac{P_1 - P_2 + t}{2t}] - F_s$. Maximizing π_1 and π_2 with respect to P_1 and P_2 and imposing symmetry yields $P_1^* = P_2^* = c(\theta) + t$. Each of the firms faces a demand equal to $\frac{1}{2}$ and earns a profit of $\frac{t}{2} - F_s$ each.

Most profitable deviation: Suppose firm 1 deviates by producing quality 0 and not applying for certification. Imposing $\theta^e_{1,r} = 0$ and $\theta^e_1 = 0$, the indifferent consumer is given by $\hat{d} = \frac{P_2 - P_1^D + t - V(\theta) + V(0)}{2t}$. Therefore, the demand faced by firm 1 is $D_1 = \frac{P_2 - P_1^D + t - V(\theta) + V(0)}{2t}$ and that of firm 2 is $D_2 = 1 - \hat{d} = \frac{P_1^D - P_2 + t + V(\theta) - V(0)}{2t}$. The profit functions of firms 1 and 2 will be $\pi_1^D = [P_1^D - c(0)][\frac{P_2 - P_1^D + t - V(\theta) + V(0)}{2t}]$ and $\pi_2 = [P_2 - c(\theta)][\frac{P_1^D - P_2 + t + V(\theta) - V(0)}{2t}] - F_s$ respectively. Solving for P_1^D and P_2 from the two first order conditions, I obtain $P_1^D = \frac{2c(0) + c(\theta) + 3t + V(0) - V(\theta)}{3}$ and $P_2 = \frac{2c(\theta) + c(0) + 3t - V(0) + V(\theta)}{3}$. The deviational profit for firm 1 will be $\pi_1^D = \frac{(3t + [V(0) - c(0)] - [V(\theta) - c(\theta)])^2}{18t}$. This deviation will not be profitable for firm 1 if

$$\frac{t}{2} - F_s \geq \frac{(3t + [V(0) - c(0)] - [V(\theta) - c(\theta)])^2}{18t}$$

$$\iff F_s \leq \frac{t}{2} - \frac{(3t + [V(0) - c(0)] - [V(\theta) - c(\theta)])^2}{18t}$$

$$\iff F_s \leq \frac{([V(\theta) - c(\theta)] - [V(0) - c(0)])(6t - ([V(\theta) - c(\theta)] - [V(0) - c(0)]))}{18t}$$

$$= \frac{w(\theta)(6t - w(\theta))}{18t}.$$

If F_s satisfies above inequality, no firm has an incentive to deviate, which completes the proof.

Proof of Proposition 3: Suppose one firm (say firm 2) produces quality θ and applies for certification and the other (firm 1) does not apply for certification. As firm 1 does not opt for certification, its quality is not observed by firm 2 and by consumers. Therefore, firm 1's best possible quality choice would be to produce quality 0. Firm 1 and firm 2 set prices P_1 and P_2 respectively. Let a consumer located at point $\hat{d}$ be indifferent between buying from firm 1 and firm 2. Then,

$$V(0) - P_1 - t\hat{d}^2 = V(\theta) - P_2 - t(1 - \hat{d})^2 \iff \hat{d} = \frac{P_2 - P_1 + t - V(\theta) + V(0)}{2t}.$$

The demand faced by firm 1 is given by $D_1 = \frac{P_2 - P_1 + t - V(\theta) + V(0)}{2t}$ and that of firm

2 is $D_2 = 1 - \hat{d} = \frac{P_2 - P_1 + t + V(\theta) - V(0)}{2t}$. The profit functions of firms 1 and 2 are $\pi_1 = [P_1 - c(0)][\frac{P_2 - P_1 + t - V(\theta) + V(0)}{2t}]$ and $\pi_2 = [P_2 - c(\theta)][\frac{P_2 - P_1 + t + V(\theta) - V(0)}{2t}] - F_a$ respectively. Maximizing π_1 and π_2 with respect to P_1 and P_2 respectively, I obtain $P_1^* = \frac{2c(0)+c(\theta)+3t+V(0)-V(\theta)}{3}$ and $P_2^* = \frac{2c(\theta)+c(0)+3t-V(0)+V(\theta)}{3}$. Hence, the profits will be $\pi_1^* = \frac{(3t+[V(0)-c(0)]-[V(\theta)-c(\theta)])^2}{18t}$ and
$\pi_2^* = \frac{(3t+[V(\theta)-c(\theta)]-[V(0)-c(0)])^2}{18t} - F_a$.

Most profitable deviation for the certified firm: Suppose firm 2 deviates by producing quality 0 and not applying for certification. Then both the firms produce quality 0 and do not apply for certification. Imposing the belief of each firm regarding the quality choice of its rival and that of consumers, the deviational price will be $P_2^D = c(0) + t$, each of the firms will face a demand equal to $\frac{1}{2}$, and $\pi_2^D = \frac{t}{2}$. To have this deviation not profitable for firm 2, we must have

$$\frac{(3t + [V(\theta) - c(\theta)] - [V(0) - c(0)])^2}{18t} - F_a \geq \frac{t}{2}$$

$$\iff F_a \leq \frac{([V(\theta) - c(\theta)] - [V(0) - c(0)])(6t + [V(\theta) - c(\theta)] - [V(0) - c(0)])}{18t}. \tag{2.5}$$

Most profitable deviation for the non-certified firm: Suppose firm 1 deviates by producing quality θ and applying for certification. In this case, both the firms produce quality θ and apply for certification. So each of the firms will charge prices equal to $c(\theta) + t$, each facing a demand equal to $\frac{1}{2}$, and will earn a profit of $\frac{t}{2} - F_a$. This deviation will not be profitable for firm 1 if

$$\frac{(3t + [V(0) - c(0)] - [V(\theta) - c(\theta)])^2}{18t} \geq \frac{t}{2} - F_a$$

$$\iff F_a \geq \frac{([V(\theta) - c(\theta)] - [V(0) - c(0)])(6t - ([V(\theta) - c(\theta)] - [V(0) - c(0)]))}{18t}. \tag{2.6}$$

Combining conditions (4.5) & (4.6), I obtain

$$\frac{([V(\theta)-c(\theta)]-[V(0)-c(0)])(6t-([V(\theta)-c(\theta)]-[V(0)-c(0)]))}{18t} \leq F_a$$

$$\leq \frac{([V(\theta)-c(\theta)]-[V(0)-c(0)])(6t+[V(\theta)-c(\theta)]-[V(0)-c(0)])}{18t}.$$

If F_a satisfies the above inequalities, neither firm has an incentive to deviate, which completes the proof.

Maximization problem for the for-profit certifier: The certifier announces a certification policy which states that it will certify qualities θ_1 and θ_2 with $0 \leq \theta_1 \leq \theta_2 \leq \bar{\theta}$, and to apply for certification, a firm has to pay fees F_1(for quality θ_1) and F_2 (for quality θ_2), with $F_1 \leq F_2$. Given the certification policy, suppose one firm (say firm 1) produces quality θ_1 and applies for certification by paying the fee F_1, whereas the other firm (firm 2) produces quality θ_2 and applies for certification by paying the fee F_2. Let a consumer located at point $\hat{d}$ be indifferent between buying from firm 1 and firm 2. Then,

$$V(\theta_1)-P_1-t\hat{d}^2 = V(\theta_2)-P_2-t(1-\hat{d})^2 \Longleftrightarrow \hat{d} = \frac{P_2-P_1+t-V(\theta_2)+V(\theta_1)}{2t}.$$

The demands of firm 1 and 2 are given by $D_1 = \frac{P_2-P_1+t-V(\theta_2)+V(\theta_1)}{2t}$ and $D_2 = \frac{P_1-P_2+t+V(\theta_2)-V(\theta_1)}{2t}$ respectively. Therefore, their profits will be $\pi_1 = [P_1 - c(\theta_1)][\frac{P_2-P_1+t-V(\theta_2)+V(\theta_1)}{2t}] - F_1$ and $\pi_2 = [P_2 - c(\theta_2)][\frac{P_1-P_2+t+V(\theta_2)-V(\theta_1)}{2t}] - F_2$ respectively. Maximizing π_1 and π_2 with respect to P_1 and P_2 respectively, I obtain $P_1^* = \frac{2c(\theta_1)+c(\theta_2)+3t+V(\theta_1)-V(\theta_2)}{3}$ and $P_2^* = \frac{2c(\theta_2)+c(\theta_1)+3t-V(\theta_1)+V(\theta_2)}{3}$. Hence the profits are $\pi_1^* = \frac{(3t+[V(\theta_1)-c(\theta_1)]-[V(\theta_2)-c(\theta_2)])^2}{18t} - F_1$ and
$\pi_2^* = \frac{(3t+[V(\theta_2)-c(\theta_2)]-[V(\theta_1)-c(\theta_1)])^2}{18t} - F_2$.

Most profitable deviations for firm 1: Suppose firm 1 deviates by producing quality θ_2 and applying for certification. In that case, the profits of firm 1 and 2 will be $\pi_1^D = [P_1^D - c(\theta_2)][\frac{P_2 - P_1^D + t}{2t}] - F_2$ and $\pi_2 = [P_2 - c(\theta_2)][\frac{P_1^D - P_2 + t}{2t}] - F_2$. Firm 1 will maximize π_1^D with respect to P_1^D and firm 2 will maximize π_2 with respect to P_2. The two first order conditions and the symmetry yield $P_1^D = P_2 = c(\theta_2) + t$. Each of the firms faces a demand equal to $\frac{1}{2}$ and earns a profit of $\frac{t}{2} - F_2$ each. This deviation will not be profitable for firm 1 if

$$\frac{(3t + [V(\theta_1) - c(\theta_1)] - [V(\theta_2) - c(\theta_2)])^2}{18t} - F_1 \geq \frac{t}{2} - F_2$$

$$\Longleftrightarrow \frac{(3t + S(\theta_1) - S(\theta_2))^2}{18t} - F_1 \geq \frac{t}{2} - F_2. \tag{2.7}$$

Suppose firm 1 deviates by producing quality 0 and not applying for certification. Imposing $\theta_{1,2}^e = 0$ and $\theta_1^e = 0$, the profit functions of firms 1 and 2 are $\pi_1^D = [P_1 - c(0)][\frac{P_2 - P_1^D + t - V(\theta_2) + V(0)}{2t}]$ and $\pi_2 = [P_2 - c(\theta_2)][\frac{P_1^D - P_2 + t + V(\theta_2) - V(0)}{2t}] - F_2$. Maximizing π_1^D and π_2 with respect to P_1^D and P_2 respectively, I obtain $P_1^D = \frac{2c(0) + c(\theta_2) + 3t + V(0) - V(\theta_2)}{3}$ and $P_2 = \frac{2c(\theta_2) + c(0) + 3t - V(0) + V(\theta_2)}{3}$. The deviational profit for firm 1 will be $\pi_1^D = \frac{(3t + [V(0) - c(0)] - [V(\theta_2) - c(\theta_2)])^2}{18t}$. This deviation not profitable for firm 1, we must have

$$\frac{(3t + [V(\theta_1) - c(\theta_1)] - [V(\theta_2) - c(\theta_2)])^2}{18t} - F_1 \geq \frac{(3t + [V(0) - c(0)] - [V(\theta_2) - c(\theta_2)])^2}{18t}$$

$$\Longleftrightarrow F_1 \leq \frac{(S(\theta_1) - S(0))(6t + S(\theta_1) + S(0) - 2S(\theta_2))}{18t} = f_1(\theta_1, \theta_2). \tag{2.8}$$

Most profitable deviations for firm 2: Similarly, it will not be profitable for firm 2 to deviate by producing quality θ_1 and to apply for certification if

$$\frac{(3t + S(\theta_2) - S(\theta_1))^2}{18t} - F_2 \geq \frac{t}{2} - F_1. \tag{2.9}$$

Imposing similar out of equilibrium beliefs ($\theta^e_{2,1} = \theta^e_2 = 0$), the deviation to produce quality 0 and not apply for certification will not be profitable for firm 2 if

$$F_2 \leq \frac{(S(\theta_2) - S(0))(6t + S(\theta_2) + S(0) - 2S(\theta_1))}{18t} = f_2(\theta_1,\, \theta_2). \tag{2.10}$$

The certifier wants to

$$\underset{\theta_1,\,\theta_2}{maximize}\,(f_1(\theta_1,\, \theta_2) + f_2(\theta_1,\, \theta_2))$$

subject to the following conditions

$$\frac{(3t + S(\theta_1) - S(\theta_2))^2}{18t} - f_1(\theta_1,\, \theta_2) - \frac{t}{2} + f_2(\theta_1,\, \theta_2) \geq 0,$$

$$\frac{(3t + S(\theta_2) - S(\theta_1))^2}{18t} - f_2(\theta_1,\, \theta_2) - \frac{t}{2} + f_1(\theta_1,\, \theta_2) \geq 0,$$

$$f_2(\theta_1,\, \theta_2) - f_1(\theta_1,\, \theta_2) \geq 0,$$

$$0 \leq \theta_1,\, \theta_2 \leq \bar{\theta},$$

$$\theta_2 - \theta_1 \geq 0.$$

Let us call the above maximization problem $M1$. Now maximizing $f_1(\theta_1,\, \theta_2) + f_2(\theta_1,\, \theta_2)$ with respect to θ_1 and θ_2 subject to the above five constraints is equivalent to maximizing $18t(f_1(\theta_1,\, \theta_2) + f_2(\theta_1,\, \theta_2))$ with respect to θ_1 and θ_2 subject to the above five constraints. Now consider the following maximization problem $M2$:

$$\underset{\theta_1,\,\theta_2}{maximize}\,18t(f_1(\theta_1,\, \theta_2) + f_2(\theta_1,\, \theta_2))$$

subject to $\theta_1 \geq 0$, $\theta_1 \leq \bar{\theta}$, $\theta_2 \leq \bar{\theta}$, $\theta_1 \leq \theta_2$.

Note that, due to having fewer constraints, the maximum of $M2$ will be less than or equal to the maximum of $M1$.

Let us define the following:

$x(\theta_1) = [V(\theta_1) - c(\theta_1)] - [V(0) - c(0)]$, and

$y(\theta_2) = [V(\theta_2) - c(\theta_2)] - [V(0) - c(0)]$.

Clearly $x(\theta_1)$ is increasing in θ_1 and $y(\theta_2)$ is increasing in θ_2, and $x(\theta_1),\ y(\theta_2) \in [0,\ K]$. I denote $x(\theta_1)$ as x and $y(\theta_2)$ as y. Therefore $M2$ is equivalent to the following maximization problem $M3$:

$$\underset{x,\,y}{maximize}\; x(6t + x - 2y) + y(6t + y - 2x)$$

subject to $x \geq 0,\ x \leq K,\ y \leq K,\ x \leq y$.

I solve $M3$ using Kuhn-Tucker conditions. The Lagrangian is

$L(x,\ y,\ \mu_1,\ \mu_2,\ \mu_3,\ \mu_4) = x(6t + x - 2y) + y(6t + y - 2x) + \mu_1 x + \mu_2(K - x) + \mu_3(K - y) + \mu_4(y - x)$,

which gives the following optimality conditions:

$6t - 4y + 2x + \mu_1 - \mu_2 - \mu_4 = 0,\ 6t - 4x + 2y - \mu_3 + \mu_4 = 0,\ \mu_1 x = 0,$
$\mu_2(K - x) = 0,\ \mu_3(K - y) = 0,\ \mu_4(y - x) = 0,\ x \geq 0,\ K - x \geq 0,$
$K - y \geq 0,\ y - x \geq 0,\ \mu_1,\ \mu_2,\ \mu_3,\ \mu_4 \geq 0.$

Since there are four complementarity conditions, I need to consider sixteen cases:

1. $\mu_1,\ \mu_2,\ \mu_3,\ \mu_4 = 0:$ gives $x = 3t,\ y = 3t$, provided $K \geq 3t$ and the maximum value is $18t^2$.

2. $\mu_1 \neq 0 \Rightarrow x = 0,\ \mu_2,\ \mu_3,\ \mu_4 = 0:$ gives $y = -3t$, not feasible.

3. $\mu_1 \neq 0 \Rightarrow x = 0,\ \mu_2 \neq 0 \Rightarrow x = K$, not possible. So I rule out all four cases where $\mu_1,\ \mu_2 \neq 0$.

4. $\mu_1 \neq 0 \Rightarrow x = 0,\ \mu_2 = 0,\ \mu_3 \neq 0 \Rightarrow y = K,\ \mu_4 \neq 0 \Rightarrow y = x$, not possible.

5. $\mu_1 \neq 0 \Rightarrow x = 0,\ \mu_2 = 0,\ \mu_3 \neq 0 \Rightarrow y = K,\ \mu_4 = 0:$ gives $\mu_1 = 4K - 6t > 0 \Rightarrow K > \frac{3t}{2},\ \mu_3 = 6t{+}2K > 0$. The maximum value is $K(6t{+}K) = 6tK{+}K^2$.

6. $\mu_1 \neq 0 \Rightarrow x = 0,\ \mu_2 = 0,\ \mu_3 = 0,\ \mu_4 \neq 0 \Rightarrow y = x = 0:$ gives $\mu_1 = -12t$ and $\mu_4 = -6t$, not feasible.

7. $\mu_1 = 0$, $\mu_2 \neq 0 \Rightarrow x = K$, μ_3, $\mu_4 = 0$: gives $y = 2K - 3t \geq 0 \Rightarrow K \geq \frac{3t}{2}$, $\mu_2 = 18t - 6k > 0 \Rightarrow K < 3t$, $y - x \geq 0 \Rightarrow K \geq 3t$, not possible as the last two conditions cannot hold simultaneously.

8. μ_1, $\mu_2 = 0$, $\mu_3 \neq 0 \Rightarrow y = K$, $\mu_4 = 0$: gives $x = 2K - 3t \geq 0 \Rightarrow K \geq \frac{3t}{2}$, $\mu_3 = 18t - 6K > 0 \Rightarrow K < 3t$, $y - x \geq 0 \Rightarrow K \leq 3t$. Combining together I obtain, for $\frac{3t}{2} \leq K < 3t$, $x = 2K - 3t$, $y = K$, and the maximum value is $18Kt - 9t^2 - 3K^2$.

9. μ_1, μ_2, $\mu_3 = 0$, $\mu_4 \neq 0 \Rightarrow y = x$: gives $x = y = 3t$, but $\mu_4 = 0$, which contradicts $\mu_4 \neq 0$. So this is not possible.

10. $\mu_1 = 0$, $\mu_2 \neq 0 \Rightarrow x = K$, $\mu_3 \neq 0 \Rightarrow y = K$, $\mu_4 = 0$: gives $\mu_2 = \mu_3 = 6t - 2K > 0 \Rightarrow K < 3t$. The maximum value is $2K(6t - K) = 12Kt - 2K^2$.

11. $\mu_1 = 0$, $\mu_2 \neq 0 \Rightarrow x = K$, $\mu_3 = 0$, $\mu_4 \neq 0 \Rightarrow y = x = K$: gives $\mu_2 = 6t - 2K > 0 \Rightarrow K < 3t$, $\mu_3 = 2K - 6t > 0 \Rightarrow K > 3t$, not possible simultaneously.

12. $\mu_1 = 0$, $\mu_2 \neq 0 \Rightarrow x = K$, $\mu_3 \neq 0 \Rightarrow y = K$, $\mu_4 \neq 0 \Rightarrow y = x = K$: gives $\mu_2 + \mu_3 = 12t - 4K > 0 \Rightarrow K < 3t$ and $\mu_4 = 6t - 2K - \mu_2$.There are many possible values of μ_2, μ_3 and μ_4 which satisfy the two conditions, one of which is $\mu_2 = 3t - K$, $\mu_3 = 9t - 3K$, $\mu_4 = 3t - K$. The maximum value is $2K(6t - K) = 12Kt - 2K^2$.

13. μ_1, $\mu_2 = 0$, $\mu_3 \neq 0 \Rightarrow y = K$, $\mu_4 \neq 0 \Rightarrow y = x = K$: gives $\mu_4 = 6t - 2K > 0 \Rightarrow K < 3t$, $\mu_3 = 12t - 4K > 0 \Rightarrow K < 3t$. The maximum value is $2K(6t - K) = 12Kt - 2K^2$.

Combining all the above, I obtain

for $K \geq 3t$, $x = y = 3t$, maximum value $18t^2$,

for $K > \frac{3t}{2}$, $x = 0$, $y = K$, maximum value $6tK + K^2$,

for $\frac{3t}{2} \leq K < 3t$, $x = 2K - 3t$, $y = K$, maximum value $18Kt - 9t^2 - 3K^2$, for $K < 3t$, $x = y = K$, maximum value $12Kt - 2K^2$.

Now it is easy to check that $6tK + K^2 > 18Kt - 9t^2 - 3K^2$ and $12Kt - 2K^2 > 18Kt - 9t^2 - 3K^2$.

As $6tK + K^2 > 12Kt - 2K^2 \iff K > 2t$,

for $K < 2t$, $x = y = K$, is optimal.

And as $6tK + K^2 > 18t^2 \iff K > 3t(\sqrt{3} - 1)$, which is smaller than $3t$,

for $K \geq 2t, x = 0$, $y = K$, is optimal.

Therefore the optimal solution of the $M3$ is:

for $K < 2t$, $x = y = K$, the maximum value is $12Kt - 2K^2$,

for $K \geq 2t$, $x = 0$, $y = K$, the maximum value is $6tK + K^2$.

Therefore the optimal solution of $M2$ is :

for $K < 2t$, $\hat{\theta}_1 = \hat{\theta}_2 = \bar{\theta}$, the maximum value is $12Kt - 2K^2$,

for $K \geq 2t$, $\hat{\theta}_1 = 0$, $\hat{\theta}_2 = \bar{\theta}$, the maximum value is $6tK + K^2$.

It can be verified that the solution for $M2$ also satisfies all the constraints for $M1$. Therefore the optimal solution of $M1$ is

for $t > \frac{K}{2}$, $\hat{\theta}_1 = \hat{\theta}_2 = \bar{\theta}$, $f_1\widehat{(\theta_1, \theta_2)} = f_2\widehat{(\theta_1, \theta_2)} = \frac{6Kt - K^2}{18t}$, the maximum value is $\frac{12Kt - 2K^2}{18t}$,

for $t \leq \frac{K}{2}$, $\hat{\theta}_1 = 0$, $\hat{\theta}_2 = \bar{\theta}$, $f_1\widehat{(\theta_1, \theta_2)} = 0$, $f_2\widehat{(\theta_1, \theta_2)} = \frac{6tK + K^2}{18t}$, the maximum value is $\frac{6tK + K^2}{18t}$.

3 To Sell or Provide a Subscription?

3.1 Introduction

Selling and providing subscriptions are phenomena seen across various durable goods markets. When a firm sells its product, consumers buy and use it for a longer period of time. Subscriptions are time specific, i.e., monthly, annual, etc. When a firm offers a subscription for its product, consumers buy the subscription and use the product for a limited period of time. As soon as the subscription period expires, consumers need to subscribe again in order to use the product in the next period. Traditionally, most firms would sell their products. However, in recent times, more and more firms are offering subscriptions. For example, most cloud service providers, software companies, and entertainment platforms such as Netflix, Amazon, etc., are offering subscriptions for their products. For the last ten years, even Microsoft has been offering subscriptions for its Microsoft Office suite. However, selling is still prevalent in many markets. For example, Microsoft still sells its Windows operating system.

We know from the existing durable good monopolist literature that a monopolist would prefer leasing (or renting) to selling. This is due to the Coase conjecture. If a monopolist sells the same product in two consecutive periods, it will not be able to commit credibly to setting the same price for both periods. Having sold the product in the first period to some consumers, the monopolist has an incentive to lower the price in the second period in order to sell the product to the remaining consumers who did not buy in the first period. Rational consumers can foresee this behavior, and will wait in order to buy in the second period at a lower price. However, a monopolist does not face this problem if it leases its products.

Nonetheless, leasing has its own problems, e.g., moral hazard. When consumers opt for leasing, they often do not pay attention to proper maintenance of the leased products, as after the leasing period the firm takes them back. However, the above issue of moral hazard does not arise in technological markets. There are several ways of evading the Coase problem, e.g., the introduction of new consumers in each period, having a third party arbitrator, or when the monopolist wants to maintain its reputation or offering a money-back guarantee scheme, etc. All these ways ensure that the monopolist can credibly commit to a fixed price and that it has no incentive to lower its price in the second period.

In the above context of durable goods, the monopolist offers the same product in both periods. However, one important aspect, the quality of the product, has not been taken into account. Usually, over time a monopolist introduces a new product in the market which is of better quality than the previous one, although the extent of such a quality increment can be small or large, and its exact level is difficult to measure. Over time, Microsoft has introduced and sold upgraded versions for its Windows operating systems. In the markets where subscription is more common, e.g., with most software products, cloud services, etc., we also see newer products with better quality over time. To the best of my knowledge, this chapter is the first of its kind to take into consideration the aspect of product quality that is offered in the future periods in a durable good monopoly framework, and it seeks to answer the following question — for a given level of quality improvement for the future product, will a monopolist sell or offer a subscription?

To answer the above question, I have taken a model where a monopolist offers a product with a certain quality level in the first period and one of better quality by incuring an additional cost in the second period. Consumers are heterogeneous in how they value quality of the product. The quality levels are exogenously given and consumers know these quality levels at the beginning of the first period. As by offering a subscription, the firm can charge respective monopoly prices in each period, it may seem that the firm will opt to offer a subscription rather than selling its products. Interestingly, I will show that this is not always the case. The level of quality improvement between two periods and the associated cost for this quality improvement play a pivotal role in a firm's pricing strategy. When the quality improvement is small (I term this small innovation), I find that the monopolist will choose to offer a subscription over selling, provided the cost of such an innovation is not very high. However, when the quality improvement is very big (I term this large innovation), I find that the firm will choose to sell rather than to offer a subscription, provided the cost of this innovation is sufficiently high.

In the large innovation case, when the firm decides to sell, some consumers buy in the first period. Out of these, low valuation consumers will keep consuming the product in the next period, whereas the remaining consumers who value higher quality more will buy in the next period. On the one hand, as consumers who buy a product in the first period can also keep consuming it in the next period, this reduces the demand for the product with higher quality in the next period. This reduction in demand in the next period puts pressure on the firm to charge a much lower price in the next period as compared to the respective subscription price. On the other hand, the firm exploits the consumers' higher willingness to pay in the first period as they can consume the product in the next period, and thus charges a very high price in the first period. In both subscription and selling, the firm experiences the same demand in the first period, but the selling price in the first period is very high as compared to the respective subscription price. As a result, the profit earned by the firm in the first period from selling becomes so high that it compensates for the lost profit (due to the lower demand and lower price in the next period) in the next period as compared to the subscription.

In the small innovation case, when the firm decides to sell, there is a screening of consumers over time, that is, some consumers buy only in the first period, some consumers buy only in the second period and some consumers buy in both periods. As in the large innovation case, the firm's profit in the first period from selling is higher than its first period subscription profit. However, through subscription the firm earns a respective monopoly profit in the second period. For a low cost of innovation, the firm not only sells to high valuation consumers who buy in both periods but also to some low valuation consumers who buy only in the second period. This significantly drives down the second period selling price as compared to the second period subscription price. For a low cost of innovation, the second period profit of the firm from subscription is high compared to the profit from selling such that it compensates for the lost profit from the first period. As a result, the firm chooses to offer a subscription.

When the cost of innovation becomes sufficiently high, the firm's second period profit from subscription decreases and the firm chooses to sell as it can earn a significantly high profit in the first period from selling. Nonetheless, the firm will choose to sell in the case of a large innovation with a high associated cost, where its profit in the second period is smaller than its profit from subscription. In the case of a small innovation with a small associated cost, the firm chooses subscription over selling and its second period profit from subscription is higher than selling. Hence, one can argue that subscription gives a firm an incentive to look for smaller innovations with a small associated cost. This is probably one of the reasons why more and more firms are opting to offer subscriptions for their products where the levels of quality improvements in their future products are relatively small.

Coase (1972) argues that a durable good monopolist is unable to exercise its monopoly power due to its incentive to exploit residual demand in subsequent periods. If a monopolist sells the same product in consecutive periods, then it is unable to commit credibly to set the same price for both periods. For any price it sets in the first period, only those consumers will buy in the first period whose willingness to pay is higher than the price that is set by the monopolist. However, in the second period, the monopolist has a strict incentive to lower the price in order to sell to some of the consumers who did not buy in the first period. Rational consumers can foresee this behavior of the monopolist in the second period. As a result, none of the consumers will buy in the first period, preferring to buy in the second period at a much lower price. Schmalensee (1979) provides a comprehensive literature survey on durable good monopolists. However, in the case of leasing or providing subscriptions, the monopolist does not face this issue of credibility, and can charge the same monopoly price in each period. Bulow (1982) shows that if there are no obstacles to renting, a monopolist will earn a higher profit through renting than selling due to its inability to announce credibly in the first period that it will not exploit the residual demand in the second period. I introduce the aspect of product quality into this framework and analyze how it influences the monopolist's choice of pricing strategy. In my model, for small innovations with a small associated cost, the monopolist will offer a subscription, whereas when the quality improvement is very high with a high associated cost, it will sell.

Waldman (1996) analyzes the interaction between a monopolist's choice of price and the durability of a product in a setting where the choice of durability controls the speed with which the quality of the product deteriorates. In such a set up, he finds that the price depends on the one in the second hand market. He also finds that due to the linkage between prices for old and new units, the durability stays below the socially optimal level and the monopolist has an incentive to reduce durability in order to eliminate the second hand market. Bond and Samuelson (1984) explore the interaction between replacement sales and the depreciation of goods. Bulow (1986) analyzes the issue of 'planned obsolescence' in the context of monopoly and oligopoly. In this chapter I do not consider secondary markets, as their presence is less common in the technological market. In my model, the only way the monopolist reduces the durability of the product offered in the first period is by offering a subscription.

The chapter is organized as follows: In the next section I will describe the model, and in section 3 I will analyze the model. I will start with subscription-based pricing followed by the large innovation and small innovation cases respectively. The final section concludes.

3.2 Model

Consider a market where a monopolist firm sells products in two periods ($t = 0$ and $t = 1$). In period 0, the firm sells a product of quality q_0 and in period 1 it sells a product of quality q_1, where $q_1 > q_0 > 0$. The marginal costs of producing qualities q_0 and q_1 are 0 and c respectively, where $q_1 > c > 0$. The market consists of a unit mass of heterogenous consumers. Heterogeneity is considered in terms of how consumers value quality. In each period, a consumer with quality premium parameter θ is willing to pay θq_0 for quality q_0 and θq_1 for quality q_1 where θ follows some cumulative distribution function F. For simplicity, I assume that $\theta \sim U(0, 1)$ which gives a nice linear form of the demand function. Also, by assuming uniformity, I omit the possibility that the results of this chapter are dependent on whether there is a larger mass of consumers who value high quality more (or less) than the others. All consumers and the firm have the same discount factor δ where $\delta \in (0, 1)$. In each period, a consumer derives utility by consuming one type of product at most, that is, either a product with quality q_0 or one with quality q_1. In principle, one can assume different discount factors for consumers and the firm, but that would make the model complicated. If both the firm and consumers belong to the same economy, it is also reasonable to assume that they all have the same discount factor. There is no secondary market for the goods which are traded in this primary market.

In real markets, firms usually invest in order to produce better products in the future. The exact level of quality that is produced in the future period is not known ex-ante, but both the firm and the consumers know that the firm will produce a better quality product in the future. To keep the model simple, I assume that the quality of the product is exogenous where both the firm and the consumers know ex-ante the exact level of quality that is produced in the future period. Hence, there is no informational asymmetry with respect to product quality in this model.

3.2.1 Firm's pricing strategy

The firm can choose one of the following pricing strategies:

1. Subscription-based pricing: In this pricing strategy, the monopolist does not sell the products. Instead, it sells subscriptions for its products to consumers. More specifically, it sets subscription prices p_0 for quality q_0 in period 0 and p_1 for quality q_1 in period 1. By paying price p_0 in period 0, a consumer buys a subscription to use the product with quality q_0 for the period 0. In period 1, the consumer can no longer use this product. If he decides to use a product in period 1, he will have to pay p_1 to buy a subscription for the product with quality q_1 for the period 1.

2. Selling without any commitment for future price: In this case, at the beginning of each period, the monopolist sets prices for products in the respective periods. In period 0, it sells its products by setting prices p_0 for the product with quality q_0 and in period 1 it sets price p_1 for the product with quality q_1. By paying price p_0 in period 0, a consumer buys the product with quality q_0. A consumer, who buys in period 0, can also use the product in period 1. If a consumer decides to switch to the product with quality q_1, he pays p_1 and buys the product q_1 in period 1.

3.2.2 The game

1. At first, the monopolist decides whether to sell or provide a subscription.

2. Depending on its decision in stage 1, it sets a price p_0^* for the product with quality q_0 in period 0.

3. Consumers observe (p_0^*, q_0), form an expectation (p_1^e) about price p_1 for quality q_1 in period 1 and decide whether to buy/subscribe in period 0 or not.

4. In period 1, the monopolist sets a price p_1^* for the product with quality q_1.

5. Consumers observe p_1^* and decide whether to buy/subscribe in period 1 or not.

In equilibrium, it must be the case that $p_1^e = p_1^*$. The equilibrium concept I use in this game is sub-game perfection.

3.2.3 Utilities: subscription

The utility of a consumer (with the quality premium parameter θ) from buying a subscription in period 0 is $\theta q_0 - p_0$ and from buying a subscription in period 1 is $\theta q_1 - p_1$. A consumer will buy a subscription in a period if his utility of buying the subscription in the respective period is non-negative.

3.2.4 Utilities: selling

Let $u_0^e(\theta,\, p_0,\, p_1^e)$ denote the ex-ante expected utility of a consumer (θ) at the beginning of period 0 who buys only in period 0. Similarly, $u_1^e(\theta,\, p_0,\, p_1^e)$ and $u_{0,\,1}^e(\theta,\, p_0,\, p_1^e)$ are the ex-ante expected utilities of the consumer (θ) at the beginning of period 0 who buys only in period 1 and who buys in both periods respectively. For simplicity, I omit prices p_0 and p_1^e from the notations of utilities. u_0^e, u_1^e and $u_{0,\,1}^e$ are the respective ex-ante expected utilities of the consumer with $\theta = 1$. Hence,

$$u_0^e(\theta) = u_0(\theta) = (\theta q_0 - p_0) + \delta\theta q_0 = (1+\delta)\theta q_0 - p_0 \tag{3.1}$$

$$u_1^e(\theta) = \delta(\theta q_1 - p_1^e) \tag{3.2}$$

$$u_{0,\,1}^e(\theta) = (\theta q_0 - p_0) + \delta(\theta q_1 - p_1^e) = (q_0 + \delta q_1)\theta - (p_0 + \delta p_1^e) \tag{3.3}$$

If θ_0^* is the consumer who is indifferent between buying only in period 0 and not buying at all, then $u_0(\theta_0^*) = 0 \iff \theta_0^* = \frac{p_0}{(1+\delta)q_0}$.

Similarly, if θ_1^{e*} is the consumer who is indifferent between buying only in period 1 and not buying at all, then $u_1^e(\theta_1^{e*}) = 0 \iff \theta_1^{e*} = \frac{p_1^e}{q_1}$.

A consumer (θ) would weakly prefer to buy only in period 1 over buying in both periods if $u_1^e(\theta) \geq u_{0,1}^e(\theta) \iff \theta \leq \frac{p_0}{q_0}$. Similarly, a consumer (θ) would weakly prefer to buy only in period 0 over buying in both periods if $u_0(\theta) \geq u_{0,1}^e(\theta) \iff \theta \leq \frac{p_1^e}{q_1 - q_0}$.

Let m_0, m_1 and $m_{0,1}$ be the respective slopes of $u_0^e(\theta)$, $u_1^e(\theta)$ and $u_{0,1}^e(\theta)$. It is easy to check that for $\delta q_1 \geq (1+\delta)q_0$, $m_{0,1} = (q_0 + \delta q_1) > \delta q_1 = m_1 \geq (1+\delta)q_0 = m_0$, whereas for $\delta q_1 < (1+\delta)q_0$, $m_{0,1} = (q_0 + \delta q_1) > (1+\delta)q_0 = m_0 > \delta q_1 = m_1$.

The two figures below show examples of the ex-ante expected utilities in these two different cases, and the separation of consumer types in terms of their buying behavior.

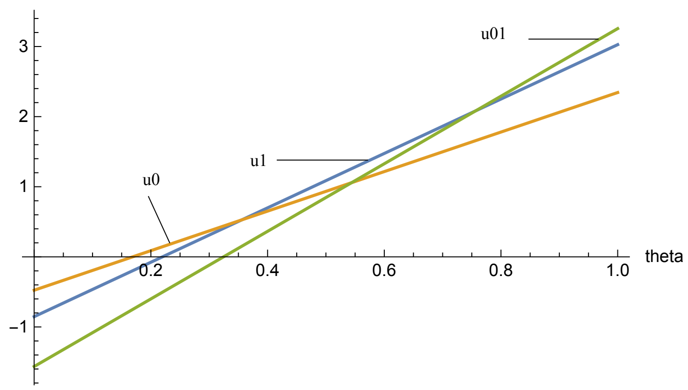

Figure 3.1: Ex-ante expected utilities for $\delta q_1 \geq (1+\delta)q_0$

In Figure 3.1, we see that $u_{0,1}^e(\theta)$ has the steepest slope, followed by $u_1^e(\theta)$ and $u_0^e(\theta)$ respectively. From the above figure, we see that the lowest valuation consumers do not buy. As the consumers' willingness to pay increases, there is a group of consumers who will buy only in period 0, followed by a group of higher valuation consumers who will buy only in period 1. The highest valuation consumers will buy in both periods.

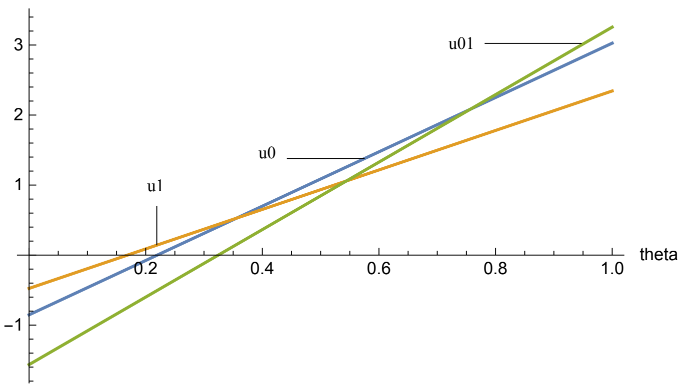

Figure 3.2: Ex-ante expected utilities for $\delta q_1 < (1+\delta)q_0$

Figure 3.2 shows that $u^e_{0,1}(\theta)$ has the steepest slope, followed by $u^e_0(\theta)$ and $u^e_1(\theta)$ respectively. Similar to the previous case, the lowest valuation consumers do not buy. As the consumers' willingness to pay increases, a group of consumers buy only in period 1, followed by a group of higher valuation consumers who buy only in period 0. The highest valuation consumers buy in both periods. It is worth mentioning that the above figures are just examples. Any changes in the discount factor or quality levels impact the slopes of the curves, whereas any change in prices results in parallel shifts of the respective utility curves. In equilibrium, the separation of consumer types can be completely different from those depicted here.

3.3 Analysis

In order to analyze the model, I first analyze the game starting from stage 2. First, I find the stage 2 equilibrium when the firm chooses subscription. Then I find the stage 2 equilibrium when the firm chooses to sell. Based on the stage 2 equilibria, I find the stage 1 equilibrium which is the equilibrium of the whole game.

3.3.1 Subscription-based pricing

Let θ_0^* be the consumer who is indifferent between buying the subscription in period 0 and not buying the subscription in period 0. Let θ_1^* be the consumer who is indifferent between buying the subscription in period 1 and not buying the subscription in period 1. Then, we must have

$$\theta_0^* q_0 - p_0 = 0 \Rightarrow \theta_0^* = \frac{p_0}{q_0}$$

and

$$\theta_1^* q_1 - p_1 = 0 \Rightarrow \theta_1^* = \frac{p_1}{q_1}.$$

The above two equations imply the following:

1. Consumers with $\theta \geq \theta_0^*$ will buy a subscription in period 0 and the other consumers will not buy.
2. Consumers with $\theta \geq \theta_1^*$ will buy a subscription in period 1 and the other consumers will not buy.

Hence, the profit function of the firm will be

$$\pi(p_0,\, p_1) = (1 - \theta_0^*)p_0 + \delta(1 - \theta_1^*)(p_1 - c)$$

$$\Longleftrightarrow \pi(p_0,\, p_1) = (1 - \frac{p_0}{q_0})p_0 + \delta(1 - \frac{p_1}{q_1})(p_1 - c). \tag{3.4}$$

Maximizing (3.4) with respect to p_0 and p_1 gives $p_0 = \frac{q_0}{2}$ and $p_1 = \frac{q_1+c}{2}$. Note that these two prices are the monopoly prices in the respective periods. The firm's profit by taking this subscription-based pricing strategy will be $\frac{q_0}{4} + \frac{\delta(q_1-c)^2}{4q_1}$.

3.3.2 Selling without any commitment on future price

In this case, I split the analysis into two cases. In the first, I consider $\delta q_1 \geq (1+\delta)q_0$, which I term large innovation and the second is $\delta q_1 < (1+\delta)q_0$, which I term small innovation. For each of these two cases, I will first find the stage 2 equilibrium, assuming that at stage 1 the monopolist decides to sell. Then I will find the stage 1 equilibrium or the equilibrium of the whole game by comparing the stage 2 equilibrium profit to the profit from subscription. In this case, the firm sells goods with qualities q_0 and q_1 in period 0 and period 1 respectively. It sets prices p_0 and p_1 for quality q_0 and q_1 at the beginning of period 0 and period 1 respectively. In period 0, the consumers observe price p_0 for quality q_0 and know that in the next period (period 1) they can buy quality q_1. In period 0, as consumers do not observe price p_1 for quality q_1, they form an expectation (p_1^e) about price p_1. Based on this actual information (q_0, p_0, q_1) and their expectation (p_1^e), consumers make their buying decisions at the beginning of period 0, that is, whether to buy only in period 0 or to buy only in period 1 or to buy in both periods or not to buy at all. They observe the actual price p_1 only at the beginning of period 1. In equilibrium, it must be the case that $p_1^e = p_1$.

Large innovation ($\delta q_1 \geq (1+\delta)q_0$)

In this case, the quality that is offered in period 1 is significantly higher than the quality that is offered in period 0 ($q_1 \gg q_0$). In particular, at the beginning of period 0, the present discounted value of the quality that is offered in the future period is higher than the present discounted value of the quality that is offered in period 0.

Lemma 3. ***In the case of large innovation, an equilibrium cannot consist of the consumers' behavior in which some consumers buy only in period 0 and some buy only in period 1.***

Proof. See Appendix. □

Following Lemma 3 and as the slopes are such that $m_{0,1} > m_1 > m_0$, the only option which can be part of the stage 2 equilibrium is where some consumers buy only in period 0, some consumers buy in both periods and the rest of the consumers do not buy at all. It is clear that all other options where some consumers buy only in period 0 and the rest of the consumers do not buy or some consumers buy only in period 1 and the rest of consumers do not buy cannot be part of an equilibrium. In the remaining option, of the consumers who buy in period 0, some will also keep on using the product with quality q_0 in period 1. Of the remaining consumers who value high quality q_1 more will switch and buy the new product in period 1. To make this consumer behavior part of an equilibrium of the game, we need to find prices (p_0^*, p_1^*) such that the prices support this consumer behavior and the firm has no incentive to deviate from these prices.

Let (p_0, p_1^e) be the prices such that some consumers buy only in period 0 and some consumers buy in both periods. Based on the utility functions, consumers with $\frac{p_0}{(1+\delta)q_0} \leq \theta \leq 1$ will buy in period 0 at price p_0. Of these consumers who have bought in period 0, consumers with $\frac{p_1^e}{q_1-q_0} \leq \theta \leq 1$ will also buy in period 1. In order to support this behavior of consumers, it must be the case that the intersection point between utility curves $u_1^e(\theta)$ and $u_0(\theta)$ should be higher than the intersection point between the utility curves $u_0(\theta)$ and $u_{0,1}^e(\theta)$, which implies

$$\frac{\delta p_1^e - p_0}{\delta q_1 - (1+\delta)q_0} > \frac{p_1^e}{q_1 - q_0}$$

$$\iff q_0 p_1^e - p_0(q_1 - q_0) > 0. \tag{3.5}$$

The profit function for the firm in period 1 will be $(1 - \frac{p_1}{q_1-q_0})(p_1 - c)$. Maximizing it with respect to p_1 gives $p_1^* = \frac{q_1-q_0+c}{2}$.

The profit function for the firm in period 0 will be $(1 - \frac{p_0}{(1+\delta)q_0})p_0$. Maximizing it with respect to p_0 gives $p_0^* = \frac{(1+\delta)q_0}{2}$.

In equilibrium, it must be the case that $p_0 = p_0^*$ and $p_1^e = p_1^*$ and these prices should satisfy condition (3.5) which implies

$$q_0 p_1^* - p_0^*(q_1 - q_0) > 0$$

$$\Longleftrightarrow c > \delta(q_1 - q_0).$$

Hence, for any $\delta \in (0,\, 1)$, and $c > \delta(q_1 - q_0)$, the above constitutes the stage 2 equilibrium which is described in the proposition given below.

Proposition 5. ***In the case of a large innovation with*** $c > \delta(q_1 - q_0)$, ***the stage 2 equilibrium where the firm chooses to sell is characterized as follows:***

The firm sets price $p_0^* = \frac{(1+\delta)q_0}{2}$ ***in period 0 and price*** $p_1^* = \frac{q_1 - q_0 + c}{2}$ ***in period 1. Consumers with*** $\theta < \frac{1}{2}$ ***do not buy; consumers with*** $\frac{1}{2} \leq \theta \leq \frac{q_1 - q_0 + c}{2(q_1 - q_0)}$ ***buy in period 0 only; and consumers with*** $\frac{q_1 - q_0 + c}{2(q_1 - q_0)} < \theta \leq 1$ ***buy in both periods.***

In the above equilibrium, the firm's profit in period 0 is $\frac{(1+\delta)q_0}{4}$ and that in period 1 is $\frac{(q_1 - q_0 - c)^2}{4(q_1 - q_0)}$. Hence, the firm's total profit from selling is $\frac{(1+\delta)q_0}{4} + \delta\frac{(q_1 - q_0 - c)^2}{4(q_1 - q_0)}$ whereas the firm's profit from providing a subscription is $\frac{q_0}{4} + \frac{\delta(q_1 - c)^2}{4q_1}$. It is easy to check that

$$\frac{(1+\delta)q_0}{4} + \delta\frac{(q_1 - q_0 - c)^2}{4(q_1 - q_0)} > \frac{q_0}{4} + \frac{\delta(q_1 - c)^2}{4q_1}$$

$$\Longleftrightarrow c^2 q_0 > 0.$$

Hence, we obtain Proposition 6.

Proposition 6. ***In the case of a large innovation with*** $c > \delta(q_1 - q_0)$, ***the firm will choose to sell rather than to offer a subscription and the equilibrium is as described in Proposition 5.***

Further comparison shows that the demands in period 0 are same in both cases, although the price in period 0 of selling ($\frac{(1+\delta)q_0}{2}$) is strictly higher than the respective price ($\frac{q_0}{2}$) under subscription. In period 1, the selling price ($\frac{q_1-q_0+c}{2}$) is strictly smaller than the respective price ($\frac{q_1+c}{2}$) under subscription and the demand in period 1 for selling ($\frac{q_1-q_0-c}{2(q_1-q_0)}$) is also strictly smaller than the respective demand ($\frac{q_1-c}{2q_1}$) under subscription.

At first glance it may seem that the firm can earn more profit from providing a subscription, as it can charge the respective monopoly prices in each period. But this is not the case, although the firm earns more profit in period 1 under subscription compared to what it earns in period 1 in selling. In both cases, the firm experiences the same monopoly demand in period 0. But in selling, the price in period 0 is much higher than the respective price under subscription. The reason is as follows: In the case of selling, consumers who buy in period 0 are willing to pay more for the product offered in period 0 as they can derive utility from consuming the product also in period 1, which is not possible in the case of a subscription. The willingness of consumers (who buy in period 0) to keep consuming the product in period 1 puts pressure on the firm to lower the price for the new product in period 1 and also reduces the demand in period 1. However, the firm exploits this behavior of consumers in period 0 and charges a very high price in period 0. As a result, the firm's profit in period 0 is so high that it compensates more than the lost profit in period 1 as compared to subscription. The firm will choose to sell rather than to offer a subscription in this case, but we must consider that the firm's profit in period 1 from selling is smaller than that from offering a subscription. Hence, by examining what happens in period 1, it seems that selling does not give a firm a sufficient incentive to invest in a large innovation.

Small innovation ($(1+\delta)q_0 > \delta q_1$)

In this case, the quality that is offered in period 1 is also higher than the quality that is offered in period 0 ($q_1 > q_0$), but the improvement in quality level in period 1 over the quality in period 0 is relatively small. In particular, at the beginning of period 0, the present discounted value of the quality that is offered in period 0 is higher than the present discounted value of the quality that is offered in the future period.

In this case, there is a screening of consumers over time, that is, some consumers buy only in the first period, some consumers buy only in the second period and some consumers buy in both periods. Mathematical derivation is given in the Appendix.

For any given p_0, demand in period 1 is given by $(1 - \frac{p_1}{q_1 - q_0} + \frac{p_0 - \delta p_1}{(1+\delta)q_0 - \delta q_1} - \frac{p_1}{q_1})$. The firm's profit function in period 1 is

$$(1 - \frac{p_1}{q_1 - q_0} + \frac{p_0 - \delta p_1}{(1+\delta)q_0 - \delta q_1} - \frac{p_1}{q_1})(p_1 - c). \tag{3.6}$$

Maximizing (3.6) with respect to p_1 gives $p_1 = \frac{c}{2} + \frac{1 + \frac{p_0}{x}}{2y}$ where $x = (1+\delta)q_0 - \delta q_1$ and $y = (\frac{1}{q_1 - q_0} + \frac{\delta}{x} + \frac{1}{q_1})$.

Given p_1, the firm's profit function for period 0 is given by

$$(1 - \frac{p_0 - \delta p_1}{(1+\delta)q_0 - \delta q_1})p_0. \tag{3.7}$$

Plugging $p_1 = \frac{c}{2} + \frac{1 + \frac{p_0}{x}}{2y}$ into (3.7) and maximizing it with respect to p_0 gives $p_0 = \frac{x(2xy + \delta cy + \delta)}{2(2xy - \delta)}$. Substituting p_0 in the expression for $p_1$gives $p_1 = \frac{c}{2} + \frac{6xy + \delta cy - \delta}{4y(2xy - \delta)}$. Given the consumers' behavior, prices (p_0, p_1) are optimal for the firm and given prices (p_0, p_1) consumers' behavior is optimal if the following conditions are satisfied:

$$(1+\delta)q_0 > \delta q_1 \tag{3.8}$$

$$\frac{p_1}{q_1 - q_0} < 1 \tag{3.9}$$

$$\frac{p_0 - \delta p_1}{(1+\delta)q_0 - \delta q_1} > \frac{p_1}{q_1} \tag{3.10}$$

$$\frac{p_1}{q_1 - q_0} > \frac{p_0 - \delta p_1}{(1+\delta)q_0 - \delta q_1} \tag{3.11}$$

If condition (3.9) is violated, none of the high valuation consumers will switch to q_1 from q_0 in period 1. If condition (3.10) is violated, no consumers will only buy q_1. If condition (3.11) is violated, no consumer will only buy q_0. For the parameter region in which the above conditions are satisfied, it is difficult to have a proper analytical comparison between the firm's profit in this case and the profit from subscription. Hence, I proceed with a numerical simulation which provides some interesting results. The Shaded regions in the Figures 3.3 - 3.5 show the parameter region (c, δ), in which conditions (3.8) - (3.11) are satisfied. Hence, the shaded region gives us the parameter region in c and δ where the above constitutes the stage 2 selling equilibrium that is described in Proposition 7. In all three figures below, I set $q_0 = 2$ and observe how this shaded region changes as the quality increment $(q_1 - q_0)$ increases from 0.5 to 2. Figures 3.4 and 3.5 also show the firm's indifference curves where the firm's stage 2 equilibrium profits from selling and from offering a subscription are equal. For a small innovation with $q_1 - q_0 = 0.5$, we see that the firm's profit from subscription is higher for the entire parameter region in which the stage 2 selling equilibrium exists. However, as the quality increment increases, we see that for a higher cost of such an innovation, the stage 2 selling equilibrium profit dominates the subscription profit. Summarizing all these facts, I obtain Proposition 7.

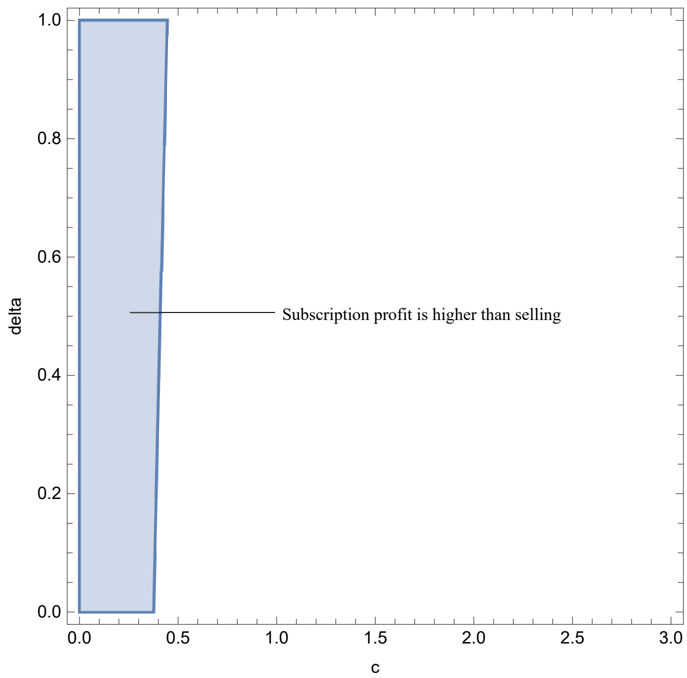

Figure 3.3: Shaded region showing the parameter region in which the proposed stage 2 selling equilibrium exists, for a given $q_0 = 2$ and $q_1 - q_0 = 0.5$

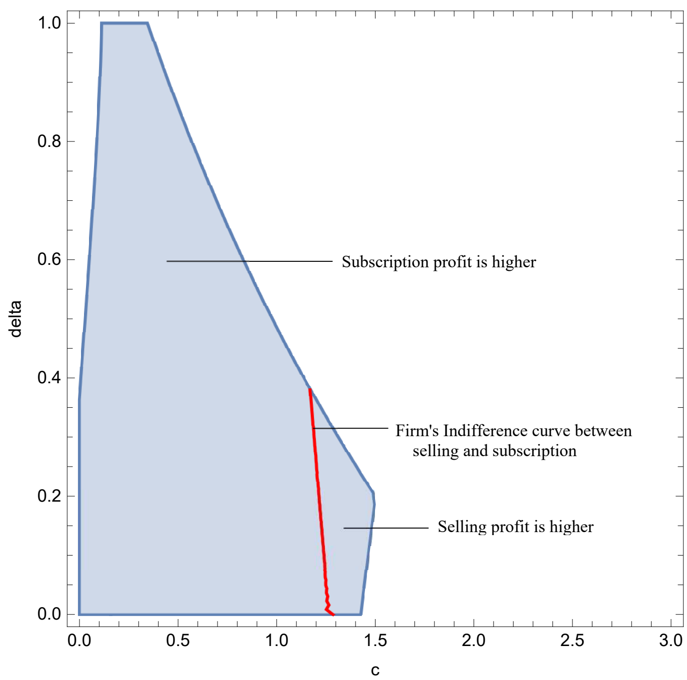

Figure 3.4: Shaded region showing the parameter region in which the proposed stage 2 selling equilibrium exists, for a given $q_0 = 2$ and $q_1 - q_0 = 1.5$

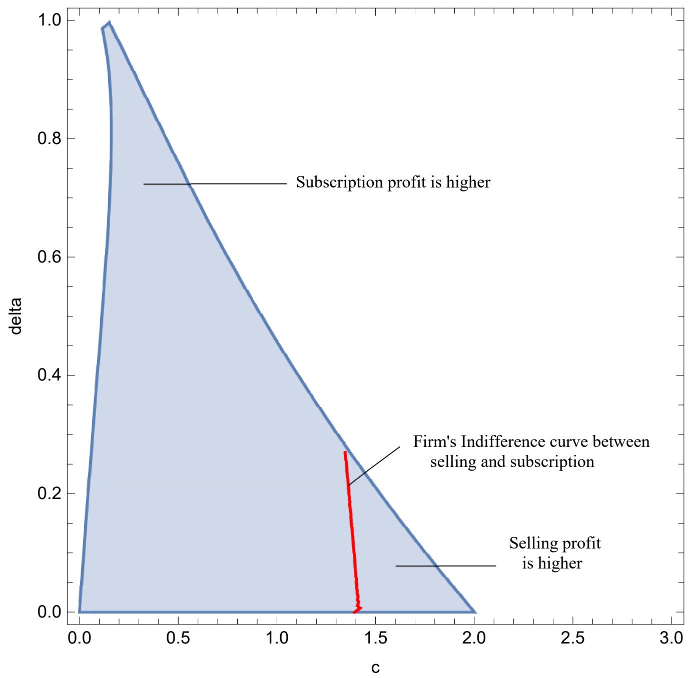

Figure 3.5: Shaded region showing the parameter region in which the proposed stage 2 selling equilibrium exists, for a given $q_0 = 2$ and $q_1 - q_0 = 2$

Proposition 7. ***In the case of a small innovation, for any*** c ***there exists*** δ ***such that the stage 2 equilibrium in which the firm chooses to sell is characterized as follows:***

The firm sets price $p_0^* = \frac{x(2xy+\delta cy+\delta)}{2(2xy-\delta)}$ ***in period 0 and price*** $p_1^* = \frac{c}{2} + \frac{6xy+\delta cy-\delta}{4y(2xy-\delta)}$ ***in period 1 where*** $x = (1+\delta)q_0 - \delta q_1$ ***and*** $y = (\frac{1}{q_1-q_0} + \frac{\delta}{x} + \frac{1}{q_1})$***. Consumers with*** $\theta < \frac{p_1^*}{q_1}$ ***do not buy; consumers with*** $\frac{p_1^*}{q_1} \leq \theta \leq \frac{p_0^*-\delta p_1^*}{(1+\delta)q-\delta q_1}$ ***buy in period 1 only; consumers with*** $\frac{p_0^*-\delta p_1^*}{(1+\delta)q-\delta q_1} < \theta \leq \frac{p_1^*}{q_1-q_0}$ ***buy in period 0 only; and consumers with*** $\frac{p_1^*}{q_1-q_0} < \theta \leq 1$ ***buy in both periods.***

In Figures 3.4 and 3.5, the region on the left of the shaded region is where the condition (3.11) is violated, that is, no consumer buys only in period 0. Hence, in this region, the above obtained prices (p_0, p_1) are not optimal for the firm. In this parameter region, consumers with $\theta \in (\frac{p_1}{q_1}, \frac{p_0}{q_0})$ will only buy in period 1, consumers with $\theta \in (\frac{p_0}{q_0}, 1)$ will buy in both periods and the rest do not buy at all. Hence, the firm's demand in period 1 will be $(1 - \frac{p_1}{q_1})$ and in period 0 will be

$(1 - \frac{p_0}{q_0})$. The firm's profit function in period 0 will be $(1 - \frac{p_0}{q_0})p_0$ and in period 1 will be $(1 - \frac{p_1}{q_1})(p_1 - c)$. The firm will maximize these two profit functions subject to the constraint that no consumer will buy in period 0 only, which is given by $\frac{p_0 - \delta p_1}{(1+\delta)q_0 - \delta q_1} > \frac{p_0}{q_0}$. Now these profit functions are the same as in the subscription case but the maximization problem in this case is constrained, whereas in the subscription case it is unconstrained. Hence, the firm's profit in this case cannot be higher than the subscription profit. Therefore, in this parameter region, the firm will choose subscription over selling.

In Figures 3.4 and 3.5, the region on the right of the shaded region is where subscription is better, condition (3.10) is violated, that is no consumer buys only in period 1. Hence, in this region, the above obtained prices (p_0, p_1) are not optimal for the firm. In this parameter region, consumers with $\theta \in (\frac{p_0}{(1+\delta)q_0}, \frac{p_1}{q_1 - q_0})$ will buy only in period 0, consumers with $(\frac{p_1}{q_1 - q_0}, 1)$ will buy in both periods and the rest do not buy at all. This is the same as already discussed in the large innovation case. Hence, the firm will not provide a subscription provided the cost of producing high qualty is sufficiently high.

In Figure 3.6, the marked regions A, B, C and D represent the following:

A: In this region, the selling prices are such that in the stage 2 selling equilibrium, some consumers buy in both periods, some consumers buy only in period 1 and the rest of the consumers do not buy. In this parameter region, the firm's profit from subscription dominates its profit from selling.

B: In this region, the stage 2 selling equilibrium as described in Proposition 7 exists. In this parameter region, the firm's profit from subscription dominates its profit from selling.

C: In this region, the selling prices are such that in the stage 2 selling equilibrium some consumers buy in both periods, some consumers buy only in period 0 and the rest of the consumers do not buy. In this parameter region, the firm's profit from subscription dominates its profit from selling.

D: In this region, the stage 2 selling equilibrium as described in Proposition

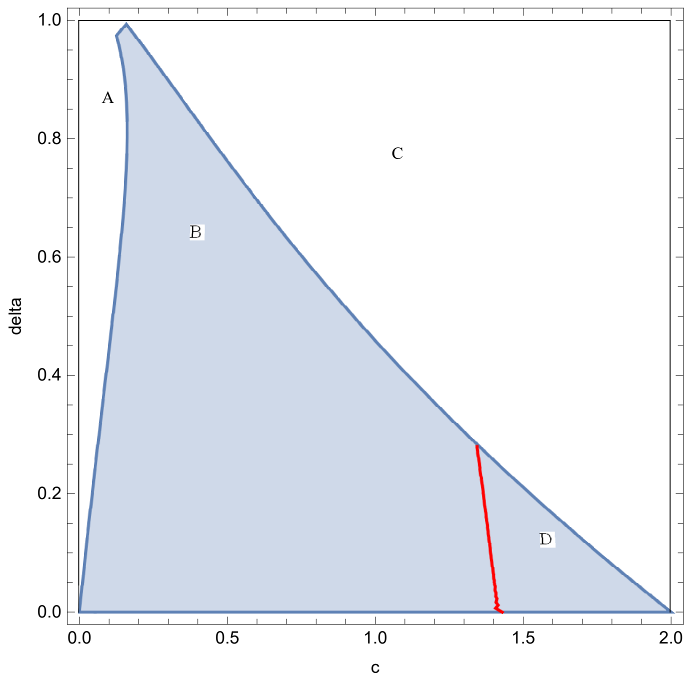

Figure 3.6: Separation of parameter region according to different types of equilibria for a given $q_0 = 2$ and $q_1 - q_0 = 2$

7 exists. This is the region in which the cost of innovation is sufficiently high. However, unlike region B, in this parameter region, the firm's profit from selling dominates its profit from subscription.

Thus, I obtain Proposition 8.

Proposition 8. ***In a small innovation with a given*** $q_1 - q_0$, ***there exists*** c^* ***such that for all*** $c < c^*$***there exists*** δ ***for which the monopolist firm will choose to offer a subscription rather than to sell.***

Figures 3.7 - 3.9 provide a comparative view of prices, and the firm's profits between subscription and selling. Figure 3.7 shows the movements of the subscription prices and the stage 2 selling prices which are described in proposition 7 as the cost of innovation increases for a given $q_0 = 2$, $q_1 - q_0 = 1.5$ and $\delta = 0.3$. For a low cost of innovation, the firm not only sells to high valuation consumers who buy in both periods but also to some low valuation consumers who buy only in period 1. As a result, the second period selling price is significantly lower than the second period subscription price.

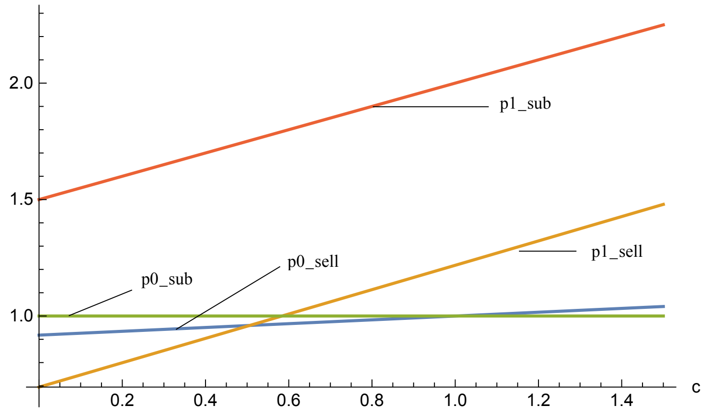

Figure 3.7: Subscription prices versus Selling prices for a given $q_0 = 2$, $q_1 - q_0 = 1.5$, $\delta = 0.3$

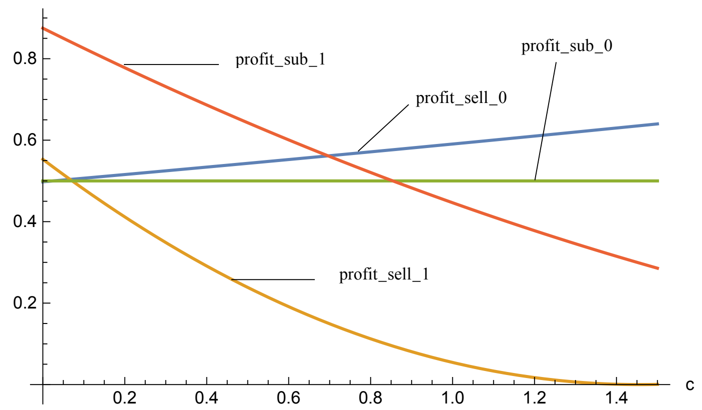

Figure 3.8: Subscription profit versus Selling profit in each period for a given $q_0 = 2$, $q_1 - q_0 = 1.5$, $\delta = 0.3$

Figure 3.8 shows the movements of the firm's profits in each period from subscription and the stage 2 selling equilibrium that is described in Proposition 8 as the cost of innovation increases for a given $q_0 = 2$, $q_1 - q_0 = 1.5$ and $\delta = 0.3$. One can see that the subscription profit in period 1, which is the monopoly profit, is always significantly higher than the respective selling profit. However, as the cost of innovation increases, the firm's profit from selling in period 0 becomes higher and higher as compared to the respective subscription profit. This is because as the cost of innovation increases, the prices in period 1 also increase. However, as the selling price in period 1 increases, fewer consumers buy only in period 1 and fewer consumers buy in both periods. This increases the demand in period 0, which in turn increases the firm's profit from selling in period 0.

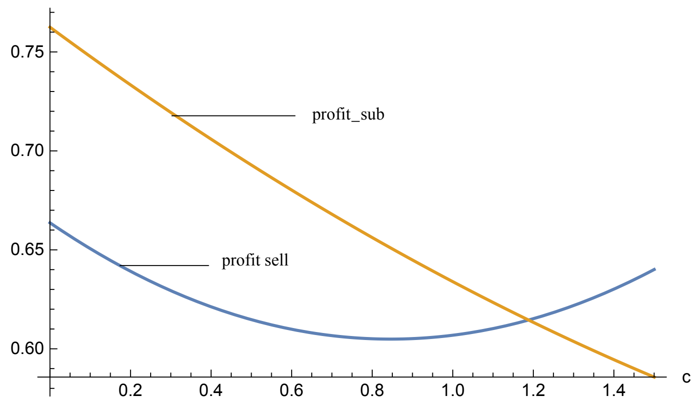

Figure 3.9: Subscription profit versus Selling profit for a given $q_0 = 2$, $q_1 - q_0 = 1.5$, $\delta = 0.3$

Figure 3.9 shows that the firm's profit from selling is smaller than its profit from the subscription for a smaller cost $(c < c^*)$ of producing q_1. For $c < c^*$, the subscription price p_1 is so high that the profit in period 1 is much higher than the respective profit in selling. Also, for small c, the subscription price p_0 is higher than the respective selling price. As c increases, the selling price p_0 increases and for high c it dominates the respective subscription price. However, the profit from selling in period 0 increases as c increases and dominates the profit from subscription in period 0. This increase in profit in period 0 from selling reduces the profit-gap in period 1 between subscription and selling. For a sufficiently high c, the selling profit dominates the subscription profit. The reason is the following: as c increases, both the subscription price and selling price in period 1 increase, which reduces the respective demand in period 1. However, the increase in the selling price in period 1 increases the selling demand in period 0, which is not the case under subscription. Under subscription, the demand in period 0 is fixed and is independent of the subscription price p_1. When the quality improvement between the two periods is small and the related cost of such an improvement is also small, the firm will choose to offer a subscription which is driven by the fact that the firm's profiit from subscription in period 1 is much higher. Hence, it seems that subscription incentivizes smaller innovation with a smaller cost of such an innovation. This is probably one of the reasons why more or more firms are now opting for subscription-based pricing where the level of quality improvement between two consecutive periods is relatively small.

3.4 Conclusion

In this chapter, I have analyzed a two-period model with a monopolist firm where the firm provides a product with a certain quality in one period and in the next period it offers the same product with better quality which is costlier to produce. I have split the quality improvement into two cases - large innovation and small innovation. As by offering a subscription, the firm can charge respective monopoly prices in each period, it may seem that the firm will indeed opt to offer a subscription rather than selling its products. Interestingly, as I have shown, this is not always the case. The level of quality improvement between two periods and the associated cost for this quality improvement play a pivotal role in a firm's pricing strategy. When the quality improvement is small, I find that the monopolist will indeed choose to offer a subscription over selling, provided the cost of this innovation is not very high. However, when the quality improvement is very large, I find that the firm will choose to sell rather than to offer a subscription, provided the cost of this innovation is sufficiently high.

In the large innovation case, when the firm decides to sell, some consumers buy in the first period. Of the consumers who buy in the first period, consumers with low valuation for higher quality keep consuming the product from the first period in the next period, whereas the remaining consumers who value higher quality more than the others switch to the higher quality product by buying in the next period. On the one hand, as consumers who buy a product in the first period can also keep consuming it in the next period, this reduces the demand for the product with higher quality in the next period. This reduction in demand in the next period puts pressure on the firm to charge a much lower price in the next period as compared to the respective subscription price. On the other hand, the firm exploits the consumers' higher willingness to pay in the first period as they can consume it in the next period, and it charges a very high price in the first period. In both subscription and selling, the firm experiences the same demand in the first period, but the selling price in the first period is very high as compared to the respective subscription price. As a result, the profit earned by the firm in the first period from selling becomes so high that it compensates for the lost profit (due to less demand and lower price in the next period) in the next period as compared to the subscription.

In the small innovation case, when the firm decides to sell, there is a screening of consumers over time, that is, some consumers buy only in the first period, some consumers buy only in the second period and some consumers buy in both periods. As in the large innovation case, the firm's profit in the first period from selling is higher than its first period subscription profit. However, through subscription, the firm earns a respective monopoly profit in the second period. For a low cost of innovation, the firm not only sells to high valuation consumers who buy in both periods but also to some low valuation consumers who buy only in the second period. This significantly drives down the second period selling price as compared to the second period subscription price. For a low cost of innovation, the second period profit of the firm from subscription is high as compared to the respective profit from selling, such that it compensates for the lost profit from the first period. As a result, the firm chooses to offer a subscription.

To the best of my knowledge, this chapter is the first one to provide important insights into how the quality of a product can influence a firm's pricing strategy for a durable good. In particular, when the quality of the product offered in the future period is significantly higher than in the current period, as well as when the cost of producing it is high, it is better for the monopolist to sell its products rather than to offer a subscription. However, when the level of quality improvement is small and also the cost of this improvement is small, the firm will indeed offer a subscription. The results seem to suggest that small innovation and subscription go hand in hand and this is probably one of the reasons why more and more firms are offering subscriptions for their products. However, this result is based on the fact that in my model quality levels are exogenously given. In terms of future research, it would be interesting to see if we endogenize quality levels and the firm commits to offer a subscription, whether the firm will choose to opt for small innovation or not.

Appendix

Proof of Lemma 3:

As $\delta q_1 \geq (1+\delta)q_0$, the slope of $u_1^e(\theta)$ is steeper or equal to the slope of $u_0(\theta)$. Suppose the prices (p_0^*, p_1^e) are such that some consumers buy only in period 0 and some buy only in period 1. Given these prices, $u_1^e(\theta) \geq u_{0,1}^e(\theta)$ implies a consumer with $\theta = \frac{\delta p_1^e - p_0}{\delta q_1 - (1+\delta)q_0}$ will be indifferent between buying only in period 0 and buying only in period 1. Consumers with $\theta > \frac{\delta p_1 - p_0}{\delta q_1 - (1+\delta)q_0}$ would strictly prefer to buy in period 1 only and consumers with $\theta < \frac{\delta p_1 - p_0}{\delta q_1 - (1+\delta)q_0}$ would strictly prefer to buy in period 0 only. But in period 1, the firm can marginally increase the price p_1^e, and can get the same demand and make more profit. With this marginal increase in price, the indifferent consumer would have strictly preferred to buy in period 0. This is true for any pair of prices (p_0^*, p_1^e) . Hence, it cannot be part of an equilibrium.

Derivation of screening in case of small innovation:

At the beginning of period 0, the expected utility of a consumer (θ) by buying the product in period 0 and also consuming it in period 1 is $(1+\delta)\theta q_0 - p_0$, whereas if the consumer decides not to buy the product in period 0 and buys the product with quality q_1 in period 1, his expected ex-ante utility at the beginning of period 0 will be $\delta(\theta q_1 - p_1^e)$. Hence, at the beginning of period 0, a consumer (θ) will buy in period 0 if and only if $(1+\delta)\theta q_0 - p_0 > \delta(\theta q_1 - p_1^e) \iff \theta > \frac{p_0 - \delta p_1^e}{(1+\delta)q_0 - \delta q_1}$.

Consumers with $\theta < \frac{p_0 - \delta p_1^e}{(1+\delta)q_0 - \delta q_1}$ who do not buy in period 0, will buy in period 1 if and only if $\theta q_1 - p_1 > 0 \iff \theta > \frac{p_1}{q_1}$. Hence, consumers with $\theta \in (\frac{p_1}{q_1}, \frac{p_0 - \delta p_1^e}{(1+\delta)q_0 - \delta q_1})$ will only buy in period 1. In period 1, the utility of a consumer by keeping consuming quality q_0 is θq_0 whereas the utility of buying the product with quality q_1 is $\theta q_1 - p_1$. Hence, consumers with $\theta > \frac{p_0 - \delta p_1^e}{(1+\delta)q_0 - \delta q_1}$ who buy in period 0 will buy the product with q_1 in period 1 if and only if $\theta q_1 - p_1 > \theta q_0 \iff \theta > \frac{p_1}{q_1 - q_0}$. Combining all these conditions, consumers with $\theta \in (\frac{p_1}{q_1}, \frac{p_0 - \delta p_1^e}{(1+\delta)q_0 - \delta q_1})$ will buy in period 1 only, consumers with $\theta \in (\frac{p_0 - \delta p_1^e}{(1+\delta)q_0 - \delta q_1}, \frac{p_1}{q_1 - q_0})$ will buy in period 0 only, consumers with $\theta \in (\frac{p_1}{q_1 - q_0}, 1)$ will buy in both periods and the rest of the consumers do not buy at all. In equilibrium, it must be the case that $p_1^e = p_1$.

4 Price-Quality Competition and Consumer Search

4.1 Introduction

The phenomenon of price dispersion has been widely studied in both the theoretical and empirical literatures on consumer search[1]. Following Wolinsky (1986) and Anderson and Renault (1999), there is quite a bit of literature on search for horizontal product attributes (match value) which I will dscuss later. However, there has been much less research thus far for quality attributes. Consumer search is based not only on price but also on quality. After visiting a store, a consumer not only finds out that firm's product price but often its quality, through product feature specifications. It is thus important to study the relationship between search friction and firms' quality choices.

Wildenbeest (2011) provides an estimation technique of search costs for vertically differentiated products, although he bypasses the aspect of firms' quality choice. Under some assumptions, he converts the problem of firms competing in terms of price and quality into a problem where firms compete directly in terms of utility. He shows that firms randomize their prices in response to search friction in the market, but some firms persistently charge higher prices than others because of vertical differentiation among themselves. Among many other interesting empirical findings in his paper, he notes that if one estimates a homogeneous goods model without

[1]Following seminal contributions by Reinganum (1979), Varian (1980), Burdett and Judd (1983), Stahl (1989), some theoretical papers are Dana Jr (1994), Baye and Morgan (2001), Janssen et al. (2005), Armstrong et al. (2009), Janssen et al. (2011), etc. Some of the empirical papers are Hortaçsu and Syverson (2004), Lach (2007), Moraga-González and Wildenbeest (2008), De los Santos et al. (2012) etc.

taking into account the aspect of vertical differentiation, one may overestimate the search cost. In this chapter, I show that the existence of homogeneous products in a market depends on choices made by firms. When the search costs are small or the proportion of consumers who search costlessly (shoppers) is large, firms have incentives to differentiate themselves vertically. If search costs are large and the proportion of shoppers is small, this incentive does not exist. Hence, with large search costs and a low proportion of shoppers, one may expect firms to produce homogeneous goods. Wildenbeest's finding shows that one may find large search costs if one assumes a homogeneous good's market, whereas I show that large search costs imply goods are homogeneous in the market.

In this chapter, I consider a standard model with vertical differentiation where each firm simultaneously chooses its product quality and price. High quality products are more costly to produce. Consumers have identical valuation for the low quality product but they differ with respect to the extra premium they are willing to pay for the high quality product. Consumers also differ in terms of their search costs. One group of consumers (shoppers) incur no search cost, whereas the rest of the consumers (non-shoppers) incur a positive search cost for visiting a firm. Consumers search sequentially. After visiting a firm, consumers observe both the quality and price set by that firm. I find that the symmetric equilibrium where both firms produce high quality or both produce low quality exists if the search cost is sufficiently large and the proportion of shoppers in the market is sufficiently small. Small search costs or a large number of shoppers give firms incentives to deviate and differentiate themselves. Recent technological advancements have lowered consumers' search costs. In this context, the results suggest that with a decrease in search cost, we should expect some degree of product differentiation among firms. The results of this chapter are similar to what Kuksov (2004) finds in his paper on product design. He looks into spatial product differentiation between duopolists where consumers search for prices but know their product preferences and firms' product designs in advance. In my model, it is only after visiting a firm that a consumer observes the quality of that firm's product along with the price set by that firm.

In the literature, following Wolinsky (1986) and Anderson and Renault (1999), many papers have examined horizontal differentiation. Armstrong et al. (2009) study the effects of prominence in search markets but their paper does not allow firms to choose differing types of quality. Wolinsky (1983) does allow firms to choose types of quality but consumers observe all prices charged by the firms costlessly and incur a search cost to obtain imperfect information about the quality of the firm's product. Fishman and Simhon (2000) study a monopolist firm's incentive to invest in high quality, which is risky, and where the firm has private information about the outcome of such a risky investment. Consumers observe the price costlessly but can learn the firm's product quality after incurring a positive cost. They find that the firm's incentive to invest in improving quality is greater the less costly it is for consumers to become informed. In my model, I introduce competition between firms: it is only after visiting a firm that a consumer observes quality perfectly and the price set by that firm. This is also the set-up in Fishman and Levy (2011) except that there (*i*) consumers have downward sloping demand, and (*ii*) the outcome of investing in high quality is risky. They show that reducing the search cost leads to higher quality if the initial level of the search cost is sufficiently high but may lead to lower quality if the initial level of the search cost is sufficiently low. In my model, consumers have unit demand and there is no uncertainty about the outcome of investing in high quality.

This chapter relates closely to the literature on provision of services in various markets. Janssen and Ke (2020) show that when there are search frictions in a market, firms may provide services if the cost of such service provision is not too large. While the first best outcome is to have at most one firm providing services, in equilibrium service provision is either over-provided (two or more firms provide services) or under-provided (no firm provides services). Janssen and Ke consider horizontal product differentiation, assuming that all consumers incur a positive search cost and that this cost differs: one for a service-providing firm and the other for a non-service-providing firm. In my model, I consider a vertical differentiation model and assume that some consumers search costlessly and the remaining consumers have a positive search cost, with the first search being free. Also, in my model I do not distinguish search costs to find a high or low quality product. Moraga-González and Sun (2019) show that higher search costs may lead to less investment in quality. They find that, in equilibrium, quality is over-provided or under-provided from a socially optimum level if and only if the equilibrium number of searches increases or decreases in quality. This chapter shows that the symmetric equilibrium in which both firms produce either high or low quality can only be sustained when the search friction is sufficiently large and the proportion of shoppers is sufficiently small.

The chapter is organized as follows. Section 2 describes the model and Section 3 provides analysis. I first consider the case when the search cost is small, followed by the case when the search cost is large. The last section concludes with a discussion.

4.2 Model

I consider the model of oligopolistic competition and sequential consumer search as in Stahl (1989) but I allow each firm to choose the quality of its product. The model consists of two firms which compete on quality and price. Each individual firm's choice of quality is a binary decision - it can produce either low quality (0) at zero marginal cost or high quality ($\bar{q}$) at a unit marginal cost c. There is a unit mass of consumers with each consumer having unit demand. Firms choose qualities and prices simultaneously and each firm's choices are not observed by the other firm. Consumers need to visit a firm in order to find out the choices made by that firm. Consumers search sequentially and once a consumer visits a firm, he observes the quality[2] and the price set by that firm. There is $\lambda \in (0,\, 1)$ proportion of consumers (shoppers) who observe both firms' qualities and prices costlessly. But the remaining $(1-\lambda)$ proportion of consumers (non-shoppers) have a positive search cost (s) to visit a firm, with the first search being free. I assume costless perfect recall. Consumer i's willingness to pay for quality q is given by $x + \theta_i q$, where $\theta_i \sim U\,[0,\, 1]$, $q \in \{0,\, \bar{q}\}$, and $\bar{q} > c$. Hence, all consumers have the same maximum willingness to pay x for low quality but they all differ in terms of the extra premium they are willing to pay for high quality.

Firms and consumers play the following game: each firm chooses its quality and sets a price, taking all the quality and price choices of its rival firm as well as consumers' search behavior as given. Each consumer forms opinions about the quality and price choices made by the firms and decides on his optimal search strategy.

I restrict the analysis to symmetric Nash equilibria, assuming that consumers always possess passive beliefs about equilibrium qualities and prices.

4.3 Analysis

As I will show that the magnitude of search cost plays a key role in having a symmetric Nash equilibrium, I split the analysis of the model into two sub-sections: when the search cost is small and when the search cost is large.

[2]If consumers do not observe the quality choice made by the firm, then the good becomes a Credence good. For such a product, consumers either have to rely on the honesty of the seller or to rely on certifications provided by the certification intermediaries, as firms have incentives to cheat. On this topic, Das (2016) analyzes the influence of (exogenous) horizontal differentiation between firms on equilibrium certification policy, when firms compete in terms of price and quality. Kerschbamer and Sutter (2017) provides an overview of recent laboratory and field experiments on Credence goods.

4.3.1 Search cost is small

I consider the following two candidates for a symmetric Nash equilibrium: either both firms produce high quality, or both firms produce low quality. In the first case, each firm chooses its random pricing strategy according to the same distribution function $F_H(.)$ over the support $[\underline{p_H}, \overline{p_H}]$, whereas in the second case each firm randomizes its price over the support $[\underline{p_L}, \overline{p_L}]$ according to the same distribution function $F_L(.)$.

Case (a): *Symmetric equilibrium with high quality*

The expected payoff to firm i from choosing high quality and charging price p_i when its rival chooses high quality and chooses a random pricing strategy (Janssen and Moraga-González (2004)) according to the cumulative distribution $F_H(.)$ is

$$\pi_i(\bar{q},\, p_i,\, F_H(p_i)) = \{\lambda(1 - F_H(p_i)) + \frac{1-\lambda}{2}\}(p_i - c). \tag{4.1}$$

Firm i attracts all the shoppers when it charges a price that is lower than its rival's price, which happens with probability $(1 - F_H(p_i))$. It also sells to $(1 - \lambda)$ non-shoppers whenever they visit its store, which occurs with probability $\frac{1}{2}$. As in equilibrium, a firm must be indifferent between charging any price in support of $F_H(.)$, any price in support of $F_H(.)$ must satisfy $\pi_i(\bar{q},\, p_i,\, F_H(p_i)) = \pi_i(\overline{p_H})$, i.e.,

$$\{\lambda(1 - F_H(p_i)) + \frac{1-\lambda}{2}\}(p_i - c) = \{\frac{1-\lambda}{2}\}(\overline{p_H} - c). \tag{4.2}$$

Solving equation (4.2) for the price distribution yields

$$F_H(p) = 1 - \frac{(1-\lambda)((\overline{p_H} - p))}{2\lambda(p - c)}. \tag{4.3}$$

Since $F_H(.)$ is a distribution function, it must be the case that $F_H(\underline{p_H}) = 0$. Solving for $\underline{p_H}$ one obtains the lower bound of the price distribution $\underline{p_H} = \frac{(1-\lambda)\overline{p_H}+2\lambda c}{1+\lambda}$.

The cumulative distribution (Eqn. (4.3)) represents each firm's optimal pricing strategy given that it chooses high quality and its rival firm does the same.

To study optimal consumer behavior, consider a non-shopper with quality premium parameter θ_i who has observed high quality at a given price p. This consumer will continue to search if the expected benefit from searching further exceeds the search cost. If ρ_H is the reservation price of a consumer for the high quality product, this is the price that makes the consumer indifferent between accepting the high quality product at the given price and searching again. Hence, it must satisfy the following:

$$x + \theta_i \bar{q} - \rho_H = x + \theta_i \bar{q} - s - (1 - F_H(\rho_H))\rho_H - F_H(\rho_H) E_H(p' \mid p' < \rho_H)$$

$$\Longleftrightarrow \rho_H = E_H(p' \mid p' < \rho_H) + \frac{s}{F_H(\rho_H)}. \tag{4.4}$$

Note that the Eqn. (4.4) does not depend on θ_i. Hence, all the non-shoppers will have the same reservation price ρ_H. No firm will charge a price above ρ_H, otherwise non-shoppers will continue to search (Stahl (1989)). As a result, the upper bound $\overline{p_H} = \rho_H$ and the expression for ρ_H becomes the following:

$$\rho_H = E_H(p) + s. \tag{4.5}$$

After calculating $E_H(p)$ (calculations are shown in the Appendix) and by plugging it in Eqn. (4.5), I get

$$\rho_H = c + \frac{s}{1 - \alpha}, \tag{4.6}$$

where $\alpha = \frac{ln|1+a|}{a}$, $a = \frac{2\lambda}{1-\lambda} > 0$ with $\alpha \in [0, 1)$.

Hence, the profit of each firm will be

$$\pi^* = \frac{1 - \lambda}{2}(\rho_H - c) = \frac{(1 - \lambda)s}{2(1 - \alpha)}.$$

For consumers' behavior to be optimal, it must be the case that $\rho_H \leq x$.

The proposed strategies will form an equilibrium if the equilibrium profit is greater than the most profitable deviation profit. Note that the proposed equilibrium is constructed in such a way that a deviation in price only is never profitable for either of the firms. A firm could only profitably deviate by producing low quality and charging a price p^D. Suppose firm 1 deviates. This deviation will be observed by all the shoppers and by non-shoppers who first visit firm 1, as consumers have passive beliefs. The worst outcome for the deviating firm is that the non-deviating firm charges the lowest price ($\underline{p_H}$) in the price support. In such a case, a shopper (θ_i) will buy from the deviating firm if

$$x - p^D \geq x + \theta_i \bar{q} - \underline{p_H}$$

$$\Longleftrightarrow p^D \leq \underline{p_H} - \theta_i \bar{q}.$$

Hence, for a given $\underline{p_H}$, the deviating firm can set a price equal to $\underline{p_H} - \theta_i^* \bar{q}$ such that it sells to all the shoppers with $\theta_i \leq \theta_i^*$. In that case, the deviating firm's profit would be at least $\lambda \theta_i^* (\underline{p_H} - \theta_i^* \bar{q})$. This deviation would be profitable for the deviating firm if

$$\lambda \theta_i^* (\underline{p_H} - \theta_i^* \bar{q}) \geq \frac{(1-\lambda)s}{2(1-\alpha)}. \tag{4.7}$$

The above inequality (4.7) holds when s is sufficiently small or λ is sufficiently large (the analytical proof is given in the Appendix). Therefore, when the search costs are sufficiently small, firms have incentives to differentiate themselves.

Case (b): *Symmetric equilibrium with low quality*

The analysis is similar to case (a), except for the fact that in this case $c = 0$. The respective price distribution (Janssen and Moraga-González (2004)) is given by

$$F_L(p) = 1 - \frac{(1-\lambda)(\overline{p_L} - p)}{2\lambda p}. \tag{4.8}$$

The lower bound of the price distribution $\underline{p_L}$ is $\frac{(1-\lambda)\overline{p_L}}{1+\lambda}$. If ρ_L denotes the reservation price of a consumer for the low quality product, this is the price which makes the consumer indifferent between accepting the low quality product at the given price and searching again. Hence, it must satisfy the following:

$$x - \rho_L = x - s - (1 - F_L(\rho_L))\rho_L - F_L(\rho_L)E_L(p' \mid p' < \rho_L)$$

$$\Longleftrightarrow \rho_L = E_L(p' \mid p' < \rho_L) + \frac{s}{F_L(\rho_L)}.$$

Note that the expression for ρ_L does not depend on θ_i. Hence, all the non-shoppers will have the same reservation price ρ_L. No firm will charge a price above ρ_L, otherwise non-shoppers will continue to search (Stahl (1989)). As a result, the upper bound $\overline{p_L} = \rho_L$ and the expressions for ρ_L and $E_L(p)$ become the following:

$$\rho_L = E_L(p) + s. \tag{4.9}$$

After calculating $E_L(p)$ (calculations are shown in the Appendix) and by plugging it in Eqn. (4.9), I get

$$\rho_L = \frac{s}{1-\alpha}.$$

Hence, the profit of each firm will be

$$\pi^* = \frac{1-\lambda}{2}\rho_L = \frac{(1-\lambda)s}{2(1-\alpha)}.$$

For consumers' behavior to be optimal, it must be the case that $\rho_L \leq x$.

The proposed strategies will form an equilibrium if the equilibrium profit is greater than the most profitable deviation profit. As in the previous case, the proposed equilibrium is constructed in such a way that a deviation in price only is never profitable for either of the firms. A firm could only profitably deviate by producing high quality and charging a price p^D. Suppose firm 1 deviates. This deviation will be observed by all the shoppers and by non-shoppers who first visit firm 1, as consumers have passive beliefs. The worst outcome for the deviating firm is that the non-deviating firm charges the lowest price ($\underline{p_L}$) in the price support. In such a case, a shopper (θ_i) will buy from firm 1 if the following holds:

$$x + \theta_i \bar{q} - p^D \geq x - \underline{p_L}$$

$$\Longleftrightarrow p^D \leq \theta_i \bar{q} + \underline{p_L}.$$

Hence, for a given $\underline{p_L}$, the deviating firm can set a price equal to $\theta_i^* \bar{q} + \underline{p_L}$ such that it sells to all the shoppers with $\theta_i \geq \theta_i^*$. In that case, the deviating firm's profit would be at least $\lambda(1 - \theta_i^*)(\theta_i^* \bar{q} + \underline{p_L})$. The deviation would be profitable for the deviating firm if

$$\lambda(1 - \theta_i^*)(\theta_i^* \bar{q} + \underline{p_L} - c) \geq \frac{(1-\lambda)s}{2(1-\alpha)}. \tag{4.10}$$

The above inequality (4.10) holds when s is sufficiently small or λ is sufficiently large (the analytical proof is given in the Appendix). Therefore, when search costs are sufficiently small, firms have incentives to differentiate themselves.

Hence, we have the following proposition:

Proposition 9. *For any finite $\bar{q}$, c, and λ, there exists sufficiently small search cost (s^*) such that for all $s \leq s^*$ there does not exist any symmetric equilibrium where both firms produce either high or low quality.*

The above proposition is in line with a Betrand result under perfect competition and is relatively intuitive. In my model, as the search cost goes to zero or the proportion of shoppers goes to one, each firm's profit under the proposed symmetric strategies also goes to zero. As consumers value high quality more than low quality, for a sufficiently small search cost or for a sufficiently large proportion of shoppers, firms have strict incentives to deviate from the proposed symmetric strategies and differentiate themselves. By deviating and creating vertical differentiation in the market, a firm can at least sell to some shoppers such that it generates more profit than the profit it gets from the earlier discussed symmetric strategies. In this way, firms soften the degree of competition in the market.

4.3.2 Search cost is large

This is the case in which the non-shoppers have a large search cost which is incurred through a significant amount of effort and time by the non-shoppers to find their desired products. In such a framework, the underlying market is far less competitive than the market where non-shoppers have a small search cost. Having a search cost that is large enough provides each firm with some degree of monopoly power.

Case (a): *Symmetric equilibrium with high quality*

The proposed strategies in the previous sub-section will form an equilibrium if the equilibrium profit is greater than the most profitable deviation profit. Suppose firm 1 deviates by producing low quality and charging a price p^D. This deviation will be observed by all the shoppers and by non-shoppers who first visit firm 1, as consumers have passive beliefs. I consider the highest possible profit for the deviating firm. Therefore, while analyzing the behavior of the non-shoppers, I do not consider the expected price of the non-deviating firm. Instead, I consider the price of the non-deviating firm, which gives an upper bound of the profit from the deviating firm. The upper bound of the profit of the deviating firm can be obtained if the non-deviating firm sets the highest price ($\overline{p_H}$) in the price support.

In such a case, a shopper (θ_i) who visits firm 1 first will buy from firm 1 if the following holds:

$$x - p^D \geq x + \theta_i \bar{q} - \overline{p_H}$$

$$\iff \theta_i \leq \frac{\overline{p_H} - p^D}{\bar{q}}. \tag{4.11}$$

Out of half of the $(1-\lambda)$ non-shoppers who visit firm 1 first, the utility of a non-shopper (θ_i) from buying from firm 1 is $x - p^D$, whereas the utility from buying from firm 2 is $x + \theta_i \bar{q} - \overline{p_H} - s$. The non-shopper will buy from firm 1 if

$$x - p^D \geq x + \theta_i \bar{q} - \overline{p_H} - s$$

$$\iff \theta_i \leq \frac{\overline{p_H} - p^D - s}{\bar{q}}.$$

Therefore, the deviational profit for firm 1 would be

$$\pi^D(p^D) = [\lambda Prob(\theta_i \leq \frac{\overline{p_H} - p^D}{\bar{q}}) + \frac{(1-\lambda)}{2} Prob(\theta_i \leq \frac{\overline{p_H} - p^D - s}{\bar{q}})]p^D$$

$$= [\lambda(\frac{\overline{p_H} - p^D}{\bar{q}}) + \frac{(1-\lambda)}{2}(\frac{\overline{p_H} - p^D - s}{\bar{q}})]p^D.$$

Maximizing $\pi^D(p^D)$ with respect to p^D, we get

$$p^D = \frac{\overline{p_H}}{2} - \frac{(1-\lambda)s}{2(1+\lambda)}. \tag{4.12}$$

By plugging in p^D in $\pi^D(p^D)$ and simplifying, we get

$$\pi^D = (\frac{1+\lambda}{8\bar{q}})[c + \frac{s}{(1-\alpha)} - \frac{(1-\lambda)s}{(1+\lambda)}]^2.$$

This deviation will not be profitable for firm 1 if

$$\pi^* = \frac{(1-\lambda)s}{2(1-\alpha)} \geq \pi^D = (\frac{1+\lambda}{8\bar{q}})[c + \frac{s}{(1-\alpha)} - \frac{(1-\lambda)s}{(1+\lambda)}]^2. \tag{4.13}$$

The above π^* is a linear function of s and λ. As s increases, π^*increases, whereas as λ increases, π^* decreases. However, π^D is a parabolic function in s which has an opening towards the positive. It can be shown that the inequality (4.13) holds for sufficiently large values of value of s provided λ is sufficiently small (proof is given in the Appendix).

For a small enough search cost, π^D dominates π^*. For a sufficiently high search cost, π^D intersects with π^*, and beyond that intersecting point of search cost π^*dominates π^D. However, whether π^D will intersect with π^* or not depends on the value of λ. For sufficiently small λ, π^D intersects with π^*. As λ becomes large enough, π^D does not intersect with $\pi^*$and π^D dominates π^* for all values of s.

Figures 4.1 and 4.2 show graphs of the above two profits as functions of s for a given $\bar{q}$ and c, and for two different values of λ(small and large).

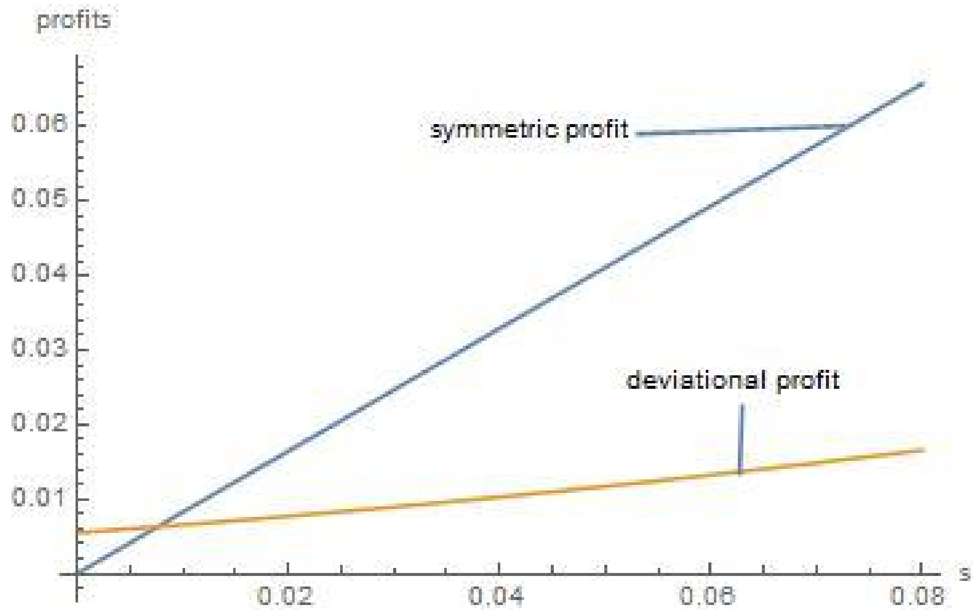

Figure 4.1: Firm's profits for $\bar{q} = 0.5$, $c = 0.25$, $\lambda = 0.4$

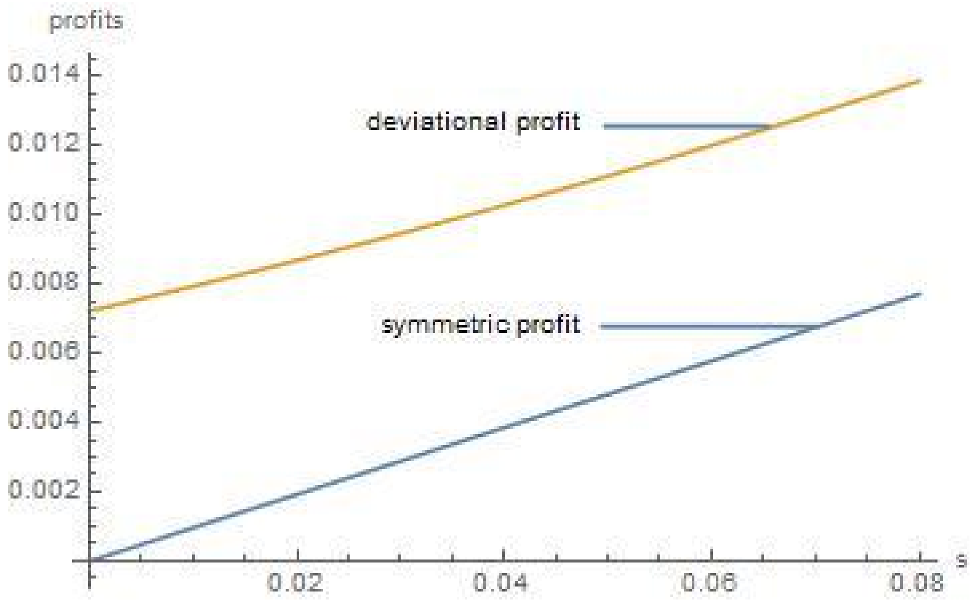

Figure 4.2: Firm's profits for $\bar{q} = 0.5$, $c = 0.25$, $\lambda = 0.85$

Figure 4.1 shows that for sufficiently small search cost it is profitable for the firm to deviate from the proposed symmetric strategies. When the search cost is large and the proportion of shoppers is sufficiently small, the proposed symmetric strategies form an equilibrium. However, from Figure 4.2 we see that as the proportion of shoppers becomes sufficiently large, the symmetric equilibrium breaks down.

Case (b): *Symmetric equilibrium with low quality*

The proposed strategies in the previous sub-section will form an equilibrium if the equilibrium profit is greater than the most profitable deviation profit. Suppose firm 1 deviates by producing high quality and charging a price p^D. This deviation will be observed by all the shoppers and by non-shoppers who first visit firm 1, as consumers have passive beliefs. Similar to the previous case, I consider the highest possible profit for the deviating firm. Therefore, while analyzing the behavior of the non-shoppers, I do not consider the expected price of the non-deviating firm. Instead, I consider the price of the non-deviating firm, which gives an upper bound of the profit from the deviating firm. The upper bound of the profit of the deviating firm can be obtained if the non-deviating firm sets the highest price ($\overline{p_L}$) in the price support.

In such a case, a shopper (θ_i) who visits firm 1 first, will buy from firm 1 if the following holds:

$$x + \theta_i \bar{q} - p^D \geq x - \overline{p_L}$$

$$\iff \theta_i \geq \frac{p^D - \overline{p_L}}{\bar{q}}. \tag{4.14}$$

Out of half of the $(1-\lambda)$ non-shoppers who visit firm 1 first, the utility of a non-shopper (θ_i) from buying from firm 1 is $x + \theta_i \bar{q} - p^D - s$ whereas the utility from buying from firm 2 is $x - \overline{p_L}$. The non-shopper will buy from firm 1 if

$$x + \theta_i \bar{q} - p^D \geq x - \overline{p_L} - s$$

$$\iff \theta_i \geq \frac{p^D - \overline{p_L} - s}{\bar{q}}.$$

The deviational profit for firm 1 would be

$$\pi^D(p^D) = [\lambda Prob(\theta_i \geq \frac{p^D - \overline{p_L}}{\bar{q}}) + \frac{(1-\lambda)}{2} Prob(\theta_i \geq \frac{p^D - \overline{p_L} - s}{\bar{q}})](p^D - c)$$

$$= [\lambda(\frac{\bar{q} + \overline{p_L} - p^D}{\bar{q}}) + \frac{(1-\lambda)}{2}(\frac{\bar{q} + \overline{p_L} + s - p^D}{\bar{q}})](p^D - c).$$

Maximizing $\pi^D(p^D)$ with respect to p^D gives

$$p^D = \frac{(\bar{q} + \overline{p_L} + c)}{2} + \frac{s(1-\lambda)}{2(1+\lambda)}. \tag{4.15}$$

By plugging in p^D in $\pi^D(p^D)$ and simplifying, we get

$$\pi^D = \frac{(1+\lambda)}{8\bar{q}}[\bar{q} - c + \frac{s}{(1-\alpha)} + \frac{(1-\lambda)s}{(1+\lambda)}]^2.$$

This deviation will not be profitable for firm 1 if

$$\pi^* = \frac{(1-\lambda)s}{2(1-\alpha)} \geq \pi^D = \frac{(1+\lambda)}{8\bar{q}}[\bar{q} - c + \frac{s}{(1-\alpha)} + \frac{(1-\lambda)s}{(1+\lambda)}]^2. \tag{4.16}$$

The above π^* is a linear function of s and λ. As s increases, π^*increases, whereas as λ increases, π^* decreases. However, π^D is a parabolic function in s which has an opening towards the positive. It can be shown that the inequality (4.16) holds for sufficiently large values of value of s provided λ is sufficiently small (proof is given in the Appendix).

For a small enough search cost, π^D dominates π^*. For a sufficiently high search cost, π^D intersects with π^*, and beyond that intersecting point of search cost π^*dominates π^D. However, whether π^D will intersect with π^* or not depends on the value of λ. For sufficiently small λ, π^D intersects with π^*. As λ becomes large enough, π^D does not intersect with $\pi^*$and π^D dominates π^* for all values of s.

Figures 4.3 and 4.4 show graphs of the above two profits as functions of s for a given $\bar{q}$ and c, and for two different values of λ(small and large). Figure 4.3 shows that for sufficiently small search cost it is profitable for the firm to deviate from the proposed symmetric strategies. When the search cost is sufficiently large and the proportion of shoppers is sufficiently small, the proposed symmetric strategies form an equilibrium. However, from Figure 4.4 we see that as the proportion of shoppers becomes sufficiently large, the symmetric equilibrium breaks down.

Proposition 10. *Symmetric equilibria where both firms produce either high quality or low quality and randomize their prices following the same distribution exist if the search cost is sufficiently large and the proportion of shoppers is sufficiently small.*

We see that the symmetric equilibria where both firms produce either high or low

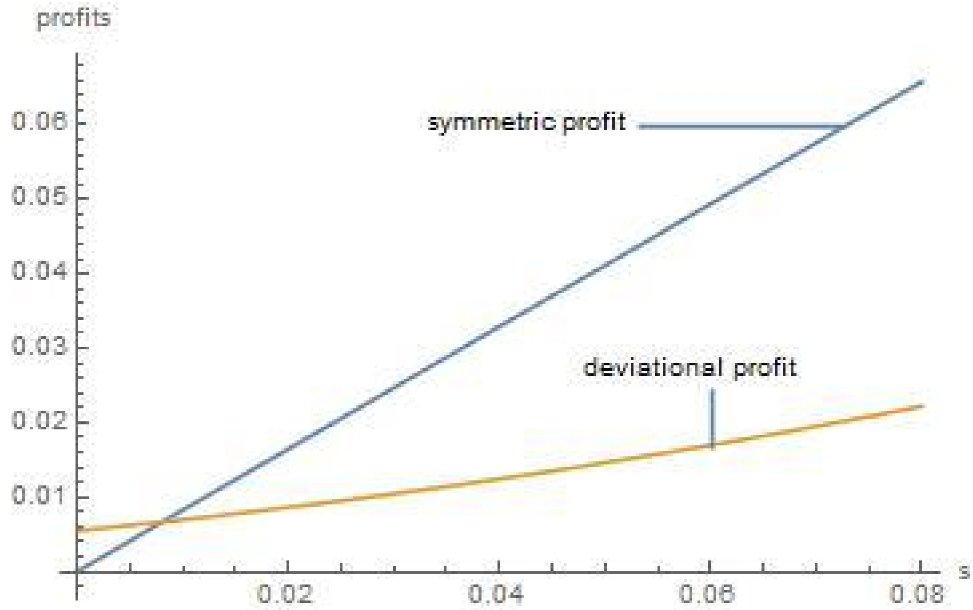

Figure 4.3: Firm's profits for $\bar{q} = 0.5$, $c = 0.25$, $\lambda = 0.4$

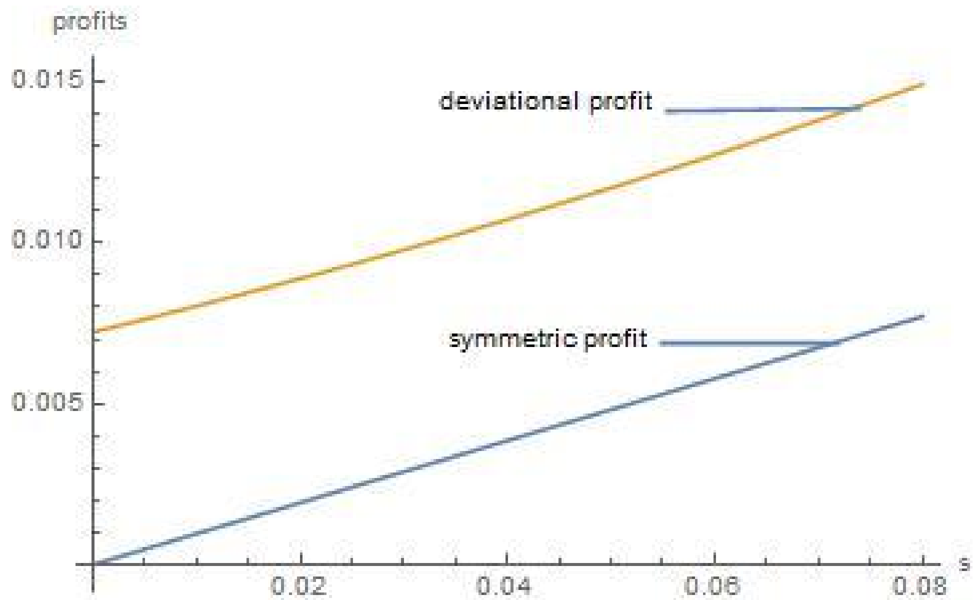

Figure 4.4: Firm's profits for $\bar{q} = 0.5$, $c = 0.25$, $\lambda = 0.85$

quality can exist only when the search cost is sufficiently large and the proportion of shoppers is sufficiently small. For a sufficiently small search cost or for a sufficiently large proportion of shoppers, firms tend to differentiate themselves vertically. As a low search cost and a large proportion of shoppers increase competition between firms, firms cannot sustain any one of the above mentioned symmetric equilibria, as each firm has an incentive to produce the other quality level. Recall the sequential consumer search model for homogeneous goods (e.g., Stahl (1989)) and note the difference. In the homogeneous goods model, a symmetric equilibrium does not exist. As the search cost decreases, the consumer's reservation price also decreases. In the limit, the price becomes equal to the marginal cost and firms make zero profit. But in this case, each firm has the option to deviate from the symmetric strategies by producing a different quality in order to counter the high degree of competition and make a positive profit.

4.4 Discussion and Conclusion

In this chapter, I investigate the relationship between vertical differentiation and search friction in a market. More specifically, how does search friction affect a firm's decisions on both price and quality? I have considered a duopoly market where each firm simultaneously chooses its product quality and price. High quality products are more costly to produce. Consumers have identical valuation for the low quality product but they differ with respect to the extra premium they are willing to pay for high quality. Some consumers search costlessly, whereas the rest of the consumers have a positive search cost. The results of this chapter show that the existence of homogeneous products in a market depends on choices made by firms. If the search costs are small or the proportion of consumers who search costlessly (shoppers) is large, firms have incentives to differentiate themselves vertically. On the other hand, if the search costs are large and the proportion of shoppers is small, this incentive does not exist. In such a case, I show that a symmetric equilibrium exists where both firms produce either high or low quality.

In this chapter, I have shown that if search costs are small or the proportion of shoppers is large, firms have incentives to differentiate themselves vertically. As the analysis gets extremely complicated, I have not characterized any equilibrium for small search cost or large proportion of shoppers. In my opinion, two potential equilibrium candidates would merit further research. The first would be to check if there exists any asymmetric equilibrium where one firm produces high quality and the other firm produces low quality, as I have shown that for small serach cost or large proportion of shoppers firms have incentives to differentiate themselves vertically. In such a case, as there is search friction in the market, one firm will randomize its price over a support, and the other firm will randomize its price over a different support. In the limit, as the search cost becomes zero or all consumers become shoppers, the two supports should converge to the respective degenerating prices of the standard vertical differentiation model without any search cost. Another possible equilibrium candidate would be the symmetric one in which both firms randomize both quality and price. By randomizing over qualities, firms can create some degree of uncertainty in terms of quality provision in the market which softens the competition between firms. This would also give rise to an element of vertical differentiation in the market.

Appendix

Calculation of $E_H(p)$ for Symmetric equilibrium with high quality

To calculate $E_H(p)$, I solve Eqn. (4.3), which gives

$$p = \frac{\rho_H + ac(1 - F_H(p))}{1 + a(1 - F_H(p))},$$

where $a = \frac{2\lambda}{1-\lambda} > 0$.

By changing variables, I get $E_H(p) = \int_0^1 pdz$ and substituting p from the above equation gives

$$E_H(p) = \rho_H \int_0^1 \frac{dz}{1 + az} + ac \int_0^1 \frac{z}{1 + az} dz$$

$$= \rho_H \frac{ln \mid 1 + a \mid}{a} + ac[\frac{1}{a} - \frac{ln \mid 1 + a \mid}{a^2}]$$

$$= \alpha \rho_H + (1 - \alpha)c,$$

where $\alpha = \frac{ln|1+a|}{a}$, with $\alpha \in [0,\, 1)$.

Proof of inequality (4.7)

Inequality (4.7) states the following:

$$\lambda \theta_i^* (\underline{p_H} - \theta_i^* \bar{q}) \geq \frac{(1-\lambda)s}{2(1-\alpha)}.$$

We have $\underline{p_H} = \frac{(1-\lambda)\rho_H + 2\lambda c}{1+\lambda}$ and $\rho_H = c + \frac{s}{1-\alpha}$.

Plugging in ρ_H in $\underline{p_H}$ and subsequently $\underline{p_H}$ in inequality (4.7) we have

$$\lambda \theta_i^* (c + \frac{(1-\lambda)s}{(1+\lambda)(1-\alpha)} - \theta_i^* \bar{q}) \geq \frac{(1-\lambda)s}{2(1-\alpha)}.$$

Now, as $s \to 0$ or $\lambda \to 1$,both $\frac{(1-\lambda)s}{(1+\lambda)(1-\alpha)}$ and $\frac{(1-\lambda)s}{2(1-\alpha)} \to 0$.

Hence, assuming s is sufficiently small or λ is sufficiently large, maximizing the left hand side of the above inequality with respect to θ_i^* gives $\theta_i^* = \frac{c}{2\bar{q}}$. Using this θ_i^* in the left hand side of the above inequality and assuming that s is sufficiently small or λ is sufficiently large, one gets $\frac{\lambda c^2}{4\bar{q}}$ as a lower bound of the left hand side of the above inequality, which is strictly positive.

Hence, for all finite $\bar{q}$ and c, with λ sufficiently large or s sufficiently small, the inequality (4.7) holds.

Calculation of $E_L(p)$ for Symmetric equilibrium with low quality

To calculate $E_L(p)$, I solve Eqn. (4.8), which gives

$$p = \frac{\rho_L}{1 + a(1 - F_L(p))},$$

where $a = \frac{2\lambda}{1-\lambda} > 0$.

By changing variables, we obtain $E_L(p) = \int_0^1 pdz$ and substituting p from the above equation we have

$$E_L(p) = \rho_L \int_0^1 \frac{dz}{1+az}$$

$$= \rho_L \frac{ln \mid 1+a \mid}{a}$$

$$= \alpha \rho_L,$$

where $\alpha = \frac{ln|1+a|}{a}$, with $\alpha \in [0,\, 1)$.

Proof of inequality (4.10)

Inequality (4.10) states the following:

$$\lambda(1-\theta_i^*)(\theta_i^* \bar{q} + \underline{p_L} - c) \geq \frac{(1-\lambda)s}{2(1-\alpha)}.$$

We have $\underline{p_L} = \frac{(1-\lambda)\rho_L}{1+\lambda}$ and $\rho_L = \frac{s}{1-\alpha}$.

Plugging in ρ_L in $\underline{p_L}$ and subsequently $\underline{p_L}$ in inequality (4.10) we have

$$\lambda(1-\theta_i^*)(\theta_i^* \bar{q} + \frac{(1-\lambda)s}{(1+\lambda)(1-\alpha)} - c) \geq \frac{(1-\lambda)s}{2(1-\alpha)}.$$

Now, as $s \to 0$ or $\lambda \to 1$, both $\frac{(1-\lambda)s}{(1+\lambda)(1-\alpha)}$ and $\frac{(1-\lambda)s}{2(1-\alpha)} \to 0$.

Hence, assuming s is sufficiently small or λ is sufficiently large, maximizing the left hand side of the above inequality with respect to θ_i^* gives $\theta_i^* = \frac{\bar{q}+c}{2\bar{q}}$. Using this θ_i^* in the left hand side of the above inequality, and assuming that s is sufficiently small or λ is sufficiently large, one gets $\frac{\lambda(\bar{q}-c)^2}{4\bar{q}}$ as a lower bound of the left hand side of the above inequality, which is strictly positive. The demand term $\lambda(1-\theta_i^*) = \frac{\bar{q}-c}{2\bar{q}}$ would be positive if $\bar{q} > c$, which is the case in my model.

Hence, for all finite $\bar{q}$ and c, if λ is sufficiently large or s is sufficiently small, the inequality (4.10) holds.

Proof of inequality (4.13)

Inequality (4.13) states the following:

$$\frac{(1-\lambda)s}{2(1-\alpha)} \geq (\frac{1+\lambda}{8\bar{q}})[c + \frac{s}{(1-\alpha)} - \frac{(1-\lambda)s}{(1+\lambda)}]^2$$

$$\iff (\frac{1+\lambda}{4\bar{q}})[c + \frac{s}{(1-\alpha)} - \frac{(1-\lambda)s}{(1+\lambda)}]^2 - \frac{(1-\lambda)s}{(1-\alpha)} \leq 0$$

$$\iff f(s) \leq 0. \tag{4.17}$$

Now the second order derivative of $f(s)$ with respect to s is given by

$$(\frac{1+\lambda}{2\bar{q}})(\frac{1}{1-\alpha} - \frac{1-\lambda}{1+\lambda})^2.$$

The function $f(s)$ is continuous in s and it is easy to check that the second order derivative of $f(s)$ exists and is positive for $s > 0$ and $\lambda \in (0, 1]$. Hence, the function $f(s)$ is convex in s.

If the equation $f(s) = 0$ has positive roots s_1 and s_2 with $s_1 < s_2$, then the inequality (4.17) is satisfied for $s \in (s_1, s_2)$. The reservation price ρ_H is increasing in s. For very high values of s, ρ_H becomes equal to x which I do not consider. I focus on those values of s with $s \in (s_1, s_2)$ such that $\rho_H < x$.

Figures 4.5 and 4.6 shows the graphs of $f(s)$ for a given $\bar{q}$ and c, and for two different values of λ(small and large). It is visible from the Figure 4.6 that the equation $f(s) = 0$ does not have positive roots for λ sufficiently large.

Therefore, the inequality (4.13) is satisfied for sufficiently large values of s provided λ is sufficiently small.

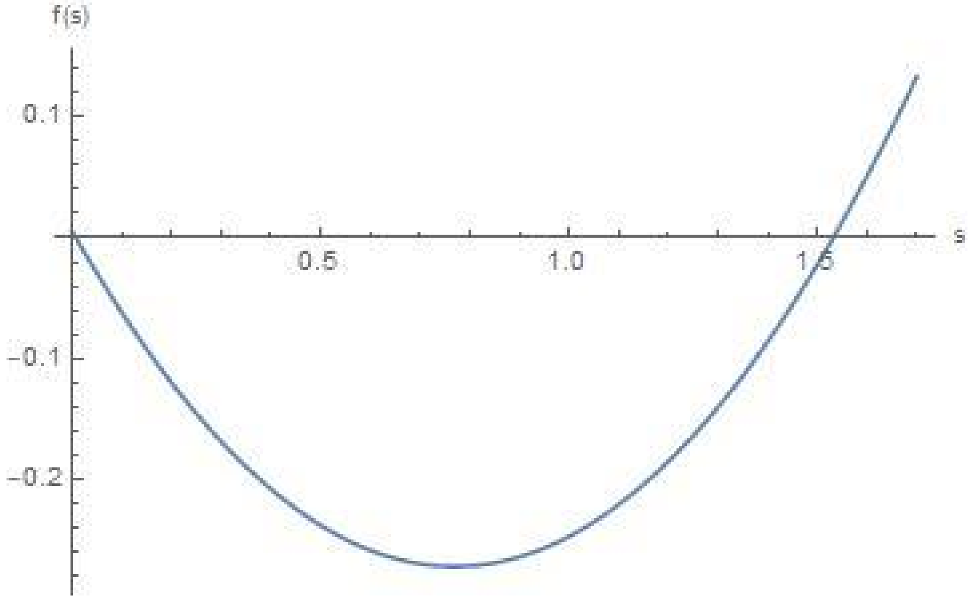

Figure 4.5: Graph of $f(s)$ for $\bar{q} = 0.5$, $c = 0.25$, $\lambda = 0.4$

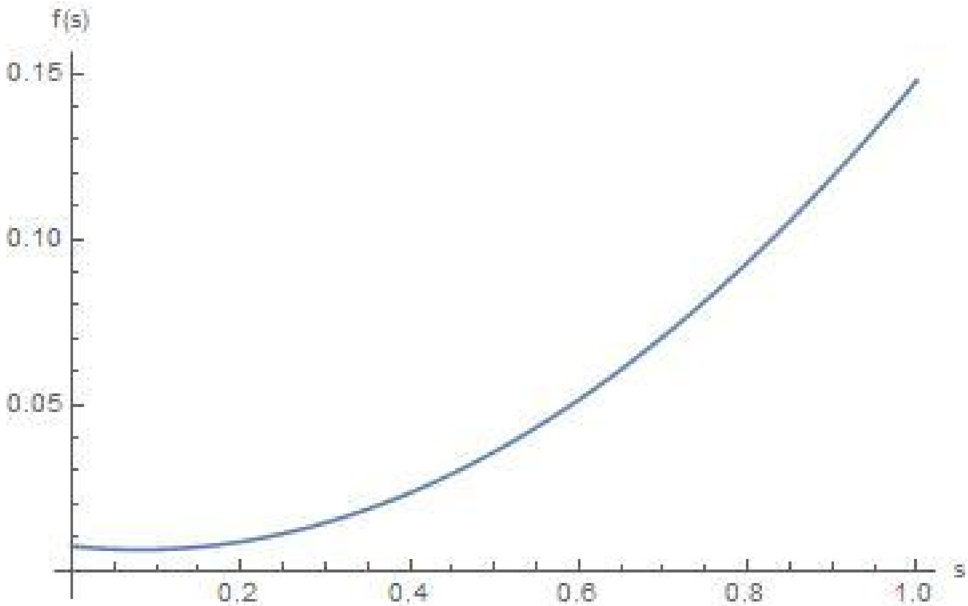

Figure 4.6: Graph of $f(s)$ for $\bar{q} = 0.5$, $c = 0.25$, $\lambda = 0.85$

Proof of inequality (4.16)

Inequality (4.16) states the following:

$$\frac{(1-\lambda)s}{2(1-\alpha)} \geq \frac{(1+\lambda)}{8\bar{q}}[\bar{q} - c + \frac{s}{(1-\alpha)} + \frac{(1-\lambda)s}{(1+\lambda)}]^2$$

$$\iff (\frac{1+\lambda}{4\bar{q}})[\bar{q} - c + \frac{s}{(1-\alpha)} + \frac{(1-\lambda)s}{(1+\lambda)}]^2 - \frac{(1-\lambda)s}{(1-\alpha)} \leq 0$$

$$\iff g(s) \leq 0. \tag{4.18}$$

Now the second order derivative of $f(s)$ with respect to s is given by

$$(\frac{1+\lambda}{2\bar{q}})(\frac{1}{1-\alpha} + \frac{1-\lambda}{1+\lambda})^2.$$

The function $g(s)$ is continuous in s and it is easy to check that the second order derivative of $g(s)$ exists and is positive for $s > 0$ and $\lambda \in (0, 1]$. Hence, the function $g(s)$ is convex in s.

If the equation $g(s) = 0$ has positive roots s_1 and s_2 with $s_1 < s_2$, then the inequality (4.18) is satisfied for $s \in (s_1, s_2)$. The reservation price ρ_L is increasing in s. For very high values of s, ρ_L becomes equal to x which I do not consider. I focus on those values of s with $s \in (s_1, s_2)$ such that $\rho_L < x$.

Figures 4.7 and 4.8 shows the graphs of $g(s)$ for a given $\bar{q}$ and c, and for two different values of λ(small and large).

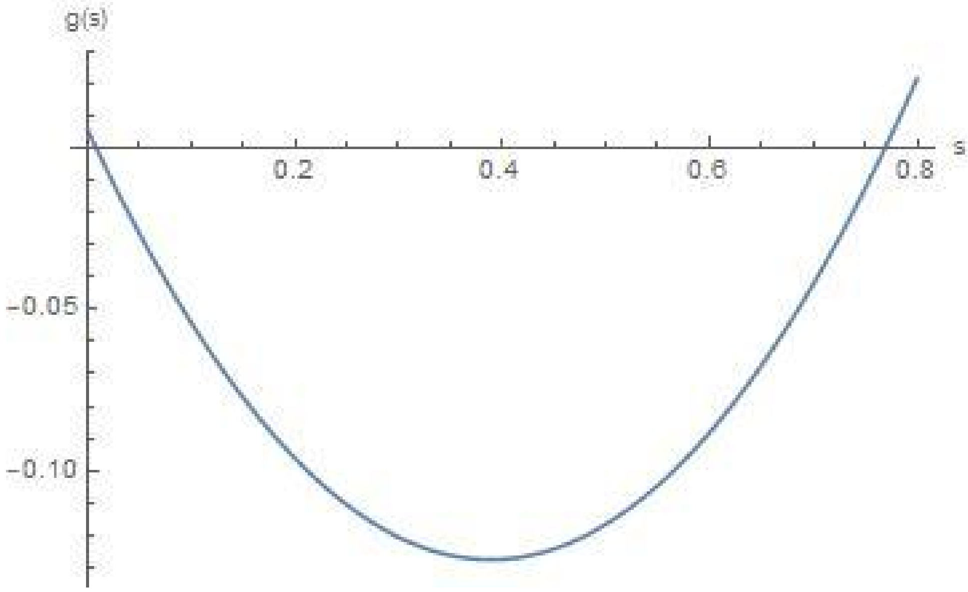

Figure 4.7: Graph of $f(s)$ for $\bar{q} = 0.5$, $c = 0.25$, $\lambda = 0.4$

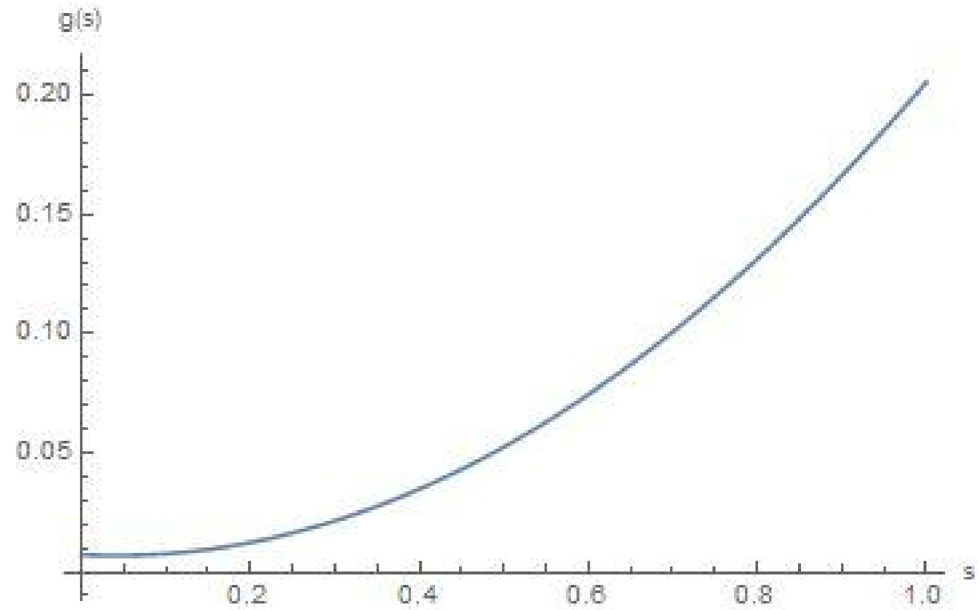

Figure 4.8: Graph of $f(s)$ for $\bar{q} = 0.5$, $c = 0.25$, $\lambda = 0.85$

It is visible from the Figure 4.8 that the equation $f(s) = 0$ does not have positive roots for λ sufficiently large.

Therefore, the inequality (4.16) is satisfied for sufficiently large values of s provided λ is sufficiently small.

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Abstract

In this dissertation, I discuss how firms deal with the issue of price-quality competition in three different markets.

In the first essay, I analyze a symmetric duopolistic market where each firm's choice regarding certain quality attributes such as the environmental friendliness of its product is its own private information. I find that the extent of horizontal differentiation between firms plays a crucial role in a certifier's optimal certification policy. Under a non-profit certifier it is always the case that both firms produce the highest quality and opt for certification. This is also the case under a for-profit certifier, but only when the degree of horizontal differentiation is sufficiently high. When horizontal differentiation is low, the for-profit certifier, by charging a very high certification fee, creates maximum vertical differentiation between firms. As a result, only one firm produces the highest quality and opts for certification whereas the other firm produces the lowest quality and does not opt for certification. This asymmetry under a for-profit certifier makes the market inefficient, which provides one possible explanation for the existence of mostly non-profit certifiers in such markets.

In the second essay, I analyze the interaction between the level of quality improvement of a product and a monopolist firm's pricing strategy where the firm can either sell or offer a subscription. When the level of quality improvement and the cost of such an improvement are small, the firm will offer a subscription. However, when the quality of the product that is offered in the future period is significantly higher than in the current period, and the cost of producing it is also high, it is better for the monopolist to sell its products rather than to offer a subscription.

In the third essay I show that the existence of homogeneous products in a market depends on choices made by firms. If search costs are small or the proportion of consumers who search costlessly (shoppers) is large, firms have incentives to differentiate themselves vertically. On the other hand, if search costs are large and the proportion of shoppers is small, this incentive does not exist. Therefore, with large search costs and a small proportion of shoppers, firms produce homogeneous products.

Zusammenfassung

In dieser Dissertation untersuche ich, wie Unternehmen mit dem Thema Preis-Qualitäts-Wettbewerb auf drei verschiedenen Märkten umgehen.

Im ersten Teil analysiere ich einen symmetrischen duopolistischen Markt, auf dem die Entscheidung jedes Unternehmens hinsichtlich bestimmter Qualitätsmerkmale - wie die Umweltfreundlichkeit des Produkts - seine private Information ist. Die Studie findet heraus, dass der Grad der horizontalen Differenzierung zwischen den Unternehmen eine entscheidende Rolle bei der optimalen Zertifizierungspolitik spielt. Bei einem nicht gewinnorientierten Zertifizierer ist es immer der Fall, dass beide Firmen die höchste Qualität produzieren und sich für die Zertifizierung entscheiden. Bei einem gewinnorientierten Zertifizierer ist dies hingegen nur dann der Fall, wenn der Grad der horizontalen Differenzierung ausreichend hoch ist. Wenn die horizontale Differenzierung gering ist, schafft der gewinnorientierte Zertifizierer maximale vertikale Differenzierung zwischen den Unternehmen, indem er eine sehr hohe Zertifizierungsgebühr erhebt. Dadurch produziert in diesem Fall nur eine Firma die höchste Qualität und entscheidet sich für die Zertifizierung, während die andere Firma die niedrigste Qualität herstellt und keine Zertifizierung beschließt. Die Asymmetrie unter dem gewinnorientierten Zertifizierer führt zur Marktineffizienz, was eine mögliche Erklärung für die Existenz von nicht gemeinnützigen Zertifizierern in solchen Märkten ist.

Im zweiten Teil analysiere ich die Interaktion zwischen dem Grad der Qualitätsverbesserung eines Produkts und der Preisstrategie eines monopolistischen Un-ter-neh-men-s, wobei das Unternehmen das Produkt entweder verkaufen oder ein Abonnement anbieten kann. Wenn das Niveau und die Kosten der Qualitätsverbesserung gering sind, bietet die Firma ein Abonnement an. Wenn jedoch die Qualität des

Produkts in der Zukunft deutlich höher ist als in der Gegenwart und zudem die Kosten für die Herstellung hoch sind, ist es für den Monopolisten lukrativer, seine Produkte zu verkaufen.

Im dritten Teil zeige ich, dass die Existenz homogener Produkte auf einem Markt von den Entscheidungen der Unternehmen abhängt. Wenn die Suchkosten gering sind oder der Anteil der Verbraucher, die kostenlos suchen (Shopper), groß ist, haben die Unternehmen Anreize sich vertikal zu differenzieren. Wenn die Suchkosten hingegen hoch sind und der Anteil der Shopper gering ist, besteht dieser Anreiz nicht. Demzufolge führen hohe Suchkosten und ein niedriger Anteil an Shoppern zur Produktion homogener Produkte.