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1. Synopsis

1 Introduction

The participation of the consumer in the price determination process stands out as one of the most innovative and interesting developments in pricing in recent years. Although consumers have been involved in setting the price since as early as ancient times, when haggling and auctions were the most popular method of price determination (Phillips 2012), this topic was surprisingly largely neglected by economics and marketing researchers for a long time. It was not until the early 2000s that researchers considered participative pricing worthy of scholarly attention. Driven by the new possibilities of online pricing and some very successful offline examples (Chandran & Morwitz, 2005), several studies have begun to examine how these pricing mechanisms work from the perspective of a firm as well as that of consumers.

This dissertation seeks to shed light on two approaches to engaging the consumer in pricing – the participative pricing scheme 'pay-what-you-want' (PWYW) and the pricing tactic 'gambled price promotions'. The following synopsis will provide an overview of the extant literature on PWYW that serves as a background for articles 1-3 and a brief overview of gambled promotions, which serves as a background for article 4. Finally, it will offer an overview of all four contributions.

2 Participative pricing and PWYW

2.1 Classifications of participative pricing mechanisms

While fixed pricing is the dominant pricing mechanism in practice, participative pricing has gained increasing attention over the last 20 years (Spann et al., 2017). The term participative pricing describes a pricing structure that lets the consumer have a say in the determination of the final price level (Chandran & Morwitz, 2005). From a strategic point of view, a firm that adopts a participative pricing mechanism limits its pricing decision on the price structure and leaves the price level up to the consumer. There are different ways in which the consumer can be involved in the price-setting process. Four primary classifications have been proposed in participatory pricing.

The first classification distinguishes between structured and unstructured pricing mechanisms (Chandran & Morwitz, 2005). Structured pricing mechanisms include auctions in which buyers compete for the seller's product by bidding and only the winners of the auction obtain access to actually purchasing the product. Meanwhile, reverse auctions involve multiple sellers competing to carry out the buyer's offer by bidding lower prices. Then there are formal negotiations, wherein buyers and sellers alternatingly exchange bids until they arrive at an agreement (e.g., when buying a house). Unstructured participative pricing mechanisms include consumer-to-consumer interactions such as garage sales and haggling.

The second classification distinguished between mechanisms with horizontal interaction (auction, reverse auction, and exchange) and one-to-one price mechanisms like PWYW, name-your-own-price (NYOP), and negotiations (Kim, Natter, & Spann, 2009). In addition to the mechanisms in the first classification, this includes exchanges, PWYW, and NYOP. In exchanges, multiple sellers and buyers submit their offers and bids. In PWYW, buyers can set any price, and the seller cannot reject it. The price can even be zero. NYOP is a two-step pricing scheme where the seller first sets a secret minimum price, followed then by the buyer’s bid. Whenever the bid is higher than the minimum, the buyer can then buy the product for this bidding price.

Bertini and Koenigsberg (2014) categorized pricing mechanisms in a third classification according to consumer participation as being a) company imposed (fixed price, personalized prices, price menus), b) collaborative (auction, name-your-own-price, negotiation), or c) consumer-imposed (PWYW). Company-imposed pricing mechanisms give the consumer no or little power to influence the price. In the case of fixed pricing and personalized prices, the seller presents a take-it-or-leave-it offer that is either equal for all consumers (fixed pricing) or different for each consumer (personalized prices). In price menus, the seller proposes multiple options, e.g., different quality levels at different prices, and the buyer can select the most suitable offer. Collaborative tools require mutual understanding and, therefore, both parties have to be satisfied with the conditions of the interaction. PWYW marks the most extreme instance of consumer participation and relies solely on the goodwill of the buyer.

A fourth categorization, proposed by Spann et al. (2017), places the nucleus of the classification on two dimensions – the competition among the buyers and the interactivity between the buyer and the seller post bidding. Auctions are classified as featuring competition among buyers. They can be further distinguished into auctions where the seller can set a hidden minimum price and auctions that have a public minimum price. NYOP, bargaining, and PWYW do not involve competition among buyers. Furthermore, NYOP and bargaining allow the seller to influence the final price by deciding whether to accept the buyer’s offer, while the consumer decides independently on the price in PWYW.

Several lines of evidence suggest that participative pricing mechanisms yield various advantages for the seller. Consumer participation can allow the seller to distinguish him/herself from the competition (Fay, 2004; Schmidt, Spann, & Zeithammer, 2015), achieve price differentiation (Krämer, Schmidt, Spann, & Stich, 2017; Schmidt et al., 2015; Terwiesch, Savin, & Hann, 2005), increase market penetration (Krämer et al., 2017; Schmidt et al., 2015), evoke positive feelings and therein influence the overall company image (Fay & Laran, 2009). In sum, participative pricing mechanisms present an additional choice and can be more profitable than posted prices (Kim et al., 2009; Shapiro, 2011).

Having defined what is meant by participative pricing, we will now move on to discuss one form in detail, namely PWYW, which is the thrust of the first three contributions of this dissertation.

2.2 Payment drivers in PWYW

When PWYW was introduced as a business model at the end of the first decade of the 21st century, it gained widespread attention. The notion that sellers effectively charge a price of zero and that consumers are still willing to voluntarily pay prices that keep the seller in business was perceived to contradict common standards and economic principles, which, in turn, led to the intense mass media coverage of this topic (Kim, Natter, & Spann, 2010; Tyrangiel, 2007).

Since then, PWYW has also received increased attention in academic literature. The pioneering scientific articles on PWYW were published in leading academic journals. First, Kim et al. (2009) introduced the price mechanism to the scientific literature. They showed that consumers pay more than they have to and that PWYW can even become more profitable than traditional pricing. Furthermore, they investigated payment factors and showed that consumer's fairness, satisfaction, price consciousness, and income all influence the price which the buyer sets. Second, Regner and Barria (2009) investigated PWYW with a minimum price and showed that it can be a profitable pricing scheme, that consumers pay more than they have to, and that reciprocity is a payment driver in their setting. Third, Gneezy, Gneezy, Nelson, and Brown (2010) showed that using PWYW along a charitable component can even enhance the profitability of PWYW and that the combination of PWYW and charitable contributions can even outperform traditional pricing models by a considerable margin.

Following these initial contributions, an increasing amount of literature has honed in on PWYW, replicating the notion that it can be an effective payment scheme (Gerpott, 2018). Furthermore, numerous studies have attempted to explain why consumers prefer to pay something instead of nothing in PWYW (for comprehensive reviews of the empirical literature see Chung, 2017; Gerpott, 2018; Natter & Kaufmann, 2015; Stegemann, 2014). In the following, we outline these findings.

The consumer's payment motives can be classified into three groups: a) egoistic, b) other-regarding, and c) image preferences. In what follows, we give a brief overview of empirical results with these preferences. For a more detailed discussion of the empirical results, we point to the previously mentioned literature reviews.

a) *Egoistic payment preferences*: Consumers might pay because they have a preference for the consumption of the product. In this case, they pay for egoistic or strategic reasons. For example, consumers with egoistic preferences might want to keep the seller in business to profit from the PWYW offer in future. If they were to pay too little for their current consumption, the seller would go bankrupt and they would not be able to consume in the future. Normally, a single buyer alone can hardly pay enough to sustain the PWYW offer of a seller. However, when several buyers cooperate and pay voluntarily, PWYW could be sustainable for the seller and cheaper for the buyers than a fixed price seller. The price which a consumer with such preferences chooses depends on his/her preference for current and future consumption. Similarly, consumers might believe that paying fair prices ensures better treatment by the seller (Lynn & Grassman, 1990).

   Several studies illustrate that consumers have (partly) egoistic motives when they decide on the price in PWYW. Kunter (2015) finds that 15% of the consumers indicate that they determine their price based on the economic attractiveness of the offer. Individual monetary motives such as price consciousness have shown to be consistent negative payment drivers (Kim et al., 2009; Kunter, 2015; Schons et al., 2014), pointing to egoistic preferences as important drivers in PWYW. On that note, Mak, Zwick, Rao, and Pattaratanakun (2015) and Schmidt et al. (2015) have consistently shown support for the notion that egoistic motives play an important role in laboratory PWYW experiments and that consumers pay and coordinate to keep the seller in business.

b) *Other-regarding preferences:* Other-regarding preferences might drive consumers' payment motives, i.e., consumers not only care about their own consumption but also about the outcome of the seller. The literature describes different kinds of other-regarding preferences that can determine the underlying payment motives. Despite the lack of a consistent terminology (Dhami, 2016), these preferences may be divided into three main sub-categories: i) altruism, ii) reciprocity and iii) equity concerns.

i) *Altruism* is a form of unconditional kindness. An altruist does not expect anything from the other in return for being kind. Altruism can be further divided into pure and impure altruism. Purely altruistic preferences imply that the consumer contributes to benefit the other out of empathy and without getting anything in return (Batson & Shaw, 1991; Becker, 1974). Impure altruism, sometimes also called warm glow altruism, describes kind acts out of egoistic motives, i.e., to relieve feelings of guilt and pity or because the consumer wants to feel good about him/herself (Andreoni, 1989, 1990). Previous empirical studies provide evidence that altruism is positively correlated with PWYW payments, suggesting that the consumers' altruistic preferences play a role in PWYW pricing (Gahler, 2016; Huber, Lenzen, Meyer, & Appelmann, 2015; Jung, Nelson, Gneezy, & Gneezy, 2017; Kim et al., 2009; Roy, Rabbanee, & Sharma, 2016; Schmidt et al., 2015).

ii) *Reciprocity* describes an individual's tendency to reward the kind actions of others while punishing unkind actions. An extended concept of reciprocity is intention-based reciprocity. In intention-based reciprocity, people consider not only the other's actions but also their intentions (for an overview of different kinds of reciprocity, cf. Dhami, 2016). In this case, people who want to be kind are rewarded and people with negative intentions are punished (Dufwenberg & Kirchsteiger, 2004; Falk & Fischbacher, 2006; Rabin, 1993). If a consumer pays the seller because he/she perceives the seller's PWYW offer as a kind act, this points to reciprocal preferences. Several empirical studies point to the importance of reciprocity as a payment motive in PWYW (Gravert, 2017; Kim et al., 2009; León, Noguera, & Tena-Sanchez, 2012; Regner, 2015; Regner & Barria, 2009).

iii) Approaches on *equity and fairness motives* concentrate on the preferences for the distribution of the payoffs between different individuals, i.e., in PWYW, buyer and seller. A popular approach includes inequity-averse preferences. In this case, consumers prefer equitable outcomes, i.e., outcomes that are considered to be fair, over inequitable outcomes, i.e., outcomes that favor one party disproportionally. To establish equity, privileged individuals are willing to share some of their material payoffs with others (Fehr & Schmidt, 1999). Note that inequity aversion does not necessarily require an equal distribution between buyer and seller but rather a "fair distribution" of the outcome (Starmans, Sheskin, & Bloom, 2017).

The terms fairness and equity are sometimes used interchangeably in a PWYW context. Marketing has a long history of research on price fairness perceptions (Xia, Monroe, & Cox, 2004). To judge whether a price is fair, consumers compare the seller's prices to a benchmark price. This price might follow the dual entitlement principle that argues that firms are entitled to a reference profit, and consumers are entitled to a reference consumer rent (Kahneman, Knetsch, & Thaler, 1986b). Other approaches posit that consumers compare the price to a 'typical' or reference price (Mazumdar, Raj, & Sinha, 2005; Monroe, 1973). In fixed pricing, the seller's price is typically above the benchmark price and, hence, perceived as unfair (Thaler, 1983).

This fair benchmark is also an important determinant in judging the fairness of the PWYW price. However, in PWYW, consumers set their own pieces. Hence, they might choose a price that is below the benchmark price and therefore behave unfairly on their part. If consumers long for equity, they will adjust their prices accordingly. Several studies show that equity and fairness preferences are associated with PWYW payments (Jang & Chu, 2012; Kim, Kaufmann, & Stegemann, 2014; Schmidt et al., 2015; Schons et al., 2014).

Two studies have compared which of the above other-regarding preferences were empirically dominating. Schmidt et al. (2015) used a laboratory environment and compared egoistic, altruistic, reciprocal, and equity explanations. Their results were consistent with egoistic, altruistic, and inequity aversion preferences but not with reciprocity. Jung et al. (2017) examine a PWYW setting where part of the revenues goes to charity and compare explanations of reciprocity, impure altruism, and equity and find that impure altruism was the most consistent explanation. It is, however, essential to note that these preferences are not mutually exclusive and that consumers could be motivated by more than one preference.

c) *Image preferences:* Consumers might pay because they have image concerns. Image concerns describe that individuals care about how their actions appear to themselves or others. The literature examines two different types of image concerns: social image concerns and self-image concerns.

If individuals care about their *social image,* they will take actions that make them look good in the eyes of others as they feel pressure to behave according to norms and want to signal their socially appropriate actions to others (Batson & Powell, 2003). In PWYW, paying fair prices that satisfy or even exceed social norms should increase one's social image. In line with this finding, various studies show that consumers spend more when their payment is more visible to others and less when their payment has been made anonymously (e.g. Kim et al., 2014; Riener & Traxler, 2012).

The concept of *self-image* implies that people draw conclusions about their own kindness from their actions. They compare their behavior with what they believe to be morally correct and aim to align their actions with the identities they want to uphold themselves (Baumeister, 1998; Bem, 1977; Bénabou & Tirole, 2006). Applied to PWYW, consumers who have self-image concerns like to see themselves as kind and pay the seller accordingly. Gneezy, Gneezy, Riener, and Nelson (2012) analyze three field experiments and find that self-image concerns are important payment drivers in PWYW. Furthermore, Kunter (2015) finds that consumers pay to relieve their feelings of shame or guilt, which is closely related to these self-image concerns. Gravert (2017) also offers self-image as a potential determinant of PWYW payments.

2.3 Foundations of PWYW models

In sum, the empirical literature demonstrates that PWYW can work and why consumers pay. Additionally, it shows from a seller's perspective, that PWYW can be more successful than fixed prices. However, fixed pricing is still the dominant pricing mechanism in practice. Moreover, examples abound for unsuccessful PWYW applications (e.g., the American coffee chain Panera Bread (Patton, 2019) and the Austrian airline People's Viennaline (Schürmann, 2017) discontinued their offer after reporting many non-paying consumers). In such cases, PWYW proved to be of little avail, and consumers paid very low prices or nothing at all resulting in enormous losses for the seller (Gautier & van der Klaauw, 2012; León et al., 2012).

For the seller to become profitable, prices need to cover at least variable costs. To that extent, with respect to the consumer, egoistic, other-regarding, and image preferences need to be sufficient, and with respect to the seller, costs should not be too high. Hence, the success of a PWYW system depends on the interplay of (at least) these factors. However, empirical studies do not allow precise guidelines for sellers to choose between PWYW and other mechanisms from the pricing toolkit. In pursuit of an analytical assessment of the advantages of PWYW over fixed prices, theoretical models that include the different consumer and seller parameters simultaneously seek to fill this void.

2.3.1 The standard microeconomic model applied to PWYW

In traditional economic price theory, the price is determined by supply and demand. The seller assesses the consumers' demand and sets the price. Afterwards, the consumers make a take-it-or-leave-it decision considering that price. Whenever the utility from obtaining the good for the seller's price is above 0, a consumer buys; otherwise, he/she refrains from purchasing. Economists strive to build models that make normative predictions and guidelines on how to optimize profits by choosing the right pricing mechanism and prices. To do so, the microeconomic theory relies on several assumptions about the consumers and the seller.

The consumers are assumed to have rational preferences. Hence, they can consistently express which products they like or dislike and behave in such a way that they achieve the best result for themselves. These preferences are then represented by a utility function that encodes the magnitude of liking in numerical values. In the simplest case of a consumer-seller-interaction, the utility function, u_i, of consumer i is given by

$$u_i = \begin{cases} r_i - p & \text{if the consumer buys} \\ 0 & \text{if the consumer does not buy} \end{cases} \quad (1)$$

where r_i describes consumer i's benefit from consuming the product and p describes the price the consumer has to pay in order to obtain the good. Thus, the consumer's utility increases, the more he/she prefers to consume the good, and decreases with higher prices. If the consumer does not buy, the utility will be 0. Note that utility functions are cardinal concepts. For that reason, monotonic transformations should be valid. However, there is no natural zero point (Varian, 2010). To include the participation constraint in the model, we assume that the outside option (i.e., not buying) gives zero utility. To find their best outcome, consumers maximize utility, i.e.,

$$\max u_i. \quad (2)$$

Thus, for

$$r_i - p > 0$$

$$r_i > p$$

the consumer will buy. Otherwise, the consumer will not buy. This model is rooted in the idea of a 'homo oeconomicus.' The utility function only considers preferences for the consumption of the good and its monetary costs. The consumer does not care about the seller's profits or losses or whether his or her actions are fair or image-boosters or not.

The preferences of the different consumers are heterogeneous, i.e., the extent to which consumers like a good varies among them. To model this heterogeneity, we assume that r_i is distributed to some distribution $\phi(r_i)$.

The seller in classic price theory is assumed to be profit-maximizing. For every good that is sold, the seller will receive the price p from the buyer and has to bear the costs for the good, c. Thus, in the simplest form, the seller's profits, π, will be given by

$$\pi = (p - c)q \tag{3}$$

with p the price the seller sets, c the costs and q the quantity sold. In the classic model, the seller has the full scope of information on the utility functions of the buyers. The quantity sold is given by the demand of the consumers, i.e., all consumers with a positive utility function, $u_i > 0$. All consumers whose preference for consumption is above the seller's price will buy. The seller can therefore calculate the demand of the buyers,

$$q = D(p) = \int_{p \geq c} \phi(r_i) dr_i. \tag{4}$$

This allows us to define the seller's profits,

$$\pi = \int_{p \geq c} (p - c)\phi(r_i) dr_i. \tag{5}$$

Here, we assume that the seller can set the price (i.e., the seller is a monopolist with pricing power). As the seller is profit-maximizing, he/she is trying to find the optimal price. Hence,

$$\max_p \pi. \tag{6}$$

For computational simplicity, researchers often assume that r_i is distributed according to a uniform distribution over the domain $[0,1]$ without loss of generality. Thus, $\phi(r_i) = 1$ and, therefore, (5) simplifies to

$$\begin{aligned} \pi &= \int_{p \geq c}^{1} (p - c)\phi(r_i) dr_i \\ &= \int_{p \geq c}^{1} (p - c) 1 dr_i \\ &= (1 - p)(p - c). \end{aligned} \tag{7}$$

We can solve (6) for p,

$$\frac{\partial \pi}{\partial p} = 1 + c - 2p,$$

and, therefore, the profit-maximizing price, p^*, is given by

$$p^* = \frac{1 + c}{2}. \tag{8}$$

The price is profit-maximizing as $\frac{\partial^2 \pi}{\partial p^2} = -2 < 0$.

Now the seller can calculate its optimal profits, π^*, by inserting (*8*) in (*7*),

$$\pi^* = \frac{(1 - c)^2}{4}. \tag{9}$$

If we apply this reasoning to PWYW, standard microeconomic logic predicts that the seller will fail to make profits. In this case, the utility function of a consumer would be given by,

$$u_i = \begin{cases} r_i - p_i & \text{if the consumer buys} \\ 0 & \text{if the consumer does not buy} \end{cases}. \tag{10}$$

The consumer would set his/her individual price, p_i, that maximizes the utility.

Therefore,

$$\max_{p_i} u_i \tag{11}$$

and consequently, by the linear optimization theorem, the result is a corner solution; the consumer will pay the smallest feasible price. As negative prices are not possible in PWYW, i.e., the seller will not pay the consumer to buy the good. The utility-maximizing price from a rational consumer in PWYW is thus

$$p_i^* = 0. \tag{12}$$

This holds true for all consumers. Furthermore, all buyers will obtain the good. In this case, the seller cannot choose the price, and the seller's profits are simply given by inserting (*12*) in (*7*),

$$\pi = -c. \tag{13}$$

These results are in marked contrast to the empirical literature on PWYW (see Section 2.2). In this vein, the following section provides an overview of how PWYW could be modeled more successfully.

2.3.2 Behavioral economic models of consumer behavior

The standard microeconomic utility function in (*1*) does not leave room for the payment motives and preferences discussed in Section 2.2. The consumer only cares about the extent to which he/she values the product and the price he/she pays. However, over the last decades, behavioral economists and marketing scholars have begun to incorporate additional motives and preferences in the consumer's utility function.

An early example of such a utility function is the seminal study by Thaler (1983). He proposes a utility function that not only includes the value of the product but also the consumers' perceptions or happiness with the outcome of the transaction. Thaler (1983) models consumers who have a preference for making a 'good deal,' i.e., for buying the product cheaper than the price they expect to pay, i.e. their reference price. Doing so generates additional utility. Similarly, consumers lose utility when they make a 'bad deal,' i.e., when the price is above their reference price. The total utility of the consumer when buying can be broken down into the following: the standard utility component denoted 'acquisition utility' that corresponds to (*1*) and a non-standard utility component, 'transaction utility,' that corresponds to the perception of the outcome of the transaction. That is, the value the consumer assigns to consuming the good, r_i, is different than the overall utility from a purchase. Hence, the utility when the consumer buys is given by

$$u_i = \underbrace{r_i - p}_{\text{acquisition utility}} + \underbrace{v(p_f - p)}_{transaction\ utility} \tag{14}$$

where p_f represents the fair reference price and v the extent to which the consumer cares about transaction utility. The acquisition/transaction-utility approach does not capture PWYW behavior very well, as the consumer always prefers lower prices. This would imply that consumers want to pay as little as possible. However, it helps to illustrate how additional preferences, in this case– "making a deal" – can enter the utility function.

Using a similar approach, economists sought to model why people behave prosocially by adding non-standard utility components to utility functions. Most of this work has been done in the context of laboratory experiments where consumers play stylized games. One of the most prominent examples is a dictator game (Kahneman, Knetsch, & Thaler, 1986a). In this setup, two individuals are paired randomly; one plays the role of the dictator, and the other player is passive and has no influence on the outcome of the game. In the first stage, the dictator receives an endowment from the experimenter (e.g., $10 cash). The passive player does not receive anything. In the second stage, the dictator is asked to divide the endowment between him/herself and the passive player. A ‘homo oeconomicus’ dictator that only cares about his/her payoff would take the entire endowment. However, the key finding of these games has been that individuals care about the other’s payoff in addition to their share, on average, about 30 percent of the endowment (Engel, 2011). To understand the dictators’ behavior, their utility functions need to include a term that considers the payoff of the passive players.

$$u_i = v_{1,i}(\text{own payoff}) + v_{2,i}(\text{other's payoff}) \qquad (15)$$

where $v_{1,i}$ and $v_{2,i}$ describe how the individual values its own, $v_{1,i}$, and the other player’s, $v_{2,i}$, payoffs. Note that the shift from egoistic to social preferences is gradual; the consumer not only values his/her own payoffs, but relies solely on the other’s outcomes (André, Bureau, Gautier, & Rubel, 2017; Mauss, 1925).

Contingent on the type of prosocial preference, the second part of (*15*) differs with respect to whether it monotonously increases in the other’s payoffs (as in the case of altruism), or whether it depends on a fair allocation (as in the case of inequity models) or on the other’s behavior and intentions (as in the case of reciprocity). For an overview of different types of prosocial utility functions, cf. Fehr and Schmidt (2006) and Clavien and Chapuisat (2016). Similarly, (social and self-) image effects can be modeled by introducing a utility component that mirrors the preference for image-increasing outcomes.

The additional preferences might be conceptualized positively or negatively, i.e., consumers could be interpreted to value fair outcomes (e.g., through an additional self-image gain) or to devalue unfair ones (e.g., through the disapproval of the unfair allocation of resources). As utility functions do not have a natural zero point, positive and negative transformations are straightforward (Varian, 2010).

Most of the existing literature that models consumer behavior in PWYW is based on utility functions that resemble a combination of (*14*) and *(15)*,

$$u_i = r_i - p_i \pm \text{additional utility component.} \qquad (16)$$

Based on this premise, deriving optimal profits is similar to the classical microeconomic approach (see Section 2.3.1). In the case of fixed prices, the seller considers the additional preferences, derives demand, and sets the profit-maximizing price. The consumer then decides whether or not to buy. The seller’s profits result from the revenues achieved at the profit-maximizing price and the variable costs to be incurred by production. Recently, the additional utility component in participative pricing was also termed “process utility” (Spann et al., 2017).

In PWYW, the consumers decide on the price by maximizing their utility function and solving for p_i. The seller's profits are then given by the sum of prices minus the respective variable costs.

This flexible approach also allows us to analyze hybrid structures where the seller tries to influence the buyer, e.g., with the help of a minimum price or a suggested price. In this case, the seller determines this optimal (minimum or suggested) price. This price, in turn, influences prosocial perceptions. Based on these perceptions, the consumer maximizes his/her utility by deciding whether to make a purchase or not and by paying his/her utility-maximizing price. The seller's profits would then result from the aggregation of prices set by the consumers minus the costs.

A seller can choose between different pricing schemes (fixed pricing, PWYW, PWYW with a suggested price) and selects the one with highest profits.

In the following section, we will summarize the results of extant approaches to model PWYW. Three sub-areas are analyzed: consumer behavior in PWYW, the firm's profitability in PWYW, and modifications of PWYW, i.e., design choices (suggested and minimum price).

2.4 PWYW models

In pursuit of a comprehensive representation of the current state of theoretical work on PWYW, a literature search was conducted to identify relevant papers. We identified relevant papers by searching Google Scholar, Web of Science, EBSCO, Emerald, JSTOR, Sage, Springer, and Wiley for the keywords "pay what/as you want/can/like/wish".

In a second step, we scanned through each paper to identify whether it contains an analytical microeconomic model pertaining to PWYW. Papers with a purely empirical or conceptual approach were excluded from the review. Furthermore, we used a snowballing procedure to identify potentially missing papers by scanning through the reference lists of the papers identified. This left us with 14 peer-reviewed articles. We included all articles in our review.

In order to compare the results from different models, we will use the term "prosocial preference" to summarize the additional preference component in (*16*) that provides additional (dis)utility to the buyer. While acknowledging the contextual differences of the different concepts (other-regarding, image, or not explicitly specified preferences), this aims at simplifying and comparing the results from different models. Furthermore, to avoid bulky notation when summarizing the results from the different models, we will refer to the papers by their number in Tables 1-4. For each area investigated, we give recommendations for future research.

2.4.1 The scope of the studies

The scope of the studies is presented in Table 2. Out of the 14 papers analyzed, four papers focus solely on buying behavior (#4, 6, 10, 13). These papers concentrate on analyzing conditions for which consumers buy or do not buy, consumers' prices in PWYW, and the construction of the benchmark price. A single paper (#14) focusses exclusively on the seller and takes the buyer's voluntary payments as given. In this contribution, Tudón (#14) examines on the effect of demand uncertainty on PWYW and shows that higher demand uncertainty can make PWYW more beneficial than traditional pricing. The remaining nine papers analyze PWYW simultaneously for buyer and seller (#1, 2, 3, 5, 7, 8, 9, 11, 12). These papers concentrate on finding conditions for which PWYW is profitable, compare it to posted prices and examine PWYW in competition.

Seven papers discuss modifications of the PWYW mechanism, e.g., whether and how the seller should try to influence consumer's payments (#3, 4, 6, 7, 9, 10, 11). These contributions focus on consumer behavior and the firm's profitability under optimal suggested and minimum prices.

In PWYW, consumers' choices and the seller's profitability are two sides of the same coin. Only when the consumers' preferences induce sufficiently high prices, will PWYW become a promising pricing mechanism from the seller's perspective. Furthermore, the consumer can decide on prices only when the seller offers PWYW. That is, an isolated perspective on either the consumer or the seller risks analyzing theoretically possible but practically unlikely scenarios (e.g., while it is interesting whether consumers behave prosocially when purchasing a high value good such as a Porsche 911, sellers will most probably not take the risk and sell such goods under PWYW). Similarly, PWYW might be an effective pricing mechanism from the seller's perspective when all consumers are extremely prosocial and share their entire consumer rent with the seller; however, such a scenario is highly unlikely. To ensure that models study relevant scenarios from the buyers' and seller's perspective, thus, future research should try to integrate both sides.

No	Author	Consumer preferences [1]	Utility function [2]
1	Chao, Fernandez, and Nahata (2015)	ORP	$u_i = r_i - p_i - \mu(p_f - p_i)^2$
2	Chao, Fernandez, and Nahata (2019)	ORP	$u_i = r_i - p_i - \mu \max\{p_f - p_i, 0\}$
3	Chen, Koenigsberg, and Zhang (2017)	Inequity aversion	$u_i = r_i - p_i - \gamma_i \max\{p_i - p_f, 0\} - \beta_i \max\{p_f - p_i, 0\}$
4	Christopher and Machado (2019)	Fairness, image, cognitive costs	$u_i = r_i - p_i - \gamma_i[(p_{f,1} - p_i)^2 + \chi] + \iota_i(p_i - p_{f,2})$
5	El Harbi, Grolleau, and Bekir (2014)	ORP	$u = r - p + \mu r$
6	Gautier and van der Klaauw (2012)	Social image, self-image	$u_i = r_i - p_i + \iota_{self,i}(p_i, r_i) + \iota_{social,i}(p_i, p_f)$
7	Isaac, Lightle, and Norton (2015)	Warm glow altruism	$u_i = r_i - p_i + \alpha_i(p_i, p_f)$
8	Kahsay and Samahita (2015)	Self-image	$u_i = r_i - p_i \pm \iota_i(p_i, p_f)$
9	Mak et al. (2015)	Egoistic, forward-looking	$u_{i,T} = (r_i - p_i) + \delta(r_i - p_i) + \delta^2(r_i - p_i) + \delta^3(r_i - p_i) + \cdots$
10	Park, Nam, and Lee (2017)	ORP	$u_i = r_i - p_i - \mu_{1,i}(p_f - p_i)^{\mu_{2,i}}$
11	Regner and Barria (2009)	Intention-based Reciprocity	$u_i = r_i - p_i + \rho_i \kappa_{consumer} \kappa_{firm}$
12	Samahita (2019)	ORP	$u_i = r_i - p_i - \mu(p_f - p_i)$
13	Schmidt et al. (2015)	Pure altruism	$u_i = r_i - p_i + \alpha_i(\pi)$
14	Tudón (2015)	Altruism	N/A

(1) ORP: not explicitly specified other-regarding preference

(2) α parameter for altruistic concerns, β parameter for inequity concerns, γ parameter for inequity concerns, δ discount factor for future consumption, ι parameter for image concerns, κ parameter for kindness, μ parameter for not explicitly specified social concerns, π seller's profits, ρ parameter for reciprocity concerns, χ parameter for cognitive costs, p_f benchmark price, N/A not applicable

Table 1: Consumer preference in PWYW models

No	Author	Level of analysis	Minimum price	Suggested price
1	Chao et al. (2015)	Consumer and seller	N/A	N/A
2	Chao et al. (2019)	Consumer and seller	N/A	N/A
3	Chen et al. (2017)	Consumer and seller	Yes	Yes
4	Christopher and Machado (2019)	Consumer	Yes	Yes
5	El Harbi et al. (2014)	Consumer and seller (indirect distribution)	N/A	N/A
6	Gautier and van der Klaauw (2012)	Consumer	N/A	Yes
7	Isaac et al. (2015)	Consumer and seller	Yes	Yes
8	Kahsay and Samahita (2015)	Consumer and seller	N/A	N/A
9	Mak et al. (2015)	Consumer and seller	N/A	Yes
10	Park et al. (2017)	Consumer	N/A	Yes
11	Regner and Barria (2009)	Consumer and seller	Yes	Yes
12	Samahita (2019)	Consumer and seller	N/A	N/A
13	Schmidt et al. (2015)	Consumer	N/A	N/A
14	Tudón (2015)	Seller	N/A	N/A

Table 2: Level and scope of analysis of PWYW models

No	Author	Drivers of benchmark prices	Drivers of PWYW prices
1	Chao et al. (2015)	Consumption utility, external anchor (typical price, costs)	Consumption utility, costs, ORP, benchmark price
2	Chao et al. (2019)	Consumption utility, external anchor (typical price)	ORP, benchmark price
3	Chen et al. (2017)	Consumption utility, costs, suggested price	Generosity, benchmark price, advantageous inequity aversion, suggested price, minimum price
4	Christopher and Machado (2019)	Consumption utility, internal reference price, suggested price	Fairness, image, cognitive costs, consumption utility, reference price
5	El Harbi et al. (2014)	N/A	N/A
6	Gautier and van der Klaauw (2012)	Consumption utility, external anchor (typical price)	Benchmark price, reputational concerns (social and self-image)
7	Isaac et al. (2015)	Consumption utility, suggested price	Consumption utility, benchmark price
8	Kahsay and Samahita (2015)	Exogenous fair price	Image, benchmark price
9	Mak et al. (2015)	N/A	Consumption utility, low discount factor
10	Park et al. (2017)	Exogeneous typical price	Social preferences, reference price
11	Regner and Barria (2009)	N/A	Enough reciprocity concerns
12	Samahita (2019)	Consumption utility, costs, shared surplus	Generosity, consumption utility, costs
13	Schmidt et al. (2015)	N/A	Altruism, consumption utility, seller characteristics
14	Tudón (2015)	N/A	N/A

Table 3: Consumer behavior in PWYW models

No	Author	Firm preferences	Costs	Comparison to fixed pricing	Conditions for Profitability
1	Chao et al. (2015)	Profit maximization	Yes	Yes	Low costs, high prosocial preferences
2	Chao et al. (2019)	Profit maximization	Yes	Yes	Low costs, high prosocial preferences, competition uses fixed pricing
3	Chen et al. (2017)	Profit maximization	Yes	Yes	Low costs, low share of freeloaders, high generosity
4	Christopher and Machado (2019)	N/A	N/A	N/A	N/A
5	El Harbi et al. (2014)	Profit maximization	Yes	Yes	High average prices, cross-selling potential
6	Gautier and van der Klaauw (2012)	N/A	N/A	N/A	N/A
7	Isaac et al. (2015)	Profit maximization	Yes	Yes	In the absence of freeloaders always, otherwise N/A
8	Kahsay and Samahita (2015)	Profit maximization	0	Yes	High prosocial preferences, low benchmark prices
9	Mak et al. (2015)	Achievement of a profit goal	0	N/A	N/A
10	Park et al. (2017)	N/A	N/A	N/A	N/A
11	Regner and Barria (2009)	Payoff maximization with reciprocity	N/A	Yes	Sufficient reciprocity concerns
12	Samahita (2019)	Profit maximization	Yes	Yes	Low share of freeloaders, high generosity, market incumbent charges posted prices
13	Schmidt et al. (2015)	N/A	Yes	N/A	N/A
14	Tudón (2015)	Profit maximization	Yes	Yes	High altruism, low costs High demand uncertainty

Table 4: The seller in PWYW models

2.4.2 The consumer in PWYW models

In the following, we will present the findings from PWYW models for consumer behavior. We will concentrate on how to model different consumer preferences, benchmark prices, and the consumers’ payments. The results are summarized in Table 3.

2.4.2.1 Consumer preferences

In this section, we will discuss the general setup of utility functions in PWYW models, the consumer preferences in different pricing mechanisms, and the consideration of consumer heterogeneity.

All models include additional preferences with a non-standard utility component for explaining consumer behavior. In the presentation of the review, we group the articles by the type of non-standard utility which is assumed. This gives us one article that models egoistic, time-consistent preferences (#9), three articles that use altruism (#7, 13, 14), one article that uses reciprocity (#11), one article that uses inequity aversion (#4), two articles that use image concerns (#6, 8), five articles that use a not explicitly specified prosocial preference parameter (#1, 2, 5, 10, 12) and one article that explicitly models multiple factors (fairness and image concerns) (#6). The corresponding utility functions are summarized in Table 1. To ensure consistent presentation and to compare the models with each other, the notation used in the respective papers is standardized, simplified, and adapted to mirror (*16*).

Although the consumers’ underlying preferences differ across models, the different approaches may be divided into two main categories: models with positive additional utility components and models with disutility components.

In the models of the first category, the respective authors assume that consumers who behave according to their prosocial preferences receive additional utility. Under these circumstances, the preferences for altruism (#7, 13), reciprocity (#11), image (#4 (partially), 6), or general social preferences (#5) reward prosocial behavior. Typically, the marginal utility from acting prosocial is concave and decreases with increasing payments. In these cases, by maximizing the utility function, consumers seek a utility-maximizing price that balances acquisition utility and prosocial preferences. When, however, the consumers do not behave prosocially, they likewise fail to obtain additional utility.

In the models of the second category, authors assume that behaving in violation of prosocial preferences decreases the consumers’ overall utility. This is the case for models that use equity/fairness concerns (#3, 4 (partially)), and general social preferences (#1, 2, 10, 12). Under these circumstances, consumers typically compare their price to a benchmark price (cf. Section 2.4.2.2). If consumers do not behave prosocially, they suffer a utility loss in the 'additional utility component' part of the utility function (cf. (*16*)). When they have strong social preferences, this utility loss exceeds the utility gain from saving money. Consequently, consumers prefer to pay the benchmark price and to behave fairly or to refrain from purchasing altogether. When consumers have weak social preferences, either the monetary benefits from paying nothing or the low prices exceed the disutility from prosocial preferences, and the consumers do not pay enough and or proceed to freeload, i.e., pay nothing and take the good for free.

One paper combines the two approaches. When consumers pay above a benchmark price, they receive additional utility, and when they pay below the benchmark, they lose utility (#8).

Another difference in the representation of preferences pertains to the consistency of utility functions in PWYW and other pricing mechanisms. Ideally, consumers should have similar preferences in PWYW and posted pricing. Consumers who behave prosocially in PWYW most likely also have such preferences in posted pricing. It is unlikely that they accept a seller behaving very selfishly without experiencing antipathy, and consequently a utility loss. Hence, focusing on prosocial preferences solely in PWYW while assuming traditional utility in fixed pricing might be a problematic assumption (as in #1, 2, 3 (partially), 8, 14).

We discuss this issue in greater depth in Articles 1 and 2 of this dissertation. There we will show the consequences of neglecting prosocial preferences in posted pricing and in PWYW with a minimum price.

Another difference in the models lies in the inclusion of the heterogeneity of consumer preferences. Traditional microeconomic models derive demand based on heterogeneous consumption utilities (cf. equation (*4*)). Consumers with high consumption utilities are willing to pay higher (fixed) prices than consumers with lower consumption utilities. In addition to the heterogeneity in consumption utilities, models in PWYW assume that consumer preferences are also heterogeneous for the additional utility component. However, models differ in the way they introduce this heterogeneity. We can observe the following four approaches.

In the first approach, consumers are truly heterogeneous with respect to their prosocial preferences (#3 (partially), 7, 8). This allows the authors to specify a distribution of prosocial preferences in their models and determine consumer behavior consistently. The second approach is to introduce different consumer segments that differ with respect to their prosocial preferences. Typically, these consumers are divided into a group that has strong prosocial preferences and one that has no or low prosocial preferences and always freeloads (#1 (partially), 3 (partially), 13). Articles 1 and 2 of this dissertation discuss the limitations of this approach. In the third approach, authors do not introduce heterogeneity for prosocial preferences (#1 (partially), 2). In these cases, all consumers are equally prosocial. However, as consumers have different consumption utilities and benchmark prices, their buying behavior and prices can differ widely despite constant prosocial preferences. The fourth approach assumes that all consumers are equally prosocial and do not differ with respect to benchmark prices. These consumers then all behave alike, paying the same price. Of course, this simplified approach is only suitable when the authors focus on the seller (#5, 9, 14).

Future PWYW models should continue to reflect realistic consumer behavior. Hence, additional work is needed in the way of assessing and comparing the empirical observations of the different theoretical formulations of utility functions. In line with empirical studies, the models should identify different consumer segments and further research the interplay of consumption utility and prosocial preferences. Future work should also continue to allow some consumers to pay little or nothing and others to behave prosocially and pay substantial prices.

2.4.2.2 Benchmark prices

As discussed in the previous sections, some models use a benchmark that serves as a basis for comparison of whether the price paid is appropriate. This benchmark has different names, e.g., equitable price (#3), reference price (#1, 2, 4, 10), or fair price (#3, 4, 8). However, the reasoning is very similar across papers. Prices below this benchmark are considered to be non-prosocial, i.e., unfair, embarrassing, or non-altruistic, etc. Depending on the type of model, prices above the benchmark can either be considered generous and increase overall utility, e.g., when the consumer gains prestige or self-image for paying more (e.g., #6, 8), or considered to be excessive and, therefore, decrease prosocial utility, as in the case of inequity aversion (e.g., #3). In these cases, the benchmark prices also stand at the upper bounds for the actual price.

Benchmark prices are common features in PWYW models. Out of the 14 examined papers, eight papers use a benchmark against which the price is compared (#1, 2, 3, 4, 6, 7, 8, 10). In one paper, the benchmark price is not used explicitly but is part of the PWYW prices (#12). Although benchmark prices are used similarly across models, they draw on different contextual concepts and can be classified into exogenously-influenced and consumer-specific.

Five papers assume a (partially) exogenously-given benchmark price (#1, 3, 7, 8, 10). In this case, the benchmark price takes the form of an anchor price. The seller or the society signal a benchmark price and can therefore directly influence perceptions on the consumers' actual prices. Park et al. (#10) suggest that the seller might influence consumers' benchmark prices by changing the design of the PWYW offer, e.g., by donating parts of the revenues to charity or by adding a suggested price. However, when benchmark prices take the form of anchor prices, they are identical for all consumers. Yet, empirical research has shown that benchmark prices vary widely across consumer segments.

Five papers (#2, 3, 4, 6, 12) assume that consumers build individual benchmark prices. Hence, their benchmark prices resemble the internal reference price. The benchmark price consists of a combination of two or more model parameters and divides the total surplus of the transaction between seller and buyer such that buyers are entitled to some consumer rent and the seller to make a certain profit. That is, when constructing a benchmark price, the more the consumer takes the payoffs of the seller into account, i.e., the more they weigh the outcome of the seller, ergo the more generous they are (#3). A stylized version of such a benchmark price is given by

$$p_f = \lambda \text{ Consumption utility} + (1-\lambda) \text{ Seller's costs} \tag{17}$$

with $\lambda \in [0,1]$ the weighting factor between seller and buyer. When consumption utility is above costs – which is a precondition for a positive surplus of the transaction – a higher λ corresponds to a more generous the consumer. Note that anchor prices might also play a role as they can influence the internal reference price.

Four papers (#2, 3, 6, 12) assume that consumer decision-making proceeds based on two model parameters in the construction of their benchmark price. In these cases, consumption utilities, r_i, act as the baseline for the consumer rent. However, the models differ in their definition for the seller's minimum profit level. Two papers use the seller's costs as a baseline for profits (#3, 12). Note that the authors need to assume that the buyers know the seller's costs. Two articles use an external anchor price (i.e., the typical price in fixed pricing) as a minimum baseline (#2, 6). One paper constructs the benchmark price using three components: consumption utility, typical prices, and the seller's price suggestion (#4).

Furthermore, six authors assume that the benchmark price has an upper (#1, 2) or lower ceiling (#3, 7, 8, 12), i.e., a maximum or minimum level beyond which each payment is inappropriate. Consumption utility and the external anchor price are proposed as an upper ceiling (#1, 2). The lower ceiling is presumed to be the seller's costs (#3, 12), the suggested donation (#7), and the buyer's consumption utility (#7).

The benchmark price is an essential feature of most PWYW models and an important driver for PWYW prices (cf Section 2.4.2.3). Depending on the magnitude of the benchmark price, identical prices might either be considered appropriate or inappropriate. Hence, with a rise in the benchmark price, consumers might react by increasing their actual prices, dropping out of the market, or by deciding to freeload. To improve PWYW models, further research should examine which type of benchmark prices most closely relates to behavioral benchmarks and whether these benchmarks differ between PWYW and fixed prices. Future research might also compare the influence of different conceptualizations of benchmark prices on the findings of different models.

2.4.2.3 Individual price in PWYW

An exciting challenge for researchers modeling consumer behavior in PWYW is to explain why consumers pay voluntarily and to predict their actual prices. 12 of the 14 models (#1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13) use the previously established utility functions to derive utility-maximizing prices for the consumers. The remaining two papers assume that consumers will pay but do not derive their payments from their utility functions (#10, 14). While the approaches vary in detail, the general idea of determining individual prices is similar across models. Just as in basic microeconomic models (cf. (*11*)-(*12*)), the price can be derived by maximizing utility. However, in contrast to the simplified approach sketched above, additional utility components (cf. (*16*)) come into play in these models and all authors find prices above zero. Consistent with the reasoning above, three main drivers of the PWYW prices can be identified: namely, their consumption utilities, their prosocial preferences, and the benchmark price. Most models share a number of key features –the prices typically increase with consumption utilities, stronger social preferences, or higher benchmark prices. Since the benchmark price is, in turn, determined by different factors (see Section 2.4.2.2), the PWYW price, p_i, is also indirectly co-determined by these factors. As an example, the benchmark price in Chao et al. (#1) is influenced by the seller's costs and, therefore, the PWYW prices also increase with costs.

With respect to the absolute magnitude of prices, most authors (e.g., #3, 7) find that prices in PWYW are lower than the seller-set price in posted pricing. An exception to this rule is given by Mak et al. (#9). In their model, consumers might pay more than their consumption utility when they hope to preserve the offer for the future.

Taken together, previous research successfully explained why consumers pay in PWYW and also how this matches empirical observations. Future research modeling PWYW behavior might want to look at the interplay of different preferences and how these differences determine prices.

2.4.3 The firm in PWYW models

Ten of the 14 papers analyze the seller's perspective in PWYW (#1, 2, 3, 5, 7, 8, 9, 11, 12, 14). These papers try to delineate conditions in which the seller can operate successfully in PWYW. The results are summarized in Table 4.

2.4.3.1 Firm preferences

In contrast to the buyers, sellers are considered to be behaving rationally and, therefore, to be profit-maximizing. Consequently, the seller tries to find the optimal pricing scheme for the market in which he/she is operating. There are only two exceptions to this general case. First, Mak et al. (#9) consider a seller with a profit goal that is not necessarily profit-maximizing and show that such a seller can cover costs in PWYW. Second, Regner and Barria (#11) assume that the seller also has prosocial concerns and, therefore, prefers reciprocal markets. In both cases, PWYW can be a profitable pricing scheme for the seller.

When examining the profitability of PWYW from a profit-maximizing seller's perspective, all relevant papers find cases in which PWYW is profitable. Eight papers compare PWYW to traditional pricing and find conditions in which PWYW can even outperform fixed pricing (#1, 2, 3, 7, 8, 12, 14). We will discuss these in the following.

2.4.3.2 Average prices and demand in PWYW

The average price per buyer in PWYW depends on two elements: the share of buyers who make a payment and the magnitude of their payments. Evidently, these factors are determined by the aggregate choices of the consumers and, therefore, by the distribution of the consumers' preferences. From a seller's perspective, the more consumers have strong prosocial preferences, the higher the share of paying consumers and the higher the prices these consumers pay. PWYW works as market expansion force, as more buyers can buy from the seller while the average price is lower than in posted pricing. Hence, the seller faces a tradeoff between more demand and lower prices. Consequently, several authors find that PWYW can only be offered when prosocial preferences are strong and, therefore, prices are sufficiently high (#1, 2, 3, 5, 7, 11, 12, 14).

In some models, benchmark prices are important drivers for the prices ultimately paid by consumers –the higher the benchmark price, the higher the prices. Determinants of the benchmark price thus indirectly influence the PWYW prices and, consequently, the seller's profits. As discussed above, the benchmark price is a weighted average of the consumers' consumption utilities, the seller's costs (#1, 3, 12), and the suggested price (#3, 4, 7). With respect to costs, this implies that higher costs induce a higher average PWYW price, i.e., a positive relation between prices and costs. However, an upper limit enters into the equation of the effect of the benchmark price on prices paid and on the share of consumers who actually pay. When benchmark prices become too high, behaving fairly might become too costly for the consumer. In this case, they might switch to freeloading or to the no-purchase option (#3, 7).

Most models predict that PWYW firms sell a higher quantity than a respective fixed-price firm. The absence of a fixed price only excludes consumers with high prosocial concerns and low consumption utilities from the market, while fixed pricing excludes all consumers with small consumption utilities. Some models even predict full market coverage in PWYW, i.e., that all consumers buy (# 1, 2, 5, 11, 9, 13, 14). Comparing these results to empirical observations, we find that PWYW sellers can expand on the market (e.g., in the famous PWYW offer by the British band Radiohead, sales increased significantly (Tyrangiel, 2007)). However, empirical papers seldom observe full market coverage. To that extent, these theoretical predictions of total market coverage seem to exaggerate the potential of PWYW.

2.4.3.3 The effect of costs on the seller's profits

Consistent with microeconomic conventions, all studies in question neglect fixed costs. Besides their indirect influence on prices via benchmark prices (see above), variable costs also have a direct impact on the seller's profits. In general, variable costs have a diametral influence on profit (#1, 2, 3, 5, 7, 12, 13, 14). Furthermore, when comparing PWYW with fixed pricing, high cost levels inhibit the implementation of PWYW as the seller has to cover full costs for freeloaders without any compensation. Even if the remaining share of the consumers behaves fairly, they will not be able to offset the losses from freeloaders (#3, 7, 12).

Note, however, that some papers do not account for costs in case of PWYW or assume that the seller's costs are negligible and set them to zero (#8, 9). This limits the applicability of PWYW to very specific areas. In sum, the effect of costs on profits is well documented. Future studies should continue to build informative models that include non-zero costs when accounting for the seller's profitability.

2.4.3.4 Other aspects that support the introduction of PWYW

In addition to the general considerations on the suitability and profitability of PWYW, specific seller and market characteristics might support the introduction of this mechanism. Two scenarios might encourage the seller to adopt PWYW: markets in which the seller has difficulties finding the correct fixed price and markets in which the seller has to fight product piracy such as the record industry.

First, PWYW might be advantageous for sellers who have difficulties finding the right posted price in traditional selling. Two models show that when price setting is difficult (#14) or costly (#1), PWYW can offer a viable alternative as consumers decide on the prices. When uncertainty about the demand in the market surfaces and the seller wants to maximize profits, setting the optimal price is nontrivial. The seller will either choose a price that is too high or too low. When the seller sets a price that is too high, some consumers will not buy, despite the opportunity of being served profitably at a lower price. When prices are too low, the seller forgoes profit potential as consumers would have been willing to pay more. In PWYW, consumers choose their price; hence, these inefficiencies should not exist. That is, if consumers are prosocial enough, the seller will be better off using PWYW as compared to posted price (#14). Furthermore, the size of the market might influence price-setting costs (#1). When the seller only serves a small market with costly price setting, offering PWYW might stand as a more profitable pricing strategy than posted pricing where the seller would incur costs for setting the price and would pass these costs on to the buyer. However, in PWYW, the buyer would set the price and, therefore, even for rather low levels of prosocial preferences, employing PWYW might be optimal.

Second, in the record industry, traditional pricing becomes difficult to enforce as many consumers engage in product piracy. Given high prosocial preferences, El Harbi et al. (#5) show that PWYW can be an attractive choice for musicians who traditionally depend on music labels. When the label shares little of its revenues with the artist, PWYW becomes a viable alternative for the artist. PWYW allows the artist to cut out the intermediary, increase their fan-base, and generate cross-selling effects.

Similarly, Regner and Barria (#11) show that PWYW can be more successful than traditional pricing when prosocial preferences are high as music producers and consumers do not have to worry about protection against piracy. In addition to a prosocial utility gain, PWYW is beneficial for both parties. Consumers get the product without copyright protection and should have higher consumption utilities. The seller saves money on copyright protection.

While PWYW can be a profitable pricing scheme, posted pricing remains the pricing mechanism of choice for most sellers. Future research should continue to model why PWYW is especially prevalent in some markets (e.g., restaurants, and hotels) and less so in others (e.g., supermarkets, and transportation).

2.4.3.5 PWYW in competition

While the above results hold for profit-maximizing monopolists, three papers examine PWYW under competition (#2, 3, 13) and discuss setups in which PWYW can be successful when competing with at least one other firm. The papers agree that there is a minimum level of social preferences and that costs must be low for the adoption of PWYW. However, the predictions on the competitive structure differ. While two papers conclude that PWYW is best used when the competition is using fixed pricing (# 2, 12), the other group of authors recommends PWYW for all sellers in the market (#3). Furthermore, disagreement persists as to whether product differentiation leads to more or fewer firms adopting PWYW.

Chen et al. (#3) consider a duopoly in the absence of freeloaders. They show that when consumers are generous, and product differentiation is low, PWYW is chosen by both firms. However, when consumers have little prosocial preferences, and differentiation is high, both firms opt for fixed pricing.

Chao et al. (#2) investigate Bertrand competition in a duopoly. Traditionally, Bertrand competition does not allow any seller to make profits when both firms offer fixed pricing. However, if one seller offers PWYW and the other offers fixed pricing, both sellers will be profitable when costs are low, and consumers are fair-minded. In this case, some fair-minded buyers will support the PWYW seller, while the fixed-pricing seller can continue to charge high prices.

Samahita (#12) finds that product differentiation and higher levels of prosocial preferences support the introduction of PWYW in competition. In particular, when the seller enters a market in which fixed pricing is the dominating pricing scheme, PWYW offers a way to be profitable in a competitive market.

Using PWYW models might be especially significant for answering the question of the right pricing choice in competition. Due to the complexity of competitive markets, empirical and experimental research becomes very challenging and has not yet been attempted as a result. Given this background, theoretical models might drive knowledge-building in this area.

However, the contradictory findings in this area call for additional studies on the suitability of PWYW in competitive markets.

2.4.4 Design choices

A seller that implements PWYW plays itself fully into the hands of the buyers. Yet, empirical examples of PWYW show that sellers try to influence the buyer by setting a suggested price or try to limit their risk by adding a minimum price. However, as empirical results of the effectiveness of such modifications are mixed (Johnson & Cui, 2013; Jung, Perfecto, & Nelson, 2016), theoretical models sought to understand whether a suggested or minimum price can be successful and how the seller can set optimal suggested or minimum prices.

2.4.4.1 Suggested price

Seven papers (#3, 4, 6, 7, 9, 10, 11) model the effects of a suggested price in PWYW. However, the concept of the suggested price and, thus, its effectiveness differ greatly from contribution to contribution.

Mak et al. (#9) propose an indirect form of price suggestion. In their model, the seller posts an overall profit goal, and the consumers derive their individual price based on that goal. In turn, the sum of consumer payments adds up to the profit. This helps the coordination of buyers and makes PWYW profitable. That is, the seller's profitability depends on arriving at the appropriately suggested profit goal. However, the authors do not give guidelines on the optimal goal as their paper focusses on consumer behavior.

The other authors posit an indirect effect of the suggested price on actual prices (#3, 4, 6, 7, 10). The suggested price alters the benchmark price, and as consumers adjust to this change, they will adapt their prices accordingly.

In Isaac et al. (#7), the suggested price is the minimum payment for getting utility from warm glow altruism. Consumers who pay at least the suggested price receive benefits from their prosocial actions. In their baseline models, all consumers accept the suggested price as the benchmark and have strong prosocial preferences. Consumers with consumption utilities above the suggested price will buy from the seller. Otherwise, they will refrain from purchasing. It follows, then, that the seller must find the optimal price suggestion that maximizes profits. In this case, the suggested price is crucial so that PWYW can outperform fixed pricing.

In Chen et al. (#3), the suggested price is only recognized by some consumers. For those consumers, it increases their benchmark prices. Consumers that are affected by this suggested price will pay more than without the price suggestion. Overall, however, consumers who are freeloading without a suggested price cannot be convinced by the suggestion and will continue to freeload. Nevertheless, when set optimally, the suggested price always increases profits for the seller as compared to PWYW without a price suggestion.

Furthermore, a suggested price reduces the probability of consumers opting out of the market and increases payments (#4). Fair-minded consumers with high consumption utilities accept the price suggestion. Hence, the seller misses out on a potential revenue source by setting the suggested price as consumers would otherwise be willing to pay more. However, it is not yet clear how this will be reflected in the seller's profits.

In summary, suggested prices are double-edged swords. They extract higher prices from some of the consumers, while others who would pay lower prices are driven out of the market altogether. The optimal suggested price is below the profit-maximizing price of a fixed price monopolist (#3, 7). However, the models are contradictory in defining who will be affected by the suggestion. In particular, Isaac et al. (#7) find that suggested prices extract higher prices from consumers with low and high consumption utilities. Chen et al. (#3) predict that only fair-minded consumers with intermediate consumption utilities are affected by the suggested price. Christopher and Machado (#4) argue that fair-minded consumers with high consumption utilities will pay less when a suggested price is present.

Future research might synthesize these findings to model the suggested price. As we discuss in Articles 1 and 2 of this dissertation, the derivations of Chen et al. (#3) are flawed. To that extent, we will give an updated view of the profitability of the suggested price in these contributions.

2.4.4.2 Minimum price

Four papers model the minimum price in PWYW. Two of these articles include the minimum price as a lower bound for the price (#11) or investigate conditions in which this lower bound is paid (#4). The other two remaining contributions aim at giving guidance to the seller for finding the optimal minimum price (#3, 7).

From a consumer perspective, the minimum price helps to determine the price (#4). From a seller's perspective, the minimum price offers a powerful tool to exclude freeloaders as they do not receive the good when paying below the minimum price (#3, 7). Finding the profit-maximizing minimum price is an optimization problem similar to finding the price in monopoly pricing. The minimum price is portrayed very positively from a seller's perspective. It only has positive effects on PWYW prices and profitability and works as an enhancement of PWYW.

That is, prosocial consumers are still contributing more, and freeloaders are excluded from the market (#3).

In contrast to these predictions, previous empirical results (Jang & Chu, 2012; Johnson & Cui, 2013) are less optimistic and show that minimum prices induce a downward shift towards the minimum. The above models do not account for this potentially detrimental effect. Future research should therefore concentrate on disentangling the positive and negative effects of the minimum price. We offer an update of the results of Chen et al. (#3) with respect to the potentially negative aspects of the minimum price in Articles 1 and 2 of this dissertation.

3 Gambled price promotions

Involving the consumers in the price-setting process allows sellers to distinguish themselves from the competition and to enable the consumer to experience something novel and exciting. However, engaging the consumer in price-setting is far from limited to the continuous adoption of the PWYW mechanism. Even for short-term price promotions, retailers can choose between involving the consumer in price-setting or fixed discounts. An example of a promotional application of PWYW is the mark-off-your-own-price strategy, where the consumers choose a discount level rather than a price (Schröder, Lüer, & Sadrieh, 2015). As seen in PWYW, running a short-term price promotion increases consumer involvement and media coverage (León et al., 2012). However, the promotional use of PWYW might lead to lower profits (Kim et al., 2014).

To that extent, in the domain of short-term promotions, other pricing tactics have gained increased interest over the past decade – namely, price promotions for which the outcome of the promotion (i.e., the discount the buyer receives) is unclear at the time of the purchase (Ailawadi, Gedenk, Langer, Ma, & Neslin, 2014). These promotions are random discounts (e.g., every tenth purchase is for free) or discounts conditioned on the realization of an uncertain event (e.g., Austria winning the UEFA European Championship). Often, these promotions make use of consumer participation. In this case, the promotion is framed as a gamble such as a game of chance, a dice game, or a wheel of fortune. These promotions are termed gambled price promotions (Alavi, Bornemann, & Wieseke, 2015). By playing a game with the seller, the buyer can win a discount.

As compared to traditional discounts, price promotions with unknown discounts feature several advantages. They help boost intention to purchase and the overall purchase rate (Dhar, González-Vallejo, & Soman, 1995; Dhar, González-Vallejo, & Soman, 1999; Hock, Bagchi, &

Anderson, 2020). Consumers overestimate the financial attractiveness of such offers and, thus, have a lower price sensitivity (Mazar, Shampanier, & Ariely, 2017). Furthermore, consumers are less likely to decrease their internal reference price when getting a discount that is determined by chance as compared to a traditional discount (Alavi et al., 2015). For a more detailed literature overview on gambled price promotions, cf. Article 4 of this dissertation.

Similar to PWYW, gambled price promotions build on consumer participation. In that way, gambled price promotions might be novel and exciting to consumers. The consumer nevertheless exercises no real influence on the price as the outcome of the gamble determines the discount. Furthermore, these promotions do not pose a significant risk for the seller, as – similar to a casino – by the law of large numbers, multiple replications ensure that discounts approach the expected value of the promotion. This makes gambled price promotions an interesting alternative to fixed prices and PWYW.

In Article 4 of this dissertation, we study whether gambled price promotions can have a positive influence on the consumers' perceptions of the seller, similar to PWYW. In particular, we examine empirically whether word-of-mouth (WOM) intention and customer satisfaction are higher than in fixed price promotions.

4 Overview of Articles

The remainder of this dissertation consists of four articles (Table 5). Articles 1-3 focus on the modeling of PWYW and its extrapolations (minimum price, suggested price, multiple products). Article 4 offers an empirical study of the effectiveness of gambled price promotions. In the following, we will provide an outline of each article.

Article	Reference
1	Akbari, K., Wagner, U. (2020). Comments and Refinements on the Pay as you wish Model by Chen et al. (2017), (submitted to Marketing Science, 2nd review round).
2	Wagner, U., Akbari, K. (2020). Supplementary Appendix to "Comments and Refinements on the Pay as you wish Model by Chen et al. (2017)". Forschungsberichte des Instituts für Betriebswirtschaftslehre der Universität Wien.
3	Akbari, K., Wagner, U. (2018). Pay-What-You-Want Pricing for Multiple Goods. Conference Proceedings EMAC 2018. Glasgow (United Kingdom). 29.05.-01.06.2018. (electronic proceedings USB).
4	Akbari, K., Wagner, U. (2020). When paying is fun, I'll tell others: Outcome effects of gambled price promotions (submitted to Schmalenbach Business Review, 2nd review round).

Table 5: Overview of articles

4.1 Article 1

The purpose of Article 1 is to develop a model that consistently portrays consumer behavior in PWYW and allows us to understand conditions under which the seller can operate profitably under PWYW. Furthermore, it aims to explain the circumstances under which the seller can increase profits by introducing a suggested and a minimum price.

This article builds on Chen et al. (2017), who published a similar analysis in *Marketing Science* and who attempted to model PWYW behavior and firm profitability in PWYW, posted pricing, PWYW with a minimum price and PWYW with a suggested price. The basic features of their model are based on inequity averse consumers and a profit-maximizing seller. Based on their analysis, three consumer segments emerge in PWYW: (1) consumers with high fairness preferences who buy the product and pay the fair price, (2) consumers with high fairness preferences who do not buy because their fairness concerns exceed consumption utility and (3) consumers with low fairness concerns who freeload whenever possible. Concerning the firm's profitability, the seller can successfully use PWYW and be more profitable than under fixed prices when the share of consumers with low fairness concerns is small, costs are low, and fair prices are high. PWYW with the minimum price is successful when the fair prices are high, and the share of freeloaders is also high, as the seller can exclude these freeloaders by instituting a minimum price. Moreover, PWYW with a suggested price is profitable when the share of freeloaders is small, but the consumers' fair prices are low.

However, the derivations in Chen et al. (2017) suffer from some inaccuracies and errors that lead to a (partial) misinterpretation of the profitability of PWYW. This article therefore has three objectives: (1) to uncover the inconsistencies in Chen et al. (2017), (2) to show that these inconsistencies are influential, and (3) to offer a model that does not suffer from these inconsistencies and that predicts the consumers' and firm's behavior and outcomes under PWYW more consistently.

The rework of their model divides the segment of consumers with low fairness concerns into two sub-segments. It disperses with the notion that consumers with low fairness concerns invariably hurt the seller. In addition to the freeloading consumers, a consumer segment appears who maintains low fairness concerns and however does not freeload. These results also increase the seller's profitability in PWYW and PWYW with the suggested price. Regarding the minimum price, we apply a more comprehensive inequity aversion concept that also takes into account the buyer's exploitation by the seller. Furthermore, our derivations on the minimum price and the suggested price clear up some inconsistent results and formulate more realistic, optimal suggested and minimum prices.

Additionally, the article contains a simulation analysis that compares the updated results of this article to the findings by Chen et al. (2017) and shows differences between 10 and 30 percent in the seller's profits between the two models. Beyond the microeconomic model, Article 1 proves the existence of the additional consumer segment in a field experiment from an Austrian supermarket. During the PWYW condition, we could show that the segment of less fair-minded consumers indeed needs to be separated into consistent freeloaders and partial freeloaders.

4.2 Article 2

Article 2 contains a technical report on PWYW modeling in Article 1. In addition to the content-related aspects that emerge when modeling PWYW, this research report provides details on the mathematical aspects of modeling and the derivations of the model.

This report contains additional and more explicit analyses of the inconsistencies in the study by Chen et al. (2017), compares important model scenarios using what-if analyses, and attempts to uncover the underlying reasons for the inconsistencies in Chen et al. (2017).

4.3 Article 3

Most previous models on PWYW have investigated a buyer-seller interaction with a single product. However, most product offerings include multiple products and product bundles. A seller with two products (A and B) who can choose between PWYW and fixed pricing already has four options: (1) selling both products under PWYW, (2) selling both products with fixed pricing, (3) selling one product (A) in PWYW and the other with fixed pricing (B), and (4) selling the other product (B) in PWYW and the first product (A) under fixed pricing. Besides, sellers have further developed PWYW and combined ideas from bundling with PWYW. Consumers can obtain the bundle of both products when they pay above a threshold price. Otherwise, only product A is sold under PWYW conditions. This pricing scheme is termed multi-tier PWYW (MTPWYW).

To develop guidelines for the sellers to choose the right pricing mechanism, we assume inequity averse buyers enter into the equation, just as in Chen et al. (2017), and offer a simulation analysis at a wide parameter range. The results of this analysis imply that consumers have to be sufficiently inequity averse in order for the seller to consider using PWYW as part of its pricing strategy.

In this case, we can formulate the following guidelines: when costs of both products are low, pure PWYW is the most profitable pricing scheme. When the costs of one product are low and the costs of the second product are intermediate, PWYW for one product and fixed pricing for the other product is the optimal choice. When the costs of one product are low and high for the other product, MTPWYW is optimal.

4.4 Article 4

Article 4 deals with consumer behavior in gambled price promotions. This type of rebate policy allows the consumer to participate in determining the price without the consumer having the opportunity of directly influencing the price. Instead, the consumer plays a promotional game, and the result of this game determines the final price. Previous studies have shown that this type of price promotion is advantageous from the seller’s point of view, as consumers bought more when a gambled promotion was running.

This article proposes that the entertainment value and the consumers' assessment of the outcome of promotion might influence customer satisfaction with the store and the consumers' word-of-mouth (WOM) intentions. When compared to traditional price promotions with the same expected value, letting the consumer play a promotional game could have two effects. First, playing a game might be viewed as entertaining and, therefore, might increase the consumer's assessment of the seller's service, which should increase consumer satisfaction and WOM intention. Second, consumers might assess the outcome of the game as favorable or unfavorable. When the consumers have favorable assessments of the outcome of the game, they tend to be more satisfied with the seller regardless of the entertainment value of the promotion. However, when consumers do not perceive the promotion as entertaining, consumers who are more satisfied with the discount might still be even more satisfied with the seller than otherwise. Furthermore, consumers who find the promotion entertaining and who are overall more satisfied with the seller, might therefore be more willing to engage in WOM. Despite these potential advantages, a negative side to this equation enters into play as well. When consumers perceive the gamble as boring or do not approve of the outcome of the gamble, customer satisfaction and WOM intentions tend to suffer.

This article tests these relations in two field experiments. Consumers' responses to gambled price promotions were assessed and compared to fixed and non-promotional setups.

Three significant findings arise from this study: (1) consumers are in favor of gambled price promotions and are more willing to engage in word-of-mouth if they participate in such a campaign. The effect of gambled promotions on customer satisfaction is mixed. (2) Entertainment value mediates the positive relationship between gambled price promotions and customer satisfaction and WOM intentions. (3) If consumers are dissatisfied with the discount received, the entertainment value could nevertheless compensate for the effect on customer satisfaction in a low-stakes environment.

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Glossary of prices

The synopsis above shows that price setting in PWYW is complex and that consumers and sellers draw on different concepts to determine the price. Therefore, we offer a glossary on the various kinds of prices used in the text.

actual price: The price a consumer decides to in PWYW. 22, 23, 24, 29

anchor price: An externally announced price that could influence the perception of the actual price. In PWYW models, anchor prices could be used as *benchmark prices* and *suggested prices*. *See also external reference price*. 18, 23

average prices: Average prices of all customers from a seller's perspective. 19

benchmark price: A price that acts as a comparison to the *actual price*. In PWYW models, the construction of benchmark prices is not consistently agreed on. 6, 14, 16, 18, 19, 20, 21, 22, 23, 24, 26, 29

equitable price: *See fair price.* 22

external reference price: *See anchor price.* 18, 23

fair price: A hypothetical price that is considered to be fair. In PWYW models, it could be a *benchmark price*. 18, 22, 33

fixed price: *See posted price.* 1, 2, 4, 6, 7, 8, 13, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36

individual price: The price a consumer decides to in PWYW. These prices differ among consumers. *See actual price*. 10, 29

internal reference price: The price in the consumer's memory. In PWYW models, it could be a *benchmark price.* 18, 23, 32

minimum price: A price that is added as a lower boundary in PWYW. Consumers can only buy the product if they pay at least the minimum price. 2, 3, 4, 13, 18, 21, 29, 30, 31, 33, 34

optimal price: Profit-maximizing price from a seller's perspective. 9, 27, 29

personalized prices: When a seller allows prices to be different among consumers. 2

posted price: Prices in a *traditional pricing mechanism*. The seller announces a price. The buyer then accepts or rejects this price to make a purchase decision. 3, 14, 19, 21, 24, 25, 27, 28, 33

price differentiation: A pricing strategy that charges different prices for the same product or service. 3

reference price: Standards against which the purchase price of a product is judged. Usually divided into two different types of reference prices 6, 11, 18, 22, 23, 32

suggested price: A recommendation by the seller that aims at influencing the consumer's payments in PWYW. In PWYW models, it could be an *anchor price* or *benchmark price*. 13, 18, 23, 26, 29, 30, 33, 34

threshold price: *Minimum price* a consumer has to pay to get both goods in multi-tier PWYW. 35

traditional price: A pricing scheme that uses a *posted price/fixed price*. 36

typical price: The price a consumer expects to pay. Often the regular *posted price*. In PWYW models, it could be a *benchmark price*. 18, 23

utility-maximizing price: The price that is optimal from the consumer's perspective and would be paid in PWYW. Could be the *actual price*. 10, 13, 20

2. Article 1

Akbari, K., Wagner, U. (2020). Comments and Refinements on the Pay as you wish Model by Chen et al. (2017), (submitted to Marketing Science, 2nd review round).

MARKETING SCIENCE

Comments and Refinements on the Pay as you wish Model by Chen et al. (2017)

Journal:	*Marketing Science*
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Comments and Refinements on the Pay as you wish Model by Chen et al. (2017)

Chen et al. [Marketing Science 36(5):780–791 (2017)] published a model to describe consumer behavior under pay as you wish (PAYW) pricing. This paper identifies inconsistencies in that model. First, the paper points to a new segment of consumers who were previously unconsidered. They are characterized by a decision not to buy a good under a PAYW pricing policy, even if they can get it for free, and are not very averse with respect to advantageous inequity. Second, the paper incorporates the effect of disadvantageous inequity aversion on PAYW with the minimum price. Third, the paper discusses the profitability of PAYW with a suggested price. Fourth, the paper offers updated guidelines on how a seller should choose the optimal pricing policy. Finally, the paper shows that revised results differ considerably compared to those of Chen et al. (2017).

Keywords: pay what you want; pay as you wish; participative pricing, inequity aversion, pricing

1 Introduction

In the last decade, there has been increasing interest in participatory pricing mechanisms that give the consumer power over the pricing decision. Pay as you wish (PAYW) pricing is probably the most disrupting and extreme form giving the buyer (almost) no limit on how to set the price. Recently, Chen et al. (2017) published an analytical model in *Marketing Science* that aims to describe consumer behavior under this pricing scheme. Their model draws on inequity aversion and offers an excellent and comprehensive approach in describing why consumers pay voluntarily in PAYW.

However, their paper contains inconsistencies that influence the profitability of PAYW and might mislead firms and researchers drawing on their model. First, Chen et al. (2017) fail to correctly identify a consumer segment that does not buy a good under a PAYW pricing policy, even if they can get it for free, and are not very averse with respect to advantageous inequity. This also leads to incorrect results for PAYW with a suggested price. Second, Chen et al. (2017) omit disadvantageous inequity aversion under PAYW with the minimum price although the minimum price might induce reactance behavior for some buyers. Third, the calculation of the optimal choice of the pricing policy is substantially flawed. Furthermore, we conduct a simulation study that shows that Chen et al.'s (2017) results differ from the correct results by between 10 to 20 percent and that a seller who would rely on Chen et al.'s (2017) results would choose the wrong pricing policy in about 20 percent of all cases.

Given these inconsistencies, here we provide provides refinements of Chen et al.'s (2017) derivations concerning the share of freeloaders in the market, the seller's profits under PAYW, conditions for which the seller should choose PAYW over traditional pricing, the optimal minimum price, the optimal suggested price and how to manage profitability under (variants of) PAYW. This paper attempts to address and revise these issues and their consequences and offer solutions that help to understand PAYW pricing better and lead toward a more realistic model of consumer behavior in PAYW.

This paper begins by introducing the initial PAYW model by Chen et al. (2017). The third chapter deals with the consequences of freeloading behavior on the overall profitability of PAYW and under which conditions a manager should choose PAYW over 'traditional' pricing. The fourth and fifth section discusses modifications of PAYW such as adding a minimum or a suggested price to the initial PAYW offer and how disadvantageous inequity aversion and freeloading behavior influence these pricing mechanisms. Based on these updated results, Section 6 delineates the appropriate pricing mechanism according to the respective prevailing circumstances and shows how Chen et al.'s (2017) results differ from the results derived in this paper. A discussion section concludes.

2 The model for pay as you wish pricing

As a starting point, Chen et al. (2017) – CKZ hereafter – seek to model buyer's reactions and seller's profits under PAYW pricing and traditional pricing. Under PAYW, customers are offered a product or service and are free to choose any price they want, including zero (obtaining the good for free) (Kim et al. 2009). CKZ assume inequity averse buyers (Fehr and Schmidt 1999) to explain why people pay under PAYW and also why people refrain from buying because they would experience inequity otherwise.

CKZ introduce the customer's (i) utility (u_i) in their Eq. (1) as:

$$u_i = r_i - p_i - \beta_i \max\{p_i - r_{i0}, 0\} - \gamma_i \max\{r_{i0} - p_i, 0\}, \;\; \beta_i, \gamma_i \geq 0 \tag{1}$$

The consumption utility (r_i) describes the customer's benefit from consuming the good, disregarding any transaction utility (Thaler 1983). The term $\gamma_i \max\{r_{i0} - p_i, 0\}$ captures disutility from advantageous inequity; i.e., if the price (p_i) is below the consumer's perceived fair price (r_{i0}). Higher values of γ_i correspond to a stronger aversion to advantageous inequity; i.e., characterize more fair-minded customers. Thus, the higher γ_i, the more the customer dislikes being over-privileged. Correspondingly, the term $\beta_i \max\{p_i - r_{i0}, 0\}$ captures disutility from disadvantageous inequity; i.e., if the actual price is above the consumer's perceived fair price. This term becomes relevant only if the seller sets the price. Similarly, higher β_i's indicate that customers have stronger opposition when the seller is over-privileged; i.e., charges high prices.

CKZ define the perceived fair price as

$$r_{i0} = \begin{cases} c & r_i \leq c \\ \lambda r_i + (1-\lambda)c & r_i > c \end{cases}, 0 \leq \lambda \leq 1 \tag{2}$$

where c describes the seller's costs and λ describers the seller's equitable share of the total surplus. CKZ assume that the consumer knows the seller's costs and compares them to her consumption utility. When the costs are higher than the consumption utility, the consumer's fair price is equal to the seller's costs, as the consumer knows that lower prices will cause a loss for the seller. When the consumption utility is higher than the costs, the transaction creates a positive surplus ($r_i - c > 0$) and CKZ further assume that the consumer is willing to split this surplus proportionally (i.e., λ: $(1-\lambda)$). The parameter λ describes the generosity of the consumer. The higher λ, the stronger the consumer's conviction that she should pay to the seller in order to be fair. By convention, CKZ restrict, without loss of generality, the domain of r_i to $[0,1]$. This results in $p_h = c + \lambda(1-c)$ as the highest possible fair price (cf. (2)).

Initially, CKZ distinguish between two scenarios for setting prices. (i) The customer sets the price; i.e., PAYW: She will never pay more than the fair price because she strives to avoid disadvantageous inequity (β_i-term in (1): $p_i \leq r_{i0}$). (ii) The seller sets the price: CKZ show that a profit-maximizing firm will always charge prices above the perceived fair price. CKZ denote this pricing scheme as "pay as asked" pricing (PAAP). They derive on p. 782 (their equation (3)) the optimal price under PAAP as[1]

$$p^*_{PAAP} = \frac{1 + c + \beta(\lambda + 2c - c\lambda)}{2(1+\beta)} = c + \frac{1 + \beta\lambda}{2(1+\beta)}(1 - c). \quad (3)$$

When comparing the highest possible fair price to the price under PAAP, we find that for

$$\lambda > 1/(2+\beta), \quad (4)$$

the price in PAAP is lower than p_h. As will be shown later, this relation is relevant under several scenarios. This implies that generosity λ and disadvantageous inequity aversion β are not completely independent from each other. Rather, when allowing for $\beta > 0$, the feasible domain for λ increases;[2] i.e., consumers are also less generous when opposing to a privileged seller.

3 Not so inequity averse, but still no freeloader

3.1 Conceptual remarks

In the following three subsections, we concentrate on the PAYW case. Therefore, (1) simplifies to

$$u_i = r_i - p_i - \gamma_i\,(r_{i0} - p_i) = r_i - \gamma_i r_{i0} - (1 - \gamma_i)p_i. \quad (1a)$$

This utility function is linear in p_i and under PAYW $0 \leq p_i \leq r_{i0}$ holds.

In the sequel, we define consumer reactions under PAYW. Figure 1 summarizes the results. The horizontal axis defines the consumers' level of advantageous inequity aversion while the vertical axis defines the consumers' consumption utilities. Thus, we find the following reactions:

1) A less fair-minded consumer; i.e., a customer with $\gamma_i \leq 1$, will never pay more than zero ($p_i = 0$), thus (1a) simplifies further:

$$u_i = r_i - \gamma_i r_{i0}. \quad (1b)$$

Next, we substitute (2) in (1b) and arrive at:

$$u_i = \begin{cases} r_i - \gamma_i c & r_i \leq c \\ r_i - \gamma_i(\lambda r_i + (1-\lambda)c) & r_i > c \end{cases}. \quad (1c)$$

[1] In the interest of a consistent notation, we use p^*_{PAAP} instead of p_U (which is used by CKZ).
[2] In their Appendix B CKZ note that $\lambda > 1/2$ for PAYW to be optimal.

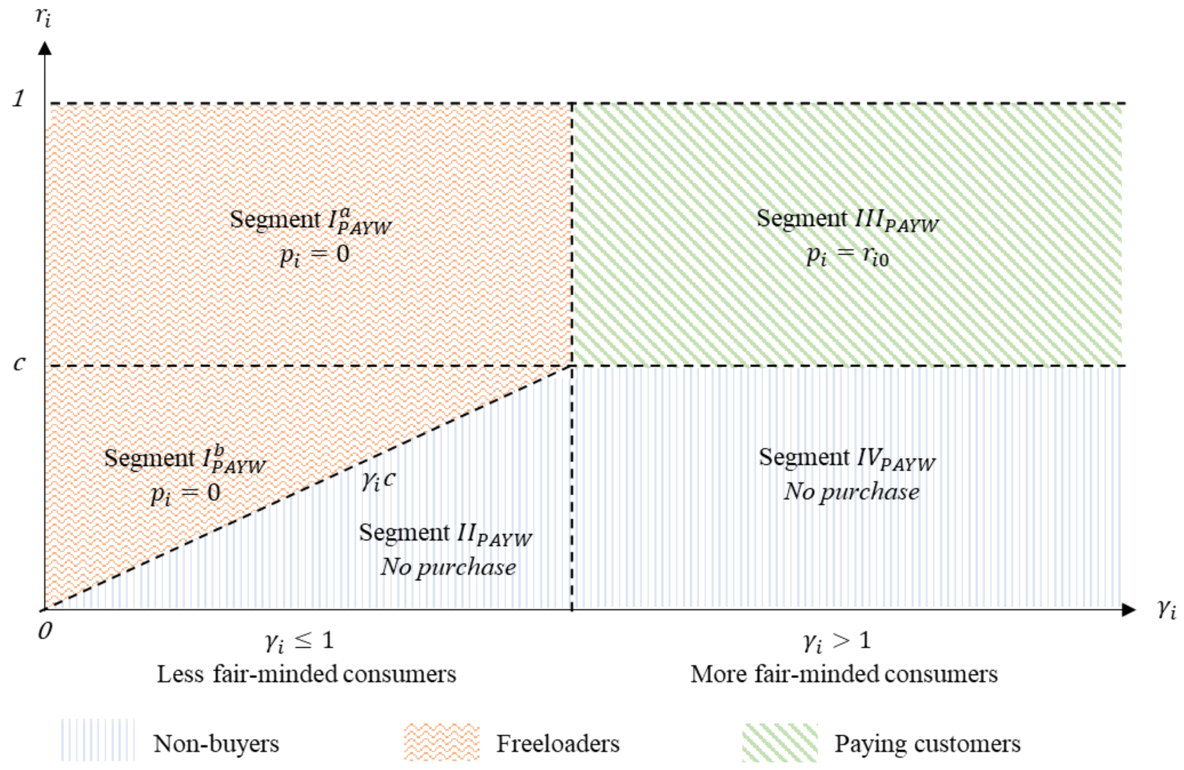

Figure 1: Customer Segments in PAYW

a) For a customer with $r_i > c$ we rearrange (1c) to $u_i = r_i(1 - \gamma_i\lambda) - (\gamma_i - \gamma_i\lambda)c$ and find that $u_i > 0$. For this customer, obtaining the good for free always leads to a positive utility. Thus, she will always take the good and pay nothing (Segment I^a_{PAYW} in Figure 1). This behavior is termed freeriding or freeloading.

b) For a customer with $r_i \leq c$, $u_i > 0$ only, if $r_i > \gamma_i c$. In this case, she also takes the good for free (Segment I^b_{PAYW} in Figure 1). If her consumption utility is low (i.e., $r_i \leq \gamma_i c$), however, she is better off not buying the product (Segment II_{PAYW} in Figure 1).

2) For a more fair-minded consumer; i.e., for a customer with $\gamma_i > 1$, the upper bound of the domain for p_i is relevant and utility is maximized for $p_i = r_{i0}$; thus $u_i = r_i - r_{i0}$. Because of (2) (i.e., $c \leq r_{i0} \leq r_i$) the utility is not negative when $r_i > c$; in this case, the customer buys the good and pays her perceived fair price r_{i0} (Segment III_{PAYW} in Figure 1). The utility is not positive when $r_i \leq c$ and, therefore, the customer does not purchase the good (Segment IV_{PAYW} in Figure 1).

Contrary to Figure 1, CKZ, in their Eq. (4), state that under PAYW *"... consumers with valuation $r_i < c$ pay zero dollars if $\gamma_i \leq 1$ and do not buy if $\gamma_i > 1$"* (p. 783). Given the derivation above, we suggest adapting this statement by highlighting that there might be customers with $r_i < c$ and $\gamma_i \leq 1$ who also do not buy, instead of obtaining the good with a payment of zero (i.e., Segment II_{PAYW} in Figure 1). The next subsections analyze whether this refinement has consequences for other results derived by CKZ.

3.2 Consequences of the refinement on the proportion of freeloaders

CKZ introduce heterogeneity by assuming that r_i and γ_i are distributed independently over the population according to density functions $\phi(r)$ and $h(\gamma)$, respectively. Note that CKZ assume uniformly distributed consumption utilities with $\phi(r) = 1$.[3] Thus, their proportion of freeloaders (θ^{CKZ}) is given by

$$\theta^{CKZ} = \int_0^1 \int_0^1 \phi(r)h(\gamma)dr\,d\gamma = \int_0^1 h(\gamma)d\gamma,$$

which is equal to Segments I^a_{PAYW}, I^b_{PAYW}, and II_{PAYW} in Figure 1. Following our suggested refinement, we would need to exclude Segment II_{PAYW} (i.e., customers who refrain from purchasing because they do not want to damage the seller, δ) from the share of freeloaders. Therefore, $\theta = \theta^{CKZ} - \delta$ with

$$\theta = \int_0^1 \left[\int_{\gamma c}^{c} \phi(r)dr + \int_c^1 \phi(r)dr\right] h(\gamma)d\gamma = \int_0^1 \int_{\gamma c}^{1} \phi(r)h(\gamma)dr\,d\gamma \qquad (5)$$

$$\delta = \int_0^1 \int_0^{\gamma c} \phi(r)h(\gamma)drd\gamma\,. \qquad (6)$$

For calculating δ, (6) aggregates over all less fair-minded customers whose utility would be negative when they take the good and pay nothing. One of the issues that emerges from (5) is that unlike in CKZ, the number of freeloaders does not only depend on the distribution of γ but also on the distribution of consumption utilities r, which also affects the other measures provided by CKZ (i.e., profits, critical costs, and the suggested price). Next, we demonstrate the effects on the firm's profit π_{PAYW}.[4]

[3] Therefore, we omit index i for this subsection.
[4] In the interest of a consistent notation, we use π_{PAYW} instead of π_p (which is used by CKZ).

The firm's profits in PAYW are given by

$$\pi_{PAYW} = \int_c^1 (1-\delta-\theta)(r_0 - c)\phi(r)dr - c\theta = \int_c^1 (1-\delta-\theta)\lambda(r-c)\phi(r)dr - c\theta. \qquad (7)$$

The first term of the profit function includes all paying customers. They pay their perceived fair price and the costs c incur at the firm level. The second term of the right-hand side of (7) describes the firm's costs that result from freeloaders. When checking the corresponding Eq. (4) in CKZ,

$$\pi_{PAYW}^{CKZ} = \int_c^1 (1-\theta^{CKZ})\lambda(r-c)\phi(r)dr - c\theta^{CKZ} \qquad (7a)$$

it becomes obvious that $\Delta\pi = \pi_{PAYW} - \pi_{PAYW}^{CKZ} = c(\theta^{CKZ} - \theta) = c\delta \geq 0$. This implies that (7a) underestimates profits under a PAYW policy.

Next, we investigate the conditions under which $\Delta\pi$ might be substantial. This requires postulating assumptions about the distributions of r and γ in order to solve the integral in (6). Remember that we follow CKZ by specifying $\phi(r)$ as a density function of a uniform distribution over the $[0,1]$ domain. Little empirical evidence is available supporting the choice of an appropriate distribution of γ and, therefore, we conduct some conceptual considerations. If there are no customers with $\gamma > 1$, then nobody would pay for the good and, therefore, PAYW cannot be profitable. These segments (Segment III_{PAYW} and IV_{PAYW}), however, do not affect $\Delta\pi$, and we assume this proportion to be $(1-\omega)$. For further reference, we label this part of the market as more fair-minded consumers. As a result, there also exist some customers with small $\gamma \leq 1$. These consumers (Segment I_{PAYW}^a, I_{PAYW}^b, and II_{PAYW}) will make up a proportion of ω of the market and match to the less fair-minded consumers that will never pay. Figure 1 highlights that Segment II_{PAYW} creates the difference between the CKZ model and our refinement.

We postulate γ to be distributed according to some distribution $h_{[0,1]}(\gamma)$ in the domain [0,1]. Therefore (cf. (6)),

$$\delta = \int_0^1 \left[\omega h_{[0,1]}(\gamma) \int_0^{\gamma c} dr\right] d\gamma = c\omega\bar{\gamma}_{[0,1]} \qquad (6a)$$

with $\bar{\gamma}_{[0,1]}$: mean of γ in [0,1].

The determination of δ permits the identification of the drivers of $\Delta\pi = c\delta$. Clearly, (i) the higher the seller's costs (c); (ii) the larger the proportion of less fair-minded consumers (ω; i.e., more customers in the critical area $0 \leq \gamma \leq 1$); and, (iii) the higher the mean of γ in this area, the higher the difference between the two profit functions. Thus, for higher costs and higher $\bar{\gamma}_{[0,1]}$, more customers will decide not to freeload as their disutility from advantageous inequity aversion will exceed their acquisition utility ($r_i - p_i$). These results possess face validity.

Previous studies on dictator games show that some otherwise selfish participants prefer to give up a monetary payoff to avoid showing unfair behavior to their co-participants (Dana et al. 2006, Lazear et al. 2012). This refinement incorporates such actions into PAYW: even customers with relatively low values of advantageous inequity aversion can (at least in some cases) decide not to purchase at all. To show that this behavior also exists in real PAYW situations outside the lab, we report on a PAYW field experiment in Appendix A that shows that the share of δ-customers accounts for about 20 percent of all promotional customers in our sample. Thus, empirically, this error might be highly relevant.

3.3 Consequences of the refinement on Proposition 1

This section explains how the misspecification affects CKZ's Proposition 1. First, note that CKZ's θ^{CKZ} is equivalent to our ω, which is the share of less fair-minded consumers. This share is independent of the cost level c. This allows CKZ to determine the critical cost level, c^+,[5] above which PAYW cannot be more profitable than PAAP.

However, as we see from (5), the correct share of freeloaders θ depends on c. Thus, inferring conditions on c^+ from the share of freeloaders, θ, is not viable. Therefore, the correct region in which PAYW is optimal is given by[6]

5 CKZ name this threshold c^*. However, to avoid confusion with the asterisks that denote optimal values, we will use the "+"-superscript.

6 See Appendix B for the proof.

$$c^{+} < 1 - \frac{2B\left(1 - 2\bar{\gamma}_{[0,1]}\right) - 2\sqrt{B\left(B - A\left(1 - \bar{\gamma}_{[0,1]}\right)\right)}}{A - 4B\bar{\gamma}_{[0,1]}}$$

$$=: c^{+}(\beta, \lambda, \omega, \bar{\gamma}_{[0,1]}) \tag{8}$$

with $A = 1 - \lambda\left(2 + \beta - 2\omega(1 + \beta)\right)$, $B = \omega(1 + \beta)$.

$$c^{+} > 0 \text{ if } \omega \leq 1/2 \text{ and } \lambda > 1/(2 + \beta)$$

Like in CKZ, the critical cost, c^{+}, depends on the degree of disadvantageous inequity aversion, β, generosity, λ, and on the share of less fair-minded customers, θ^{CKZ}; i.e., ω. However, in contrast to CKZ, the critical costs also depend on $\bar{\gamma}_{[0,1]}$, the average degree of advantageous inequity aversion of less fair-minded customers.

Thus, we summarize the results in an updated Proposition 1:

Updated Proposition 1: For any given β, ω, and $\bar{\gamma}_{[0,1]}$ and for sufficiently large λ, the critical cost c must be sufficiently small for a firm to choose PAYW over PAAP. However, low costs are not a sufficient condition. Even at zero cost, PAYW will not dominate PAAP as a pricing mechanism if too few consumers are willing to pay ($c^{+}\left(\beta, \lambda, \omega > 1/2, \bar{\gamma}_{[0,1]}\right) \leq 0$) or if their generosity is too low ($c^{+}\left(\beta, \lambda \leq 1/(2 + \beta), \omega = 1/2, \bar{\gamma}_{[0,1]}\right) \leq 0$).

Our refinement (8) allows us to derive conditions under which the cost c must be sufficiently small for a firm to choose PAYW over PAAP. In doing so, we need to consider the share of less fair-minded consumers, ω, and incorporate the mean of the distribution of advantageous inequity aversion for those customers, $\bar{\gamma}_{[0,1]}$, in our model. Furthermore, we need to consider the level of disadvantageous inequity aversion, β, and generosity, λ. This contrasts with CKZ's Proposition 1 that derives c^{+} based on θ^{CKZ}, β, and λ. Thus, instead of relying on the share of freeloaders, we need to take the share of less fair-minded consumers and their advantageous inequity aversion into consideration.

The second part of Proposition 1 states that PAYW is not preferable over PAAP when (more than) half of the consumers are less fair-minded or generosity is too low. This is in contrast to CKZ, who only claim that a firm will not operate more profitably under PAYW than under PAAP when it faces exclusively freeloaders. This is not a strong conjecture as, compared to a PAYW firm that faces only freeloaders, a firm using PAAP would already be better off if it does not sell anything. However, we show that it is not necessarily the number of freeloaders that make PAYW unattractive but rather the absence of paying and less generous customers.

4 PAYW and the minimum price

4.1 Conceptual remarks

In the previous sections, we identified an inconsistency in the definition of the size of the freeloader segment in PAYW. In PAYW with a minimum price (abbreviated as PAYW-MP in the sequel), this inconsistency does not influence the profitability of the seller, as a minimum price screens out potential freeloaders and obliges them to pay this minimum price, $\underline{p}$, as shown below.

However, we are going to argue that CKZ miss a substantive aspect by neglecting disadvantageous inequity aversion for most of their considerations (i.e., by assuming $\beta = 0$ on page 783). This assumption carries over to PAYW-MP. Setting $\beta = 0$ implies that consumers' utilities are unaffected by a seller setting prices above the fair price. CKZ claim that doing so only hampers the profitability of PAAP. They suggest that this will simply make PAAP more profitable than PAYW as the firm's profits are independent of β when the customers set the prices. Consequently, they disregard disadvantageous inequity for their subsequent analyses which implies that PAYW pricing schemes are unaffected by this.

Yet, this procedure is merely viable in the case of "pure" PAYW and PAYW with a suggested price. Under these circumstances, the consumers can set any price, including zero, and they can indeed avoid disadvantageous inequity. However, as soon as a minimum price is introduced, consumers can experience disadvantageous inequity (if $r_i \geq \underline{p} > r_{i0}$). Note that in this case consumer i's utility function (1) simplifies to[7]

$$u_i = r_i - \underline{p} - \beta\,(\underline{p} - r_{i0}). \tag{1d}$$

[7] We follow CKZ and choose a constant β for all consumers; i.e., $\beta_i = \beta$.

All consumers will buy the product and pay the minimum price if their consumption utility is higher than the minimum price and the disutility from disadvantageous inequity (i.e., $u_i > 0$). We denote the critical consumption utility that must be exceeded in order to buy the product as critical threshold[8],

$$r^+ = \underline{p} + \frac{\beta(1-\lambda)\left(\underline{p} - c\right)}{1 + \lambda\beta}. \tag{9}$$

However, despite $\beta > 0$, (1) implies γ_i and β to be independent of each other. Therefore, consumers still experience advantageous inequity if $\gamma_i > 0$ and $r_i > r_{i0} > \underline{p}$.

In setting $\beta = 0$, CKZ (implicitly) claim that consumers do not care that the seller introduces a minimum price if the minimum price is higher than their fair price. This means that even if the seller sets an exploitative minimum price and if buyers feel that the seller tries to scam most consumers, they will without any grief accept the price as long as it is below their consumption utility.

Moreover − following the set-up of CKZ − more fair-minded consumers are assumed to pay even more than the minimum price voluntarily, provided their consumption utilities are sufficiently large. In other words, instead of retaliating, the consumers would turn the other cheek. However, many empirical studies show that (i) the introduction of a minimum price lowers overall prices (Johnson and Cui 2013, Jung et al. 2016); that (ii) consumers do indeed care for disadvantageous inequity; and that (iii) some consumers are driven out of the market when a minimum price is set.

Allowing for $\beta > 0$ adds behavioral realism to the PAYW model, for example, a notion of reciprocity, a payment motive that has been shown to play an important role in PAYW pricing systems (Kim et al. 2009, León et al. 2012). If the seller behaves nicely towards the buyer, e.g., by offering PAYW pricing, the buyer will pay the fair price. However, if the seller behaves unfairly by asking an exploiting minimum price, the buyers might decide to omit the consumption opportunity instead. This can then be interpreted as a reciprocal way to punish the seller by decreasing the seller's profits.

[8] For consistency, we name this threshold r^+ instead of r^* as CKZ do in PAAP.

In sum, abstracting from disadvantageous inequity aversion appears to be an oversimplification that affects the analysis of the seller's profitability under PAYW-MP and skews the results towards this pricing policy.

4.2 Consumers' responses when considering disadvantageous inequity aversion

Domain of $\underline{p}$: A minimum price below cost; i.e., $\underline{p} < $ c, does not affect fair-minded consumers (i.e., $\gamma_i > 1$), as they never buy below the fair price which is always above c. Less fair-minded consumers (i.e., $\gamma_i < 1$) will either pay $\underline{p}$ or do not buy. This either turns freeloaders into buyers or relieves the seller from freeloaders. Therefore, considering profits, there is no benefit from setting $\underline{p}$ below c.

For a minimum price above the highest possible fair price; i.e., $\underline{p} > p_h$, nobody will pay more than this which then resembles to PAAP. As a result, $[c, p_h]$ is the domain for $\underline{p}$ and $\lambda > 1/(2+\beta)$ (cf. (4)).

Customer segments: When determining the (optimal) minimum price, the seller considers all potential circumstances for the consumer's consumption utilities (i.e., $0 \le r_i \le 1$). The horizontal axis of Figure 2 presents these potential settings (discussed in detail below); the vertical axis shows the corresponding prices. The notation by CKZ does not distinguish between utilities and prices which at some points impede ease of presentation; for this reason, we use $p_f = \lambda r + (1-\lambda)c$ instead of r_0 for the perceived fair price if $r > c$ (cf. (2)); for notational convenience, we also drop the index i. We denote the consumption utility that equals the minimum price as $\underline{p}^u$. This threshold indicates that even consumers without disadvantageous inequity aversion will not buy if their consumption utility is below the minimum price; i.e., $r < \underline{p}^u$. Moreover, $\underline{r}$ denotes the critical consumption utility for which $\underline{p} = p_f$ and is determined by solving (2) for $\underline{r}$; i.e., $\underline{r} = \left(\underline{p} - (1-\lambda)c\right)/\lambda$ (cf. Figure 2).

As in PAYW, we distinguish between different types of customers and summarize our results graphically in Figures 2 and 3. Figure 3a delineates the consumer segments according to consumption utilities (outer vertical axis) and advantageous inequity aversion (horizontal axis). Furthermore, the corresponding fair prices are given on the inner vertical axis. Figure 3b considers the same (inner and outer) vertical axes but with disadvantageous inequity aversion on the horizontal axis.

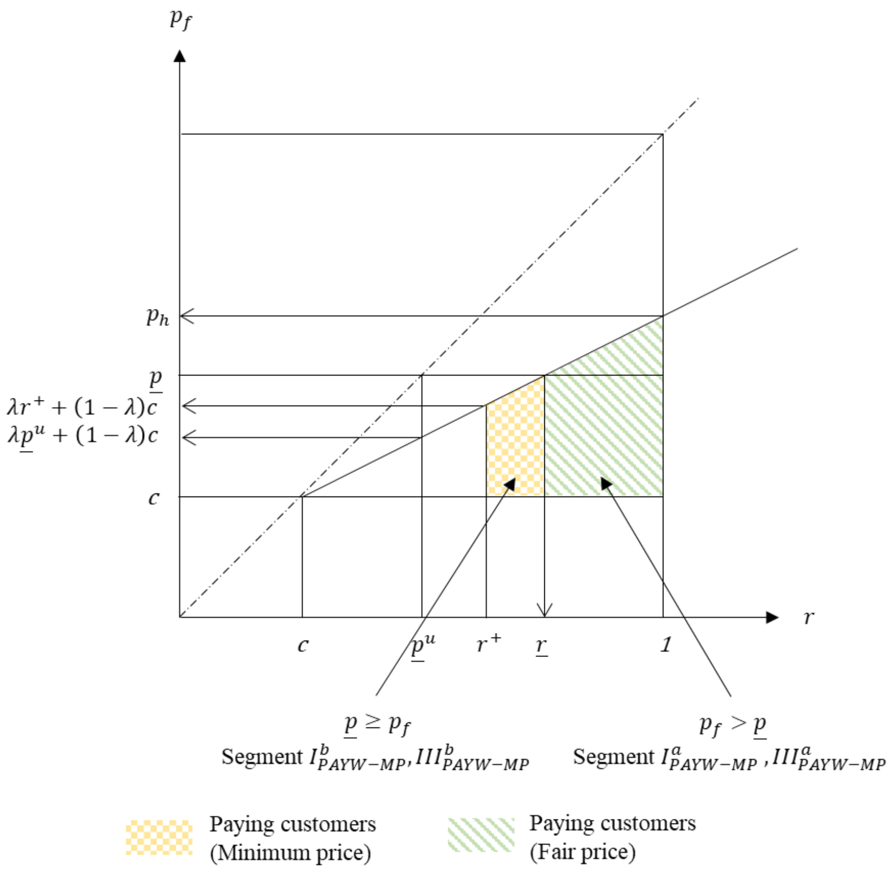

Figure 2: The basic relationship between consumption utilities and prices for $\gamma > 1$ via (2) for PAYW-MP

All potential relationships between $r, \underline{p}^u$, r^+ and $\underline{r}$ have to be taken into consideration.

a) $r \leq \underline{p}^u \leq r^+$ (cf. Figure 2, horizontal axis)

As the consumption utility is below the consumption utility of the minimum price, the consumers will never buy the product (Segments $II^b_{PAYW-MP}$and $IV^b_{PAYW-MP}$ in Figure 3).

b) $\underline{p}^u \leq r \leq r^+$ (cf. Figure 2, horizontal axis)

The consumption utility is above that of the minimum price but below the threshold of purchasing the product (Equation (9)). Therefore, consumers do not purchase the product (Segments $II^a_{PAYW-MP}$ and $IV^a_{PAYW-MP}$ in Figure 3).

c) $\underline{p}^u \leq r^+ \leq r \leq \underline{r}$ (cf. Figure 2, horizontal axis)

In this case, the consumption utility exceeds the threshold for purchasing the product. Therefore, consumers buy the product. However, as their consumption utility is below the critical limit corresponding to a perceived fair price, they suffer from disadvantageous inequity and pay the minimum price only (Segments $I^b_{PAYW-MP}$ and $III^b_{PAYW-MP}$ in Figure 3).

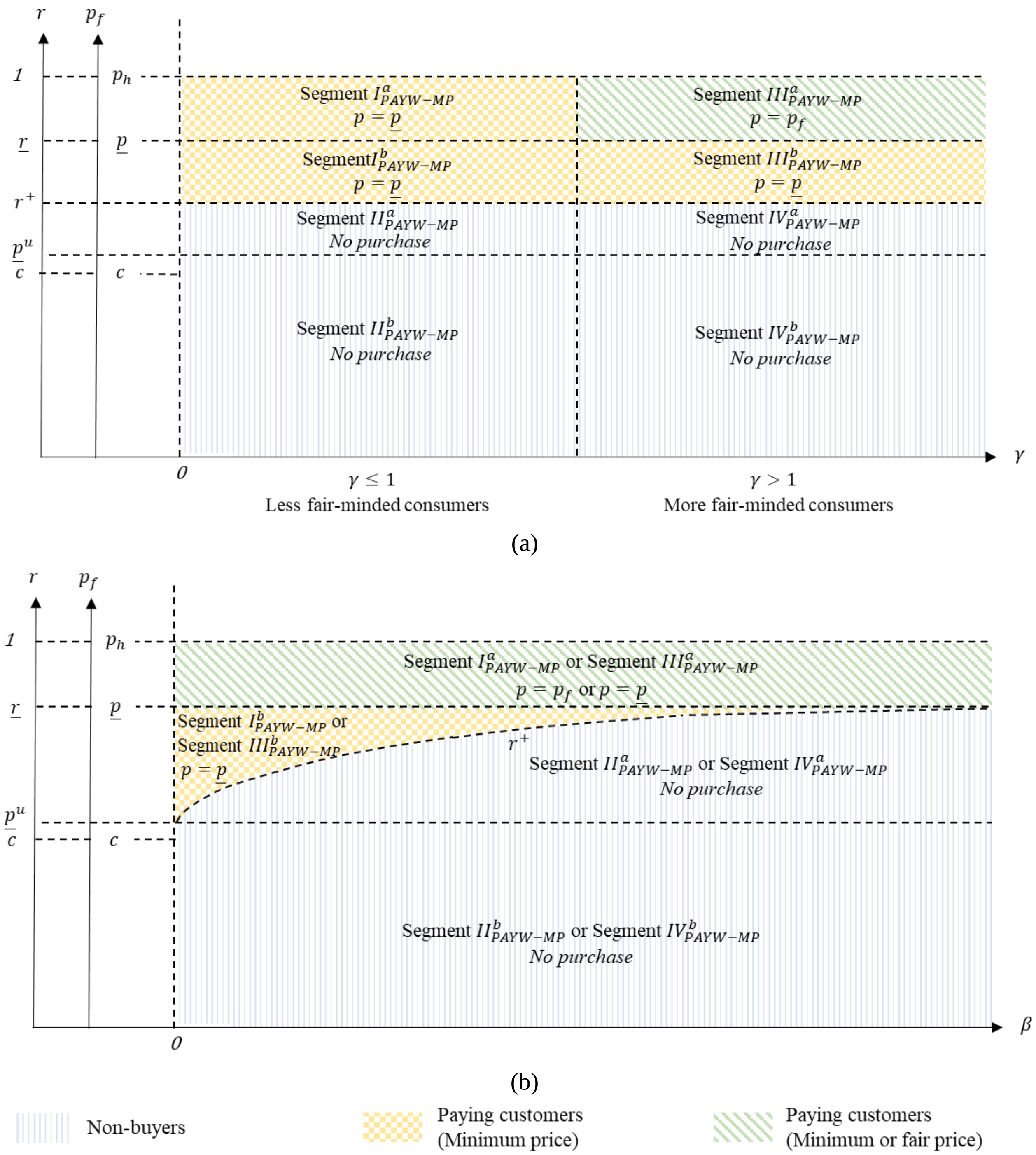

Figure 3: Customer Segments in PAYW-MP

d) $\underline{p}^u \leq r^+ \leq \underline{r} \leq r$ (cf. Figure 2, horizontal axis)

The consumers' consumption utility is above the threshold for purchasing the product and the critical limit corresponding to a perceived fair price. Therefore, the more fair-minded consumers maximize their utility by buying the product and paying their fair price in order not to suffer from advantageous inequity aversion (Segment $III^a_{PAYW-MP}$in Figure 3). Less fair-minded consumers maximize their utility by buying the good and paying the minimum price withstanding the advantageous inequity aversion this behavior creates (Segment $I^a_{PAYW-MP}$ in Figure 3).

4.3 Optimal prices and the firm's profits under PAYW-MP

Distinguishing between customer segments above, the firm's profits are as follows:[9]

$$\pi_{PAYW-MP} = (1-\omega)\left(\int_{r^+}^{\underline{r}} \left(\underline{p} - c\right)\phi(r)\,dr + \int_{\underline{r}}^{1} \lambda(r-c)\,\phi(r)dr\right) + \omega\int_{r^+}^{1}\left(\underline{p}-c\right)\phi(r)\,dr. \tag{10}$$

The first term pertains to the profits made from more fair-minded consumers ($(1-\omega)$ in size). The first integral determines revenues of more fair-minded consumers with consumption utilities above the critical threshold but below the critical limit corresponding to a perceived fair price (i.e., $r^+ \leq r \leq \underline{r}$). These consumers buy the good and pay the minimum price. The second integral determines the revenues from the consumers with high consumption utilities (i.e., $r \geq \underline{r}$). These consumers buy the good and pay their fair price. The third integral describes the revenues from less fair-minded consumers (ω in size). These consumers buy the good and pay the minimum price. For all consumers, the seller incurs costs c.

To maximize profits, the firm chooses the optimal minimum price (see Appendix C for proofs). The optimal minimum price for the above equation is

9 For consistency, we use $\pi_{PAYW-MP}$ instead of π_p^{2m} (which is used by CKZ).

$$\underline{p}^* = c + (1-c)k\lambda$$

$$\text{with } k = \begin{cases} 1 & \lambda \le \frac{1}{2+\beta} \\ \frac{\omega(1+\beta\lambda)}{(2+\beta(1+\omega))\lambda - 1 + \omega} & \lambda > \frac{1}{2+\beta} \end{cases} \tag{11}$$

Rewriting the minimum price according to CKZ,

$$\underline{p}^{*CKZ} = c + \frac{\omega\lambda(1-c)}{2\lambda - 1 + \omega} \quad \text{for } \lambda \ge \frac{1}{2} \tag{12}$$

it becomes obvious that $\underline{p}^*$ and $\underline{p}^{*CKZ}$ are identical for $\beta = 0$ but the new formulation extends the feasible range of λ and β. From a behavioral point of view, this implies that we can allow for less generous customers if at the same time we allow for more pronounced disadvantageous inequity aversion. This extension is in line with the situation for PAAP (cf. (3), (4)).

From this starting point, we explore the effect of the inclusion of disadvantageous inequity aversion on CKZ's Proposition 2 and summarize our results in an updated proposition.

Updated Proposition 2. Under PAYW-MP, a profit-maximizing firm's optimal minimum price when consumers' valuations are distributed uniformly is $\underline{p}^*$. The optimal minimum price always increases with cost c ($\delta\underline{p}^*/\delta c \ge 0$). Furthermore, the optimal minimum price increases with the proportion of less fair-minded consumers ω ($\delta\underline{p}^*/\delta\omega \ge 0$) and decreases with disadvantageous inequity aversion β ($\delta\underline{p}^*/\delta\beta \le 0$). No general conclusions with respect to generosity can be made since $\delta\underline{p}^*/\delta\lambda$ might be either less than, equal to, or larger than 0 (depending on β, λ, and ω; see Appendix C for details).

Conceptionally, PAYW-MP is a mix between PAAP and PAYW. The minimum price allows the firm to screen out freeloaders while benefiting from the additional consumption utility of fair-minded consumers with a high consumption utility. However, as in PAAP, there is a dark side for a minimum price that exceeds the perceived fair price of the consumers. These customers will be driven out of the market although they would contribute to the seller's profits when paying their fair price.

The proposition states, in line with CKZ, that the higher the costs and the higher the share of less fair-minded consumers, the higher the optimal minimum price. However, in addition to CKZ, we find that the optimal minimum price is kept in check by the disadvantageous inequity aversion in the market, β. If the minimum price is perceived as being unfairly high, consumers with high levels of disadvantageous inequity aversion will shy away from purchasing the product leading to a foregone profit opportunity for the seller. Therefore, the more disadvantageous inequity averse the buyers the lower the optimal minimum price $\underline{p}^*$.

CKZ find that the optimal minimum price “decreases with the generosity of fair-minded consumers λ” (p. 784). However, the above derivations paint a more nuanced picture. For low levels of generosity ($\lambda \leq \frac{1}{2+\beta}$), the seller’s minimum price will be p_h. As p_h increases linearly with λ, the optimal minimum price also rises with higher generosity. This allows the seller to extract more profits from Segments $I^b_{PAYW-MP}$ and $III^b_{PAYW-MP}$. For high levels of generosity ($\lambda > \frac{1}{2+\beta}$), the optimal minimum price depends on ω and β (cf. Figure C.1, Table C.1 in Appendix C). If the share of less fair-minded consumers and the degree of disadvantageous inequity aversion are low, the minimum price decreases with higher generosity. In this case, the seller wants to expand on Segments $I^b_{PAYW-MP}$ and, particularly, on Segment $III^b_{PAYW-MP}$ by converting Segment $II^a_{PAYW-MP}$ and $IV^a_{PAYW-MP}$ consumers into buyers. However, in markets with high levels of disadvantageous inequity aversion and a high share of less fair-minded consumers, the firm increases the minimum price with higher generosity. If β is high, Segments $I^b_{PAYW-MP}$ and $III^b_{PAYW-MP}$ are small (cf. Figure 3b). Hence, if ω is high, the seller will increase the minimum price in case of higher generosity levels in order to increase revenues from less fair-minded consumers with high consumption valuations (Segments $I^a_{PAYW-MP}$).

Summing up, we expand CKZ’s analysis on PAYW-MP. We find that disadvantageous inequity aversion influences the seller’s minimum price. Furthermore, we extend CKZ’s Proposition 2 to now include the effect of disadvantageous inequity aversion on the optimal minimum price; as a result, generosity might increase the optimal minimum price under certain conditions.

5 PAYW with a suggested price

5.1 Conceptual remarks

This section considers the effects of the proposed refinement of the less fair-minded consumer segments, ω, on CKZ's derivations when accounting for a suggested price p_s (abbreviated as PAYW-SP in the sequel; Section 3.2. of CKZ). CKZ claim that the suggested price does not affect freeloaders (p. 785). However, as shown below, a higher suggested price also deters the purchase of some of these freeloaders.

According to CKZ, buyers pay attention to a suggested price with a probability of $(1 - z)$. In CKZ, less fair-minded customers always freeload. Yet, if less fair-minded customers maximize their utility correctly following (1b) and are influenced by a price suggestion above their perceived fair price, freeloading initiates more advantageous inequity aversion than without this price suggestion (i.e., they replace their level of comparison c by p_s). Therefore, cf. (1c), the utility for a less fair-minded consumer is

$$u_i = \begin{cases} r_i - \gamma_i p_s & r_i \leq p_s \\ r_i - \gamma_i(\lambda r_i + (1-\lambda)p_s) & r_i > p_s \end{cases} \text{ with a probability of } (1-z). \quad \text{(1e)}$$

Intuitively, this approach has face validity. Less fair-minded consumers who observe a fair price but still freeload might feel more embarrassed as they, even more obviously in the presence of a suggested price ($p_s \geq c$), reveal their true character to the seller in blatantly disobeying social norms. Therefore, considering the suggested price, a higher share of less fair-minded consumers might decide not to buy if the disutility from the embarrassment will outweigh consumption utility.

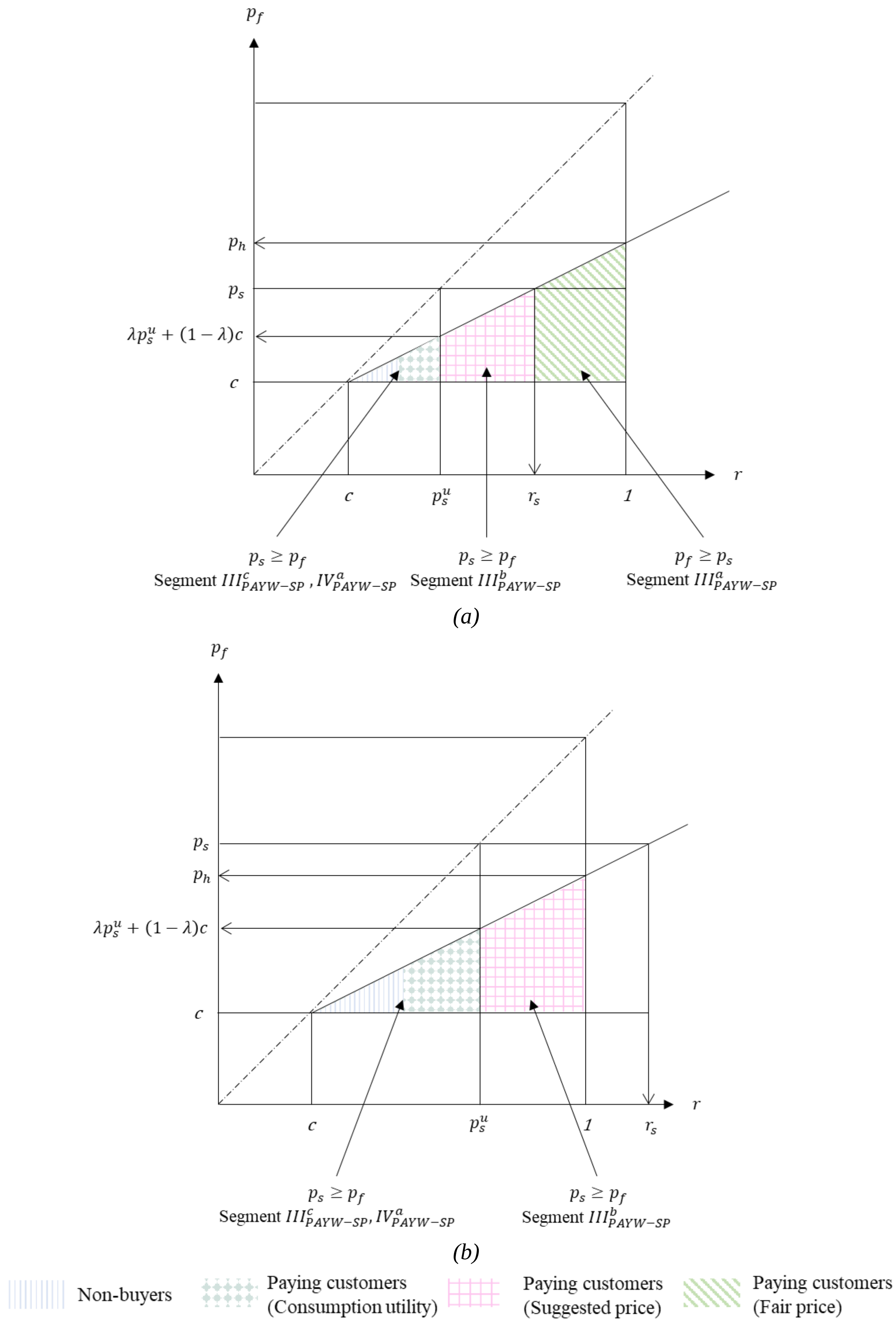

Figure 4: The basic relationship between consumption utilities and prices for $\gamma > 1$ via (2) for PAYW-SP

5.2 Consumers' responses to PAYW-SP

Thus, for a holistic view of PAYW-SP, many different types of consumer behavior have to be distinguished. Again, for notational convenience, we drop the index i and make use of the notation p_f to identify the perceived fair price if $r > c$ (cf. (2)) and, the critical consumption utility $r_s = (p_s - (1-\lambda)c)/\lambda$ for which $p_s = p_f$. We distinguish the following consumer segments:

(1) Customers who ignore the suggested price (with probability z)

These customers correspond to the situation analyzed in pure PAYW (Section 3 and Figure 1). Thus, the previous derivations still apply.

(2) Customers who consider the suggested price p_s (with probability $1 - z$)

Relationships between four interrelated (cf. (2)) variables (c, r, p_f, and p_s) have to be considered here. Please note that p_s is set by the seller without knowing the utility structure of the consumer (i.e., her perceived fair price p_f) but the seller considers all potential circumstances for the consumer's consumption utilities (i.e., $0 \leq r_i \leq 1$). Thus, we distinguish two sub-cases: a) $p_s \leq p_f$; b) $p_s > p_f$; these cases are highlighted by Figures 4a and 4b. As before, the horizontal axis represents consumption utility, and the vertical axis prices. Again, we use p_s^u to denote the critical consumption utility below which consumers will never pay the suggested price (or more). Figure 5 complements Figure 1 by additionally accounting for these $(1 - z)$ customers who pay attention to the suggested price. As before, Figure 5 employs two verticalaxes: one scale for $0 \leq r \leq 1$, and one scale for $0 \leq p_f \leq p_h$. Given the linear relationship between these two variables, Figures 4a and 4b should facilitate interpretation.

a) $p_s \leq p_h$ (cf. Figure 4a, vertical axis)

Please note, that $c \leq p_s \leq 1$: The seller would never suggest a price below costs, as every consumer who purchases for this suggested price would result in a loss.

(i) $r \leq c \leq p_s^u$ (cf. Figure 4a, horizontal axis)

CKZ do not explicitly discuss this case. As we are in the scenario in which the consumers consider the seller's suggestion, the perceived fair price p_f is set to p_s. Fair-minded consumers abstain from buying (Segment $IV^b_{PAYW-SP}$ in Figure 5); for less fair-minded buyers we find freeloaders (Segment $I^{d}_{PAYW-SP}$ in Figure 5) or non-purchasers (Segment $II^{b}_{PAYW-SP}$). Note, that compared to 'pure' PAYW, a higher share of less fair-minded buyers is driven out of the market, as their fair prices and their extent of advantageous inequity are higher than without a price suggestion ($c \leq p_s$).

(ii) $c < r \leq p^{u}_{s}$ (cf. Figure 4a, horizontal axis)

CKZ assume that with a probability of 0.5, the customer's fair price p_f is either equal to r or to p_s. The intuition behind this assumption is that a more fair-minded customer might feel it is justifiable to set her fair price to r and make a purchase (Segment $III^{c}_{PAYW-SP}$in Figure 5) or, alternatively, she may feel embarrassed for not paying p_s and decides not to purchase at all (Segment $IV^{a}_{PAYW-SP}$ in Figure 5) (CKZ, p. 790, FN 9). This, however, does not apply for less fair-minded customers. They will never pay, their perceived fair price is still the suggested price p_s and they will either freeload (Segment $I^{c}_{PAYW-SP}$ in Figure 5) or do not purchase (Segment $II^{a}_{PAYW-SP}$ in Figure 5). Please note that this segment is empty for $\gamma p_s \leq c$.

(iii) $c < p^{u}_{s} < r \leq r_s$ (cf. Figure 4a, horizontal axis)

Solving $p_f \leq p_s$ we find $r_s = (p_s - (1-\lambda)c)/\lambda$. More fair-minded consumers purchase the good and pay p_s (Segment $III^{b}_{PAYW-SP}$ in Figure 5). For less fair-minded consumers we have to keep $\gamma p_s < p_s$ in mind. Thus, all less fair-minded buyers freeload (Segment $I^{b}_{PAYW-SP}$ in Figure 5).

(iv) $c < p^{u}_{s} < r_s < r \leq 1$ (cf. Figure 4a, horizontal axis)

More fair-minded consumers purchase the good and pay p_f (Segment $III^{a}_{PAYW-SP}$ in Figure 5), less fair-minded free ride (Segment $I^{a}_{PAYW-SP}$ in Figure 5).

b) $p_s > p_h$ (cf. Figure 4b, vertical axis)

Again we would need to consider different segments. Please note, that there are no differences with respect to (i), (ii) and (iii); but (iv) is empty because $r_s < 1$ is not feasible.

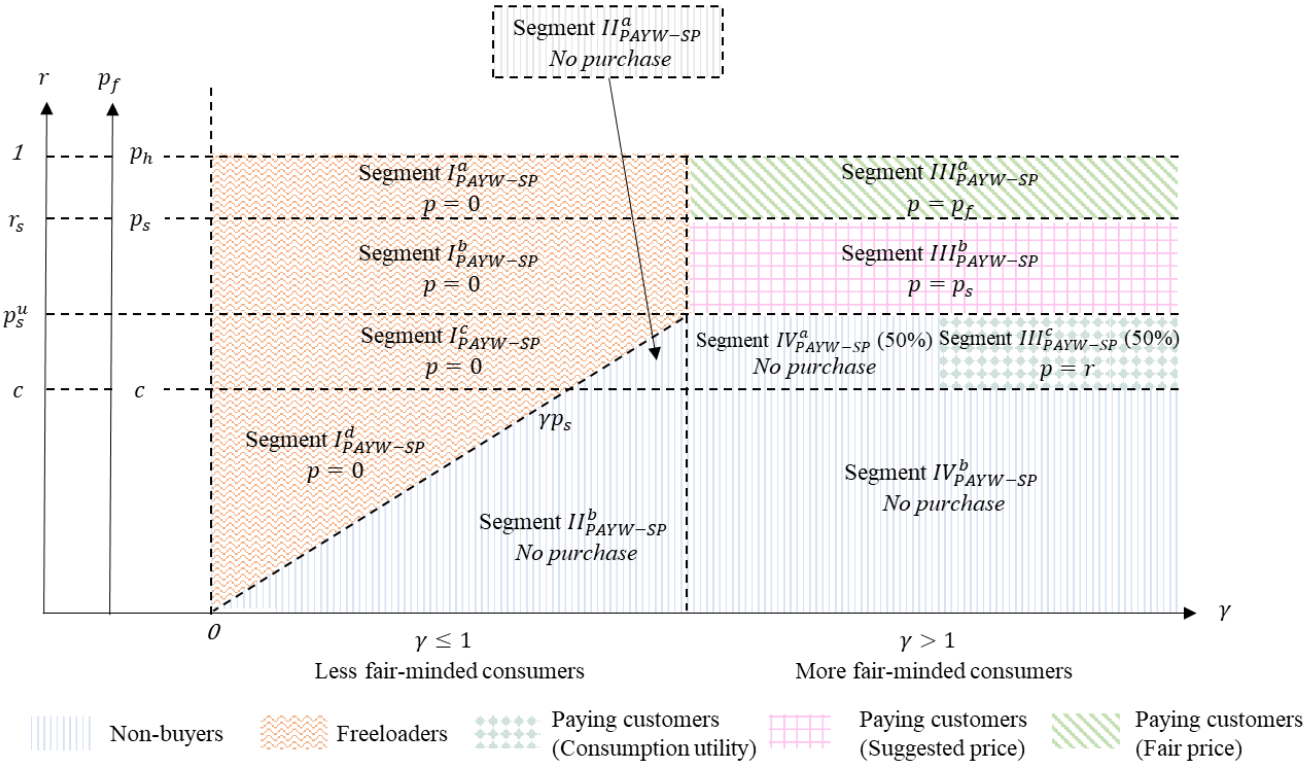

Figure 5: Customer Segments in PAYW for consumers who pay attention to the suggested price

Summing up, the share of freeloaders (Segments $I^a_{PAYW-SP}$, $I^b_{PAYW-SP}$, $I^c_{PAYW-SP}$ and $I^d_{PAYW-SP}$ in Figure 5) when a price has been suggested is for those, who ignore this suggestion (cf. (5))

$$\theta_s^z = z\int_0^1\int_{\gamma c}^1 \phi(r)h(\gamma)dr\,d\gamma = z\omega\left(1 - c\bar{\gamma}_{[0,1]}\right) \tag{5a}$$

and for those who consider the suggestion

$$\theta_s^{1-z} = (1-z)\int_0^1\int_{\gamma p_s}^1 \phi(r)\,h(\gamma)dr\,d\gamma = (1-z)\omega\left(1 - p_s\bar{\gamma}_{[0,1]}\right). \tag{5b}$$

By comparing θ_s^z with θ_s^{1-z} it becomes obvious that suggesting a price p_s results in a reduction of the share of freeloaders only if p_s is greater than c; i.e., $\bar{\gamma}_{[0,1]}p_s > \bar{\gamma}_{[0,1]}c$.

In a similar way, the segments of customers refraining from purchasing (Segment $II^a_{PAYW-SP}$and $II^b_{PAYW-SP}$) are (cf. (6))

$$\delta_s^z = z\int_0^1\int_0^{\gamma c} \phi(r)h(\gamma)dr\,d\gamma = zc\omega\bar{\gamma}_{[0,1]} \tag{6a}$$

and

$$\delta_s^{1-z} = (1-z)\int_0^1 \int_0^{p_s} \phi(r)h(\gamma)dr\, d\gamma = (1-z)p_s\omega\bar{\gamma}_{[0,1]}\,. \tag{6b}$$

5.3 Optimal prices and the firm's profits under PAYW-SP

Finally, we turn to the seller's profit (cf. (7)) and distinguish between sub-cases (a) and (b) as above. For sub-case (a) (of all segments described above, cf. enumeration (1) and (2)) we find:[10]

$$\begin{aligned}\pi_{PAYW-SP}^{(1)} = (1-z)\Bigg((1-\omega)\left(\frac{1}{2}\int_c^{p_s}(r-c)\,\phi(r)dr + \int_{p_s}^{r_s}(p_s-c)\,\phi(r)dr\right.\\ \left.+\int_{r_s}^{1}\lambda(r-c)\,\phi(r)dr\right)\Bigg) + z(1-\omega)\int_c^1 \lambda(r-c)\,\phi(r)dr\\ -\,c(\theta_s^{1-z}+\theta_s^z)\end{aligned} \tag{13a}$$

Only paying consumers ($(1-\omega)$ in size) generate sales. For them, the first integral corresponds to profits from consumers who consider the price suggestion but have a consumption utility below the price suggestion. However, they buy and pay their consumption utility (Segment $III_{PAYW-SP}^{c}$). The second integral corresponds to consumers who consider the price suggestion, whose consumption utility is above that of the price suggestion and who, therefore, pay the suggested price (Segment $III_{PAYW-SP}^{b}$). The third integral describes sales from consumers whose consumption utility is higher than the price suggestion and pay their perceived fair price (Segment $III_{PAYW-SP}^{a}$). The fourth integral refers to consumers ignoring the price suggestion (cf. (5)). Losses caused by freeloaders are subtracted.

In a similar vein, we consider profits for sub-case (b):

$$\begin{aligned}\pi_{PAYW-SP}^{(2)} = (1-z)\left((1-\omega)\left(\frac{1}{2}\int_c^{p_s}(r-c)\,\phi(r)dr + \int_{p_s}^{1}(p_s-c)\,\phi(r)dr\right)\right)\\ +\,z(1-\omega)\int_c^1 \lambda(r-c)\,\phi(r)dr - c(\theta_s^{1-z}+\theta_s^z).\end{aligned} \tag{13b}$$

[10] In the interest of a consistent notation, we use $\pi_{PAYW-SP}^{(1)}$ and $\pi_{PAYW-SP}^{(2)}$ instead of π_s^1 and π_s^2 (which is used by CKZ).

In PAYW-SP, the firm maximizes its profits by finding the optimal suggested price p_s^* (cf. Appendix D). The results of this maximization problem are given by

$$p_s^* = \begin{cases} 1 & \text{if} & & m > 1/2 \\ \left(2 + c + 2m(1-c)\right)/3 & \text{if} & 0 \leq \lambda \leq 2(1+m)/3 \quad \text{and} & m \leq 1/2 \\ c + 2\lambda m(1-c)/(3\lambda - 2) & \text{if} & 2(1+m)/3 \leq \lambda \leq 1 \quad \text{and} & m \leq 1/2 \end{cases} \tag{14}$$

$$\text{with } m = c\omega\bar{\gamma}_{[0,1]}/\left((1-c)(1-\omega)\right)$$

The corresponding results for the suggested price are summarized in our updated Proposition 3.

> **Updated Proposition 3**. Under PAYW-SP, a profit-maximizing firm's optimal suggested price when consumers' valuations are distributed uniformly, p_s^*, is given by (14). The optimal suggested price increases with the cost c ($\delta p_s^*/\delta c \geq 0$). Furthermore, the optimal suggested price increases with the proportion of less fair-minded consumers ω ($\delta p_s^*/\delta\omega \geq 0$) and the mean of the advantageous inequity aversion in $[0,1]$, $\bar{\gamma}_{[0,1]}$ ($\delta p_s^*/\delta\bar{\gamma}_{[0,1]} \geq 0$). In addition, a higher price is suggested if fair-minded consumers are not sufficiently generous ($\delta p_s^*/\delta\lambda \leq 0$).

Our Updated Proposition 3 postulates that the higher the seller's costs, the higher the optimal suggested price. This is expected as it does not make sense for the seller to set a suggested price lower than its costs, as this will only lower payments and hurt profitability. This fundamental result is in line with CKZ.

Furthermore, our proposition states that the higher the share of less fair-minded consumers the higher the optimal suggested price. By raising the suggested price, the seller influences the inequity perceptions of these less fair-minded customers. The higher the suggested price, the more inequitable and, therefore, the more psychologically costly it becomes to freeload. Therefore, if there is a higher share of less fair-minded consumers in the market, raising the suggested price enhances profitability. The stronger the advantageous inequity aversion of less fair-minded consumers, the more of these non-payers decide to refrain from purchasing as compared to freeloading. Therefore, the seller will set a higher suggested price if less fair-minded consumers have a higher degree of advantageous inequity aversion. If the share of potential freeloaders is very high, the firm will try to discourage them from buying by increasing the suggested price.

This is particularly true when the costs, c, the share of less fair-minded consumers, ω, and the distribution of advantageous inequity aversion, $\bar{\gamma}_{[0,1]}$ are simultaneously high. In this case ($m > 1/2$), the seller should focus on deterring the freeloading of less fair-minded consumers and set the highest possible suggested price; i.e., $p_s^* = 1$.

On this matter, our results deviate from CKZ's conclusions. CKZ argue that "the optimal suggested price is independent of θ [more accurately: θ^{CKZ}]. This is expected, as the suggested price does not affect the behavior of freeloaders".

As shown above, the correct share of freeloaders θ is given by the sum of (5a) and (5b). However, (5b) depends on p_s. Thus, the seller's suggested price influences the behavior of freeloaders. Moreover, in setting the suggested price, the seller must balance two opposing effects: On the one hand, the positive effect of a higher suggested price on non-payers, as described above. On the other hand, a higher suggested price will drive some of the fair-minded customers out of the market. Therefore, the more generous the customers (the higher λ) the lower the suggested price. Thus, instead of focusing on $\theta,$ we need to analyze the effect of the share of less fair-minded consumers, ω, (which is equivalent to θ^{CKZ}) and the distribution of advantageous inequity aversion for these consumers (represented by $\bar{\gamma}_{[0,1]}$) (cf. Figure D.1).

Furthermore, CKZ argue that the seller will set a suggested price of c when λ is large enough; i.e., $\lambda > \frac{2}{3}$. This effectively converts PAYW-SP to "pure" PAYW as it does not alter the buying behavior of paying customers. However, as we indicate above, this simplification does not hold when we consider non-buyers within of the freeloader segment. Our updated Proposition 3 demonstrates that a firm will always set a suggested price that is higher than the cost level in order to deter freeloaders.

Overall, this is consistent with anecdotal observations of PAYW-SP illustrating that the suggested price also influences freeloaders. As negative online reviews of popular PAYW restaurants like "Der Wiener Deewan" or "Weinerei" show, consumers (who would like to pay little or nothing) complain about the seller's price suggestions and promise not to return as the suggestion violates their freedom to exploit the PAYW offer and makes them feel immoral.

6 Choosing the right pricing mechanism

6.1 Set-up of the simulation study

Using the previous profit functions and optimal prices, CKZ move on to give normative guidelines for the seller's optimal pricing mechanism. In short, they find that

- when fair-minded consumers are not sufficiently generous; i.e., $\lambda \leq 2/3$,
  - the firm should adopt PAYW-SP when there are a few freeloaders,
  - PAAP when the number of freeloaders is middle-sized and
  - PAYW-MP, when the number of freeloaders is high;
- when consumers are sufficiently generous; i.e., $\lambda > 2/3$, the firm should adopt PAYW-MP.

CKZ calculate thresholds for the optimal pricing decision. However, this comparison is flawed for various reasons (i.e., consecutive faults of the correct specification of freeloaders and the omission of β, the upper bound of $\underline{p}^*$ is not accounted for in the calculation of optimal PAYW-MP profits, the definition of the optimal pricing scheme is incomplete, and calculation errors when comparing PAYW-SP to PAYW-MP; a detailed discussion of CKZ's results is available upon request from the authors).

In contrast to CKZ results, our model refinements add two additional parameters ($\bar{\gamma}_{[0,1]}$ and β), thereby resulting in a more complex picture concerning optimal prices and corresponding profits. This complicates the determination of the best pricing policy. In the sequel, we provide recommendations for the dominating pricing policy given certain market conditions. These recommendations are based on analytic considerations and a comprehensive simulation study. In addition, real world cases substantiate that such pricing policies are applied in practice.

The functional forms of the optimal prices under PAYW-MP (cf. (11)) and PAYW-SP (cf. (14)) depend on generosity λ which highlights the particular importance of this parameter. For the purpose of illustration, Figure 6 compares different pricing policies (PAAP, PAYW, PAYW-MP, and PAYW-SP) with generosity on the horizontal axis and corresponding profits on the vertical axes (for

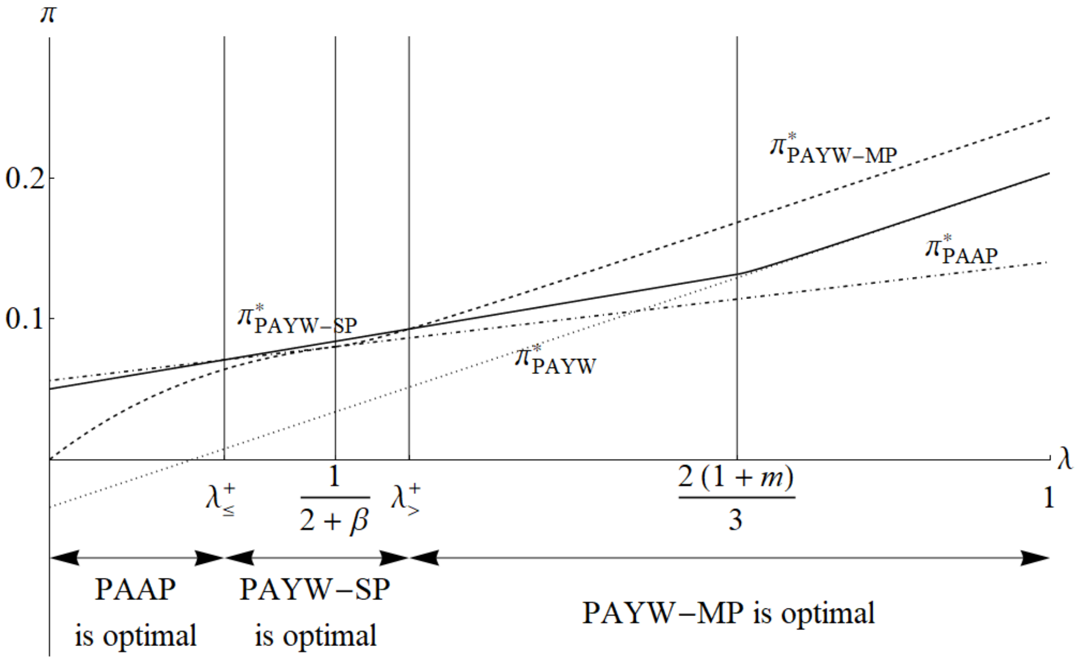

Figure 6: Optimal profits for different levels of generosity with $\beta = 1.5$, $\bar{\gamma}_{[0,1]} = .5$, $c = .25$, $\omega = .15$, $z = .5$, $m = 1/34$

presentational convenience the other parameters, $\beta, \bar{\gamma}_{[0,1]}, c, \omega$, and z, are kept constant at rather "typical" values). As a first remark, we note that all profit functions increase with generosity λ and that – tentatively – PAAP dominates for small λ, PAYW-SP for intermediate λ, and PAYW-MP for large λ. Analytical considerations provided in the sequel allow adding rigor to this remark. Figure 7 provides a more comprehensive picture and determines the optimal pricing mechanism for a variety of different parameter constellations.

- Generosity, λ, and share of less fair-minded customers, ω, are of particular importance and, therefore, Figure 7 offers a full enumeration of $0 \leq \lambda, \omega \leq 1$. The inner horizontal/vertical axes of the mappings in Figure 7 correspond to λ/ω, respectively.
- The domain of costs, c, is assumed to be $[0,1)$. Small c might occur for digital goods. However, because consumption utilities r cannot exceed 1, profits are only possible for $c < 1$. Therefore, the outer horizontal axis of Figure 7 considers $c \in \{0; .25; .5; .75\}$.
- We follow CKZ (pp. 789f.) and consider disadvantageous inequity aversion, $\beta \in$

$\{0; 1.5; 4; 6.5; 9\}$ on the outer vertical axis of Figure 7.

- Different shadings of the mappings represent these regions for which a certain pricing policy dominates the other policies (i.e., PAAP – green shading, PAYW-MP – red, PAYW-SP – blue). Partitions on the inner horizontal axis represent different levels of generosity λ; $2(1+m)/3$ depends on ω which results in the curvilinear progression; for $\lambda = 1$, $2(1+m)/3$ intersects with $m = 1/2$ (blue horizontal line in each mapping). The left column of Figure 7 corresponds to mappings with $c = 0$ and, therefore, $2(1+m)/3 \equiv 2/3$ and $m = 1/2$ is displayed at $\omega = 1$ which make these mappings appearing less "crowded". Finally, the "red" line corresponds to the threshold λ^{+} which separates regions for which PAYW-SP or PAAP ($\lambda^{+}_{\leq}$) and PAYW-SP or PAYW-MP are optimal ($\lambda^{+}_{>}$).

- In order not to overload Figure 7 and because of their minor impact on profits, the mean of advantageous inequity aversion of less fair-minded customers, $\bar{\gamma}_{[0,1]}$, and the probability of paying attention to the price suggestion, z, are kept constant (i.e., both equal to .5).

6.2 Results

6.2.1 Result 1 (focus on PAYW)

As shown by Proposition 1, PAYW can be more profitable than PAAP for small c, sufficiently small ω and sufficiently large λ. A customer's utility as induced by a standard homo oeconomicus model (i.e., neglecting both types of inequity aversion) will never be smaller than the utility according to (1). From the perspective of the seller, however, profits according to (1) might be larger, if there is a sufficient share of fair-minded customers who are willing to contribute. This implies a small ω and $\lambda > 0$. From an analytical point of view, PAYW (A.3) matches PAAP (A.1) if $\beta = 0$, $\omega = 0$, and $\lambda = 1/2$. As a general guideline: the seller should consider (variants of) PAYW if customers are sufficiently fair (i.e., $(1-\omega)$ substantial, λ high) and PAAP otherwise (cf. Figure 7).

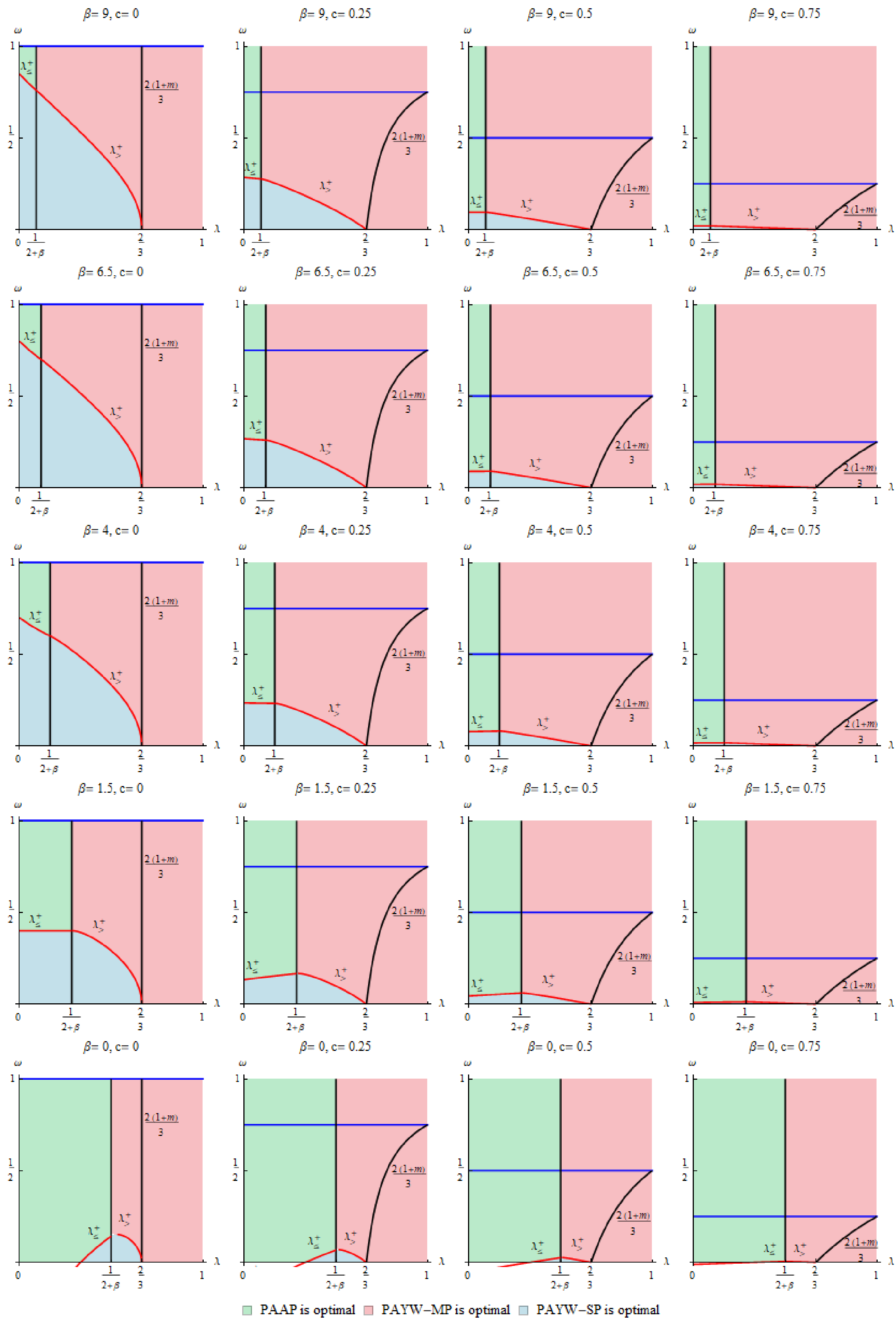

Figure 7: Choice of optimal pricing mechanisms depending on costs (c, outer horizontal axis), disadvantageous inequity aversion (β, outer vertical axis), generosity (λ, inner horizontal axes) and share of potential freeloaders (ω, inner vertical axes) for fixed $\bar{\gamma}_{[0,1]} = .5$ and $z = .5$.

The case study described in Appendix A (as an example for anonymous grocery shopping) provides empirical evidence for a market situation in which PAAP is optimal: consumers primarily care about their own consumption utility and are not admonished to ethical behavior by social pressure.

If the minimum price is set to zero, PAYW-MP corresponds to PAYW. Therefore, PAYW is nested within PAYW-MP and profits under PAYW-MP cannot be less than profits under PAYW. Thus, PAYW-MP is always preferred against PAYW if $\omega > 0$ (cf. A.3, A.8).

The retailer of digital goods, *stacksocial.com,* is a real-world example for the use of PAYW-MP in a high share of less fair-minded consumers and high disadvantageous inequity aversion case. This merchant, who most probably faces a high number of freeloaders (because of the anonymity of the internet) and only low to moderate levels of generosity, sells parts of its offer under PAYW-MP. Offering these goods under PAAP appears to be difficult as digital goods are often also easily available for free from dubious sources, which might also serve as a reference for a fair price.

When comparing PAYW to PAYW-SP, making allowance for consumers who ignore the price suggestion causes the nested structure of PAYW (cf. A.3, A.15) within PAYW-SP. PAYW-SP is identical to PAYW for $z = 1$ and dominates PAYW for $z < 1$ (cf. Figure 1, 5); differences in profits between PAYW-SP and PAYW decrease with increasing generosity λ (cf. Figure 6).

Consequently, we will exclude PAYW in favor of PAYW-MP and PAYW-SP from further analysis of the optimal pricing mechanism.

6.2.2 Result 2 (focus on cost c)

$(1 - c)^2$ is a multiplicative element of all profit functions and thus important for the absolute magnitude of profits rather than for discriminating between different pricing policies; the structure of the different mappings of Figure 7 is similar. However, because of freeloading behavior, PAYW-SP suffers from high unit cost c and is not recommended in such cases. For higher costs, the seller must safeguard against freeloading by choosing PAYW-MP or PAAP. This is in line with empirical findings, as Kim et al. (2014) also report that higher costs typically rule out PAYW and PAYW-SP.

$\pi^*(\beta,\lambda,\omega,\bar{\gamma}_{[0,1]})$			$\lambda=0$	$\lambda=\frac{1}{2+\beta}$	$\lambda=\frac{2}{3}$	$\lambda=1$	Row
PAAP		$\beta^{min}=0$	←	$\frac{(1-c)^2}{4}$		→	1
		$\beta^{max}\to\infty$	0	0	$\frac{(1-c)^2}{6}$	$\frac{(1-c)^2}{4}$	2
PAYW-MP	(1)	$\beta^{min}=0$	0	$\frac{(1-c)^2}{4}$			3
		$\beta^{max}\to\infty$	0	0			4
	(2)	$\beta^{min}=0$		$\frac{(1-c)^2}{4}$	$\frac{(1-c)^2(1+2\omega)}{3(1+3\omega)}$	$\frac{(1-c)^2}{2(1+\omega)}$	5
		$\beta^{max}\to\infty$		0	$\frac{(1-c)^2}{3(1+\omega)}$	$\frac{(1-c)^2}{2(1+\omega)}$	6
PAYW-SP	(1)[a, b]	$\bar{\gamma}_{[0,1]}^{min}=\frac{(1-c)(1-\omega)}{2c\omega}$	←	$\frac{(1-\omega)(1-c)(3-c)}{4}-c\omega$		→	7
		$\bar{\gamma}_{[0,1]}^{max}=1$	←	$\frac{(1-\omega)(1-c)^2}{4}$		→	8
	(2)[a, c]	$\bar{\gamma}_{[0,1]}^{min}=0$	←		$\frac{(1-\omega)(1-c)^2}{3}-c\omega$		9
		$\bar{\gamma}_{[0,1]}^{max}=\frac{(1-c)(1-\omega)}{2c\omega}$	←	$\frac{(1-\omega)(1-c)(3-c)}{4}-c\omega$		⇢ (dashed)	10
	(3)[c]	$\bar{\gamma}_{[0,1]}^{min}=0$			$\frac{(1-\omega)(1-c)^2}{3}-c\omega$	$\frac{(1-\omega)(1-c)^2}{2}-c\omega$	11
		$\bar{\gamma}_{[0,1]}^{max}=\frac{(1-c)(1-\omega)}{2c\omega}$			$\frac{(1-\omega)(1-c)}{2}-c\omega$	$\frac{(1-\omega)(1-c)(3-c)}{4}-c\omega$	12

Note: $z=0$

[a] independent of λ

[b] the restriction $m>\frac{1}{2}$ makes sure that the lower bound for $\pi_{PWYW-SP}$ does not exceed the upper bound; $\bar{\gamma}_{[0,1]}^{min}$ follows from solving $m>\frac{1}{2}$ for $\bar{\gamma}_{[0,1]}$

[c] the restriction $m\leq\frac{1}{2}$ makes sure that the lower bound for $\pi_{PWYW-SP}$ does not exceed the upper bound; $\bar{\gamma}_{[0,1]}^{max}$ follows from solving $m\leq\frac{1}{2}$ for $\bar{\gamma}_{[0,1]}$

Solid arrows indicate that the value holds for the respective range, dashed arrows indicate that the value depends on $m(c,\omega)$

Table 1: Optimal profits for selected levels of generosity (cf. sub-section 6.2.3)

6.2.3 Result 3 (focus on fairness λ, disadvantageous inequity aversion β, share of less fair-minded consumers ω)

Equations (11) and (14) reinforce that three different levels of generosity have to be distinguished. Accordingly, Table 1 evaluates the profit functions for the three different pricing policies for the corresponding generosity boundary values.[11] Because of continuity and monotonicity (in λ, β, ω), these evaluations provide estimates for the profit functions' domains.

6.2.3.1 Result 3a (Generosity λ is small: $\lambda \leq 1/(2+\beta)$)

Table 1 provides evidence that PAAP dominates PAYW-MP (rows 1-4; i.e., $\pi_{PAAP}(\lambda = 0) > \pi_{PAYW-MP}(\lambda = 0)$). The domain of π_{PAAP} and the domain of $\pi_{PAYW-SP}$ might overlap (in particular for small ω, intermediate λ) which requires a more detailed investigation provided in Appendix E.[12,13]

For low levels of disadvantageous inequity aversion, β, PAYW-SP is more profitable than PAAP if generosity, λ, exceeds a critical threshold $\lambda > \lambda^+$. When β is low, the slope of $\lambda^+(\omega)$, is positive ($\left.\frac{\partial \lambda^+(\omega)}{\partial \omega}\right|_{\lambda^+} > 0$) (e.g., mapping in column 1, row 5 of Figure 7). In this case, the positive effect of generosity on the preference for PAYW-SP relative to PAAP is higher than the negative effect of less fair-minded consumers. On the contrary, PAAP is the optimal pricing system when the

[11] To facilitate argumentation, we set $z = 0$ for PAYW-SP; this implies that all consumers consider the price suggestion and, therefore, favors this pricing policy. This is a tentative not a structural advantage only because $z > 0$ adds a segment of customers opting for PAYW. Formal derivations provided in Appendix E consider $z > 0$.

[12] In essence, Appendix E analyzes analytically under which conditions the difference between two profit functions $\pi_1 - \pi_2$ is positive / negative which implies that policy 1 is preferred over policy 2 or vice versa. Profit functions depend on a set of parameters which makes formal derivations tedious. Because of the prominent role of generosity λ, we determine the critical threshold λ^+ for which $\pi_1(\lambda^+) = \pi_2(\lambda^+)$; this implies that for $\lambda < \lambda^+$ policy 1 dominates policy 2 and vice versa for $\lambda > \lambda^+$. Since the comparison between profit functions apply in each case to a certain domain of λ (small, intermediate, large), the respective domain is also effective for λ^+. However, only for small λ, $\lambda^+ \notin [0; 1/(2+\beta)]$ might occur.

[13] It is also only for small λ that the decision whether policy 1 dominates policy 2 requires futher investigation. In this case $\lambda^+_{\leq}(\omega)$ is considered a function depending on ω (and $\beta, c, \bar{\gamma}_{[0,1]}, z$) rather than a calculated threshold value $\lambda^+_{\leq}$ (for a specific ω). In particular, the algebraic sign of $\left.\frac{\partial \lambda^+(\omega)}{\partial \omega}\right|_{\lambda^+_{\leq}}$ helps to determine the dominant policy. From an interpretative point of view, the threshold $\lambda^+_{\leq}$ refers to the amount of consumers' generosity, $\left.\frac{\partial \lambda^+(\omega)}{\partial \omega}\right|_{\lambda^+_{\leq}}$ to its (local) dependence on consumers' fair mindedness.

share of less fair-minded consumers (i.e., the share of potential freeloaders), ω, is high. *For higher levels of disadvantageous inequity aversion β*, consumers who dislike being duped by the seller drop out of the market and limit the seller's prices which in turn hurts profitability in PAAP (the green area in Figure 7 diminishes for larger β). The profitability under PAYW-SP, however, is not affected by a change in disadvantageous inequity aversion β. Therefore, PAYW-SP becomes optimal at the expense of PAAP for large β.

An example from practice for the use of PAYW-SP in such a low generosity, low share of less fair-minded consumers' set-up might be a self-cutting flower field where we often observe consumer's underpayment (Schlüter and Vollan 2015). PAYW sellers typically set a suggested price in these cases. Furthermore, disadvantageous inequity aversion is probably very high as consumers who cut the flowers by themselves might not accept 'unfair' (and given) prices making PAAP less profitable than PAYW-SP.

6.2.3.2 Result 3b (Intermediate generosity: $\frac{1}{2+\beta} < \lambda \leq 2(1+m)/3$)

The domain of $\pi_{PAYW-SP}$ does not depend on λ here. Therefore, as outlined by Figure 6 and proven in Appendix E the dominance of PAYW-SP over PAAP continues (for intermediate λ, small ω, small β). Contrariwise, rows 1and 2 and 5 and 6 of Table 1 demonstrate the dominance of PAYW-MP over PAAP. At the same time, the domains of $\pi_{PAYW-MP}$ and of $\pi_{PAYW-SP}$ might overlap which requires a more detailed investigation provided in Appendix E.

When there are few less fair-minded consumers, ω, the firm can obtain higher prices from more fair-minded consumers while also deterring some potential freeloaders by the suggested price. As soon as the share of less fair-minded consumers, ω, increases, the firm should decide for excluding all freeloaders and extracting higher payments from more fair-minded consumers by implementing PAYW-MP.

Additionally, increasing generosity levels make the price suggestion obsolete as the perceived fair price is high already and allows the firm to charge a profitable minimum price without triggering disadvantageous inequity aversion at the consumer side.

In the field, this situation corresponds to restaurants and bars that let the consumers decide the prices for their meals and drinks. In generous markets where the share of potential freeloaders is low, we typically observe PAYW-SP (e.g., Die Weinerei, a restaurant which is located in a privileged area of Berlin), for markets with higher levels of potential freeloaders we observe PAYW-MP (e.g., Weine und Geflügel, a restaurant which is located in a less privileged neighborhood in Berlin).

6.2.3.3 Result 3c (Generosity λ is high: $\lambda > 2(1+m)/3$)

The dominance of PAYW-MP over PAAP continues and in addition, the upper bound for $\pi_{PAYW-SP}$ ($\frac{(1-\omega)(1-c)^2}{4}$) is smaller than the lower bound for $\pi_{PAYW-MP}$ ($\frac{(1-c)^2}{3(1+\omega)}$). Thus, PAYW-MP is the preferred choice in this case.

This is in line with several fundraising campaigns: donors are typically very generous; the seller sets a minimum price, but buyers often still largely overpay. For instance, some private schools operate on a solidarity pricing scheme, similar to PAYW. Generosity and the community spirit are high, and parents must pay a minimum fee but are asked to pay more if they can afford it.

6.2.4 Result 4 (focus on $\bar{\gamma}_{[0,1]}$ and z)

The mean of advantageous inequity aversion of less fair-minded consumers, $\bar{\gamma}_{[0,1]}$, only effects PAYW-SP; i.e., an increasing $\bar{\gamma}_{[0,1]}$ increases profits (cf. A.15). Therefore, PAYW-SP might dominate at the expense of PAAP and PAYW-MP for large $\bar{\gamma}_{[0,1]}$. Obviously, there is also some interdependency between ω and $\bar{\gamma}_{[0,1]}$: a large $\bar{\gamma}_{[0,1]}$ diminishes losses due to large values of ω (A.15).[14] Jung et al. (2017) report an example of such behavior. In a field experiment, the authors sold reusable grocery bags and doughnuts under PAYW with or without a charitable component added to the PAYW pricing system. As a consequence, freeloading became more despicable when parts of the revenues were donated to charity. Therefore, even less fair-minded consumers should experience some advantageous inequity aversion. In fact, the purchase rate decreased in the presence (vs. absence) of a charitable component because some consumers abstained from freeloading.

[14] A more detailed analysis for different levels of $\bar{\gamma}_{[0,1]}$ is available upon request from the authors.

With increasing z; i.e., when more consumers ignore the suggested price and, thus, the segment of PAYW consumers increases, PAAP and PAYW-MP dominate at the expense of PAYW-SP (cf. Figure 6). This result is intuitively appealing because only PAYW-SP is affected by this probability z.[15]. As an example from practice, we refer to the Metropolitan Museum of Art in New York. This institution switched from PAYW-SP to PAAP because the number of visitors who paid the suggested price declined by 73 percent in a 13-year span (Weiss 2018).

Taken together, this allows us to state the following proposition:

Updated Proposition 4. PAYW is never the preferred pricing policy.

(i) Low levels of generosity $\left(0 \leq \lambda \leq \frac{1}{2+\beta}\right)$:[16]

if $\lambda_{\lessgtr}^{\pm} < 0$ then PAAP is optimal;

if $\lambda_{\lessgtr}^{\pm} \in \left[0; \frac{1}{2+\beta}\right]$ and $\left.\frac{\partial \lambda_{\lessgtr}^{\pm}(\omega)}{\partial \omega}\right|_{\lambda_{\lessgtr}^{\pm}} > 0$ PAAP is optimal for $\lambda < \lambda_{\lessgtr}^{\pm}$ but PAYW-SP for $\lambda > \lambda_{\lessgtr}^{\pm}$;

if $\lambda_{\lessgtr}^{\pm} \in \left[0; \frac{1}{2+\beta}\right]$ and $\left.\frac{\partial \lambda_{\lessgtr}^{\pm}(\omega)}{\partial \omega}\right|_{\lambda_{\lessgtr}^{\pm}} < 0$ PAYW-SP is optimal;

if $\lambda_{\lessgtr}^{\pm} > \frac{1}{2+\beta}$ PAAP is optimal for $\left.\frac{\partial \lambda_{\lessgtr}^{\pm}(\omega)}{\partial \omega}\right|_{\lambda_{\lessgtr}^{\pm}} > 0$ but PAYW-SP for $\left.\frac{\partial \lambda_{\lessgtr}^{\pm}(\omega)}{\partial \omega}\right|_{\lambda_{\lessgtr}^{\pm}} < 0$.

(ii) Intermediate levels of generosity $\left(\frac{1}{2+\beta} < \lambda \leq \frac{2(1+m)}{3}\right)$:[17]

if $\lambda \in \left[\frac{1}{2+\beta}; \lambda_{>}^{\pm}\right]$ then PAYW-SP is optimal;

if $\lambda \notin \left[\frac{1}{2+\beta}; \lambda_{>}^{\pm}\right]$ then PAYW-MP is optimal.

(iii) High levels of generosity $\left(\frac{2(1+m)}{3} < \lambda \leq 1\right)$: PAYW-MP is optimal.

6.3 Comparison between results of CKZ and revised and extended version of the model

Our results differ from CKZ in a number of important ways. However, the algebraic formulations tend to mask the consequences of the misspecifications by CKZ. To demonstrate the effects of CKZ's misspecifications, we calculate the extent to which our results numerically differ from the results of CKZ for PAYW, PAYW-MP, and PAYW-SP.

15 A more detailed analysis for different levels of z is available upon request from the authors.

16 (A.17) and (A.18) specify $\lambda_{\lessgtr}^{\pm}$ and $\left.\frac{\partial \lambda_{\lessgtr}^{\pm}(\omega)}{\partial \omega}\right|_{\lambda_{\lessgtr}^{\pm}}$

17 (A.19) specifies $\lambda_{>}^{\pm}$.

As CKZ's profit formulations are inconsistent, we consider CKZ's optimal prices for PAYW-MP and PAYW-SP (of their Propositions 2 and 3), calculate the seller's profits using (10), (13a), and (13b) and compare them to the optimal profits, (A.8) and (A.15). For PAYW, the seller does not choose a price. However, in order to compare our results, we use CKZ's profit function (CKZ.4).

To capture the whole parameter range, we analyze 6400 different scenarios. As before, we study $c \in \{0; .25; .5; .75\}$ and $\beta \in \{0; 1.5; 4; 6.5; 9\}$. Furthermore, we need to discretize $\bar{\gamma}_{[0,1]}, \lambda, \omega$, and z. With the aim of being comprehensive, we proceed in increments of 0.25. Thus, we take $\bar{\gamma}_{[0,1]} \in \{0; .25; .5; .75\}$, $\lambda \in \{0; .25; .5; .75; 1\}$, $\omega \in \{0; .25; .5; .75\}$, and $z \in \{0; .25; .5; .75\}$.To quantify the difference between CKZ's and our results, we treat CKZ's results as a forecast for the actual results obtained by the correct formulation. This allows us to use measures of forecast accuracy to determine the mean percentage error of CKZ's results. The canonical choice would be the mean absolute percentage error (MAPE). However, the MAPE is not feasible in our case as it provides very high or undefined values when the seller's profits are very low or zero. This occurs in some scenarios.
As an alternative, we calculate the iMAPE, the MAPE for all cases in which the actual profit is not equal to zero. Another alternative includes even these very small values but after an appropriate transformation. Kim and Kim (2016) propose using the arctangent function on the relative differences. This gives us the mean arctangent absolute percentage error (MAAPE),

$$MAAPE = \frac{1}{N}\sum_{i=1}^{N} \arctan\left(\left|\frac{A_i - F_i}{A_i}\right|\right),$$

where N is the number of data points, A_i are the actual observations and F_i are the forecast values.

	PAAP	PAYW	PAYW-MP	PAYW-SP	Overall
iMAPE	-	0.29	0.24	0.10	
MAAPE	-	0.19	0.21	0.07	
Erroneous decision	0	-	0.29	0.07	0.19

Table 2: Percentage error between results

Table 2 indicates that the discrepancy in terms of percentage errors of CKZ and the revised model is about 20 percent for PAYW, and PAYW-MP, and about 10 percent for PAYW-SP. Finally, we determine the number of cases in which the seller would choose the wrong pricing scheme when applying the rules of CKZ. We find that in 19 percent of the 6400 scenarios analyzed, the seller would make a different choice under our specification as compared to CKZ's specification. A more detailed analysis finds that if PAYW-MP is the correct choice, CKZ disagree in 29 percent of all cases and that if PAYW-SP is the correct choice, CKZ disagree in 7 percent of all cases. However, CKZ always choose PAAP when it is optimal according to our model. This implies that CKZ underestimate the profitability of PAYW-MP and PAYW-SP.

7 Discussion and conclusion

The aim of the present research was to examine the effect of some inconsistencies in the article of CKZ and their propositions and conclusions regarding the applicability of PAYW as a novel and alternative pricing policy. By doing so, this paper contributes to the literature on four aspects.

From a conceptual point of view, we identified two types of consumer segments that have been neglected so far. First, there are customers who are not very advantageous inequity averse but still do not freeload when a seller offers participatory pricing because they perceive the corresponding consumption utility as insufficient. For the PAYW seller, the existence of such a segment will decrease her costs. In addition, this behavior is also relevant under PAYW-SP because a higher suggested price (partially) discourages consumers from freeloading. Therefore, PAYW and PAYW-SP are more profitable than previously assumed. An empirical study indicated that this segment exists and accounts about 30 percent of the promotional consumers observed.

Second, there are buyers who are characterized by substantial disadvantageous inequity aversion. Neglecting them would result in an unwarranted overestimation of PAYW-MP as the minimum price might screen out otherwise paying consumers. These findings add behavioral realism to the PAYW model. Such persons are also present in the field. Neglecting their existence does not only oversimplify consumer behavior in PAYW but also leads to an erroneous assessment of the profitability of (variants of) PAYW.

Third, from a technical point of view, this research eliminated multiple incorrect results and tried to provide analytical representations that are more easily comprehensible. Modelling PAYW pricing with Fehr-Schmidt-preferences results in a more sophisticated picture of consumption behavior than previously thought. In our effort to be more concise and extensive, we uncover different consumer segments and sub-cases that have so far been masked. The effects of this adjustment are substantial. A numerical investigation of the discrepancies between CKZ's and the revised model identified percentage errors of about 20 percent for PAYW and PAYW-MP and of about 10 percent for PAYW-SP. Furthermore, CKZ's deviations would lead to an erroneous choice of the pricing policy in about 20 percent of the analyzed cases.

Fourth, from a managerial perspective, guidelines for optimal minimum and suggested prices are updated and conditions for the optimal participating pricing policy have been sharpened. With respect to the minimum price, the consideration of disadvantageous inequity aversion suggests that sellers need to charge the highest fair price when generosity is low. However, when generosity is high, sellers need to charge a lower minimum price, allowing more fair-minded consumers to enter the market. Furthermore, in contrast to CKZ, we determine a lower minimum price to account for disadvantageous inequity aversion.

With respect to the suggested price, we find that the seller can deter less fair-minded consumers from entering the market by setting a high suggested price. Thus, when less fair-minded consumers are numerous, care for advantageous inequity aversion, and costs are high, and the seller should set the maximum feasible price as the price suggestion. Moreover, also in other cases, setting a higher suggested price deters less fair-minded consumers from entering and is beneficial for the seller. Here we contradict CKZ who advise the seller to only set a meaningful suggested price in case of low generosity.

When choosing the optimal pricing scheme, in disagreement with CKZ, PAYW-MP and PAYW-SP are more effective than previously determined. In contrast to CKZ, we observe that increasing disadvantageous inequity aversion leads on the one hand, to the choice of PAYW-MP over PAAP and, on the other hand, to PAYW-SP over PAYW-MP and PAAP.

Taken together, these results suggest that modeling PAYW is more complex than previously assumed. However, the findings also become more realistic and explain some of the contradictory empirical observations of the profitability of PAYW.

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Appendix A: Empirical study

Purpose

This section provides empirical evidence that Segment II_{PAYW} (Figure 1) exist (i.e., $\delta > 0$) and that $\Delta\pi$ might indeed be greater than zero (i.e., profits considering the proposed refinements are greater than profits under the CKZ scenario). The following study discriminates empirically between the segments described in Figure 1. From a managerial standpoint, customers in the Segment I_{PAYW} segment freeload in a PAYW setting; i.e., they take everything that is offered and pay the minimum price (i.e., nothing). Conversely, customers in Segment II_{PAYW} only freeload if their utility from taking the good and paying nothing is higher than their disutility from advantageous inequity. However, they never pay to obtain a product in PAYW. On the contrary, fair customers (Segment III_{PAYW}) are customers who pay their fair price (i.e., more than the absolute minimum) to get the product offered under PAYW.

Setup

The management of a European grocery chain supported a collaboration to conduct this field experiment. During the experimental condition (EG), the store manager permitted our team to run a PAYW campaign in his store. The store was located in a socially disadvantaged suburb of a European capital. Over a three-day period (Monday to Wednesday), customers could choose their own prices for nondurables whose date of expiry was approaching. The offering consisted of a variety of perishable groceries (e.g., refrigerated products, frozen products, and pasta). The manager excluded alcohol, meat, fruits, and fresh vegetables from this promotion. These PAYW products were offered on separated shelf spaces; e.g., refrigerated and frozen products close to the regular cooler, other products near the check-out counters. Such shelf spacing corresponded to the retailer's regular strategy for promoting groceries that neared their expiration dates and ran under the well-established slogan: "Food is precious." We observed 5278 shopping carts during the experimental condition.

Conducting the PAYW condition required several implementation measures because this was not an established promotional campaign within this store. Plain price tags were provided near all PAYW products. However, no regular, unreduced prices were displayed. Shoppers put the product of their choice into their shopping baskets and wrote the price they were willing to pay on these plain price tags. The grocery store's accounting department required two minor modifications to the PAYW pricing scheme. (i) A minimum price threshold of 0.10€ was determined. This was necessary to warrant scanning UPCs at the checkout, an action that also provided data for inventory management and automatic reordering. Additionally, 0.10€ was the minimum amount the computerized system could register. (ii) The maximum number of products that could be purchased under PAYW was set to ten. Accountants strived to avoid excessive freeloading behavior. For pragmatic reasons (e.g., long lines at checkout) this modification was not enforced consistently by the store management.

The control condition (CG) also spanned three days (Monday to Wednesday). An in-store promotion offered products with an approaching expiration date at half-off. Customers were very familiar with this type of campaign. We observed 5193 shopping carts during the control condition. The range of products on offer for a 50 percent discount was very similar to the range of products under PAYW during the experimental condition: mean regular retail prices were 2.90€ (CG) vs. 3.14€ (EG).

Data were collected by means of observation: store management provided shopping basket data as collected at the checkout counter. When interpreting the results, we have to keep in mind that the retail store offered about 20,000 store keeping units, but only purchases of products approaching the date of expiry were under promotion (CG: 50 percent discount; EG: PAYW) and thus relevant for this study. Thus, no purchase might either be the result of a lack of demand for the product categories considered or of disapproval of the prices or the promotional campaign. This prevents us from estimating the size of Segment IV_{PAYW} (Figure 1).

Results

The results of the experiment are presented in Table A.1. First, Table A.1 distinguishes between the two experimental conditions (CG, EG). Promotional customers are defined as those who purchased products approaching the date of expiry offered at reduced prices; i.e., 6% during CG and 5% during EG (cf. grey shaded rows of Table A.1). Shop assistants reported that under PAYW some shoppers felt alienated by this new type of promotion because they did not know how much they should pay. As a consequence, they might have abstained from buying these products. For CG, customers bought on average 2.6 discounted products and paid, on average, 1.82€ for them, which approximately matches the 50 percent discount (not all products were offered at half price). PAYW customers utilized their power to set prices more extensively; on average they bought 4.1 products on sale and only paid an average of .89€ (which corresponds to a discount of 77%). Cross-category sales seemed to be more pronounced under CG than under EG because of larger basket sizes (14.9 vs. 11.6) and basket values (20.03€ vs. 12.59€). We note that these differences are much smaller for non-promotional customers (9.9 vs. 10.2 and 15.42€ vs. 17.17€).

Second, Table A.1 distinguishes between different types of customers for EG. We admit that this segmentation is based on conceptual reasoning rather than on statistical analysis because only observational data were available.

1. "Freeloaders" (Segment I^{a}_{PAYW} customers) were defined as shoppers who fully capitalized on PAYW: they put (even more than) the maximum of ten products (i.e., 12.1) into their shopping basket and paid 0.10 € or slightly more (i.e., prices up to 0.20 € in order to account for mock prices such as 0.13 €) for each product. On average they requested a discount of 94% by paying 0.12€ per product. They are the smallest group; the size of this segment is 12% (of all promotional customers).
2. "Partial freeloaders" (Segment $I^{b}_{I_{PAYW}}$ or Segment II_{PAYW} customers) also paid close to nothing but did not take full advantage of the offer by buying less than ten products (i.e., 3.8). They freeloaded for some products but abstained from doing so for other products. This behavior resulted in an average discount of 92%; again, they only paid 0.12€ for each individual product. The size of this segment is 30% (of all promotional customers).
3. "Fair customers" (Segment III_{PAYW} customers) paid higher prices than 0.20€ for at least one product (i.e., 2.5 on average) but still utilized their price power substantially (i.e., by claiming an average discount of 66% by paying 1.45€ per product). The size of this segment is 58% (of all promotional customers).

Table A.1 further highlights that these three segments also differ with respect to their purchasing behavior of non-promotional goods. Basket sizes and basket values (columns 5 and 6 of Table A.1) vary considerably. In EG, fair customers bought the highest number of regularly priced products, which resulted in the highest basket value.

Discussion

Evaluation of the PAYW campaign. From a purely monetary perspective,[18] PAYW was not successful for the seller. In actuality, store management decided to terminate the promotion earlier than planned for three reasons. First, demand exceeded expectations and resulted in an out-of-stock situation. Second, some customers took heavy advantage of the PAYW offer, coming in groups of three or four to circumvent the ten-item limit while paying (almost) nothing, making it very unprofitable for the seller. Third, the seller observed the unethical behavior by some of these PAYW customers who tried to determine their own prices for regular products as well as for PAYW products. On the positive side, management's experiences with PAYW reinforced their belief that the established 50 percent discount campaign on expiring products was a superior approach.

Size of Segment II_{PAYW} (i.e., an empirical estimate of δ). Segment II_{PAYW} consumers are nested within the "partial freeloader" segment. These consumers were not willing to pay more than the absolute minimum, thus $p_i = 0$. However, they only freeloaded on some goods and decided not to purchase the rest. Tentatively, we argue that for every ten possible items, these customers chose to freeload in 3.8 cases (i.e., they bought on average 3.8 discounted products) but abstained from doing so in 6.2 cases. As a rough estimate, we proportionally split this segment into Segment I_{PAYW} (12%) and Segment II_{PAYW} (18%) customers (for a certain product category).

Taken together, for the empirical data analyzed, we distinguish between different segments of promotional buyers (cf. Figure 1) and calibrated the sizes of these segments as 24% for Segment I^a_{PAYW} and I^b_{PAYW} (i.e., θ), 58% for Segment III_{PAYW}, and 18% for Segment II_{PAYW} (i.e., δ). We want to emphasize that it is not the exact size of Segment II_{PAYW} customers what matters in this context, but merely the existence of this category of customers. Of course, we acknowledge the exploratory nature of this study. Nevertheless, it demonstrates that our proposed refinement is also relevant from an empirical perspective.

[18] Other aspects of this campaign could be considered as well: e.g. salvage costs of the products if not sold before expiration date, evaluation of a new marketing tool, generation of positive image because of an attempt to reduce spoiling food. However, this is beyond the scope of this paper.

Experimental condition	Tentative segment label	Average price paid (€)	Average discount received (%)	Basket size (number of products)	Basket value (€)	Average number of discounted products	Segment size (frequency)	Relative segment size PAYW (%)	Relative segment size all customers (%)
EG	Freeloader (Segment I_{PAYW}^{a})	0.12	94	18.0	4.44	12.1	35	12	
	Partial freeloader (Segment I_{PAYW}^{b} or II_{PAYW})	0.12	92	8.6	7.05	3.8	84	30	
	Fair customers (Segment III_{PAYW})	1.45	66	11.5	17.20	2.5	163	58	
	(Sum of) Promotional Customers	0.89	77	11.6	12.59	4.1	282	100	5
	Non-promotional Customers (Segment IV_{PAYW})			10.2	17.17	0	4996		95
CG	Promotional Customers	1.82	47	14.9	20.03	2.6	318		6
	Non-promotional Customers			9.9	15.42	0	4875		94

Table A.1: Customer segments and descriptive measures

Appendix B: Proof of Updated Proposition 1

This appendix deals with the effects of the proposed refinement on Proposition 1 of CKZ; i.e., the conditions under which PAYW is more profitable than PAAP. We will apply the above specification of the firm's profits in PAYW, π_{PAYW} (7) and derive new conditions for which they outperform the firm's profits under PAAP, π_{PAAP}.

The optimal profits under PAAP are given by CKZ: Eq. (3):

$$\pi^*_{PAAP} = \frac{(1-c)^2(1+\lambda\beta)}{4(1+\beta)}. \tag{A.1}$$

Using the uniform distribution $\phi(r) = 1$ in (7a), CKZ derive their Eq. (5)

$$\pi^{CKZ}_{PAYW} = \frac{\lambda(1-\theta^{CKZ})(1-c)^2}{2} - c\theta^{CKZ}. \tag{A.2}$$

Using (6a) and the uniform distribution $\phi(r) = 1$ in (7), we arrive at

$$\pi_{PAYW} = \frac{\lambda(1-\omega)(1-c)^2}{2} - c\omega\big(1 - c\bar{\gamma}_{[0,1]}\big). \tag{A.3}$$

CKZ determine the critical cost level c^{+CKZ} such that $\pi^{CKZ}_{PAYW} \geq \pi^*_{PAAP}$; i.e., PAYW is more profitable than PAAP. Since $\frac{\partial \pi^{CKZ}_{PAYW}}{\partial c} = (c-1)\lambda(1-\theta^{CKZ}) - \theta^{CKZ} < 0$ for $c < 1$, π^{CKZ}_{PAYW} monotonically decreases in the domain of interest. Furthermore, $\Delta\pi = \pi_{PAYW} - \pi^{CKZ}_{PAYW} \geq 0$. Therefore, the critical cost level c^+ (taking the refinement into account) must be greater than c^{+CKZ}. In particular, after some algebraic manipulation, we find that

$$c^+ < 1 - \frac{2B\big(1-2\bar{\gamma}_{[0,1]}\big) - 2\sqrt{B\left(B - A\big(1-\bar{\gamma}_{[0,1]}\big)\right)}}{A - 4B\bar{\gamma}_{[0,1]}} \tag{A.4}$$

=

$$A = 1 - \lambda\big(2 + \beta - 2\omega(1+\beta)\big)$$

$$B = \omega(1+\beta). \tag{8}$$

Equation (A.4) is a relevant threshold for c^+only if $c^+ \geq 0$. Therefore, we require $c^+ \geq 0$ and solve (A.4) for ω. This results in

$$\omega \leq \frac{1}{2} + \frac{\lambda - 1}{2\lambda(1+\beta)}.$$

As $\lambda - 1 \leq 0$, we see that $\omega \leq \frac{1}{2}$ to ensure $c^{+} \geq 0$.

Again, this threshold is only relevant if $\omega \geq 0$ which requires $\lambda \geq \frac{1}{2+\beta}$ to ensure $\omega \geq 0$.

Summing up, PAYW can be more profitable than PAAP only if $\omega \leq 1/2$ and $\lambda \geq 1/(2+\beta)$.

Finally, we note that π^{*}_{PAAP} increases monotonically in λ, decreases monotonically in β, and, therefore,

$$\frac{(1-c)^2\lambda}{4} \leq \pi^{*}_{PAAP} \leq \frac{(1-c)^2}{4}. \tag{A.5}$$

Appendix C: Proof of Updated Proposition 2

For the uniform distribution in (10), we get

$$\begin{aligned}\pi_{PAYW-MP} = (1-\omega)\left(\left(\underline{p}-c\right)^2 \frac{1-\lambda}{\lambda(1+\beta\lambda)} - \frac{\left(\underline{p}-c\right)^2}{2\lambda} + \frac{\lambda(1-c)^2}{2}\right) \\ + \omega\left(\left(\underline{p}-c\right)\left(1-\underline{p}-\left(\underline{p}-c\right)\frac{\beta(1-\lambda)}{1+\beta\lambda}\right)\right)\end{aligned} \tag{A.6}$$

When solving the first-order condition of $\pi_{PAYW-MP}$ with respect to $\underline{p}$ ($\delta\pi_{PAYW-MP}/\delta\underline{p}$), we need to distinguish several cases, depending on λ, second order conditions, and feasible domain of $\underline{p}$.

$$\underline{p}^{*} = c + (1-c)k\lambda \tag{A.7}$$

$$\text{with } k = \begin{cases} 1 & \lambda \leq \frac{1}{2+\beta} \\ \frac{\omega(1+\beta\lambda)}{(2+\beta(1+\omega))\lambda - 1 + \omega} & \lambda > \frac{1}{2+\beta} \end{cases} \tag{11}$$

=

Similarly, CKZ require $\lambda \geq 1/2$ for $\underline{p}^{*}$ to be lower than the highest possible fair price:

$$\underline{p}^{*CKZ} = c + \frac{\omega\lambda(1-c)}{2\lambda - 1 + \omega} \Rightarrow \frac{\omega}{2\lambda - 1 + \omega} \leq 1 \Rightarrow \lambda \geq \frac{1}{2}$$

Figure C.1 offers an exemplary overview of the optimal minimum price for different λ.

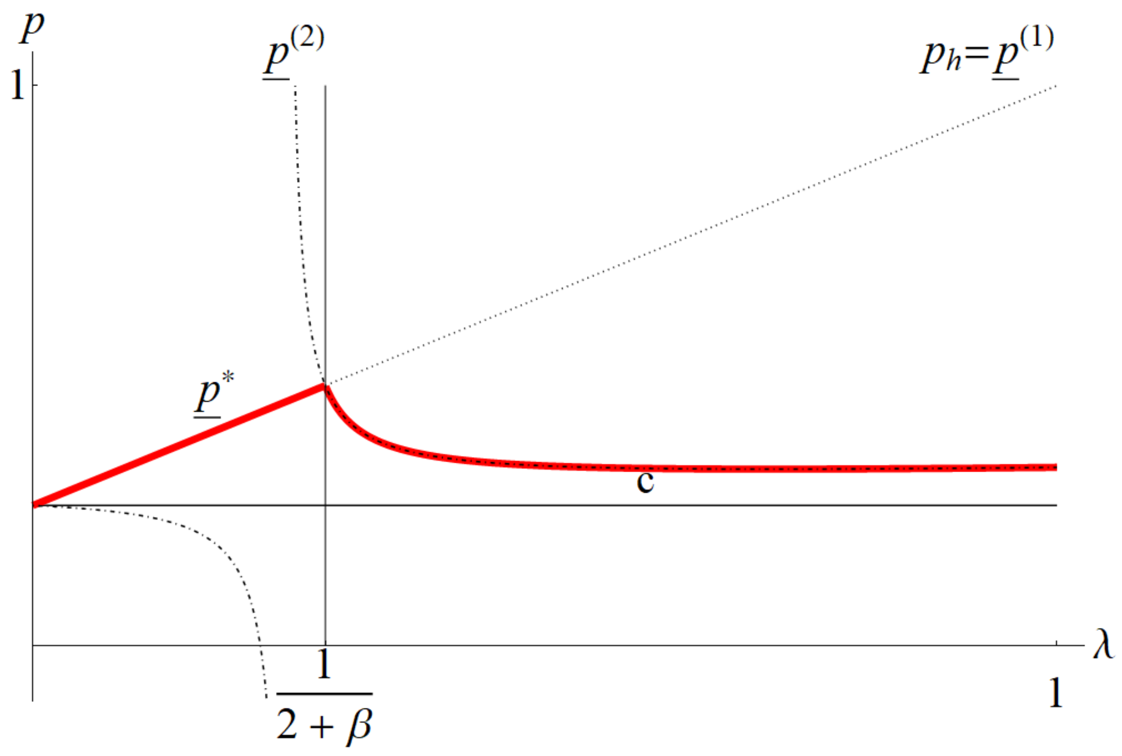

Figure C.1: Composition of $\underline{p}^*$ for different λ ($\beta = 1.5, \omega = .1, c = .25$)

Therefore, our model extends CKZ for including β. Next, we investigate the influence of c, ω, λ, and β on $\underline{p}^*$.

	$\lambda \leq 1/(2+\beta)$	$\lambda > 1/(2+\beta)$
$\frac{\partial \underline{p}^*}{\partial c}$	$1-\lambda \geq 0$	$\frac{a+b(1-\lambda)}{a+b} > 0$
$\frac{\partial \underline{p}^*}{\partial \omega}$	0	$\frac{(1+\beta\lambda)}{a+b} > 0$
$\frac{\partial \underline{p}^*}{\partial \lambda}$	$1-c \geq 0$	$\frac{(1-c)(-\omega+\beta\lambda\omega(a-1)+b^2)}{(a+b)^2}$
$\frac{\partial \underline{p}^*}{\partial \beta}$	0	$\frac{-\lambda^2\omega(1-c)(3+\beta\lambda-2\lambda)}{(a+b)^2} < 0$

with: $a = (2+\beta)\lambda - 1$, $b = \omega(1+\beta\lambda)$

Table C.1: The influence of c, ω, λ, and β on $\underline{p}^*$

Therefore, the optimal minimum price increases with the unit cost and share of less fair-minded customers but decreases with the degree of disadvantageous inequity aversion. The situation with respect to generosity is more complex but Table C.2 provides tentative results. The optimal minimum price increases with increasing generosity but only if ω, β meet certain conditions (right columns of Table C.2); these conditions become less restrictive with increasing generosity. These results are face valid.

$\frac{\partial \underline{p}^*}{\partial \lambda}$ will be positive, if		Restriction on ω	Restriction on β
$\lambda = 1/(2+\beta)$		$\omega > (2+\beta)/2(1+\beta)$	$\beta > 2(1-\omega)/(2\omega-1) \ \wedge \ \omega > 1/2$
↓ λ increases	conditions on ω, β become less restrictive	$\omega > \frac{1+\beta\lambda(2-\lambda(2+\beta))}{(1+\beta\lambda)^2}$	$\beta > \frac{1-\lambda-\omega+\sqrt{2(1-\omega)+\lambda(\lambda+2\omega-2)}}{\lambda(1+\omega)}$
$\lambda = 1$		$\omega \geq \max\{0, (1-\beta)/(1+\beta)\}$	$\beta \geq (1-\omega)/(1+\omega)$

Table C.2: Requirements for $\frac{\partial \underline{p}^*}{\partial \lambda}$ *to be positive.*

The optimal profit amounts to

$$\pi^*_{PAYW-MP} = \begin{cases} \dfrac{(1-c)^2\lambda(1-\lambda)}{1+\beta\lambda} & \lambda \leq 1/(2+\beta) \\ \dfrac{(1-c)^2\lambda}{2}\left(\dfrac{\lambda(2+\beta)+2\omega(1-\lambda)-1}{\big(2+\beta(1+\omega)\big)\lambda-1+\omega}\right) & \lambda > 1/(2+\beta) \end{cases}. \qquad \text{(A.8)}$$

Finally, we note that $\pi^*_{PAYW-MP}$ increases monotonically in λ, decreases monotonically in β, and, therefore,

$$\begin{gathered} 0 \leq \pi^*_{PAYW-MP} \leq (1-c)^2\lambda(1-\lambda) \leq \frac{(1-c)^2}{4} \wedge \lambda \leq \frac{1}{2} \\ \frac{(1-c)^2\lambda}{2} \cdot \frac{1}{1+\omega} \leq \pi^*_{PAYW-MP} \leq \frac{(1-c)^2\lambda}{2} \cdot \frac{2\lambda+2\omega(1-\lambda)-1}{2\lambda-1+\omega} \wedge \lambda > \frac{1}{2}. \end{gathered} \qquad \text{(A.9)}$$

Appendix D: Proof of Updated Proposition 3

First, we find the optimal suggested prices for maximizing $\pi_{PAYW-SP}^{(1)}$ and $\pi_{PAYW-SP}^{(2)}$. Since the price suggestion is ignored by some consumers (with probability z), they are not considered here.

$$\pi_{PAYW-SP}^{(1)} = (1-z)\left((1-\omega)\left(\frac{1}{2}\int_{c}^{p_s}(r-c)\,\phi(r)dr + \int_{p_s}^{r_s}\left(p_s - c\right)\phi(r)dr \right.\right.$$
$$\left.\left. + \int_{r_s}^{1}\lambda(r-c)\,\phi(r)dr\right)\right) - c\theta_s^{1-z} \tag{A.10}$$

When solving the first-order condition of $\pi_{PAYW-SP}^{(1)}$ with respect to p_s ($\partial\pi_{PAYW-SP}^{(1)}/\partial p_s$), we need to distinguish several cases, depending on λ and second-order conditions:

$$p_s^{(1)} = \begin{cases} c & \text{if} \quad \lambda < 2/3 \\ (2+c)/3 & \text{if} \quad \lambda = 2/3 \\ \left(2+c+2m(1-c)\right)/3 & \text{if} \quad 2/3 \le \lambda \le 2(1+m)/3 \text{ and } m \le 1/2 \\ c + 2\lambda m(1-c)/(3\lambda-2) & \text{if} \quad 2(1+m)/3 \le \lambda \le 1 \text{ and } m \le 1/2 \end{cases} \tag{A.11}$$

with $m = c\omega\bar{\gamma}_{[0,1]}/\left((1-c)(1-\omega)\right)$.

Similarly, for $\pi_{PAYW-SP}^{(2)}$,

$$\pi_{PAYW-SP}^{(2)} = (1-z)\left((1-\omega)\left(\frac{1}{2}\int_{c}^{p_s}(r-c)\,\phi(r)dr + \int_{p_s}^{1}(p_s-c)\,\Phi(r)dr\right)\right) - c\theta_s^{1-z}$$
$$= (1-z)\left((1-\omega)\left(\frac{-3}{4}p_s^2 + p_s\left(1+\frac{c}{2}\right) + \frac{c^2}{4} - c\right) - c\omega\left(1 - p_s\bar{\gamma}_{[0,1]}\right)\right) \tag{A.12}$$

and

$$p_s^{(2)} = \begin{cases} 1 & \text{if} \quad m > 1/2 \\ \left(2+c+2m(1-c)\right)/3 & \text{if} \quad \lambda \le 2(1+m)/3 \text{ and } m \le 1/2 \end{cases} \tag{A.13}$$

For $\lambda \le 2/3$ we determine the maximum of $\pi_{PAYW-SP}^{(1)}\left(p_s^{(1)}\right), \pi_{PAYW-SP}^{(2)}\left(p_s^{(2)}\right)$ and find the optimal suggested price as:

$$p_s^* = \begin{cases} 1 & \text{if} \quad m > 1/2 \\ \left(2+c+2m(1-c)\right)/3 & \text{if} \quad 0 \le \lambda \le 2(1+m)/3 \text{ and } m \le 1/2 \\ c + 2\lambda m(1-c)/(3\lambda-2) & \text{if} \quad 2(1+m)/3 \le \lambda \le 1 \text{ and } m \le 1/2 \end{cases} \tag{A.14)=(14}$$

Figure D.1 offers an exemplary overview of the optimal suggested price for different λ.

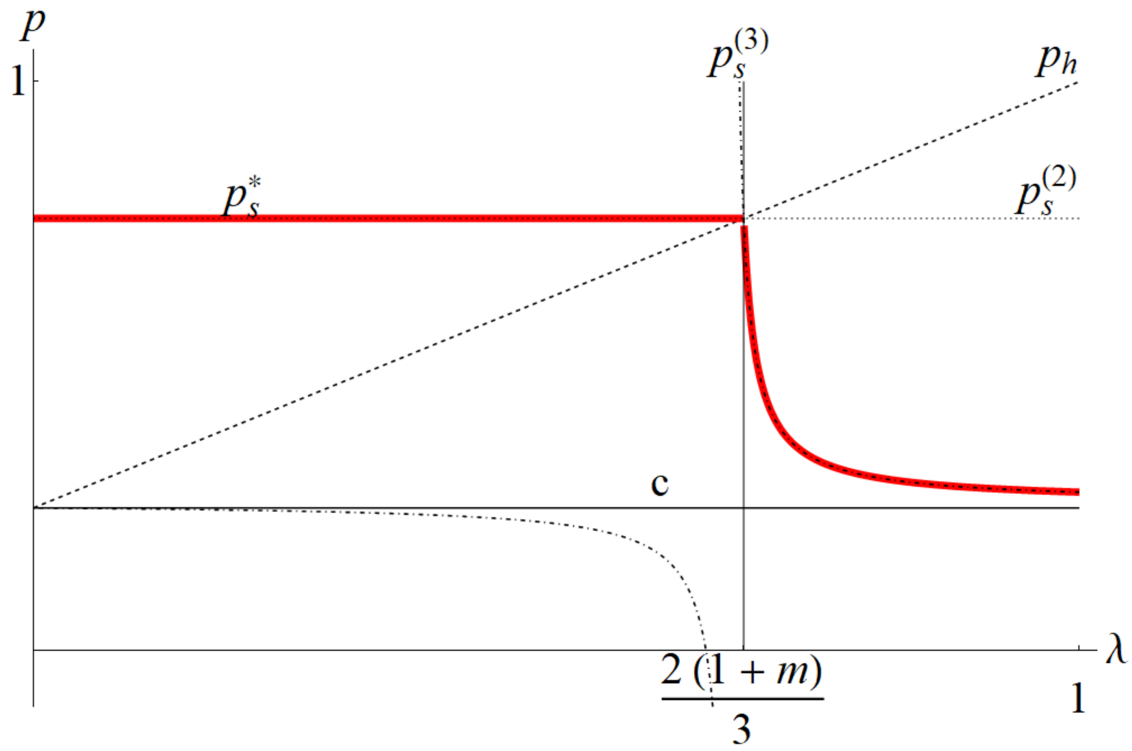

Figure D.1: Composition of p_s for different λ ($\omega = .1, c = .25, \bar{\gamma}_{[0,1]} = .5, z = .5, m = \frac{1}{54}$), $p_s^{(2)}$ represents the second line of (A.14), $p_s^{(3)}$ the third

Again, our model extends CKZ for including less fair-minded consumers who nevertheless abstain from freeloading and do not purchase. Next, we investigate the influence of c, ω, λ, and $\bar{\gamma}_{[0,1]}$ on p_s^*.

	$m > 1/2$	$m \leq 1/2 \wedge$ $0 \leq \lambda \leq 2(1+m)/3$	$m \leq 1/2 \wedge$ $2(1+m)/3 \leq \lambda \leq 1$
$\frac{\partial p_s^*}{\partial c}$	0	$\left(1+\bar{\gamma}_{[0,1]}\omega/(1-\omega)\right)/3 > 0$	$1+2\bar{\gamma}_{[0,1]}\lambda\omega/\left((3\lambda-2)(1-\omega)\right) > 0$
$\frac{\partial p_s^*}{\partial \omega}$	0	$2c\bar{\gamma}_{[0,1]}/(3(1-\omega)^2) \geq 0$	$2c\bar{\gamma}_{[0,1]}\lambda/\left((3\lambda-2)(1-\omega)^2\right) \geq 0$
$\frac{\partial p_s^*}{\partial \lambda}$	0	0	$-4c\,\bar{\gamma}_{[0,1]}\omega/\left((3\lambda-2)^2(1-\omega)\right) \leq 0$
$\frac{\partial p_s^*}{\partial \bar{\gamma}_{[0,1]}}$	0	$2c\omega/\left(3(1-\omega)\right) \geq 0$	$2c\lambda\omega/\left((3\lambda-2)(1-\omega)\right) \geq 0$

Table D.1: Influence of model variables on the optimal suggested price p_s^*

Thus, suggested prices are higher if costs are higher. Optimal suggested prices increase with an increasing share of less fair-minded consumers. Generosity does not influence the optimal suggested price (cf. Figure D.1) if λ is small. For large λ optimal suggested prices decrease with higher generosity. Again, optimal suggested prices increase with an increasing mean of advantageous inequity aversion in [0,1].

The optimal profit amounts to

$$\pi^*_{PAYW-SP} = (1-\omega)(1-c)^2\left(\frac{\lambda z}{2} + g_{1\lambda}(1-z)\right) - c\omega\left(1 - \bar{\gamma}_{[0,1]}\left(cz + g_{2\lambda}(1-z)\right)\right) \quad \text{(A.15)}$$

with

$$\begin{cases} g_{1\lambda} = 1/4 \wedge g_{2\lambda} = 1 & \text{if} \quad m > 1/2 \\ g_{1\lambda} = (1-m^2)/3 \wedge g_{2\lambda} = \left(2 + c + 2m(1-c)\right)/3 & \text{if} \quad 0 \le \lambda \le 2(1+m)/3 \text{ and } m \le 1/2 \\ g_{1\lambda} = \lambda(-m^2/(3\lambda - 2) + 1/2) \wedge g_{2\lambda} = c + \left(2\lambda m(1-c)\right)/(3\lambda - 2) & \text{if} \quad 2(1+m)/3 \le \lambda \le 1 \text{ and } m \le 1/2 \end{cases}$$

Finally, we note that $\pi_{PAYW-SP}$ increases monotonically in λ and for $z = 0$ one finds

$$\begin{aligned} &\frac{(1-\omega)(1-c)^2}{3} - c\omega \le \pi^*_{PAYW-SP} \le \frac{(1-\omega)(1-c)(3-c)}{4} - c\omega \wedge m > \frac{1}{2}, c, \omega > 0 \\ &\frac{(1-c)^2(1-\omega)\lambda}{2} - c\omega \le \pi^*_{PAYW-SP} \le \frac{(1-c)(1-\omega)}{2}\left(\frac{3}{2}\lambda(1-c) + c\right) - c\omega \wedge m \le \frac{1}{2}. \end{aligned} \quad \text{(A.16)}$$

Appendix E: Proof of Updated Proposition 4.

We derive the critical λ^+ where profits from PAAP and PAYW-MP equal PAYW-SP (cf. Footnote 12).

Small λ ($\lambda \le 1/(2+\beta)$)

We compare PAAP (A.1) to PAYW-SP (A.15) (for $m > 1/2$ and $m \le 1/2$) and solve for λ. This results in

$$\lambda^+_{\le} = \frac{g_{\le}}{f_{\le}} \quad \text{(A.17)}$$

with

$$f_{\leq} = (1-c)^2\left(\frac{\beta}{4(1+\beta)} - \frac{(1-\omega)z}{2}\right)$$
$$g_{\leq} = (1-c)^2\left((1-z)(1-\omega)g_{1\lambda} - \frac{1}{4(1+\beta)}\right) - c\omega\left(1 - \bar{\gamma}_{[0,1]}(cz + (1-z)g_{2\lambda})\right).$$

By means of implicit differentiation we find

$$\left.\frac{\partial\lambda_{\leq}^{+}(\omega)}{\partial\omega}\right|_{\lambda_{\leq}^{+}} = \frac{g_{\leq}^{'} - \lambda_{\leq}^{+} f_{\leq}^{'}}{f_{\leq}} \tag{A.18}$$

Large* $\boldsymbol{\lambda}$ *(*$\boldsymbol{\lambda > 1/(2+\beta)}$*)

We compare PAYW-MP (A.8) to PAYW-SP (A.15) (for $m > 1/2$ and $m \leq 1/2$) and solve for λ.

This results in

$$\lambda_{>\,1,2}^{+} = \frac{-g_{>} \pm \sqrt{g_{>}^2 - 4f_{>}j_{>}}}{2f_{>}} \tag{A.19}$$

with

$$f_{>} = (1-c)^2\left(\frac{z\beta\omega^2}{2} + (1-z)\left(\frac{2+\beta}{2} - \omega\right)\right)$$

$$g_{>} = (1-c)^2\left(\omega^2\left(\frac{z}{2} + \beta(1-z)\mathrm{g}_{1\lambda}\right) - (1-z)\left(\frac{1}{2} + (2+\beta-2\omega)\mathrm{g}_{1\lambda}\right)\right.$$
$$\left. - (1+z)\omega\right) + c\omega(2+\beta+\beta\omega)\left(1 - \bar{\gamma}_{[0,1]}\left(cz + \mathrm{g}_{2\lambda}(1-z)\right)\right)$$

$$j_{>} = (1-c)^2(1-\omega)^2(1-z)\mathrm{g}_{1\lambda} - c\omega(1-\omega)\left(1 - \bar{\gamma}_{[0,1]}(cz + (1-z)\mathrm{g}_{2\lambda})\right).$$

Infeasible solutions have to be discarded.

3. Article 2

Wagner, U., Akbari, K. (2020). Supplementary Appendix to “Comments and Refinements on the Pay as you wish Model by Chen et al. (2017)”. Forschungsberichte des Instituts für Betriebswirtschaftslehre der Universität Wien.

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Supplementary Appendix to “Comments and Refinements on the Pay as you wish Model by Chen et al. (2017)”

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Lehrstuhl für Marketing

Juni 2020

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Supplementary Appendix to “Comments and Refinements on the Pay as you wish Model by Chen et al. (2017)”

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June 2020

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Table of Contents

1 Introduction and setup

In Akbari and Wagner (2020) we discuss extensions of the Pay as you wish (PAYW) model by Chen et al. (2017) (CKZ hereafter). The setup of the mathematical derivations of the model leads to complicated and long expressions. Due to readability considerations and the page limit for the submission to *Marketing Science*, these derivations are omitted from the paper and the appendices of the paper. However, reproducibility concerns call for the disclosure of our calculation paths. Therefore, this report aims to support the reader of our work in retracing the results in the paper and the appendices. Accordingly, this report can be regarded as an 'appendix to the appendices.'

In addition to disclosing our calculation paths, we also try to reconstruct the results of CKZ in this report. Although parts of these derivations have already been discussed in the original CKZ model, their derivations are not always accessible and sometimes flawed.

In Akbari and Wagner (2020) we run a simulation analysis to quantify the consequences of the misspecifications in CKZ. This article also presents the code and extended results on this simulation in Section 6.

1.1 Notation conventions

This report strives for consistent notation. However, the results discussed in this report are based on two different sources, Akbari and Wagner (2020) and CKZ. Hence, it is necessary to find notational compromises between consistency, compactness and comparability within this report.

Therefore, the following conventions are established in this report:

- Auxiliary variables are typically denoted in uppercase letters and numbered alphabetically. The letter C is usually omitted to avoid confusion with costs (c). This might lead to a different notation in the report than in the paper. In some cases, the original notation from the manuscript is retained. When deviating from these rules, we point to the difference in notation between the sources.
- The sequencing starts anew in each chapter (and, in some cases, subchapter). Subscripts indicate the respective (sub-)section. For compactness, subscripts of auxiliary variables are abbreviated for PAYW with a minimum price (PAYW-MP becomes MP) and PAYW with a suggested price (PAYW-SP becomes s).

- As profit functions play an important role in Chapter 5, their subscripts always include the entire name of the pricing policy instead of the abbreviation.
- Superscripts are used to indicate special conditions of parameters (e.g., $\beta = 0$) and to distinguish different versions by different authors.
- An asterisk ($*$) as a superscript denotes an optimal value. A plus ($+$) denotes a critical value.
- We drop the index for consumers, i, when we take the sellers perspective.
- CKZ sometimes use different notation conventions (e.g., they use asterisks to denote both, optimal values and critical values). In this case we align their notation with ours. This might lead to different labels. To avoid confusion, please confer to the list of abbreviations.
- When referring to equations in the main body of our manuscript, we will use the abbreviation AW followed by the respective number of the equation (e.g., (1) will become (AW.1)). When referring to equations of the appendices in our manuscript, we will use AW.A. and the respective number (e.g., (A.1) will become (AW.A.1)), and when referring to equations in CKZ, we add “CKZ” in front of the equation (e.g., (1) will become (CKZ.1)). These equations are not necessarily notationally equivalent, however, they have a similar meaning.

1.2 Fundamentals in CKZ’s PAYW model

In this section, we summarize the most important assumptions in CKZ’s model. In their paper, CKZ model consumer behavior based on two important equations, the consumer’s utility function and the perceived fair price.

The consumer’s utility function includes advantageous, disadvantageous inequity aversion, consumption utility, and the perceived fair price,

$$u_i = r_i - p_i - \beta_i \max\{p_i - r_{i0}, 0\} - \gamma_i \max\{r_{i0} - p_i, 0\}, \;\; \beta_i, \gamma_i \geq 0. \tag{1}$$

(CKZ.1)

(AW.1)

This utility function originates from Fehr and Schmidt (1999). However, CKZ contribute to the literature by introducing the perceived fair price, which is given by

$$r_{i0} = \begin{cases} c & r_i \leq c \\ \lambda r_i + (1-\lambda)c & r_i > c \end{cases}. \tag{2}$$

(AW.2)

Furthermore, CKZ make assumptions on the distribution of the model parameters. They assume that

- the consumption utilities, r, are uniformly distributed,
- advantageous inequity aversion, γ, follows some distribution function $h(\gamma)$, and that
- generosity, λ, and disadvantageous inequity aversion, β, are constant for all consumers.

2 Derivations of Proposition 1 (PAAP, PAYW, and critical costs)

This section is constructed as follows. First, we retrace CKZ's calculations for PAAP. Second, we derive the firm's profit in PAYW and compare our results to the results of CKZ. Third, we derive the critical costs level that equates profits in PAAP and PAYW.

2.1 Derivation and properties of PAAP

2.1.1 Derivation of optimal prices and profits in PAAP

For their Proposition 1 in their Section 2, CKZ derive the firms profits under "pay as asked" pricing (PAAP) and PAYW pricing.

First, CKZ show by contradiction that a seller with pricing power will always charge a posted price, p^*_{PAAP} above the marginal consumer's fair price. They denote the consumption utility for the marginal consumer as $\tilde{r}$. If the seller's price is lower than the marginal consumer's fair price, r_{i0}, the consumer would experience advantageous inequity aversion. Thus, the marginal buyer's utility would be given by

$$\begin{aligned} u_{\tilde{r}} &= \tilde{r} - p_{PAAP} - \gamma_i(r_{i0} - p_{PAAP}) \\ &= \tilde{r} - p_{PAAP} - \gamma_i(\lambda\tilde{r} + (1-\lambda)c - p_{PAAP}). \end{aligned}$$

Furthermore, as there must be advantageous inequity,

$$\lambda\tilde{r} + (1-\lambda)c > p_{PAAP}.$$

As the seller is a profit maximizing monopolist, one can assume that the seller will charge prices above costs, i.e., $p_{PAAP} > c$.

Next, $u_{\tilde{r}}$ is set to zero and solved for $\tilde{r}$ to find the critical consumption utility of the marginal consumer.

$$\tilde{r} - p_{PAAP} - \gamma_i(\lambda\tilde{r} + (1-\lambda)c - p_{PAAP}) = 0$$

$$\tilde{r}(1-\lambda\gamma_i) = p_{PAAP}(1-\gamma_i) + \gamma_i c(1-\lambda)$$

$$\tilde{r} = \frac{p_{PAAP}(1-\gamma_i) + \gamma_i c(1-\lambda)}{(1-\lambda\gamma_i)} \coloneqq \tilde{r}^+$$

for all $\gamma_i < \frac{1}{\lambda}$ as $(1-\lambda\gamma_i) > 0$.

As the marginal consumer will buy,

$$\tilde{r}^+ > p_{PAAP}.$$

$$\frac{p_{PAAP}(1-\gamma_i) + \gamma_i c(1-\lambda)}{(1-\lambda\gamma_i)} > p_{PAAP}$$

$$\gamma(1-\lambda)(-p+c) > 0$$

$$c > p_{PAAP}.$$

This is a contradiction. Hence, there cannot be advantageous inequity for the buyer if the seller sets the price.

Therefore, in PAAP, the consumers will experience disadvantageous inequity, $0 \leq r_{i0} \leq p_{PAAP}$. Thus, the marginal consumer will only buy the product if the consumption utility exceeds the price and the disutility from disadvantageous inequity. In this case (1) will simplify to

$$u_i = r_i - p_{PAAP} - \beta(p_{PAAP} - r_{i0}).$$

By substituting r_{i0}, we get

$$u_i = r_i - p_{PAAP} - \beta\big(p_{PAAP} - (\lambda r_i + (1-\lambda)c)\big). \tag{3}$$

Note that we are ultimately interested in paying consumers. Therefore, we only need to observe consumers with $r_i > c$ (hence $r_{i0} = (\lambda r_i + (1-\lambda)c)$) and can neglect consumers with $r_i \leq c$ (hence $r_{i0} = c$).

The marginal consumer will make a purchase if $u_i \geq 0$. Rearranging (3) and solving for r_i, CKZ arrive at r_i for the marginal consumer

$$r_i = p_{PAAP} + \frac{\beta(1-\lambda)(p_{PAAP} - c)}{1+\lambda\beta} := r^+.$$

All consumers with a consumption utility above this threshold, $r^+ \leq r_i \leq 1$, will make a purchase. The firm will earn the price, p_{PAAP}, and has to bear the costs, c. Hence, the seller's profits are given by

$$\pi_{PAAP} = \int_{r^+}^{1} (p_{PAAP} - c)\phi(r)dr. \tag{4}$$

(CKZ.2)

As r_i follows a uniform distribution, $\phi(r) = 1$, this is equivalent to

$$\begin{aligned}\pi_{PAAP} &= \int_{r^+}^{1} (p_{PAAP} - c)dr \\ &= \frac{(c - p_{PAAP})(p_{PAAP}(1+\beta) - c\beta(1-\lambda) - \beta\lambda - 1)}{1+\beta\lambda}.\end{aligned}$$

Note that we drop index i when we take the seller's perspective (i.e., for segment and profit calculations).

For finding the optimal prices and profits, we take the first derivative, set it to 0 and solve for p_{PAAP}.

$$\pi_{PAAP} \rightarrow Max$$

$$\begin{aligned}\frac{\partial \pi_{PAAP}}{\partial p_{PAAP}} &= \frac{(c - p_{PAAP})(1+\beta)}{1+\beta\lambda} - \frac{-1 + p_{PAAP}(1+\beta) - c\beta(1-\lambda) - \beta\lambda}{1+\beta\lambda} \\ &= \frac{-2p_{PAAP}(1+\beta) + c(1+\beta) + 1 + c\beta(1-\lambda) + \beta\lambda}{1+\beta\lambda} \stackrel{!}{=} 0.\end{aligned}$$

Hence, the optimal price is

$$p^*_{PAAP} = c + \frac{1+\beta\lambda}{2(1+\beta)}(1-c). \tag{5}$$

(CKZ.3)

(AW.3)

This is a maximum, as the second derivative,

$$\frac{\partial^2 \pi_{PAAP}}{\partial (p_{PAAP})^2} = -\frac{2(1+\beta)}{1+\beta\lambda} < 0$$

is always negative.

Substituting p^*_{PAAP} in π_{PAAP}, we find the optimal profits

$$\begin{aligned}\pi_{PAAP}(p^*_{PAAP}) &= -\frac{1-c}{2(1+\beta)}\left(c(1+\beta) + \frac{1+\beta\lambda}{2}(1-c) - c\beta - \beta\lambda(1-c) \right. \\ &\quad \left. - 1\right) \\ &= \frac{(1-c)^2(1+\beta\lambda)}{4(1+\beta)} \coloneqq \pi^*_{PAAP}.\end{aligned} \tag{6}$$

(AW.A1)

(CKZ.3)

Later, CKZ set $\beta = 0$. In this case the optimal profits reduce to

$$\pi_{PAAP}^{\beta=0,*} = \frac{(1-c)^2}{4}. \tag{7}$$

This corresponds to standard monopolistic pricing.

2.1.2 Properties of PAAP

The derivation of the optimal price and profits allows us to determine properties of PAAP with inequity averse consumers.

When we consider p_{PAAP}^* as a function of λ, we find that the price increases linearly in λ,

$$p_{PAAP}^*(\lambda) = \frac{1 + c(1+2\beta) + \beta\lambda(1-c)}{2(1+\beta)}$$

$$p_{PAAP}^*(\lambda = 0) = \frac{1 + c(1+2\beta)}{2(1+\beta)} \leq p_{PAAP}^*(\lambda = 1) = \frac{1+c}{2}$$

$$\frac{\partial p_{PAAP}^*(\lambda)}{\partial \lambda} = \frac{(1-c)\beta}{2(1+\beta)}.$$

When we consider p_{PAAP}^* as a function of β, we find that the price decreases monotonically with β

$$p_{PAAP}^*(\beta) = \frac{1 + c(1+2\beta) + \beta\lambda(1-c)}{2(1+\beta)}$$

$$p_{PAAP}^*(\beta = 0) = \frac{1+c}{2}$$

$$\lim_{\beta\to\infty} p_{PAAP}^*(\beta) = \lim_{\beta\to\infty} (c + \frac{1/\beta + \lambda}{2(1/\beta + 1)} (1 - \mathrm{c})$$

$$= c + \frac{\lambda(1-c)}{2}$$

$$\frac{\partial p_{PAAP}^*(\beta)}{\partial \beta} = \frac{(1-c)(\lambda-1)}{2(1+\beta)^2} < 0.$$

With respect to the optimal profits, π_{PAAP}^*, we find that profits increase monotonically in λ,

$$\frac{\partial \pi_{PAAP}^*}{\partial \lambda} = \frac{(1-c)^2\beta}{4(1+\beta)} > 0$$

and decreases monotonically in β,

$$\frac{\partial \pi_{PAAP}^*}{\partial \beta} = \frac{(1-c)^2}{4} \cdot \frac{\lambda - 1}{(1+\beta)^2} \leq 0.$$

Furthermore, this permits us to observe π_{PAAP}^* at some critical values,

$$\pi^*_{PAAP}(\lambda = 0) = \frac{(1-c)^2}{4(1+\beta)}$$

$$\pi^*_{PAAP}\left(\lambda = \frac{1}{2+\beta}\right) = \frac{(1-c)^2 2(1+\beta)}{4(1+\beta)(2+\beta)}$$

$$= \frac{(1-c)^2}{2(2+\beta)}$$

$$\pi^*_{PAAP}\left(\lambda = \frac{2}{3}\right) = \frac{(1-c)^2(1+2\beta/3)}{4(1+\beta)}$$

$$\pi^*_{PAAP}(\lambda = 1) = \frac{(1-c)^2}{4}$$

$$\lim_{\beta\to\infty} \pi^*_{PAAP} = \frac{(1-c)^2\lambda}{4}.$$

Taken together, this allows us to determine the rage of profits as

$$\frac{(1-c)^2\lambda}{4} \le \pi^*_{PAAP} \le \frac{(1-c)^2}{4}. \tag{8}$$

2.2 Derivation and properties of PAYW

2.2.1 Derivation of optimal individual prices and profits in PAYW

CKZ show that in PAYW, a consumer can always prevent disadvantageous inequity which is caused by a price above the perceived fair price by paying a lower price. Thus, the buyer's utility function will be given by

$$u_i = r_i - p_i - \gamma_i\,(r_{i0} - p_i) \tag{9}$$

$$= r_i - \gamma_i r_{i0} - (1-\gamma_i)p_i. \tag{AW.1a}$$

Note: As in CKZ, we denote the consumer's individual price as p_i instead of $p_{PAYW,i}$. This little inconsistency is in the interest of a sparse notation. A consumer maximizes its utility by setting the optimal price. The consumer's utility function, u_i, is a linear function in the price, p_i. Thus, the derivative can be positive or negative depending on γ_i. As this is linear in p_i, we find a corner solution.

For $\gamma_i \le 1$, the consumer maximizes her utility function by paying as little as possible. As negative prices are not feasible, we observe a corner solution: $[p_i^*|\gamma_i \le 1] = 0$.

For $\gamma_i > 1$, the consumer maximizes her utility function by paying as much as possible. Prices above r_{i0} create disadvantageous inequity which means that our utility function would be invalid. Thus, the consumer pays $[p_i^*|\gamma_i > 1] = r_{i0}$.

Thus, the optimal price is given by

$$p_i^* = \begin{cases} 0 & \gamma_i \leq 1 \\ r_{i0} & \gamma_i > 1 \end{cases}.$$

Next, we need to check whether the consumer would purchase with these prices (participation constraint), i.e., whether optimal prices lead to positive utilities. A consumer who would experience negative utility when buying for their optimal price, p_i, chooses the outside option, abstains from buying and receives a utility of $u_i = 0$.

Using the above prices, the utility function simplifies to

$$u_i = \begin{cases} r_i - \gamma_i r_{i0} - (1-\gamma_i) \cdot 0 & \gamma_i \leq 1 \\ r_i - \gamma_i r_{i0} - (1-\gamma_i) \cdot r_{i0} & \gamma_i > 1 \end{cases} \tag{10}$$

$$= \begin{cases} r_i - \gamma_i r_{i0} & \gamma_i \leq 1 \\ r_i - r_{i0} & \gamma_i > 1 \end{cases}. \tag{AW.1c}$$

The consumers will maximize this utility function

$$u_i \rightarrow Max.$$

We proceed in a stepwise manner.

1) We investigate less fair-minded consumers, i.e., $0 \leq \gamma_i \leq 1$. Consumers with a positive utility decide to make a purchase

$$u_i = r_i - \gamma_i r_{i0} \geq 0 \tag{11}$$

$$r_i \geq \gamma_i r_{i0}. \tag{AW.1b}$$

In contrast to PAAP, we need to observe r_{i0} for both $r_i \geq c$ and $r_i < c$.

a) If $1 \geq r_i > c$, $r_{i0} = \lambda r_i + (1-\lambda)c$ and

$$r_i \geq \gamma_i(\lambda r_i + (1-\lambda)c) \geq 0$$

$$r_i \geq -\frac{(1-\lambda)c}{\lambda}.$$

This always the case. Thus, $u_i^* \geq 0$, the consumer takes the good and freeloads.

The size of this segment of consumers can be defined as

$$\theta_{PAYW,1} = \int_0^1 \int_c^1 \phi(r) dr\, h(\gamma) d\gamma,$$

with $h(\gamma)$ the distribution of advantageous inequity. Note that we again drop index i when we take the seller's perspective (i.e., for segment and profit calculations).

b) If $r_i \leq c \leq 0$, it follows that $r_{i0} = c$. Thus,

$$u_i = r_i - \gamma_i c \geq 0$$
$$r_i \geq \gamma_i c.$$

Thus, the consumers freeload if $r_i \geq \gamma_i c$, i.e., if their consumption utility is high, or if their disutility from advantageous inequity aversion is low. Thus, this segment of freeloaders can be defined as

$$\theta_{PAYW,2} = \int_0^1 \int_{\gamma c}^{c} \phi(r)dr\, h(\gamma)d\gamma.$$

If this does not hold, i.e., $0 \leq r_i < \gamma_i c$, consumers do not buy. This segment can be defined as

$$\delta_{PAYW} = \int_0^1 \int_0^{\gamma c} \phi(r)h(\gamma)drd\gamma\,.$$

The total segment of freeloaders is defined as

$$\begin{aligned}\theta_{PAYW} &= \theta_{PAYW,1} + \theta_{PAYW,2} \\ &= \int_0^1 \left[\int_{\gamma c}^{c} \phi(r)dr + \int_c^1 \phi(r)dr\right] h(\gamma)d\gamma \\ &= \int_0^1 \int_{\gamma c}^{1} \phi(r)h(\gamma)dr\,d\gamma.\end{aligned} \qquad \begin{matrix}(12)\\(\text{AW.5})\end{matrix}$$

We name the share of less fair-minded consumers $\omega \in [0,1]$.

$$\begin{aligned}\omega &= \theta_{PAYW} + \delta_{PAYW} \\ &= \int_0^1 h(\gamma)d\gamma.\end{aligned} \qquad (13)$$

Furthermore, we assume that γ_i is distributed according to some distribution $h_{[0,1]}(\gamma_i)$ in the domain $[0,1]$. Again, we drop index i. This allows us to estimate δ and θ more accurately,

$$\delta_{PAYW} = \int_0^1 \left[\omega h_{[0,1]}(\gamma) \int_0^{\gamma c} dr\right] d\gamma \qquad \begin{matrix}(14)\\(\text{AW.6})\end{matrix}$$

$$= c\omega\bar{\gamma}_{[0,1]} \qquad (\text{AW.6a})$$

with $\bar{\gamma}_{[0,1]}$: mean of γ in $[0,1]$ and

$$\theta_{PAYW} = \int_0^1 \left[\omega h_{[0,1]}(\gamma) \int_{\gamma c}^1 dr\right] d\gamma \\ = \omega\left(1 - c\bar{\gamma}_{[0,1]}\right). \tag{15}$$

2) Next, we investigate, more fair-minded consumers $\gamma_i > 1$. In this case,

$$u_i = r_i - r_{i0} \geq 0$$
$$r_i \geq r_{i0}.$$

a) Again, if $r_i > c$, $r_{i0} = \lambda r_i + (1-\lambda)c$ and, therefore,

$$r_i - (\lambda r_i + (1-\lambda)c) \geq 0$$
$$(1-\lambda)r_i - (1-\lambda)c \geq 0$$
$$(1-\lambda)(r_i - c) > 0$$
$$(r_i - c) > 0.$$

This is always the case. Hence, the consumer takes the good and pays her fair price.

b) If $r_i \leq c$, it follows that $r_{i0} = c$. Thus,

$$u_i = r_i - c \geq 0.$$

A contradiction. Hence, the consumer does not buy.

The size of all more fair-minded consumers is given by $(1-\omega)$.

Having defined different segments, we can move on to the firm's profits from all segments. The profits are composed of:

- Revenues and costs from fair-minded consumers ($(1-\omega)$ in size): These consumers pay their fair price r_{i0} and the firm incurs cost, c.
- Costs of freeloaders (θ_{PAYW}): The firm incurs costs of c for all freeloading consumers but does not get any revenue.

This allows us to arrive at the seller's profits,

$$\pi_{PAYW} = \int_c^1 (1-\omega)(r_0 - c)\phi(r)dr - c\theta_{PAYW} \tag{16}$$
$$= \int_c^1 (1-\omega)\lambda(r-c)\phi(r)dr - c\omega\left(1 - c\bar{\gamma}_{[0,1]}\right). \tag{AW.7}$$

As the seller cannot influence prices, the optimal profits are given when consumers behave utility maximizing and not require any optimization from the seller. We can solve the integral and arrive at

$$\pi_{PAYW} = \frac{\lambda(1-\omega)(1-c)^2}{2} - c\omega\left(1 - c\bar{\gamma}_{[0,1]}\right) \coloneqq \pi^*_{PAYW}. \tag{17}$$

(AW.A.3)

2.2.2 Properties of PAYW

This allows us to determine how the sellers profit change with changes in the model parameters,

$$\frac{\partial \pi^*_{PAYW}}{\partial \bar{\gamma}_{[0,1]}} = c^2\omega > 0$$
$$\frac{\partial \pi^*_{PAYW}}{\partial \lambda} = \frac{1}{2}(1-c)^2(1-\omega) > 0$$
$$\frac{\partial \pi^*_{PAYW}}{\partial \omega} = -c\left(1 - c\bar{\gamma}_{[0,1]}\right) - \frac{1}{2}(1-c)^2\lambda < 0.$$

Profits of PAYW are increasing with generosity, λ, and the mean of advantageous inequity aversion in $[0,1]$, $\bar{\gamma}_{[0,1]}$, and decreasing in the share of less fair-minded consumer, ω.

Furthermore, this allows us to observe π^*_{PAYW} at some critical values,

$$\pi^*_{PAYW}\left(\bar{\gamma}_{[0,1]} = 0\right) = \frac{\lambda(1-\omega)(1-c)^2}{2} - c\omega$$
$$\pi^*_{PAYW}\left(\bar{\gamma}_{[0,1]} = 1\right) = \frac{\lambda(1-\omega)(1-c)^2}{2} - c\omega(1-c)$$
$$\pi^*_{PAYW}(\omega = 0) = \frac{\lambda(1-c)^2}{2}$$
$$\pi^*_{PAYW}(\omega = 1) = -c\left(1 - c\bar{\gamma}_{[0,1]}\right)$$
$$\pi^*_{PAYW}\left(\omega = 1, \bar{\gamma}_{[0,1]} = 0\ \right) = -c.$$

Thus, profits are in the range of

$$-c \leq \frac{\lambda(1-\omega)(1-c)^2}{2} - c\omega \leq \pi^*_{PAYW} \leq \frac{\lambda(1-\omega)(1-c)^2}{2} - c\omega(1-c) \tag{18}$$
$$\leq \frac{\lambda(1-c)^2}{2}.$$

(AW.A.5)

2.2.3 Excursus: CKZ's profit calculation

The results derived in Section 2.2.1 deviate from CKZ in Case 1b) of Section 2.2.1. In their paper, CKZ ignore the δ segment and assume that all consumers behave like in Case 1a). Thus, all less fair-minded consumers freeload.

Therefore, they derive their profits as

$$\pi_{PAYW}^{CKZ^*} = \int_c^1 (1-\omega)(r_0 - c)\phi(r)dr - c\omega \quad (19)$$

$$= \frac{\lambda(1-\omega)(1-c)^2}{2} - c\omega. \quad \text{(AW.7a) (CKZ.4)}$$

CKZ use the term “freeloader” instead of “less fair-minded consumer” for consumers with $\gamma_i \leq 1$. When sticking to their notation and assuming that all less fair-minded consumers are freeloaders, $\omega = \theta^{CKZ}$, we get

$$\pi_{PAYW}^{CKZ^*} = \int_c^1 (1-\theta^{CKZ})(r_0 - c)\phi(r)dr - c\theta^{CKZ} \quad (20) \text{ (AW.A.2)}$$

$$= \frac{\lambda(1-\omega)(1-c)^2}{2} - c\theta^{CKZ}. \quad \text{(CKZ.5) (CKZ.A.1)}$$

2.3 Comparing PAAP to PAYW

When comparing PAAP to PAYW, we can derive the critical cost level above which PAAP will always be better than PAYW. First, we offer our calculations that include the correct share of freeloaders. Second, we retrace CKZ’s calculations.

2.3.1 Deriving the new c^+

In order to derive the new critical cost level, c^+ for which PAYW is better than PAAP, we compare our updated optimal profit function π_{PAYW}^* to the profits under fixed prices and solve for c:

$$\pi_{PAYW}^* > \pi_{PAAP}^*$$

$$\pi_{PAYW}^* - \pi_{PAAP}^* > 0$$

$$\frac{\lambda(1-\omega)(1-c)^2}{2} - c\omega\big(1 - c\bar{\gamma}_{[0,1]}\big) - \frac{(1-c)^2(1+\beta\lambda)}{4(1+\beta)} > 0$$

$$(1-c)^2\big(2\lambda(1+\beta)(1-\omega) - (1+\lambda\beta)\big) - 4c\omega(1+\beta)\big(1 - c\bar{\gamma}_{[0,1]}\big) > 0$$

$$-A_{COMP}(1-c)^2 - 4cB_{COMP}\big(1 - c\bar{\gamma}_{[0,1]}\big) > 0$$

with $A_{COMP} = 1 - \lambda\big(2 + \beta - 2\omega(1+\beta)\big)$ and $B_{COMP} = \omega(1+\beta)$.

$$A_{COMP} - 2A_{COMP}c + A_{COMP}c^2 + 4cB_{COMP} - 4c^2B_{COMP}\bar{\gamma}_{[0,1]} < 0$$

$$c^2\big(A_{COMP} - 4B_{COMP}\bar{\gamma}_{[0,1]}\big) - 2c(A_{COMP} - 2B_{COMP}) + A_{COMP} < 0. \quad (21)$$

We solve using the reduced quadratic equation form,

$$c_{1,2} = \frac{2(A_{COMP} - 2B_{COMP}) \pm \sqrt{4(A_{COMP} - 2B_{COMP})^2 - 4A_{COMP}\left(A_{COMP} - 4B_{COMP}\bar{\gamma}_{[0,1]}\right)}}{2\left(A_{COMP} - 4B_{COMP}\bar{\gamma}_{[0,1]}\right)}$$

$$= \frac{2(A_{COMP} - 2B_{COMP}) \pm \sqrt{16B_{COMP}\left(B_{COMP} - A_{COMP}\left(1 - \bar{\gamma}_{[0,1]}\right)\right)}}{2\left(A_{COMP} - 4B_{COMP}\bar{\gamma}_{[0,1]}\right)}$$

$$= \frac{A_{COMP} - 2B_{COMP} \pm 2\sqrt{B_{COMP}\left(B_{COMP} - A_{COMP}\left(1 - \bar{\gamma}_{[0,1]}\right)\right)}}{A_{COMP} - 4B_{COMP}\bar{\gamma}_{[0,1]}}$$

$$= 1 - \frac{2B_{COMP}\left(1 - 2\bar{\gamma}_{[0,1]}\right) \mp 2\sqrt{B_{COMP}\left(B_{COMP} - A_{COMP}\left(1 - \bar{\gamma}_{[0,1]}\right)\right)}}{A_{COMP} - 4B_{COMP}\bar{\gamma}_{[0,1]}}$$

$$c_1 = 1 - \frac{2B_{COMP}\left(1 - 2\bar{\gamma}_{[0,1]}\right) - 2\sqrt{B_{COMP}\left(B_{COMP} - A_{COMP}\left(1 - \bar{\gamma}_{[0,1]}\right)\right)}}{A_{COMP} - 4B_{COMP}\bar{\gamma}_{[0,1]}}$$

$$c_2 = 1 - \frac{2B_{COMP}\left(1 - 2\bar{\gamma}_{[0,1]}\right) + 2\sqrt{B_{COMP}\left(B_{COMP} - A_{COMP}\left(1 - \bar{\gamma}_{[0,1]}\right)\right)}}{A_{COMP} - 4B_{COMP}\bar{\gamma}_{[0,1]}}.$$

We could also solve (21) by completing the square

$$\begin{aligned} c^2(A_{COMP} - 4B_{COMP}\gamma) - 2c(A_{COMP} - 2B_{COMP}) + A_{COMP} \\ = (A_{COMP} - 4B_{COMP}\gamma)(c - c_1)(c - c_2) < 0 \end{aligned} \tag{22}$$

as

$$c_1 = 1 - \frac{(a - b)}{-d} = 1 + \frac{a - b}{d}$$

$$c_2 = 1 - \frac{(a + b)}{-d} = 1 + \frac{a + b}{d}.$$

$c_1 < c_2$.

a) When $A_{COMP} = 4B_{COMP}\bar{\gamma}_{[0,1]}$, this does not yield a solution in $c \in [0,1]$ as

$$-2c \cdot 2B_{COMP}\left(2\bar{\gamma}_{[0,1]} - 1\right) + 4B_{COMP}\bar{\gamma}_{[0,1]} < 0$$

$$c\left(1 - 2\bar{\gamma}_{[0,1]}\right) + \bar{\gamma}_{[0,1]} < 0.$$

Hence for

1) $\bar{\gamma}_{[0,1]} = \frac{1}{2}$, dividing by 0, wid.

2) $\bar{\gamma}_{[0,1]} > \frac{1}{2}$, $c > -\frac{\bar{\gamma}_{[0,1]}}{1-2\bar{\gamma}_{[0,1]}} = 1 + \frac{-1+\bar{\gamma}_{[0,1]}}{1-2\bar{\gamma}_{[0,1]}} > 1$

3) $\bar{\gamma}_{[0,1]} < \frac{1}{2}, c < -\frac{\bar{\gamma}_{[0,1]}}{1-2\bar{\gamma}_{[0,1]}} < 0.$

b) When $A_{COMP} - 4B_{COMP}\bar{\gamma}_{[0,1]} < 0$, the second part of (22) must be positive, i.e., $(c - c_1)(c - c_2) > 0$, so that the inequality holds.

Therefore, $(c - c_1) > 0$ and $(c - c_2) > 0$, and $c_2 > 0$ as a feasible solution or $(c - c_1) < 0$ and $(c - c_2) < 0$, and, therefore, $c < c_1$. as are interested in the lower cost level, i.e., $c < c_1$. Hence, we can discard c_2 as a possible solution.

Thus,

$$c^+ < 1 - \frac{2B_{COMP}\left(1 - 2\bar{\gamma}_{[0,1]}\right) - 2\sqrt{B_{COMP}\left(B_{COMP} - A_{COMP}\left(1 - \bar{\gamma}_{[0,1]}\right)\right)}}{A_{COMP} - 4B_{COMP}\bar{\gamma}_{[0,1]}} \tag{23}$$

(AW.8)

$$A_{COMP} = 1 - \lambda\left(2 + \beta - 2\omega(1 + \beta)\right) \tag{AW.A.4}$$

$$B_{COMP} = \omega(1 + \beta).$$

To find the maximum share of less fair-minded consumers a PAYW seller can tolerate before switching to PAAP, we set c^+ to 0 (costs below 0 are not tolerable).

$$c^+ = 0$$

$$1 - \frac{2B_{COMP}\left(1 - 2\bar{\gamma}_{[0,1]}\right) - 2\sqrt{B_{COMP}\left(B_{COMP} - A_{COMP}\left(1 - \bar{\gamma}_{[0,1]}\right)\right)}}{A_{COMP} - 4B_{COMP}\bar{\gamma}_{[0,1]}} = 0$$

$$A_{COMP} - 4B_{COMP}\bar{\gamma}_{[0,1]}$$

$$= 2B_{COMP}\left(1 - 2\bar{\gamma}_{[0,1]}\right) - 2\sqrt{B_{COMP}\left(B_{COMP} - A_{COMP}\left(1 - \bar{\gamma}_{[0,1]}\right)\right)}$$

$$A_{COMP}^2 = 4A_{COMP}B_{COMP}\bar{\gamma}_{[0,1]}.$$

As $A_{COMP} = 4B_{COMP}\bar{\gamma}_{[0,1]}$ does not yield a meaningful solution for c_1, this only has a solution for $A_{COMP} = 0$. Therefore,

$$1 - \lambda(2 + \beta) + 2\lambda\omega(1 + \beta) = 0.$$

Thus, our critical threshold is given by

$$\begin{aligned} \omega &= \frac{\lambda(2 + \beta) - 1}{2\lambda(1 + \beta)} \\ &= \frac{1}{2} + \frac{\lambda - 1}{2\lambda(1 + \beta)} \coloneqq \omega^+. \end{aligned} \tag{24}$$

Thus, even for zero costs, for PAYW to dominate PAAP, $\omega \leq \omega^{+}$. As $\lambda - 1 < 0$, ω^{+} is always smaller than $1/2$.

Furthermore, we can use this expression to check conditions for λ so that $\omega^{+} > 0$.

$$\omega^{+} > 0$$
$$\frac{\lambda(1+\beta)+\lambda-1}{2\lambda(1+\beta)} > 0$$
$$\lambda \geq \frac{1}{2+\beta}.$$

Thus, as a necessary condition, λ must be greater than $\lambda \geq \frac{1}{2+\beta}$ for PAYW to be more profitable than PAAP.

2.3.2 CKZ's c^{+}

In this section, we retrace CKZ's results. CKZ derive the critical cost level below which PAYW is better than PAAP, c^{+}, by comparing the profits of both pricing schemes and solving for c. In their paper, CKZ denote this threshold as c^{*}. However, as noted above, we reserve *-superscripts for optimal values. Thus, we return to $c^{CKZ^{+}}$.

$$\pi_{PAYW}^{CKZ,*} > \pi_{PAAP}^{*}$$

$$\pi_{PAYW}^{CKZ,*} - \pi_{PAAP}^{*} > 0$$
$$\frac{\lambda(1-\theta^{CKZ})(1-c)^2}{2} - c\theta^{CKZ} - \frac{(1-c)^2(1+\beta\lambda)}{4(1+\beta)} > 0$$
$$\frac{(1-c)^2}{2}\left(\lambda - \lambda\theta^{CKZ} - \frac{1+\beta\lambda}{2(1+\beta)}\right) - c\theta^{CKZ} > 0$$
$$A_{COMP}^{CKZ}(1-c)^2 - c\theta^{CKZ} > 0$$

with $A_{COMP}^{CKZ} = \frac{1}{2}\left(\lambda - \lambda\theta^{CKZ} - \frac{(1+\beta\lambda)}{2(1+\beta)}\right)$. Furthermore, $A_{COMP}^{CKZ} > 0$ in order to get a solution for this inequality.

$$A_{COMP}^{CKZ} - 2A_{COMP}^{CKZ}c + A_{COMP}^{CKZ}c^2 - c\theta^{CKZ} > 0.$$

We solve this quadratic equation by using the reduced quadratic equation form,

$$A_{COMP}^{CKZ}c^2 + c(-2A_{COMP}^{CKZ} - \theta^{CKZ}) + A_{COMP}^{CKZ} = 0$$
$$A_{COMP}^{CKZ}c^2 + cB_{COMP}^{CKZ} + A_{COMP}^{CKZ} = 0,$$

with $B_{COMP}^{CKZ} = -2A_{COMP}^{CKZ} - \theta^{CKZ} < 0$. Thus,

$$c_{1,2} = \frac{-B_{COMP}^{CKZ} \pm \sqrt{{B_{COMP}^{CKZ}}^2 - 4{A_{COMP}^{CKZ}}^2}}{2A_{COMP}^{CKZ}}$$

$$= \frac{2A_{COMP}^{CKZ} + \theta^{CKZ} \pm \sqrt{\theta^{CKZ}(4A_{COMP}^{CKZ} + \theta^{CKZ})}}{2A_{COMP}^{CKZ}}$$

$$c_1 = 1 + \frac{\theta^{CKZ}}{2A_{COMP}^{CKZ}} + \frac{\sqrt{\theta^{CKZ}(4A_{COMP}^{CKZ} + \theta^{CKZ})}}{2A_{COMP}^{CKZ}} > 1$$

$$c_2 = 1 + \frac{\theta^{CKZ}}{2A_{COMP}^{CKZ}} - \frac{\sqrt{\theta^{CKZ}(4A_{COMP}^{CKZ} + \theta^{CKZ})}}{2A_{COMP}^{CKZ}}.$$

Hence, $c_1 > c_2$ and as c_1 is outside of the parameter range, it is not a feasible solution.

We set $D_{COMP}^{CKZ} = 1 + \frac{\theta^{CKZ}}{2A_{PAYW}} > 1$ for this examination. Thus, we get

$$c_2 = D_{COMP}^{CKZ} - \sqrt{{D_{COMP}^{CKZ}}^2 - 1} \geq 0$$

as a feasible solution as long as it does not exceed the upper threshold for costs ($c = 1$).

Thus, for CKZ PAYW is optimal if

$$0 < c < \min(1, c_2)$$

c_2 is equivalent to CKZ's critical threshold:

$$c^{CKZ,+} = 1 - \frac{\left(2\theta^{CKZ}(1+\beta) - 2\sqrt{\theta^{CKZ}(1+\beta)(\theta^{CKZ}(1+\beta)(1-2\lambda) + \lambda(2+\beta) - 1)}\right)}{1 - \lambda\left(2 + \beta - 2\theta^{CKZ}(1+\beta)\right)}. \quad (25)\ (\text{CKZ.6})\ (\text{CKZ.A.2})$$

Furthermore, by solving $A_{COMP}^{CKZ} > 0$ for λ, we can derive the minimum level of generosity that is needed for PAYW being better than PAAP.

$$\frac{1}{2}\left(\lambda - \lambda\theta^{CKZ} - \frac{(1+\beta\lambda)}{2(1+\beta)}\right) > 0$$

$$\lambda > \frac{1}{2 + \beta - 2\theta^{CKZ} - 2\beta\theta^{CKZ}}.$$

Thus, even when the share of freeloaders minimal, $\theta^{CKZ} = 0$, generosity must exceed $A_{COMP}^{CKZ} > \frac{1}{2+\beta}$ for PAYW to be preferred to PAAP.

Furthermore, this also allows us to define the maximum level of freeloaders by setting $A_{COMP}^{CKZ} > 0$ or $c^{CKZ,+} > 0$ and solving for θ^{CKZ}.

$$\frac{1}{2}\left(\lambda - \lambda\theta^{CKZ} - \frac{(1+\beta\lambda)}{2(1+\beta)}\right) > 0$$

$$\theta^{CKZ} < \frac{2\lambda + \beta\lambda - 1}{2\lambda + 2\beta\lambda}.$$

If we account for the maximum amount of generosity, $\lambda = 1$, we see that the highest tolerable level of freeloaders is

$$\theta^{CKZ} < \frac{1+\beta}{2+2\beta}$$

$$\theta^{CKZ} < \frac{1}{2}.$$

Thus, PAYW can only be better than PAAP if less than half of the consumers freeride.

3 Derivations of Proposition 2 (PAYW-MP)

In this part we first retrace CKZ's calculations with respect to PAYW with the minimum price (PAYW-MP) and show an inconsistency in the derivation of the properties of the minimum price. Second, we elaborate on the extension of PAYW-MP with disadvantageous inequity aversion.

This section introduces a few notational inconsistencies with respect to our symbolic conventions:

- We use $\underline{p}$ to denote the minimum price instead of the longer $p_{PAYW\text{-}MP}$.
- Auxiliary variables use a shortened form for PAYW-MP (i.e., MP).
- The upper part of the fair price r_{i0} ($r_i > c$) is denoted p_f, i.e., $p_f = \lambda r_i + (1-\lambda)c$.

3.1 Derivations and properties for Proposition 2 with $\beta = 0$

In this section, we retrace CKZ's calculations for PAYW-MP. In their paper, they only consider the special case of $\beta = 0$.

3.1.1 Derivation of PAYW-MP for $\beta = 0$

When ruling out disadvantageous inequity, consumer i's utility function will be given by

$$u_i = r_i - p_i - \gamma_i \max\{p_f - p_i, 0\}, \;\; \gamma_i \geq 0. \tag{26}$$

In case the consumer pays the seller's minimum price, $\underline{p}$, the consumer's utility would be given by

$$u_i = \begin{cases} r_i - \underline{p} & \underline{p} \geq p_f \\ r_i - \underline{p} - \gamma_i(p_f - \underline{p}) & p_f > \underline{p} \end{cases}. \tag{27}$$

We analyze consumption behavior in a stepwise manner:

1) When $\underline{p} > p_f$: Consumers with a positive utility will buy as

$$u_i = r_i - \underline{p} \geq 0$$
$$r_i \geq \underline{p}.$$

   Furthermore, they will pay their utility-maximizing price. As this is a linear function with a negative slope in $\underline{p}$, the extremum will be at the lower bound. Therefore, consumers will pay as little as possible, which is the minimum price, $\underline{p}$.

   All other consumers choose the outside option and will refrain from purchasing.

2) When $\underline{p} < p_f$: Whenever the consumer's fair price exceeds the minimum price and the consumer pays this minimum price, she will experience advantageous inequity aversion. However, the consumer can choose to set a higher price and evade disutility from advantageous inequity aversion. In this case, the consumer's utility will be similar to PAYW (9):

$$u_i = r_i - p_i - \gamma_i\,(p_f - p_i)$$
$$= r_i - \gamma_i p_f - (1 - \gamma_i)p_i.$$

   In maximizing this their utility with respect to the optimal price, we have a linear function again and, therefore, find a corner solution:

   - Consumers with $\gamma_i \leq 1, \frac{\partial u_i}{\partial p_i} \leq 0$, will pay as little as possible, which is the minimum price, and bear advantageous inequity aversion. Thus, their utility is given by

$$u_i = r_i - \underline{p} - \gamma_i\left(p_f - \underline{p}\right)$$

$$= r_i - \underline{p} - \gamma_i\left((\lambda r_i + (1-\lambda)c) - \underline{p}\right) \geq 0$$

$$r^+ = \frac{\underline{p}(1-\gamma_i) + c\gamma_i(1-\lambda)}{1-\gamma_i\lambda} \geq 0.$$

This is always lower than the $\underline{p}$ because

$$r^+ - \underline{p} = \frac{\underline{p}(1-\gamma_i - 1 + \gamma_i\lambda) + c\gamma_i(1-\lambda)}{1-\gamma_i\lambda} = \frac{\left(c - \underline{p}\right)\gamma_i(1-\lambda)}{1-\gamma_i\lambda} \leq 0,$$

as the seller will not set the minimum price below costs, $\underline{p} \geq c$.

Consumers can only obtain the good when paying at least the minimum price, $\underline{p}$. Therefore, only consumers with

$$r_i \geq \underline{p}$$

buy and pay $\underline{p}$. All others refrain from purchasing.

- For consumers with $\gamma_i > 1$, $\frac{\partial u_i}{\partial p_i} > 0$, therefore, their optimal price is the highest feasible price, i.e., the fair price. Thus, their utility will be given by

$$u_i = r_i - p_f$$

$$r_i - (\lambda r_i + (1-\lambda)c) \geq 0.$$

As shown before, this is always positive.

Thus, the buyer's utility is given by

$$u_i = \begin{cases} r_i - \underline{p} & \underline{p} \geq p_f \\ r_i - \underline{p} - \gamma_i\left(p_f - \underline{p}\right) & p_f > \underline{p} \wedge \gamma_i \leq 1. \\ r_i - p_i - \gamma_i(p_f - p_i) & p_f > \underline{p} \wedge \gamma_i > 1 \end{cases} \tag{28}$$

The critical consumption utility, $\underline{r}$, that distinguishes consumers who experience advantageous inequity aversion from those who do not (because of $\beta = 0$) can be derived by comparing the minimum price to the fair price and solving for r_i

$$\begin{aligned} &\underline{p} = p_f. \\ &\underline{p} = \lambda r_i + (1-\lambda)c \\ &r_i = \frac{\underline{p} - (1-\lambda)c}{\lambda} := \underline{r}. \end{aligned} \tag{29}$$

For reasons of simplification, we set

$$\underline{r} = \frac{\underline{p}}{\lambda} - cA_{MP}^{\beta=0}$$

with (30)

$$A_{MP}^{\beta=0} = \frac{1-\lambda}{\lambda}.$$

Using this notation, we can set up the profit function for the firm. First, we distinguish between consumers with $\gamma_i > 1$, $(1-\omega)$ in size and consumers with $\gamma_i \leq 1$, ω in size.

Note: we use ω instead of θ^{CKZ}. This follows from CKZ use of their θ^{CKZ} to describe all consumers with $\gamma_i \leq 1$ instead of freeloaders. However, in PAYW-MP the seller can exclude freeloaders. Thus, these consumers should be labeled less fair-minded consumers, ω.

As discussed before, consumers with $\gamma_i \leq 1$ pay the minimum price when $r_i \geq \underline{p}$ and refrain from purchasing otherwise. Consumers with $\gamma_i > 1$ pay their fair price when $r_i > \underline{r}$, the minimum price when $\underline{r} > r_i \geq \underline{p}$ and refrain from purchasing otherwise, $r_i < \underline{r}$. The firm incurs costs for all buying consumers. As before, we drop the index i when taking the seller's perspective. Thus,

$$\pi_{PAYW\text{-}MP}^{\beta=0} = (1-\omega)\left(\int_{\underline{p}}^{\underline{r}} \left(\underline{p}-c\right)\phi(r)dr + \int_{\underline{r}}^{1} (p_f - c)\phi(r)dr\right) + \omega \int_{\underline{p}}^{1} \left(\underline{p}-c\right)\phi(r)dr$$

$$= (1-\omega)\left(\int_{\underline{p}}^{\underline{r}} \left(\underline{p}-c\right)\phi(r)dr + \int_{\underline{r}}^{1} \lambda(r - c)\phi(r)dr\right) + \omega \int_{\underline{p}}^{1} \left(\underline{p}-c\right)\phi(r)dr.$$

To solve the integrals, we use the uniform distribution $\phi(r)$ for r. Thus, $\phi(r) = 1$.

Thus, we get

$$\pi_{PAYW-MP}^{\beta=0} = (1-\omega)\left(\left(\underline{p}-c\right)^2 A_{MP}^{\beta=0} + \frac{-\left(\underline{p}-c\right)^2}{2\lambda} + \frac{\lambda(1-c)^2}{2}\right) + \omega\left(\underline{p}-c\right)\left(1-\underline{p}\right)$$

$$= \underline{p}^2\left(\frac{B_{MP}^{\beta=0}}{2} - \omega\right) + \underline{p}\left(-cB_{MP}^{\beta=0} + \omega(1+c)\right)$$

$$+ \frac{(1-\omega)}{2}\left(\frac{1-2\lambda}{\lambda}c^2 + \lambda(1-c)^2\right) - c\omega$$

$$\pi_{PAYW-MP}^{\beta=0} = \frac{-\omega \underline{p}^2}{2\lambda D_{MP}^{\beta=0}} + \underline{p}\left(\frac{c\omega}{\lambda D_{MP}^{\beta=0}} + \omega(1-c)\right) + \frac{(1-\omega)}{2}\left(\frac{1-2\lambda}{\lambda}c^2 + \lambda(1-c)^2\right) - c\omega \tag{31}$$

with (CKZ.7)

$$B_{MP}^{\beta=0} = (1-\omega)\frac{(1-2\lambda)}{\lambda}$$
$$D_{MP}^{\beta=0} = \frac{\omega}{2\lambda+\omega-1}$$

and hence

$$B_{MP}^{\beta=0} - 2\omega = \frac{1-2\lambda-\omega}{\lambda} = -\frac{\omega}{\lambda D_{MP}^{\beta=0}}$$

The profit function is a quadratic/linear function in $\underline{p}$. As discussed before, the optimal minimum price cannot be below costs, $\underline{p} > c$. Furthermore, the minimum price must not be above the highest possible fair price,

$$p_f(r_i = 1) = \lambda + (1-\lambda)c = c + (1-c)\lambda \coloneqq p_h$$

which serves as an upper boundary for $\underline{p}$. Otherwise, $\pi_{PAYW-MP}^{\beta=0}$ will be incorrectly defined.

We observe two cases.

1) For $\frac{B_{MP}^{\beta=0}}{2} - \omega = 0$, the profit function is linear in $\underline{p}$. Note that $\frac{B_{MP}^{\beta=0}}{2} - \omega = 0$ implies $\omega = 1-2\lambda$ and, therefore, $B_{MP}^{\beta=0} = 2\omega$. Profits are linearly increasing in $\underline{p}$, the maximum is at the bound and the firm maximizes its profits by setting the highest possible price which is p_h. A higher price would yield infeasible results in the objective function. Thus,

$$\underline{p}_1^{\beta=0,*} = c + (1-c)\lambda. \tag{32}$$

2) For $\frac{B_{MP}^{\beta=0}}{2} - \omega \neq 0$, the objective function has a quadratic term. Taking the first derivative yields

$$\frac{\partial \pi_{PAYW-MP}^{\beta=0}}{\partial \underline{p}} = -\frac{\omega \underline{p}}{\lambda D_{MP}^{\beta=0}} + \frac{c\omega}{\lambda D_{MP}^{\beta=0}} + \omega(1-c) \overset{!}{=} 0 \tag{33}$$

$$\underline{p} = c + D_{MP}^{\beta=0}\lambda(1-c) \tag{AW.12}$$

$$= c + \frac{(1-c)\lambda\omega}{2\lambda + \omega - 1} \coloneqq \underline{p}_2^{\beta=0,*}.$$

Next, we need to check whether this is a local minimum or maximum

$$\frac{\partial^2 \pi_{PAYW-MP}^{\beta=0}}{\partial^2 \underline{p}} = B_{MP}^{\beta=0} - 2\omega \begin{cases} > 0 \rightarrow \text{Minimum} & \underline{p}_2^* < c \\ < 0 \rightarrow \text{Maximum} & \underline{p}_2^* > c \end{cases}.$$

Thus, for $B_{MP}^{\beta=0} > 0$ the price $\underline{p}_2^{\beta=0,*}$ would lead to minimal profits and the seller is better of charging p_h.

Thus, we can derive conditions for which $\underline{p}_2^{\beta=0,*}$ is the optimal price. First, the price must be below the highest possible price,

$$\underline{p}_2^{\beta=0,*} \leq p_h. \tag{34}$$

Second, the price must be a local maximum

$$\frac{\partial^2 \pi_{PAYW-MP}^{\beta=0}}{\partial^2 \underline{p}} < 0. \tag{35}$$

Solving (34) for λ gives us $\lambda \geq \frac{1}{2}$.

Furthermore, solving (35) for λ, we find $\lambda > \frac{1-\omega}{2}$. This is always true for $\lambda \geq \frac{1}{2}$. For $\frac{1-\omega}{2} < \lambda \leq \frac{1}{2}$, $\underline{p}_1^{\beta=0,*} = p_h$ as the optimal price.

We combine $\underline{p}_1^{\beta=0,*} = p_h$ and $\underline{p}_2^{\beta=0,*}$ in a single equation

$$\underline{p}^{\beta=0,*} = c + (1-c)E_{MP}^{\beta=0}\lambda$$

$$\text{where } E_{MP}^{\beta=0} = \begin{cases} 1 & \lambda \leq \frac{1}{2} \\ \omega/(2\lambda + \omega - 1) & \lambda > \frac{1}{2} \end{cases}. \tag{36}$$

Using the optimal minimum price, we can calculate the seller's optimal profits.

$$\pi^{\beta=0}_{PAYW\text{-}MP}\left(\underline{p}^{\beta=0,*}\right)$$

$$= \underline{p}^{\beta=0,*^2}\left(\frac{B^{\beta=0}_{MP}}{2} - \omega\right) + \underline{p}^{\beta=0,*}\left(-cB^{\beta=0}_{MP} + \omega(1+c)\right)$$

$$+ \frac{(1-\omega)}{2}\left(\frac{1-2\lambda}{\lambda}c^2 + \lambda(1-c)^2\right) - c\omega$$

$$= -\frac{1}{2}\underline{p}^{\beta=0,*^2}\left(2\omega - B^{\beta=0}_{MP}\right) + \underline{p}^{\beta=0,*}\left(c\left(2\omega - B^{\beta=0}_{MP}\right) + \omega(1-c)\right)$$

$$+ \frac{(1-\omega)}{2}\left(\frac{1-2\lambda}{\lambda}c^2 + \lambda(1-c)^2\right) - c\omega$$

$$= \underline{p}^{\beta=0,*}\left(2\omega - B^{\beta=0}_{MP}\right)\left(-\frac{\underline{p}^{\beta=0,*}}{2} + c\right) + \underline{p}^{\beta=0,*}\omega(1-c)$$

$$+ \frac{(1-\omega)}{2}\left(\frac{1-2\lambda}{\lambda}c^2 + \lambda(1-c)^2\right) - c\omega$$

$$= \left(c + (1-c)E^{\beta=0}_{MP}\lambda\right)\left(2\omega - B^{\beta=0}_{MP}\right)\left(-\frac{\left(c + (1-c)E^{\beta=0}_{MP}\lambda\right)}{2} + c\right)$$

$$+ \left(c + (1-c)E^{\beta=0}_{MP}\lambda\right)\omega(1-c) + \frac{(1-\omega)}{2}\left(\frac{1-2\lambda}{\lambda}c^2 + \lambda(1-c)^2\right) - c\omega$$

$$= \frac{1}{2}\left(c + (1-c)E^{\beta=0}_{MP}\lambda\right)\left(2\omega - B^{\beta=0}_{MP}\right)\left(c - (1-c)E^{\beta=0}_{MP}\lambda\right) + c\omega(1-c-1)$$

$$+ (1-c)^2\left(E^{\beta=0}_{MP}\lambda\omega + \frac{(1-\omega)\lambda}{2}\right) + \frac{(1-\omega)}{2}\left(\frac{1-2\lambda}{\lambda}c^2\right)$$

$$= \frac{1}{2}\left(c^2 - (1-c)^2\left(E^{\beta=0}_{MP}\right)^2\lambda^2\right)\left(2\omega - B^{\beta=0}_{MP}\right) - c^2\omega + (1-c)^2\left(E^{\beta=0}_{MP}\lambda\omega + \frac{(1-\omega)\lambda}{2}\right)$$

$$+ \frac{(1-\omega)}{2}\left(\frac{1-2\lambda}{\lambda}c^2\right)$$

$$= c^2\left(\omega - \frac{B^{\beta=0}_{MP}}{2} - \omega + \frac{(1-\omega)}{2}\frac{1-2\lambda}{\lambda}\right)$$

$$+ (1-c)^2\lambda\left(-\frac{\left(E^{\beta=0}_{MP}\right)^2\lambda}{2}\cdot\frac{2\lambda+\omega-1}{\lambda} + E^{\beta=0}_{MP}\omega + \frac{1-\omega}{2}\right).$$

For $E^{\beta=0}_{MP} = 1$, we get

$$\pi^{\beta=0}_{PAYW\text{-}MP}(c + (1-c)\lambda) = (1-c)^2\lambda\left(-\frac{2\lambda+\omega-1}{2} + \frac{1+\omega}{2}\right)$$
$$= (1-c)^2\lambda(1-\lambda) \coloneqq \pi^{\beta=0,*}_{PAYW\text{-}MP,1}. \quad (37)$$

For $E_{MP}^{\beta=0} = \frac{\omega}{(2\lambda+\omega-1)}$, we get

$$\begin{aligned}
\pi_{PAYW\text{-}MP}^{\beta=0}&\left(c + \frac{(1-c)\lambda\omega}{2\lambda+\omega-1}\right) \\
&= (1-c)^2\lambda\left(-\frac{\omega^2\lambda}{2}\frac{2\lambda+\omega-1}{\lambda(2\lambda+\omega-1)^2} + \frac{\omega^2}{(2\lambda+\omega-1)} + \frac{1-\omega}{2}\right) \\
&= (1-c)^2\lambda\left(\frac{\omega^2}{2(2\lambda+\omega-1)} + \frac{(1-\omega)(2\lambda+\omega-1)}{2(2\lambda+\omega-1)}\right) \\
&= \frac{(1-c)^2\lambda(2\lambda+2\omega(1-\lambda)-1)}{2(2\lambda+\omega-1)} := \pi_{PAYW\text{-}MP,2}^{\beta=0,*}.
\end{aligned} \tag{38}$$

Thus, we can summarize,

$$\pi_{PAYW\text{-}MP}^{\beta=0,*} = \begin{cases} (1-c)^2\lambda(1-\lambda) & \lambda \leq \frac{1}{2} \\ \frac{(1-c)^2\lambda(2\lambda+2\omega(1-\lambda)-1)}{2(2\lambda+\omega-1)} & \lambda > \frac{1}{2} \end{cases}. \tag{39}$$

3.1.2 Properties of PAYW-MP for $\beta = 0$

The derivation of the optimal minimum price allows us to determine the properties of PAYW-MP with $\beta = 0$.

As in CKZ, we observe how the optimal minimum price changes with respect to marginal costs, c, the share of less fair-minded consumers ω and generosity, λ. We proceed in a stepwise manner.

First, we consider $\lambda \leq \frac{1}{2}$,

$$\frac{\partial \underline{p}_1^{\beta=0,*}}{\partial c} = 1 - \lambda \geq 0$$

$$\frac{\partial \underline{p}_1^{\beta=0,*}}{\partial \omega} = 0$$

$$\frac{\partial \underline{p}_1^{\beta=0,*}}{\partial \lambda} = 1 - c > 0.$$

Second, we consider $\lambda > \frac{1}{2}$,

$$\frac{\partial \underline{p}_2^{\beta=0,*}}{\partial c} = 1 + \frac{\lambda\omega}{1-2\lambda-\omega} > 0$$

$$\frac{\partial \underline{p}_2^{\beta=0,*}}{\partial \omega} = \frac{(1-c)\lambda(2\lambda-1)}{(1-2\lambda-\omega)^2} > 0$$

$$\frac{\partial \underline{p}_2^{\beta=0,*}}{\partial \lambda} = \frac{(1-c)(-1+\omega)\omega}{(1-2\lambda-\omega)^2} < 0.$$

3.1.3 Excursus: CKZ's optimal prices and profits in PAYW-MP

CKZ present these calculations in their Proofs of Proposition 2 (derivation of the optimal price) and Proposition 4 (derivation optimal profits). These results are notationally somewhat different than our results above.

In their Proof to Proposition 2, they define profits as

$$\pi_{PAYW\text{-}MP}^{CKZ} = (1-\omega)\left(\frac{\lambda(1-c)^2}{2} + \frac{(1-2\lambda)\left(\underline{p}-c\right)^2}{2\lambda}\right) + \theta^{CKZ}\left(\underline{p}-c\right)\left(1-\underline{p}\right), \quad (40)$$

which corresponds to (31). This allows them to derive optimal minimum prices as

$$\begin{aligned} \underline{p}^{CKZ,*} &= \frac{(\theta^{CKZ}\lambda - (1-\theta^{CKZ}-2\lambda+\theta^{CKZ}\lambda)c)}{1-\theta^{CKZ}-2\lambda} \\ &= c - \frac{\theta^{CKZ}\lambda(1-c)}{1-2\lambda-\theta^{CKZ}}, \end{aligned} \quad (41)$$

which corresponds to (33). In the paper, they name this result $\underline{p}^*$ but later denote it $\underline{p}$. This is then used to define the optimal minimum price as

$$\underline{p}^{CKZ,*} = \max\left[c, \min\left[\underline{p}^{CKZ}, \lambda + (1-\lambda)c\right]\right]$$

with (42)

$$\underline{p}^{CKZ} = \frac{\theta^{CKZ}\lambda(1-c)}{2\lambda-1+\theta^{CKZ}} + c.$$

This result is similar to (36). However, the $\max/\min$ combination does not allow the reader to distinguish between high and low generosity settings.

Furthermore, they derive the optimal profits in PAYW-MP in their Proof to Proposition 4,

$$\pi_{PAYW\text{-}MP}^{CKZ,*} = \frac{(1-c)^2\lambda[1-2\theta^{CKZ}(1-\lambda)-2\lambda]}{2(1-\theta^{CKZ}-2\lambda)}. \quad (43)$$

As PAYW-MP allows the seller to exclude all freeloaders at no cost for $\beta = 0$, the misspecification of the freeloader segment discussed in Section 2.2 does not matter and $\theta^{CKZ} = \omega$, we can also write

$$\pi_{PAYW\text{-}MP}^{CKZ,*} = \frac{(1-c)^2\lambda[1-2\omega(1-\lambda)-2\lambda]}{2(1-\omega-2\lambda)}. \tag{44}$$

This corresponds to $\pi_{PAYW\text{-}MP,2}^{\beta=0,*}$, i.e., the profits for $\lambda > 1/2$. Unfortunately, results for $\lambda \leq 1/2$ are omitted by CKZ. This hinders an effective comparison.

3.1.4 Excursus: CKZ's properties of PAYW-MP

In their Poof of Proposition 2, CKZ observe properties of the minimum price by taking the derivatives with respect to c, ω (resp. θ^{CKZ}), and λ. However, they do so only with respect to

$$\underline{p}_2^* = c - \frac{\omega\lambda(1-c)}{1-2\lambda-\omega},$$

i.e., only for $\lambda > 1/2$. This leads to erroneous conclusions: In their Proposition 2, CKZ note "Furthermore, the optimal minimum price […] decreases with the generosity of fair-minded consumers λ ($\frac{\partial \underline{p}^*}{\partial\lambda} < 0$)" (CKZ, p. 784). However, this is only partially true. Even for their optimal price (42), they should get

$$\frac{\partial c}{\partial\lambda} = 0$$
$$\frac{\partial(\lambda+(1-\lambda)c)}{\partial\lambda} = 1-c > 0.$$

These finding changes how we need to look at the minimum price, as there are some cases in which a higher generosity allows a higher minimum price. Unfortunately, CKZ's notation does not allow us to determine conditions under which this is the case. However, as shown above (cf. Section 3.1.2) note that for $\lambda \leq \frac{1}{2}$, $\frac{\partial \underline{p}^*}{\partial\lambda} > 0$. Obviously, this not a borderline case but holds for half of the considered parameter range. In these cases, increasing generosity allows the seller to increase the minimum price.

3.2 Derivations and properties for Proposition 2 with $\beta \geq 0$

3.2.1 Derivation of PAYW-MP for $\beta \geq 0$

In case of $\beta \geq 0$, we allow for disutility from disadvantageous inequity aversion. Thus, the consumers utility function is given by (1).

In line with CKZ, we assume β_i to be the same for all consumers. Again, the minimum price might be higher or lower than the consumer's fair price. Thus, (1) can also be written as a piecewise function

$$u_i = \begin{cases} r_i - \underline{p} - \beta(\underline{p} - p_f) & \underline{p} > p_f \\ r_i - \underline{p} - \gamma_i\left(p_f - \underline{p}\right) & p_f > \underline{p} \wedge \gamma_i \leq 1. \\ r_i - p_i - \gamma_i(p_f - p_i) & p_f > \underline{p} \wedge \gamma_i > 1 \end{cases} \tag{45}$$

(AW.1d)

Thus, for consumers with $\underline{p} > p_f$, the purchase decision also depends on their disadvantageous inequity aversion. As in PAAP, we calculate the threshold for which the consumer buys as $u_i \geq 0$ and solve for r_i.

$$r_i - \underline{p} - \beta\left(\underline{p} - (\lambda r_i + (1-\lambda)c)\right) \geq 0$$
$$r_i \geq \frac{\underline{p} - c\beta + \underline{p}\beta + c\beta\lambda}{1 + \beta\lambda} := r^+. \tag{46}$$

(AW.9)

This gives us the critical r^+ for which the consumer still buys with the minimum price $\underline{p}$.

For further reference, we write more compactly,

$$r^+ = \underline{p} + A_{MP}\left(\underline{p} - c\right)$$
$$= \underline{p}(1 + A_{MP}) - A_{MP}c$$

with $A_{MP} = \frac{\beta(1-\lambda)}{1+\lambda\beta}$.

Thus, consumers with $r_i \geq r^+$ buy the product.

The derivations for consumers with $p_f > \underline{p}$ are equivalent to the derivations in Section 3.1.1. Thus, the profit function can be set up as,

$$\pi_{PAYW\text{-}MP} = (1-\omega)\left(\int_{r^+}^{\underline{r}}\left(\underline{p} - c\right)\phi(r)\,dr + \int_{\underline{r}}^{1}\lambda(r-c)\,\phi(r)dr\right)$$
$$+\,\omega\int_{r^+}^{1}\left(\underline{p} - c\right)\phi(r)\,dr. \tag{47}$$

(AW.10)

Again, we use the uniform distribution, $\phi(r) = 1$, for consumption utilties to solve the integrals

$$\pi_{PAYW\text{-}MP} = (1-\omega)\left(\left(\underline{p} - c\right)^2(B_{MP} - A_{MP}) - \frac{\left(\underline{p} - c\right)^2}{2\lambda} + \frac{\lambda(1-c)^2}{2}\right)$$
$$+\,\omega\left(\left(\underline{p} - c\right)\left(1 - \underline{p}(1 + A_{MP}) + A_{MP}c\right)\right)$$

$$= \underline{p}^2\left((1-\omega)\left(B_{MP}-A_{MP}-\frac{1}{2\lambda}\right)-\omega(1+A_{MP})\right)$$
$$+\underline{p}\left((1-\omega)\left(-2c(B_{MP}-A_{MP})+\frac{c}{\lambda}\right)+\omega(1+c(1+A_{MP})+A_{MP}c)\right)$$
$$+(1-\omega)\left(c^2\left(B_{MP}-A_{MP}-\frac{1}{2\lambda}\right)+\frac{\lambda}{2}(1-c)^2\right)+\omega c(-1-cA_{MP})$$
$$=\frac{\underline{p}^2}{2\lambda(1+\beta\lambda)}\left(-D_{MP}(1-\omega)-2\lambda\omega(1+\beta)\right)$$
$$+\frac{\underline{p}}{\lambda(1+\beta\lambda)}\left((1-\omega)cD_{MP}+E_{MP}\lambda+c\lambda\omega(1+2\beta-\beta\lambda)\right)+F_{MP}$$
$$=\frac{\underline{p}^2}{2\lambda(1+\beta\lambda)}(-D_{MP}-E_{MP})$$
$$+\frac{\underline{p}}{\lambda(1+\beta\lambda)}\left(D_{MP}c+E_{MP}\lambda+c\omega(1+\beta\lambda)(1-\lambda)\right)+F_{MP}$$

$$\pi_{PAYW\text{-}MP}=\frac{\underline{p}^2}{2\lambda(1+\beta\lambda)}(-D_{MP}-E_{MP})$$
$$+\frac{\underline{p}}{\lambda(1+\beta\lambda)}\left(D_{MP}c+E_{MP}\left(\lambda+c(1-\lambda)\right)\right)+F_{MP}$$

with (48)

$$B_{MP}=\frac{1-\lambda}{\lambda}=A_{MP}^{\beta=0} \qquad \text{(AW.A.6)}$$
$$D_{MP}=(2+\beta)\lambda-1$$
$$E_{MP}=\omega(1+\beta\lambda)$$
$$F_{MP}=(1-\omega)\left(\frac{-D_{MP}c^2}{2\lambda(1+\beta\lambda)}+\frac{\lambda(1-c)^2}{2}\right)+\omega c(-1-cA_{MP}).$$

To derive the optimal minimum price, we again distinguish between the linear and the quadratic case of $\pi_{PAYW\text{-}MP}$.

The profit, $\pi_{PAYW\text{-}MP}$, is linearly increasing in $\underline{p}$ if $D_{MP}+E_{MP}=0$. Hence optimal profits will be at the corners. Thus, the seller will set the highest possible fair price,

$$p_h=c+(1-c)\lambda:=\underline{p}_1^*. \qquad (49)$$

For$D_{MP}+E_{MP}\neq 0$, $\pi_{PAYW-MP}$ is a quadratic function. Thus, to find the global maximum,

$$\frac{\partial\pi_{PAYW\text{-}MP}}{\partial\underline{p}}\overset{!}{=}0$$

$$\begin{aligned}\underline{p} &= \frac{D_{MP}c + E_{MP}p_h}{D_{MP} + E_{MP}} \\ &= \frac{(D_{MP} + E_{MP})c + E_{MP}(-c + c + (1-c)\lambda)}{D_{MP} + E_{MP}} \\ &= c + \frac{E_{MP}(1-c)\lambda}{D_{MP} + E_{MP}} \coloneqq \underline{p}_2^*.\end{aligned} \tag{50}$$

Furthermore, we check the second derivative to find the local maximum

$$\frac{\partial^2 \pi_{PAYW-MP}}{\partial^2 \underline{p}} = \frac{-(D_{MP} + E_{MP})}{\lambda(1+\beta\lambda)} \begin{cases} > 0 & \rightarrow \text{Minimum} \quad \underline{p}_2^* < c \\ < 0 & \rightarrow \text{Maximum} \quad c \leq \underline{p}_2^* \end{cases}.$$

Thus, for $D_{MP} + E_{MP} < 0$, $\underline{p}_2^*$ will be a local minimum. As we are looking for a maximum, the seller chooses the highest feasible price $\underline{p}_1^* = p_h$.

For $D_{MP} + E_{MP} > 0$, we will find a local maximum. Thus, the seller will charge $\underline{p}_2^*$. However, the seller needs to take the upper boundary of p_h into consideration. Thus, $\underline{p}_2^*$ is only valid for

$$\underline{p}_2^* \leq p_h = c + (1-c)\lambda$$

$$c + \frac{E_{MP}(1-c)\lambda}{D_{MP} + E_{MP}} \leq c + (1-c)\lambda$$

$$\frac{E_{MP}}{D_{MP} + E_{MP}} \leq 1.$$

As $D_{MP} \geq 0$,

$$\lambda \geq \frac{1}{2+\beta}.$$

Hence $\underline{p}_2^*$ is valid only for high λ. Thus, for $\frac{1-\omega}{(2+\beta+\beta\omega)\lambda} < \lambda \leq \frac{1}{2+\beta}$ the optimal price is $\underline{p}_2^* = p_h$.

Next, we can summarize the optimal minimum price a single piecewise function:

$$\underline{p}^* = c + (1-c)G_{MP}\lambda \tag{51}$$

$$\text{with } G_{MP} = \begin{cases} 1 & \lambda \leq 1/(2+\beta) \\ E_{MP}/(D_{MP} + E_{MP}) & \lambda > 1/(2+\beta) \end{cases}. \quad \text{(AW.11)} \quad \text{(AW.A.7)}$$

Note that $\frac{E_{MP}}{(D_{MP}+E_{MP})} = \frac{\omega(1+\beta\lambda)}{(2+\beta(1+\omega))\lambda - 1 + \omega}$.

Finally, we compute the optimal profits under PAYW-MP.

$$
\begin{aligned}
&\pi_{PAYW\text{-}MP}\left(\underline{p}^*\right)\\
&= (1-c)^2\left((1-\omega)\left({G_{MP}}^2\lambda^2\left(B_{MP}-A_{MP}-\frac{1}{2\lambda}\right)+\frac{\lambda}{2}\right)\right.\\
&\left.+\,\omega G_{MP}\lambda\big(1-G_{MP}\lambda(1+A_{MP})\big)\right)\\
&= (1-c)^2\lambda\left((1-\omega)\left({G_{MP}}^2\frac{1-\lambda(2+\beta)}{2(1+\beta\lambda)}+\frac{1}{2}\right)\right.\\
&\left.+\,G_{MP}\omega\big(1-G_{MP}\lambda(1+A_{MP})\big)\right)\\
&= \frac{(1-c)^2\lambda}{2}\left(\frac{(1-\omega){G_{MP}}^2\big(1-\lambda(2+\beta)\big)-2\omega {G_{MP}}^2\lambda(1+\beta)}{1+\beta\lambda}+1-\omega\right.\\
&\left.+\,2G_{PAYW\text{-}MP}\omega\right)\\
&= \frac{(1-c)^2\lambda}{2}\left(\frac{{G_{MP}}^2}{1+\beta\lambda}\big(1-\omega-\lambda(2+\beta)+\lambda\omega(2+\beta)-2\lambda\omega(1+\beta)\big)+1\right.\\
&\left.+\,\omega(2G_{MP}-1)\right)\\
&= \frac{(1-c)^2\lambda}{2}\left(\frac{{G_{MP}}^2}{1+\beta\lambda}\big(1-\lambda(2+\beta)-\omega(1+\beta\lambda)\big)+1+\omega(2G_{MP}-1)\right)\\
&= \frac{(1-c)^2\lambda}{2}\left(\frac{{G_{MP}}^2\big(1-\lambda(2+\beta)\big)}{1+\beta\lambda}+1-\omega\big({G_{MP}}^2-2G_{MP}+1\big)\right)\\
&= \frac{(1-c)^2\lambda}{2}\left(\frac{{G_{MP}}^2\big(1-\lambda(2+\beta)\big)}{1+\beta\lambda}+1-\omega(G_{MP}-1)^2\right) \coloneqq \pi^*_{PAYW\text{-}MP}.
\end{aligned}
$$

We set $G_{MP} = 1$ and get

$$
\begin{aligned}
\pi_{PAYW\text{-}MP,1}\left(\underline{p}_1^*\right) &= \frac{(1-c)^2\lambda}{2}\left(\frac{\big(1-\lambda(2+\beta)\big)}{1+\beta\lambda}+1\right)\\
&= \frac{(1-c)^2\lambda(1-\lambda)}{1+\beta\lambda} \coloneqq \pi^*_{PAYW\text{-}MP,1}.
\end{aligned} \tag{52}
$$

For $G_{PAYW\text{-}MP} = \frac{E_{MP}}{(D_{MP}+E_{MP})}$, we get

$$\begin{aligned}\pi_{PAYW\text{-}MP,2}\left(\underline{p}_2^*\right) &= \frac{(1-c)^2\lambda}{2}\left(\frac{-D_{MP}(E_{MP}/(D_{MP}+E_{MP}))^2}{E_{MP}/\omega}\right.\\ &\qquad\left. + 1 - \omega\left(\frac{E_{MP}-D_{MP}-E_{MP}}{D_{MP}+E_{MP}}\right)^2\right)\\ &= \frac{(1-c)^2\lambda}{2}\left(\frac{-D_{MP}\omega}{D_{MP}+E_{MP}}+1\right)\\ &= \frac{(1-c)^2\lambda}{2}\left(\frac{(1-(2+\beta)\lambda)\omega}{\big(2+\beta(1+\omega)\big)\lambda-1+\omega}+1\right)\\ &= \frac{(1-c)^2\lambda}{2}\left(\frac{\lambda(2+\beta)+2\omega(1-\lambda)-1}{\big(2+\beta(1+\omega)\big)\lambda-1+\omega}\right) \coloneqq \pi^*_{PAYW\text{-}MP,2}.\end{aligned} \tag{53}$$

Thus, the optimal profits are given by

$$\pi^*_{PAYW-MP} = \begin{cases} \dfrac{(1-c)^2\lambda(1-\lambda)}{1+\beta\lambda} & \lambda \leq 1/(2+\beta) \\ \dfrac{(1-c)^2\lambda}{2}\left(\dfrac{\lambda(2+\beta)+2\omega(1-\lambda)-1}{\big(2+\beta(1+\omega)\big)\lambda-1+\omega}\right) & \lambda > 1/(2+\beta) \end{cases}. \tag{54}\ \text{(AW.A.8)}$$

3.2.2 Properties of PAYW-MP for $\beta \geq 0$

3.2.2.1 Properties of the optimal minimum price $\underline{p}^*$

We derive the derivations of $\underline{p}^*$ with respect to the model parameters c, β, ω and λ.

First, we consider $G_{MP} = 1$.

$$\frac{\partial \underline{p}_1^*}{\partial c} = 1 - \lambda > 0$$

$$\frac{\partial \underline{p}_1^*}{\partial \beta} = 0$$

$$\frac{\partial \underline{p}_1^*}{\partial \omega} = 0$$

$$\frac{\partial \underline{p}_1^*}{\partial \lambda} = (1-c) > 0.$$

Second, we consider $G_{MP} = \frac{E_{MP}}{(D_{MP}+E_{MP})} = \frac{\omega(1+\beta\lambda)}{\big(2+\beta(1+\omega)\big)\lambda-1+\omega}$.

$$\frac{\partial \underline{p}_2^*}{\partial c} = \frac{D_{MP}+E_{MP}(1-\lambda)}{D_{MP}+E_{MP}} > 0$$

$$\frac{\partial \underline{p}_2^*}{\partial \beta} = \frac{-\lambda^2 \omega (1-c)(3+\beta\lambda-2\lambda)}{(D_{MP}+E_P)^2} < 0$$

$$\frac{\partial \underline{p}_2^*}{\partial \omega} = \frac{(1+\beta\lambda)(1-c)\lambda}{(D_{MP}+E_{MP})^2}(\lambda(2+\beta)-1) > 0$$

$$\frac{\partial \underline{p}_2^*}{\partial \lambda} = \frac{(1-c)\left(-2\omega\lambda(1+\beta)+D_{MP}E_{MP}+{E_{MP}}^2\right)}{(D_{MP}+E_{MP})^2}$$

$$= \frac{(1-c)\left(-2\lambda\omega-2\beta\lambda\omega+D_{MP}\omega+D_{MP}\beta\lambda\omega+{E_{MP}}^2\right)}{(D_{MP}+E_{MP})^2}$$

$$= \frac{(1-c)\left(\omega(D_{MP}-2\lambda)+\beta\lambda\omega(D_{MP}-2)+{E_{MP}}^2\right)}{(D_{MP}+E_{MP})^2}$$

$$= \frac{(1-c)(\omega(\beta\lambda-1)-\beta\lambda\omega+\beta\lambda\omega(a-1)+b^2)}{(D_{MP}+E_{MP})^2}$$

$$= \frac{(1-c)\left(-\omega+\beta\lambda\omega(D_{MP}-1)+{E_{MP}}^2\right)}{(D_{MP}+E_{MP})^2}.$$

To determine conditions for $\frac{\partial \underline{p}^*}{\partial \lambda}$ to be positive or negative, we need to consider the nominator as a function of $\beta, \lambda,$ and ω,

$$f(\beta,\lambda,\omega) = (1-c)\left(-\omega+\beta\lambda\omega(D_{MP}-1)+{E_{MP}}^2\right)$$

$$= (1-c)\left(-\omega+\beta\lambda\omega\left(\left((2+\beta)\lambda-1\right)-1\right)+\left(\omega(1+\beta\lambda)\right)^2\right)$$

$$= (1-c)\left(-\omega+\omega\beta\lambda\left((2+\beta)\lambda-2\right)+\omega^2(1+\beta\lambda)^2\right).$$

Thus, after cancelling out two terms that are always positive, $(1-c)$ and ω, we obtain:

$$f(\beta,\lambda,\omega) = \left(-1+\beta\lambda\left((2+\beta)\lambda-2\right)+\omega(1+\beta\lambda)^2\right).$$

This function f is continuous in β, λ, and ω and increases monotonically with β, λ, and ω. Therefore, we observe the sign of f at the boundaries of λ ($\frac{1}{\beta+2} \leq \lambda \leq 1$). For the lower boundary, we find

$$f\left(\beta,\frac{1}{\beta+2},\omega\right) = (1+\beta\lambda)\left(-1+\omega(1+\beta\lambda)\right).$$

As $(1+\beta\lambda)$ is always positive, we must inspect $\left(-1+\omega(1+\beta\lambda)\right)$ only. Solving this term for β and ω we find that the term is positive if

$$\omega > 1/(1+\beta\lambda)$$

$$\beta > \frac{1-\omega}{\lambda\omega}.$$

For the upper boundary, we find

$$f(\beta, 1, \omega) = (1+\omega)\left(\beta - \frac{1-\omega}{1+\omega}\right)(\beta+1),$$

As $(1+\omega)$ and $(\beta+1)$ are always positive, we must inspect $\left(\beta - \frac{1-\omega}{1+\omega}\right)$ only. This will be positive for large β and large ω, i.e.,

$$\omega > \frac{1-\beta}{1+\beta}$$

$$\beta > \frac{1-\omega}{1+\omega}.$$

3.2.2.2 Properties of the optimal profits $\pi^*_{PAYW-MP}$

We examine the profits with respect to model variables β, λ.

First, we concentrate on disadvantageous inequity aversion, β, and inspect $\lambda \leq 1/(2+\beta)$. Thus, take the first derivative,

$$\frac{\partial \pi^*_{PAYW-MP}}{\partial \beta} = \frac{(1-c)^2\lambda}{2} \cdot \frac{2(\lambda-1)\lambda}{(1+\beta\lambda)^2} \leq 0$$

which is monotonically decreasing in β. This also allows to inspect critical values,

$$\pi^*_{PAYW-MP}(\beta=0) = (1-c)^2\lambda(1-\lambda)$$

$$\lim_{\beta\to\infty} \pi^*_{PAYW-MP} = \frac{(1-c)^2\lambda}{2} \cdot 0 = 0.$$

Second, we turn to $\lambda > 1/(2+\beta)$ and take the first derivative,

$$\frac{\partial \pi^*_{PAYW-MP}}{\partial \beta} =$$

$$\frac{(1-c)^2\lambda}{2} \cdot \frac{\Big(\big(2+\beta(1+\omega)\big)\lambda - 1 + \omega\Big)\lambda - \big(\lambda(2+\beta) + 2\omega(1-\lambda) - 1\big)\lambda(1+\omega)}{\Big(\big(2+\beta(1+\omega)\big)\lambda - 1 + \omega\Big)^2}$$

$$= \frac{(1-c)^2\lambda}{2} \frac{\lambda}{\Big(\big(2+\beta(1+\omega)\big)\lambda - 1 + \omega\Big)^2} \Big(2\lambda + \lambda\beta(1+\omega) - 1 + \omega - 2\lambda(1+\omega)$$

$$- \lambda\beta(1+\omega) - \big(2\omega(1-\lambda) - 1\big)(1+\omega)\Big)$$

$$= \frac{(1-c)^2\lambda^2}{2\Big(\big(2+\beta(1+\omega)\big)\lambda - 1 + \omega\Big)^2} \big(-2\lambda\omega - 1 + \omega - 2\omega(1-\lambda)(1+\omega) + 1 + \omega\big)$$

$$= \frac{(1-c)^2\lambda^2}{2\left(\left(2+\beta(1+\omega)\right)\lambda - 1 + \omega\right)^2}\left(2\omega(-\lambda+1) - 2\omega(1-\lambda)(1+\omega)\right)$$

$$= \frac{(1-c)^2\lambda^2}{2\left(\left(2+\beta(1+\omega)\right)\lambda - 1 + \omega\right)^2} 2\omega(1-\lambda)(-\omega) \leq 0.$$

Which is monotonically decreasing. Once again, we observe the boundaries.

$$\pi^*_{PAYW-MP}(\beta = 0) = \frac{(1-c)^2\lambda}{2} \cdot \frac{2\lambda + 2\omega(1-\lambda) - 1}{2\lambda - 1 + \omega}$$

$$\lim_{\beta\to\infty} \pi^*_{PAYW-MP} = \frac{(1-c)^2\lambda}{2} \cdot \frac{1}{1+\omega}.$$

Now, we turn to λ.

First, $\lambda \leq 1/(2+\beta)$,

$$\frac{\partial \pi^*_{PAYW-MP}}{\partial \lambda} = \frac{(1-c)^2}{(1+\beta\lambda)^2}\left((1+\beta\lambda)(1-2\lambda) - (\lambda - \lambda^2)\beta\right)$$

$$= \frac{(1-c)^2}{(1+\beta\lambda)^2}(1 - 2\lambda - \beta\lambda^2)$$

$$= \frac{(1-c)^2}{(1+\beta\lambda)^2}\left(1 - \lambda(2+\beta)\right) \geq 0.$$

Profits are increasing in λ.

Second, $\lambda > 1/(2+\beta)$,

$$\pi^*_{PAYW-MP} = \frac{H_{MP}\lambda(I_{MP}\lambda - 2\lambda\omega + 2\omega - 1)}{I_{MP}\lambda + \beta\lambda\omega - 1 + \omega}$$

$$= \frac{H_{MP}(J_{MP}\lambda^2 + L_{MP}\lambda)}{K_{MP}\lambda + M_{MP}}$$

with $H_{MP} = \frac{(1-c)^2}{2} > 0$, $I_{MP} = (2+\beta)$, $J_{MP} = b - 2\omega > 0$, $K_{MP} = b + \beta\omega > 0$, $L_{MP} = 2\omega - 1$, $M_{MP} = \omega - 1$.

this allows us to take the derivative,

$$\frac{\partial \pi^*_{PAYW-MP}}{\partial \lambda} = H_{MP}\left(\frac{(K_{MP}\lambda + M_{MP})(2J_{MP}\lambda + L_{MP})}{(K_{MP}\lambda + M_{MP})^2} - \frac{(J_{MP}\lambda^2 + L_{MP}\lambda)K_{MP}}{(K_{MP}\lambda + M_{MP})^2}\right)$$

$$= \frac{H_{MP}}{(K_{MP}\lambda + M_{MP})^2}(J_{MP}K_{MP}\lambda^2 + 2J_{MP}M_{MP}\lambda + L_{MP}M_{MP}).$$

The first term, $\frac{H_{MP}}{(K_{MP}\lambda+M_{MP})^2}$ is always positive. The second term $(J_{MP}K_{MP}\lambda^2 + 2J_{MP}M_{MP}\lambda + L_{MP}M_{MP})$ is positive for $\lambda > 1/(2+\beta)$, as the term takes the form of a U-shaped quadratic function that is open at the top as $J_{MP}K_{MP} > 0$. As the vertex is in the region $\lambda \leq 1/(2+\beta)$ as $\frac{\partial \pi^*_{PAYW-MP}}{\partial \lambda}\left(\lambda = \frac{1}{2+\beta}\right) > 0$, as

$$\frac{H_{MP}}{(K_{MP}\lambda + M_{MP})^2} > 0$$

$$(J_{MP}K_{MP}\lambda^2 + 2J_{MP}M_{MP}\lambda + L_{MP}M_{MP})_{\lambda=1/(2+\beta)}$$
$$= 1 - \frac{2\omega}{2+\beta} + \frac{\beta\omega}{2+\beta} - \frac{2\beta\omega^2}{(2+\beta)^2} + \omega - 1 + \frac{2\beta\omega^2}{2+\beta} - \frac{2\beta\omega}{2+\beta}$$
$$= \frac{\omega}{2+\beta}(-2+\beta+2+\beta-2\beta) + \frac{2\beta\omega^2}{(2+\beta)^2}(-1+2+\beta)$$
$$= \frac{2\beta\omega^2}{(2+\beta)^2}(\beta+1) \geq 0.$$

Thus, $\frac{\partial \pi^*_{PAYW-MP}}{\partial \lambda} > 0$ for $\lambda > 1/(2+\beta)$.

For a better understanding of the profit function, we consider critical thresholds. With respect to λ, for $\lambda \leq \frac{1}{2+\beta}$,

$$\pi^*_{PAYW-MP}(\lambda = 0) = 0$$
$$\pi^*_{PAYW-MP}\left(\lambda = \frac{1}{2+\beta}\right) = \frac{(1-c)^2}{2(2+\beta)},$$

and for $\lambda > \frac{1}{2+\beta}$, we find

$$\pi^*_{PAYW\text{-}MP}\left(\lambda = \frac{1}{2+\beta}\right) = \frac{(1-c)^2}{2(2+\beta)}$$
$$\frac{(1-c)^2}{3(1+\omega)} \leq \pi^*_{PAYW\text{-}MP}\left(\lambda = \frac{2}{3}\right) \leq \frac{(1-c)^2(1+2\omega)}{3(1+3\omega)}$$
$$\pi^*_{PAYW\text{-}MP}(\lambda = 1) = \frac{(1-c)^2}{2(1+\omega)}.$$

Furthermore, with respect to ω,

$$\pi^*_{PAYW\text{-}MP}(\omega = 0) = \frac{(1-c)^2\lambda}{2}.$$

Thus, optimal profits are in the range of

$$0 \leq \pi^*_{PAYW-MP} \leq (1-c)^2\lambda(1-\lambda) \leq \frac{(1-c)^2}{4} \qquad \lambda \leq \frac{1}{2} \tag{55}$$

$$\frac{(1-c)^2\lambda}{2} \cdot \frac{1}{1+\omega} \leq \pi^*_{PAYW-MP} \leq \frac{(1-c)^2\lambda}{2} \cdot \frac{2\lambda + 2\omega(1-\lambda) - 1}{2\lambda - 1 + \omega} \qquad \lambda > \frac{1}{2}. \tag{AW.A.9}$$

4 Derivations of Proposition 3 (PAYW-SP)

The setup of CKZ and AW in PAYW-SP differ with respect to the freeloader segment, i.e., the misspecification discussed in Section 2.2. The more nuanced discussion of the behavior of less fair-minded consumers in AW adds computational complications. Therefore, we start by retracing CKZ's solutions followed by a discussion of AW's solutions.

4.1 CKZ's derivations and properties of Proposition 3

4.1.1 Derivation of PAYW-SP for CKZ

To point to the difference between AW and CKZ which stems from the misspecification of the freeloader segment, we use θ^{CKZ} instead of ω in this subsection.

In PAYW-SP, the consumer cannot be forced to pay something. With a probability of z, the consumer behaves exactly as in PAYW. However, with a probability of $(1-z)$ the consumer adapts the suggested price[1] p_s as her fair price. However, if the perceived price, p_f is higher than p_s, the consumers will continue to hold p_f as their fair price.

Thus, the perceived fair price in PAYW-SP is (with a probability of $(1-z)$)

$$p_{f,s} = \begin{cases} p_s & p_s^u \geq p_f \\ p_f & p_s^u < p_f \end{cases}. \tag{56}$$

The variable p_s^u corresponds to p_s, however, the superscript u serves to distinguish the consumer's evaluation of the suggested price from the seller set suggested price.

As before, the consumer will not experience disadvantageous inequity and the consumers utility simplifies to

$$u_i = r_i - p_i - \gamma_i(p_{f,s} - p_i). \tag{57}$$

Maximizing this utility function, we once again find a corner solution. The consumer's price can be the maximum or the minimum depending on γ_i.

[1] Note that we use the subscript s instead of $PAYW\text{-}SP$ for compactness.

For $\gamma_i \leq 1$, the consumer maximizes her utility function by paying as little as possible. As negative prices are not feasible, we observe a corner solution: $p_i^* = 0$. As CKZ misinterpret the freeloader segment they assume that all consumers buy. However, as shown below this is not correct. Just as in PAYW, some consumers will drop out of the market.

For $\gamma_i > 1$, the consumer maximizes her utility by paying the highest feasible price. Prices above $p_{f,s}$ create disadvantageous inequity which is not captured by the utility function (57). Thus, for $p_s^u \geq p_f$ the consumer pays the maximum feasible price which is $p_i^* = p_s$. For $p_s^u < p_f$ the consumer again pays her maximum feasible price which is $p_i^* = p_f = \lambda r_i + (1-\lambda)c$. When paying less she would experience disutility from advantageous inequity aversion that outweighs the utility gain from the lower price.

Consumers will only buy if $u_i \geq 0$.

For $p_s \geq p_f$,

$$\begin{aligned} u_i &\geq 0 \\ r_i - p_s - \gamma_i(p_s - p_s) &\geq 0 \\ r_i &\geq p_s. \end{aligned}$$

Thus, as the consumption utility exceeds the suggested price, this consumer segment buys and pays p_s.

For $p_s < p_f$, we are finding the marginal consumer (i.e., the last consumer that still buys) by setting the utility function to zero and solving for r_i,

$$\begin{aligned} u_i &\geq 0 \\ r_i - p_f - \gamma_i\left(p_f - p_f\right) &\geq 0 \\ r_i &\geq \lambda r_i + (1-\lambda)c \\ r_i &\geq \frac{(1-\lambda)c}{1-\lambda} \\ r_i &> c. \end{aligned}$$

This is always the case, as the seller will never suggest a price that is below costs and $p_f > p_s \geq c$ holds in this case. Thus, these consumers always buy and pay p_f.

To distinguish these two cases, we set $p_f \geq p_s$ and solve for r_i

$$\begin{aligned} p_f &\geq p_s \\ \lambda r_i + (1-\lambda)c &\geq p_s \end{aligned}$$

$$r_i > \frac{p_s - (1-\lambda)c}{\lambda} := r_s.$$

Thus, consumers with $\gamma_i > 1$ will pay the suggested price if $p_s \leq r \leq r_s$ and the fair price if $r_s < r \leq 1$.

However, if $p_s > p_h$, nobody will pay more than the suggested price. Therefore, for the analysis of the suggested price, we must distinguish two cases,

1) $p_s < p_h$ and
2) $p_s \geq p_h$.

Before turning to the profit calculations, we need to consider an additional assumption for consumers $c < r \leq p_s^u$, as introduced by CKZ. They assume that disadvantageous inequity aversion is not possible in PAYW-SP. Still, these consumers with low consumption utilities will accept the suggested price only with probability $\frac{1}{2}$ and will set the fair price at r also with probability $\frac{1}{2}$. This gives the following fair price.

$$p_{f,\,s,\;c<r\leq p_s^u} = \begin{cases} p_s & \text{with probability } 1/2 \\ r_i & \text{with probability } 1/2 \end{cases}. \tag{58}$$

The corresponding utility is given by

$$u_i = r_i - p_i - \gamma_i\left(p_{f,\,s,\;c<r\leq p_s^u} - p_i\right). \tag{59}$$

For consumers who set p_s as their fair price, the above specifications hold. As postured by CKZ, we let consumers with $\gamma_i \leq 1$ buy. Consumers with $\gamma_i > 1$ will refrain from purchasing as

$$\begin{aligned} u_i &> 0 \\ r_i - p_s - \gamma_i(p_s - p_s) &> 0 \\ r_i &> p_s, \end{aligned}$$

a contradiction as we are in the region $c < r \leq p_s^u$.

For a consumer who sets r_i as her fair price, maximizing her utility,

$$u_i = r_i - p_i - \gamma_i(r_i - p_i),$$

yields another linear function. Thus, the maximum is at the bounds. This can be positive or negative depending on γ_i.

For $\gamma_i \leq 1$ the consumer maximizes her utility function by paying as little as possible. As negative prices are not feasible, we observe a corner solution: $p_i^* = 0$. The consumer will always buy as

$$u_i \geq 0$$
$$r_i - 0 - \gamma_i(r_i - 0) \geq 0$$
$$r_i(1 - \gamma_i) \geq 0.$$

For $\gamma_i > 1$ the consumer maximizes her utility by paying as much as possible. Prices above $p_{f,\,s,\;c<r\leq p_s^u} = r_i$ create disadvantageous inequity which means that our utility function would be invalid. Thus, $p_i^* = r_i$. These consumers will buy as the utility

$$u_i \geq 0$$
$$r_i - r_i - \gamma_i(r_i - r_i) \geq 0,$$

is positive.

Now, we can turn to CKZ's profit calculation. To retrace, we adopt their assumption assumption that all less fair-minded consumers freeload ($\omega = \theta^{CKZ}$).

First, we turn to $p_s < p_h$,

$$\pi_{PAYW\text{-}SP,1}^{CKZ} = (1-\theta^{CKZ})(1-z)\left(\frac{1}{2}\int_c^{p_s}(r-c)\phi(r)dr + \int_{p_s}^{r_s}(p_s-c)\phi(r)dr \right.$$
$$\left. + \int_{r_s}^{1}\left(p_f-c\right)\phi(r)dr\right) + (1-\theta^{CKZ})z\int_c^1\left(p_f-c\right)\phi(r)dr - \theta^{CKZ}c$$
$$= (1-\theta^{CKZ})(1-z)\left(\frac{1}{2}\int_c^{p_s}(r-c)dr + \int_{p_s}^{r_s}(p_s-c)dr \;+ \int_{r_s}^{1}\left(p_f-c\right)dr\right)$$
$$+ (1-\theta^{CKZ})z\int_c^1\left(p_f-c\right)dr - \theta^{CKZ}c$$
$$= (1-\theta^{CKZ})(1-z)\left(\frac{1}{2}\int_c^{p_s}(r-c)dr + \int_{p_s}^{r_s}(p_s-c)dr \;+ \int_{r_s}^{1}\left((\lambda r + (1-\lambda)c) - c\right)dr\right)$$
$$+ (1-\theta^{CKZ})z\int_c^1\left((\lambda r + (1-\lambda)c) - c\right)dr - \theta^{CKZ}c$$
$$= (1-\theta^{CKZ})(1-z)\left(\frac{1}{2}\int_c^{p_s}(r-c)dr + \int_{p_s}^{r_s}(p_s-c)dr \;+ \int_{r_s}^{1}\lambda(r-c)dr\right)$$
$$+ (1-\theta^{CKZ})z\int_c^1\lambda(r-c)dr - \theta^{CKZ}c$$

$$= (1-\theta^{CKZ})(1-z)\left(\frac{1}{2}\left(\frac{p_s^2}{2} - cp_s + \frac{c^2}{2}\right) + \left(\frac{1-\lambda}{\lambda}(p_s - c)^2\right)\right.$$

$$\left. + \left(\lambda\left(\frac{1}{2} - c + \frac{1}{2\lambda^2}(-p_s^2 + 2cp_s - (1-\lambda^2)c^2)\right)\right)\right)$$

$$+ (1-\theta^{CKZ})z\left(\frac{\lambda}{2} - c\lambda + \frac{c^2\lambda}{2}\right) - \theta^{CKZ}c$$

$$= (1-\theta^{CKZ})(1-z)\left(p_s^2\frac{-3\lambda+2}{4\lambda} + cp_s\frac{3\lambda-2}{2\lambda} + \frac{c^2}{4} + \frac{\lambda}{2} - c\lambda + \frac{(1-\lambda)^2c^2}{2\lambda}\right)$$

$$+ (1-\theta^{CKZ})z\lambda\frac{1}{2}(1-c)^2 - \theta^{CKZ}c.$$

To find the optimal suggested price, we take the first derivative with respect to p_s set this to zero and solve for p_s

$$\frac{\partial\pi_{PAYW\text{-}SP,1}^{CKZ}}{\partial p_s} = (1-z)(1-\theta^{CKZ})\left(\frac{p_s(2-3\lambda)}{2\lambda} + \frac{c(-2+3\lambda)}{2\lambda}\right) \overset{!}{=} 0$$

$$p_s = c \coloneqq p_{s,1}^{CKZ^*}. \tag{60}$$

To check whether this is a minimum or a maximum, we need to check the second derivative,

$$\frac{\partial^2\pi_{PAYW-SP,1}^{CKZ}}{\partial(p_s)^2} = \frac{(1-z)(1-\theta^{CKZ})(2-3\lambda)}{2\lambda}.$$

As $(1-z)$ and $(1-\omega)$ are always positive, we must check $\frac{(2-3\lambda)}{2\lambda}$.

$$\frac{(2-3\lambda)}{2\lambda}\begin{cases} > 0 \rightarrow \text{Minimum} & \lambda \leq \frac{2}{3} \\ < 0 \rightarrow \text{Maximum} & \lambda > \frac{2}{3} \end{cases}.$$

Thus, if $\lambda < \frac{2}{3}$, the optimal suggested price should be the minimum, i.e., c. Otherwise, the suggested price should be the maximum, i.e., 1. Yet, because of $p_s < p_h$ suggested prices above p_h are not feasible. However, CKZ do not account for this solution. Thus, their optimal profit is given by

$$
\begin{aligned}
\pi_{PAYW\text{-}SP,1}^{CKZ}&\left(p_s = p_{s,1}^{CKZ^*}\right) \\
&= \frac{1}{2}(1-c)^2(1-z)(1-\theta^{CKZ})\lambda + \frac{1}{2}(1-c)^2 z\lambda(1-\theta^{CKZ}) \\
&- c\theta^{CKZ} \\
&= \frac{1}{2}(1-c)^2(1-\theta^{CKZ})\lambda - c\theta^{CKZ} \coloneqq \pi_{PAYW-SP,1}^{CKZ^*}.
\end{aligned}
\tag{61}
$$

Now, we turn to $p_s \geq p_h$. In this case, no consumer pays more than the suggested price. Thus,

$$
\begin{aligned}
\pi_{PAYW\text{-}SP,2}^{CKZ} &= (1-\theta^{CKZ})(1-z)\left(\frac{1}{2}\int_c^{p_s}(r-c)\phi(r)dr + \int_{p_s}^1 (p_s - c)\phi(r)dr\right) \\
&+ (1-\theta^{CKZ})z\int_c^1 \left(p_f - c\right)\phi(r)dr - \theta^{CKZ}c \\
&= (1-\theta^{CKZ})(1-z)\left(-c + \frac{c^2}{4} + \left(1+\frac{c}{2}\right)p_s - \frac{3p_s^2}{4}\right) + (1-\theta^{CKZ})z\lambda\frac{1}{2}(1-c)^2 - \theta^{CKZ}c.
\end{aligned}
$$

We take the first derivative and solve for p_s

$$
\frac{\partial\pi_{PAYW\text{-}SP,2}^{CKZ}}{\partial p_s} = (1-\theta^{CKZ})(1-z)\left(1 + c - 2p_s + \frac{1}{2}(-c+p_s)\right) \overset{!}{=} 0
$$

$$
p_s = \frac{2+c}{3} \coloneqq p_{s,2}^{CKZ^*}. \tag{62}
$$

We check the second derivative to see whether this is a maximum or a minimum,

$$
\frac{\partial^2\pi_{PAYW\text{-}SP,2}^{CKZ}}{\delta(p_s)^2} = -\frac{3}{2}(1-z)(1-\theta^{CKZ}).
$$

This is always negative. Thus, $p_{s,2}^{CKZ^*}$ is a maximum.

$$
\begin{aligned}
\pi_{PAYW\text{-}SP,2}^{CKZ}&\left(p_s = p_{s,2}^{CKZ^*}\right) \\
&= (1-z)(1-\theta^{CKZ})\left(-c + \frac{c^2}{4} + \frac{1}{3}\left(1+\frac{c}{2}\right)(2+c) - \frac{1}{12}(2+c)^2\right) \\
&+ (1-\theta)z\frac{1}{2}(1-c)^2\lambda - c\theta^{CKZ} \\
&= \frac{1}{3}(1-c)^2(1-z)(1-\theta^{CKZ}) + \frac{1}{2}(1-c)^2 z(1-\theta)\lambda - c\theta^{CKZ} \\
&= \frac{1}{6}(1-c)^2(1-\theta^{CKZ})\left(2 + z(-2+3\lambda)\right) - c\theta^{CKZ} \coloneqq \pi_{PAYW\text{-}SP,2}^{CKZ^*}.
\end{aligned}
\tag{63}
$$

To find whether the firm should charge $p_{S,1}^{CKZ^*}$ or $p_{S,2}^{CKZ^*}$ we check when $\pi_{PAYW\text{-}SP,1}^{CKZ^*}$ is higher than $\pi_{PAYW\text{-}SP,2}^{CKZ^*}$.

$$\pi_{PAYW-SP,1}^{CKZ^*} - \pi_{PAYW-SP,2}^{CKZ^*} > 0$$

$$\frac{1}{2}(1-c)^2(1-z)(1-\theta^{CKZ})\lambda + \frac{1}{2}(1-c)^2 z\lambda(1-\omega) - c\theta^{CKZ}$$
$$-\left(\frac{1}{3}(1-c)^2(1-z)(1-\theta^{CKZ}) + \frac{1}{2}(1-c)^2 z(1-\theta^{CKZ})\lambda - c\theta^{CKZ}\right) > 0$$

$$\frac{1}{2}(1-c)^2(1-z)(1-\theta^{CKZ})\lambda - \frac{1}{3}(1-c)^2(1-z)(1-\theta^{CKZ}) > 0$$

$$\frac{1}{2}\lambda - \frac{1}{3} > 0$$

$$\lambda > \frac{2}{3}.$$

Thus, for $\lambda > \frac{2}{3}$, $p_{S,1}^{CKZ^*}$ and for $\lambda \leq \frac{2}{3}$, $p_{S,2}^{CKZ^*}$. Hence, we can collect,

$$p_S^{CKZ^*} = \begin{cases} \frac{2+c}{3} & \lambda \leq \frac{2}{3} \\ c & \lambda > \frac{2}{3} \end{cases} \tag{64}$$

4.1.2 Properties of PAYW-SP for CKZ

CKZ offer two additional insights in their Proposition 3:

1) $p_S^{CKZ^*}$ increases with costs
2) $p_S^{CKZ^*}$ is higher when fair-minded consumers are not sufficiently generous.

It is easy to show that this is true.

Ad 1) The first derivative,

$$\frac{dp_S^{CKZ^*}}{dc} = \begin{cases} \frac{1}{3} & \lambda \leq \frac{2}{3} \\ 1 & \lambda > \frac{2}{3} \end{cases}$$

is always larger 0.

Ad 2) we check

$$p_{S,2}^{CKZ^*} > p_{S,1}^{CKZ^*}$$

$$\frac{2+c}{3} > c$$
$$1 > c$$

which is always the case.

Due to CKZ's condensed notation in their Proof of Proposition 3, we cannot retrace their exact calculations. However, their main results, $p_s^{CKZ^*}$ and $\pi_{PAYW-SP}^{CKZ*}$ correspond to our results.

4.2 The solution with the correct share of freeloaders

4.2.1 Derivations of PAYW-SP

Our calculations do not differ from those of CKZ for consumers with $\gamma_i > 1$. As discussed before, for less fair-minded consumers ($\gamma_i \leq 1$) who consider the price suggestion (which is the case with probability $(1-z)$) we find $p_i = 0$.

For $p_s \geq p_f$, consumers maximize their utility,

$$u_i \geq 0$$
$$r_i - 0 - \gamma_i(p_s - 0) \geq 0$$
$$r_i \geq \gamma_i p_s.$$

Thus, consumers with $r_i \geq \gamma_i p_s$ freeload. Consumers with $r_i < \gamma_i p_s$ do not purchase.

For $p_s < p_f$,

$$u_i \geq 0$$
$$r_i - 0 - \gamma_i\left(p_f - 0\right) \geq 0$$
$$r_i \geq \frac{(1-\lambda)c}{1-\lambda}$$
$$r_i > c.$$

This is always the case as $c \leq p_s < p_f$, therefore all consumers in this region freeload (cf. Akbari and Wagner 2020).

Thus, all consumers with $r_i \geq \gamma_i p_s$ freeload and we can define the segment of freeloaders for those consumers who consider the price suggestion as:

$$\theta_s^{1-z} = (1-z)\int_0^1 \int_{\gamma p_s}^1 \phi(r)\, h(\gamma) dr\, d\gamma = (1-z)\omega\big(1 - p_s \bar{\gamma}_{[0,1]}\big). \tag{65}$$

(AW.5b)

The share of freeloaders for consumers who ignore the price suggestion is the same as in PAYW,

$$\theta = \int_0^1 \int_{\gamma c}^1 \phi(r) h(\gamma) dr d\gamma.$$

Thus, the size of the freeloader segment for those consumers who ignore the price suggestion is given by

$$\theta_s^z = z\int_0^1 \int_{\gamma c}^1 \phi(r) h(\gamma) dr d\gamma = z\omega\big(1 - c\bar{\gamma}_{[0,1]}\big). \tag{66}$$

(AW.5a)

The total freeloader segment is given by

$$\theta_s = \theta_s^{1-z} + \theta_s^z.$$

This allows us to determine profits. As before, we have to distinguish two cases:

1) $p_s < p_h$ and
2) $p_s \geq p_h$.

First, we discuss the case $p_s \leq p_h$.

$$\pi_{PAYW\text{-}SP,1} = (1-z)\left((1-\omega)\left(\frac{1}{2}\int_c^{p_s}(r-c)\,\phi(r)dr + \int_{p_s}^{r_s}(p_s-c)\,\phi(r)dr + \int_{r_s}^1 \lambda(r-c)\,\phi(r)dr\right)\right) + z(1-\omega)\int_c^1 \lambda(r-c)\,\phi(r)dr - c\theta_s$$

$$\pi_{PAYW\text{-}SP,1} = (1-z)\left((1-\omega)\left(\frac{1}{2}\int_c^{p_s}(r-c)\,\phi(r)dr + \int_{p_s}^{r_s}(p_s-c)\,\phi(r)dr + \int_{r_s}^1 \lambda(r-c)\,\phi(r)dr\right)\right) + z(1-\omega)\int_c^1 \lambda(r-c)\,\phi(r)dr - c(\theta_s^{1-z} + \theta_s^z). \tag{67}$$

(AW.13a)

Using the uniform distribution, we get

$$\pi_{PAYW\text{-}SP,1} =$$

$$(1-z)\left((1-\omega)\left(\frac{1}{2}\left(\frac{p_s^2}{2} - cp_s + \frac{c^2}{2}\right) + \left(\frac{1-\lambda}{\lambda}(p_s - c)^2\right) + \lambda\left(\frac{1}{2} - c - \frac{r_s^2}{2} + cr_s\right)\right)\right.$$
$$\left. - c\omega\big(1 - p_s\bar{\gamma}_{[0,1]}\big)\right) + z\left((1-\omega)(1-c)^2\frac{\lambda}{2} - c\omega\big(1 - c\bar{\gamma}_{[0,1]}\big)\right).$$

To find the optimal suggested price, we omit the terms that do not include p_s, i.e., all cases in which $z > 0$,

$$\pi_{PAYW\text{-}SP,1}^{z=0} = (1-\omega)\left(\frac{1}{2}\left(\frac{p_s^2}{2} - cp_s + \frac{c^2}{2}\right) + \left(\frac{1-\lambda}{\lambda}(p_s - c)^2\right)\right.$$
$$\left. + \lambda\left(\frac{1}{2} - c - \frac{r_s^2}{2} + cr_s\right)\right) - c\omega\big(1 - p_s\bar{\gamma}_{[0,1]}\big) \tag{68}$$

(AW.A.10)

$$= (1-\omega)\left(p_s^2\frac{-3\lambda + 2}{4\lambda} + cp_s\frac{3\lambda - 2}{2\lambda} + \frac{c^2}{4} + \frac{\lambda}{2} - c\lambda + \frac{(1-\lambda)^2c^2}{2\lambda}\right)$$
$$- c\omega\big(1 - p_s\bar{\gamma}_{[0,1]}\big).$$

Again, we can make a case distinction. We get a linear term by setting $\lambda = \frac{2}{3}$. For $\lambda \neq \frac{2}{3}$, $\pi_{PAYW\text{-}SP,1}$ has a quadratic form.

First, we examine the linear case.

$$\frac{\pi_{PAYW\text{-}SP,1}^{z=0}\left(\lambda = \frac{2}{3}\right)}{\partial p_s} = c\bar{\gamma}_{[0,1]}\omega.$$

In this case, the seller should set the maximum suggested price. As p_s cannot exceed r_s in this setting, the seller will use the maximum $r_s = 1$ and solve for p_s.

$$\begin{aligned} r_s &= 1 \\ \frac{p_s - (1-\lambda)c}{\lambda} &= 1 \\ p_s &= c + (1-c)\lambda \\ &= c + \frac{2}{3}(1-c) \\ &= \frac{2+c}{3} \coloneqq p_{s,1a}^*. \end{aligned} \tag{69}$$

In case $\lambda \neq \frac{2}{3}$, we take the first derivative,

$$\frac{\partial \pi^{z=0}_{PAYW\text{-}SP,1}}{\partial p_s} = \left(\frac{p_s(2-3\lambda)}{2\lambda} + \frac{c(-2+3\lambda)}{2\lambda}\right)(1-\omega) + c\bar{\gamma}_{[0,1]}\omega \stackrel{!}{=} 0$$

And solve for p_s

$$\begin{aligned} p_s &= \frac{2\lambda c\omega\bar{\gamma}_{[0,1]}}{(3\lambda-2)(1-\omega)} \\ &= c + \frac{2\lambda A_s}{3\lambda-2} \coloneqq p^*_{s,1b}. \end{aligned} \tag{70}$$

with $A_s = \frac{c\omega\bar{\gamma}_{[0,1]}}{1-\omega}$. To check whether we have a local minimum or maximum, we take the second derivative:

$$\frac{\partial^2 \pi_{PAYW\text{-}SP,1}}{\partial^2 p_s} = \frac{(2-3\lambda)(1-\omega)}{2\lambda}.$$

Again, as $(1-\omega)$ is always positive, we must check $\frac{(2-3\lambda)}{2\lambda}$.

$$\frac{(2-3\lambda)}{2\lambda}\begin{cases} > 0 \rightarrow \text{Minimum} & \lambda \leq \frac{2}{3} \\ < 0 \rightarrow \text{Maximum} & \lambda > \frac{2}{3} \end{cases}.$$

Thus, if $\lambda \leq \frac{2}{3}$, the optimal suggested price should be the corner solution, c.

If $\lambda > \frac{2}{3}$, the optimal suggested price should be $p^*_{s,1b}$ as

$$\begin{aligned} c &\leq p_s \leq p_h \\ c &\leq c + \frac{2\lambda A_s}{3\lambda-2} \leq c + (1-c)\lambda. \end{aligned}$$

As proof, we check the lower bound

$$c \leq c + \frac{2\lambda A_s}{3\lambda-2}.$$

This is always positive as $c\omega\bar{\gamma}_{[0,1]} \geq 0$ and $A_s > 0$ for $\lambda > \frac{2}{3}$.

Next, we check the upper bound,

$$\begin{aligned} c + \frac{2\lambda A_s}{3\lambda-2} &\leq c + (1-c)\lambda \\ \frac{2A_s}{1-c} &\leq 3\lambda - 2 \\ \lambda &\geq \frac{2}{3}\left(1 + \frac{A_s}{1-c}\right). \end{aligned}$$

Thus, for large λ, $p^*_{s,1b}$ is optimal.

For $\frac{2}{3} \leq \lambda \leq \frac{2}{3}\left(1 + \frac{A_s}{1-c}\right)$ the optimal suggested price will be

$$p_s = p_h \\ = c + (1-c)\lambda \coloneqq p^*_{s,1c}. \tag{71}$$

Note: we have to ensure $\lambda \leq \frac{2}{3}\left(1+\frac{A_s}{1-c}\right) \leq 1$ and, thus,

$$\left(1+\frac{A_s}{1-c}\right) \leq \frac{3}{2}$$
$$B_s = \frac{A_s}{1-c} < \frac{1}{2}.$$

Thus, the optimal price for $p_s > p_h$ is

$$p^*_{s,1} = \begin{cases} c & \lambda < \frac{2}{3} \\ \frac{2+c}{3} & \lambda = \frac{2}{3} \\ c+(1-c)\lambda & \frac{2}{3} \leq \lambda \leq \frac{2}{3}(1+B_s) \wedge B_s < \frac{1}{2} \\ c+\frac{2\lambda A_s}{3\lambda-2} & \lambda > \frac{2}{3}(1+B_s) \end{cases}. \tag{72}$$

(AW.A.11)

This allows us to determine optimal profits under $\pi_{PAYW\text{-}SP,1}$ (consumers who ignore the price suggestion are still disregarded)

$$\begin{aligned} \pi^{z=0}_{PAYW\text{-}SP,1}(c) &= (1-\omega)\left(c^2\frac{-3\lambda+2}{4\lambda} + c^2\frac{3\lambda-2}{2\lambda} + \frac{c^2}{4} + \frac{\lambda}{2} - c\lambda + \frac{(1-\lambda)^2c^2}{2\lambda}\right) \\ &\quad - c\omega\left(1-c\bar{\gamma}_{[0,1]}\right) \\ &= (1-\omega)\left(\frac{c^2}{4\lambda}(-3\lambda+2+6\lambda-4+\lambda+2-4\lambda+2\lambda^2) + \frac{\lambda}{2} - c\lambda\right) \\ &\quad - c\omega\left(1-c\bar{\gamma}_{[0,1]}\right) \\ &= (1-\omega)\left(\frac{c^2\lambda}{2} + \frac{\lambda}{2} - c\lambda\right) - c\omega\left(1-c\bar{\gamma}_{[0,1]}\right) \\ &= (1-\omega)\frac{\lambda}{2}(1-c)^2 - c\omega\left(1-c\bar{\gamma}_{[0,1]}\right). \end{aligned} \tag{73}$$

The profits with, $\pi^{z=0}_{PAYW\text{-}SP,1}(c+(1-c)\lambda)$ are given by

$$\begin{aligned} &\pi^{z=0}_{PAYW\text{-}SP,1}(c+(1-c)\lambda) \\ &= \frac{1}{4\lambda}\Big((-1+c)\big(\lambda(-4+(10-3\lambda)\lambda) \\ &+ c(-2+\lambda)(2+\lambda(-4+3\lambda))\big)(-1+\omega) \\ &- 4c\lambda(1+c\gamma(-1+\lambda)-\gamma\lambda)\omega\Big). \end{aligned} \tag{74}$$

For $2(1+B_s)/3 \leq \lambda$ only $\pi_{PAYW\text{-}SP,1}$ is valid. This is given by

$$\pi_{PAYW\text{-}SP,1}\left(c+\frac{2\lambda A_s}{3\lambda-2}\right) \tag{75}$$

$$= (1-\omega)\left(-\frac{D_s}{2}\left(c+\frac{A_s}{D_s}\right)^2 + c\left(c+\frac{A_s}{D_s}\right)D_s + \frac{c^2}{4} + \frac{\lambda}{2} - c\lambda + \frac{(1-\lambda)^2c^2}{2\lambda}\right)$$

$$- c\omega\left(1-\left(c+\frac{A_s}{D_s}\right)\bar{\gamma}_{[0,1]}\right)$$

$$= (1-\omega)\left(-\frac{{A_s}^2}{2D_s} + \frac{c^2}{4}\left(2D_s + 1 + \frac{2(1-\lambda)^2}{\lambda}\right) + \frac{\lambda}{2} - c\lambda\right)$$

$$- c\omega\left(1-\left(c+\frac{A_s}{D_s}\right)\bar{\gamma}_{[0,1]}\right)$$

$$= (1-\omega)\left(-\frac{\lambda {A_s}^2}{3\lambda-2} + \frac{\lambda}{2}(1-c)^2\right) - c\omega\left(1-\left(c+\frac{2\lambda A_s}{3\lambda-2}\right)\bar{\gamma}_{[0,1]}\right)$$

with $D_s = (3\lambda-2)/(2\lambda)$.

Next, we discuss the case $p_h < p_s$:

$$\pi_{PAYW\text{-}SP,2} = (1-z)\left((1-\omega)\left(\frac{1}{2}\int_c^{p_s}(r-c)\,\phi(r)dr + \int_{p_s}^1 (p_s-c)\,\phi(r)dr\right)\right) \tag{76}$$

$$+ z(1-\omega)\int_c^1 \lambda(r-c)\,\phi(r)dr - c(\theta_s^{1-z}+\theta_s^z). \tag{AW.13b}$$

For the uniform distribution of r, $\phi(r) = 1$, we get

$$\pi_{PAYW\text{-}SP,2} = (1-z)\left((1-\omega)\left(\frac{1}{2}\left(\frac{p_s^2}{2} - cp_s + \frac{c^2}{2}\right) + (p_s-c)(1-p_s)\right) - c\omega(1-p_s\bar{\gamma})\right)$$

$$+ z\left((1-\omega)(1-c)^2\frac{\lambda}{2} - c\omega\big(1-c\bar{\gamma}_{[0,1]}\big)\right).$$

For finding the optimal price, we again only consider the terms that include p_s (i.e., set z to zero). This gives us

$$\pi_{PAYW\text{-}SP,2}^{z=0} = (1-\omega)\left(\frac{1}{2}\left(\frac{p_s^2}{2} - cp_s + \frac{c^2}{2}\right) + (p_s-c)(1-p_s)\right) \tag{77}$$

$$- c\omega\big(1-p_s\bar{\gamma}_{[0,1]}\big) \tag{AW.A.12}$$

$$= (1-\omega)\left(\frac{-3}{4}p_s^2 + p_s\left(1+\frac{c}{2}\right) + \frac{c^2}{4} - c\right) - c\omega\big(1-p_s\bar{\gamma}_{[0,1]}\big).$$

To find the optimum, we take the first derivative and set it to zero,

$$\frac{\partial \pi_{PAYW\text{-}SP,2}^{z=0}}{\partial p_s} = (1-\omega)\left(1+\frac{c}{2}-\frac{3p}{2}\right) + c\bar{\gamma}_{[0,1]}\omega \stackrel{!}{=} 0.$$

Next, we solve for p_s

$$\begin{aligned} p_s &= \frac{2}{3}\left(1+\frac{c}{2}+\frac{c\omega\bar{\gamma}_{[0,1]}}{1-\omega}\right) \\ &= \frac{1}{3}(2+c+2A_s) \coloneqq p_{s,2a}^*. \end{aligned} \tag{78}$$

The second derivative indicates that the extremum is a maximum,

$$\frac{\partial^2 \pi_{PAYW\text{-}SP,2}^{z=0}}{\partial p_s^2} = \frac{-3}{2} < 0.$$

However, we must

a) make sure that $r_s \geq 1$ (otherwise, we would be in condition $\pi_{PAYW-SP}^1$) and
b) that the suggested price is not higher than the highest possible price, i.e., $p_{s,2}^* \leq 1$.

Ad a)

$$r_s = \frac{1}{\lambda}(p_s - (1-\lambda)c) \geq 1$$

$$\frac{1}{\lambda}\left(\frac{1}{3}(2+c+2A_s) - (1-\lambda)c\right) \geq 1$$

$$\lambda \leq \frac{2}{3}\left(1+\frac{A_s}{(1-c)}\right).$$

Thus, for $\lambda \leq \frac{2}{3}\left(1+\frac{A_s}{(1-c)}\right) \leq 1$, the optimal price is given by $p_{s,2a}^*$.

Ad b) Furthermore, we have to check the upper boundary,

$$p_{s,2a}^* \leq 1$$

$$\frac{1}{3}(2+c+2A_s) \leq 1$$

$$A_s \leq \frac{1-c}{2}.$$

Thus, for

$$A_s > \frac{1-c}{2},$$

the seller would like to set an infeasible suggested price, $p^*_{s,2a} > 1$. Therefore, the price is set at the upper limit,

$$p^*_{s,2b} = 1. \tag{79}$$

This also ensures that we do not exceed the upper boundary for λ,

$$\lambda \leq 1$$
$$\frac{2}{3}\left(1 + \frac{A_s}{(1-c)}\right) \leq 1$$
$$A_s \leq \frac{(1-c)}{2}.$$

We can collect

$$p_s^{(2)} = \begin{cases} 1 & \text{if} \quad B_s > 1/2 \\ \big(2 + c + 2B_s(1-c)\big)/3 & \text{if} \quad \lambda \leq 2(1+B_s)/3 \text{ and } B_s \leq 1/2 \end{cases}. \tag{80}$$

(AW.A.13)

Thus, we can define optimal profits under $\pi^{z=0}_{PAYW\text{-}SP,2}$ (consumers who ignore the price suggestion are still disregarded, $z = 0$),

$$\pi^{z=0}_{PAYW\text{-}SP,2}\left(\frac{1}{3}(2 + c + 2A_s)\right)$$
$$= (1-\omega)\left(-\frac{\big((2+c)^2 + 4(2+c)A_s + 4{A_s}^2\big)}{12} + \frac{\big((2+c) + 2A_s\big)(2+c)}{6} + \frac{c^2}{4} - c\right) - c\omega\left(1 - \frac{(2 + c + 2A_s)\bar{\gamma}_{[0,1]}}{3}\right)$$
$$= (1-\omega)\frac{\big(-(2+c)^2 - 4(2+c)A_s - 4{A_s}^2 + 2(2+c)^2 + 4(2+c)A_s + 3c^2 - 12c\big)}{12} - c\omega\left(1 - \frac{(2 + c + 2A_s)\bar{\gamma}_{[0,1]}}{3}\right)$$
$$= (1-\omega)\left(\frac{1}{12}\big(4 - 8c + 4c^2 - 4{A_s}^2\big)\right) - c\omega\left(1 - \frac{(2 + c + 2A_s)\bar{\gamma}_{[0,1]}}{3}\right)$$

$$\pi^{z=0}_{PAYW\text{-}SP,2}\left(\frac{1}{3}(2 + c + 2A_s)\right) = (1-\omega)\left(\frac{(1-c)^2}{3} - \frac{{A_s}^2}{3}\right) - c\omega\left(1 - \frac{(2 + c + 2A_s)\bar{\gamma}_{[0,1]}}{3}\right). \tag{81}$$

For $B_s > 1/2$

$$\pi_{PAYW\text{-}SP,2}^{z=0}\big(p_{S,2b}^{*}=1\big)=\left((1-\omega)\left(\frac{-3}{4}+\left(1+\frac{c}{2}\right)+\frac{c^2}{4}-c\right)-c\omega\big(1-\bar{\gamma}_{[0,1]}\big)\right) \quad (82)$$
$$=\left(\frac{1}{4}(1-\omega)(1-c)^2-c\omega\big(1-\bar{\gamma}_{[0,1]}\big)\right).$$

Next, we check which pricing scheme, $\pi_{PAYW\text{-}SP,1}^{z=0}$ or $\pi_{PAYW\text{-}SP,2}^{z=0}$ is optimal for the seller. We follow a piecewise procedure.

For $\lambda < 2/3$, we have to compare $\pi_{PAYW\text{-}SP,1}^{z=0}(p_s = c)$ to $\pi_{PAYW\text{-}SP,2}^{z=0}\left(p_s = \frac{1}{3}(2+c+2A_s)\right)$.

Now, we compare the two profit functions

$$\pi_{PAYW\text{-}SP,1}^{z=0}(c)-\pi_{PAYW\text{-}SP,2}^{z=0}\left(\frac{2+c+2A_s}{3}\right)$$
$$=\left((1-\omega)\frac{\lambda}{2}(1-c)^2-c\omega\big(1-c\bar{\gamma}_{[0,1]}\big)\right)$$
$$-\left((1-\omega)\left(\frac{(1-c)^2}{3}-\frac{{A_s}^2}{3}\right)-c\omega\left(1-\frac{(2+c+2A_s)\bar{\gamma}_{[0,1]}}{3}\right)\right)$$
$$=(1-\omega)\left((1-c)^2\left(\frac{\lambda}{2}-\frac{1}{3}\right)+\frac{{A_s}^2}{3}\right)-c\omega\left(1-c\bar{\gamma}_{[0,1]}-1+\frac{(2+c+2A_s)\bar{\gamma}_{[0,1]}}{3}\right)$$
$$=(1-\omega)\left((1-c)^2\frac{3\lambda-2}{6}+\frac{{A_s}^2}{3}\right)-\frac{2}{3}A_s(1-\omega)\big((1-c)+A_s\big)$$
$$=(1-\omega)\left((1-c)^2\frac{3\lambda-2}{6}-\frac{{A_s}^2}{3}-\frac{2}{3}A_s(1-c)\right)<0.$$

This is the case as $\lambda < 2/3$ and therefore $3\lambda - 2 < 0$.

Next, we check which profit is optimal at $\lambda = 2/3$.

The profits for $\pi_{PAYW\text{-}SP,2}^{z=0}$ are given above (74).

Thus, comparing $\pi_{PAYW\text{-}SP,1}^{z=0}$ to $\pi_{PAYW\text{-}SP,2}^{z=0}$, we get

$$\pi_{PAYW\text{-}SP,1}^{z=0}\left(\frac{2+c}{3}\right)-\pi_{PAYW\text{-}SP,2}^{z=0}\left(\frac{2+c+2A_s}{3}\right)$$
$$=(1-\omega)\left(\frac{{A_s}^2}{3}\right)-c\omega\left(-\frac{2+c}{3}\bar{\gamma}_{[0,1]}+\frac{(2+c+2A_s)\bar{\gamma}_{[0,1]}}{3}\right)$$
$$=(1-\omega)\left(\frac{{A_s}^2}{3}\right)-A(1-\omega)\frac{2A_s}{3}$$

$$= (1-\omega)\frac{{A_s}^2}{3}(1-2) < 0.$$

Thus, $\pi^{z=0}_{PAYW\text{-}SP,2}$ is always preferred.

For $2(1+B_s)/3 \le \lambda$, only $\pi^{z=0}_{PAYW\text{-}SP,1}$ is valid.

Contrarily, for $A_s > \frac{1-c}{2}$, only $\pi^{z=0}_{PAYW\text{-}SP,2}$ is valid.

Thus, we can conclude,

$$p_s^* = \begin{cases} 1 & \text{if} & & B_s > 1/2 \\ \big(2+c+2B_s(1-c)\big)/3 & \text{if} & 0 \le \lambda \le 2(1+B_s)/3 \quad \text{and} & B_s \le 1/2 \\ c+2\lambda B_s(1-c)/(3\lambda-2) & \text{if} & 2(1+B_s)/3 \le \lambda \le 1 \quad \text{and} & B_s \le 1/2 \end{cases} \tag{83}$$

(AW.14)

with $B_s = c\omega\bar{\gamma}_{[0,1]}/\big((1-c)(1-\omega)\big)$. (AW.A.14)

Combining equations, we can collect the final optimal profits under PAYSW-SP. In contrast to previous deviations, we also need to take the consumers who ignore the price suggestion into account.

For $B_s > 1/2$, we use (89)

$$\pi_{PAYW\text{-}SP,1a}(p_s = 1) = (1-\omega)(1-c)^2\left(\frac{\lambda z}{2} + \frac{1}{4}(1-z)\right) - c\omega\left(1 - \bar{\gamma}_{[0,1]}\big(cz + (1-z)\big)\right).$$

For $0 \le \lambda \le 2(1+B_s)/3$ and $B_s \le 1/2$, we use (81)

$$\begin{aligned} \pi_{PAYW\text{-}SP,2}\big(p_s &= \big(2+c+2B_s(1-c)\big)/3\big) \\ &= (1-\omega)(1-c)^2\left(\frac{\lambda z}{2} + \left(\frac{1-{B_s}^2}{3}\right)(1-z)\right) \\ &\quad - c\omega\left(1 - \bar{\gamma}_{[0,1]}\left(cz + \frac{\big(2+c+2B_s(1-c)\big)}{3}(1-z)\right)\right). \end{aligned}$$

Finally, for $2(1+B_s)/3 \le \lambda \le 1$ and $B_s \le 1/2$, profits are given by using (75)

$$\pi_{PAYW\text{-}SP,1}\left(p_s = c + \frac{2\lambda B_s(1-c)}{3\lambda-2}\right)$$
$$= (1-\omega)(1-c)^2\left(\frac{\lambda z}{2} + \lambda\left(-\frac{{B_s}^2}{3\lambda-2} + \frac{1}{2}\right)(1-z)\right)$$
$$- c\omega\left(1 - \bar{\gamma}_{[0,1]}\left(cz + \left(c + \frac{2\lambda B_s(1-c)}{3\lambda-2}\right)(1-z)\right)\right).$$

Thus, we can collect,

$$\pi^*_{PAYW\text{-}SP} = (1-\omega)(1-c)^2\left(\frac{\lambda z}{2} + g_{1\lambda}(1-z)\right) - c\omega\left(1 - \bar{\gamma}_{[0,1]}\big(cz + g_{2\lambda}(1-z)\big)\right) \tag{84}$$

with (AW.A.15)

$$\begin{cases} g_{1\lambda} = 1/4 \wedge g_{2\lambda} = 1 & \text{if} \quad B_s > 1/2 \\ g_{1\lambda} = (1-{B_s}^2)/3 \wedge g_{2\lambda} = \big(2 + c + 2B_s(1-c)\big)/3 & \text{if} \quad 0 \le \lambda \le 2(1+B_s)/3 \text{ and } B_s \le 1/2. \\ g_{1\lambda} = \lambda\big(-{B_s}^2/(3\lambda-2) + 1/2\big) \wedge g_{2\lambda} = c + \big(2\lambda B_s(1-c)\big)/(3\lambda-2) & \text{if} \quad 2(1+B_s)/3 \le \lambda \le 1 \text{ and } B_s \le 1/2 \end{cases}$$

4.2.2 Properties of PAYW-SP

4.2.2.1 Properties of the optimal suggested price p_s^*

How does the price change with the change in the model parameters? In order to answer this question, we take the partial derivatives of the optimal prices. p_s^*.

When $B_s > 1/2$, $p_s^* = 1$. Thus, the partial derivatives are given by

$$\frac{\partial p_s^*}{\partial c} = 0$$
$$\frac{\partial p_s^*}{\partial \lambda} = 0$$
$$\frac{\partial p_s^*}{\partial \omega} = 0$$
$$\frac{\partial p_s^*}{\partial \bar{\gamma}_{[0,1]}} = 0.$$

When $0 \le \lambda \le 2(1+B_s)/3$ and $B_s \le 1/2$, $p_s^* = \frac{1}{3}\left(2 + c + 2\frac{c\omega\bar{\gamma}_{[0,1]}}{1-\omega}\right)$. Thus, the partial derivatives are given by

$$\frac{\partial p_s^*}{\partial c} = \frac{1}{3}\left(1 + \frac{2\omega\bar{\gamma}_{[0,1]}}{1-\omega}\right) > 0$$
$$\frac{\partial p_s^*}{\partial \lambda} = 0$$

$$\frac{\partial p_s^*}{\partial \omega} = \frac{2c\bar{\gamma}_{[0,1]}}{3} \frac{1}{(1-\omega)^2} > 0$$
$$\frac{\partial p_s^*}{\partial \bar{\gamma}_{[0,1]}} = \frac{2c\omega}{3(1-\omega)} > 0.$$

When $2(1+B_s)/3 \le \lambda \le 1$ and $B_s \le 1/2$, $p_s^* = c + \frac{2\lambda c\omega\bar{\gamma}_{[0,1]}}{(3\lambda-2)(1-\omega)}$. Thus, the partial derivatives are given by

$$\frac{\partial p_s^*}{\partial c} = 1 + \frac{2\lambda\omega\bar{\gamma}_{[0,1]}}{(3\lambda-2)(1-\omega)} > 0$$
$$\frac{\partial p_s^*}{\partial \lambda} = \frac{2c\omega\bar{\gamma}_{[0,1]}}{(1-\omega)} \frac{-2}{(3\lambda-2)^2} < 0$$
$$\frac{\partial p_s^*}{\partial \omega} = \frac{2\lambda c\bar{\gamma}_{[0,1]}}{(3\lambda-2)} \frac{1}{(1-\omega)^2} > 0$$
$$\frac{\partial p_s^*}{\partial \bar{\gamma}_{[0,1]}} = \frac{2\lambda c\omega}{(3\lambda-2)(1-\omega)} > 0.$$

4.2.2.2 Properties of the optimal profits $\pi^*_{PAYW\text{-}SP}$

Next, we examine the profits with respect to model variables $\bar{\gamma}_{[0,1]}$ and λ. As we are interested in the change due to the suggested price, we set $z = 0$, all consumers consider the suggested price.

1) First, we consider $B_s > \frac{1}{2}$. Therefore, there are some restrictions on parameters in the model,

$$B_s = \frac{c\omega\bar{\gamma}_{[0,1]}}{(1-c)(1-\omega)} > \frac{1}{2}$$
$$1 \ge \bar{\gamma}_{[0,1]} > \frac{(1-c)(1-\omega)}{2c\omega} \land c, \omega > 0$$

and consequently,

$$2c\omega > (1-c)(1-\omega).$$

Therefore,

$$\pi^*_{PAYW\text{-}SP}(p_s^* = 1) = (1-\omega)(1-c)^2 \frac{1}{4} - c\omega\left(1 - \bar{\gamma}_{[0,1]}\right).$$

This is linear in $\bar{\gamma}_{[0,1]}$, thus, we find a corner solution: profits increase in $\bar{\gamma}_{[0,1]}$.

Profits will be lowest for $\bar{\gamma}_{[0,1]}^{\min} = \frac{(1-c)(1-\omega)}{2c\omega}$ and highest for $\bar{\gamma}_{[0,1]}^{\max} = 1$. Thus, profits will be in the region,

$$\frac{(1-\omega)(1-c)^2}{4} - c\omega\left(1 - \frac{(1-c)(1-\omega)}{2c\omega}\right) \leq \pi^*_{PAYW\text{-}SP}(p_s^*) \leq \frac{(1-\omega)(1-c)^2}{4}$$

$$\frac{(1-\omega)(1-c)^2}{4} - c\omega + \frac{(1-c)(1-\omega)}{2} \leq \pi^*_{PAYW\text{-}SP}(p_s^*) \leq \frac{(1-\omega)(1-c)^2}{4} \quad (85)$$

$$\frac{(1-\omega)(1-c)(3-c)}{4} - c\omega \leq \pi^*_{PAYW\text{-}SP}(p_s^*) \leq \frac{(1-\omega)(1-c)^2}{4}$$

2) Second, we consider $0 \leq \lambda \leq 2(1+B_s)/3 \wedge B_s \leq 1/2$, and check how optimal profits change in $\bar{\gamma}_{[0,1]}$, the profits are given by

$$\pi^*_{PAYW\text{-}SP}(p_s^*) = (1-\omega)(1-c)^2 \frac{\left(1-{B_s}^2\right)}{3} - c\omega\left(1 - \bar{\gamma}_{[0,1]} \frac{2+c+2B_s(1-c)}{3}\right)$$

as $B_s = \frac{c\omega\bar{\gamma}_{[0,1]}}{(1-c)(1-\omega)}$ contains $\bar{\gamma}_{[0,1]}$, we rearrange terms,

$$\pi^*_{PAYW\text{-}SP}\left(\bar{\gamma}_{[0,1]}\right) = E_s\left(1 - {F_s}^2{\bar{\gamma}_{[0,1]}}^2\right) - c\omega + G_s\bar{\gamma}_{[0,1]} + H_s{\bar{\gamma}_{[0,1]}}^2$$

with

$$E_s = \frac{(1-\omega)(1-c)^2}{3}$$

$$F_s = \frac{c\omega}{(1-c)(1-\omega)}$$

$$G_s = \frac{c\omega(2+c)}{3}$$

$$H_s = \frac{c\omega}{3}\frac{2c\omega(1-c)}{(1-c)(1-\omega)} = \frac{2c^2\omega^2}{3(1-\omega)} = F_s\frac{2c\omega(1-c)}{3}.$$

Thus, we can take the first derivative with respect to $\bar{\gamma}_{[0,1]}$

$$\frac{\partial\pi^*_{PAYW\text{-}SP}}{\partial\bar{\gamma}_{[0,1]}} = -2E_s{F_s}^2\bar{\gamma}_{[0,1]} + G_s + 2H_s\bar{\gamma}_{[0,1]}$$

$$= \left(-2E_s{F_s}^2 + 2H_s\right)\bar{\gamma}_{[0,1]} + G_s \geq 0.$$

This is positive as both terms are positive,

$$G_s = \frac{c\omega(2+c)}{3} \geq 0$$

and

$$-2E_s{F_s}^2 + 2H_s = 2F_s\left(-\frac{(1-\omega)(1-c)^2}{3}\frac{c\omega}{(1-c)(1-\omega)} + \frac{2c\omega(1-c)}{3}\right)$$

$$= 2F_s \frac{c\omega(1-c)}{3}(-1+2)$$
$$= \frac{2}{3}\frac{c^2\omega^2}{(1-\omega)} \geq 0.$$

Thus, $\pi^*_{PAYW\text{-}SP}$ is monotonically increasing in $\bar{\gamma}_{[0,1]}$.

Consequently, minimum/maximum profits for $\bar{\gamma}^{\min}_{[0,1]} = 0$ and $\bar{\gamma}^{\max}_{[0,1]} = \frac{(1-c)(1-\omega)}{2c\omega}$ are given by

$$\pi^*_{PAYW\text{-}SP}\left(\bar{\gamma}_{[0,1]} = 0; B_s = 0\right) = \frac{(1-\omega)(1-c)^2}{3} - c\omega$$
$$\pi^*_{PAYW\text{-}SP}\left(\bar{\gamma}_{[0,1]} = \frac{(1-c)(1-\omega)}{2c\omega}; B_s = \frac{1}{2}\right) = \frac{(1-\omega)(1-c)^2}{4} - c\omega + \frac{(1-\omega)(1-c)}{2}$$
$$= \frac{(1-\omega)(1-c)(3-c)}{4} - c\omega$$

and, thus,

$$\frac{(1-\omega)(1-c)^2}{3} - c\omega \leq \pi^*_{PAYW\text{-}SP}(p_s^*) \leq \frac{(1-\omega)(1-c)(3-c)}{4} - c\omega. \tag{86}$$

3) Third, we consider $2/3 < 2(1+B_s)/3 \leq \lambda \leq 1 \wedge B_s \leq 1/2$ and rearrange

$$\pi^*_{PAYW\text{-}SP} = (1-\omega)(1-c)^2\lambda\left(-\frac{B_s^2}{3\lambda-2}+\frac{1}{2}\right) - c\omega\left(1-\bar{\gamma}_{[0,1]}\left(c+\frac{2\lambda B_s(1-c)}{3\lambda-2}\right)\right)$$

$$0 \leq \frac{B_s}{3\lambda-2} \leq \frac{1}{2}$$

$$\pi^*_{PAYW\text{-}SP} = (1-\omega)(1-c)^2\lambda\left(-I_sB_s+\frac{1}{2}\right) - c\omega\left(1-\bar{\gamma}_{[0,1]}(c+2I_s\lambda(1-c))\right)$$

with

$$I_s = \frac{B_s}{3\lambda-2}$$

and as

$$\frac{2}{3} \leq \frac{2(1+B_s)}{3} \leq \lambda \leq 1$$
$$0 \leq 2B_s \leq 3\lambda - 2 \leq 1$$
$$0 \leq I_s \leq \frac{1}{2}.$$

We rearrange further,

$$\pi^*_{PAYW\text{-}SP} = -\lambda I_s c\omega\bar{\gamma}_{[0,1]}(1-c) + \frac{(1-\omega)(1-c)^2\lambda}{2} - c\omega + c\omega\bar{\gamma}_{[0,1]}\big(c + 2I_s\lambda(1-c)\big)$$
$$= c\omega\bar{\gamma}_{[0,1]}\big(-\lambda I_s(1-c) + c + 2I_s\lambda(1-c)\big) + \frac{(1-\omega)(1-c)^2\lambda}{2} - c\omega$$
$$= c\omega\bar{\gamma}_{[0,1]}(\lambda I_s(1-c) + c) + \frac{(1-\omega)(1-c)^2\lambda}{2} - c\omega.$$

Again, this is linearly increasing in $\bar{\gamma}_{[0,1]}$. Thus, minimum/maximum profits for $\bar{\gamma}^{\min}_{[0,1]} = 0$ and $\bar{\gamma}^{\max}_{[0,1]} = \frac{(1-c)(1-\omega)}{2c\omega}$ and $I_S^{\min} = 0$ and $I_S^{\max} = 1/2$,

$$\pi^*_{PAYW\text{-}SP}\big(\bar{\gamma}_{[0,1]} = 0; I_s = 0\big) = \frac{(1-\omega)(1-c)^2\lambda}{2} - c\omega$$
$$\pi^*_{PAYW\text{-}SP}\left(\bar{\gamma}_{[0,1]} = \frac{(1-c)(1-\omega)}{2c\omega}; I_s = \frac{1}{2}\right)$$
$$= \frac{(1-\omega)(1-c)}{2}\left(\frac{\lambda}{2}(1-c) + c\right) + \frac{(1-\omega)(1-c)^2\lambda}{2} - c\omega$$
$$= \frac{(1-c)(1-\omega)}{2}\left(\frac{3}{2}\lambda(1-c) + c\right) - c\omega.$$

Thus, profits range in

$$\frac{(1-\omega)(1-c)^2\lambda}{2} - c\omega \le \pi^*_{PAYW\text{-}SP} \le \frac{(1-c)(1-\omega)}{2}\left(\frac{3}{2}\lambda(1-c) + c\right) - c\omega.$$

Furthermore, we examine $\pi^*_{PAYW\text{-}SP}$ with respect to λ

$$\frac{\partial\pi^*_{PAYW\text{-}SP}}{\partial\lambda} = \frac{c\omega\bar{\gamma}_{[0,1]}B_s(-2)}{(3\lambda-2)^2} + \frac{(1-\omega)(1-c)^2}{2}$$
$$= \frac{c^2\omega^2\bar{\gamma}^2_{[0,1]}}{(1-\omega)}\left(-\frac{2}{(3\lambda-2)^2} + \frac{(1-\omega)^2(1-c)^2}{2c^2\omega^2\bar{\gamma}^2_{[0,1]}}\right)$$
$$= \frac{2c^2\omega^2\bar{\gamma}^2_{[0,1]}}{(1-\omega)}\left(-\frac{1}{(3\lambda-2)^2} + \frac{1}{4B_s}\right) \ge 0$$

as $\frac{2(1+B_s)}{3} \le \lambda$.

The minimum and maximum λ values are given by $\lambda = \frac{2}{3}$ and $\lambda = 1$ as we are in case of $2/3 < 2(1+B_s)/3 \le \lambda \le 1 \land B_s \le 1/2$. Therefore, use these values in the profit range determined before,

$$\frac{(1-\omega)(1-c)^2\lambda}{2} - c\omega \leq \pi^*_{PAYW\text{-}SP} \qquad (87)$$

$$\leq \frac{(1-c)(1-\omega)}{2}\left(\frac{3}{2}\lambda(1-c)+c\right) - c\omega. \qquad \text{(AW.A.16)}$$

For $\lambda = \frac{2}{3}$

$$\frac{(1-\omega)(1-c)^2}{3} - c\omega \leq \pi^*_{PAYW\text{-}SP} \leq \frac{(1-c)(1-\omega)}{2} - c\omega \qquad (88)$$

and for $\lambda = 1$

$$\frac{(1-\omega)(1-c)^2}{2} - c\omega \leq \pi^*_{PAYW\text{-}SP} \leq \frac{(1-c)(1-\omega)(3-c)}{4} - c\omega. \qquad (89)$$

(AW.A.16)

Thus, overall, the profits in PAYW-SP span

$$\frac{(1-\omega)(1-c)^2}{3} - c\omega \leq \pi^*_{PAYW\text{-}SP} \leq \frac{(1-c)(1-\omega)(3-c)}{4} - c\omega.$$

Note that this corresponds to (86).

5 Derivations of Proposition 4 (Comparisons between the pricing mechanisms)

5.1 Setup

In their calculation of optimal pricing regimes, CKZ use equations (7), (39), (61), and (63) to calculate the regions in which each pricing regime PAAP, PAYW-MP, PAYW-SP optimal. As they build on their previous results, their outcomes differ from our outcome because they

(a) disregard the screening out of freeloaders in PAYW-SP (cf. Section 4),
(b) abstract from disadvantageous inequity aversion (cf. Sections 2.1 and 3)

Furthermore, in their calculations of optimal pricing regions, three additional errata occur. CKZ

(c) make calculating errors when deriving the optimal profit difference between PAYW-MP and PAYW-SP when $\lambda \leq \frac{2}{3}$,
(d) do not account for the upper bound of the minimum price in their profit comparisons, and
(e) leave an area undefined regarding the definition of the optimal pricing scheme.

To unveil these inconsistencies, we first retrace their calculations by using their profit functions and to derive consistent optimal pricing schemes. Note, these results still suffer from (a) and (b) above but should not be affected by (c), (d) and (e). Second, we compare these consistent schemes to CKZ's results and try to uncover and interpret the differences. Third, we use CKZ's optimal minimum and suggested prices to derive the profits a seller would make with inconsistent prices. These prices are later used in our simulation analysis. Fourth and finally, we derive the optimal pricing schemes using the correct prices and correct profit functions.

5.2 Profit comparisons for CKZ's results

CKZ compare their profits in Proposition 4. However, the calculations are performed in the proofs in their Appendix D. They use their respective profit functions which we derived in (7), (39), (61), and (63).

5.2.1 Comparing pricing schemes for $\lambda \geq 2/3$

First, CKZ compare optimal profits for $\lambda \geq \frac{2}{3}$. We follow their reasoning and start by comparing the optimal profits for PAYW-MP to the optimal profits under PAYW-SP

$$\pi_{PAYW\text{-}MP}^{CKZ,*} - \pi_{PAYW\text{-}SP,1}^{CKZ^*}$$

$$= \frac{(1-c)^2\lambda[1-2\theta^{CKZ}(1-\lambda)-2\lambda]}{2(1-\theta^{CKZ}-2\lambda)} - \left(\frac{\lambda}{2}(1-\theta^{CKZ})(1-c)^2 - c\theta\right)$$

$$= \frac{\theta^{CKZ}(\theta^{CKZ}\lambda + c^2\theta^{CKZ}\lambda + c(-2-2\theta^{CKZ}(-1+\lambda)+4\lambda))}{2(-1+\theta^{CKZ}+2\lambda)}$$

$$= \frac{\theta^{CKZ}[\lambda[4c+\theta^{CKZ}(1-c)^2]-2c(1-\theta^{CKZ})]}{2(-1+\theta^{CKZ}+2\lambda)}.$$

To check the sign of the above expression, we start by inspecting whether the sign of the denominator is positive,

$$2(-1+\theta^{CKZ}+2\lambda) > 0$$

$$(-1+\theta^{CKZ}+2\lambda) > 0$$

$$2\lambda > 1-\theta^{CKZ}.$$

As $\lambda > \frac{2}{3}$. and $0 \leq \theta^{CKZ} \leq 1$, this is always the case. The denominator is always positive. Second, we test the nominator

$$\theta^{CKZ}[\lambda[4c+\theta^{CKZ}(1-c)^2]-2c(1-\theta^{CKZ})] > 0$$

$$\lambda[4c + \theta^{CKZ}(1-c)^2] - 2c(1-\theta^{CKZ}) > 0$$

$$\lambda[4c + \theta^{CKZ}(1-c)^2] > 2c(1-\theta^{CKZ})$$

$$4c + (1-c)^2\theta^{CKZ} > \frac{2c - 2c\theta^{CKZ}}{\lambda}$$

$$\theta^{CKZ}\left((1-c)^2 + \frac{2c}{\lambda}\right) > -4c + \frac{2c}{\lambda}$$

$$\theta^{CKZ} > \frac{c(2-4\lambda)}{2c + (1-c)^2\lambda} < 0.$$

The nominator is also positive. Thus, the whole expression $\pi^{CKZ,*}_{PAYW\text{-}MP} - \pi^{CKZ^*}_{PAYW\text{-}SP,1}$ is always positive. Therefore, PAYW-MP is always better than PAYW-SP when $\lambda > \frac{2}{3}$.

Next, we compare PAYW-MP to PAAP,

$$\pi^{CKZ,*}_{PAYW\text{-}MP} - \pi^{\beta=0,*}_{PAAP} = \frac{(1-c)^2\lambda[1 - 2\theta^{CKZ}(1-\lambda) - 2\lambda]}{2(1-\theta^{CKZ} - 2\lambda)} - \frac{(1-c)^2}{4}$$

$$= \frac{(1-c)^2(1-\theta^{CKZ})(1-2\lambda)^2}{4(-1+\theta^{CKZ}+2\lambda)}.$$

To inspect whether PAAP can dominate PAYW-MP in this case, thus, whether this difference can become negative, we first inspect the denominator

$$4(-1+\theta^{CKZ}+2\lambda) > 0$$

$$(-1+\theta^{CKZ}+2\lambda) > 0$$

$$2\lambda > 1 - \theta^{CKZ}.$$

As before, the denominator is always positive for $\lambda > \frac{1}{2}$. As $\lambda > \frac{2}{3}$ this is the case. Next, we move to the nominator.

$$(1-c)^2(1-\theta^{CKZ})(1-2\lambda)^2 > 0.$$

This is always positive as $\theta^{CKZ} \leq 1$. Thus, $\pi^{CKZ,*}_{PAYW\text{-}MP} - \pi^{\beta=0,*}_{PAAP}$ is positive and PAYW-MP is always better than PAAP when $\lambda > \frac{2}{3}$. This result corresponds to CKZ.

5.2.2 Comparing pricing schemes for $1/2 \leq \lambda < 2/3$

Next, we consider $\frac{1}{2} \leq \lambda < \frac{2}{3}$. First, we compare PAYW-MP to PAAP,

$$\pi^{CKZ,*}_{PAYW\text{-}MP} - \pi^{\beta=0,*}_{PAAP}$$

$$= \frac{(1-c)^2\lambda[1-2\theta^{CKZ}(1-\lambda)-2\lambda]}{2(1-\theta^{CKZ}-2\lambda)} - \left(-c\theta^{CKZ} + \frac{[2+z(3\lambda-2)](1-c)^2(1-\theta^{CKZ})}{6}\right)$$

$$= c\theta^{CKZ} + \frac{(1-c)^2\lambda[1-2\theta^{CKZ}(1-\lambda)-2\lambda]}{2(1-\theta^{CKZ}-2\lambda)} - \frac{[2+z(3\lambda-2)](1-c)^2(1-\theta^{CKZ})}{6}.$$

CKZ solve this term for θ^{CKZ} to find the critical threshold of "freeloaders" for which PAYW-MP is better than PAYW-SP. To compare, we do the same and solve $\pi_{PAYW\text{-}MP}^{CKZ,*} - \pi_{PAAP}^{\beta=0,*}$ for θ^{CKZ}. This gives us

$$\theta_{RE,1,2}^{CKZ} = \frac{A_{RE}^{CKZ} \pm \sqrt{B_{RE}^{CKZ} C_{RE}^{CKZ}}}{D_{RE}^{CKZ}}$$

with

$$\begin{aligned}
A_{RE}^{CKZ} &= 2 - 2z + c(-1 - 2c(-1+z) + 4z) - 5\lambda + \big((4-5c)c + 5(1-c)^2 z\big)\lambda \\
&\qquad + 3(1-c)^2(1-z)\lambda^2 \\
&= 2 - c(1 - 2c(1-z) - 4z) - 2z - 5\lambda + ((4-5c)c + 5(-1+c)^2 z)\lambda \\
&\qquad + 3(1-c)^2(1-z)\lambda^2 \\
B_{RE}^{CKZ} &= 3c^2 + 2(-1 + c(2 + c(-4+z) - 2z) + z)\lambda + 3(1-c)^2(1-z)\lambda^2 \\
C_{RE}^{CKZ} &= 3 + 2(-4 + (-2+c)c(-1+z) + z)\lambda - 3(-1+c)^2(-1+z)\lambda^2 \\
D_{RE}^{CKZ} &= 2(1 + c + c^2 - z(1-c)^2) + 3z\lambda(1-c)^2.
\end{aligned} \tag{90}$$

Note, CKZ derive similar results and denote their auxiliary variables A, B, C, D. Therefore, we exceptionally also use C with a capital letter as an auxiliary variable in this context.

First, check the feasibility of both solutions $\theta_{RE,1}^{CKZ}$ and $\theta_{RE,2}^{CKZ}$.

We check whether the denominator D_{RE}^{CKZ} can become negative

$$D_{RE}^{CKZ} < 0$$

$$2(1 + c + c^2 - z(1-c)^2) + 3z\lambda(1-c)^2 < 0.$$

The second term, $3z\lambda(1-c)^2$, is always positive. Therefore, we only inspect the first term

$$2(1 + c + c^2 - z(1-c)^2) < 0$$

$$2(1 - z + c + c^2(1-z) + 2cz) < 0$$

$$1 + c + c^2(1-z) + 2cz < z.$$

A contradiction as $z \in [0;1]$ can never become larger than 1. Thus, the denominator, D_{RE}^{CKZ}, is always positive.

Second, we inspect the nominator for the first solution

$$\theta_{RE,1}^{CKZ} = \frac{A_{RE}^{CKZ} - \sqrt{B_{RE}^{CKZ} C_{RE}^{CKZ}}}{D_{RE}^{CKZ}}.$$

For a viable solution in $\theta_{RE,1}^{CKZ}$, we need numerator to be positive as well. Thus,

$$A_{RE}^{CKZ} - \sqrt{B_{RE}^{CKZ} C_{RE}^{CKZ}} > 0$$

$$A_{RE}^{CKZ} > \sqrt{B_{RE}^{CKZ} C_{RE}^{CKZ}}$$

$${A_{RE}^{CKZ}}^2 > B_{RE}^{CKZ} C_{RE}^{CKZ}$$

$$\begin{aligned} 0 > \big(&2 - 2z + c(-1 - 2c(-1+z) + 4z) - 5\lambda + \big((4-5c)c + 5(-1+c)^2 z\big)\lambda \\ &- 3(-1+c)^2(-1+z)\lambda^2\big)^2 - (3c^2 + 2(-1 + c(2 + c(-4+z) - 2z) + z)\lambda \\ &- 3(-1+c)^2(-1+z)\lambda^2)(3 + 2(-4 + (-2+c)c(-1+z) + z)\lambda \\ &- 3(-1+c)^2(-1+z)\lambda^2. \end{aligned}$$

However, this is not the case as

$$\begin{aligned} &\big(2 - 2z + c(-1 - 2c(-1+z) + 4z) - 5\lambda + \big((4-5c)c + 5(-1+c)^2 z\big)\lambda \\ &- 3(-1+c)^2(-1+z)\lambda^2\big)^2 - (3c^2 + 2(-1 + c(2 + c(-4+z) - 2z) + z)\lambda \\ &- 3(-1+c)^2(-1+z)\lambda^2)(3 + 2(-4 + (-2+c)c(-1+z) + z)\lambda \\ &- 3(-1+c)^2(-1+z)\lambda^2 \end{aligned}$$

$$= \underbrace{(1-c)^2}_{>0}\underbrace{(1-z)}_{>0}\underbrace{(2-7\lambda+6\lambda^2)}_{<0}\underbrace{\left(\underbrace{2 + \underbrace{z(-2+3\lambda)}_{>-\frac{1}{2}}}_{>0} + c\underbrace{\big(2 + z(4-6\lambda)\big)}_{>0} + \underbrace{c^2\left(2 + \underbrace{z(-2+3\lambda)}_{>-\frac{1}{2}}\right)}_{>0}\right)}_{>0}$$

$$\leq 0.$$

Thus, the nominator is always smaller 0 and, therefore, $\theta_1$does not yield a meaningful solution. Second, we test whether

$$\theta_{RE,2}^{CKZ} = \frac{A_{RE}^{CKZ} + \sqrt{B_{RE}^{CKZ} C_{RE}^{CKZ}}}{D_{RE}^{CKZ}}$$

yields a meaningful solution. Therefore, we consider $\theta_{RE,2}^{CKZ}$ to be a function of λ, z and c and inspect the boundaries to test $(\exists \lambda, c, z)\theta_{RE,2}^{CKZ}(\lambda, c, z) \in [0,1]$.

$$\theta_{RE,2}^{CKZ}(\lambda, c, z) =$$

$$\theta_{RE,2}^{CKZ}\left(\frac{1}{2}, c, z\right) = \frac{-1 + (-2 + c)c(-1 + z) - \sqrt{(1 - c)^4(1 - z)^2} + z}{-8(1 + c + c^2) + 2(1 - c)^2 z}$$
$$= \frac{-1 + (-2 + c)c(-1 + z) - (1 - c)^2(1 - z) + z}{-8(1 + c + c^2) + 2(1 - c)^2 z}$$
$$= \frac{(1 - c)^2(1 - z)}{4(1 + c + c^2) - (1 - c)^2 z}.$$

We see that the nominator

$$1 > (1 - c)^2(1 - z) \geq 0,$$

as $1 > c, z \geq 0$.

For the denominator we find,

$$4(1 + c + c^2) - (1 - c)^2 z > 0.$$

Thus, for $\lambda = \frac{1}{2}$, $0 \leq \theta_{RE,2}^{CKZ} < 1$.

We also inspect the upper boundary,

$$\theta_{RE,2}^{CKZ}\left(\frac{2}{3}, c, z\right) = \frac{-c + \sqrt{c^2}}{2(1 + c + c^2)}$$
$$= 0.$$

Thus, $\theta_{RE,2}^{CKZ}$ is a viable solution. Hence, PAYW-MP and PAYW-SP are possible for $\frac{1}{2} \leq \lambda < \frac{2}{3}$. Therefore, we define

$$\theta_{RE,2}^{CKZ} \coloneqq \theta_{RE,\frac{1}{2}<\lambda<\frac{2}{3}}^{CKZ,+} \tag{91}$$

as the critical threshold indicating whether PAYW-MP or PAYW- SP are optimal. As shown in the previous chapter PAYW-MP is always better than PAAP for $\lambda > \frac{1}{2}$. Therefore, a comparison between PAYW-SP and PAAP is redundant.

5.2.3 Comparing pricing schemes for $\lambda < 1/2$

First, we compare PAAP to PAYW-MP

$$\pi_{PAYW\text{-}MP}^{CKZ,*} - \pi_{PAAP}^{*} = (1 - c)^2(1 - \lambda)\lambda - \frac{(1 - c)^2}{4}$$

$$= (1-c)^2\left((1-\lambda)\lambda - \frac{1}{4}\right).$$

We only need to consider the second term

$$(1-\lambda)\lambda - \frac{1}{4} < 0$$

$$(1-\lambda)\lambda < \frac{1}{4}.$$

This is true, because $\lambda < 1/2$. Thus, PAAP is always preferred to PAYW-MP in this interval. Therefore, comparing PAYW-MP to PAYW-SP would be redundant.

Consequently, we only need to compare PAYW-SP to PAAP.

$$\pi^{CKZ,*}_{PAYW\text{-}SP} - \pi^*_{PAAP} = 1/6\,(1-c)^2(1-\theta^{CKZ})\big(2+z(-2+3\lambda)\big) - c\theta^{CKZ} - \frac{(1-c)^2}{4}.$$

To find the critical level of freeloaders, we set to zero and solve for θ^{CKZ}. This gives us

$$\theta^{CKZ} \le \frac{(1-c)^2(1+z(-4+6\lambda))}{4(1+c+c^2-(1-c)^2 z)+6(-1+c)^2 z\lambda} := \theta^{CKZ,+}_{RE,\lambda\le\frac{1}{2}}. \quad (92)$$

Thus, we can conclude:

- For $\lambda \ge 2/3$, PAYW-MP is optimal.
- For $\frac{1}{2} < \lambda < 2/3$,
    - PAYW-MP is optimal if $\theta^{CKZ} \ge \theta^{CKZ,+}_{RE,\frac{1}{2}<\lambda<\frac{2}{3}}$ and
    - PAYW-SP is optimal if $\theta^{CKZ} < \theta^{CKZ,+}_{RE,\frac{1}{2}<\lambda<\frac{2}{3}}$
- For $\lambda \le \frac{1}{2}$
    - PAAP is optimal if $\theta^{CKZ,+}_{RE,\lambda\le\frac{1}{2}}$ and
    - PAYW-SP is optimal if $\theta^{CKZ,+}_{RE,\lambda\le\frac{1}{2}}$.

Figure 1 describes the results graphically.

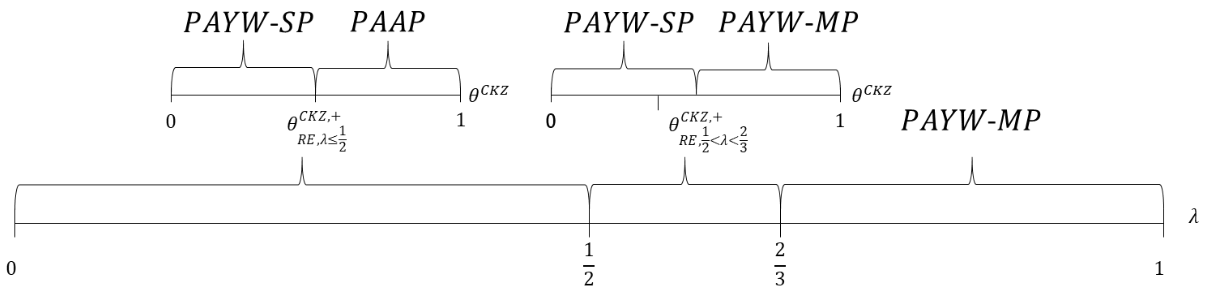

Figure 1: Optimal pricing schemes when using CKZ's profit functions

5.3 CKZ's results

The above calculations use CKZ's optimal prices and the resulting optimal profit functions. These functions do not take less fair-minded consumers or disadvantageous inequity aversion into account and therefore draw a misleading picture. However, our results above are different than the solution of CKZ in their Proposition 4 and in the corresponding proofs. In the following, we are going to compare our results to those of CKZ. As the process of calculation in the original paper is not always transparent, we only speculate about the reasons for deviation.

In their Proposition 4, CKZ's recommendations for the firm's pricing choice are:

- For $\lambda > 2/3$, PAYW-MP is optimal.
- For $\lambda \leq 2/3$, CKZ define
  - PAYW-MP to be optimal if $\theta^{CKZ} > \max\{1-2\lambda, \theta^{CKZ,+}(z,c,\lambda)\}$
  - PAYW-SP to be optimal if $\theta^{CKZ} \leq \min\{\theta^{CKZ,+}(z,c,\lambda), E^{CKZ}_{ORIG}\}$ and
  - PAAP to be optimal if $E^{CKZ}_{ORIG} \leq \theta^{CKZ} \leq \min\{1-2\lambda, \theta^{CKZ,+}(z,c,\lambda)\}$,

with

$$\theta^{CKZ,+}_{ORIG}(z,c,\lambda) = \frac{A^{CKZ}_{ORIG} + \sqrt{B^{CKZ}_{ORIG} C^{CKZ}_{ORIG}}}{D^{CKZ}_{ORIG}} \tag{93}$$

$$A^{CKZ}_{ORIG} = 2 - 2z - c[1 - 2c(1-z)] - 4z - 5\lambda - \lambda(c(4-5c) + 5z(1-c)^2) + 3\lambda(1-z)(1-c)^2$$

$$B^{CKZ}_{ORIG} = 3c^2 - 2\lambda\left(1 - z - c\big(2 - 2z - c(4-z)\big)\right) + 3\lambda 2(1-c)2(1-z)$$
$$= 3c^2 + 12(1-c)(1-z)\lambda - 2(1 - c(2 - c(4-z) - 2z) - z)\lambda$$

$$C^{CKZ}_{ORIG} = 3 - 2\lambda\big(4 - z - c(2-c)(1-z)\big) + 3\lambda 2(1-c)2(1-z)$$
$$= 3 + 12(1-c)(1-z)\lambda - 2(4 - (2-c)c(1-z) - z)\lambda$$

$$D_{ORIG}^{CKZ} = 2(1 + c + c^2 - (1-c)^2 z) + 3(1-c)^2 z\lambda$$

$$E_{ORIG}^{CKZ} = \frac{(1-c)^2(1 - z(4 - 6\lambda))}{4(1 + c + c^2 - (1-c)^2 z) + 6(1-c)^2 z\lambda}.$$

Our results partially differ from CKZ's. While we also find that PAYW-MP should be optimal for $\lambda \geq 2/3$, we find a different structure for value for $\lambda \leq 2/3$. In particular, we do not find $\theta^{CKZ,+}(z, c, \lambda)$ and the boundary $1 - 2\lambda$. However, their lower boundary for PAAP to be more profitable than PAYW, E, is equal to our boundary $\theta_{RE,\lambda \leq \frac{1}{2}}^{CKZ,+}$.

In the following subsections, we first try to disentangle how and why the critical thresholds of "freeloaders" differ between our results in Section 5.2 and the results by CKZ. Second, we will discuss the different boundaries in Section 5.2 and CKZ.

5.3.1 Differences in θ^+

When we inspect $\theta_{ORIG}^{CKZ,+}(z, c, \lambda)$, we find a similar mathematical structure for the results derived in Section 5.2

$$\theta_{RE,\frac{1}{2}<\lambda<\frac{2}{3}}^{CKZ,+} = \frac{A_{RE}^{CKZ} + \sqrt{B_{RE}^{CKZ} C_{RE}^{CKZ}}}{D_{RE}^{CKZ}}$$

and CKZ's results

$$\theta_{ORIG}^{CKZ,+} = \frac{A_{ORIG}^{CKZ} + \sqrt{B_{ORIG}^{CKZ} C_{ORIG}^{CKZ}}}{D_{ORIG}^{CKZ}}.$$

However, the boundaries are different,

$$\theta_{RE,\frac{1}{2}<\lambda<\frac{2}{3}}^{CKZ,+} \neq \theta_{ORIG}^{CKZ,+}.$$

This similarity allows us to compare A_{RE}^{CKZ} to A_{ORIG}^{CKZ}, B_{RE}^{CKZ} to B_{ORIG}^{CKZ}, C_{RE}^{CKZ} to C_{ORIG}^{CKZ} and D_{RE}^{CKZ} to D_{ORIG}^{CKZ}. However, only the denominator is equivalent in both equations.

$$D_{RE}^{CKZ} = D_{ORIG}^{CKZ}$$

$$2(1 + c + c^2 - z(1-c)^2) + 3z\lambda(1-c)^2 = 2(1 + c + c^2 - (1-c)^2 z) + 3(1-c)^2 z\lambda.$$

This is surprising since we use the same starting point as CKZ in their Proposition 4. We test several potential explanations for this difference.

5.3.1.1 Potential explanation: Calculation mistake

A potential reason for the difference between $\theta^{CKZ,+}_{RE,\frac{1}{2}<\lambda<\frac{2}{3}}$ and $\theta^{CKZ,+}_{ORIG}(z,c,\lambda)$ is a calculation mistake or typo in the comparison between the profits of PAYW-MP and PAYW-SP for $\lambda \leq \frac{2}{3}$ in the Proof to Proposition 4 in CKZ which is given by

$$\begin{aligned}\pi^{CKZ,*}_{PAYW\text{-}MP} - \pi^{CKZ,*}_{PAYW\text{-}SP,2} &\\ = c\theta^{CKZ} &+ \frac{\big(2+z(3\lambda-2)\big)(1-c)^2(1-\theta^{CKZ})}{6} \\ &+ \frac{(1-c)^2\lambda[1-2\theta^{CKZ}(1-\lambda)-2\lambda]}{2(1-\theta^{CKZ}-2\lambda)}.\end{aligned} \tag{94}$$

However, using their optimal profits, the "revenue part" of PAYW-SP ($\frac{(2+z(3\lambda-2))(1-c)^2(1-\theta^{CKZ})}{6}$) should be subtracted from (instead of added to) the profits in PAYW-MP

$$\begin{aligned}\pi^{CKZ,*}_{PAYW\text{-}MP} - \pi^{CKZ,*}_{PAYW\text{-}SP,2} &\\ = c\theta^{CKZ} &- \frac{\big(2+z(3\lambda-2)\big)(1-c)^2(1-\theta^{CKZ})}{6} \\ &+ \frac{(1-c)^2\lambda[1-2\theta^{CKZ}(1-\lambda)-2\lambda]}{2(1-\theta^{CKZ}-2\lambda)}.\end{aligned}$$

In order to check whether this is the cause for the difference between (92) and (93), we use (94) set it to zero and solve for θ^{CKZ}. However, we find

$$\theta^{CKZ,+'}_{ORIG} = \frac{A^{CKZ\,'}_{ORIG} \pm \sqrt{B^{CKZ\,'}_{ORIG}}}{D^{CKZ\,'}_{ORIG}}$$

$$\begin{aligned}A^{CKZ\,'}_{ORIG} = {} & 2(-2+c(7+2c(-1+z)-4z)+2z-\lambda-(c(4+c)+5(-1+c)^2z)\lambda \\ & +3(-1+c)^2(1+z)\lambda^2)\end{aligned}$$

$$\begin{aligned}B^{CKZ\,'}_{ORIG} = {} & 4\Big((2-2z+c(-7-2c(-1+z)+4z)+\lambda+(c(4+c)+5(-1+c)^2z)\lambda \\ & -3(-1+c)^2(1+z)\lambda^2)^2 \\ & +(-1+c)^2(-1+2\lambda)(2-2z+2c(-5+c+2z-cz)+3(-1+c)^2z\lambda)\big(2 \\ & +3\lambda+z(-2+3\lambda)\big)\Big)\end{aligned}$$

$$D^{CKZ\,'}_{ORIG} = -4(1+(-5+c)c)+4(-1+c)^2z-6(-1+c)^2z\lambda.$$

This does not correspond to $\theta^{CKZ,+}_{RE,\frac{1}{2}<\lambda<\frac{2}{3}}$ or its components (A^{CKZ}_{RE}, B^{CKZ}_{RE}, C^{CKZ}_{RE}, and D^{CKZ}_{RE}).

Therefore, we suspect that the erroneous minus sign was a typo in CKZ's Proof to Proposition 4 and did not bias the calculation of $\theta^{CKZ,+}_{ORIG}$.

5.3.1.2 Potential explanation: Missing superscripts

A close inspection of B_{ORIG}^{CKZ} and C_{ORIG}^{CKZ} suggest that there might be a simple typographic or formatting error in CKZ's paper.

$$B_{ORIG}^{CKZ} = 3c^2 - 2\lambda\left(1 - z - c\big(2 - 2z - c(4 - z)\big)\right) + 3\lambda 2(1 - c)2(1 - z)$$
$$C_{ORIG}^{CKZ} = 3 - 2\lambda\big(4 - z - c(2 - c)(1 - z)\big) + 3\lambda 2(1 - c)2(1 - z)$$

The second term of both equations

"$3\lambda 2(1 - c)2(1 - z)$"

exhibits an unusual pattern of three integers that could be multiplied and displayed as a single number. As two of these numbers are 2's, we suspect that these are supposed to be (quadratic) superscripts. Thus, we compare our results with these updated values $B_{ORIG}^{CKZ}{}'$ and $C_{ORIG}^{CKZ}{}'$.

$$\begin{aligned} B_{ORIG}^{CKZ}{}'' &= 3c^2 - 2\lambda\left(1 - z - c\big(2 - 2z - c(4 - z)\big)\right) + 3\lambda^2(1 - c)^2(1 - z) \\ &= 3c^2 - 2(1 - c(2 - c(4 - z) - 2z) - z)\lambda + 3(1 - c)^2(1 - z)\lambda^2 \\ C_{ORIG}^{CKZ}{}'' &= 3 - 2\lambda\big(4 - z - c(2 - c)(1 - z)\big) + 3\lambda^2(1 - c)^2(1 - z) \\ &= 3 - 2(4 - (2 - c)c(1 - z) - z)\lambda + 3(1 - c)^2(1 - z)\lambda^2. \end{aligned}$$

If we compare the product of $B_{ORIG}^{CKZ}{}''$ and $C_{ORIG}^{CKZ}{}''$ to the counterpart in our solution, the product of B_{RE}^{CKZ} and C_{RE}^{CKZ}, we find equivalence:

$$B_{RE}^{CKZ} C_{RE}^{CKZ} = B_{ORIG}^{CKZ}{}'' C_{ORIG}^{CKZ}{}''.$$

Thus, the deviation between B_{RE}^{CKZ} and B_{ORIG}^{CKZ} and C_{RE}^{CKZ} to C_{ORIG}^{CKZ} is probably due to a formatting error.

However, we still find no equivalence with respect to A_{RE}^{CKZ} and A_{ORIG}^{CKZ}.

$$\begin{aligned} A_{RE}^{CKZ} - A_{ORIG}^{CKZ} &= (2 - 2z - c[1 - 2c(1 - z)] - 4z - 5\lambda - \lambda(c(4 - 5c) + 5z(1 - c)^2) \\ &+ 3\lambda(1 - z)(1 - c)^2) \\ &- (2 - c(1 - 2c(1 - z) - 4z) - 2z - 5\lambda + ((4 - 5c)c + 5(-1 + c)^2 z)\lambda \\ &+ 3(1 - c)^2(1 - z)\lambda^2) \\ &= -4(1 + c)z + (3 - 14c + 13c^2 - 13(-1 + c)^2 z)\lambda \\ &+ 3(-1 + c)^2(-1 + z)\lambda^2 \\ &\neq 0. \end{aligned}$$

Unfortunately, we could not discover the cause for this difference. Therefore, the reason for $\theta^{CKZ,+}_{RE,\frac{1}{2}<\lambda<\frac{2}{3}} \neq \theta^{CKZ,+}_{ORIG}$ remains unclear.

5.3.1.3 *Potential explanation: Wrong $\pi_{PAYW\text{-}MP}$ for $\lambda < 1/2$*

In their comparison of boundaries for $\lambda < 1/2$, CKZ compare PAYW-MP to PAAP and PAYW-SP. However, they do not use the right minimum price for optimal profits. As shown in (39), profits in PAYW with the minimum price are given by

$$\pi^{\beta=0*}_{PAYW\text{-}MP,1} = \begin{cases} (1-c)^2\lambda(1-\lambda) & \lambda \leq \frac{1}{2} \\ \frac{(1-c)^2\lambda(2\lambda + 2\theta^{CKZ}(1-\lambda) - 1)}{2(2\lambda + \theta^{CKZ} - 1)} & \lambda > \frac{1}{2} \end{cases}.$$

However, in their comparison between PAYW-MP and PAYW-SP as well as PAYW-MP and PAAP for $\lambda < 2/3$, CKZ use

$$\pi^{CKZ,*}_{PAYW\text{-}MP} = \frac{(1-c)^2\lambda[1 - 2\theta^{CKZ}(1-\lambda) - 2\lambda]}{2(1-\theta^{CKZ} - 2\lambda)}.$$

Using the correct form (37) and comparing PAYW-MP to PAAP would give

$$\pi^{\beta=0*}_{PAYW\text{-}MP,1} - \pi^{*}_{PAAP} = (1-c)^2\lambda(1-\lambda) - \frac{(1-c)^2}{4} < 0.$$

Thus, PAAP is always greater than PAYW-MP for $\lambda < 1/2$. Therefore, CKZ's boundaries $\theta^{CKZ,+}_{ORIG}$ should not be considered for $\lambda < 1/2$.

5.3.2 Different boundaries

In addition to the previous errors, CKZ's formulation of boundaries is inconsequential for $\lambda \leq 2/3$. As noted above, they assume

- PAYW-MP to be optimal if $\theta^{CKZ} > \max\{1-2\lambda, \theta^{CKZ,+}(z,c,\lambda)\}$
- PAYW-SP to be optimal if $\theta^{CKZ} \leq \min\{\theta^{CKZ,+}(z,c,\lambda), E^{CKZ}_{ORIG}\}$ and
- PAAP to be optimal if $E^{CKZ}_{ORIG} \leq \theta^{CKZ} \leq \min\{1-2\lambda, \theta^{CKZ,+}(z,c,\lambda)\}$,

This implies that $1-2\lambda \geq \theta^{CKZ,+}(z,c,\lambda)$ and $1-2\lambda \leq \theta^{CKZ,+}(z,c,\lambda)$ can be the case. Furthermore, $\theta^{CKZ,+}(z,c,\lambda) \geq E^{CKZ}_{ORIG}$ or $\theta^{CKZ,+}(z,c,\lambda) \leq E$.

Imagine the following scenario, $\theta^{CKZ,+}(z,c,\lambda) \geq E^{CKZ}_{ORIG}$, $1-2\lambda \leq \theta^{CKZ,+}(z,c,\lambda)$. Furthermore, we assume $1-2\lambda \geq E^{CKZ}_{ORIG}$. In this case, they predict that

- PAYW-MP is optimal if $\theta^{CKZ} > \theta^{CKZ,+}(z,c,\lambda)$
- PAYW-SP is optimal if $\theta^{CKZ} \leq E^{CKZ}_{ORIG}$ and
- PAAP is optimal if $E^{CKZ}_{ORIG} \leq \theta^{CKZ} \leq 1 - 2\lambda$.

Consequently, in the area between $1 - 2\lambda$ and $\theta^{+}(z,c,\lambda)$ the optimal pricing scheme is undefined. This problem prevails for other relations between the boundaries. Consider Figure 2 for an overview of all possible combinations. Note the blank areas where optimal pricing is undefined.

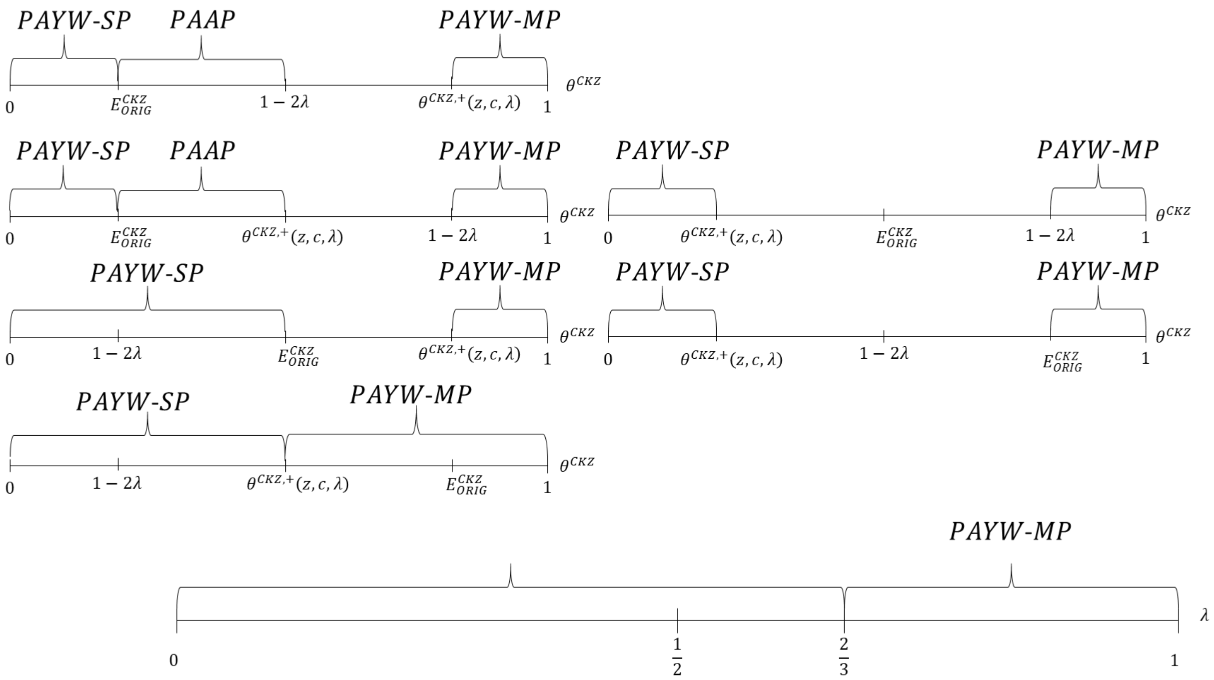

Figure 2: CKZ's boundary formulation

5.4 CKZ’s prices and AW profit functions

In the simulation in Section 6.3 in the paper, we offer a comparison between our profits and CKZ’s profits. However, as shown above, CKZ’s formulation is inconsistent. However, to estimate the magnitude of their misspecification, we calculate CKZ’s profits with the “correct” but not optimal profit functions and use the prices derived by CKZ. PAYW and PAAP prices do not need to be recalculated as the seller does not set a price in PAYW and as the PAAP price was calculated correctly by CKZ.

5.4.1 Profits for PAYW-MP

We use (48) as profit function and insert CKZ’s optimal price (36). This gives us

$$\pi_{PAYW\text{-}MP}\left(\underline{p}^{CKZ,*}\right) \tag{95}$$

$$= \frac{1}{2\lambda(1+\beta\lambda)}\Bigg(\Big(2c\lambda(1+\beta\lambda)(\lambda(-1+\omega)-\omega)$$
$$-\lambda^2(1+\beta\lambda)(-1+\omega)$$
$$+c^2(-1+\lambda)\left(-1+\omega+\lambda\big(1-\omega+\beta(1+\lambda+\omega-\lambda\omega)\big)\right)$$
$$+\max\left(c,\min\left(c+\lambda-c\lambda, c-\frac{(-1+c)\lambda\omega}{-1+2\lambda+\omega}\right)\right)\Big(2c(-1$$
$$+(2+\beta)\lambda)-2(c(-1+\lambda)-\lambda)(1+\beta\lambda)\omega$$
$$-\big(-1+\omega$$
$$+\lambda(2+\beta$$
$$+\beta\omega)\big)\max\left(c,\min\left(c+\lambda-c\lambda, c-\frac{(-1+c)\lambda\omega}{-1+2\lambda+\omega}\right)\right)\Big)\Big)\Bigg)$$
$$\coloneqq \pi_{PAYW\text{-}MP}^{CKZnew,*}.$$

5.4.2 Profits for PAYW-SP

We use (67) and (76) as profit functions. Optimal prices are given by (64). This leads us to the following profit function

$$\pi_{PAYW\text{-}SP(p_s^{CKZ,*})} \tag{96}$$

$$= \begin{cases} \frac{1}{6}(-1+c)^2\big(2+z(-2+3\lambda)\big)+\frac{1}{6}\omega(-3(1-c)^2 z\lambda)+ \\ \frac{1}{3}\omega\left(-1+z+c\left(-1+2\bar{\gamma}_{[0,1]}-2z\big(1+\bar{\gamma}_{[0,1]}\big)+c\big(-1+z+\bar{\gamma}_{[0,1]}+2z\bar{\gamma}_{[0,1]}\big)\right)\right) & \lambda \le \frac{2}{3} \\ \frac{1}{2}(-(1-c)^2\lambda(-1+\omega)+2c(-1+c\gamma)\omega) & \lambda > \frac{2}{3} \end{cases}$$
$$\coloneqq \pi_{PAYW\text{-}SP}^{CKZnew,*}.$$

Note: $\frac{2+c}{3} > p_h$ for $\lambda < 2/3$

5.5 Finding the optimal pricing scheme

In this section we offer the optimal pricing scheme choice using our optimal prices and the correct formulation of the profit function. We analyze analytically under which conditions the difference between two profit functions $\pi_1 - \pi_2$ is positive / negative which implies that policy 1 is preferred over policy 2 or vice versa.

5.5.1 Comparing PAAP to PAYW-MP

First, we want to consider the circumstances for which PAAP is chosen over PAYW-MP.

$$\pi^*_{PAAP} - \pi^*_{PAYW\text{-}MP} > 0.$$

As previously discussed, the optimal profits under PAYW-MP are given by a piecewise function:

$$\frac{(1-c)^2(1+\lambda\beta)}{4(1+\beta)} - \begin{cases} \dfrac{(1-c)^2\lambda(1-\lambda)}{1+\beta\lambda} & \lambda \leq 1/(2+\beta) \\ \dfrac{(1-c)^2\lambda}{2}\left(\dfrac{\lambda(2+\beta)+2\omega(1-\lambda)-1}{\big(2+\beta(1+\omega)\big)\lambda-1+\omega}\right) & \lambda > 1/(2+\beta) \end{cases} > 0.$$

First, we consider $\lambda \leq 1/(2+\beta)$.

$$\begin{aligned}\pi^*_{PAAP} - \pi^*_{PAYW\text{-}MP,1} &= \frac{(1-c)^2(1+\lambda\beta)}{4(1+\beta)} - \frac{(1-c)^2\lambda(1-\lambda)}{1+\beta\lambda} \\ &= \frac{(1-c)^2(1-(2+\beta)\lambda)^2}{4(1+\beta)(1+\beta\lambda)}.\end{aligned}$$

We inspect the nominator,

$$(1-c)^2(1-(2+\beta)\lambda)^2 \geq 0$$

This is always positive. Second, we move to the denominator,

$$4(1+\beta)(1+\beta\lambda) > 0.$$

Thus, the term is always positive. Hence, PAAP is always better for $\lambda \leq 1/(2+\beta)$.

Next, we consider the $\lambda > 1/(2+\beta)$,

$$\begin{aligned}\pi^*_{PAAP} - \pi^*_{PAYW\text{-}MP,2} &= \frac{(1-c)^2(1+\lambda\beta)}{4(1+\beta)} - \frac{(1-c)^2\lambda}{2}\left(\frac{\lambda(2+\beta)+2\omega(1-\lambda)-1}{\big(2+\beta(1+\omega)\big)\lambda-1+\omega}\right) \\ &= \frac{-(1-c)^2\big(1-\lambda(2+\beta)\big)^2(1-\omega)}{4(1+\beta)\big(1-\omega-\lambda(2+\beta+\beta\omega)\big)}.\end{aligned}$$

The nominator,

$$(1-c)^2\big(1-\lambda(2+\beta)\big)^2(1-\omega) \geq 0,$$

is always positive as $0 \leq \omega \leq 1$.

Next, we move to the denominator,

$$\left(2+\beta(1+\omega)\right)\lambda - 1 + \omega \geq (2+\beta)\lambda - 1 + \omega > 1 - 1 + \omega \geq 1.$$

Therefore, the whole expression is negative. Thus, PAYW-MP is always better for $\lambda > 1/(2+\beta)$.

Next, we derive the critical λ^{+} where profits from PAAP and PAYW-MP equal PAYW-SP. Due to the prominent role of generosity λ, we determine the critical threshold λ^{+} for which $\pi_1(\lambda^{+}) = \pi_2(\lambda^{+})$; this implies that for $\lambda < \lambda^{+}$ policy 1 dominates policy 2 and vice versa for $\lambda > \lambda^{+}$. Since the comparison between profit functions apply in each case to a certain domain of λ (small, intermediate, large), the respective domain is also effective for λ^{+}. However, only for small λ, $\lambda^{+} \notin [0; 1/(2+\beta)]$ might occur. This stands in contrast to CKZ who derive the optimal pricing scheme based on the critical threshold $\theta^{CKZ,+.}$

5.5.2 Comparing PAAP to PAYW-SP

We compare PAAP to PAYW-SP (for $B_s > 1/2$ and $B_s \leq 1/2$) and solve for λ. Thus, we first collect terms for a more compact computation,

$$\begin{aligned}\pi^{*}_{PAAP} &= \frac{(1-c)^2(1+\beta\lambda)}{4(1+\beta)}\\ &= A_{AW} + \beta\lambda A_{AW}\end{aligned}$$

with

$$A_{AW} = \frac{(1-c)^2}{4(1+\beta)}$$

and

$$\begin{aligned}\pi^{*}_{PAYW-SP} &= (1-\omega)(1-c)^2\left(\frac{\lambda z}{2} + g_{1\lambda}(1-z)\right) - c\omega\left(1 - \bar{\gamma}_{[0,1]}\left(cz + g_{2\lambda}(1-z)\right)\right)\\ &= B_{AW}\lambda - B_{AW}\lambda\omega + D_{AW} - \omega D_{AW} - c\omega E_{AW}\end{aligned}$$

with $B_{AW} = \frac{(1-c)^2 z}{2}$, $D_{AW} = (1-c)^2(1-z)g_{1\lambda}$, $E_{AW} = \left(1 - \bar{\gamma}_{[0,1]}\left(cz + g_{2\lambda}(1-z)\right)\right)$.

We compare the profit functions,

$$\pi^{*}_{PAYW} = \pi^{*}_{PAYW-SP}$$

$$\begin{aligned}A_{AW} + \beta\lambda A_{AW} &= B_{AW}\lambda - B_{AW}\lambda\omega + D_{AW} - \omega D_{AW} - c\omega E_{AW}\\ \lambda(A_{AW}\beta - B_{AW} + B_{AW}\omega) &= D_{AW} - \omega D_{AW} - c\omega E_{AW} - A_{AW}\end{aligned}$$

we solve for λ

$$\lambda = \frac{D_{AW} - \omega D_{AW} - c\omega E_{AW} - A_{AW}}{A_{AW}\beta - B_{AW} + B_{AW}\omega} \coloneqq \lambda_{\leq}^{+}. \tag{97}$$

We resubstitute and get

$$\lambda_{\leq}^{+} = \frac{g_{\leq}}{f_{\leq}}$$

with

$$f_{\leq} = (1-c)^2 \left(\frac{\beta}{4(1+\beta)} - \frac{(1-\omega)z}{2}\right) \tag{98}$$

(AW.A.17)

$$g_{\leq} = (1-c)^2 \left((1-z)(1-\omega)g_{1\lambda} - \frac{1}{4(1+\beta)}\right) - c\omega\left(1 - \bar{\gamma}_{[0,1]}(cz + (1-z)g_{2\lambda})\right).$$

In this case $\lambda_{\leq}^{+}(\omega)$ is considered a function depending on ω (and $\beta, c, \bar{\gamma}_{[0,1]}, z$) rather than a calculated threshold value $\lambda_{\leq}^{+}$ (for a specific ω). In particular, the algebraic sign of $\left.\frac{\partial \lambda^{+}(\omega)}{\partial \omega}\right|_{\lambda_{\leq}^{+}}$ helps to determine the dominant policy. From an interpretative point of view, the threshold $\lambda_{\leq}^{+}$ refers to the amount of consumers' generosity, $\left.\frac{\partial \lambda^{+}(\omega)}{\partial \omega}\right|_{\lambda_{\leq}^{+}}$ to its (local) dependence on consumers' fair-mindedness.

We treat $f_{\leq}$ and $g_{\leq}$ as rational functions of ω.

$$\lambda^{+} = \frac{g_{\leq}(\omega)}{f_{\leq}(\omega)}$$

This allows us to use implicit differentiation,

$$f_{\leq}' \frac{\partial \lambda_{\leq}^{+}}{\partial \omega} \lambda^{+\prime}{}_{\leq} + \lambda_{\leq}^{+} f_{\leq}' = g_{\leq}',$$

which gives us

$$\left.\frac{\partial \lambda_{\leq}^{+}(\omega)}{\partial \omega}\right|_{\lambda_{\leq}^{+}} = \frac{f_{\leq} g_{\leq}' - g_{\leq} f_{\leq}'}{f_{\leq}^{2}}. \tag{99}$$

(AW.A.18)

Which in turn is the derivation of λ with respect to ω at a specific value of $\lambda_{\leq}^{+}$.

5.5.3 Comparing PAYW-MP to PAYW-SP

We only need to compare large λ ($\lambda > 1/(2+\beta)$), as PAYW-MP is otherwise dominated by PAAP.

We compare PAYW-MP (54) to PAYW-SP (84) and solve for λ.

Again, we need to collect terms to facilitate computation, this results in

$$\pi^*_{PAYW\text{-}MP,2} = \frac{(1-c)^2\lambda}{2}\left(\frac{\lambda(2+\beta)+2\omega(1-\lambda)-1}{\big(2+\beta(1+\omega)\big)\lambda-1+\omega}\right)$$
$$= \frac{G_{AW}\lambda^2F_{AW}-2\omega\lambda^2F_{AW}-2\omega\lambda F_{AW}-\lambda F_{AW}}{2(G_{AW}\lambda+\beta\omega\lambda+\omega-1)}$$

With $F_{AW} = (1-c)$, $G_{AW} = 2+\beta$.

for PAYW-MP and

$$\pi^*_{PAYW\text{-}SP} = (1-\omega)(1-c)^2\left(\frac{\lambda z}{2}+g_{1\lambda}(1-z)\right)-c\omega\left(1-\bar{\gamma}_{[0,1]}\big(cz+g_{2\lambda}(1-z)\big)\right)$$
$$= H_{AW}\lambda + I_{AW}$$

with $H_{AW} = \frac{(1-\omega)(1-c)^2z}{2}$, $I_{AW} = (1-\omega)(1-c)^2(1-z)g_{1\lambda} - c\omega\left(1-\bar{\gamma}_{[0,1]}(cz + (1-z)g_{2\lambda})\right)$ for PAYW-SP.

We set the two functions equal to each other,

$$\frac{G_{AW}\lambda^2F_{AW}-2\omega\lambda^2F_{AW}-2\omega\lambda F_{AW}-\lambda F_{AW}}{2(G_{AW}\lambda+\beta\omega\lambda+\omega-1)} = H_{AW}\lambda + I_{AW}$$

and collect the λ-terms

$$\frac{1}{2}G_{AW}\lambda^2F_{AW}-\omega\lambda^2F_{AW}-\omega\lambda F_{AW}-\frac{1}{2}\lambda F_{AW} = (H_{AW}\lambda+F)(G_{AW}\lambda+\beta\omega\lambda+\omega-1)$$

$$\left(\frac{F_{AW}G_{AW}}{2}-F_{AW}\omega-H_{AW}G_{AW}-H_{AW}\beta\omega\right)\lambda^2$$
$$+\left(H_{AW}-I_{AW}G_{AW}-\beta\omega I_{AW}-\omega F_{AW}-\frac{1}{2}F_{AW}-H_{AW}\omega\right)\lambda+I_{AW}-\omega I_{AW}$$
$$= 0.$$

We solve the quadratic equation

$$\lambda = \frac{-g_> \pm\sqrt{g_>^2-4f_>j_>}}{2f_>} := \lambda^+_{>\ 1,2} \tag{100}$$

(AW.A.19)

with

$$f_> = H_{AW} - I_{AW}G_{AW} - \beta\omega I_{AW} - \omega F_{AW} - \frac{1}{2}F_{AW} - H_{AW}\omega$$

$$j_> = (1-\omega)I_{AW}.$$

Finally, we resubstitute. This results in

$$\lambda^{+}_{>\,1,2} = \frac{-g_> \pm \sqrt{g_>^2 - 4f_> j_>}}{2f_>}$$

with

$$f_> = (1-c)^2\left(\frac{z\beta\omega^2}{2} + (1-z)\left(\frac{2+\beta}{2} - \omega\right)\right) \quad (101)$$

$$g_> = (1-c)^2\left(\omega^2\left(\frac{z}{2} + \beta(1-z)g_{1\lambda}\right) - (1-z)\left(\frac{1}{2} + (2+\beta-2\omega)g_{1\lambda}\right) - (1+z)\omega\right) + c\omega(2+\beta+\beta\omega)\left(1 - \bar{\gamma}_{[0,1]}\big(cz + g_{2\lambda}(1-z)\big)\right) \quad \text{(A.19)}$$

$$j_> = (1-c)^2(1-\omega)^2(1-z)g_{1\lambda} - c\omega(1-\omega)\left(1 - \bar{\gamma}_{[0,1]}(cz + (1-z)g_{2\lambda})\right).$$

Thus, we can collect the optimal pricing choice as in Proposition 4:

Updated Proposition 4. PAYW is never the preferred pricing policy.

(i) Low levels of generosity $\left(0 \le \lambda \le \frac{1}{2+\beta}\right)$:

if $\lambda^{+}_{\lesseqgtr} < 0$ then PAAP is optimal;

if $\lambda^{+}_{\lesseqgtr} \in \left[0; \frac{1}{2+\beta}\right]$ and $\left.\frac{\partial \lambda^{+}_{\lesseqgtr}(\omega)}{\partial \omega}\right|_{\lambda^{+}_{\lesseqgtr}} > 0$ PAAP is optimal for $\lambda < \lambda^{+}_{\lesseqgtr}$ but PAYW-SP for $\lambda > \lambda^{+}_{\lesseqgtr}$;

if $\lambda^{+}_{\lesseqgtr} \in \left[0; \frac{1}{2+\beta}\right]$ and $\left.\frac{\partial \lambda^{+}_{\lesseqgtr}(\omega)}{\partial \omega}\right|_{\lambda^{+}_{\lesseqgtr}} < 0$ PAYW-SP is optimal;

if $\lambda^{+}_{\lesseqgtr} > \frac{1}{2+\beta}$ PAAP is optimal for $\left.\frac{\partial \lambda^{+}_{\lesseqgtr}(\omega)}{\partial \omega}\right|_{\lambda^{+}_{\lesseqgtr}} > 0$ but PAYW-SP for $\left.\frac{\partial \lambda^{+}_{\lesseqgtr}(\omega)}{\partial \omega}\right|_{\lambda^{+}_{\lesseqgtr}} < 0$.

(ii) Intermediate levels of generosity $\left(\frac{1}{2+\beta} < \lambda \le \frac{2(1+B_S)}{3}\right)$:

if $\lambda \in \left[\frac{1}{2+\beta}; \lambda^{+}_{>}\right]$ then PAYW-SP is optimal;

if $\lambda \notin \left[\frac{1}{2+\beta}; \lambda^{+}_{>}\right]$ then PAYW-MP is optimal.

(iii) High levels of generosity $\left(\frac{2(1+B_S)}{3} < \lambda \le 1\right)$: PAYW-MP is optimal.

For an interpretation of these results please cf. to Akbari and Wagner (2020).

6 Simulation analysis

We run a simulation analysis in our submitted manuscript. The procedure was as follows,

1) Simulate the data for different points of the parameter space,
2) estimate the effect of the misspecifications of CKZ.

To allow reproducibility, the code for data generation and analysis follows as part of this section.

6.1 Data generation

The data was generated in Mathematica 11.0. The following code generates an Excel spreadsheet with profit calculations for all combinations of $\omega \in \{0,.25,.50,0.75\}$, $\lambda \in \{0,.25,.5,0.75,1\}$, $\beta \in \{0,1.5,4,6.5,9\}$, $c \in \{0,.25,.5,.75\}$, $\bar{\gamma}_{[0,1]} \in \{0,.25,.5,.75\}$, and $z \in \{0,.25,0.5,.75\}$ for all pricing schemes, and for our profit functions (6), (17), (54), and (84) and the adapted profit functions by CKZ (7), (19), (43), (61), (63), (95), and (96).

The profits following our profit functions are calculated by

$$\text{PAAP}[\beta\_,\ \omega\_,\ \lambda\_,\ c\_,\ \gamma\_,\ z\_] := \frac{(1-c)^2 (1+\beta\lambda)}{4(1+\beta)}$$

$$\text{AWPAYW}[\beta\_,\ \omega\_,\ \lambda\_,\ c\_,\ \gamma\_,\ z\_] := (\lambda (1-\omega)(1-c)\text{^}2)/2 - c\,\omega\,(1-c\,\gamma)$$

$$\text{AWPAYWMP}[\beta\_,\ \omega\_,\ \lambda\_,\ c\_,\ \gamma\_,\ z\_] := \begin{cases} \frac{(1-c)^2(1-\lambda)\lambda}{1+\beta\lambda} & \lambda \leq 1/(2+\beta) \\ \frac{(1-c)^2\lambda(-1+(2+\beta)\lambda+2(1-\lambda)\omega)}{2(-1+\omega+\lambda(2+\beta(1+\omega)))} & \lambda > 1/(2+\beta) \\ 0 & \text{True} \end{cases}$$

$$\text{AWPAYWSP}[\beta\_,\ \omega\_,\ \lambda\_,\ c\_,\ \gamma\_,\ z\_] :=$$

$$\begin{cases} (1-c)^2\left(\frac{1-z}{4}+\frac{z\lambda}{2}\right)(1-\omega) - c\left(1-(1-z+c\,z)\,\gamma\right)\omega & \frac{c\gamma\omega}{(1-c)(1-\omega)} > \frac{1}{2} \\ (1-c)^2(1-\omega)\left(\frac{z\lambda}{2}+\frac{1}{3}(1-z)\left(1-\frac{c^2\gamma^2\omega^2}{(1-c)^2(1-\omega)^2}\right)\right) - \\ \quad c\,\omega\left(1-\gamma\left(c\,z+\frac{1}{3}(1-z)\left(2+c+\frac{2c\gamma\omega}{1-\omega}\right)\right)\right) & \frac{c\gamma\omega}{(1-c)(1-\omega)} \leq \frac{1}{2} \,\&\&\, \lambda \leq \frac{2}{3}\left(1+\frac{c\gamma\omega}{(1-c)(1-\omega)}\right) \\ (1-c)^2(1-\omega) \\ \quad \left(\frac{z\lambda}{2}+(1-z)\lambda\left(\frac{1}{2}-\frac{c^2\gamma^2\omega^2}{(1-c)^2(-2+3\lambda)(1-\omega)^2}\right)\right) - \\ \quad c\,\omega\left(1-\gamma\left(c\,z+(1-z)\left(c+\frac{2c\gamma\lambda\omega}{(-2+3\lambda)(1-\omega)}\right)\right)\right) & \frac{c\gamma\omega}{(1-c)(1-\omega)} \leq \frac{1}{2} \,\&\&\, \lambda > \frac{2}{3}\left(1+\frac{c\gamma\omega}{(1-c)(1-\omega)}\right) \\ 0 & \text{True} \end{cases}$$

The profits with the prices derived by CKZ in the correct profit functions are calculated by

```
CKZPAYWMPinAW[β_, ω_, λ_, c_, γ_, z_] =
 - 1/(2 λ (1 + β λ)) (λ^2 (1 + β λ) (-1 + ω) - 2 c λ (1 + β λ) (-λ + (-1 + λ) ω) +
    c^2 (-1 + λ) (1 - λ (1 + β + β λ) - ω + (1 + β (-1 + λ)) λ ω) +
    2 (c - c (2 + β) λ + (c (-1 + λ) - λ) (1 + β λ) ω) Max[c, Min[c + λ - c λ, c - ((-1 + c) λ ω)/(-1 + 2 λ + ω)]] +
                                                        größte···  kleinstes Element
    (-1 + ω + λ (2 + β + β ω)) Max[c, Min[c + λ - c λ, c - ((-1 + c) λ ω)/(-1 + 2 λ + ω)]]^2)
CKZPAYWSPinAW[β_, ω_, λ_, c_, γ_, z_] =
 { ((1 + 1/3 (-2 - c)) (-c + (2+c)/3) + 1/2 (c^2/2 - 1/3 c (2 + c) + 1/18 (2 + c)^2)) (1 - z) (1 - ω) +     λ ≤ 2/3
    z (λ/2 - c λ + (c^2 λ)/2) (1 - ω) - c (z (1 - c γ) ω + (1 - z) (1 - 1/3 (2 + c) γ) ω)
   1/2 (-(-1 + c)^2 λ (-1 + ω) + 2 c (-1 + c γ) ω)                                                          λ > 2/3
   0                                                                                                        True
```

The profits with CKZs profit functions are calculated by

```
CKZPAYW[β_, ω_, λ_, c_, γ_, z_] = (λ (1 - ω) (1 - c)^2) / (2) - c ω
CKZPAYWMP[β_, ω_, λ_, c_, γ_, z_] = ((1 - c)^2 λ (1 - 2 ω (1 - λ) - 2 λ)) / (2 (1 - ω - 2 λ))
                                   { -c ω + ((2 + z (3 λ - 2)) (1 - c)^2 (1 - ω)) / 6   λ ≤ 2/3
CKZPAYWSP[β_, ω_, λ_, c_, γ_, z_] = { (λ / 2) (1 - ω) (1 - c)^2 - c ω                  λ > 2/3
                                   { 0                                                  True
```

Data generation will be performed as follows:

```
fObs = Flatten[Table[{
       ebne ein  Tabelle
    PAAP[β, ω, λ, c, γ, z],
    AWPAYW[β, ω, λ, c, γ, z],
    AWPAYWMP[β, ω, λ, c, γ, z],
    AWPAYWSP[β, ω, λ, c, γ, z],
    CKZPAYWMPinAW[β, ω, λ, c, γ, z],
    CKZPAYWSPinAW[β, ω, λ, c, γ, z],
    CKZPAYW[β, ω, λ, c, γ, z],
    CKZPAYWMP[β, ω, λ, c, γ, z],
    CKZPAYWSP[β, ω, λ, c, γ, z],
    β,
    ω,
    λ,
    c,
    γ,
    z}, {ω, {0, .25, .50, 0.75}},
   {λ, {0.0001, .250001, .500001, 0.750001, 1}}, {β, {9, 6.5, 4, 1.5, 0}},
   {c, {0, .25, .5, .75}}, {γ, {0, .25, .5, .75}}, {z, {0, .25, 0.5, .75}}], 5];
```

Finally, we export the data to an excel spreadsheet.

```
Export[NotebookDirectory[] <> "profits_AW_CKZaw_CKZ.xls",
exporti··· Notebook-Verzeichnis
 Map[NumberForm[N@#, {4, 3}] &, fObs, {2}]]
 w··· Zahlenform     numerischer Wert
```

6.2 Data analysis

Data analysis was performed in R 3.6.2. In short, we load the previously generated data and measure forecast accuracy using different approaches. The final manuscript only makes use of the MAAPE and iMAPE.

6.2.1 Setup

First, the necessary data and R packages are loaded.

```
library(dplyr)
library(readxl)
library(tidyverse)
library(knitr)
library(kableExtra)
library(xtable)
```

Second, the data is loaded, and the variables are renamed.

```
path <- "\\FILEPATH\\profits_AW_CKZaw_CKZ.xls"

profits <- read_excel(path,
                      col_names = FALSE)

profits <- profits %>%
    rename(PAAP = X__1,
           AWPAYW = X__2,
           AWPAYWMP = X__3,
           AWPAYWSP = X__4,
           CKZinAWPAYWMP = X__5,
           CKZinAWPAYWSP = X__6,
           CKZPAYW = X__7,
           CKZPAYWMP = X__8,
           CKZPAYWSP = X__9,
           beta = X__10,
           omega = X__11,
           lambda = X__12,
```

```
        c = X__13,
        gamma = X__14,
        z = X__15)
```

6.2.2 Measuring the forecast error

Next, different alternatives for measuring forecast accuracy are defined.

```
imape <- function(a, f) {
  data <- data.frame(a, f) %>%
    filter(a != 0)
  data$e <-  data$a - data$f
  data$p <- data$e / data$a
  mape <- mean(abs(data$p))
  mape
}

maape <- function(a, f){
  aape <- atan(abs((a - f)/ a))
  maape <- mean(aape)
  maape
}

smape <- function(a, f){
  ape <- abs((a - f)/((a + f)/2))
  smape <- mean(ape)
  smape
}

madmean <-  function(a, f){
  ad <- abs(a - f)
  mad <- mean(ad)
  madmean <- mad/(mean(a))
  madmean
}


smdape <- function(a, f){
  ape <- abs((a - f)/((a + f)/2))
  smdape <- median(ape)
  smdape
```

```
}
```

6.2.3 Calculation of differences and relative differences

Differences between the profits after CKZ and the correct formulation are calculated.

```
#----------------- Calculation of differences ----------------- #
profits_diff <- profits %>%
    mutate(PAYWDiff = AWPAYW - CKZPAYW,
           PAYWMPDiffinAW = AWPAYWMP - CKZinAWPAYWMP,
           PAYWSPDiffinAW = AWPAYWSP - CKZinAWPAYWSP,
           PAYWMPDiffPiCKZ = AWPAYWMP - CKZPAYWMP,
           PAYWSPDiffPiCKZ = AWPAYWSP - CKZPAYWSP
           ) %>%
    mutate(PAYWRelDiff = PAYWDiff/(CKZPAYW),
           PAYWMPRelDiffinAW = PAYWMPDiffinAW/CKZinAWPAYWMP,
           PAYWSPRelDiffinAW =PAYWSPDiffinAW/CKZinAWPAYWSP,
           PAYWMPRelDiff = PAYWMPDiffPiCKZ/CKZPAYWMP ,
           PAYWSPRelDiff = PAYWSPDiffPiCKZ/CKZPAYWSP)
```

6.2.4 Calculation of optimal choices

In this step, the calculation of optimal choices for the seller is performed for both of CKZ's specifications and the correct specification by finding the maximum profit for each scenario.

```
profits_aw <-  tibble(PAAP = profits_diff$PAAP,
                      PAYWMP = profits_diff$AWPAYWMP,
                      PAYWSP = profits_diff$AWPAYWSP)


profits_CKZ_inAW <-  tibble(PAAP = profits_diff$PAAP,
                            PAYWMP = profits_diff$CKZinAWPAYWMP,
                            PAYWSP = profits_diff$CKZinAWPAYWSP)


profits_CKZ_own_function <-  tibble(PAAP = profits_diff$PAAP,
                                    PAYWMP = profits_diff$CKZPAYWMP,
                                    PAYWSP = profits_diff$CKZPAYWSP)


AW_choice <- colnames(profits_aw)[apply(profits_aw,1,which.max)]
CKZ_inAW_choice <- colnames(profits_CKZ_inAW)[apply(profits_CKZ_inAW,1,which.max)]
CKZ_own_function_choice <- colnames(profits_CKZ_own_function)[apply(profits_CKZ_own
_function,1,which.max)]
```

This gives us the following interim results. Following the correct specification, we find the that seller decides in favor of pricing schemes as follows:

```
table(AW_choice) %>%
    kable() %>%
  kable_styling()
```

AW_choice	Freq
PAAP	1212
PAYWMP	3805
PAYWSP	1383

Using CKZ's prices in correct profit function would yield the following optimal pricing decisions:

```
table(CKZ_inAW_choice)   %>%
    kable() %>%
  kable_styling()
```

CKZ_inAW_choice	Freq
PAAP	2238
PAYWMP	2782
PAYWSP	1380

Using CKZ's initial profit functions would yield the following decisions:

```
table(CKZ_own_function_choice)  %>%
    kable() %>%
  kable_styling()
```

CKZ_own_function_choice	Freq
PAAP	1104
PAYWMP	4160

CKZ_own_function_choice	Freq
PAYWSP	1136

The following tables show differences between the different models:

```
 disagreement_inAW <- table(CKZ_inAW_choice, AW_choice)
disagreement_inAW %>%
    kable() %>%
    add_header_above(c("CKZ in AW", "AW"=3)) %>%
  kable_styling()
```

CKZ in AW	AW		
	PAAP	PAYWMP	PAYWSP
PAAP	1212	1026	0
PAYWMP	0	2686	96
PAYWSP	0	93	1287

6.2.5 Calculation of forecast error

Now we can define the forecast error for the different setups:

6.2.5.1 CKZ in our profit functions

```
tabelle <- cbind(
    c(
        imape(profits_diff$AWPAYW, profits_diff$CKZPAYW),
        maape(profits_diff$AWPAYW, profits_diff$CKZPAYW),
        smape(profits_diff$AWPAYW, profits_diff$CKZPAYW),
        madmean(profits_diff$AWPAYW, profits_diff$CKZPAYW)
    ),

    c(imape(profits_diff$AWPAYWMP, profits_diff$CKZinAWPAYWMP),
      maape(profits_diff$AWPAYWMP, profits_diff$CKZinAWPAYWMP),
      smape(profits_diff$AWPAYWMP, profits_diff$CKZinAWPAYWMP),
      madmean(profits_diff$AWPAYWMP, profits_diff$CKZinAWPAYWMP)
    ),
```

```
    c(imape(profits_diff$AWPAYWSP, profits_diff$CKZinAWPAYWSP),
      maape(profits_diff$AWPAYWSP, profits_diff$CKZinAWPAYWSP),
      smape(profits_diff$AWPAYWSP, profits_diff$CKZinAWPAYWSP),
      madmean(profits_diff$AWPAYWSP, profits_diff$CKZinAWPAYWSP)
    ))

colnames(tabelle) <- c("PAYW", "PAYW-MP", "PAYW-SP")
rownames(tabelle) <- c("iMAPE", "MAAPE", "sMAPE", "MAD/Mean")


kable(tabelle, digits = 2)%>%
  kable_styling()
```

	PAYW	PAYW-MP	PAYW-SP
iMAPE	0.29	0.24	0.10
MAAPE	0.19	0.21	0.07
sMAPE	0.20	0.35	0.11
MAD/Mean	-0.84	0.08	-1.95

6.2.5.2 *CKZ's profit functions*

```
tabelle <- cbind(
    c(
        imape(profits_diff$AWPAYW, profits_diff$CKZPAYW),
        maape(profits_diff$AWPAYW, profits_diff$CKZPAYW),
        smape(profits_diff$AWPAYW, profits_diff$CKZPAYW),
        madmean(profits_diff$AWPAYW, profits_diff$CKZPAYW)
    ),

    c(imape(profits_diff$AWPAYWMP, profits_diff$CKZPAYWMP),
      maape(profits_diff$AWPAYWMP, profits_diff$CKZPAYWMP),
      smape(profits_diff$AWPAYWMP, profits_diff$CKZPAYWMP),
      madmean(profits_diff$AWPAYWMP, profits_diff$CKZPAYWMP)
    ),

    c(imape(profits_diff$AWPAYWSP, profits_diff$CKZPAYWSP),
      maape(profits_diff$AWPAYWSP, profits_diff$CKZPAYWSP),
```

```
      smape(profits_diff$AWPAYWSP, profits_diff$CKZPAYWSP),
      madmean(profits_diff$AWPAYWSP, profits_diff$CKZPAYWSP)
    ))

colnames(tabelle) <- c("PAYW", "PAYW-MP", "PAYW-SP")
rownames(tabelle) <- c("iMAPE", "MAAPE", "sMAPE", "MAD/Mean")


kable(tabelle, digits = 2) %>%
  kable_styling()
```

	PAYW	PAYW-MP	PAYW-SP
iMAPE	0.29	6792.04	0.72
MAAPE	0.19	0.36	0.29
sMAPE	0.20	149.04	0.47
MAD/Mean	-0.84	3866.55	-14.93

References

Akbari K, Wagner U (2020) Comments and Refinements on the Pay as you wish Model by Chen et al. (2017).

Chen Y, Koenigsberg O, Zhang ZJ (2017) Pay-as-You-Wish Pricing. *Marketing Science*. 36(5):780–791.

List of Symbols

The following symbols are a collection and comparison of symbols in Akbari and Wagner (2020), CKZ and this report. The symbols are listed in order of appearance.

Section 2: PWYW and critical costs

Akbari and Wagner (2020)	CKZ	This Report	Explanation
i		i	Index of individual consumer
u_i		u_i	Consumer i's utility
r_i		r_i	Consumer i's consumption utility
p_i		p_i	Consumer i's price
β_i		β_i	Consumer i's disadvantageous inequity aversion parameter
γ_i		γ_i	Consumer i's advantageous inequity aversion parameter
r_{i0}		r_{i0}	Consumer i's perceived fair price
c		c	Firm's costs
λ		λ	Generosity
p_h		p_h	Highest possible fair price
p^*_{PAAP}		p^*_{PAAP}	Optimal price in PAAP
$PAAP$		$PAAP$	Pay as asked pricing
AW		AW	Akbari and Wagner (2020)
		$u_{\tilde{r}}$	Marginal consumer's utility
		$\tilde{r}$	Consumption utility for the marginal consumer with advantageous inequity aversion
		p_{PAAP}	Price in PAAP
		$\tilde{r}^+$	Critical consumption utility of the marginal consumer
		β	Disadvantageous inequity aversion parameter
		r^+	Consumption utility marginal consumer with disadvantageous inequity
		π_{PAAP}	Profits under PAAP
		π^*_{PAAP}	Optimal profits under PAAP
		$\pi^{\beta=0,*}_{PAAP}$	Optimal profits under PAAP with $\beta = 0$

Akbari and Wagner (2020)	CKZ	This Report	Explanation
$Segment\ I^{a}_{PAYW}$			Freeloader segment in PAYW (high consumption utility)
$Segment\ I^{b}_{PAYW}$			Freeloader segment in PAYW (low consumption utility)
$Segment\ II_{PAYW}$			Non-buyer segment in PAYW (less fair-minded consumers)
$Segment\ III_{PAYW}$			Paying customers segment in PAYW
$Segment\ IV_{PAYW}$			Non-buyer segment in PAYW (more fair-minded consumers)
$\phi(r)$		$\phi(r)$	Density function of consumption utilities
$h(\gamma)$		$h(\gamma)$	Density function of advantageous inequity aversion
θ^{CKZ}		θ^{CKZ}	CKZ's share of freeloaders
θ		θ_{PAYW}	Share of freeloaders
δ		δ_{PAYW}	Share of less fair-minded consumers who remain non-buyers in PAYW
π_{PAYW}		π^{*}_{PAYW}	Profits under PAYW
π^{CKZ}_{PAYW}		$\pi^{*,CKZ}_{PAYW}$	Profits under PAYW in CKZ
$\Delta\,\pi$		$\Delta\,\pi$	Profit difference in PAYW between AW and CKZ
ω		ω	Less fair-minded consumers
$(1-\omega)$		$(1-\omega)$	More fair-minded consumers
$\bar{\gamma}_{[0,1]}$		$\bar{\gamma}_{[0,1]}$	Mean of γ_i in [0,1]
$h_{[0,1]}$		$h_{[0,1]}$	Distribution of γ_i in [0,1]
c^{+}		c^{+}	Critical cost level above which PAYW cannot be profitable
A		A_{COMP}	Auxiliary variable in c^{+}
B		B_{COMP}	Auxiliary variable in c^{+}
		p^{*}_{i}	Consumers optimal price in PAYW
		D_{COMP}	Auxiliary variable in c^{+}
		E_{COMP}	Auxiliary variable in c^{+}
		ω^{+}	Critical threshold of freeloaders
		A^{CKZ}_{COMP}	Auxiliary variable in c^{+} for CKZ
		B^{CKZ}_{COMP}	Auxiliary variable in c^{+} for CKZ
		D^{CKZ}_{COMP}	Auxiliary variable in c^{+} for CKZ
	c^{*}	$c^{CKZ,+}$	Critical threshold of costs for PAYW to dominate PAAP

Section 3: Minimum price

Akbari and Wagner (2020)	CKZ	This Report	Explanation
$\underline{p}$		$\underline{p}$	Minimum price
$PAYW\text{-}MP$		MP	PAYW with a minimum price
r^{+}		r^{+}	Critical consumption utility that a consumer buys in PAYW-MP
p_f		p_f	Perceived fair price when $r_i > c$
$\underline{p}^{u}$		$\underline{p}^{u}$	Consumption utility at the minimum price
$\underline{r}$		$\underline{r}$	Critical consumption utility for which minimum price equal fair price
$Segment\ I^{a}_{PAYW\text{-}MP}$			Freeloader segment in PAYW-MP (suffering from advantageous inequity)
$Segment\ I^{b}_{PAYW\text{-}MP}$			Freeloader segment in PAYW-MP (suffering from disadvantageous inequity)
$Segment\ II^{a}_{PAYW\text{-}MP}$			Non-buyer segment in PAYW-MP because of disadvantageous inequity aversion (less fair-minded consumers)
$Segment\ II^{b}_{PAYW\text{-}MP}$			Non-buyer segment in PAYW-MP because of low consumption utility (less fair-minded consumers)
$Segment\ III^{a}_{PAYW\text{-}MP}$			Paying customers segment in PAYW-MP (fair price)
$Segment\ III^{b}_{PAYW\text{-}MP}$			Paying customers segment in PAYW-MP (minimum price)
$Segment\ IV^{a}_{PAYW\text{-}MP}$			Non-buyer segment in PAYW-MP because of disadvantageous inequity aversion (more fair-minded consumers)
$Segment\ IV^{b}_{PAYW\text{-}MP}$			Non-buyer segment in PAYW-MP because of low consumption utility (more fair-minded consumers)
$\pi_{PAYW\text{-}MP}$		$\pi_{PAYW\text{-}MP}$	Profits under PAYW-MP
$\underline{p}^{*}$		$\underline{p}^{*}$	Optimal minimum price
$\underline{p}^{CKZ,*}$		$\underline{p}^{CKZ,*}$	Optimal minimum price in CKZ
	k	G_{MP}	Auxiliary variable distinguishing the optimal minimum price for high/low values of generosity, λ
	a		Auxiliary variable for minimum price
	b		Auxiliary variable for minimum price
		$\pi^{*}_{PAYW\text{-}MP}$	Optimal profits under PAYW-MP
		$A^{\beta=0}_{MP}$	Auxiliary variable for minimum price when $\beta = 0$

Akbari and Wagner (2020)	CKZ	This Report	Explanation
		$\pi_{PAYW-MP}^{\beta=0}$	Profits in PWYW-MP with no disadvantageous inequity aversion
		$B_{MP}^{\beta=0}$	Auxiliary variable for minimum price when β=0
		$D_{MP}^{\beta=0}$	Auxiliary variable for minimum price when β=0
		p_h	Highest possible fair price
		$E_{MP}^{\beta=0}$	Auxiliary variable for minimum price
		$\pi_{PAYW-MP}^{\beta=0,*}$ $\pi_{PAYW-MP}^{CKZ}$	Profits in PWYW-MP for CKZ
		$\underline{p}^{\beta=0,*}$ $\underline{p}^{CKZ,*}$	Optimal minimum price in PWYW-MP for CKZ
		$\pi_{PAYW-MP}^{CKZ,*}$	Optimal profits in PWYW-MP for CKZ
		A_{MP}	Auxiliary variable for minimum price
		B_{MP}	Auxiliary variable for minimum price
		D_{MP}	Auxiliary variable for minimum price
		E_{MP}	Auxiliary variable for minimum price
		F_{MP}	Auxiliary variable for minimum price
		G_{MP}	Auxiliary variable for minimum price
		$\pi_{PAYW-MP}^{*}$	Optimal profits in PWYW-MP
		H_{MP}	Auxiliary variable for minimum price
		I_{MP}	Auxiliary variable for minimum price
		J_{MP}	Auxiliary variable for minimum price
		K_{MP}	Auxiliary variable for minimum price
		L_{MP}	Auxiliary variable for minimum price
		M_{MP}	Auxiliary variable for minimum price

Section 4: Suggested price

Akbari and Wagner (2020)	CKZ	This Report	Explanation
$PAYW\text{-}SP$		s	PAYW with a suggested price
p_s		p_s	Suggested price
$(1-z)$		$(1-z)$	Probability that consumers pay attention to the suggested price
z		z	Probability that consumers ignore the suggested price
r_s		r_s	Critical consumption utility for which suggested price equal fair price
$Segment\ I^a_{PAYW\text{-}SP}$			Freeloader segment in PAYW-SP (very high consumption utility)
$Segment\ I^b_{PAYW\text{-}SP}$			Freeloader segment in PAYW-SP (high consumption utility)
$Segment\ I^c_{PAYW\text{-}SP}$			Freeloader segment in PAYW-SP (low consumption utility)
$Segment\ I^d_{PAYW\text{-}SP}$			Freeloader segment in PAYW-SP (very low consumption utility)
$Segment\ II^a_{PAYW\text{-}SP}$			Non-buyer segment in PAYW-SP with high consumption utility (less fair-minded consumers)
$Segment\ II^b_{PAYW\text{-}SP}$			Non-buyer segment in PAYW-SP with low consumption utility (less fair-minded consumers)
$Segment\ III^a_{PAYW\text{-}SP}$			Paying customers segment in PAYW-SP (fair price)
$Segment\ III^b_{PAYW\text{-}SP}$			Paying customers segment in PAYW-SP (suggested price)
$Segment\ III^c_{PAYW\text{-}SP}$			Paying customers segment in PAYW-SP (paying consumption utility)
$Segment\ IV^a_{PAYW\text{-}SP}$			Non-buyer segment in PAYW-SP with high consumption utility (more fair-minded consumers)
$Segment\ IV^b_{PAYW\text{-}SP}$			Non-buyer segment in PAYW-SP with low consumption utility (more fair-minded consumers)
θ^z_s		θ^z_s	Share of freeloaders that ignore price suggestion in PAYW-SP
θ^{1-z}_s		θ^{1-z}_s	Share of freeloaders that consider price suggestion in PAYW-SP
δ^z_s		δ^z_s	Share of less fair-minded consumers who remain non-buyers in PAYW-SP and ignore the price suggestion
δ^{1-z}_s		δ^{1-z}_s	Share of less fair-minded consumers who remain non-buyers in PAYW-SP and consider the price suggestion
$\pi^1_{PAYW\text{-}SP}$		$\pi^1_{PAYW\text{-}SP}$	Profits under PAYW-SP when suggested price is low
$\pi^2_{PAYW\text{-}SP}$		$\pi^2_{PAYW\text{-}SP}$	Firm's profits under PAYW-SP when suggested price is high
p^*_s		p^*_s	Optimal suggested price

Akbari and Wagner (2020)	CKZ	This Report	Explanation
m		B_s	Auxiliary variable in PAYW-SP, threshold that determines whether costs, the share of fair-minded consumers and their fair-mindedness is high
$g_{1\lambda}$		$g_{1\lambda}$	Auxiliary variable in PAYW-SP
$g_{2\lambda}$		$g_{2\lambda}$	Auxiliary variable in PAYW-SP
		$p_{f,s}$	Fair price in the presence of a suggested price
		p_s^u	Utility at the suggested price
		$p_{f,\,s,\ c<r\leq p_s^u}$	Fair price for high consumption utilities
		$p_{s,1}^{CKZ^*}$	Optimal suggested price in CKZ
		$\pi_{PAYW-SP,1}^{CKZ}$	Profits under PAYW-SP with suggested price in CKZ
		$\pi_{PAYW-SP,1}^{CKZ^*}$	Optimal profits under PAYW-SP with suggested price in CKZ
		$\pi_{PAYW\text{-}SP,2}^{CKZ}$	Profits under PAYW-SP with suggested price in CKZ
		$p_{s,2}^{CKZ^*}$	Optimal suggested price in CKZ
		$\pi_{PAYW\text{-}SP,2}^{CKZ^*}$	Optimal profits under PAYW-SP with suggested price in CKZ
		$p_s^{CKZ^*}$	Optimal suggested price in CKZ
		$\pi_{PAYW\text{-}SP,1}$	Profits under PAYW-SP with suggested price
		$\pi_{PAYW\text{-}SP,1}^{z=0}$	Profits under PAYW-SP with suggested price
		$p_{s,1a}^*$	Optimal suggested price in CKZ
		$p_{s,1b}^*$	Optimal suggested price in CKZ
		A_s	Auxiliary variable in PAYW-SP
		$p_{s,1}^*$	Optimal suggested price in CKZ
		D_s	Auxiliary variable in PAYW-SP
		$\pi_{PAYW\text{-}SP,2}$	Profits under PAYW-SP with suggested price
		$\pi_{PAYW\text{-}SP,2}^{z=0}$	Profits under PAYW-SP with suggested price
		$p_{s,2a}^*$	Optimal suggested price in CKZ
		$p_{s,2b}^*$	Optimal suggested price in CKZ
		$\pi_{PAYW\text{-}SP}^*$	Optimal profits under PAYW-SP with suggested price
		E_s	Auxiliary variable in PAYW-SP
		F_s	Auxiliary variable in PAYW-SP

Akbari and Wagner (2020)	CKZ	This Report	Explanation
		G_s	Auxiliary variable in PAYW-SP
		H_s	Auxiliary variable in PAYW-SP
		$\bar{\gamma}_{[0,1]}^{\min}$	Minimum feasible $\bar{\gamma}_{[0,1]}$
		$\bar{\gamma}_{[0,1]}^{\max}$	Maximum feasible $\bar{\gamma}_{[0,1]}$
		I_s	Auxiliary variable in PAYW-SP

Section 5: Profit comparisons

Akbari and Wagner (2020)	CKZ	This Report	Explanation
λ^+		λ^+	Threshold that separates regions in which PAYW-SP or other pricing mechanism is optimal
$\lambda^+_{\leq}$		$\lambda^+_{\leq}$	Threshold that separates regions in which PAYW-SP or PAAP are optimal
$\lambda^+_{>}$		$\lambda^+_{>}$	Threshold that separates regions in which PAYW-SP or PAYW-SP are optimal
$\bar{\gamma}_{[0,1]}^{\min}$		$\bar{\gamma}_{[0,1]}^{\min}$	Minimum feasible $\bar{\gamma}_{[0,1]}$
$\bar{\gamma}_{[0,1]}^{\max}$		$\bar{\gamma}_{[0,1]}^{\max}$	Maximum feasible $\bar{\gamma}_{[0,1]}$
$\beta^{\min}$		$\beta^{\min}$	Minimum feasible β
$\beta^{\max}$		$\beta^{\max}$	Maximum feasible β
$\left.\frac{\partial\lambda^+_{\leq}(\omega)}{\partial\omega}\right\rvert_{\lambda^+_{\leq}}$		$\left.\frac{\partial\lambda^+_{\leq}(\omega)}{\partial\omega}\right\rvert_{\lambda^+_{\leq}}$	Amount of consumers' generosity to its local dependence on consumers' fair mindedness
A_i			Actual observations (AW's results)
F_i			Forecast values (CKZ's results)
		θ_{RE}^{CKZ+}	Critical threshold of freeloaders when following CKZ’s
		A_{RE}^{CKZ}	Auxiliary variable for θ_{RE}^{CKZ+}
		B_{RE}^{CKZ}	Auxiliary variable for θ_{RE}^{CKZ+}
		C_{RE}^{CKZ}	Auxiliary variable for θ_{RE}^{CKZ+}
		D_{RE}^{CKZ}	Auxiliary variable for θ_{RE}^{CKZ+}
		$\theta^{CKZ,+}_{RE,\frac{1}{2}<\lambda<\frac{2}{3}}$	Threshold for optimality

Akbari and Wagner (2020)	CKZ	This Report	Explanation
		$\theta^{CKZ,+}_{RE,\lambda\leq\frac{1}{2}}$	Threshold for optimality
$\theta^{*}(z,\lambda,c)$		$\theta^{CKZ,+}_{ORIG}(z,c,\lambda)$	Threshold for optimality
	A	A^{CKZ}_{ORIG}	Auxiliary variable in determination for optimal choice of pricing scheme in CKZ
	B	B^{CKZ}_{ORIG}	Auxiliary variable in determination for optimal choice of pricing scheme in CKZ
	C	C^{CKZ}_{ORIG}	Auxiliary variable in determination for optimal choice of pricing scheme in CKZ
	D	D^{CKZ}_{ORIG}	Auxiliary variable in determination for optimal choice of pricing scheme in CKZ
	E	E^{CKZ}_{ORIG}	Auxiliary variable in determination for optimal choice of pricing scheme in CKZ
		${\theta^{CKZ,+}_{ORIG}}'$	Threshold for optimal choice of pricing scheme in CKZ
		${A^{CKZ}_{ORIG}}'$	Auxiliary variable in determination for optimal choice of pricing scheme in CKZ
		${B^{CKZ}_{ORIG}}'$	Auxiliary variable in determination for optimal choice of pricing scheme in CKZ
		${D^{CKZ}_{ORIG}}'$	Auxiliary variable in determination for optimal choice of pricing scheme in CKZ
		${B^{CKZ}_{ORIG}}''$	Auxiliary variable in determination for optimal choice of pricing scheme in CKZ
		${C^{CKZ}_{ORIG}}''$	Auxiliary variable in determination for optimal choice of pricing scheme in CKZ
		$\pi^{CKZnew,*}_{PAYW\text{-}MP}$	Optimal profits in PWYW-MP
		$\pi^{CKZnew,*}_{PAYW\text{-}SP}$	Optimal profits in PWYW-SP
		A_{AW}	Auxiliary variable in determination for optimal choice of pricing scheme
		B_{AW}	Auxiliary variable in determination for optimal choice of pricing scheme
		D_{AW}	Auxiliary variable in determination for optimal choice of pricing scheme
		E_{AW}	Auxiliary variable in determination for optimal choice of pricing scheme
$g_{\leq}$		$g_{\leq}$	Auxiliary variable in optimal choice of PAYW-SP vs. PAAP
$f_{\leq}$		$f_{\leq}$	Auxiliary variable in optimal choice of PAYW-SP vs. PAAP
$g_{>}$		$g_{>}$	Auxiliary variable in optimal choice of PAYW-SP vs. PAYW-MP
$f_{>}$		$f_{>}$	Auxiliary variable in optimal choice of PAYW-SP vs. PAYW-MP
$j_{>}$		$j_{>}$	Auxiliary variable in optimal choice of PAYW-SP vs. PAYW-MP
		F_{AW}	Auxiliary variable in determination for optimal choice of pricing scheme
		G_{AW}	Auxiliary variable in determination for optimal choice of pricing scheme
		H_{AW}	Auxiliary variable in determination for optimal choice of pricing scheme
		I_{AW}	Auxiliary variable in determination for optimal choice of pricing scheme

Akbari and Wagner (2020)	CKZ	This Report	Explanation
$MAPE$			Mean absolute percentage error, Measure of prediction accuracy of a forecasting method
$iMAPE$			Mean absolute percentage error excluding 0 actual profits, Measure of prediction accuracy of a forecasting method
$MAAPE$			Mean arctangent absolute percentage error

4. Article 3

Akbari, K., Wagner, U. (2018). Pay-What-You-Want Pricing for Multiple Goods. Conference Proceedings EMAC 2018. Glasgow (United Kingdom). 29.05.-01.06.2018. (electronic proceedings USB).

Pay-What-You-Want Pricing for Multiple Goods

The adoption of the participatory pricing system Pay What You Want (PWYW) changes the seller's pricing strategy and the buyer's position in the purchasing process fundamentally. The literature in this area has largely focused on a seller offering all of her goods either at posted prices or at a PWYW system, limiting the applicability of PWYW to low-cost goods. However, limited attention has been given to the seller's option of offering only parts of her product portfolio under participatory pricing, although this is a common approach in the field. In this paper, we model the seller's profitability of traditional prices, PWYW and combinations between the two options. The results of our simulation analysis suggest that if both goods have high costs, posted prices are the most profitable strategy; if both goods have low marginal costs PWYW is optimal. If costs of one good are low but medium or high for the other good, combinations between PWYW and posted prices might also be optimal.

Keywords: *Participatory pricing, bundling, modeling*

Track*: Methods, Modelling & Marketing Analytics*

1. Introduction

Pay What You Want (*PWYW*) offers an alternative to a seller's conventional pricing strategy, and some businesses attribute their success story to the adoption of this pricing system. In this participatory pricing system, the buyer engages in the price setting and is free to set any nonnegative price for a good, the seller has to accept this price and has to sell the good to the buyer (Kim et al., 2009). Most research on this pricing system has focused on a seller offering the entire product portfolio either under *PWYW* or at posted prices (*PP*).

Imagine a seller of two goods (*A* and *B*) considering the adoption of a pricing system. She knows about traditional prices and *PWYW*. Combining these different systems, there are four options at her disposal: (i) The seller sets a price for both goods. For each of these goods, the buyer will decide whether she purchases or does not purchase. (ii) The seller offers both of her goods under *PWYW* and, again, the customer chooses for each of these goods to buy or not to buy. In addition, she sets a price for her purchase. (iii) The seller simultaneously offers *A* under *PWYW* and *B* under *PP*. The buyer decides on her purchases and chooses a price for *A*. Thus, she is able to offset the price for *B* against the *PWYW* price for *A*. (iv) The seller bundles the products. She sells *A* (the basic product) under *PWYW* and *B* only if the buyer's price exceeds a certain price threshold. The buyer decides whether to buy *A* or the bundle. Furthermore, she decides which price to pay for *A* and whether to accept or even to overbid the price threshold for the bundle.

In practice, we observe all options (i) – (iv). An example of pure *PWYW* (i.e., (ii)) is Panera Bread, a U.S. bakery chain. In 2013, it adopted *PWYW* pricing in five of its branches. Customers were free to choose their price for every item on the menu. Conversely, many sellers offer only parts of their product range under PWYW (*https://goo.gl/FqB52e*) (i.e., (iii)). A classic example is the Pakistani restaurant Der Wiener Deewan (*www.deewan.at*) in Vienna (Riener & Traxler, 2012). This restaurant offers an all you can eat buffet where customers can pay whatever the like. Yet, a closer look reveals that parts of the restaurant's product portfolio – the beverages – are still sold at *PP*. Further examples for the mix of *PP* and *PWYW* are found online where sellers of digital goods offer the basic version of a bundle under *PWYW* while additional benefits are only unlocked if the buyer's price exceeds a publicly announced threshold price (*https://goo.gl/tHZEzD*, *https://goo.gl/Pkojh3*) (i.e., (iv)). This paper aims to explore the profitability of these different pricing systems and to give advice to sellers on which system to choose depending environmental conditions (i.e., cost structure, consumer preferences).

2. Model

In a recent study, Chen et al. (2017) model buyers' reactions to single-product *PWYW*. Their model describes how a monopolist will choose between *PWYW* and *PP* for one product only. This paper extends their model and investigates a situation in which the seller's supply consists of more than one product. Consequently, she has the four options outlined above at her disposal. In accordance with Chen et al. (2017), the seller strives for static profit maximization.

2.1. The seller

We consider a market in which a monopolist sells two goods ($j \in \{A, B\}$), assume a linear cost structure with marginal costs c_j and neglect fixed costs. For presentational convenience, the range of marginal costs is restricted to $[0,1]$ and the following notation is used for the four pricing strategies S outlined above: (i) posted prices (PP); (ii) pay what you want for both products (*PWYW*); (iii) combination of *PWYW* (for *A*) and *PP* (for *B*) ($PWYW\&PP$); if the buyer chooses both goods (*A*+*B*) we assume that this bundle is perceived as an aggregated whole with additive costs and additive prices; (iv) it will become reasonable later on that bundling makes sense particularly in the case of a two-tiers product assortment; *A* is the lower and *B* the higher tier product, *A* is sold under *PWYW* and there is a price threshold for the bundle (*A*+*B*); it is not possible, however, to purchase only *B*. This strategy is called multi-tier *PWYW* ($MTPWYW$).

2.2. The buyers

The market potential corresponds to M consumers. We restrict the analysis to a binary purchase decision (buying or not buying a single unit of the goods). Thus, the consideration set under PP, $PWYW$, and $PWYW\&PP$ corresponds to $j \in \{A, B, (A+B), \emptyset\}$, under $MTPWYW$ to $j \in \{A, (A+B), \emptyset\}$; $\emptyset$ denotes no purchase.

In line with Chen et al. (2017) we assume that buyers are inequality averse. In the behavioral economics literature, the term inequality aversion is defined by the notion that an individual's utility does not only depend on her own monetary payoff but also on the payoff of others taking part in the transaction. More precisely, individuals compare their own payoff to a fair benchmark. If their payoff is above this fair benchmark, they will perceive negative feelings because they are overprivileged. Thus, their utility decreases. If their payoff is below this benchmark, they feel envious and will again have a utility loss from the transaction (Fehr & Schmidt, 1999). Such preferences are commonly called Fehr-Schmidt preferences. Various economic experiments, most notably dictator and ultimatum games, have shown that individuals behave in accordance with these preferences (Eckel & Gintis, 2010).

We apply this behavioral economic model to our case of a buyer-seller interaction with two goods. Each buyer i has a net valuation r_{ij} for each product j (for reasons of tractability, the range of r_{ij} is restricted to $[0,1]$). This net valuation is independent of the seller's pricing and describes the buyer's benefit from consuming j. The buyer's payoff corresponds the difference between this net evaluation and the price p_{ij} (again restricted to $[0,1]$) a buyer pays $(r_{ij} - p_{ij})$. This is the consumer surplus in standard economic theory and will be denoted as acquisition utility in the rest of the paper (Thaler, 2008). The buyers are inequality averse. Therefore, deviations from the fair benchmark, which in our model is represented by the perceived fair price r_{i0j}, create disutility:

a) If the price paid (p_{ij}) is lower than the perceived fair price (r_{i0j}), the buyer will experience disutility because this price is perceived as being too low which disrespects the seller offers. Thus, sufficiently inequality averse buyers can try to minimize this discrepancy by paying more. Money-wise, however, this inequality puts the buyer in a favorable position. Therefore, it is refered to as advantageous inequality.
b) If the price paid (p_{ij}) is higher than the perceived fair price the buyer will also experience disutility from inequality aversion. She feels exploited by the seller because of the price being too high. Thus, in addition to decreasing the acquisition utility, the high prices will lower the transaction utility. As this type of inequality puts the buyer in an inferior position money-wise, it is refered to as disadvantageous inequality (cf. *Figure 1*).

Formally, buyer i's Fehr-Schmidt transaction utility u_{ij} is given by

$$u_{ij} = \underbrace{(r_{ij} - p_{ij})}_{\text{acquisition utility}} \underbrace{-\beta_i \max\{p_{ij} - r_{i0j}, 0\}}_{\substack{\text{disutility from} \\ \text{disadvantageous inequality}}} \underbrace{-\gamma_i \max\{r_{i0j} - p_{ij}, 0\}}_{\substack{\text{disutility from} \\ \text{advantageous inequality}}} \geq 0 \ \forall i, j \qquad (1)$$

where $\beta_i \geq 0$ and $\gamma_i \geq 0$ measure the buyer's inequality aversion. The β_i parameter specifies the extent of disadvantageous inequality aversion (bullet point b)), the γ_i parameter specifies advantageous inequality aversion (bullet point a)). Higher values represent stronger inequality aversion. Using the max operator in equation (1) ensures that only one, either advantageous or disadvantageous inequality, can be positive for a given setting.

The central construct representing the buyer's perception of fairness constitutes the reference of a fair transaction – the perceived fair price r_{i0j} (Chen et al., 2017). To evaluate this price, the buyer assesses the overall economic surplus of the transaction which is given by her net valuation and the seller's cost $(r_{ij} - c_j)$. Then she decides on a fair allocation of this surplus between herself and the seller. We set $\lambda_i \in [0,1]$ to be the fair proportion of the economic surplus a buyer allocates to the seller and arrive at $r_{i0j} = c_j + \lambda_i(r_{ij} - c_j)$ as the fair price, which is also represented in *Figure* 1. Note: this implies that the buyer can assess the economic surplus of the transaction and that she consequently knows the seller's costs or can approximate them accurately (Chen et al., 2017).

Given the corresponding consideration sets (see above) and – if applicable – the choice of deciding on prices the buyer maximizes the Fehr-Schmidt utility function (1). Since j might be equal $\emptyset$, $u_{ij} \geq 0$. If a buyer is indifferent between buying and not buying, we assume she buys. Furthermore, we assume that a buyer evaluates the two goods independently of each other.

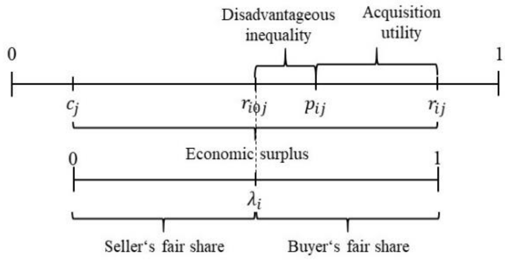

Figure 1: A buyer's assessment of fairness in with disadvantageous inequality

2.3. The seller's profits

Due to limitations in space, we can only outline on how we solved the above-stated optimization problem analytically.

(i) (PP): the M buyers are confronted with the seller's prices and decide whether or not to buy (one of) the products. As Chen et al. (2017) show, the seller – maximizing her profits – will never choose prices below the buyers' fair prices. Thus, the buyers will always suffer from disadvantageous inequality. In particular, she only buys if the sum of the seller's price and this type of disutility is smaller than her net valuation (cf. equation (1)). Rearranging terms we find a linear combination between seller's costs and prices with β_i, λ_i as parameters. As is standard, the seller maximizes her profits (i.e., prices minus costs times units sold) and such determines optimal prices.

(ii) ($PWYW$): A buyer purchases if her net valuation for a good is higher than the price and the disutility from inequality aversion. Disadvantageous inequality is not feasible in $PWYW$, as the buyer can always set a lower price and increase her utility (Chen et al., 2017). However, advantageous inequality can result from setting prices below a buyer's fair price. Buyers with a low level of advantageous inequality aversion ($\gamma_i \leq 1$) will always take the good for a price of zero (i.e., corner solution for optimizing u_{ij} which is linear in p_{ij}). Buyers with high levels of advantageous inequality aversion ($\gamma_i > 1$) only buy the good if their net valuation is higher than their perceived fair price. The seller's profits result from the aggregated differences between individual prices paid minus costs for each unit sold.

(iii) ($PWYW\&PP$): The buyer compares her utilities for each of the options in the consideration set and chooses the one with the highest utility. As in (ii), she purchases A if her net valuation is higher than the price and the disutility from (advantageous) inequality aversion. For B the situation analytically resembles (i): the buyer purchases if her net valuation is higher than the price and the disutility from disadvantageous inequality aversion. The buyer will purchase A+B only if her net valuation is higher than the price for B and the disutility from inequality aversion. This can be either disadvantageous or advantageous inequality. Correspondingly, we again end up with the same structural situation: (i) the buyer will either pay a price which is a linear combination between seller's costs and prices (for A+B) or ((ii) and low level of advantageous inequality aversion) the seller's price p_B (which resembles to the freeloader behavior of above) or ((ii) and high level of advantageous inequality aversion) her perceived fair price for A+B. The seller maximizes her profits by setting the optimal price for B.

(iv) ($MTPWYW$): Formally, this is a special case of (iii) with a restricted consideration set (cf. sub-section 2.2).

3. Setup of simulation analysis

Our objective is to determine circumstances under which each of the above pricing systems is optimal. Due to the complexity and multidimensionality of the model, we did not obtain a closed form solution. Therefore, we use numerical methods to solve the seller's problem. This is a common procedure in bundling research (Venkatesh & Kamakura, 2003).

For each possible combination of the seller's costs, we simulate her profits of each pricing system. To this end, we create synthetic heterogeneous buyers which differ in their net valuations r_{ij}, inequality aversion (β_i, γ_i) and assessment of their fair share (λ_i). We let them making their purchase decisions, setting their prices and then calculate the seller's profits. Next, we choose the pricing system which results in the highest profits for each cost combination.

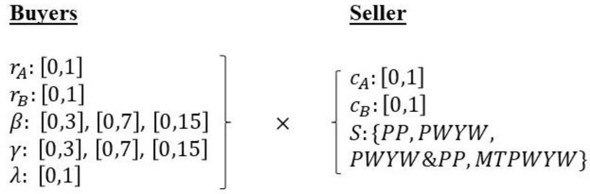

Figure 2: Simulated values for the buyers and the seller

The demand side of the market structure postulated above is covered by a 5-dimensional grid $(r_A, r_B, \beta, \gamma, \lambda)$. The left side of *Figure 2* shows the parameters' domains. Within these intervals discrete values for r_A, r_B and λ differ by 0.1 increments. Such a type of setup is commonly used when modeling bundling and participatory pricing.

We examine three domains for both parameters representing inequality aversion (β and γ): $[0, 3]$, $[0, 7]$ and $[0, 15]$ (within these intervals we again simulate 11 equidistant interpolation values). These three domains reflect populations having low, middle and high levels of inequality aversion. For instance, for γ this allows the following interpretation: In the "low" condition, 66% of the consumer have a high level of advantageous inequality aversion (i.e., $\gamma_i > 1$). These buyers pay their fair price to the seller, while 33% take the product for free. In the "middle" condition, 80% of the buyers have $\gamma_i > 1$ but 20% are freeloaders. In the "high" condition the respective percentages are 93% and 7%. These high proportions of a population willing to pay positive prices are by no means of only theoretical relevance. Riener & Traxler (2012) report that in their case (the Pakistani restaurant described in section 1) only 0.53% of the guests are freeloaders.

The supply side spans 484 ($11 \times 11 \times 4$) scenarios ($c_A \times c_B \times S$) regarding the seller's costs in steps of 0.1 from (0,0) to (1,1) and the different pricing schemes (S) (see right side of *Figure 2*). For each scenario, prices (p_A, p_B, p_{A+B}) vary in the feasible domain in steps of 0.1. This feasible domain results for prices at and above costs and extends up to 1 (for p_A, p_B) in PP and $PWYW\&PP$ and to 2 (for p_{A+B}, representing the maximum costs of the bundle) in $MTPWYW$. For each of these scenarios, complete enumeration determines optimal prices and seller's profits. The analysis carried out in section 4 compares the seller's profits with an optimal price structure across different pricing strategies for given costs.

4. Optimal Pricing Strategies and Conclusions

In this section, we discuss the findings of the simulation and summarize the results qualitatively. The seller's choice on which pricing strategy to use always depends on prevailing environmental conditions. Therefore, we present the results along four dimensions, cost structure (c_A, c_B) and inequality aversion (β, γ). To facilitate interpretation we provide some intuitive reasoning on the interrelations expected.

Intuition 1: *When inequality aversion in the population is low, participatory pricing systems are not likely to be profitable.*

In most cases, *PP* is the optimal pricing system and our results support this finding. This is the normal case: most sellers do not use participatory pricing systems. However, whenever participatory pricing is optimal, a sufficient degree of inequality aversion is necessary to sustain this form of pricing. Intuitively, if there are not enough buyers who are willing to pay voluntarily, revenues will be too low, and the seller is better off setting *PP*. Thus, the higher the level of advantageous inequality aversion in the population, the more the participative pricing systems ($PWYW$, $PWYW\&PP$, and $MTPWYW$) become profitable. The second driver of higher profitability of the participative pricing systems is disadvantageous inequality aversion. If more buyers experience disutility because of the seller's unfair prices, they will refrain from purchasing under posted pricing. This will lower the seller's revenues and profits under posted prices, making participatory pricing systems more attractive as they are not affected by disutility from disadvantageous inequality aversion.

Figure 3 provides "phase diagrams" showing regions of optimality for the different pricing strategies in a four-dimensional space. The outer dimensions represent the three levels of both types of inequality aversion – advantageous (outer horizontal-axis) and disadvantageous (outer vertical-axis) inequality aversion. In each panel I-IX, the horizontal/vertical-axis represents costs for *A*/*B*. For low levels of advantageous and disadvantageous inequality aversion, *PP* always dominate participatory pricing systems. The higher the level of inequality aversion in the population, the more participatory pricing systems become profitable. Furthermore, we find that the effect of advantageous inequality aversion on the choice of participatory pricing is higher than for disadvantageous inequality aversion (compare panel III and VII in *Figure 3*). This is in line with evidence of many examples from practice: participatory pricing crucially depends on the buyers' readiness to contribute to the seller (León et al., 2012).

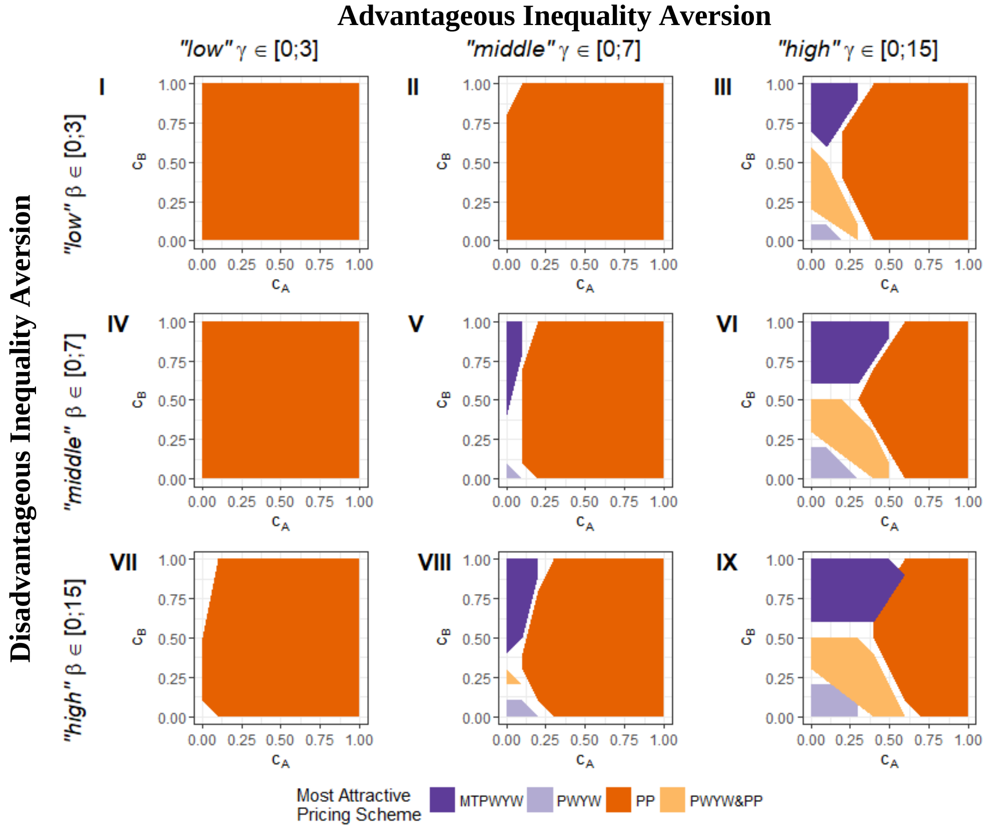

Figure 3: Phase Diagrams on Relative Attractiveness of the Different Payment Strategies, depending on Seller's Costs and Level of Inequality Aversion in the Population. Blank regions are not identified by the rather rough simulation approach.

Intuition 2: *When inequality aversion in the population is sufficiently high:*

a) *PP is the most profitable pricing system when costs of both goods are medium to high.*
b) *PWYW is the most profitable pricing system when costs for both goods are low.*
c) *PWYW&PP is the most profitable pricing system when costs for one good are low and medium for the other good.*
d) *MTPWYW is the most profitable pricing system when costs for the basic good A are low and high for B.*

a): If costs are high, every freeloader causes non-substantial costs for the seller which cannot be compensated by the contributing buyers. Consider Panel IX in *Figure 3*: In this case, both types of inequality aversion are very high. However participatory pricing is still not profitable for most cost combinations. This is in line with observations in practice: one rarely finds participatory pricing for high-cost products.

b): In the case of low marginal costs for both products, inequality averse buyers will pay their fair price for each of the goods. Some of the buyers pay prices which are higher than the seller's price in *PP.* In addition, the seller achieves a high market penetration and skims higher prices. Even if the prices paid are low, the seller still obtains a positive profit from most of them, as the costs for each buyer are low. Furthermore, freeloaders will not significantly hurt the seller's profitability. Panels V, VIII, III, VI, and IX in *Figure 3* nicely replicate these arguments. Findings are is in line with many *PWYW* sellers having low marginal costs such as the bakery Panera bread mentioned in the introduction. Another example is the German news page taz.de which offers every single article for a *PWYW* price. Marginal prices for every article are close to zero. Therefore, *PWYW* appears to be optimal.

c): If costs for A are low and medium for B, $PWYW\&PP$ is the most profitable strategy (cf. Panels III, V, VI, VIII, and IX in *Figure 3*). The seller can achieve high market penetration by selling A to buyers just as in $PWYW$. Buyers with a sufficient degree of inequality aversion ($\gamma_i > 1$) will pay their fair price. However, for B potential freeloaders have to pay the posted price. Therefore, the seller's losses from B can be limited. Consider the Pakistani restaurant from the introduction: at the buffet, every additional eater causes only low marginal costs and prices are set by the buyer (A). Beverages with typically higher marginal costs are sold separately for posted prices (B).

d): If costs for A are low and high for B, *MTPWYW* is the optimal pricing systems (cf. Panels III, VI, VIII, and IX in *Figure 3*). *MTPWYW* dominates *PWYW* because the negative effect of freeloaders for B does not take effect, while it still generates higher prices from sufficiently inequality averse buyers. It is more successful than *PWYW&PP* because it allows high threshold prices and as the products are bundled, the buyer evaluates both products simultaneously which can justify the higher threshold price. It is also more successful than *PP* prices as the seller receives different prices from heterogeneous buyers and as some inequality averse buyers pay more than the threshold price. Thus, the prices payed by some buyers are higher than the optimal price in *PP*. This is also reflected in practice by Humble Bundle who offers online games in the basic tier but sets a fixed threshold price for more costly physical goods such as fan t-shirts, mugs and other merchandise.

Summarising, this research helps sellers to choose the optimal pricing strategy depending on their costs and the population of their customers. We find that participatory pricing can be more profitable than *PP* even when costs are high for parts of the seller's product portfolio.

However, our research has several limitations and possibilities for extensions: First, we do not report the optimal prices and threshold prices a seller sets in our scenarios. This is due to space limitations of this conference paper. Second, we examine independently valued goods and do not study the effect of complements and substitutes on the buyers' price setting. Third, further research should assume more realistic distributions in net valuations and inequality aversions. Because of the flexibility of the numerical approach, this can be done easily and will likely result in interesting findings. Fourth, we look at a monopolist only and do not account for competition in our model. Fifth and finally, every buyer only buys one unit of each good. For many applications such as restaurants and most immaterial goods such as games and music, this is a plausible assumption. However, in other industries, e.g., in the retail sector, buyers typically purchase more than one unit of each good.

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5. Article 4

Akbari, K., Wagner, U. (2020). When paying is fun, I'll tell others:
Outcome effects of gambled price promotions (submitted to Schmalenbach Business Review, 2nd review round).

Schmalenbach Business Review

When paying is fun, I'll tell others:
Entertainment value, customer satisfaction, and word-of-mouth intention in gambled price promotions

--Manuscript Draft--

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Article Type:	Original Article
Funding Information:	
Abstract:	Gambled price promotions are discounts in which the customer's savings depend on the outcome of a game. Here we examine gambled price promotions framed as in-store events and their impact on two consequences of such campaigns: customer satisfaction with the retailer and word-of-mouth intention to endorse the retailer. The hypotheses postulate a positive relationship between gambled promotions and both, customer satisfaction and word-of-mouth ; and that this relationship is mediated by the entertainment value of the promotion. Also, discount approval is expected to moderate the entertainment value-customer satisfaction relationship. These hypotheses were tested in two field experiments; one in a candy store and the other in a furniture retail store. Three main findings emerge from this study: (1) Customers endorse gambled price promotions and are more willing to engage in word-of-mouth when participating in such a campaign. The effect on customer satisfaction is mixed. (2) The positive effects of such promotions are mediated by their entertainment value to shoppers. (3) When customers are unhappy with the discount received, entertainment value could compensate and ensure high customer satisfaction in a low stake setting.
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Response to Reviewers:	[redacted]

When paying is fun, I'll tell others:

Entertainment value, customer satisfaction, and word-of-mouth intention in gambled price promotions

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When paying is fun, I'll tell others:
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Abstract:

Gambled price promotions are discounts in which the customer's savings depend on the outcome of a game. Here we examine gambled price promotions framed as in-store events and their impact on two consequences of such campaigns: customer satisfaction with the retailer and word-of-mouth intention to endorse the retailer. The hypotheses postulate a positive relationship between gambled promotions and both, customer satisfaction and word-of-mouth; and that this relationship is mediated by the entertainment value of the promotion. Also, discount approval is expected to moderate the entertainment value-customer satisfaction relationship. These hypotheses were tested in two field experiments; one in a candy store and the other in a furniture retail store. Three main findings emerge from this study: (1) Customers endorse gambled price promotions and are more willing to engage in word-of-mouth when participating in such a campaign. The effect on customer satisfaction is mixed. (2) The positive effects of such promotions are mediated by their entertainment value to shoppers. (3) When customers are unhappy with the discount received, entertainment value could compensate and ensure high customer satisfaction in a low stake setting.

Keywords: *Gambled price promotion, probabilistic discount, price promotion, entertainment, retail, experiential value, promotional games*

JEL-Classification Code: M31

1. Introduction

The Sugar Shop is a regional icon in the German city of Heidelberg that sells a wide variety of sugary confections. When one of the authors was still a child and only had little pocket money, his schoolmates rumored about the legendary shop, which is stuffed with sweets from top to bottom. Nonetheless, more than about licorice sticks, wine gums, and chocolate, his friends reported that the quirky owner determines an item's price by rolling dice with customers. The author did not receive this tip exclusively: When Prince William and his wife Kate visited Heidelberg in 2017, a digital magazine (Kendi-Prill, 2017) asked local citizens which sights the royals should not miss; the Sugar Shop (Heidelberger Zuckerladen) was ranked in second place, after the famous Heidelberg Castle.

What differentiates the Sugar Shop from other stores is that, when items are placed on sale, customers are not offered a fixed discount percentage (e.g., 10% discount). Instead, they are offered a probabilistic discount amount (e.g., items are free-of-charge for customers who roll triple sixes). For these promotions, the savings amount is, as the name suggests, vaguely defined at the time of purchase and can differ between customers.

Recent studies have shown that in-store price promotions with unknown rewards (e.g., a gamble) can have numerous advantages over promotions with fixed outcomes. For example, companies can achieve the same result with lower costs (Dhar et al. 1995). Furthermore, customers tend to prefer discounts with unknown discounts over regular discounts when the expected value of either discount type is statistically identical (Ailawadi et al. 2014; Goldsmith and Amir 2010; Mazar et al. 2017).

On the one hand, customers who enjoy the diversion of the promotion or who enjoy gambling *per se* might find such a promotion appealing or entertaining. Unusual promotions, such as the one from the Sugar Shop, might be better retained in long-term memory, and shoppers might come to associate the seller with positive feelings. This outcome can, in turn, engender higher customer satisfaction and increased word-of-mouth (WOM). On the other hand, gambled promotions inevitably produce winners (e.g., the customer with triple sixes in the above example) and losers (e.g., the customer who rolls any other number). Thus, a small proportion of customers can profit from gambled promotions, whereas the majority of customers would have profited from the fixed outcome promotion. Consequently, gambled promotions can induce feelings of unhappiness or regret. Therefore, the positive experiential effects of the promotion might be mitigated by the outcome of the gamble.

This paper contributes to the literature by examining how customers perceive price promotions with unknown discounts and whether they transfer their corresponding experiences to the evaluation of the store and their WOM intention.

The structure of the remainder of this paper is as follows. Section 2 reviews existing research on probabilistic promotions and customer endorsement, section 3 describes two field studies, section 4 discusses the findings of these field studies, and section 5 concludes.

2. Literature review

2.1 Price promotions with unknown discounts

Although promotions with fixed outcomes are by far the most widely used discount format, companies are increasingly offering promotions with discounts that are unknown at the time of purchase (Ailawadi et al. 2014). These promotions involve risk or uncertainty and, thus, offer the customer not only the chance to win a relatively substantial discount, but also the risk of receiving a relatively low discount, or even no discount.

Most standard economic theories assume a risk-averse decision-maker, in which case a definite outcome is preferred over a probabilistic outcome when provided with two options with identical statistical expectations (Kahneman and Tversky 1979). However, some circumstances encourage risk-seeking behaviors. The possibility effect of prospect theory holds that decision-makers overestimate small probabilities and behave optimistically. Decision-makers prefer large gains with low probabilities over low-to-moderate gains with medium-to-high probabilities (Kahneman and Tversky 1979; Bar-Hillel and Budescu 1995; Bar-Hillel et al. 2008). Moreover, uncertainty can work positively and motivate consumers when they focus more on the process of reward pursuit than on the outcome of the promotion (Shen et al. 2015). These (mis)judgments can work in favor of a promotion with unknown discounts and make such promotions more attractive than they would appear under purely rational considerations.

To date, several studies have confirmed the effectiveness of promotions involving unknown discounts. Dhar et al. (1995) showed that fuzzily stated discounts in low-stakes and low-probability situations could result in increased purchase intention and greater interest in the bargain than comparable promotions with a fixed rebate. Goldsmith and Amir (2010) demonstrate that initial optimism positively affects the evaluation of uncertainty-based discounts in the same manner that the lottery is evaluated on par with its *best* possible outcome when the stakes are low.

Ailawadi et al. (2014) modeled consumer responses to discounts in which a price reduction is given conditionally on an event happening after the purchase. Based on conjoint analysis, the authors identified three groups that vary in their responsiveness to conditional rebates. The characteristics that distinguish these groups from one another are the level of perceived savings, entertainment benefits, event involvement, thinking costs, and proneness to gambling. Compared to fixed price promotions, conditional rebates appear to incur fewer costs to the seller but yield additional communicational effects. Mazar et al. (2017) demonstrate that probabilistic promotions tend to be more effective at increasing sales than are fixed price promotions with the same expected value because consumers become less price sensitive. Lee et al. (2019) show that consumers prefer probabilistic price discounts over fixed-price discounts if they can choose between them. Furthermore, probabilistic promotions seemed to lower the pain of payment among the consumers. Vries and Zhang (2020) identify uncertain promotions as powerful incentives for customers. When compared to fixed promotions, consumers more often choose the promotion that gives an unknown rather than a sure discount. This holds for both risk-seeking and risk-averse consumers.

Alavi et al. (2015) focus on changes in the internal reference price a customer holds for a given product. In retailing, rebates are standard, and, over time, customers become less willing to pay the regular price; such customers instead reduce their internal reference price. For gambled price promotions, i.e., promotions where a game of chance determines the discount, this adaptation process is less pronounced. Hock et al. (2020) document that promotional games increase conversion rates and spending when the discount is given before the purchase in the form of a coupon. Consumers that feel lucky about their win attach a positive feeling to the promotion and the seller and redeem the coupon more often than customers who receive a fixed discount coupon.

These price promotions discussed in the literature share a crucial feature: consumers do not know the discount they receive before buying the product. However, there are considerable differences in the design of these promotions. One way to think about these price promotions is across two dimensions. The first describes customer participation. On the one hand, the consumers can remain passive while the seller or an external event determines the profit. On the other hand, consumers actively participate in the promotion and take matters into their own hands by participating in a promotional game.

The second dimension distinguishes between risk and uncertainty (i.e., whether the probabilities of the outcome of the promotion are known) (Knight 1921). In the case of risky promotions, the probabilities of the outcomes can be assessed exactly before the participation in the promotion. In this case, the customer and the seller can estimate an expected discount. They are referred to as risk-involving price promotions. From a seller's perspective, these kinds of promotions are not as risky as the law of large numbers ensures that the discounts of the promotions converge to the expected value for a sufficient number of replications. In case of uncertain promotions, participants (and sometimes sellers) do not know the probabilities for the outcomes of the promotion.

This allows us to sort the terms for these kinds of price promotions into four categories:

- *Risk-involving price promotions with consumer participation:* Promotions that are risky and let the consumer participate in the determination of the discount include promotional games such as the previously mentioned casino games (e.g., dice games or wheels of fortune). Previously, these kinds of promotions have been termed gambled price promotions (cf. Alavi et al. 2015).
- *Risk-involving price promotions without consumer participation:* Examples for promotions that are risky but do not engage the consumer are promotions that give a 50:50 chance between getting a discount or not, savings based on the last digit of the receipt number, or set-ups in which every tenth customer gets the purchase for free. These promotions have been named probabilistic price promotions or uncertain incentives (Shen et al. 2019). When the consumer has a chance to get the entire purchase for free, such promotions are termed probabilistic free price promotions (cf. Mazar et al. 2017) or chance-for-free promotions (Gaertig and Simmons 2020).
- *Uncertainty-involving price promotions with consumer participation*: Promotions where the outcome is uncertain and that let the consumer participate, such as scratch cards with an unknown distribution of prizes and promotions that involve the consumer in the price setting and give a discount contingent on the successful completion of a task (e.g., hitting basketball free throws). Previous terms for these promotions include lottery promotions (Dhar et al. 1995), uncertain rewards (Shen et al. 2015), and promotional games (Hock et al. 2020; Fang and Mowen 2009; Briley et al. 2018; Ward and Hill 1991).
- *Uncertainty-involving price promotions without consumer participation:* Promotions that are uncertain and do not require consumer participation include promotions where

the discount depends on the occurrence of an external event (e.g., Germany winning the football world cup) or promotions where the buyer cannot estimate the outcome probabilities (e.g., a promotion where a consumer can get a discount between 10 and 20 percent but does not know how the actual discount is determined). Previously, these promotions were called conditional rebates (Ailawadi et al. 2014) and random discounts (Vries and Zhang 2020).

[Table 1]

2.2 Hypotheses development

The extant research has concentrated on pre-purchase metrics of promotions with unknown consequences. However, like other components in the marketing mix, promotions and prices can also influence consumer attitudes, intentions, and behaviors, which, in turn, can affect the firm's profitability in the long run. Here we consider three potential consequences: entertainment value (H1), customer satisfaction (H2, H3), and WOM intention (H4, H5) for risk-involving price promotions with consumer participation. In this research, we concentrate on risk-involving price promotions with consumer participation (i.e., gambled price promotions), as they allow us to control the risk of the unknown outcome and compare it to certain discounts. Moreover, the entertainment value of a promotion that involves the consumer will be higher than the entertainment value of a promotion without consumer involvement. Our overall research model is presented in Figure 1. In the following, we will derive our hypotheses.

[Figure 1]

2.2.1 Gambled price promotions and entertainment value

In contrast to consumers in traditional price promotions, consumers are engaged in the price setting in gambled promotions. The seller offers the consumer an experience that actively involves consumers and lets them immerse themselves in a gamble. Such framing might provide a particular appeal for the participants (Pine and Gilmore 1998). Gambling (a game of chance) is commonly associated with entertainment, distraction, and leisure (Walker 1992); thus, the primary motivation for gambling might be to have fun rather than to win money (Francis et al. 2015). This could create entertainment value[1] on the consumer side

[1] Note the different definitions of entertainment value in the literature. Pine and Gilmore (1998) characterize consumer experiences along two dimensions, customer participation (active/passive) and connection (absorption/immersion). They classify experiences that are passive and absorbing (e.g., watching TV) as entertainment. Experiences that require active participation and immersion, such as a gambled price promotion, are classified as

(Fang and Mowen 2009). In a marketing context, the enjoyableness of a promotion with unknown outcomes is an important success factor (Ward and Hill 1991). Previous studies have shown that consumers who participate in online sweepstakes value the entertainment component of such contests (Gedenk et al. 2001). Furthermore, customers indicate that the task of a contest might be more important than the prizes themselves (Teichmann et al. 2005). Therefore, we expect gambled price promotions to yield higher entertainment values than regular promotions:

H1: *Gambled price promotions result in higher entertainment value compared to non-gambled promotions or non-promotional settings.*

2.2.2 Gambled price promotions and customer satisfaction

Over recent decades, customer satisfaction has been established as a central metric in the field of marketing for researchers and practitioners alike. Ultimately, customer satisfaction leads to stronger customer loyalty and higher profitability (Barsky and Nash 2003). For companies aiming to manage customer satisfaction, gambled promotions might offer favorable outcomes. The confirmation/disconfirmation paradigm states that consumers form beliefs and expectations about the seller's quality. When the seller meets or exceeds the consumer's expectations, the consumer will be satisfied, and, conversely, consumers will be dissatisfied when the seller underperforms (Oliver 1980; Churchill and Surprenant 1982).

Gambled price promotions might provide a particular appeal for some participants. As discussed above, gambling is commonly associated with entertainment. Entertainment derived from a gambled promotion is an option to provide the customer with experiential value. Consumers increasingly focus on a shopping experience rather than on products (Pine and Gilmore 1998). Through their participation in gambled promotions, consumers undergo experiences in the store and are stimulated. This can improve the perception of the value of the purchase decision, brand, or company the consumer interacts with (Schmitt 1999). In addition to the discount that is also provided in case of fixed discounts, a seller that uses gambled pricing offers a second, emotional value component (Gee et al. 2005). Previous research shows that creating experiential value during a service encounter can increase customer satisfaction (Yuan and Wu 2008). With a specific focus on entertainment, the extant research has found that the more positive the evaluation of the entertainment component of a service encounter,

escapist. The other two prototypes of experience are esthetic (passive/immersion) and educational (active/absorption). However, they note that experience qualities are multidimensional and not mutually exclusive and that rich experiences encompass all four endpoints of the spectrum. Interestingly, they refer to gambling in a casino as such a rich experience. In this paper, we use to entertainment value to describe the consumer's perceived assessment of the experience of a promotion.

the greater the customer satisfaction (Pantano and Naccarato 2010; Söderlund and Julander 2009). Therefore, the entertainment value is a mediator in the gambled price promotion-customer satisfaction relationship. However, there are other explanations that might constitute a direct effect between gambled price promotions and customer satisfaction.

As the price constitutes a vital dimension of the seller's performance, extant research identifies the price as one determinant of customer satisfaction (Martín- Consuegra et al. 2007; Zeithaml and Bitner 2000; Oliver 1999). To a buyer, a gambled price promotion offers the possibility to obtain the purchase at a lower price than in non-promotional settings. Thus, when a gambled promotion is present, the seller can provide a better consumption experience and exceed the customer's expectations more easily than in the absence of a promotion. Gambling-prone customers will enjoy participating in promotions that involve risk, and, in turn, are likely to form more positive associations with the store than are risk-averse customers. Moreover, customers might capitalize on other promotion-specific benefits. If the promotion is framed as a game of chance, it might be new to customers (Mazar et al. 2017). As novelty increases the evaluation of different stimuli (Berlyne 1970), gambled promotions could mean variety and stimulation for the customer, which increases the preference for such promotions (Mazar et al. 2017; Sheth et al. 1991). Hence, the expectations for the seller's service offering can be exceeded when an innovative promotion is present as compared to when it is not and, therefore, customer satisfaction should be higher. Furthermore, a positive endorsement of the promotion might also cause spillover effects in the sense that the store's attributes are judged more positively when the campaign is present than when absent (Forgas 1999). Thus, we include a direct effect and assume partial mediation. Therefore, the following hypothesis is put forth:

H2: *Entertainment value partially mediates the relationship between a gambled price promotion and customer satisfaction with the store; specifically, the higher the entertainment value, the higher the customer satisfaction.*

The different outcomes of a gambled price promotion can leave some consumers to be better off than what they expected before participating and others that are worse off than expected. Buyers whose assessments of the discount are positive will consider themselves to be (lucky) winners; otherwise, customers will consider themselves to be (unlucky) losers. If assessments are rational, this should even out across all consumers when compared to a non-risky promotion of the same expected value. However, the evaluation of the results of the game of chance by the consumers may be subjective (Gilovich 1983; Gilovich and Douglas

1986), and, therefore, it is not clear who feels like a winner and who feels like a loser. Even players with sound statistical knowledge are vulnerable to cognitive biases in gambling situations (Benhsain and Ladouceur 2004).

We, thus, argue that performing above expectations in the gamble will create a "joy of winning" (Astor et al. 2013; Ding et al. 2005). Consumers who "win" the gamble might be in a better mood (Smith et al. 2009), and, consequently, evaluate the promotion and the seller more positively. Therefore, the seller can exceed the customers' expectations regardless of the entertainment value of the promotion. Therefore, even when the promotion is not entertaining, customers might be more satisfied when the discount approval is high.

Reciprocally, consumers who are disappointed by the outcome of the gamble ("frustration of losing") (Astor et al. 2013; Ding et al. 2005) will evaluate the promotion and the overall performance of the seller less favorably. For these customers, the entertainment value might be a way to distract from the disappointment of not winning the gamble. Therefore, by offering a higher entertainment value, the seller improves the consumption experience and, consistent with confirmation/disconfirmation reasoning (Oliver 1980; Churchill and Surprenant 1982), the overall performance of the seller will be evaluated more positively.

H3: *Discount approval moderates the relationship between entertainment value and customer satisfaction with the store, such that under low discount approval, the impact of entertainment value on customer satisfaction will be more positive than under high discount approval.*

2.2.3 Gambled price promotions and WOM intentions

WOM is defined as informal communication among consumers about their experiences and opinions regarding a seller's goods or services (Westbrook 1987). Companies often try to engage consumers in WOM, as this constitutes free marketing to potential new customers. We argue that gambled promotions provide an opportunity for increased WOM in favor of the seller.

The relationship between consumer satisfaction and WOM is well documented. Various researchers have detected a positive relationship between customer satisfaction and WOM (Swan and Oliver 1989; Schlesinger and Heskett 1991; Holmes and Lett 1977; Roberts 2004). Among other reasons, consumers share their satisfaction because they want to help their peers in finding a suitable product (altruism), want to appear knowledgeable and share useful information (impression management), and want to reduce cognitive dissonance (Arndt 1967; Berger 2014; Dichter 1966). This is also expected in our research.[2] Therefore,

H4: *Higher customer satisfaction results in higher WOM intentions.*

In line with the above, the relationship between gambled price promotions and WOM intention should be sequentially mediated by entertainment value and customer satisfaction. However, we also find an indication of the effect of entertainment value on WOM intentions. Participating in a gamble is likely to evoke arousal and emotions (Gee et al. 2005; Malhotra 2010). Consumers are known to share emotions with others (Heath et al. 2001; Peters and Kashima 2007). If consumers perceive a price promotion as a stimulating and positively entertaining experience, they, therefore, are expected to share these experiences. As an example, Lovett et al. (2013) show that the excitement of a brand is associated with more WOM. A second argument that supports higher WOM intentions in case of higher entertainment value is that consumers are more likely to discuss exciting topics that they find interesting; this is because they do not want to be associated with boringness (Berger 2014; Levy 1959). In line with this reasoning, consumers are more likely to discuss unusual rather than ordinary products (Berger and Iyengar 2013; Berger and Schwartz 2011), and this should also hold for interesting rather than boring promotions. Previous research shows that more differentiated sellers receive more WOM (Lovett et al. 2013). From a practitioner's point of view, Sernovitz (2006 p. 6) writes, "… nobody talks about boring companies, boring products or boring ads". Consistent with this reasoning, a higher entertainment value offers more opportunity for the buyer to talk about something interesting.

Furthermore, previous research suggests an effect of gambled price promotions on WOM intentions. For example, consumers engage in WOM when they believe that they can convey useful information to others (Rosnow 1980; Brunvand 1981). They do so because they want to appear smart or helpful (Berger 2014). When consumers consider gambled promotions as financially attractive, they should be more likely to recommend the seller that is running such a promotion as opposed to a seller who does not offer a gambled promotion. In most cases of gambled promotions, the seller does not communicate the expected value of the gamble. In contrast, consumers need to assess the attractiveness of a promotion independently. However, consumers are bad at judging probabilistic concepts. In particular, they tend to overestimate

[2] Note that there is also evidence for increased (negative) WOM for every low levels of satisfaction (Anderson 1998, Richins 1983). However, in this paper we only account for positive WOM intention.

small probabilities, such as winning a gamble (Kahneman and Tversky 1979) and, therefore, the attractiveness of a promotion. Similarly, Babad and Katz (1991) find instances of wishful thinking. Gamblers predict the occurrence of the desired outcome as more likely than that of an undesired outcome (Goldsmith and Amir 2010; Shen et al. 2015). Applying this reasoning to the current setting can, therefore, bias expectations upwards. Thus, the financial attractiveness could be overstated and, therefore, gambled promotions might spark more WOM. Consequently, we hypothesize the following:

H5: *Entertainment value through customer satisfaction partially mediates the positive relationship between gambled price promotions and WOM intentions.*

3. Empirical investigations

The empirical part of this paper reports on two quasi-experimental field studies. Study I describes a field experiment that compares entertainment value, customer satisfaction, and WOM intentions of gambled promotions with regular discounts and no discount conditions. We consider two different distributions of the discount level to test the robustness of our hypotheses. While the average discount of a risky price promotion is likely evaluated on economic considerations by the seller, its distribution might also matter. Mazar et al. (2017) show that probabilistic promotions are particularly successful if they include the chance of a 100% discount. Consumers prefer these campaigns, which leads to increased demand and store traffic. With a fixed mean, however, a 100% discount results in fewer remaining discounts; consequently, a larger proportion of customers might disapprove of their discount. In line with the above considerations (H3), promotions with more dispersed discount distributions (such as a 100% price discount) and, therefore, more consumers getting lower discounts, should lead to more customers being dissatisfied with the outcome of the promotion than in cases with more uniform distributions. This could, in turn, result in lower customer satisfaction and WOM intentions in these cases.

Study I was conducted in a candy shop whose buyers spend, on average, ~€10 per visit. The expected pay-off amounted to 5% in both risk-involving scenarios. Study II analyzes a gambled price promotion offered by a large retail furniture chain. The management of this company designed the campaign and allowed us to interview their customers. This access made it possible to investigate a high-stakes situation: the promotion offered an expected discount of 10.5% on each customer’s invoice.

3.1. Study I: Price promotion with a 'wheel of fortune'

3.1.1 Method

A small candy shop operating in a central European country served as the setting for Study I. The owner managed the store and did not hold promotions regularly; thus, gambled price promotions were entirely novel to this shop. The experiment employed a one-factor (four levels) between-subjects design with promotional campaigns as the manipulated variable; CG1: no discount; CG2: non-gambled promotion of 5% discount for all; EG1: gambled price promotion (expected discount: 5%, range: 1–10%); EG2: (gambled) probabilistic free price promotion (expected discount: 5%, range: 1–100%).

Data were collected on the shop's premises located in a downtown neighborhood of a European capital at four separate time points. The non-promotional slot began on week 1 (CG1), and the gambled price promotion began on week 2 (EG1). Then, a two-week period with no treatment followed in order to minimize spillover effects. During week 5, the candy shop offered the gambled chance-for-free price promotion (EG2). Lastly, a few weeks later, the regular price promotion with a fixed discount of 5% followed (CG2). To ensure that consumers did not participate in more than one condition, we used an individual identification code (a combination of initials and birthday). No extraneous incidents occurred during the study period that might have distorted the results of this study.

The gambled promotion was framed as a 'wheel of fortune'. Similar to roulette, the wheel carried 30 pockets, and each of these pockets corresponded to a percentage discount (cf. Figure 2). The assignment of levels of discount to these pockets resulted in an average concession of 5% under both experimental conditions. However, EG2 assigned a 100% discount to a single pocket, which was flagged by unique coloring to increase the visibility of that pocket. Accordingly, the other pockets offered relatively lower markdowns. Comparing EG2 with EG1 from a customer perspective, EG2 generates very few lucky players who receive high rebates, while most players only receive low rebates.

[Figure 2]

Research assistants surveyed all customers of this shop during this field study. For CG1, the questionnaire contained only questions on the general evaluation of the store (customer satisfaction, WOM intention) and demographics. For the other conditions, the discount campaign was carried out at the cash desk. After the customers made their purchase decision, research assistants explained the promotion in detail (EG1, EG2, and CG2) and invited shoppers to participate in the game (EG1, EG2). Before playing, respondents indicated their anticipated pay-off. After spinning the wheel and learning their discount, all respondents completed a questionnaire soliciting their opinions on the discount obtained, the entertainment value of the promotion, their satisfaction level, their WOM intention, and demographic variables (age, gender).

The busy atmosphere at the check-out area motivated the use of single-item scales to measure all of these variables in a time-efficient manner (cf. Appendix 1). The responses were recorded by itemized five-point rating scales, where 1 represented the most unfavorable evaluation, and 5 represented the most favorable evaluation. Research assistants also collected data concerning the value of the purchase and the actual discount achieved. We surveyed all customers during the observation period. The study included 51 customers in EG1 (29% male), 68 customers in EG2 (24% male), 74 customers in CG1 (32% male), and 34 customers in CG2 (35% male). The slightly smaller sample size for CG2 was due to a shortened data-collection period. All customers participated in the promotion (EG1, EG2, and CG2).

3.1.2 Results

Descriptive statistics. The upper panel of Table 2 provides descriptive statistics for Study I. As expected, a substantial discrepancy was found between expected (EG1: 36%, EG2: 21%) and realized discounts (EG1: 6%, EG2: 3%), the former being significantly higher. Consumers overestimated their expected discount in both experimental conditions. The overall realized discount approximated what had been expected (i.e., 5%). Nevertheless, subjects still held favorable impressions of the realized discount (EG1: 4.33, EG2: 3.41, CG2: 4.47) for all experimental conditions that offered a promotion (when compared to the mean of the scale [i.e., 3]). Previously held expectations seemed to have been discarded in this evaluation as we did observe a relationship between the objective outcome of the gamble and subjective approval of the discount: the higher the obtained discount, the higher the satisfaction with the discount ($r = .23$, $p < .01$). Furthermore, there is a promotional effect: Average purchase values for EG1 (13.57 €), EG2 (17.68 €), and CG2 (12.78 €) outperform their corresponding benchmark in CG1 (9.23 €) (no discount).

[Table 2]

A preliminary investigation of customer satisfaction and WOM intention in the upper panel of Table 2 shows consistently higher averages for EG1 and EG2 than for CG1 and CG2. The entertainment values in EG1 and EG2 are higher than in CG2. Discount approval is similarly high in EG1 and CG2 but lower in EG2. MANOVAs tests confirmed that these differences are statistically significant (cf. Appendix 2). Furthermore, customer satisfaction and WOM intention were positively correlated with entertainment value (cf. Table 3). This provides initial support for an effect of gambled price promotions on the endogenous variables.

[Table 3]

Testing the mediation hypotheses necessitates a focus on promotions that offer a discount (EG1, EG2, and CG2). Our model can be decomposed in a moderated mediation (gambled price promotion → entertainment value → customer satisfaction, moderator: discount approval) and a sequential mediation (gambled price promotion → entertainment value →customer satisfaction →WOM intention).

Investigating entertainment value and customer satisfaction (H1-H3). We hypothesized that entertainment value mediates the effect of a gambled price promotions (with CG2 as a reference category) on customer satisfaction. Furthermore, we hypothesized a moderating effect of discount approval on the path between entertainment value and customer satisfaction. To test for this second-stage moderated mediation model, we used Hayes (2018) PROCESS macro, Model 14 with 5000 bootstrapped samples (Holland et al. 2017; Preacher and Hayes 2004). Summary statistics of these analyses are by and large satisfactory (cf. Table 4).

[Table 4]

PROCESS results showed evidence for a significant effect of gambled price promotions on entertainment value for both conditions (Table 4, rows 1 and 2). The direct effects of gambled price promotional conditions (EG1, EG2) on entertainment value differ significantly from those of the fixed discount condition (CG2), but not from each other.

Additionally, controlling for the presence of gambled price promotions, we found a significant moderating effect of discount approval on the relationship between entertainment value and customer satisfaction (Table 4, row 5). When discount approval is low to medium, entertainment value increases customer satisfaction more than when discount approval is high (cf. Figure 3a).

Since the moderator (discount approval) was continuous, we looked for the turning points for where exactly, in the absolute value of the moderator, the effect of the mediator (entertainment value) turns from non-significance to significance (for a pre-specified significance level of 0.05). This is done using the Johnson-Neyman technique (Bauer and Curran 2005; Hayes 2018; Krishna 2016). The turning point from non-significance to significance of the effect of entertainment value is 4.36 (cf. Figure 3b). Entertainment value is associated with significantly greater customer satisfaction for values of discount approval below 4.36 (measured on a 5-point scale). When discount approval is greater than 4.36, the differences between customer satisfaction at high and low levels of entertainment values are not significant.

The direct effects of gambled price promotions on customer satisfaction were significant, indicating partial moderated mediation (Table 4, rows 1 and 2).

[Figure 3]

Investigating WOM intention (H4 and H5).

We estimated a serial mediation from gambled price promotion (with CG2 as a reference category) through entertainment value and customer satisfaction to WOM intention. Direct effects of entertainment value and customer satisfaction were positive and significant (Table 4, rows 3 and 6). However, the bootstrapping procedure of the PROCESS macro does not account for the moderation of discount approval on the relationship between entertainment value and customer satisfaction. Therefore, we assessed the significance of the indirect effects of the proposed moderated mediation with the corresponding product-of-coefficients tests that extend the work of Sobel (1982).

For the paths that involve a single mediator, we built on Iacobucci et al. (2015). Gambled price promotion had an indirect effect on WOM intention through entertainment value and through customer satisfaction (Table 4, rows 8 to 11).

For the sequential mediation, product-of-coefficients tests by Taylor et al. (2008) were used. We could establish a significant serial mediation for both gambled price conditions (Table 4, rows 12 and 13). However, the effect was only marginally significant for the promotion with more extreme outcomes (EG2). Finally, there was no direct effect of gambled price promotion on WOM intention, indicating full serial mediation with respect to the type of promotion (Table 4, rows 1 and 2).

As a side-note, we add that sociodemographic variables (e.g., gender, age) were not found to be influential.

3.1.3 Discussion

For customer satisfaction, we find support for our hypotheses. Gambled price promotions increase the entertainment value (H1). Furthermore, the relationship between entertainment value and customer satisfaction is moderated by discount approval (H2, H3). When discount approval is high, the entertainment value is not a driver for customer satisfaction. The high discount approval suffices to achieve satisfaction. However, when discount approval is low, entertainment drives satisfaction. In other words, the seller can compensate for the low discount approval with higher entertainment values. This offers the seller a possibility to maintain high customer satisfaction despite the consumers' unhappiness with the deal. In sum, partial moderated mediation is established.

With respect to WOM intentions, the positive effect of gambled promotions is mediated by entertainment value and customer satisfaction (H5). Both, entertainment value and customer satisfaction (H4) significantly positively influence WOM intentions. However, there is no direct effect of gambled promotions on WOM intentions.

Considering two different distributions of gambled discounts allowed us to observe a potential constraint for the positive effect of gambled promotions on customer satisfaction and WOM intentions. When more consumers get lower discounts, and therefore disapprove of the outcome of the gamble, the positive effects of gambled promotions becomes weaker.

3.2 Study II: Price promotion using dice

On the one hand, Study II offers a conceptual replication of Study I (Lynch et al. 2015); both studies offer a comparable setting regarding the type of promotion. By playing a game of chance, the customer takes an active role in determining the discount they receive. The result of the game is random, and the customer can only influence the outcome by playing. On the other hand, the studies differ in several aspects. First, Study II considers two experimental conditions only. Second, with mean article prices exceeding €100, risk-involving promotions in furniture stores are high-stakes gambles (Wagner and Jamsawang 2012). The high-stakes situation is a distinguishing feature of Study II because most research on risky and uncertain discounts have dealt with hypothetical scenarios or very low pay-off situations (e.g., possible gains were averaging <€5). Third, the average discount level of 10.5% is higher than the discount level in Study I (5%). Higher stakes and higher discount levels might emphasize the economic aspects of the promotion over its entertainment value. Fourth, the assessment of the expected discount might be easier this time[3], which might increase the salience of perceived wins and losses. Fifth, in contrast to the candy shop in Study I, this furniture retailer regularly

[3] Assessing the expected value is easier for calculating the average number of points achieved when rolling a dice three times vs. for a wheel of fortune with 30 pockets.

offers several kinds of traditional and gambled price promotions. Thus, customers of this retailer might have become more accustomed to gambled promotions, which reduces its novelty. Therefore, this study aims to find limiting conditions for our results in Study I.

3.2.1 Method

The study took place in a large furniture store chain operating in the same central European country. The experiment employed a one-factor (two levels) between-subjects design with exposure/no exposure to a gambled price promotion as the manipulated variable. Data were collected in an outlet located in a metropolitan area at two separate time points. Customers of the store who made purchases during the promotional week comprised the experimental group (EG); those who made purchases 5 weeks later during the non-promotional week comprised the control group (CG). Both data collection periods were comparable with respect to other relevant internal or external influences.

The store's administrative personnel invited customers to participate in a game using dice after having paid at the check-out. The shoppers rolled three dice and received a discount according to the number of points they obtained (e.g., if the three dice showed three, four, and five points, respectively, shoppers received $3 + 4 + 5 = 12\%$ off the purchase invoice). In addition to the discount, this campaign was designed to be enjoyable. A moderator contributed to the inherently event-like character of the promotion (e.g., throwing oversized dice in public) (cf. Figure 4).

[Figure 4]

Trained research assistants intercepted all customers who participated in the promotion (EG). Before the dice were rolled, shoppers indicated their expected discount level. After throwing the dice, the shoppers again answered queries on their discount approval, the entertainment value of the promotion, customer satisfaction, WOM intention, and demographics (age, gender). Research assistants observed the purchase value and the outcome of the dice game (i.e., the actual discount achieved).

No promotional events were held in this outlet 5 weeks later, at which point customers were surveyed again (CG). This time, the questionnaire contained only questions on customer satisfaction, WOM intention, and demographics. The EG comprised 99 participants (31% male), and the CG comprised 167 participants (51% male). The sampling plans were a census in EG and systematic cluster sampling in CG. The smaller sample size for EG was due to the fact that not all customers chose to participate in the dice game. This might be due to the customer's time constraints as the participation in the promotion could have included some waiting.

To keep the burden for the participants as small as possible, and to comply with the request of the manager of the retail chain, we employed single item measures as in Study I (cf. Appendix 1).

3.2.2 Results

Study II did not aim at testing the full research model. Analyses are restricted to comparing outcomes for the gambled promotion (EG) and no promotion (CG). Furthermore, the analyses of the effects of the gambled promotion on customer satisfaction and WOM intention were restricted analysis to the EG case, i.e., conditional on entertainment value.

Descriptive statistics: The lower panel of Table 2 provides descriptive statistics for Study II. A cursory inspection of the expected discount seems to imply that respondents' expectations of the discount (9%) came close to reality (i.e., 10.5%). However, this was the case for only about one-third of the participants. The large standard deviation is evidence that many shoppers expressed irrational expectations (either very low [i.e., ≤ 3%] or very high [i.e., ≥ 18%]). Given the sample size, the average realized discount corresponds to the probabilistic evidence (11%). Nonetheless, respondents were pleased with the realized discount (mean value of 3.91, when compared to the mean of the scale [i.e., 3]). Purchase values were slightly higher in the experimental condition than in the control condition. However, standard deviations are large throughout because of the wide assortment of items available in this store and the corresponding diversity of basket values.

In the EG, the average evaluation of the entertainment value (4.11) surpassed (significantly) the midpoint of the scale (i.e., 3). A comparison across studies reveals that entertainment values are of the same size. However, customer satisfaction (3.94) is not. Also, oral reports from the research assistants who interviewed customers affirmed a pleasant atmosphere throughout the event. Thus, some evidence points in the direction of a substantial entertainment value provided by this promotion. Average customer satisfaction was nearly identical in both groups (EG: 3.94, CG: 3.90); however, the substantial standard deviation for EG and the weakly significant Kruskal-Wallis test indicate that the distribution of satisfaction between the two groups was different. Customer satisfaction was more dispersed (i.e., U-shaped) for the EG but bell-shaped in the CG. A significant Kruskal-Wallis test indicates that WOM intentions were higher in the EG (2.87 on a three-point scale) than in the CG (2.57).

Investigating customer satisfaction and WOM intentions. PROCESS model 7 (Hayes 2018) analyzed the proposed moderated mediation. When testing the relevant parts of our model, we hypothesized that customer satisfaction mediates the effect entertainment value has on WOM intentions. Furthermore, we hypothesized a significant moderating effect of discount approval on the relationship between entertainment value and customer satisfaction. This model allows us to test the effect of entertainment value on customer satisfaction and WOM intention. Hence, customer satisfaction is both a focal variable for H2 and H3 and a predictor/mediator for H4 and H5.

PROCESS results did not show evidence for a significant relationship between entertainment value and customer satisfaction (Table 5, row 1). However, the relationship between discount approval and customer satisfaction was significant (Table 5, row 2). The moderating effect of discount approval (Table 5, rows 3 and 5) on the relationship between entertainment value and customer satisfaction was insignificant. Furthermore, the relationship between customer satisfaction and WOM intention was insignificant (Table 5, row 4). However, we find a direct effect of entertainment value on WOM intentions (Table 5, row 1).

[Table 5]

3.2.3 Discussion

We did not find support for our hypothesis with respect to customer satisfaction. The effect of entertainment value on customer satisfaction was insignificant, as was the moderating effect of entertainment value and discount approval. However, discount approval had a positive direct effect on customer satisfaction. This points to a potential boundary condition of entertainment value as a driver for customer satisfaction in gambled promotions. For higher expected discount levels, customer satisfaction is less driven by entertainment value and more by monetary considerations.

With respect to WOM intentions, we can partially support our hypotheses. While we did not find an indirect effect of entertainment value on WOM intention via customer satisfaction, we found a direct effect of entertainment value on WOM intention. This suggests that consumers are more likely to recommend the store when they find the promotion entertaining.

4. Discussion and conclusion

Customer satisfaction. Study I identified a positive effect of gambled promotions on customer satisfaction. Furthermore, the entertainment value of the promotion partially mediated this effect; a promising entertainment value boosted customer satisfaction. Only for very high levels of discount approval, entertainment value did not matter. The analyses accounted for other assessments of the promotion, which did not significantly explain differences in satisfaction and WOM intention. Remarkably, the gambled price promotion affected customer satisfaction favorably, despite high evaluations of the candy shop throughout all four experimental conditions. Respondents seemed to discover substantial benefits from both gambled discounts, which transferred into higher satisfaction ratings.

Study II did not reveal a positive effect for gambled price promotion on customer satisfaction. Nevertheless, the experimental and control group appeared to differ in this respect. The different distributions of customer satisfaction indicated that both customers who were more dissatisfied and those who were more satisfied existed in the EG, while satisfaction was bell-shaped for the CG. Consequently, as a result of the promotional event, more people appraised the store either very favorably or very badly. This could not be linked to entertainment benefits. However, discount approval had an influence on customer satisfaction. Consumers who were approving the discount more strongly were also more satisfied with the seller. Yet, with higher stakes, entertainment value becomes less important for customer satisfaction. Disapproving of the outcome of promotion also decreases customer satisfaction. This, in turn, might imply a need to manage consumers' assessments of the outcome of the gamble by the seller.

WOM intention: Both studies detected a positive effect of gambled price promotions on WOM intention. Furthermore, Study I supported the claim that this effect is fully mediated by the entertainment value of the promotion. In line with this evidence, Study II discovered that WOM intention correlated positively with the entertainment value of the promotion. Customers were more willing to recommend the store when they participated in a game, and especially when they considered the game enjoyable. These results underline the positive effects of gambled price promotions in terms of favorable communication by WOM.

Interestingly, these findings align well with recent evidence on promotions with unknown discounts. Hock et al. (2020) show that consumers are more likely to buy when they receive an uncertain discount before making a purchase decision. Perceived luck of getting the discount (which might be similar to discount approval) and store affective attitude (which is closely linked to hedonic purchase evaluations such as perceived entertainment) increase conversion rates and spending.

General marketing implications. The results of the two studies vary in detail; however, they are consistent in three main findings: (1) Shoppers endorsed gambled price promotions. Overall, customer satisfaction and WOM intention were higher when conducting a gambled price promotion. Furthermore, people spent more money in the presence of a risk-involving campaign in both studies; (2) Entertainment benefits were found to be drivers of the success of a gambled price promotion; and (3) Customers who believe that they won the game (i.e., who approve of the discount) are more satisfied. However, this assessment is far from objective. The consumers experienced difficulties in the assessment of probabilities and the outcomes of the game. Instead, in low stake contests, consumers did not seem to care much about the result but rather enjoyed gambling for its own sake. In the high-stake promotion, the objective outcome did not influence WOM intentions. However, subjective evaluations were linked to customer satisfaction.

Being products of field experiments, the studies show high external validity and are based upon realistic samples. The candy shop of Study I primarily targeted wealthier and older customers. The management of the furniture chain of Study II successfully addressed mass markets rather than niches, which permits the result to be generalized to more diverse populations, at least in the country of investigation.

Managerial implications. Like operating a casino and relying on probability laws, risk-involving price promotions pose no real risk to retailers if they conduct a sufficient number of replications of the gamble. The odds must be designed in accordance with the margins for the business or the allowances granted by the manufacturers. Taken together, this new element of the promotional mix appears to be a relatively inexpensive but efficient marketing instrument.

Retailers are encouraged to focus on the entertainment quality of the campaign, especially when running a low stake gamble. Our research suggests that increasing the entertainment or experiential quality of shopping by extending the fun nature of the promotional campaign (e.g., throwing the dice in a public setting) is beneficial for the seller (e.g., Pine and Gilmore 1999). Anecdotal evidence from personal interviews with retail managers and consumers supports this view. The CEO of the furniture retail chain endorsed such views by indicating that he believed that customers enjoyed the games for the sake of playing a game rather than for the potential discount. Such a characteristic engenders positive responses in customers during and after the promotion, even when they are not among the lucky winners. Content issues, such as perceived time pressure (i.e., the time available for the respective shopping task), might be of importance.

In high stake contexts, the seller should seek to alleviate the grief of some customers who feel that they have lost in the game. A remedy to this problem might be to have more consumers “winning” in the game. On the one hand, this could be achieved by paying higher discounts. However, this is costly. On the other hand, we observed that customers’ assessments of the discount could be considered as not very rational. Perceiving an outcome as “satisfactory” or “unsatisfactory” appeared to be subjective. Therefore, sellers should engage in the framing of the outcome of the promotion and try to influence the subjective assessments of discounts selling all outcomes as “wins”. Furthermore, recent findings have identified boundaries for risk and uncertainty involving price promotions. When consumers focus on details of a gamble or promotion rather than the big picture of the promotion, they have a stronger preference for risky and uncertain gambles (Duke et al. 2018). Therefore, more entertaining promotions that focus on the aspects of the game rather than the discount might be more promising.

Limitations and future research. This paper reported the results of two field studies. Study II replicated and extended Study I; by using a distinct retail category, a high stakes situation, a different framing of the campaign, and a divergent format (in particular, the 100% discount option in Study I). Furthermore, customers in Study II might have been more familiar with the type of promotion than customers in Study I.

The field study design limited the refinement of internal validity, and the complete control of many other potential drivers of customer approval of the store could not be established. The nature of field studies also accounts for the lack of a random assignment of subjects to different experimental conditions and for the moderate sample sizes. This limitation offers avenues for future research to utilize laboratory settings for better control of conditions. Furthermore, the constraints of the cooperation partners limited our research design to a brief questionnaire and, therefore, to single-item measures.

This paper investigated two specific frames of gambled price promotions (i.e., ‘wheel of fortune’ and dice game). Clearly, many alternative settings are possible. Managers might increase the gambling aspect of the promotion. As an example, scratch cards or a slot machine could be perceived as even more amusing than spinning a wheel or rolling a dice. When matching symbols determines the discount to be achieved, shoppers might even experience losses that are construed as a near-win (e.g., two matches but one mismatch), as described by Gilovich and Douglas (1986). Furthermore, other types of promotions with unknown discounts could be investigated. Some retailers run promotions where the discount is determined by external events (Ailawadi et al. 2014) or given without active consumer participation (e.g., every 10^{th} customer receives a discount von 100 percent). Future research could investigate such kinds of framing effects and to manipulate the entertainment value of the promotion experimentally.

This paper examined gambled promotions that are genuinely risky in giving each participant the same objective expected discount. As an alternative, the seller could add a skill-based component in the promotion. This could be achieved by incorporating a low-to-moderate difficulty ball game (e.g., scoring a goal, hitting a target) or card game (e.g., blackjack). Consumers who are provided with an unknown reward for performing a specific action are more likely to engage in repeat purchases (Shen et al. 2019). Further studies regarding the role of the degree of consumer participation on customer satisfaction and WOM intention would be worthwhile.

Previous studies have found that price promotions hurt brand loyalty (Gedenk and Neslin, 2000). As customer satisfaction is an important antecedent of loyalty (Oliver 1999), increasing customer satisfaction through gambled price promotions might help to mitigate the negative effect of price promotions on store loyalty. Further research should be carried out to establish these potential downstream implications.

There are also contributions that point to boundary conditions of promotions with unknown consequences. Consumers might reject these offers because they are reactant (Bertini and Aydinli 2020), because they want to maintain their freedom of choice (Briley et al. 2018), or because they engage in cognitive decision making (Laran and Tsiros 2013). Further research might investigate how these aspects influence customer satisfaction and WOM in this context.

Finally, investigating the long-term effects of such gambled promotions are likely to yield thought-provoking inquiries in this area.

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Appendix 1: Measurement and operationalization

Construct	Operationalization
Discount approval	I am satisfied with the obtained discount. [(1)]
Expected discount	What discount do you expect from the promotion? [(2)]
Realized discount	Observed by a research assistant [(2)]
Purchase value	Invoice amount observed by a research assistant [(3)]
Entertainment value	I consider the [wheel of fortune/dice] promotion to be entertaining. [(1)]
Customer satisfaction	I am satisfied with [name of the seller]. [(1)]
WOM intention	I will recommend [name of the seller] to others. [(4)]
Age	Please indicate your age.
Gender	Observed by a research assistant

(1) 5-point rating scale: 1 – do not agree, 5 – completely agree
(2) Response in %
(3) In €
(4) 5-point rating scale: 1 – do not agree, 5 – completely agree (Study I)
3-point rating scale: 1 – no intention to engage in WOM, 3 – intention to engage in WOM (Study II)

Appendix 2: MANOVA of experimental condition on entertainment value, discount approval, customer satisfaction, WOM intention

System-level	Pillai's trace = .28	$F_{8, 296}$ = 6.13		p < .01				
Dependent variable	Entertainment value		Discount approval		Customer satisfaction		WOM intention	
Omnibus test	$F_{2, 150}$	p-level	$F_{2, 150}$	p-level	$F_{2, 150}$	p-level	$F_{2, 150}$	p-level
	5.03	<.01	12.60	<.01	8.04	<.01	4.37	.01
	p-level for Tukey HSD pairwise comparisons							
	EG1	EG2	EG1	EG2	EG1	EG2	EG1	EG2
EG2	.63		<.01		.09		.43	
CG2	<.01	.04	.90	<.01	<.01	.05	<.01	.11

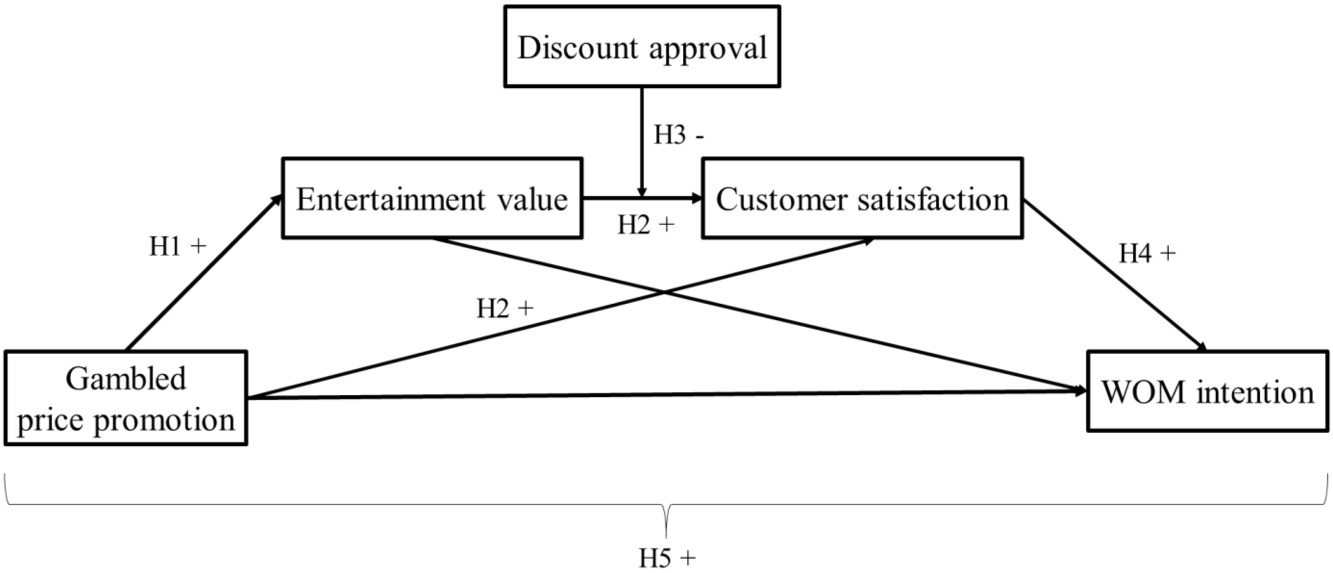
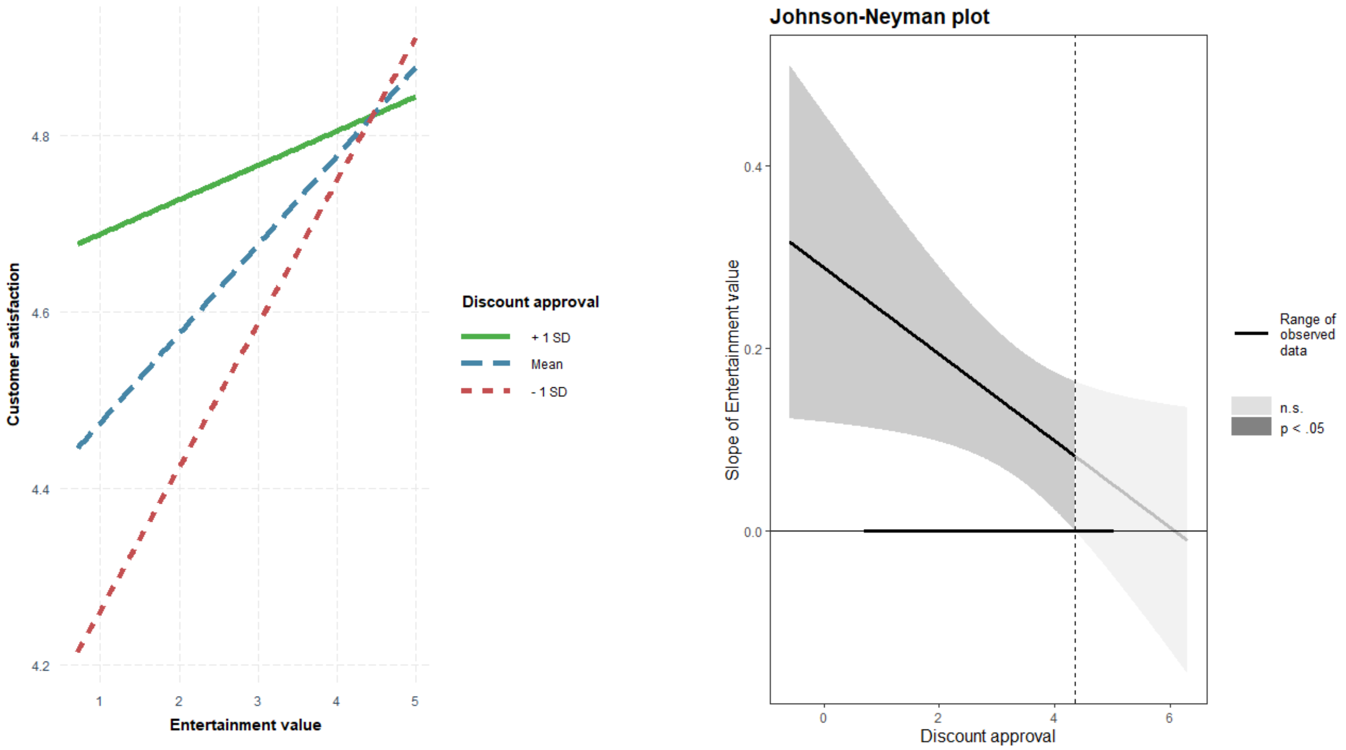


Figure 1: Conceptual model

Figure 2: Setting for Study I (EG1)



(a)

(b)

Figure 3: Results of moderation analysis

Figure 4: Setting for Study II (EG)

	Consumer participation	No consumer participation
Risk	Gambled price promotions (e.g., a game of dice, wheel of fortune)	Probabilistic price promotions (e.g., 0% discount with 50% probability, 50% discount with 50% probability) Probabilistic free price promotions / Chance-for-free promotions (e.g., 0% discount with 75%, 100% discount with 25%) Uncertain incentives
Uncertainty	Lottery promotions (e.g., scratch cards) Uncertain rewards (e.g., completing of a task) Promotional games (e.g., scratch cards, selecting one of three doors)	Conditional rebates (e.g., purchase for free if Germany wins the Soccer World Cup) Imprecisely stated discounts (e.g., random numbers generate the discount) Random discounts

Table 1: A classification of promotions with unknown discounts

Study I	*n*	Expected discount (%)		Realized discount (%)		Entertainment value (1)		Purchase value (€)	
		M	SD	M	SD	M	SD	M	SD
EG1	51	36	41	6	4	4.31	.85	13.57	12.22
EG2	68	21	37	3	12	4.16	1.20	17.68	15.34
CG1	74							9.23	8.82
CG2	34	5		5		3.53	1.19	12.78	12.76
		Discount approval (1)		Customer satisfaction (1)		WOM intention (1)			
EG1		4.33	1.08	4.96	.20	4.91	.45		
EG2		3.41	1.43	4.75	.61	4.78	.60		
CG1				4.59	1.11	4.58	1.09		
CG2		4.47	.75	4.50	.66	4.53	.79		
Study II	*n*	Expected discount (%)		Realized discount (%)		Discount approval (1)		Purchase value (€)	
EG	99	9	6	11	3	3.91	1.24	201.52	338.72
CG	167							181.33	272.23
		Entertainment value (1)		Customer satisfaction (1)		WOM intention (2)			
EG		4.11	.82	3.94	1.20	2.87	.34		
CG				3.90	.34	2.57	.62		
KW (3)				KW=3.66	*p*=.06	KW=17.91	*p*<.01		

EG1: prob. discount (average: 5%, range: 1–10%), EG2: prob. discount (average: 5%, range: 1–100%), CG1: no discount, CG2: det. discount 5%

EG: prob. discount (average: 10.5%, range: 3–18%, CG: no discount)

(1) 5-point scale

(2) 3-point scale

(3) Kruskal–Wallis test

Table 2: Descriptive statistics (Study I, II)

Correlations		Customer satisfaction		WOM intention	
Study I (EG1, EG2) [(1)]	Entertainment value	.42 [(3)]	$p < .01$	.27 [(3)]	$p < .01$
Study II (EG) [(2)]		.05 [(3)]	$p = .63$	.32 [(4)]	$p < .01$

(1) EG1: prob. discount (average: 5%, range: 1–10%), EG2: prob. discount (average: 5%, range: 1–100%)

(2) EG: prob. discount (average: 10.5%, range: 3–18%)

(3) Pearson correlation coefficient

(4) Spearman's rank correlation coefficient

Table 3: Preliminary investigation of the correlation between entertainment value and customer satisfaction and WOM intention (Study I, II)

Model 6 / 14	Entertainment value			Customer satisfaction			WOM intention			row #
	Coefficient [2]	*t*	*p*	Coefficient [2]	*t*	*p*	Coefficient [2]	*t*	*p*	
	Direct effects									
EG1 [1]	.80	3.10	<.01	.40	3.51	<.01	.05	.45	.65	1
EG2 [1]	.60	2.47	.02	.25	2.18	.03	.04	.43	.67	2
Entertainment value				.29	3.37	<.01	.11	3.16	<.01	3
Discount approval				.21	2.17	.03				4
Discount × Entert. [3]				-.05	-2.10	.03				5
Customer satisf.							.53	6.59	<.01	6
	R^2=.07 $F_{2,150}$=5.02		$p < .01$	R^2=.20 $F_{5,147}$=7.25		$p < .01$	R^2=.36 $F_{4,148}$=21.04		$p < .01$	7
							Indirect effects			
							Coefficient	*z*	*p*	
EG1→Entertainment→WOM intention [4]							.09	2.30	.03	8
EG2→Entertainment→WOM intention [4]							.07	1.74	.06	9
EG1→Satisfaction→WOM intention [4]							.19	3.07	<.01	10
EG2→Satisfaction→WOM intention [4]							.10	2.05	.04	11
EG1→Entertainment→Satisfaction→WOM intention [5]							.05	2.16	.03	12
EG2→Entertainment→Satisfaction→WOM intention [5]							.04	1.97	.05	13

(1) Reference category: 5% discount (CG2); EG1: prob. discount (average 5%, range: 1–10%), EG2: prob. discount (average 5%, range: 1–100%)
(2) Non-standardized regression coefficients
(3) Mean centered
(4) *z*-value calculated with a product-of-coefficients test according to Iacobucci et al. (2015)
(5) *z*-value calculated with a product-of-coefficients test according to Taylor et al. (2008)

Table 4: Results from moderated mediation analysis (Study I).

Model 7	**Customer satisfaction**			**WOM intention**			**row #**
	Coefficient [1]	t	p	Coefficient [1]	z	p	
Direct effects							
Entertainment value	.04	.26	.79	1.10	2.80	<.01	1
Discount approval	.23	2.37	.02				2
Discount approval × Entertainment value [2]	.22	1.79	.08				3
Customer satisfaction				.34	1.38	.17	4
	R^2=.10			*Mc Fadden* R^2=.14			
	$F_{3,95}$=3.55		p = .02	*Omnibus* χ_2^2=10.57		p = .01	

[1] Non-standardized regression coefficients

[2] Mean centered

Table 5: Results from moderated mediation analysis (Study II).

6. Summary in English

The participation of the consumer in the price determination process stands out as one of the most innovative and interesting developments in pricing in recent years. It was not until the early 2000s that researchers considered participative pricing worthy of scholarly attention. Driven by the new possibilities of online pricing and some very successful offline examples, several studies have begun to examine how these pricing mechanisms work from the perspective of a firm as well as that of consumers. This dissertation seeks to shed light on two approaches to engaging the consumer in pricing – the participative pricing scheme 'pay-what-you-want' (PWYW) and the pricing tactic 'gambled price promotions'.

The dissertation consists of four articles. The purpose of Article 1 is to develop a model that consistently portrays consumer behavior in PWYW. Furthermore, it aims at understanding the conditions under which the seller can operate profitably when using PWYW. Furthermore, it seeks to explain the circumstances under which the seller can increase profits by introducing a suggested and a minimum price. The article is a commentary on Chen et al. [*Marketing Science* 36(5):780–791 (2017)] who published a model to describe consumer behavior under PWYW pricing. Article 1 identifies inconsistencies in Chen et al.'s model and points to a new segment of consumers who were previously unconsidered. These consumers are characterized by a decision not to buy a good under a PWYW pricing policy, even if they can get it for free, and are not very averse with respect to advantageous inequity. Second, the paper incorporates the effect of disadvantageous inequity aversion on PWYW with the minimum price. The paper offers updated guidelines on how a seller should choose the optimal pricing policy and shows that revised results differ considerably compared to those of Chen et al. (2017).

Article 2 contains a research report with additional analyses on the modeling of PWYW. In addition to the content-related aspects that emerge when modeling PWYW, this technical report provides details on the mathematical aspects and the derivations of the model in Article 1.

Most previous models on PWYW have investigated a buyer-seller interaction with a single product. Article 3 models PWYW for two goods. In this paper, the seller's profitability under traditional prices, PWYW, and combinations between the two options is modeled. The results of a simulation analysis suggest that if both goods have high costs, posted prices are the most profitable strategy; if both goods have low costs, PWYW is optimal. If the costs of one good are low but medium or high for the other good, combinations between PWYW and posted prices might also be optimal.

Article 4 deals with consumer behavior in gambled price promotions. This type of rebate policy allows the consumer to participate in determining the price without having the opportunity of directly influencing the price. Instead, the consumer plays a promotional game, and the result of this game determines the final price. This article examines gambled price promotions framed as in-store events and their impact on two consequences of such campaigns: customer satisfaction and word-of-mouth intention to endorse the retailer. The hypotheses postulate a positive relationship between gambled promotions and both, customer satisfaction and word-of-mouth; and that this relationship is mediated by the entertainment value of the promotion. Also, discount approval is expected to moderate the entertainment value-customer satisfaction relationship. These hypotheses were tested in two field experiments; one in a candy store and the other in a furniture retail store. Three main findings emerge from this study: (1) Customers endorse gambled price promotions and are more willing to engage in word-of-mouth when participating in such a campaign. The effect on customer satisfaction is mixed. (2) The positive effects of such promotions are mediated by their entertainment value to shoppers. (3) When customers are unhappy with the discount received, entertainment value could compensate and ensure high customer satisfaction in a low stake setting.

7. Summary in German

Die Beteiligung von Konsumenten am Preisfindungsprozess kann als eine der innovativsten und interessantesten Entwicklungen in der Preispolitik der letzten Jahre bezeichnet werden. Angetrieben von den neuen Möglichkeiten der Online-Preisgestaltung und einigen sehr erfolgreichen Offline-Beispielen haben Autoren verstärkt untersucht, wie diese Preisbildungsmechanismen sowohl aus der Perspektive eines Unternehmens als auch aus der Perspektive der Konsumenten funktionieren. In dieser Dissertation werden zwei Ansätze beleuchtet, die den Konsumenten in die Preisgestaltung einbeziehen: das partizipative Preissystem „Bezahle was du willst“ (engl. "pay-what-you-want" (PWYW)) und die Preisgestaltung von Preisaktionen mit Glückskomponente (engl. "gambled price promotions").

Die Dissertation besteht aus vier Artikeln. Artikel 1 entwickelt ein mikroökonomisches Modell, das das Konsumentenverhalten unter PWYW-Bedingungen konsistent abbildet und es ermöglicht, die Bedingungen zu verstehen, unter denen der Verkäufer mit PWYW profitabel wirtschaften kann. Der Artikel erläutert, unter welchen Rahmenbedingungen ein PWYW-Verkäufer seine Gewinne mit der Einführung von Preisempfehlungen und Mindestpreisen steigern kann. Der Artikel ist ein Kommentar zu Chen et al. [*Marketing Science* 36(5):780-791 (2017)], die ein Modell zur Beschreibung des Konsumentenverhalten unter PWYW-Preisgestaltung veröffentlicht haben. In Artikel 1 dieser Dissertation werden Inkonsistenzen des Ursprungsmodells aufgezeigt. Der Artikel definiert ein neues Konsumentensegment, das bisher unberücksichtigt blieb. Diese Verbraucher zeichnen sich dadurch aus, dass sie sich bei einer PWYW-Preispolitik nicht für einen Kauf entscheiden, auch wenn sie diesen kostenlos tätigen können und das obwohl ihre Fairnesspräferenzen niedrig sind. Des Weiteren berücksichtigt der Artikel die Auswirkung von Fairnesspräferenzen, wenn der Verkäufer PWYW mit einem Mindestpreis anbietet. Der Beitrag bietet aktualisierte Empfehlungen, wie ein Verkäufer die optimale Preispolitik wählen sollte, und zeigt, dass sich die revidierten Ergebnisse im Vergleich zu denen von Chen et al. (2017) beträchtlich unterscheiden.

Artikel 2 ist ein Forschungsbericht mit zusätzlichen Analysen zur Modellierung von PWYW. Zusätzlich zu den inhaltlichen Aspekten, die sich bei der Modellierung von PWYW ergeben, enthält dieser Bericht Einzelheiten zu den mathematischen Aspekten und den Herleitungen des revidierten Modells aus Artikel 1.

Die meisten bestehenden Modelle zu PWYW untersuchen eine Käufer-Verkäufer-Interaktion mit einem einzelnen Produkt. Artikel 3 modelliert PWYW für zwei Produkte. In diesem Artikel wird die Profitabilität eines Verkäufers analysiert, der zwischen PWYW, festen Preisen und einer Kombination der beiden Optionen wählen kann. Die Ergebnisse der Simulationsanalyse legen nahe, dass, wenn beide Güter hohe Kosten haben, feste Preise die rentabelste Strategie sind; wenn beide Güter niedrige Kosten haben, PWYW optimal ist und wenn die Kosten des einen Gutes niedrig sind, aber die des anderen Gutes mittel oder hoch, eine Kombination zwischen PWYW und festen Preisen optimal ist.

Artikel 4 befasst sich mit dem Konsumentenverhalten bei Preisaktionen mit Glückskomponente. Diese Art von Rabatt ermöglicht es dem Verbraucher, sich an der Festlegung des Preises zu beteiligen, ohne die Möglichkeit zu haben, den Preis direkt zu beeinflussen. Stattdessen spielt der Verbraucher ein Glücksspiel, und das Ergebnis dieses Spiels bestimmt den Endpreis. Die Hypothesen postulieren eine positive Beziehung zwischen Preisaktionen mit Glückskomponente und sowohl der Kundenzufriedenheit als auch der Mund-zu-Mund-Propaganda; und dass diese Beziehung durch den Unterhaltungswert der Preisaktion mediiert wird. Außerdem wird erwartet, dass die Höhe des erspielten Gewinns die Beziehung zwischen Unterhaltungswert und Kundenzufriedenheit moderiert. Diese Hypothesen wurden in zwei Feldversuchen getestet, in einem Süßwarengeschäft und in einem Möbelhaus. Aus der Studie konnten drei Ergebnisse abgeleitet werden: (1) Kunden befürworten Preisaktionen mit Glückskomponente und sind eher bereit, Mund-zu-Mund-Propaganda zu betreiben, wenn sie an einer solchen Preisaktion teilnehmen. Die Effekte auf die Kundenzufriedenheit sind gemischt. (2) Die positiven Auswirkungen solcher Preisaktionen werden durch ihren Unterhaltungswert für die Käufer mediiert. (3) Wenn Kunden mit dem erhaltenen Rabatt unzufrieden sind, kann der Unterhaltungswert die Unzufriedenheit mit dem Gewinn kaschieren und eine hohe Kundenzufriedenheit trotz niedriger Gewinne sicherstellen.