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Chapter 1

Introduction

In economics the usual assumption is that firms maximise their absolute profit given the strategies pursued by their competitors, i.e. firms neglect the potential effect of their own strategies on the profits of rival firms. For a perfectly competitive environment, where there is no such external effect, this strategy, i.e. to maximise absolute profit, seems to be most reasonable.

However, one could raise the question, if the absolute profit maximising strategy is optimal in an imperfect competitive environment, that is in an environment in which firms possess market power. In such an environment actions of one firm do not only affect its own profit but also the profits of all other firms within the same industry.

A firm, which is aware of the negative external effect its strategy has on rival firms profits, could, for instance, pursue a strategy which harms the profits of its competitors more than its own profit, i.e. the firm could act spiteful. Against an evolutionary background, in which the Darwinian-fitness of a firm depends on the level of its absolute profit, such a spiteful strategy would be reasonable since it obviously yields a strategic advantage in the sense that the firms probability of survival within the industry increases.

Adding even another evolutionary component to this imperfect competitive environment, that is firms are able to observe and adopt best responses over the course of time, eventually could lead to a situation in which firms, which gain a strategic advantage by pursuing a spiteful strategy, dominate the whole population.

If evolution in a specific industry actually reaches this point, one can finally ask the question whether the situation, in which the whole population of firms consists of spiteful acting firms, is stable in an evolutionary sense. In other words, is the spiteful strategy an evolutionarily stable strategy if all firms adopt this strategy – evolutionarily stable in this context means that a deviant firm will not be able to gain a strategic advantage by choosing a strategy different from spiteful behaviour.

The thesis at hand now has the aim to shed some light on all these questions. A necessary prerequisite to answer these questions will be to define and introduce all the relevant theoretical concepts which can be used as instruments to analyse imperfect competitive settings – very important concepts in this respect are the theory of spiteful behaviour on the one hand and evolutionarily stable strategies on the other. The main source of reference against this background will be chapter 2 on page 6, the Literature Review.

After the reader has been acquainted with the main theoretical concepts, the line of argument outlined above will be analysed by means of modified versions of the well-known Cournot and Bertrand oligopoly models in chapter 3 on page 29 – the latter model will include a form of product differentiation simply for evaluation reasons.

The main modification in both cases will be related to the preferences of the players in the game – whereas in the classical cases all players are assumed to have independent preferences, i.e. they are solely concerend about their absolute payoff, in the alternative cases presented in this thesis a certain share of players has interdependent preferences, i.e. they are concerned about relative payoff.

In both cases it will be shown, as already indicated above, that players with interdependent preferences gain a strategic advantage by acting spiteful. Essentially this means that players with interdependent preferences act more aggressively so that these players eventually end up with a higher absolute payoff than their rivals with independent preferences. If the probability of survival depends on absolute payoff then obviously players with interdependent preferences will have a strategic advantage. Note, however, that all this observations can be made within a framework of strategic competition, where only the objective functions of the players are modified.

In addition to this, evolutionary equilibria of the modified oligopoly models will be analysed. Since the evolutionary process leading to an evolutionary equilibrium in the two models will not be modeled explicitly, the approach choosen will differ from the usual procedures in evolutionary game theory which often rely on models of imitation, inheritance, random mutation and so forth.

By means of logical reasoning it will be shown that eventually spiteful acting firms will dominate the whole population and the strategy of these firms in this situation is also evolutionarily stable. Evolutionarily stable strategies in a finite population of players often display this kind of spiteful behaviour. In particular, the finite population evolutionary equilibrium is not necessarily a Nash-equilibrium – this point will be illustrated with an example. Another result will be that evolutionary or strategic advantage can be attained at the expense of higher absolute payoffs.

The main contribution of this thesis, however, is to show that the outcome of a strategic competition, when all competitors maximise relative payoffs, is the same as in evolutionary equilibrium.

Chapter 2

Literature Review

The following sections serve to introduce and analyse the most important ideas, notions, concepts and theories in the context of the topic of this thesis. After this review the reader should have the essential understanding to follow the models in the next chapter.

2.1 Independent v. interdependent preferences

2.1.1 Profit maximisation assumption

The usual assumption in economic theory regarding the behaviour of the firm is that firms choose a strategy which will maximise their absolute profit. With regard to this profit maximisation assumption, Koçkesen et al. [7] speak about the hypothesis of *independent preferences*, thereby taking a more general view:

> "[...] agents choose their actions with the sole purpose of maximizing their own material payoffs regardless of how their actions affect the payoffs of other individuals." – i.e. they neglect the potential external effect on others inherent in their actions.

The advanced reader will anticipate the consequences of such an assumption and will therefore agree that the profit maximisation assumption can be characterised as one of the most important axioms in (micro-)economic theory. Although one should think that there are many good reasons to adhere to such an important assumption, the theoretical foundations for it are mostly intuitive by nature and arguments in favour are rather rare – for good reasons as will be seen later.

Probably the most important argument for the profit maximisation assumption is made on evolutionary grounds. Because most specifications of this argument are very similar, only two of them are outlined in the following. The first one, also known as the Friedman conjecture – brought forward by Milton Friedman in [4] – is as follows:

> Business based on "[...] rational and informed maximization of returns [...] will prosper and acquire resources with which to expand; whenever it does not, the business will tend to lose resources [...]. The process of 'natural selection' thus helps to validate the hypothesis – or, rather, given natural selection, acceptance of the hypothesis can be based largely on the judgment that it summarizes appropriately the conditions for survival."

The second specification of the argument stems from Koçkesen et al. [7] and can be summarised as follows: material sacrifices by one agent in order to affect the payoffs of other agents will result in lower wealth for the sacrificing agent, eventually leading to a situation in which the non-sacrificers dominate the environment – unfortunately the authors do not provide any arguments why exactely this situation should occur; if in addition a competitive selection process is inherent in this environment, then forgoing any sacrifices becomes a simple precondition for survival.

In other words and applied to an industrial environment, non-profit maximisers, i.e. firms which are not exclusivelly concerned with their own material payoff and which do take into account the potential effects of their actions on other industry participants, will have a lower probability to survive and therefore, over the long run, only absolute profit maximising firms will remain in the industry. As a result the optimal strategy to pursue for a firm is to maximise absolute profit.

However, Koçkesen et al. immediately emphasize in [7] and [8] that although the argument is somehow convincing in a perfectly competitive environment, i.e. an environment in which agents are not able to affect the payoffs of other agents, it does not hold for strategic settings or imperfectly competitive environments, i.e. environments in which the actions of one agent do not only affect her own payoff but also the payoffs of all other agents.

The reasons for the argument not to be generally true – specifically in strategic settings – and hence the resulting consequences are discussed in the following sections – at this point it should be mentioned that the following comments are confined to industrial environments in which the agents are firms.

2.1.2 The theory of spiteful behaviour

A very clear and intuitive explanation why the argument does not hold in strategic environments is given by Schaffer [9]:

> "[...] a firm which does not maximise its profit may still earn profits which are larger than those of its profit-maximising competitors, if the costs to itself of its deviation from maximisation are smaller than the costs it imposes on the maximising competitors."

The crucial point in Schaffer's explanation is that the non-profit maximising firm is able to impose larger costs on its rivals than on itself via its deviating strategy. This requires, as indicated above, that the firms act in a strategic environment, i.e. strategies or actions taken by one firm within the industry do not only affect its own payoff but also the payoffs of all other firms. In other words, there exists some kind of externality by which firms gain market power.

Schaffer developed his intuitive explanation by applying Hamilton's theory of spite – a theory in evolutionary biology – to an industrial environment in which firms act strategically. In evolutionary biology, a *spiteful behaviour* is described as an action by an animal which harms both the animal itself and other animals of its population. According to Hamilton, spiteful behaviour is likely to occur if it decreases an animal's Darwinian fitness, i.e. the probability of survival, less than the fitness of all other animals in the population.

Now, applying this theory to a strategic setting yields the following results: if firms have market power, i.e. firms are able to affect the payoffs of all other firms within the industry via their actions, the occurrence of a spiteful behaviour – or strategy in this case – is very likely, simply because

> "A firm which forgoes the opportunity to maximise its absolute profit may still enjoy a selective advantage over its competitors if its 'spiteful' deviation from profit-maximisation harms its competitors more than itself." (Schaffer [9]).

In other words, imperfectly competitive environments, i.e. an environment in which firms posses market power, allow firms to acquire a strategic advantage by pursuing a spiteful strategy, i.e. a strategy, different from the usual profit-maximisation behaviour, which imposes more costs on the rivals than on itself.

In this respect, Koçkesen et al. in [7] and [8] also speak about *(negatively) interdependent preferences*, as opposed to the above mentioned independent preferences, and *strategic advantage*. By (negatively) interdependent preferences the authors mean that an agent with this kind of preferences is not only concerned about absolute payoffs, but also about relative payoffs, i.e. absolute payoff in comparison with the average payoff to all agents – the term '(negatively) interdependent' implies that the objective function of a player with this kind of preferences increases with his own payoff but decreases with respect to the payoffs to other players. A strategic advantage, on the other hand, enables an agent to achieve a higher level of probability for survival during the natural selection process.

Besides these terms, the authors also give a slightly different definition of spiteful behaviour, as compared to Schaffer and Hamilton, that they call *spiteful effect*:

> "The spiteful effect occurs when it is possible for a player to deviate from a Nash equilibrium action profile in such a manner as to reduce the payoffs of other players more severely than the payoffs of the deviating player." (Koçkesen et al. [7])

The crucial difference in the two definitions is that the one given by Hamilton does not rely on a Nash equilibrium action profile.

Interconnecting all these terms with each other provides the following relationship (cp. explanation by Schaffer and Hamilton's theory of spite): In the presence of market power, negatively interdependent preferences may give rise to spiteful behaviour; this will be the case when spiteful behavior results in a strategic advantage.

In order to give the reader a much better understanding of the ideas, notions and concepts introduced above, the following section will illustrate them by means of a numerical example.

2.1.3 A numerical example

Consider a duopoly in which the decision variable is quantity – the classical Cournot case. The two firms $i = 1, 2$, setting quantity $q_i \in [0, \infty)$, both have the same cost function $C(q_i) = cq_i$, i.e. firms have constant marginal cost c and no fixed costs. Aggregate inverse demand is given by the linear function $P(Q) = a - bQ$, with $Q = q_1 + q_2 = \Sigma q_i$ (it will also be assumed that $a, b > 0$ and $a > c$). Profit to firm i can be written as $\pi_i(q_i, q_j) = (a - bQ - c)q_i$.

If both firms maximise absolute profit they solve the following maximisation problem:

$$\begin{array}{rc} \underset{q_i}{\text{argmax}} & \pi_i(q_i, q_j) = (a - bQ - c)q_i \\ [\text{foc}] & \frac{d}{dq_i}\pi_i(q_i, q_j) = -bq_i + (a - bQ - c) = 0 \\ [\text{soc}] & \frac{d^2\pi_i(q_i,q_j)}{dq_i^2} = -2b < 0 \end{array} \tag{2.1}$$

Solving the first order condition of (2.1) for q_i yields the reaction function $R_i(q_j)$ of player i – as one can see, the second order condition guarantees that the solution of the maximisation problem is going to be a maximum:

$$q_i =: R_i(q_j) = \frac{a - bq_j - c}{2b} \tag{2.2}$$

By inserting the reaction function of firm j into the reaction function of firm i and vice versa one gets the Cournot-Nash equilibrium quantities q_i^C and q_j^C – as one can see from (2.3) the solution is symmetric:

$$q_i^C = \frac{a-c}{3b} \text{ and } q_j^C = \frac{a-c}{3b} \Longrightarrow Q^C = \frac{2(a-c)}{3b} \tag{2.3}$$

Since the quantities set by the two firms in equilibrium are known, one can also calculate equilibrium profits and price:

$$\pi_i^C = \pi_j^C = \frac{(a-c)^2}{9b} \text{ and } P(Q^C) = P^C = c + \frac{(a-c)}{3} \tag{2.4}$$

In this first part of the example, both firms maximise absolute payoffs given the quantity set by the other firm. The two competing firms ignore the negative externality they impose on each other; compared to a monopolist, they produce too much from the perspective of the industry, and this is because they disregard the externality. Therefore the two firms have, according to the definition given by Koçkesen et al. in [7], independent preferences.

Now suppose that firm i does not maximise absolute profit but the difference between its own profit and the industry average profit – this behaviour could be denoted by *relative profit maximisation*. Therefore firm i solves the following maximisation problem:

$$\begin{array}{rl} \underset{q_i}{\text{argmax}} & f_i(q_i, q_j) =: \pi_i - \frac{(\pi_i + \pi_j)}{2} \\ \text{[foc]} & \frac{d}{dq_i} f_i(q_i, q_j) = a - 2bq_i - c = 0 \\ \text{[soc]} & \frac{d^2 f_i(q_i, q_j)}{dq_i^2} = -2b < 0 \end{array} \tag{2.5}$$

From (2.5) immediately follows the optimal quantity q_i^A firm i should set – again the second order condition guarantees that the solution of the maximisation problem will result in a maximum:

$$q_i^A = \frac{a-c}{2b} \tag{2.6}$$

Equation (2.6) could also be interpreted as the reaction function of the relative profit maximising firm i. The interesting thing about this reaction function is that the optimal quantity of firm i does not depend on the quantity set by firm j, the absolute profit maximiser. By inserting the reaction function of firm i into the reaction function of firm j (see equation (2.2)) one gets the Nash equilibrium quantity q_j^A for firm j:

$$q_j^A = \frac{a-c}{4b} \Longrightarrow Q^A = q_i^A + q_j^A = \frac{3(a-c)}{4b} \tag{2.7}$$

Under these conditions – firm i maximising relative profit and firm j maximising absolute profit – profits and price in equilibrium are:

$$\pi_i^A = \frac{(a-c)^2}{8b}, \ \pi_j^A = \frac{(a-c)^2}{16b} \ \text{ and } \ P(Q^A) = P^A = c + \frac{(a-c)}{4} \tag{2.8}$$

Note that $P^A < P^C$ – with P^C being the price in the standard Cournot case – and thus the total output in the industry is higher in this alternative case as compard to the standard Cournot setting.

From equation (2.8) it is obvious that $\pi_i^A > \pi_j^A$, i.e. the relative profit maximising firm makes a higher absolut profit than the absolute profit maximising firm. The intuition behind this result is as follows.

Instead of disregarding the externality, firm i is exploiting the externality to harm the opponent. By setting a higher output level, firm i reduces the payoff of firm j more than its own payoff – note that this is not totally in accordance with the theory of spiteful behaviour since the deviation of firm i does not harm itself. Instead of producing q_i^C it sets $q_i^A > q_i^C = q_j^C$ causing the market price to decrease to P^A since $Q^A > Q^C$. This spiteful behaviour by firm i, however, does more harm to firm j, the absolute profit maximiser, than to firm i since both firms are now selling at a lower price. Firm j, acting strategically, reacts to the higher quantity set by firm i by producing a lower quantity. This then results in the initially observed instance that $\pi_i^A > \pi_j^A$.

2.2 Evolution and interdependent preferences

2.2.1 Motivation

The previous part of this chapter made clear that especially in strategic environments firms which adopt interdependent preferences are able to outperform their competitors which maximise absolute profits – this has also been demonstrated by a numerical example. Futhermore it has already been indicated that by the adoption of interdependent preferences and by the resulting spiteful behaviour, a firm is able to gain a strategic advantage over its profit maximising competitors. The following part of the chapter now concentrates on this strategic advantage and the evolutionary consequences associated with it.

The main points in this respect will be emphasised by further developing the example of the previous part of this chapter. Only then will the theoretical foundations be presented. It should also be mentioned that the basis of the following example and the graphical representations have been taken from Schaffer [9], while the numerical part has been developed by the author himself.

Recall the duopoly example from the previous part of this chapter and consider the following economic selection process: after the two firms have set their quantities a selection mechanism, that determines which of the two firms will survive and therefore produces in the next period, is initiated. Firm i is selected during this process with probability $0 < p_i < 1$, i.e. there is no guarantee for survival or extinction.

The probability of survival depends on the absolute profit of firm i in the following way: $p_i > p_j$ if and only if $\pi_i > \pi_j$ • this is the so-called survival rule. This means that the Darwinian fitness of firm i is higher than the one of firm j if and only if firm i outperforms firm j in terms of absolute profit. To use the terminology from the previous part of this chapter: strategic advantage is based on absolute profits.

Now think about the following: since firm i, the relative profit maximising firm, has a strategic advantage over firm j, the absolute profit maximiser – recall that $\pi_i^A > \pi_j^A$ • it is more likely that firm i will survive the economic selection process. From the higher Darwinian fitness of firms with interdependent preferences, however, follows that the new rivals will also be endowed with such preferences, either through inheritance, imitation or any other similar process.

So if both firms maximise the relative profit – that is the difference between own profit and the industry average profit – they solve the same maximisation problem, illustrated in (2.5). It immediately follows that the optimal quantities set by the two firms are

$$q_i^* = q_j^* = Q^*/2 = \frac{a-c}{2b} \Longrightarrow Q^* = \frac{a-c}{b} \tag{2.9}$$

Under these conditions price and profits in equilibrium are:

$$\pi_i^* = \pi_j^* = 0 \quad \text{and} \quad P(Q^*) = P^* = c \tag{2.10}$$

Equation (2.10) shows a quite astonishing result, namely that if both firms have interdependent preferences, the equilibrium outcome of this 'Interdependent Duopoly' is equal to the competitive outcome $P = c$ • the graphical representation of this result is shown in Fig. 2.1.

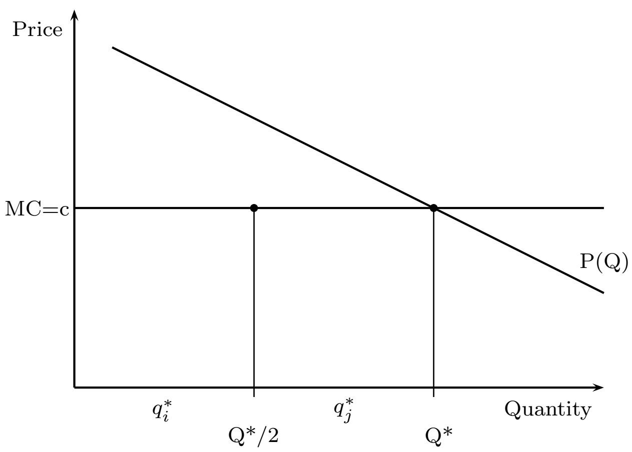

Figure 2.1

Not only are the equilibrium strategies q_i^* and q_j^* of this duopoly equal to the competitive outcome strategies but they also represent the *symmetric evolutionary equilibrium (SEE)*, as Schaffer notes in [9]. To see this consider the following case: while firm i continues to sell $Q^*/2$ – the equilibrium strategy quantity – firm j deviates and produces some other quantity. No matter what quantity firm j will produce, firm i will always make a higher profit and a lower loss than firm j respectively. Therefore firm i, due to the survival rule outlined above, will always have a higher probability of survival too.

> "In other words, the strategy of selling $Q^*/2$ units of output is analogous to the ESS [*evolutionarily stable strategy*, ed.] of evolutionary biology that a deviant ('mutant') firm which uses another strategy will always have a lower profit than a firm which sells $Q^*/2$ [...]." (Schaffer [9])

In order to illustrate the point consider Fig. 2.2 and Fig. 2.3 which represent two different cases in terms of the quantity set by firm j – the quantity set by firm i stays the same in both cases, i.e. $q_i = q_i^* = q_i^A$. In Fig. 2.2 firm j sells a smaller quantity than $Q^*/2$. This is the case, for instance, in which firm j has independent preferences and therefore maximises absolute profit (see above). Although, as can been seen from Fig. 2.2, firm j now earns a positive profit π_j, its profit is smaller than the profit of firm i, simply because firm i, the interdependent preference firm, sells a larger quantity.

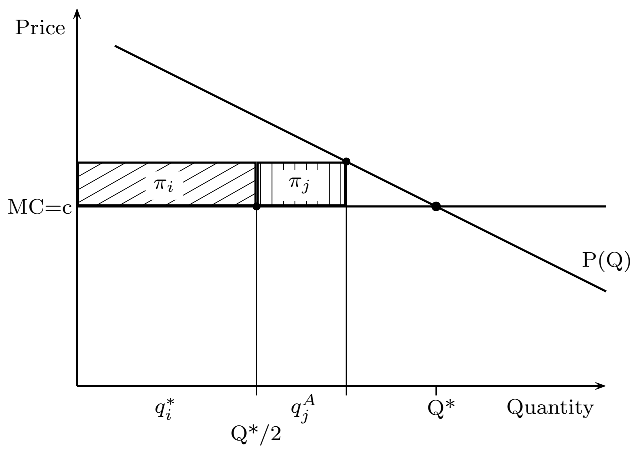

Figure 2.2

A similar pattern can be observed if firm j sells a higher quantity than $Q^*/2$ – see Fig. 2.3 on the following page. In this case, due to the high quantity sold by firm j, price is below marginal cost and both firms make a loss. However, since firm i sells a lower quantity than firm j, it suffers a smaller loss.

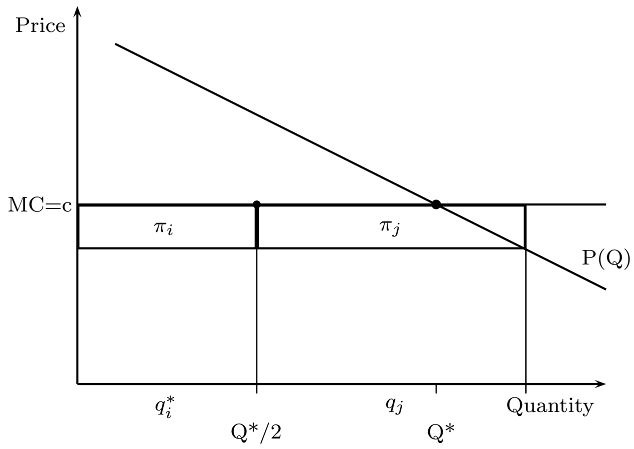

Figure 2.3

As a result, firm i always has a higher Darwinian fitness, i.e. a strategic advantage over firm j in the sense that its profit is higher or its loss is lower and therefore it has a higher probability of survival. And since deviating from the strategy $Q^*/2$ does not pay off with regard to the economic natural selection process, the strategy can be denoted as evolutionarily stable.

2.2.2 A basic evolutionary model

The first section of this literature review was concerned with the profit maximisation assumption and the evolutionary argument brought forward in favour of it. However, the last sections and especially the example in the previous one have shown that in a strategic setting, i.e. in an environment in which firms posses market power, evolutionary considerations speak in favour of interdependent preferences as opposed to independent preferences, with the latter associated with the profit maximisation assumption.

In this regard, Koçkesen et al. in [7] point out that

> "While it is conventional in economic models to posit that individuals are material payoff maximizers, there is now mounting experimental evidence that contradicts this strong 'independence' hypothesis. The theoretical plausibility of this assumption has accordingly been questioned recently by several economists on evolutionary grounds."

One of these economists is Schaffer, whose motivating example with regard to the evolutionary stability of interdependent preferences has already been presented in the previous section. In [9] he argues that if the above mentioned Friedman conjecture is to be accepted as valid, then the process of economic natural selection needs to be modelled explicitly in order to show that evolution really favours independent preferences.

As a result, Schaffer himself undertakes the task of modelling economic natural selection and in doing so he uses a concept introduced by Maynard Smith and Price, namely the so-called *evolutionarily stable strategy (ESS)*. Intuitivelly one could think of an ESS in the following way:

> "[...] an ESS is a strategy such that, if most of the members of a population adopt it, there is no 'mutant' strategy [coming into the population in a sufficiently small fraction, ed.] that would give higher reproductive fitness." (Schaffer [9])

Maynard Smith and Price defined their concept for a continuum population of individuals playing a two-person game. Schaffer modified this concept so that it can be applied to economic games with a finite population of individuals – like the two firms in the previous example – who have a large set of strategies – like the production levels of the two firms. In addition, Schaffer confines himself to looking at a finite population which is *playing the field*, i.e. all players compete with each other at the same time. The rest of this section now presents this basic evolutionary model.

Since the model is looking at an evolutionary process, G, which denotes the underlying game, is not only played once but for t periods. There are N players and N is constant for all periods t. Each player i, with $i = 1, \ldots, N$, has a strategy $s_{it} \in S$, with S being the set of strategies available in G_t. At this point Schaffer makes the simplifying assumption that $s_{it} = s_i$, i.e. player i does not change her strategy over time – one could think of inherited strategies. As usual, every player i receives a payoff π_{it} at the end of each period. π_{it} is a function of s_{it} and s_{-it}, i.e. the strategies of player i and the strategies of all other players. Droping the subscript t one can write $\pi(s_i, s_{-i})$. Note also that only symmetric games are considered, i.e. the payoffs to any player depend on her strategy and those of other players but not on the name of the player.

One last word regarding this basic setting of the model. Although, as Schaffer indicates, one could think of the players of this game as being firms, it is probably more intuitive to think of them as being the managers of the firms, especially when taking into account that the number of players is constant and they do not change strategy over time. One could, for instance, imagine a constant number of firms which hire and fire managers, whose unchangeable strategy determines the bahaviour of the firm. By replacing – or being forced to replace – the manager, firms are able to change their strategies.

In order to complete the model, three more crucial conditions are needed. The first one, denoted by Schaffer as the *entry condition*, in general sets out how players decide whether or not to enter the game. Since in the model at hand N is assumed constant, no such decision or no such condition is required, i.e. if one player, be it a firm or a manager, is selected out of the game, a new player enters.

The second one is the *strategy choice rule*, which determines the strategies choosen by the players. Modelling the strategy selection by players generally is a very difficult task due to the complex nature of this process. There are, however, some behavioural patterns, like imitation of successful strategies, which regularily appear in reality. On the other hand, of course, inherited patterns or random choices because of mutation play an important role too. For the moment, Schaffer decides to use the following strategy choice rule: if one player is selected out of the game then the new player entering the game will play any strategy $s_i \in S$ with positive probability, i.e. mutation has a positive probability – he also points out that the selection of a specific strategy choice rule has a fundamental effect on the stability of the outcome of the natural selection process.

Finally a *survival rule* – probably as important as the strategy choice rule – is defined, which determines the probability that a certain player will participate in the next round of the game. By deciding on such a rule, one determines what Darwinian fitness is or in other words, the basis upon which strategic advantage is built – thus a decision not to be underestimated.

In choosing such a rule, Schaffer goes the usual way by selecting a monotonic survival rule for his model: if p_{it} is the probability in period t that player i will participate in the game in period $t+1$ then it holds that $p_{it} > p_{jt}$ if and only if $\pi_{it} > \pi_{jt}$, i.e. player i is more likely to survive than player j if and only if the profit of player i exceeds the one of player j – it is also assumed that $0 < p_{it} < 1$, i.e. no player will survive/fail to survive with certainty – recall that the same survival rule applies for the motivating example in the previous section. Schaffer does not forget to indicate that the monotonicity assumption is not a handicap in modelling different survival rules. To strenghten his argument he gives the following, important examples: survival of player i could be dependent on absolute profit, perhaps in the form $p_{it} = f(\pi_{it}),\ f^{'} > 0$, or on relativ profit, with $p_{it} = f(\pi_{it}/\Sigma\pi),\ f^{'} > 0$.

Now that all necessary parts have been developed, the evolutionary equilibrium and the corresponding evolutionarily stable strategy (ESS) of the modell can be defined. Within this context, Schaffer notes that the ESS, i.e. the solution of the model, comes in two parts, namely an *equilibrium condition* and a *stability condition*. The difference between these two is the degree of stability of the solution, that is how many deviant players does the equilibrium tolerate without changing. While the former is stable for only one deviant player, the latter is stable for two or more deviant players – note that as a result, only the stability condition is fully consistent with the above mentioned definition of an ESS. It should also be mentioned that the ESS defined in the following is symmetric, i.e. the ESS is the same for all players, due to the composition of the model. Therefore the solution induced by the ESS can be denoted by SEE, which stand for *symmetric evolutionary equilibrium*.

Equilibrium condition

For the purpose of the above made differentiation between equilibrium condition and stability condition, first consider the equilibrium condition case, where $N-1$ players have the SEE strategy $s^{SEE} \in S$ and only one player – call her player d – chooses a different strategy $s^D \in S$. The corrsponding payoffs are π^{SEE} and π^D respectively. In a more formal way this means

$$\pi^{SEE} \equiv \pi(s^{SEE} \mid s^D, s^{SEE}, s^{SEE}, \dots), \tag{2.11}$$

$$\pi^D \equiv \pi(s^D \mid s^{SEE}, s^{SEE}, s^{SEE}. \tag{2.12}$$

By means of these two payoffs, one can now define a symmetric evolutionary equilibrium in the following way.

Definition 1. *A strong (weak) SEE is given by a strategy $s^{SEE} \in S$ which has the property that, if $N-1$ players have this strategy and one deviant* *player has some other strategy s^D, then for any deviant strategy $s^D \in S$ it holds*

$$\pi(s^{SEE} \mid s^D, s^{SEE}, s^{SEE}, \dots) > (\geq)\ \pi(s^D \mid s^{SEE}, s^{SEE}, s^{SEE}, \dots) \quad (2.13)$$

A player choosing the SEE strategy therefore has a strictly higher (or equal) payoff than with any other deviant strategy. Together with the monotonicity assumption stated above this means that those players choosing the SEE strategy have a higher probability of survival than their deviant counterparts. As a result we have reached a SEE in accordance with the definition made above.

With this basic evolutionary model Schaffer now attends himself to the question if the Friedman conjecture, i.e. profit maximisation appropriatly summarises the conditions for survival, is valid or not. In doing so he analyses the connection between SEE and SNE, the *symmetric Nash equilibrium.*

Definition 2. *One can define a strong (weak) symmetric Nash equilibrium as a strategy $s^{SNE} \in S$ which, for any alternative strategy s^D, satisfies*

$$\pi(s^{SNE} \mid s^{SNE}, s^{SNE}, \dots) > (\geq)\ \pi(s^D \mid s^{SNE}, s^{SNE}, \dots). \quad (2.14)$$

Confronting definition one and two reveals the main difference between a SEE and a SNE: while the SNE compares the payoffs of a single player if she chooses different strategies – the strategies of the other players remain unchanged – the SEE compares the payoffs of two different players with different strategies. Therefore the SNE is a comparison of the absolute payoffs of one and the same player whereas the SEE relates the payoffs of two players with different strategies, i.e. the SEE is based on relative payoffs – despite the fact that the survival rule in the model could be based upon absolute payoff – and the SNE is based on absolute payoffs.[1]

Another observation – and probably the most crucial one in the context of Schaffers attempt to proof the Friedman conjecture wrong – can be made if one formulates the SEE in a different way, namely as the Nash equilibrium of a different game.

[1]In order to guard against misunderstandings, the following should be pointed out: although both interdependent preferences and the SEE are associated with relative payoffs, this does not automatically mean that only strategies based on interdependent preferences will induce a SEE.

Definition 3. *Consider the normal-form game $G \equiv \{N, S, \pi\}$, where N, S and π are the number of players, the strategy space common to all players and the absolute payoff function to player i. The strategies $(s_1^*, \ldots, s_n^*)$ are said to be a Nash equilibrium of this game if for every player i the following holds*

$$\underset{s_i \in S_i}{\operatorname{argmax}} \ \pi(s_i \mid s_{-i}^*). \tag{2.15}$$

Basically this definition says that player i should choose the strategy which maximises her absolute profit, given that all other players choose their optimal strategies, i.e. which maximises their absolute profit. If this definition of a SNE is applied to the SEE, one has to take into account that while the former is based on absolute profits, the latter is not based on absolute but on relative profits (see remarks above). As a result, the maximisation problem of definition three has to be slightly modified: the strategy s^{SEE} is said to be a symmetric evolutionary equilibrium strategy if it solves

$$\underset{s^D \in S}{\operatorname{argmax}} \ \{\pi^D - \pi^{SEE}\}. \tag{2.16}$$

With regard to the intuition behind equation (2.16), one can think of it in the following way: players choosing the evolutionarily stable strategy get a profit of π^{SEE}. Since playing the ESS is optimal by definition, a player, call her d, who wants to maximise her profit relative to the other players – this is why the player maximises the difference – should follow suit and also play s^{SEE}. If player d does not, she will get a profit $\pi^D < \pi^{SEE}$ that does not lead to a maximum of the difference in (2.16).

Due to the fact that the SEE strategy is symmetric, instead of writing π^{SEE} one can also write the following:

$$\pi^{SEE} \equiv \pi(s_i \mid s_{-i}) \ \text{ for } \ i \neq d \qquad \equiv \frac{1}{N-1} \sum_{i \neq d}^{N} \pi(s_i \mid s_{-i}). \tag{2.17}$$

By substituting expressions of (2.17) in (2.16) one finally gets the maximisation problem

$$\underset{s^D \in S}{\operatorname{argmax}} \{\pi(s_d \mid s_{-d}) - \frac{1}{N-1}\sum_{i \neq d}^{N} \pi(s_i \mid s_{-i})\}. \tag{2.18}$$

What one can see from this last equation is that the SEE strategy solves a maximisation problem different from absolute profit maximisation.[2] By means of this model Schaffer therefore succeeds to proove the Friedman conjecture wrong – at least under the assumptions made during the development of the model, i.e. profit maximisation does not appropriatly summarises the conditions for survival. On the contrary, in this model players which maximise the difference in profits are those with the highest probability of survival and difference maximisation is evolutionarily stable too.

Stability condition

While the previous section was concerned about the equilibrium condition, i.e. the stability of the SEE if only one deviant player – deviating from the ESS – exists, the following section will address the issue of what Schaffer in [9] calls the stability condition. This condition, generally speaking, indicates how stable a SEE is if two or more players deviate from the ESS strategy.

Within this context, Schaffer speaks about *Y-stability* and *global stability* respectively:

> "[...] an SEE is Y-stable under a given strategy choice rule if, for a population with a total of anywhere from 2 to Y deviants with any deviant strategies, the payoff of an SEE player is strictly greater than the payoff of all the deviants. An SEE is globally stable if this holds for any number of deviants up to $N-1$ (since we need at least one SEE player for the definition to make sense)."

In other words, if players with the SEE strategy have a higher probability of survival – due to higher profits – than their deviant counterparts, no matter

[2]In fact, equation (2.18) is the definition of the Nash equilibrium strategy of the zero-sum 'beat-the-average' game introduced by Shubik [10]; the attentative reader will have recognised that the numeric example presented in the previous part of this chapter is such a beat-the-average game.

which deviant strategy the latter choose and no matter how many deviators there are, the SEE is called Y-stable – globally stable if $Y = N - 1$, i.e. there exits only one player with the SEE strategy.

However, as Schaffer indicates in [9], one has to take into account some crucial facts with regard to the stability of a SEE. First of all it is possible that there is no globally stable SEE or worse there is no SEE at all. Consider, for instance, the example in which 'it pays to be different'. If most players choose a certain strategy A while only a minority chooses strategy B then the latter will make a higher profit. As a result, more and more players would choose strategy B until those players with strategy A make a higher profit. In other words, deviants are always better off and there exists no SEE at all.

Two other crucial points mentioned by Schaffer are the absolute-payoff effect and that the stability of a SEE is highly dependent on the strategy choice rule choosen for the model. While the former is concerned with the fact that SEE strategy players can have a very low absolute profit, the latter refers to the increasing complexity if one chooses a random strategy choice rule instead of an imitation strategy choice rule.

2.3 Aggregative games

After this short introduction to evolutionary concepts, the remaining part of the Literature Review will deal with some general characteristics of games. One of these characteristics, which deserve closer attention, concentrates on the variables which determine the payoff of a player. Since there are many games in econcomics in which the payoff-functions exhibit this characteristic and the next chapter concentrates on this set of games, the following section introduces them.

If the payoff to any player solely depends on the players individual strategy and the sum of all strategies, the game is said to be aggregative – here the aggregate would be the sum of all players strategies; if one replaces the sum by any other combination, the game is said to be generalized aggregative. The classic example for an aggregative game would be the Cournot Oligopoly in which the payoff to player i depends on the output produced by player i and the sum of all outputs produced. The following definition has been taken from Alós-Ferrer & Ania [2].

Definition 4. *A (generalized) symmetric aggregative game with aggregate* g *is a tuple* $\Gamma \equiv (N, S, \pi)$ *where* N *is the number of players, the strategy set* S*, common to all players, is a subset of a totally ordered space* X*,* $\pi : S \times X \to \mathbb{R}$ *is a real-valued function, and* $g : S^N \to X$ *is a symmetric and monotone increasing function, such that individual payoff functions are given by* $\pi_i(s) \equiv \pi(s_i, g(s))$ *for all* $s = (s_1, \ldots, s_n) \in S^N$ *and* $i = 1, \ldots, N$*.*

Alós-Ferrer & Ania also point out in [2] that it is useful to express the aggregate g, i.e. the combination of all strategies, in a different way, namely as an aggregate g depending on the strategy of a certain player and the aggregate of the strategies of all other players.

Definition 5. *A family of symmetric aggregative games is a collection of games* $\{\Gamma^n\}_{n=1}^{\infty}$ *where* $\Gamma^n \equiv (N, S, \pi)$ *is a (generalized) symmetric aggregative game with aggregate* g^n *such that* $g^1(s) = s$ *for all* $s \in S$ *and there exists a function* $g : X \times S \to X$ *such that*

$$g^{n+1}(s_1, \ldots, s_n, s_{n+1}) = g(g^n(s_1, \ldots, s_n), s_{n+1}) \tag{2.19}$$

for all $s_1, \ldots, s_{n+1} \in S$*, and all* $n \geq 1$*.*

Using (2.19), one can write the payoff to player i in one of the following ways – recall that s_i denotes the strategy choosen by player i while s_{-i} denotes the strategies choosen by all other players:

$$\pi_i(s_i, s_{-i}) = \pi(s_i, g^n(s_i, s_{-i})) = \tilde{\pi}(s_i, g^{n-1}(s_{-i})), \tag{2.20}$$

with $\tilde{\pi} : S \times X \to \mathbb{R}$ being defined as $\tilde{\pi}(s, x) = \pi(s, g(x, s))$.

One important reason why the aggregate is divided into an individual strategy and an aggregate of all other strategies is that this step facilitates the analysis of aggregative games – e.g. the analysis of best reply correspondences (also see chapter 3 on page 29).

2.3.1 Optimal aggregate-taking strategy

Another important concept with regard to aggregative games, which will be useful in the course of this thesis, is the so-called *aggregate-taking behavior*. The main idea behind this concept is that agents in an arbitrary aggregative game ignore the effect of their individual decision/strategy on the aggregate. Alós-Ferrer & Ania in [2] give the following defintion within this context:

Definition 6. *Let* $\Gamma \equiv (N, S, \pi)$ *be a symmetric aggregative game. We say that* $s^* \in S$ *is an optimal aggregate-taking strategy (ATS) if*

$$s^* \in \underset{s}{\operatorname{argmax}} \ \pi(s, g(s^*, \ldots, s^*)). \tag{2.21}$$

A strict ATS is an ATS which is a strict maximizer of the problem.

One could interprete an ATS in the following way: given that all players choose strategy s^*, the aggregate will be $g(s^*, \ldots, s^*)$. The optimal strategy for a player facing this aggregate is again s^*.

2.3.2 Evolutionarily stable strategy in an aggregative game

As a matter of course, the concept of an evolutionarily stable strategy can be applied to aggregative games – this will be very helpful in the coming sections of this thesis. The ESS of a generalised aggregative game can be defined as follows (see Alós-Ferrer & Ania [2]):

Definition 7. *Let* $\Gamma \equiv (N, S, \pi)$ *be a symmetric aggregative game with aggregate* g*. A strategy* $s \in S$ *is said to be an evolutionarily stable strategy (ESS) if for all* $s^{'} \in S$ *it holds that*

$$\pi(s, g(s^{'}, s, \ldots, s)) \geq \pi(s^{'}, g(s^{'}, s, \ldots, s)). \tag{2.22}$$

The interpretation of (2.22) is that a deviant player with any strategy $s^{'} \in S$ performs worse than the players which choose the ESS in terms of absolute profit – note that the aggregate contains the strategies of all players.

In previous sections it has already been outlined that evolutionarily stable strategies can vary in their degree of stability. For this notion too there is an equivalent in aggregative games (see Alós-Ferrer & Ania [2]):

Definition 8. *An ESS, s, is weakly (strictly) globally stable if, for all* $s^{'} \neq s$ *and all* $1 \leq m \leq N-1$

$$\pi(s, g(\underbrace{s^{'}, \ldots, s^{'}}_{m}, s, \ldots, s)) \geq \ (>)\ \pi(s^{'}, g(\underbrace{s^{'}, \ldots, s^{'}}_{m}, s, \ldots, s)). \tag{2.23}$$

That is, players choosing the ESS will always make a higher absolute profit than deviant players regardless of the number of deviant players – note, however, that all deviant players have to choose the same strategy $s^{'}$.

Another interesting fact is that, although the two concepts of aggregative-taking strategies and evolutionarily stable strategies are different in nature, one can make two important propositions which relate the two concepts – the two propositions have been proved by Alós-Ferrer & Ania in [2]:

Proposition 1. *Let* $\Gamma \equiv (N, S, \pi)$ *be a symmetric aggregative game. Suppose* Γ *is quasisubmodular in individual strategy and the aggregate. If* $s^* \in S$ *is an ATS, then* s^* *is also an ESS and it is weakly globally stable. If* s^* *is a strict ATS, then* s^* *is the unique ESS (and hence also the unique ATS) and it is strictly globally stable.*

Proposition 2. *Let* $\Gamma \equiv (N, S, \pi)$ *be a symmetric aggregative game. Suppose* Γ *is quasisupermodular in individual strategy and the aggregate. If* $s^* \in S$ *is an ESS, the* s^* *is also an ATS. If* s^* *is a strict ESS, then* s^* *is also a strict ATS.*

2.4 Supermodular and submodular games

In general, supermodularity and submodularity respectively are a particular characteristic of a payoff-function of a game. If a certain game comprises a payoff-function with one of these two characteristics, then it is said to be a supermodular/submodular game.

The reason why the concepts of supermodular and submodular games are examined here is that these concepts seem to be in relationship with the above indicated fact that interdependent preferences potentially yield a strategic advantage over independent preferences (see section 2.1 on page 6). Koçkesen et al. in [7], for instance, conclude that

> "[...] at any intragroup symmetric equilibrium of an action-monotonic strictly supermodular game, the absolute payoffs to interdependent players are strictly greater than those to independent players [...]."

Although generally the concepts of supermodularity and submodularity respectively are relatively demanding, there also exists a more accessible and intuitive interpretation, which should be sufficient for the purposes of this thesis – especially with regard to the next chapter.[3]

2.4.1 Supermodularity

Definition 9. *A function $f : X \times Y \to \mathbb{R}$, with $X, Y \subseteq \mathbb{R}$, is supermodular, or has increasing differences, for the tuple (x, y) if for all $x^{'} \geq x$ and $y^{'} \geq y$ it holds*

$$f(x^{'}, y^{'}) - f(x, y^{'}) \geq f(x^{'}, y) - f(x, y). \tag{2.24}$$

The intuition behind this definition is as follows: a marginal increase of x leads to a greater increase of the value of the function f the more y increases.

In order to make the above mentioned definition and equation (2.24) even more clearer, consider the following game $G \equiv \{N, S, \pi\}$, where N is the number of players, S is the strategy set common to all players and π represents the payoff-function $\pi : S \times S^{N-1} \to \mathbb{R}$ (in the following s_i denotes the strategy of player i and g the aggregate of the strategies of all other players). Game G is now said to be supermodular if for all $s_i^{'} \geq s_i$ and $g^{'} \geq g$ it holds

$$\pi(s_i^{'}, g^{'}) - \pi(s_i, g^{'}) \geq \pi(s_i^{'}, g) - \pi(s_i, g), \tag{2.25}$$

i.e. the incentive for player i to increase s_i increases with the level of the aggregate g. In other words the strategies are *strategic complements*. If the

[3] For a more demanding definition/interpretation of these concepts see Koçkesen et al. [7], p. 280ff.

function π is smooth and twice differentiable then the concept of increasing differences represented in equation (2.25) can also be described using derivatives in the following way:

$$\frac{\partial^2 \pi(s_i, g)}{\partial s_i \partial g} \geq 0. \tag{2.26}$$

2.4.2 Submodularity

Definition 10. *A function $f : X \times Y \rightarrow \mathbb{R}$, with $X, Y \subseteq \mathbb{R}$, is submodular, or has decreasing differences, for the tuple (x, y) if for all $x^{'} \geq x$ and $y^{'} \geq y$ it holds*

$$f(x^{'}, y^{'}) - f(x, y^{'}) \leq f(x^{'}, y) - f(x, y). \tag{2.27}$$

This definition can be specified in the following way (cp. definition of supermodularity): a marginal increase of x leads to a lower increase of the value of the function f the more y increases.

Similar to the case of supermodularity, if one applies equation (2.27) to the game described in the previous subsection, then game G is said to be submodular if for all $s_i^{'} \geq s_i$ and $g^{'} \geq g$ it holds

$$\pi(s_i^{'}, g^{'}) - \pi(s_i, g^{'}) \leq \pi(s_i^{'}, g) - \pi(s_i, g), \tag{2.28}$$

i.e. the incentive for player i to increase s_i decreases with the level of the aggregate g.[4] In other words the strategies are *strategic substitutes*. If the function π is again smooth and twice differentiable then the concept of decreasing differences shown in equation (2.28) can also be described in the following way:

$$\frac{\partial^2 \pi(s_i, g)}{\partial s_i \partial g} \leq 0. \tag{2.29}$$

[4] See subsection 3.1.2 on page 33 for an even more concrete example.

Chapter 3

The Models

The models presented in the following sections – in fact the two models are modified versions of the classic oligopoly models, namely the Cournot and the Bertrand oligopoly models – are an application of the ideas, notions, concepts and theories of the previous chapter. Therefore they are intended to illustrate the interaction between the choice of objective function, market power and evolutionary success.

3.1 Cournot oligopoly and interdependent preferences

Section 2.1.3 on page 10 in the previous chapter already presented a Cournot duopoly in which one of the players has interdependent preferences, i.e. this player is not concerned about absolut profit but relative profit. This section now is intended to generalise this Cournot duopoly, namely to the case where the number of firms is $n \geq 2$. It is divided into two parts: the first one illustrates the Cournot oligopoly with interdependent preferences and its results, while the second one analyses these results by means of the concepts explained in the Literature Review.

3.1.1 Alternative Cournot oligopoly

Consider the alternative Cournot oligopoly, in which firms $i = 1, \ldots, n$, with $n \geq 2$, set quantities $q_i \in [0, \infty] \bullet$ q_{-i} denotes the quantities set by all other firms except firm i. All firms have the same cost function $C(q_i) = cq_i$, i.e. there are no fixed costs and marginal costs are constant at c. Aggregate inverse demand is given by the linear function $P(Q) = a - bQ$ with $Q = \sum_{i=1}^{n} q_i$. With regard to the parameters of the inverse demand function it will be assumed that $a, b > 0$ and $a > c$. Taking into account this basic setting, the profit function of firm i is given by

$$\pi_i(q_i, q_{-i}) = (a - b\sum_{i=1}^{n} q_i)q_i - cq_i. \tag{3.1}$$

As opposed to the usual Cournot oligopoly, not all firms have independent preferences in this alternative version, i.e. not all firms maximise absolute profit. Assume that there are $0 \leq k \leq n$ firms which have interdependent preferences, i.e. they are concerned about relativ profits – the remaining $n-k$ firms still have independent preferences. Firms with this kind of preferences maximise the difference between their own profit π_i and the industry average profit $\bar{\pi}$ – in the following these firms are called relative profit maximisers.[1] This difference is

$$f_i(q_i, q_{-i}) = \pi_i - \bar{\pi} = \pi_i(q_i, q_{-i}) - \frac{1}{n}\sum_{j=1}^{n} \pi_j(q_j, q_{-j}). \tag{3.2}$$

Now, given this information one can solve the maximisation problem for both, the firms with independent preferences and the firms with interdependent preferences. While for the former the problem is shown in (3.3), the problem for the latter is depicted in (3.4).

[1]Section 5.1 on page 53 in the appendix addresses the issue of whether a firm with interdependent preferences should include its own profit in $\bar{\pi}$ or not.

$$
\begin{array}{rr}
\underset{q_i}{\operatorname{argmax}} & \pi_i(q_i, q_{-i}) = (a - bQ - c)q_i \\
\text{[foc]} & \frac{d}{dq_i}\pi_i(q_i, q_{-i}) = -bq_i + (a - bQ - c) = 0 \\
\text{[soc]} & \frac{d^2\pi_i(q_i,q_{-i})}{dq_i^2} = -2b < 0
\end{array} \tag{3.3}
$$

$$
\begin{array}{rr}
\underset{q_i}{\operatorname{argmax}} & \pi_i^D(q_i, q_{-i}) = \pi_i(q_i, q_{-i}) - \frac{1}{n}\sum_{j=1}^{n} \pi_j(q_j, q_{-j}) \\
\text{[foc]} & \frac{d}{dq_i} f_i(q_i, q_{-i}) = -b(q_i - \frac{1}{n}Q) + (a - bQ - c)(1 - \frac{1}{n}) = 0 \\
\text{[soc]} & \frac{d^2 f_i(q_i,q_{-i})}{dq_i^2} = \frac{-2b(n-1)}{n} < 0
\end{array} \tag{3.4}
$$

As one can see from above, both the second order condition for absolute profit maximisers and the second order condition for difference maximisers guarantee that the solution will be a maximum. Denoting the optimal quantity set by a profit maximising firm i with q_i^P and rearranging the first order condition of (3.3) yields:

$$
q_i^P = \frac{a}{b} - \frac{c}{b} - Q. \tag{3.5}
$$

Equation (3.5) shows that all firms which maximise absolute profit will set the same quantity in the optimum. Therefore the solution q_i^P is said to be *intragroup symmetric*. If (3.5) is solved for q_i^P one gets the reaction function of an absolute profit maximising firm:

$$
q_i^P =: R_i^P(q_{-i}) = \frac{1}{2b}\left(a - c - b\sum_{j \neq i} q_j\right). \tag{3.6}
$$

Proceeding with the first order condition of (3.4) in the same way yields similar results – the quantity set by a difference maximising firm i is denoted with q_i^D:

$$
q_i^D = \left(\frac{a}{b} - \frac{c}{b}\right)\left(1 - \frac{1}{n}\right) - Q\left(1 - \frac{2}{n}\right). \tag{3.7}
$$

Again, equation (3.7) shows that the quantity set by difference maximising firms will be the same in the optimum and hence q_i^D is intragroup symmetric. The reaction function of a difference maximising firm i, on the other hand, is given by:

$$q_i^D =: R_i^D(q_{-i}) = \frac{1}{2b(n-1)} \left((n-1)(a-c) - (n-2)b \sum_{j \neq i} q_j \right). \tag{3.8}$$

It has already been mentioned that in the optimum both q_i^P, i.e. the quantity set by an absolute profit maximising firm, and q_i^D, i.e. the quantity set by a difference maximising firm, are intragroup symmetric. As a result one can also write $q_i^P = q_A^P$ and $q_i^D = q_A^D$ respectively.[2] Since there are k difference maximising firms and $n-k$ absolute profit maximising firms, the total quantity in the optimum is

$$Q_A = (n-k)q_A^P + kq_A^D. \tag{3.9}$$

Taking this into account and using equations (3.5) and (3.7) one can solve for the optimal quantities q_A^P, q_A^D and Q_A:

$$q_A^P = \frac{(n-k)(a-c)}{(n(n+1)-2k)b} \geq 0 \quad \text{and} \quad q_A^D = \frac{(2n-k-1)(a-c)}{(n(n+1)-2k)b} > 0, \tag{3.10}$$

$$Q_A = \frac{(n^2-k)(a-c)}{(n(n+1)-2k)b} > 0. \tag{3.11}$$

Bearing in mind that $a > c$, $n \geq 2$ and $0 \leq k \leq n$, one can verify that the above stated inequalities hold. Price P_A and profits – π_{abs} for absolute profit maximisers and π_{rel} for relative profit maximisers – in the optimum are then given by

$$P(Q_A) = P_A = \frac{a(n-k) + c(n^2-k)}{n(n+1)-2k} = c + \underbrace{\frac{(a-c)(n-k)}{n(n+1)-2k}}_{\geq 0}, \tag{3.12}$$

[2]The subscript A stands for ”alternative”.

$$\pi_{abs} = \frac{(a-c)^2(n-k)^2}{(n(n+1)-2k)^2 b} \geq 0 \text{ and } \pi_{rel} = \frac{(a-c)^2(n-k)(2n-k-1)}{(n(n+1)-2k)^2 b} \geq 0. \tag{3.13}$$

From the last expression in (3.12) one can already draw an important conclusion: P_A, i.e. the equilibrium price in the alternative Cournot oligopoly, will exceed marginal costs c only if $k < n$. If, however, $k = n$ – that is all firms maximise their relative profit – the equilibirum price will equal the competitive price, i.e. $P_A = c$. The implications for equilibrium profits π_{abs} and π_{rel} are straightforward: only in case that $k < n$ profits will be strictly positive.

3.1.2 Analysis of alternative Cournot oligopoly

The first part of this analysis is focused on the question whether or not the Cournot game described in the previous section is a (generalised) aggregative game or not. In the Literature Review of this thesis it has already been noted that a game is called aggregative if the payoff to any player depends on the players strategy and the sum of all strategies including the players strategy – it would be called generalised aggregative if the sum would be replaced by any combination of all strategies.

Looking at equation (3.1) yet reveals that the general, (alternative) Cournot oligopoly is an aggregative game since the payoff to firm i, denoted by π_i, depends on the strategy of firm i, which is the amount q_i in this Cournot case, and on the sum of all strategies, namely the total amount $\sum_{i=1}^{n} q_i$:

$$\pi_i(q_i, q_{-i}) = (a - b\sum_{i=1}^{n} q_i - c)q_i. \tag{3.14}$$

Rearranging the profit function (3.14) also gives:

$$\pi_i(q_i, q_{-i}) = (a - b(q_i + \sum_{j \neq i} q_j) - c)q_i. \tag{3.15}$$

Equation (3.15) reveals another important interpretation, which will be useful in the further analysis: the profit to firm i depends on the quantity set by firm i and an aggregate of the quantities set by all other firms. This aggregate, called it g^{n-1}, is defined in the following way: $g^{n-1} : S^{n-1} \longrightarrow \mathbb{R}_+$ and $g^{n-1}(q_{-i}) = \sum_{j \neq i} q_j$.[3] Therefore the profit to firm i can be written in the following form:

$$\pi_i(q_i, g^{n-1}(q_{-i})) = (a - b(q_i + \sum_{j \neq i} q_j) - c)q_i. \tag{3.16}$$

The next question of interest for this analysis is whether or not the (alternative) Cournot game at hand is supermodular/submodular. In order to answer this question, again the profit function has to be analysed. Section 2.4 on page 26 argued that these two concepts are based on the idea of increasing/decreasing differences. It has also pointed out that if the payoff-function is smooth and twice differentiable then one can use cross derivatives to check whether or not supermodularity/submodularity exists. Since the payoff-function of the analysed Cournot game is smooth and twice differentiable one can use this cross derivative check:

$$\frac{\partial \pi(q_i, g(q_{-i}))}{\partial q_i} = a - 2bq_i - b \sum_{j \neq i} q_j - c, \quad \frac{\partial^2 \pi(q_i, g(q_{-i}))}{\partial q_i \partial g(q_{-i})} = -b < 0. \tag{3.17}$$

Due to the fact that the cross derivative is negative, the Cournot game is submodular, that is it is characterised by decreasing differences in the quantity q_i and the aggregate $g(q_{-i})$. The intuition behind this result is the following: decreasing differences means that the incentive for firm i to increase its quantity q_i decreases with the aggregate $g(q_{-i})$ – in the Cournot case the latter is the sum of all quantities set by the other firms. In other words the quantity set by player i and the total quantity set by all other players are strategic subsititutes.

In order to avoid any misunderstandings regarding the submodular nature of the Cournot oligopoly consider Figure 3.1 on the following page. The illustration shows the graphic representations of two profit functions of the form specified in equation (3.15). As one can see, the only difference between these two profit functions is the level of the aggregate – the aggregate in this

[3] Compare (2.26) in section 2.4 on page 26.

case would be $\sum_{j \neq i} q_j$, which will be denoted with g in the example. Further it will be assumed that $g^{'} < g^{''}$.[4]

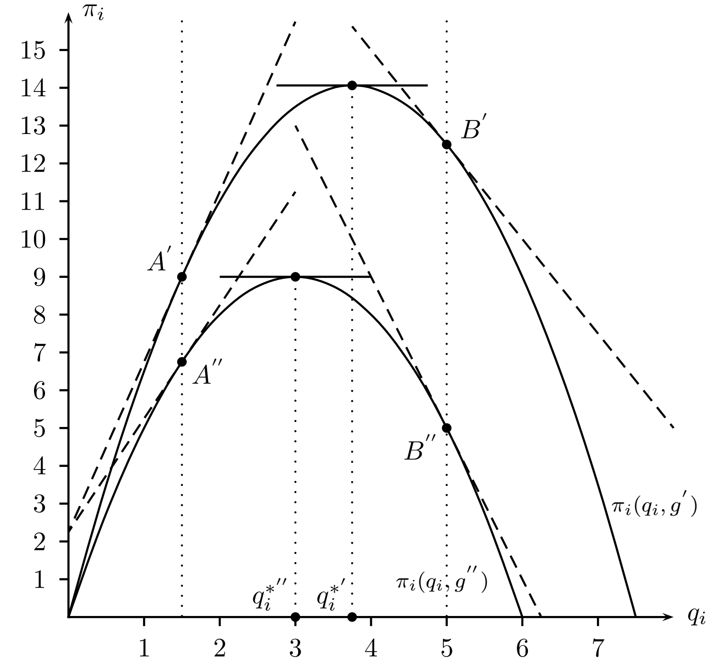

Figure 3.1

One can see that if the aggregate increases from $g^{'}$ to $g^{''}$, the optimal quantity, i.e. the quantity which maximises profits given the aggregate, decreases from $q_i^{*'}$ to $q_i^{*''}$. The same conclusion can be drawn if one looks at the reaction function of a profit maximising firm which is shown in (3.6) since this function is decreasing in the total quantity set by all other firms.

The concept of submodularity, however, goes a step further than the reaction function. Consider, for instance, a marginal increase in the quantity q_i at $q_i = 1.5$. The subsequent increase in the profit of firm i depends on the level of the aggregate. From figure 3.1 one can clearly see that if the aggregate is equal to $g^{''}$ the increase in profit will be lower than if the aggregate equals $g^{'}$ – the tangent at point $A^{''}$ has a smaller slope than the tangent at point $A^{'}$. Therefore one can conclude that the incentive to increase the quantity q_i decreases with the level of the aggregate g. The same is true for a marginal increase in the quantity q_i at $q_i = 5$: although the tangent at point $B^{''}$ is

[4]The values choosen for the two profit functions are: $a = 15$, $b = 1$, $c = 0$, $g^{'} = 7.5$ and $g^{''} = 9$.

much steeper than at $B^{'}$, i.e. the change in profits is much higher, the slopes are negative.

The next issue to be addressed is the effect of firms with interdependent preferences in this alternative Cournot game. First, however, assume that there are no difference maximisers, that is $k \rightarrow 0$. Naturally this case is equal to the classic Cournot oligopoly in which all firms maximise absolute profit – the results of this case can be found in section 5.2 on page 55 of the appendix.

Now consider the effect on quantities, price and profits shown in equation (3.10), (3.11), (3.12) and (3.13) if $k \rightarrow n$, i.e. there are only difference maximisers:

$$\lim_{k\rightarrow n} q_A^P = 0, \quad \lim_{k\rightarrow n} q_A^D = \frac{(a-c)}{bn}, \quad \lim_{k\rightarrow n} Q_A = \frac{a-c}{b}, \tag{3.18}$$

$$\lim_{k\rightarrow n} P_A = c, \quad \lim_{k\rightarrow n} \pi_{abs} = 0, \quad \lim_{k\rightarrow n} \pi_{rel} = 0. \tag{3.19}$$

As one can see from (3.18) and (3.19) respectively, if all firms have interdependent preferences, i.e. all firms are concerned about relative profit instead of absolute profit, the solution to the alternative Cournot oligopoly is equal to the competitive solution, that is price equals marginal costs and profits are zero.

Another interesting observation can be made if one compares the two quantities produced in equilibrium by absolute profit maximisers on the one hand (q_A^P) and relative profit maximisers on the other (q_A^D). One can easily show that

$$q_A^D - q_A^P = \frac{(a-c)(n-1)}{(n(n+1)-2k)b} > 0. \tag{3.20}$$

From equation (3.20) one can also see that the difference in quantities increases with k. Therefore in the general, alternative Cournot oligopoly relative profit maximising firms will always produce more than their absolut profit maximising counterparts – regardless of how many relative profit maximisers there are – and the difference between these quantities increases with the number of relative profit maximising firms.

This result suggests that firms with interdependent preferences act more aggressively and their aggressiveness increases with their number – the aggressiveness by difference maximisers can be associated with the theory of spiteful behaviour introduced in section 2.1.2 on page 8 of the Literature Review. The aggressive behaviour of relative profit maximising firms eventually leads to the result that, if the whole population consists of players with interdependent preferences, i.e. $k = n$, the equilibrium price equals the competitive price – see equation (3.12).

The rationale behind this is the following: in the alternative Cournot oligopoly it is rational for a firm to have interdependent preferences due to evolutionary reasons (see below). Now, firms with interdependent preferences are not concerned about absolute profit but about relative profit, i.e. a decrease in absolute profit is no reason to worry for them. As a result these firms set a quantity higher than the absolute profit maximising quantity – see equation (3.20) – since their preferences have a different underlying principle. The absolute profit maximisers, on the other hand, react strategically with respect to this higher (total) amount and set a lower quantity – even lower than the quantity set by firms in the usual Cournot oligopoly.

Another interesting oberservation arises if one looks at the difference of the equilibrium profits π_{rel} and π_{abs} if k converges to n. The difference in equilibrium profits is given by

$$\pi_{rel} - \pi_{abs} = \frac{(a-c)^2(n-k)(n-1)}{(n(n+1)-2k)^2 b}. \tag{3.21}$$

Evaluating this difference at $a = 25, c = 0, b = 1$ and $n = 10$ and plotting the results for all possible values of k gives the graphic representation shown in figure 3.2 on the following page.[5]

The figure clearly shows that the difference in profits is decreasing in k. This means that although relative profit maximising firms have a strategic advantage over absolute profit maximising firms – a strategic advantage could arise if the probability of survival depends on the level of absolute profit – this advantage is decreasing in k.

[5]The values have been arbitrarily choosen and one would get similar results for other feasible values for a, b, c and n.

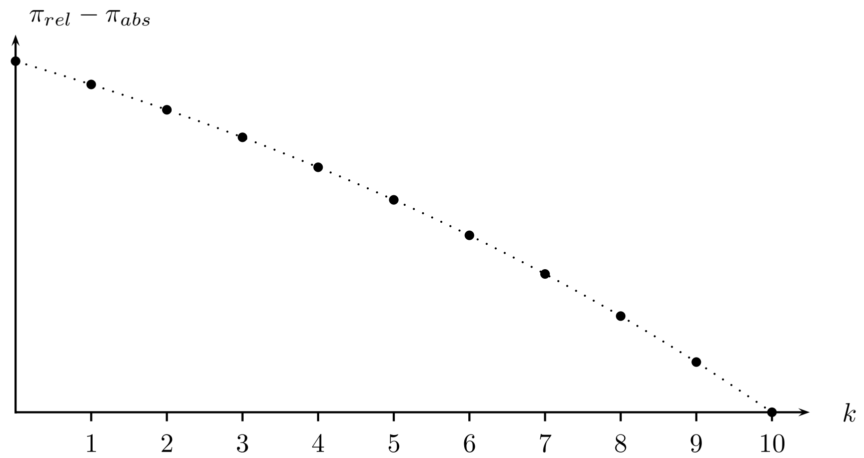

Figure 3.2

3.1.3 Evolutionary equilibrium

The final issue to be addressed with regard to the general, alternative Cournot oligopoly is the evolutionary equilibrium of this game. In order to find an evolutionarily stable strategy one has to fall back on observations made in the previous section as well as concepts introduced in the Literature Review.

First, comparing the profits of absolute profit maximiser and relative profit maximising firms already reveals an important aspect of the game:

$$\pi_{rel} - \pi_{abs} = \frac{(a-c)^2(n-k)(n-1)}{(n(n+1)-2k)^2 b} \geq 0 \quad \text{for} \quad k \leq n, \quad n \geq 2. \tag{3.22}$$

Equation (3.22) shows that except for the case where $k = n$, i.e. all firms maximise relative profit, the difference in profits between relative profit and absolute profit maximising firms respectively will be strictly positive since the numerator and the denominator are positive, i.e. relative profit maximising firms make a higher absolute profit than their rivals.

If the Darwinian-fitness of the players increases with their absolute profit and players are able to observe the profits of their rivals then it is reasonable to assume that over the course of time – and some imitation process in the background – more and more players will choose the relative maximising strategy since this behaviour obviously yields a competitive advantage.[6] Eventually this evolution implies that the number of relative profit maximising firms will equal the total number of firms, i.e. $k = n$, and prices and profits will be equal to the competitive or Walrasian outcome and all firms will produce the quantity q_A^D – in the following, q_A^D for the case of $k = n$ will be denoted by q^*.

The question now is if the strategy q^* – the Walrasian quantity – is an evolutionarily stable strategy of the general, alternative Cournot oligopoly. To answer this question one first has to recall the definition of an ESS which is given by Definition 7 on page 25: players choosing the ESS are always better off in terms of absolute profit than a deviant player choosing a different strategy.

In proving that q^* is in fact an ESS one can use the same argumentation as in the motivating example in subsection 2.2.1 on page 13:[7] first consider the case in which all players choose q^*. By now it should be clear that in this case price equals marginal costs, firms make zero profit and the total Walrasian quantity equals nq^*. Now consider the case in which a deviant player chooses q instead of q^* with $q < q^*$. As a result, the total quantity produced will be smaller than the total Walrasian quantity. Since the price is strictly decreasing in total quantity, the price will now be larger than marginal costs and all firms make a small profit. The deviant firm, however, since it produces a smaller quantity, will make a lower profit than players which produce q^*.

Finally, consider the case in which a deviant player chooses q instead of q^* with $q > q^*$. Then the total quantity produced will be larger than the total Walrasian quantity and hence the price will be smaller than marginal costs. Therefore all firms will make a small loss. The deviant firm, however, since it produces a higher quantity, will make a larger loss than the players choosing q^*.

Another question arising within this context is if the ESS q^* is globally stable. Recalling the definition of a strictly globally stable ESS which is given by Definition 8 on page 26, one can make the following observation: regard-

[6]This argumentation is linked to the so-called indirect evolutionary approach identified by Güth [6]. Of course, a more comprohensive analysis would have to dwell on the imitation process in the background.

[7]Note that for the proof one has to assume constant returns to scale; this assumption is fulfilled for the general, alternative Cournot oligopoly.

less of the number of deviant players, firms choosing q^* will always make a higher absolute profit compared to their deviant rivals – the crucial point here is that all deviant players choose the same deviant strategy. The argumentation is similar to the case in which only one deviant player exists (see above). However, one can omit the line of argument in this respect since Vega-Redondo in [12] shows in a very elegant way that q^* actually is a strictly globally stable ESS.

An alternative way to show that q^* is an ESS would be to use the fact that q^* is an optimal aggregative-taking strategy (ATS) according to Definition 6 on page 25 – in short, if all players choose q^* then it is optimal for a player to choose q^* as well if she faces the aggregate $g(q^*, \ldots, q^*)$. In addition, as has been shown in the previous subsection, the general, alternative Cournot oligopoly is a submodular game. As a result, Proposition 1 on page 26 holds in this case and one can conclude that q^* is in fact an ESS.

3.2 Bertrand oligopoly and interdependent preferences

The following section analyses the effects of the existence of firms with interdependent preferences within a Bertrand oligopoly with product differentiation.[8] Unlike in the previous section, in which a generalised, alternative Cournot model was presented, the alternative Bertrand oligopoly is not generalised in the sense that $n = 3$, i.e. there are only three firms – the reason why this analysis concentrates on this case is because of the bulky expression one gets in the generalised version; for the sake of completeness, however, section 5.3 on page 56 of the appendix presents the generalised, alternative Bertrand oligopoly with product differentiation.

This section too will be structured in the following way: the first part illustrates the Betrand oligopoly with interdependent preferences and its results, while the second one is attended to the analysis of these results.

[8]One could regard this choice as a refinement of the model in the previous section, since Cournot competition, i.e. competition in quantities, is rather rare and price or Bertrand competition is presumably predominant in todays economies and a setting with heterogenous products / product differentiation seems to be more appropriate too.

3.2.1 Product Differentiation

Before going into medias res some crucial elements of the Bertrand oligopoly with product differentiation presented here have to be explained. The consumers' utility functions underlying the demand functions in this setting have been taken from Vives [13] and are of the following form:

$$U(q) = \alpha \sum_{i=1}^{n} q_i - \frac{1}{2}\left(\beta \sum_{i=1}^{n} q_i^2 + 2\gamma \sum_{j \neq i} q_i q_j\right), \tag{3.23}$$

with $|\gamma| < \beta/2$ and $\alpha > \gamma \bullet$ n denotes the number of goods/firms from which consumers can choose while q_i is the quantity the consumer chooses from good/firm i. Differentiating (3.23) for q_i and then for q_j yields:

$$\frac{\partial^2 U(q)}{\partial q_i \partial q_j} = -\gamma. \tag{3.24}$$

The intuition behind equation (3.24) is the following: if $\gamma > 0$ then the marginal utility of consuming good i is decreasing with the consumption of good j, i.e. the goods are substitutes; on the other hand, if $\gamma < 0$ then the marginal utility of consuming good i increases with the consumption of good j, i.e. the goods are complements.

As a result, γ measures the degree of substitutability/complementarity of two different goods. This means that the closer γ is to $-\beta/2$ ($\beta/2$) the higher the degree of substitutability (complementarity) of two different goods; if, however, $\gamma = 0$ then the goods are homogenous.

Solving the consumers utility maximisation problem finally yields the following demand function for the good of firm i (see Vives [13]):

$$\begin{aligned} D_i(p_i, p_j) \quad = \quad & \frac{\alpha}{(\beta + (n-1)\gamma)} - \frac{(\beta + (n-2)\gamma)}{((\beta + (n-1)\gamma)(\beta - \gamma))} p_i \\ & + \frac{\gamma}{((\beta + (n-1)\gamma)(\beta - \gamma))} \sum_{j \neq i} p_j. \end{aligned} \tag{3.25}$$

For $n = 3$ this expression is reduced to

$$
\begin{aligned}
D_i(p_i, p_j) &= \frac{\alpha}{(\beta+2\gamma)} - \frac{(\beta+\gamma)}{((\beta+2\gamma)(\beta-\gamma))}p_i \\
&\quad + \frac{\gamma}{((\beta+2\gamma)(\beta-\gamma))}\sum_{j\neq i} p_j \\
&= \frac{\alpha}{(\beta+2\gamma)} - \frac{(\beta+\gamma)p_i + \gamma\sum_{j\neq i} p_j}{((\beta+2\gamma)(\beta-\gamma))}. \qquad (3.26)
\end{aligned}
$$

3.2.2 Alternative Bertrand oligopoly

After this preliminaries, now consider the alternative Bertrand oligopoly in which firms $i = 1, 2, 3$ set prices $p_i \in [0, \infty)$ • p_{-i} denotes the prices set by all other firms. All firms have the same cost function $C(D(p_i, p_{-i})) = cD(p_i, p_{-i})$, i.e. marginal costs are constant and there are no fixed costs – $D(p_i, p_{-i})$ is the demand function specified in equation (3.26). Taking into account this basic setting, the profit function of firm i is given by

$$\pi_i(p_i, p_{-i}) = D_i(p_i, p_{-i})(p_i - c). \qquad (3.27)$$

A firm maximising its absolute profit therefore has to solve the following maximisation problem:

$$
\begin{aligned}
&\underset{p_i}{\text{argmax}} && \pi_i(p_i, p_{-i}) \\
&[\text{foc}] && \tfrac{d}{dp_i}\pi_i(p_i, p_{-i}) = \tfrac{\alpha(\beta-\gamma)-(\beta+\gamma)c-2p_i(\beta+\gamma)+\gamma\sum_{j\neq i} p_j}{(\beta-\gamma)(\beta+2\gamma)} = 0 \\
&[\text{soc}] && \tfrac{d^2\pi_i(p_i,p_{-i})}{dp_i^2} = \tfrac{-2(\beta-\gamma)}{(\beta-\gamma)(\beta+2\gamma)} < 0 \qquad (3.28)
\end{aligned}
$$

Since the second order condition is fulfilled the solution to the maximisation problem will be a maximum. Solving the first order condition given in (3.28) yields the reaction function of an absolute profit maximising firm i:

$$p_i^P =: R_i^P(p_{-i}) = \frac{\alpha(\beta-\gamma) + (\beta+\gamma)c + \gamma\sum_{j\neq i} p_j}{2(\beta+\gamma)}. \qquad (3.29)$$

Noting that this reaction function will hold $\forall i$ maximising profit, i.e. the equilibrium strategy will be symmetric, one can also write p^P instead of p_i^P. Therefore $\sum_{j \neq i} p_j$ will be $(n-1)p^P$ with $n = 3$. Taking all this into account and solving (3.29) for p^P yields:

$$p^P = \frac{\alpha(\beta - \gamma) + (\beta + \gamma)c}{2\beta}. \tag{3.30}$$

Equilibrium demand and profit in this usual Bertrand oligopoly with product differentiation are therefore:

$$D_i(p^P) = \frac{(\alpha - c)(\beta + \gamma)}{2\beta(\beta + 2\gamma)} \quad \text{and} \quad \pi_i(p^P) = \frac{(\alpha - c)^2(\beta + \gamma)(\beta - \gamma)}{4\beta^2(\beta + 2\gamma)}. \tag{3.31}$$

Now assume, as in the previous generalised, alternative Cournot oligopoly, that not all firms maximise absolute profit but some maximise $f_i = \pi_i - \bar{\pi}$ with $\bar{\pi} = (1/n)\sum_{i=1}^{n} \pi_i$, and $n = 3$ in this case. It will be assumed that there are k such relative profit maximisers with interdependent preferences. The maximisation problem of these firms is:

$$\begin{aligned}
&\underset{p_i}{\text{argmax}} \quad f_i(p_i, p_{-i}) = D_i(p_i, p_{-i})(p_i - c) - \tfrac{1}{n}\textstyle\sum_{i=1}^{n} D_i(p_i, p_{-i})(p_i - c) \\
&[\text{foc}] \qquad \tfrac{d}{dp_i} f_i(p_i, p_{-i}) = \tfrac{2\alpha(\beta-\gamma) - 2[\beta(2p_i - c) + 2\gamma(p_i - c)] + \gamma \sum_{j \neq i} p_j}{3(\beta-\gamma)(\beta+2\gamma)} = 0 \\
&[\text{soc}] \qquad \tfrac{d^2 f_i(p_i, p_{-i})}{dp_i^2} = \tfrac{-4(\beta+\gamma)}{3(\beta-\gamma)(\beta+2\gamma)} < 0
\end{aligned} \tag{3.32}$$

As one can see from (3.32), the second order condition is fulfilled and therefore the solution of the maximisation problem will be a maximum. Solving the first order condition yields the following reaction function of a difference maximising firm i:

$$p_i^D =: R_i^D(p_{-i}) = \frac{\alpha(\beta - \gamma) + (\beta + \gamma)c + \gamma \sum_{j \neq i} p_j}{4(\beta + \gamma)}. \tag{3.33}$$

Similar to the price p_i^P set by profit maximising firms, p_i^D, i.e. the price set by a relative profit maximising firm, will also be intragroup symmetric.

Writing therefore p^D and p^P respectively, one can specify the term $\sum_{j\neq i} p_j$ in equation (3.29) and (3.33). For a profit maximising firm this term will be $(n-k-1)p^P + kp^D$, while for a difference maximising firm it will be $(n-k)p^P + (k-1)p^D$. Inserting these expressions into the corresponding reaction function and solving the resulting equations for p^D and p^P respectively yields the following equilibrium prices of the alternative Bertrand oligopoly with product differentiation ($n = 3$):

$$p^D = \frac{\alpha(\beta-\gamma)(4\beta+\gamma(k+3)) + (4\beta^2+\beta\gamma(k+11)+3\gamma^2(k+1))c}{2(4\beta^2+\beta\gamma(k+5)+\gamma^2 k)}, \quad (3.34)$$

$$p^P = \frac{\alpha(\beta-\gamma)(4\beta+\gamma(k+5)) + (4\beta^2+\beta\gamma(k+9)+\gamma^2(3k+5))c}{2(4\beta^2+\beta\gamma(k+5)+\gamma^2 k)}. \quad (3.35)$$

3.2.3 Analysis of alternative Bertrand oligopoly

The first question this analysis is going to address is whether or not the alternative Bertrand oligopoly described in the previous section is a (generalised) aggregative game, i.e. the payoff to any firm depends on the strategy choosen by this firm and a combination of all strategies of all firms.

First consider the demand function (3.26) of this Betrand oligopoly. Rewriting this function gives

$$D_i(p_i, p_j) = a - bp_i + d\sum_{j\neq i} p_j, \quad (3.36)$$

where

$$a = \frac{\alpha}{(\beta+2\gamma)}, \quad b = \frac{(\beta+\gamma)}{((\beta+2\gamma)(\beta-\gamma))} \quad \text{and} \quad d = \frac{\gamma}{((\beta+2\gamma)(\beta-\gamma))}.$$

Instead of equation (3.36) one could also write

$$D_i(p_i, p_j) = a - bp_i - dp_i + d\sum_{j=1}^{n} p_j. \tag{3.37}$$

Inserting (3.37) into the profit function of firm i finally yields:

$$\begin{aligned} \pi_i(p_i, p_j) &= D_i(p_i, p_j)(p_i - c) \\ & (a - bp_i - dp_i + d\sum_{j=1}^{n} p_j)(p_i - c), \end{aligned} \tag{3.38}$$

i.e. the profit to any firm depends on the price of that firm and the sum of all prices set by all firms. Therefore the alternative Bertrand oligopoly is an aggregative game – for the aggregate $\sum_{j=1}^{n} p_j$ one could also write $g^n : S^n \to \mathbb{R}_+$, with g^n being the aggregate.

Examining demand equation (3.36) also reveals another interesting interpretation with regard to the profit of firm i in this alternative Betrand oligopoly: since demand depends on the individual price p_i and the sum of prices set by all other firms, i.e. $\sum_{j \neq i} p_j$, also the profit to firm i depends on its individual price p_i and an aggregate $g^{n-1} : S^{n-1} \to \mathbb{R}_+$, which is the sum of all other prices. As a result, one can write

$$\pi_i(p_i, g^{n-1}) = (a - bp_i + dg^{n-1})(p_i - c). \tag{3.39}$$

Writing the profit function in this form makes it easier to answer the next question, namely if the alternative Betrand oligopoly is supermodular or submodular, i.e. whether the game has increasing or decreasing differences.[9] Since the profit function is smooth and twice differentiable, one can use the cross derivative check in order to answer this question. Differentiating (3.39) for p_i and then for g^{n-1} gives:

$$\frac{\partial \pi_i(p_i, g^{n-1})}{\partial p_i} = a - b(2p_i + c) + dg^{n-1}, \quad \frac{\partial^2 \pi_i(p_i, g^{n-1})}{\partial p_i \partial g^{n-1}} = d, \tag{3.40}$$

[9] See section 2.4 on page 26 for further explanation.

with $d = \gamma/((\beta+2\gamma)(\beta-\gamma))$. Since the denominator will always be positive, the the sign of γ, i.e. the degree of substitutability/complementarity, will determine whether the game is supermodular or submodular.

Now, if $\gamma > 0$, i.e. the goods are substitutes, then the profit function has increasing differences that is the higher the aggregate the higher the incentive to increase the price. The intuition behind this result is the following: the higher the prices for the substitute goods – and therefore the higher the aggregate – the higher the price a firm can set without loosing too much demand. In other words the strategies, in case of substitute goods, are strategic complements.

If, however, $\gamma < 0$, i.e. the goods are complements, then the profit function exhibits decreasing differences that is the higher the aggregate the lower the incentive to increase the price. One can think of this result in the following way: the higher the prices set by other firms, the lower the demand for ones own good; increasing the own price even further would decrease demand even more. In this case the strategies are strategic substitutes – like in the alternative Cournot Oligopoly.

The next part of this analysis is a comparison between the results obtained for absolute profit maximising firms on the one hand and relative profit maximising firms on the other. The main emphasis of this comparison is placed on prices, demands and profits. Consider, for instance, the difference $p^D - p^P$, where the two prices p^D and p^P are given in (3.34) and (3.35) respectively:

$$p^D - p^P = \frac{-(\alpha - c)(\beta - \gamma)\gamma}{4\beta^2 + \beta\gamma(k+5) + \gamma^2 k}. \tag{3.41}$$

Clearly, as one can see from the expression in (3.41), the sign of the difference between these two prices depends on the sign of γ, i.e. the degree of substitutability/compelementarity between the goods. If, for instance, $\gamma > 0$, that is the goods are substitutes and the profit function exhibits increasing differences, then the denominator will be positive while the numerator will be negative, regardless of k. In other words, $p^D - p^P < 0$, i.e. relative profit maximising firms will always set a lower price than their absolute profit maximising counterparts, no matter how many relative profit maximisers there are.

On the other hand, if $\gamma < 0$, that is the goods are complements and the profit function exhibits decreasing differences, the numerator will always be positive while the sign of the denominator is not clear a priori: it will be positive only if $4\beta^2 + \gamma^2 k > -\beta\gamma(k+5)$ – note that the left hand side of this inequality is positive while the right hand side is positive too in case of $\gamma < 0$. One can show, however, that this inequality holds and therefore both the numerator and the denominator are positive and hence $p^D - p^P > 0$, again for all k. This means that in the case of complements relative profit maximising firms will always set a higher price than their absolute profit maximising counterparts.

Summarising the results one can conclude that relative profit maximising firms always pursue a more aggressive pricing strategy: in the case of subsitute goods, in which strategies are strategic complements, relative profit maximising firms beat down the price, while in the case of complement goods, in which strategies are strategic subsitutes, relative profit maximising firms boost prices.

Another interesting observation can be made if one analyses the change in the difference $p^D - p^P$ with respect to the number of difference maximising firms k. By looking at equation (3.41) one can see that as k increases the denominator increases too while the numerator is not affected by an increase in the number of relative profit maximising firms.

In case of $\gamma > 0$, i.e. the goods are substitutes, the difference in prices becomes less negative if k increases. On the other hand, if $\gamma < 0$, i.e. the goods are complements, the difference in prices becomes less positive. In fact this means that the higher the number of relative profit maximising firms the more the two prices will converge to each other. Hence, relative profit maximising players become less aggressive.

The results for the difference in prices is reflected in the differences of demand. Comparing equilibrium demands for relative profit maximising firms on the one hand and absolute profit maximsing firms on the other, one obtains:

$$D^D - D^P = \frac{(\alpha - c)\gamma}{4\beta^2 + \beta\gamma(k+5) + \gamma^2 k}. \tag{3.42}$$

Since the denominator is positive (see above), the sign of the difference in (3.42) depends on the sign of γ: if the goods are substitutes ($\gamma > 0$) then the relative profit maximisers face a higher demand while in the case of complements ($\gamma < 0$) they will face a lower demand. This result is a direct implication of the pricing strategies pursued by relative profit maximising firms in the two possible cases – beating the price in case of substitutes results in higher demand while boosting the price in case of complements results in lower demand.

Finally, it is worthwhile to analyse the profits of absolute and relative profit maximising firms respectively. Without specifing the number of relative profit maximising firms – the overall number of firms still is $n = 3$ • the difference in profits is:

$$\pi_{rel} - \pi_{abs} = \frac{(\alpha - c)^2(\beta - \gamma)(2\beta + \gamma(k+1))\gamma^2}{2(\beta + 2\gamma)(4\beta^2 + \beta\gamma(k+5) + \gamma^2 k)^2}. \tag{3.43}$$

While the denominator of (3.42) will always be positive, the numerator will be positive only if $2\beta + \gamma(k+1) > 0$. Since for the number of relative profit maximising firms it holds $k \leq 3$ and one recalls that $|\gamma| < \beta/2$, this condition is fulfilled. Hence, relative profit maximising firms, regardless of their number, will always make a higher profit than absolute profit maximising firms – note that this result is true for the case of substitute goods as well as for the case of complement goods.

3.2.4 Evolutionary equilibrium

The last issue this thesis is going to address is the evolutionary equilibrium of the alternative Bertrand oligopoly. As in the previous general, alternative Cournot model, it is reasonable to assume that in the Bertrand case eventually the number of relative profit maximising firms will equal the total number of firms – recall that relative profit maximising firms always maker a higher absolute profit than their absolute profit maximising counterparts; if survival is based on absolute profit than relative profit maximising firms will have a strategic advantage and if firms can observe the profits of their rivals, k will converge to n as firms imitate relative profit maximising behaviour.

The question now is if the Nash-equilibrium price p^D – shown in (3.34) for $n = 3$ and in (5.27) in the Appendix for the general case of n players – is an evolutionarily stable strategy for $k = n$. Contrary to the previous model, the line of argument will rely on a result shown by Tanaka [11].

In this paper Tanaka shows that the evolutionarily stable strategy (ESS) is equal to what he calls globally surviving strategy (GSS). This result is then illustrated by means of a Betrand setting with product differentiation, which has the same underlying assumptions as the alternative Bertrand model presented in this thesis. The only major difference is that Tanaka uses invers demand functions of the form

$$p_i = a - x_i - b \sum_{j=1, j\neq i}^{n} x_j. \tag{3.44}$$

The invers of the demand function used in this thesis is equal to:[10]

$$P_i(q) = \alpha - \beta q_i - \gamma \sum_{j\neq i} q_j. \tag{3.45}$$

By looking at (3.44) and (3.45) respectively one can see that the invers demand functions are the same for $\alpha = a$, $\beta = 1$ and $\gamma = b$. As a result, it will be sufficient to show that the ESS Tanka identifies in the example is equal to the Nash-equilibrium price p^D for the case $k = n$ • since the example of Tanaka is not confined to the case where $n = 3$, one has to use the general expression of p^D given in equation (5.27) in the Appendix.

The ESS identified by Tanaka is equal to

$$p^* = \frac{(1-b)a + [1 + (n-1)b]c}{2 + (n-2)b}. \tag{3.46}$$

Now, substituting k for n – recall that we are looking at the reasonable case in which all firms are relative profit maximisers – and β for 1 – since one has to account for the different factors in the invers demand functions – in equation (5.27), one gets

$$p^D = \frac{\alpha(1-\gamma) + c(\gamma(n-1)+1)}{\gamma(n-2)+2}. \tag{3.47}$$

[10]see Vives [13], p. 146

Accounting for the remaining differences in the notation, one has to substitute α for a and γ for b. This step finally yields

$$p^D = \frac{a(1-b)+c(b(n-1)+1)}{b(n-2)+2}. \tag{3.48}$$

Comparing the result for the ESS identified by Tanaka (equation (3.46)) with equation (3.48) shows that p^D for the case of $k = n$ in fact is an evolutionarily stable strategy of the alternative Bertrand oligopoly. Furthermore, since the ESS identified by Tanaka is strictly globally stable, p^D also represents a stochastically stable equilibrium, that is no matter how many deviant players – all choosing the same deviant strategy – exist, players choosing p^D will always be better off.

Changing (3.48) back to the initial notation yields:

$$p^D = \frac{\alpha(1-\gamma)+c(\gamma(n-1)+1)}{\gamma(n-2)+2} = c + \frac{(\alpha-c)(1-\gamma)}{\gamma(n-2)+2}. \tag{3.49}$$

In order to compare this outcome with the perfectly competitive outcome, that is price equals marginal costs and the goods are perfect substitutes, one has to look at the case where the goods are substitutes in this Bertrand setting with product differentiation, i.e. $\gamma > 0$. Because the last term in equation (3.49) will be strictly positive in this case, one can conclude that the evolutionarily stable price in the Betrand case with substitute goods is larger than marginal costs.[11] Therefore, one can also conclude that the profits in the symmetric evolutionary equilibrium will be positive.

[11] It is not equal to marignal costs, as in the Cournot case, since the initial assumption was that goods are never perfect substitutes/complements.

Chapter 4

Conclusion

In most (micro-)economic textbooks the profit maximisation assumption is justified with the so-called Friedman conjecture. Friedman assumed that the maximisation of returns, given natural selection within an industry, appropriately summarises the condition of survival of a firm. According to his argument, every firm which forgoes to maximise absolute profits will eventually lose in the struggle for survival.

Thus, a common justification of the profit maximisation assumption is made on evolutionary grounds. However, as has been shown in this thesis and in the contributions of many other authors, the evolutionarily stable strategy, i.e. the strategy which appropriately summarises the conditions for survival, in the Cournot or the Bertrand oligopoly, for instance, is different from the absolute profit maximisation strategy.

To prove the Friedman conjecture wrong, one not even has to fall back on evolutionary reasoning, since a simple analysis of strategic competition is sufficient. Both models in this thesis have shown that in an imperfect strategic environment, in which firms possess market power, i.e. the strategy choosen by one firm exerts a negative externality on the payoffs of all competitors, firms which maximise relative profit, for instance, instead of absolute profit make higher profits than their absolute profit maximising counterparts. Therefore, as long as relative profit maximising firms exist within an industry, absolute profit maximisation is not an optimal strategy to pursue.

Of course, relative profit maximisation is only one possible alternative which yields this result in a strategic environment. Future research, however, should not try to find the optimal objective function a firm should adopt. It should rather concentrate on the factors by which alternative objective functions stand up to the usual absolute profit maximisation objective.

With regard to this aim and in order to conclude this thesis it is worthwhile to trouble Charles Darwin once more, to whom the following quotation has been ascribed to:

> "It is not the strongest nor the most intelligent that survives but rather the one most responsive to change."

Chapter 5

Appendix

5.1 Relative payoff maximisation

Provided that $\pi_D = \pi_i - \bar{\pi}$ and $\pi_D^{'} = \pi_i - \bar{\pi}_{-i}$, and given that $\bar{\pi} = \sum_{i=1}^{n} \pi_i$ and $\bar{\pi}_{-i} = \sum_{j \neq i} \pi_j$, show that

$$\underset{x_i \in X}{\text{argmax}}\ \pi_D \ = \ \underset{x_i \in X}{\text{argmax}}\ \pi_D^{'}. \tag{5.1}$$

Proof. Without loss of generality one can assume that $i = 1$. Therefore one gets:

$$\begin{aligned} \pi_D &= \pi_1 - \frac{\pi_1 + \pi_2 + \cdots + \pi_n}{n} \\ &= \frac{1}{n}\left[(n-1)\ \pi_1 - \pi_2 - \cdots - \pi_n\right] \end{aligned} \tag{5.2}$$

$$\begin{aligned} \pi_D^{'} &= \pi_1 - \frac{\pi_2 + \pi_3 + \cdots + \pi_n}{n-1} \\ &= \frac{1}{n-1}\left[(n-1)\ \pi_1 - \pi_2 - \cdots - \pi_n\right] \end{aligned} \tag{5.3}$$

Maximising π_D with respect to x_1 yields the first order condition:

$$\begin{aligned}\frac{d\pi_D}{dx_1} &= \frac{1}{n}\left[(n-1)\,\frac{d\pi_1}{dx_1} - \frac{d\pi_2}{dx_1} - \cdots - \frac{d\pi_n}{dx_1}\right] = 0 \\ &= (n-1)\,\frac{d\pi_1}{dx_1} - \frac{d\pi_2}{dx_1} - \cdots - \frac{d\pi_n}{dx_1} = 0 \end{aligned} \tag{5.4}$$

On the other hand, maximising $\pi_D^{'}$ with respect to x_1, results in the following first order condition:

$$\begin{aligned}\frac{d\pi_D^{'}}{dx_1} &= \frac{1}{n-1}\left[(n-1)\,\frac{d\pi_1}{dx_1} - \frac{d\pi_2}{dx_1} - \cdots - \frac{d\pi_n}{dx_1}\right] = 0 \\ &= (n-1)\,\frac{d\pi_1}{dx_1} - \frac{d\pi_2}{dx_1} - \cdots - \frac{d\pi_n}{dx_1} = 0 \end{aligned} \tag{5.5}$$

Comparing (5.4) with (5.5) shows that maximising $\pi_D = \pi_i - \bar{\pi}$ yields the same as maximising $\pi_D^{'} = \pi_i - \bar{\pi}_{-i}$. Finally, one has to check whether the two maximisation problems really yield a maximum.

The second order condition in maximising π_D with respect to x_1 is:

$$\frac{d^2\pi_D}{dx_1^2} = \frac{1}{n}\left[(n-1)\frac{d^2\pi_1}{d^2x_1} - \frac{d^2\pi_2}{d^2x_1} - \cdots - \frac{d^2\pi_n}{d^2x_1}\right] < 0 \tag{5.6}$$

Knowing that $\frac{1}{n} > 0$ and assuming that $\frac{d^2\pi_2}{d^2x_1} = \frac{d^2\pi_3}{d^2x_1} = \cdots = \frac{d^2\pi_n}{d^2x_1} = \frac{d^2\pi_{-1}}{d^2x_1}$ one can also write (5.6) in the following way:

$$(n-1)\frac{d^2\pi_1}{d^2x_1} < (n-1)\frac{d^2\pi_{-1}}{d^2x_1} \tag{5.7}$$

Finally, taking into account that in the case of $n \geq 2$ it also holds $(n-1) > 0$, one gets the second order condition:

$$\frac{d^2\pi_1}{d^2x_1} < \frac{d^2\pi_{-1}}{d^2x_1} \tag{5.8}$$

If one maximises $\pi_D^{'}$ with respect to x_1, one gets the same second order condition as specified by the last equation, and therefore, if (5.8) is satisfied, equation (5.1) will be fulfilled. □

5.2 General Cournot oligopoly

Consider the classic Cournot oligopoly, in which firms $i = 1, \ldots, n$ set quantities $q_i \in [0, \infty]$ – q_{-i} denotes the quantities set by all other firms except firm i. All firms have the same cost function $C(q_i) = cq_i$, i.e. there are no fixed costs and marginal costs are constant at c. Aggregate inverse demand is given by the linear function $P(Q) = a - bQ$ with $Q = \sum_{i=1}^{n} q_i$. With regard to the parameters of the inverse demand function it will be assumed that $a, b > 0$ and $a > c$. Taking into account this basic setting, the profit function of firm i is given by

$$\pi_i(q_i, q_{-i}) = (a - b\sum_{i=1}^{n} q_i)q_i - cq_i. \tag{5.9}$$

The maximisation problem of firm i is shown (5.10) – note that all players in the classic Cournot oligopoly have independent preferences, i.e. they maximise absolute payoff.

$$\begin{aligned} \underset{q_i}{\text{argmax}} \quad & \pi_i(q_i, q_{-i}) = (a - bQ - c)q_i \\ [\text{foc}] \quad & \tfrac{d\pi_i(q_i, q_{-i})}{dq_i} = -bq_i + (a - bQ - c) = 0 \\ [\text{soc}] \quad & \tfrac{d^2\pi_i(q_i, q_{-i})}{dq_i^2} = -2b < 0 \end{aligned} \tag{5.10}$$

As one can see from above, the second order condition guarantees that the solution will be a maximum. Denoting the optimal quantity set by a profit maximising firm i with q_i^P and rearranging the first order condition of (5.10) yields:

$$q_i^P = \frac{a}{b} - \frac{c}{b} - Q. \tag{5.11}$$

Equation (5.11) shows that all firms will set the same quantity in the optimum. Therefore the solution q_i^P is said to be symmetric. If (5.11) is solved for q_i^P one gets the reaction function of firm i:

$$q_i^P =: R_i^P(q_{-i}) = \frac{1}{2b}\left(a - c - b\sum_{j\neq i} q_j\right). \tag{5.12}$$

Taking into account the symmetry of the solution, and using the reaction function of firm i, one can conclude that:

$$q^P = \frac{1}{2b}\left(a - c - b(n-1)q^P\right). \tag{5.13}$$

Solving (5.13) for q^P yields the following solutions of this general Cournot oligopoly:

$$q^P = \frac{(a-c)}{b(n+1)} \quad \text{and} \quad Q^P = nq^P = \frac{(a-c)n}{b(n+1)}, \tag{5.14}$$

$$P(Q^P) = P^P = \frac{a+cn}{n+1} \quad \text{and} \quad \pi_i^P = \frac{(a-c)^2}{b(n+1)^2}. \tag{5.15}$$

5.3 General alternative Bertrand oligopoly

Consider the alternative Bertrand oligopoly in which firms $i = 1, \ldots, n$ set prices $p_i \in [0, \infty)$ • p_{-i} denotes the prices set by all other firms. All firms have the same cost function $C(D(p_i, p_{-i})) = cD(p_i, p_{-i})$, i.e. marginal costs are constant and there are no fixed costs.

The demand function $D(p_i, p_{-i})$ is specified as follows (for further explanation see section 3.2.1 on page 41):

$$
\begin{aligned}
D_i(p_i, p_{-i}) \quad &= \quad \frac{\alpha}{(\beta+(n-1)\gamma)} - \frac{(\beta+(n-2)\gamma)}{((\beta+(n-1)\gamma)(\beta-\gamma))} p_i \\
&\quad + \frac{\gamma}{((\beta+(n-1)\gamma)(\beta-\gamma))} \sum_{j \neq i} p_j, \qquad (5.16)
\end{aligned}
$$

where $p_{-i} = \sum_{j \neq i} p_j$ holds in general. In order to make things easier one can define the following variables:

$$
\sigma = \frac{\alpha}{(\beta+(n-1)\gamma)},
$$

$$
\tau = \frac{(\beta+(n-2)\gamma)}{((\beta+(n-1)\gamma)(\beta-\gamma))} \quad \text{and}
$$

$$
\psi = \frac{\gamma}{((\beta+(n-1)\gamma)(\beta-\gamma))}.
$$

As a result, the demand function specified in (5.16) is simplified to

$$
D_i(p_i, p_{-i}) = \sigma - \tau p_i + \psi \sum_{j \neq i} p_j. \qquad (5.17)
$$

Taking into account this basic setting, the objective function of firm i is given by

$$
\pi_i(p_i, p_{-i}) = D_i(p_i, p_{-i})(p_i - c). \qquad (5.18)
$$

First consider the case in which all firm maximise absolute profit. In order to obtain the the solution to this maximisation problem one has to differentiate equation (5.18) with respect to p_i – we will assume that the second order condition of the maximisation problem is fulfilled so that the result is really a maximum. Setting the first order condition equal to zero yields the following reaction function of an absolute profit maximising firm i:

$$p_i^P =: R_i^P(p_{-i}) = \frac{(\sigma + c\tau + \psi \sum_{j \neq i} p_j)}{2\tau}. \tag{5.19}$$

Noting that this reaction function will hold for all firms i maximising profit, i.e. the equilibrium strategy will be symmetric, one can also write p^P instead of p_i^P. Therefore $\sum_{j \neq i} p_j$ will be $(n-1)p^P$. Taking all this into account and solving (5.19) for p^P yields:

$$p^P = \frac{\sigma + c\tau}{2\tau - \psi(n-1)}. \tag{5.20}$$

Pluging in the expressions for σ, τ, and ψ one obtains:

$$p^P = \frac{\alpha(\beta - \gamma) + (\beta + \gamma(n-2))c}{2\beta + \gamma(n-3)}. \tag{5.21}$$

Now consider a player i who maximises the difference between her own profit π_i and the industry average profit $\tilde{\pi}$. Her objective function is then given by

$$f_i(p_i, p_{-i}) = \pi_i - \pi_i^D = D_i(p_i, p_{-i})(p_i - c) - \frac{1}{n} \sum_{i=1}^{n} D_i(p_i, p_{-i})(p_i - c). \tag{5.22}$$

Differentiating equation (5.22) with respect to p_i and solving the first order condition of this maximisation problem yields the reaction function of a difference maximising firm – again we will assume that the second order condition of the maximisation problem is fulfilled:

$$p_i^D =: R_i^D(p_{-i}) = \frac{(\sigma + \psi \sum_{j \neq i} p_j + c\tau)(1 - \frac{1}{n}) - \frac{1}{n}\psi(\sum_{j \neq i} p_j - (n-1)c)}{(2\tau - \frac{1}{n}2\tau)}. \tag{5.23}$$

Similar to the price p_i^P set by profit maximising firms, p_i^D, i.e. the price set by a difference maximising firm, will also be intragroup symmetric. Writing therefore p^D and p^P respectively, one can specify the term $\sum_{j \neq i} p_j$. For a profit maximising firm this term will be $(n-k-1)p^P + kp^D$, while for a difference maximising firm it will be $(n-k)p^P + (k-1)p^D$. Inserting these expressions into the corresponding reaction functions – equation (5.19) for absolute profit maximising firms and equation (5.23) for difference maximising firms – yields:

$$p^P = \frac{\sigma + c\tau + kp^D\psi}{2\tau + (k-n+1)\psi}, \tag{5.24}$$

$$\begin{aligned} p^D &= \frac{(n-1)\sigma + c(n-1)\tau}{2(n-1)\tau - (k-1)(n-2)\psi} \\ &\quad + \frac{[c(n-1) - (k-n)(n-2)p^P]\psi}{2(n-1)\tau - (k-1)(n-2)\psi}. \end{aligned} \tag{5.25}$$

Solving these two equations for p^D and p^P respectively and inserting the expressions for σ, τ, and ψ finally yields the equilibrium prices of the alternative Bertrand oligopoly with product differentiation:

$$\begin{aligned} p^P &= \frac{\alpha(\beta-\gamma)[2\beta(n-1) + \gamma(k + (n-2)(2n-1))]}{4\beta^2(n-1) + 2\beta\gamma(k + 3n^2 - 9n + 5) + \gamma^2(2k + 2n^2 - 7n + 3)(n-2)} \\ &\quad + \frac{[2\beta^2(n-1) + \beta\gamma(k + (n-2)(4n-3)) + \gamma^2(k(2n-3) + (n-2)^2(2n-1))]c}{4\beta^2(n-1) + 2\beta\gamma(k + 3n^2 - 9n + 5) + \gamma^2(2k + 2n^2 - 7n + 3)(n-2)}, \end{aligned} \tag{5.26}$$

$$\begin{aligned} p^D &= \frac{\alpha(\beta-\gamma)[2\beta(n-1) + \gamma(k + 2n^2 - 6n + 3)]}{4\beta^2(n-1) + 2\beta\gamma(k + 3n^2 - 9n + 5) + \gamma^2(2k + 2n^2 - 7n + 3)(n-2)} \\ &\quad + \frac{[2\beta^2(n-1) + \beta\gamma(k + 4n^2 - 10n + 5) + \gamma^2(k + n^2 - 3n + 1)(2n-3)]c}{4\beta^2(n-1) + 2\beta\gamma(k + 3n^2 - 9n + 5) + \gamma^2(2k + 2n^2 - 7n + 3)(n-2)}. \end{aligned} \tag{5.27}$$

5.4 German abstract

In der Volkswirtschaftslehre nimmt man üblicherweise an, dass Firmen ihren absoluten Profit maximieren, gegeben die Strategien der Konkurrenten, d.h. Firmen vernachlässigen den Effekt, den ihre Strategie auf die Profite der übrigen Firmen ausübt. In einem perfekt strategischen Umfeld, in dem dieser Effekt nicht existiert, ist die Strategie, den absoluten Profit zu maximieren, durchaus plausibel.

Man könnte jedoch die Frage aufwerfen, ob absolute Profitmaximierung auch in einem nicht perfekten, strategischen Umfeld, d.h. in einem Umfeld in dem Firmen Markmacht besitzen, optimal ist. In solchen Wettbewerbsumfeldern haben nämlich die Aktionen einer Firma nicht nur Auswirkungen auf den Profit der Firma sondern auch auf die Profite aller anderen Firmen innerhalb einer bestimmten Industrie, aufgrund des eben beschriebenen externen Effektes.

Eine Firma, die sich dieses negativen externen Effektes bewusst ist, könnte beispielsweise eine Strategie verfolgen, die den Profiten der Konkurrenten mehr schadet als dem eigenen Profit, d.h. sie könnte ein boshaftes Verhalten an den Tag legen. Vor einem evolutionärem Hintergrund, in dem die evolutionäre Tauglichkeit von der Höhe des absoluten Profits abhängt, könnte ein derartiges Verhalten zu einem strategischen Vorteil führen in dem Sinn, dass das boshafte Verhalten die Überlebenswahrscheinlichkeit erhöht.

Fügt man zu diesem Umfeld noch eine weitere evolutionäre Komponente hinzu, nämlich dass Firmen in der Lage sind beste Antworten zu beobachten und diese zu übernehmen, so würde man vermutlich zu einer Situation gelangen in der Firmen, die eine boshafte Strategie verfolgen und sich damit einen strategischen Vorteil sichern, die gesamte Population an Firmen dominieren.

Erreicht die Evolution schließlich diesen Punkt, kann man sich die Frage stellen, ob diese Situation, in der all Firmen boshaft handeln, evolutionär stabil ist. In andere Worten, ist das boshafte Verhalten eine evolutionär stabile Strategie wenn alle Firmen diese Strategie verfolgen – evolutionär stabil bedeutet in diesem Zusammenhang, dass eine abweichende Firma nicht in der Lage sich einen strategischen Vorteil zu sichern, indem sie eine andere Strategie als das boshafte Verhalten wählt.

Die vorliegende Diplomarbeit hat es sich nun zum Ziel gesetzt, Licht auf all diese Fragen zu werfen. Eine notwendige Voraussetzung, um diese Fragen zu beantworten, ist natürlich, alle relevanten theoretischen Konzepte in diesem Zusammenhang einzuführen und zu definieren – wichtig sind hier vor allem die Theorie des boshaften Verhaltens sowie evolutionär stabile Strategien.

Nachdem der Leser in der Literaturübersicht mit den wichtigsten theoretischen Konzepten vertraut gemacht wurde, wird die oben beschriebene Argumentationslinie anhand von modifizierten Versionen des Cournot- sowie des Bertrand Oligopol-Modells analysiert – letzteres Modell wird dabei eine Form der Produktdifferenzierung um es analytisch zugänglicher zu machen.

Die angesprochene Modifikation steht in beiden Fällen im Zusammenhang mit den Präferenzen der Spieler innerhalb des Oligopols – während in den klassischen Versionen alle Spieler sogenannte unabhängige Präferenzen besitzen, d.h. die Spieler kümmern sich nur um ihren eigenen, absoluten Profit, präsentieren die hier vorgestellten Versionen den Fall in dem ein gewisser Anteil an Spielern interdependente Präferenzen besitzen, d.h. diesen Spielern ist der relative Profit wichtig.

In beiden Fällen wird gezeigt, was oben bereits angesprochen wurde, dass nämlich Spieler mit interdependenten Präferenzen sich einen strategischen Vorteil durch boshaftes Verhalten sichern. Im Wesentlichen heißt dies, dass Spieler mit interdependenten Präferenzen aggressiver aggieren, sodass sie schließlich einen höheren Profit als ihre Rivalen mit unabhängigen Präferenzen erzielen. Ist die Überlebenswahrscheinlichkeit von der Höhe des absoluten Profits abhängig, dann eignen sich Spieler mit interdependenten Präferenzen offensichtlich einen strategischen Vorteil an. Es sollte hervorgehoben werden, dass all diese Beobachtungen bereits innerhalb eines strategischen Wettbewerbsumfeldes gemacht werden können.

Zusätzlich werden auch noch evolutionäre Gleichgewichte anhand der modifizierten Oligopol-Modelle analysiert. Da der evolutionäre Prozess, der zu diesen Gleichgewichten führt, nicht explizit modelliert wird, unterscheidet sich der hier präsentierte Ansatz von dem, der üblicherweise in der evolutionären Spieltheorie gewählt wird und oft auf Modelle von Imitationsverhalten, Vererbung und zufälligen Mutationen etc. aufbaut.

Durch logische Schlussfolgerung alleine ist es möglich zu zeigen, dass schließlich Firmen die Population dominieren werden, die ein boshaftes Verhalten an den Tag legen. Diese Strategie ist zudem evolutionär stabil in dieser Situation. Evolutionär stabile Strategien in endlichen Populationen stehen ganz allgemein in Verbindung mit boshaftem Verhalten. Insbesondere kann man auch beobachten, dass das evolutionäre Gleichgewicht in einer endlichen Population nicht notwendigerweise ein Nash-Gleichgewicht darstellt – diese Umstand wird anhand eines Beispieles verdeutlicht. Ein weiteres Resultat ist, dass ein evolutionärer bzw. strategischer Vorteil auf Kosten eines höheren Profites erreicht werden kann.

Der wichtigste Beitrag dieser Diplomarbeit ist jedoch zu zeigen, dass das Resultat des strategischen Wettbewerbs, in dem alle Spieler den relativen Profit maximieren, das gleiche ist wie im evolutionären Gleichgewicht.

Curriculum Vitae

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