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Chapter 1

Introduction

Why is economic activity spatially clustered? Nobel Prize winner Paul Krugman was the first to address and investigate this question in a full-fledged general equilibrium setup. In his seminal paper introducing the core-periphery model (Krugman (1991b)) he explains why, how and when agglomeration of productive factors emerges. Since then, numerous papers and books have contributed toward a better understanding of the spatial organization of economic activity all cumulating in the New Economic Geography (NEG) approach.[1] By focusing on the interaction of trade costs, increasing returns at the firm level, monopolistic competition and factor mobility NEG models can explain agglomeration processes due to self-reinforcing circular causality effects between factor rewards and demand patterns which are strengthened for high levels of economic integration.

As documented by the latest World Bank (2009) development report on economic geography, spatial inequalities still constitute a highly relevant topic among both economists and policy makers. Ongoing worldwide economic integration trends call for rigorous explanations of agglomeration processes that also account for recent economic developments possibly interfering and changing the standard lines of arguing developed so far. This is where the present thesis steps in by noting that countries are currently not only facing the impacts of higher economic integration but are also increas-

[1] See for example Fujita et al. (1999), Baldwin et al. (2003) or Combes et al. (2008) for an overview of the NEG literature.

ingly confronted with the economic challenges from demographic change (see e.g. the special report on aging populations of The Economist (2009)). In particular, declining fertility has caused upward shifts in the mean age of most countries' populations while simultaneously reducing their population growth rates (see Eurostat (2004) and United Nations (2007)).

Up to now, demographic change has been completely ignored in the NEG literature. This is surprising for several reasons. Most importantly, both demographic change and economic integration crucially influence demand patterns that are themselves decisive for the spatial distribution of economic activity. Analyzing agglomeration without accounting for demographic developments thus misses a crucial explanatory determinant. Similarly, the fact that NEG models are genuinely dynamic indicates that life-cycle decisions and population aging will play a prominent role in the analysis. What is thus obviously required is a framework that studies the *common* consequences of economic integration on the one hand and demographic change on the other hand on the spatial distribution of economic activity.

The present thesis provides this explanatory framework with a view to answering how demographic structures impact upon agglomeration processes. Three closely related papers, each forming one main chapter, will contribute toward analyzing different aspects of this broadly defined research question. By focusing on the effects of demographic change on both the spatial distribution of economic activity and on the linkage between growth and agglomeration, they close one substantial gap existing so far in the theoretical economics literature on agglomeration processes. While the first and third paper focus on the impact of introducing demography into the analysis of agglomeration processes by comparing a situation that fully ignores demographic structures to one that allows for them, the second contribution explicitly considers how demographic change, in particular population aging, influences the spatial distribution of economic activity.

The distinguishing characteristic of all three contributions is their methodological approach. They analyze the linkage between demography and agglomeration based on a unified theoretical model that merges two different research strands. While a NEG component forms the first building block, an overlapping generation framework, which is typically used for studying the impact of demography on macroeconomic aggregates, is incorporated to allow for intertemporal life-cycle decisions. Combining these two areas is the most immediate way to arrive at a model that can accurately analyze and explain agglomeration processes in societies that are subject to demographic change.

The first paper[2] develops this benchmark model by generalizing the constructed capital model of Baldwin (1999), which is a NEG set-up featuring catastrophic agglomeration of capital stocks due to different accumulation rates, to account for an overlapping generation structure and lifetime uncertainty à la Blanchard (1985). In particular, while Baldwin (1999) ignores demographic structures by deriving the equilibrium dynamics based on intertemporally optimized behavior of a representative individual with an infinite planning horizon, we allow for the possibility of death and thus for age-dependent heterogeneity of individuals. This makes it possible, for the first time, to establish and analyze the main linkage between demography and agglomeration processes. By equalizing the birth to the death rate and thus keeping population size constant, we, however, focus on changes in the population age structure and ignore variations in the population growth rate due to demographic change. This is relaxed in the second paper[3] that extends the above benchmark model with a view to focusing on the impacts of changes in both the population mean age and the population growth rate on agglomeration processes. To do so, it incorporates a more detailed analysis of demographic change by taking up the overlapping generation set-up of Buiter (1988) and therefore allowing for unequal birth and death rates and hence nonzero population growth. Within this framework we are able

[2]This paper presented in chapter 2, whose working paper version is also available at the WU (see Grafeneder-Weissteiner and Prettner (2009)), was jointly written with Klaus Prettner. While Klaus Prettner solved the individual utility maximization problem and calibrated the model, I derived the aggregate laws of motion for expenditures and capital and analyzed the stability properties of the resulting dynamic system.

[3]This paper is again joint work with Klaus Prettner. In particular, we together set up the model framework to arrive at the dynamic system describing the evolution of the economy. I then performed the stability analysis in order to find out how different demographic structures impact upon agglomeration processes.

to differentiate between the effects of varying birth and mortality rates and can also assess agglomeration processes for various demographic scenarios, including the one of declining fertility resulting in both population aging and slower population growth as recently faced by many industrialized countries. Moreover, it allows to identify an additional linkage between demography and agglomeration that augments the one already found in the first paper.

Finally, the third essay presents one further extension to the benchmark model of paper one by additionally accounting for endogenous growth due to learning effects. In particular, I incorporate a notion of learning as in Baldwin et al. (2001) to arrive at a NEG framework featuring both endogenous growth and demographic change. The resulting model can not only be used to again assess the impact of demography on agglomeration processes but also to analyze how lifetime uncertainty impacts upon long-run growth perspectives. In doing so, the main focus is on the linkage between growth and the spatial distribution of economic activity with a view to evaluating the pro-growth effect of spatial agglomeration in a setting that allows for demographic structures. The model's equilibrium dynamics moreover make it possible to also pay attention to the impact of lifetime uncertainty on history-versus-expectations considerations, i.e. on the question of equilibrium selection in a setting with multiple equilibria.

Each of the three papers clearly reveals that demographic structures are of crucial importance for agglomeration processes. In particular, they all confirm one main finding. Introducing overlapping generations with individuals that face a positive probability of death and differ with respect to age considerably reduces agglomeration tendencies between regions. In the first paper we establish the distributional effects from the turnover of generations as being responsible for this result. Since a higher capital stock in one region implies that dying individuals are on average richer, the distributional effects due to birth and death, i.e. the replacement of older individuals with higher consumption expenditures by newborns with lower consumption expenditures, are more severe relative to the region with the lower capital stock. The implied stronger decrease in expenditures translates into a lower capital rental rate and therefore constitutes an anti-agglomerative force by depressing home capital accumulation. Age-dependent heterogeneity of individuals is key to this turnover channel that we identify as the main linkage between demographic structures and agglomeration.

The stabilizing effect of introducing the possibility of birth and death has an important implication that is also of high relevance from an economic policy point of view: calibrations of our benchmark model show that for plausible demographic structures, agglomeration processes do not set in even if economic integration is promoted up to a high degree. Consequently, there might not be any trade-off between increased economic integration on the one hand and equal economic development on the other hand. This is in sharp contrast to Baldwin (1999) and other NEG approaches and still holds for the case of nonconstant population size considered in the second paper. Only if one allows for additional agglomeration forces - as done in the third paper by introducing knowledge spillovers - spatial concentration of economic activity is possible even for plausible demographic structures.

In the varying population size framework of paper two, the turnover effect on agglomeration is augmented by a population growth based channel. As shown in this second contribution, population growth acts as a dispersion force. In particular, it weakens the wealth and thus expenditure increase due to a higher capital stock and therefore the increase in capital rental rates that would trigger further capital accumulation and favor agglomeration tendencies. Since birth and mortality rates affect population growth differently - lower birth rates decrease the population growth rate, while lower mortality rates increase it - their impacts on agglomeration processes are also opposite. For declining fertility rates, the population growth based channel simply strengthens the pro-agglomerative effects of a lower turnover of generations, while for declining mortality rates, the implied increase in the population growth rate counteracts and even dominates the turnover channel. Consequently, population aging as represented by declining fertility strengthens agglomeration processes, whereas declining mortality weakens them. Although these findings indicate that industrialized countries with rapidly aging societies and lower population growth rates due to declines in fertility might be confronted with increased spatial concentration of economic activity, the model's calibrations suggest that they are still far away from a situation of catastrophic agglomeration.

The stabilizing effect of the turnover of generations is also found when additionally allowing for endogenous growth as in the framework of the third paper. In this case, its anti-agglomerative effects are, however, dampened by a growth-linked circular causality being present as long as interregional knowledge spillovers are not perfect. Moreover, the model's equilibrium dynamics[4] reveal that lifetime uncertainty has important implications for equilibrium selection. Higher mortality rates reduce the possibility that expectations rather than history, represented by initial conditions, are decisive with respect to the question in which region agglomeration will take place. When it comes to the effects of demographic structures on economic growth, the results are twofold. First, lifetime uncertainty is shown to decrease long-run growth since future is discounted more heavily with positive mortality rates which depresses investment in future growth prospects. Second, comparing the negative effect of the mortality rate on equilibrium growth rates in the symmetric and core-periphery equilibrium shows that, in sharp contrast to existing NEG growth models with localized knowledge spillovers, spatial agglomeration is not necessarily conducive to growth. In particular, lifetime uncertainty mitigates the pro-growth effect of agglomeration resulting from the localized nature of learning effects. This also implies that there might not be any trade-off between fostering an equal distribution of productive factors and high economic growth if one takes into account demographic structures.

The structure of the present thesis is as follows. After this motivating introduction that primarily aims at establishing the relationship between the three contributions as well as summarizing the contents and main findings of each paper, chapter 2 contains the first essay that introduces the benchmark model and identifies the turnover channel as the main linkage between agglomeration and demographic change. The second paper, which incorporates a more detailed analysis of demographic change by allowing for unequal birth

[4]Surprisingly, the introduction of endogenous growth into the benchmark model of the first paper substantially simplifies the dynamic system describing the evolution of the economy.

and mortality rates, is contained in chapter 2. Chapter 4 presents the third paper that additionally extends the benchmark model to allow for endogenous growth. The calculations originally contained in the paper appendices are finally subsumed in the joint appendix A.

Chapter 2

Agglomeration and demographic change[1]

2.1 Introduction

Over the last decades, most economies have been confronted with tremendous structural changes arising from globalization and demographic developments. Freer trade (see Sachs and Warner (1995)) has led to higher international integration meaning that goods produced in a certain region can nowadays be sold all over the world at a more competitive price. As a result, global competition for productive factors has emerged. In particular, firms have started to invest in regions where productive factors are relatively cheap and ship their goods to regions populated by consumers with high purchasing power. Deeper economic integration on the one hand and relocation of manufacturing to areas with high rates of return on capital on the other hand thus constitute two closely related recent economic developments.

At the same time, fertility rates have decreased in nearly all countries (see Eurostat (2004)) resulting in lower population growth rates and aging societies (see United Nations (2007)). These demographic developments do not only change the productivity of labor but also have crucial impacts on demand and saving patterns in the economy which in turn affect the returns of

[1]Joint work with Klaus Prettner.

productive factors. As a consequence, economic integration and demographic change are inseparably linked. Despite this fact, their common economic consequences have barely been analyzed in one single framework up to now. Our paper makes a first step toward closing the gap by introducing demography into the New Economic Geography (NEG). We are thus able to describe the effects of declining transport costs on the location of productive factors in a setting where individuals face lifetime uncertainty and differ with respect to age. In particular, our model investigates whether concentration of economic activity as emphasized by the NEG literature still takes place when allowing for plausible demographic structures.

The NEG literature pioneered by Krugman (1991b), Venables (1996) and Krugman and Venables (1995) has provided new insights for the explanation of the spatial distribution of economic activity. These models are characterized by catastrophic agglomeration meaning that for certain threshold levels of economic integration, industrial activity completely concentrates in one region. In particular, circular causality effects between factor rewards and demands for monopolistically competitive goods encourage agglomeration processes. They destabilize the symmetric equilibrium with an equal division of productive factors and turn the core-periphery outcome with all industrial activity taking place in one region into a stable equilibrium. Reciprocal liberalization between initially symmetric regions that strengthens such circular causality effects thus leads to complete deindustrialization of one region.

Puga (1999) set up a model that nested as special cases both the Krugman (1991b) framework with labor mobility between regions as well as the vertically linked-industries model of Venables (1996) and Krugman and Venables (1995) without interregional labor mobility. However, the richness of agglomeration features in these early models reduced their analytical tractability. Therefore Baldwin (1999) introduced the constructed capital framework with interregional labor and capital immobility but forward-looking agents who behave dynamically optimal. His model features catastrophic agglomeration of capital stocks explained by the difference in the capital rental rates of two regions. A higher rental rate in the home region causes home capital accumulation, whereas capital is decumulated in the foreign region. The only force fostering this agglomeration process is a demand-linked circular causality effect setting in as a higher capital stock raises capital income and thus expenditures which leads to a further increase in home rental rates. Since neoclassical growth models in the spirit of Ramsey (1928) and Solow (1956) associate capital accumulation with medium-run growth, Baldwin (1999) describes the economy accumulating capital as a growth pole, whereas the other region appears as a growth sink. His agglomeration induced growth story therefore nicely illustrates how economic integration, which strengthens the demand-linked circular causality, can lead to the development of "rust" and "boom belts".

The Ramsey (1928) framework of one single, infinitely lived representative agent, on which the constructed capital model's saving features heavily rely, does not allow to analyze demographic changes. We therefore generalize the approach of Baldwin (1999) by introducing lifetime uncertainty. In doing so, we adopt the overlapping generation structure of Blanchard (1985), where heterogeneity among individuals is due to their date of birth. While still following the lines of intertemporally optimizing agents, this results in a more comprehensive model incorporating life-cycle decisions and nesting the constructed capital set-up as a special case. In particular, it allows us to reveal in detail the linkage between demography, economic integration and agglomeration.

Our results show that ignoring the impact of demographic structures on demand and saving patterns and thus on capital rental rates misses crucial mechanisms that are fundamental for the location of productive factors. We find that the possibility of the symmetric equilibrium to be unstable is considerably reduced in a setting with overlapping generations and lifetime uncertainty. For plausible demographic structures, agglomeration processes between two regions thus do not set in even if economic integration is promoted up to a high degree. This also implies that the agglomeration induced growth story of Baldwin (1999) primarily applies in the very special case of infinitely lived individuals. The explanation for our finding is rooted in the turnover of generations which acts as an additional dispersion force against the concentration of industrial activity. Since a higher capital stock in one region implies that dying individuals are on average richer, the distributional effects due to birth and death, i.e. the replacement of older individuals with higher consumption expenditures by newborns with lower consumption expenditures, are more severe relative to the region with the lower capital stock. The implied stronger decrease in expenditures translates into a lower capital rental rate and therefore constitutes an anti-agglomerative force.

The paper proceeds as follows. Section 2.2 presents the structure of the model and derives optimal saving behavior and the equilibrium capital rental rates. Section 2.3 verifies the existence of a symmetric long-run equilibrium and characterizes its properties with respect to the economies' demographic structures, i.e. the mortality rate. Section 2.4 establishes the link between agglomeration and demographic change. To complement our analytical findings by numerical illustrations we also calibrate the model for reasonable parameter values. Finally, section 2.5 summarizes and draws conclusions for economic policy.

2.2 The model

This section describes how we integrate the overlapping generation structure of Blanchard (1985) into the constructed capital framework of Baldwin (1999). Consumption and saving behavior as well as production technologies are introduced and various intermediate results from profit maximization are presented. Combining these findings will yield aggregate laws of motion for capital and expenditures that can be used to analyze the long-run equilibrium.

2.2.1 Basic structure and underlying assumptions

The model consists of two symmetric regions or countries, referred to as H for home and F for foreign[2], with identical production technologies, preferences

[2] If further distinction is needed, foreign variables are moreover indicated by an asterisk. In particular, the superscript F denotes that a good was *produced* in the foreign region,

of individuals, labor endowments and demographic structures. Each region has three economic sectors (agriculture, manufacturing and investment) with two immobile factors (labor L and capital K) at their disposal. The homogeneous agricultural good, z, is produced in a perfectly competitive market with labor as the only input and can be traded between the two regions without any cost. Manufacturing firms are modeled as in the monopolistic competition framework of Dixit and Stiglitz (1977) and therefore produce varieties, m, with one unit of capital as fixed input and labor as the variable production factor. Since each variety exactly requires one unit of capital, a continuum of varieties $i \in (0, K]$ is produced at home, while a continuum of varieties $j \in (0, K^*]$ is manufactured in the foreign region. In contrast to the agricultural good, trade of manufactures involves iceberg transport costs such that $\varphi \geq 1$ units of the differentiated good have to be shipped in order to sell one unit abroad (see e.g. Baldwin et al. (2003)). In the Walrasian investment sector, capital, i.e. machines, are produced using labor as the only input where wages are paid out of the individuals' savings. The failure rate of a machine is assumed to be independent of the machine's age. Denoting this failure rate as $\delta > 0$, and using the law of large numbers, implies that a share δ of the capital stock depreciates at each instant (see Baldwin (1999)).

As far as the demographic structure of our model economy is concerned, we closely follow the simplified setting of Blanchard (1985). We assume that at each point in time, $\tau \in [0, \infty)$, a large cohort consisting of new individuals is born. These newborns receive no bequests and thus start their lives without any wealth. The size of this cohort is $N(\tau, \tau) = \mu N(\tau)$, where $\mu > 0$ is the constant birth rate and $N(\tau) \equiv \int_{-\infty}^{\tau} N(t_0, \tau) dt_0$ is total population at time τ with $N(t_0, \tau)$ denoting the size of the cohort born at t_0 for any given point in time τ.[3] Consequently, cohorts can be distinguished by the birth date t_0 of their members. Since there is no heterogeneity between members of the same cohort, each cohort can be described by one representative individual, who inelastically supplies her efficiency units of labor on the labor

whereas the asterisk indicates that it is *consumed* in the foreign region.

[3]In what follows the first time index of a variable will refer to the birth date, whereas the second will indicate a certain point in time.

market with perfect mobility across sectors but immobility between regions. The age of this individual is given by $a = \tau - t_0$ and her time of death is stochastic with an exponential probability density function. In particular, the instantaneous mortality rate is also pinned down by the age independent parameter μ resulting in a survival probability to age $\tau - t_0$ of $e^{-\mu(\tau - t_0)}$. Since population size is large, the frequency of dying is equal to the instantaneous mortality rate. Therefore the number of deaths at each point in time is $\mu N(\tau)$. As this equals, by assumption, the number of births, population size is constant and can be normalized to one.[4] Finally, as in Yaari (1965), a perfect life-insurance company offers actuarial notes, which can be bought or sold by each individual and are canceled upon the individual's death.

2.2.2 Individual utility optimization

Preferences over the agricultural good and a CES composite of the manufacturing varieties are Cobb-Douglas.[5] The representative individual of cohort t_0 chooses at each instant $\tau > t_0$ consumption of the agricultural good, $c_z(t_0, \tau)$, consumption of varieties produced at home, $c_m^H(i, t_0, \tau)$, and consumption of varieties produced abroad, $c_m^F(j, t_0, \tau)$, to maximize her expected lifetime utility at time t_0[6]

$$U(t_0, t_0) = \int_{t_0}^{\infty} e^{-(\rho+\mu)(\tau - t_0)} \ln \left[(c_z(t_0, \tau))^{1-\xi} (c_m^{agg}(t_0, \tau))^{\xi} \right] d\tau, \qquad (2.1)$$

[4]From now on, we will refer to μ as the mortality rate. Note, however, that μ equivalently represents the birth rate, e.g. demographic change as captured by variations in μ means that both the mortality and the birth rate change by the same amount such that population size remains constant. This also implies that we restrict attention to changes in the population age structure (lower μ implies population aging) while neglecting variations in the population growth rate due to demographic change. In particular, emphasis is put on comparing a situation that fully ignores demographic structures, i.e. where $\mu = 0$, to one that allows for them by considering nonzero mortality rates.

[5]The following discussion refers to the home region but due to symmetry, equivalent equations also hold in the foreign region.

[6]Equation (2.1) can be easily derived by calculating expected lifetime utility where the date of death is a random variable with an exponential probability density function parameterized by a constant instantaneous mortality rate μ.

where $\rho > 0$ is the pure rate of time preference, $0 < \xi < 1$ is the manufacturing share of consumption and

$$c_m^{agg}(t_0, \tau) \equiv \left[\int_0^{K(\tau)} \left(c_m^H(i, t_0, \tau) \right)^{\frac{\sigma-1}{\sigma}} di + \int_0^{K^*(\tau)} \left(c_m^F(j, t_0, \tau) \right)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}$$

represents consumption of the CES composite with $\sigma > 1$ denoting the elasticity of substitution between varieties.

Individual savings, defined as income minus consumption expenditures, are converted into capital in the investment sector with a time independent, exogenous labor input coefficient of a_i. The wealth constraint of a representative individual can be thus written as

$$\dot{k}(t_0, \tau) = \frac{w(\tau)l + \pi(\tau)k(t_0, \tau) - e(t_0, \tau)}{w(\tau)a_i} + \mu k(t_0, \tau) - \delta k(t_0, \tau), \qquad (2.2)$$

where $w(\tau)$ denotes the wage per efficiency unit of labor, l refers to the efficiency units of labor an individual supplies, $\pi(\tau)$ is the capital rental rate, $k(t_0, \tau)$ the individual capital stock and $e(t_0, \tau)$ are individual total expenditures for consumption defined as

$$\begin{aligned} e(t_0, \tau) \;\; \equiv \;\; & p_z(\tau)c_z(t_0, \tau) + \int_0^{K(\tau)} p_m^H(i, \tau)c_m^H(i, t_0, \tau)di + \\ & \int_0^{K^*(\tau)} p_{m,\varphi}^F(j, \tau)c_m^F(j, t_0, \tau)dj. \end{aligned}$$

Here $p_z(\tau)$ is the price of the agricultural good, $p_m^H(i, \tau)$ the price of a manufactured variety produced at home and $p_{m,\varphi}^F(j, \tau)$ the price of a manufactured variety produced abroad with the subscript φ indicating the dependence on transport costs.

The particular law of motion for capital given in equation (2.2) is based on the full insurance result of Yaari (1965) implying that all individuals only hold their wealth in the form of actuarial notes.[7] Therefore the market rate

[7] Two interpretations of the capital accumulation process are therefore possible. Either each individual itself converts her savings into capital and then leaves it to the insurance company or savings are immediately transferred to the insurance company which converts

of return on capital, $\frac{\pi(\tau)}{w(\tau)a_i} - \delta$, has to be augmented by μ to obtain the fair rate on actuarial notes (see Yaari (1965)).

In appendix A.1.1 we solve the individual's utility optimization problem by applying a three stage procedure. In the first stage the dynamic savings-expenditure decision is analyzed. Stage two deals with the static optimal consumption allocation between the CES composite and the agricultural good and in stage three individuals decide upon the amount of consumption they allocate to each of the manufactured varieties. Altogether this leads to the following demand functions for the agricultural good and for each of the manufactured varieties

$$
\begin{aligned}
c_z(t_0, \tau) &= \frac{(1-\xi)e(t_0, \tau)}{p_z(\tau)}, && (2.3) \\
c_m^H(i, t_0, \tau) &= \frac{\xi e(t_0, \tau)(p_m^H(i, \tau))^{-\sigma}}{\left[\int_0^{K(\tau)} (p_m^H(i, \tau))^{1-\sigma} di + \int_0^{K^*(\tau)} (p_{m,\varphi}^F(j, \tau))^{1-\sigma} dj\right]}, && (2.4) \\
c_m^F(j, t_0, \tau) &= \frac{\xi e(t_0, \tau)(p_{m,\varphi}^F(j, \tau))^{-\sigma}}{\left[\int_0^{K(\tau)} (p_m^H(i, \tau))^{1-\sigma} di + \int_0^{K^*(\tau)} (p_{m,\varphi}^F(j, \tau))^{1-\sigma} dj\right]} && (2.5)
\end{aligned}
$$

as well as to the consumption Euler equation for the representative individual of cohort t_0

$$\frac{\dot{e}(t_0, \tau)}{e(t_0, \tau)} = \frac{\pi(\tau)}{a_i w(\tau)} - \delta - \rho. \quad (2.6)$$

As first shown by Yaari (1965), the representative individual's Euler equation with fully insured lifetime uncertainty is identical to the Euler equation when no lifetime uncertainty exists, i.e. individual saving behavior is not influenced by the mortality rate and moreover does not differ across generations.

2.2.3 Aggregate expenditures and capital

Due to the overlapping generation structure our model setup does not feature *one* single representative individual. In order to be able to analyze the long-run equilibrium of the economy as well as its stability properties it is therefore necessary to derive the aggregate laws of motion of capital and consumption

them into machines by employing workers.

expenditures. The capital stock of the economy at a certain point in time t can be obtained by aggregating up the capital stocks of all cohorts. An analogous definition applies to consumption expenditures. These aggregation rules are formally given by

$$K(t) \equiv \int_{-\infty}^{t} k(t_0,t)N(t_0,t)dt_0, \tag{2.7}$$

$$E(t) \equiv \int_{-\infty}^{t} e(t_0,t)N(t_0,t)dt_0, \tag{2.8}$$

where $K(t)$ is the aggregate capital stock and $E(t)$ denotes aggregate consumption expenditures.[8] Equivalent equations hold for the foreign region.

Using the demographic assumptions described in section 2.2.1, we can exactly trace the size $N(t_0,t)$ of any particular cohort over time. A cohort born at time t_0 is of size $\mu e^{-\mu(t-t_0)}$ at time $t \geq t_0$ as the probability of surviving to time t equals $e^{-\mu(t-t_0)}$ and the initial size of the cohort is μ. Substituting for $N(t_0,t)$ in equation (2.8) therefore yields

$$E(t) \equiv \mu \int_{-\infty}^{t} e(t_0,t)e^{-\mu(t-t_0)}dt_0. \tag{2.9}$$

The "aggregate Euler equation", modified for the existence of overlapping generations and lifetime uncertainty, directly follows from equation (2.9) by differentiating it with respect to t and then substituting for $\dot{e}(t_0,t)$ from the individual Euler equation (2.6) and for $e(t,t)$ and $E(t)$ from the corresponding expressions derived in appendix A.1.2 where we describe the various aggregation steps in detail.[9] It is given by

$$\frac{\dot{E}(t)}{E(t)} = -\mu(\rho+\mu)a_i w(t)\frac{K(t)}{E(t)} + \frac{\pi(t)}{w(t)a_i} - \rho - \delta \tag{2.10}$$

$$= -\mu\frac{E(t)-e(t,t)}{E(t)} + \frac{\dot{e}(t_0,t)}{e(t_0,t)}. \tag{2.11}$$

[8]The aggregate efficiency units of labor L are equal to the individual supply l since population size is normalized to one.

[9]Those aggregation steps closely follow the ones described by Heijdra and van der Ploeg (2002) in chapter 16.

In sharp contrast to the individual Euler equation, the mortality rate plays a prominent role in the aggregate Euler equation. From equation (2.11) it follows that the difference between individual and aggregate saving behavior is captured by a correction term representing the distributional effects due to the turnover of generations (see Heijdra and van der Ploeg (2002), chapter 16). Optimal consumption expenditure *growth* is the same for all generations (see equation (2.6)) but optimal expenditure *levels* differ. In particular, allowing for lifetime uncertainty introduces heterogeneity among individuals with respect to their birth dates and, since wealth and consumption levels are age-dependent, also with respect to their expenditures. As shown in appendix A.1.2, optimal consumption expenditures $e(t_0, t)$ are proportional to total wealth with the marginal propensity to consume out of total wealth being equal to the "effective" rate of time preference $\rho+\mu$. Older individuals are wealthier due to their accumulated capital holdings and therefore have higher consumption expenditure levels than their younger counterparts. Since dying old generations are replaced by newborns with no capital holdings at each point in time, *aggregate* consumption expenditure growth is smaller than *individual* consumption expenditure growth. The correction term on the right hand side of equation (2.10) therefore describes the difference between average consumption expenditures[10] and consumption expenditures by newborns as shown in equation (2.11). Since it increases in μ, a higher mortality rate decreases aggregate consumption expenditure growth. This is intuitively clear as a higher μ implies a higher generational turnover and therefore a more pronounced (negative) distributional impact. In the case of an infinitely lived representative individual, i.e. $\mu = 0$, the turnover effect completely disappears. Since the mortality rate enters the aggregate Euler equation and thus influences aggregate saving patterns only via this turnover correction term, it is not surprising that these distributional effects due to birth and death will play a crucial role when it comes to investigating the linkage between demographic change and the forces fostering or weakening agglomeration in our model.

[10]Since we normalized population size to one, aggregate consumption expenditures $E(t)$ are equal to average consumption expenditures.

Similarly, the aggregate law of motion for the capital stock can be obtained. Rewriting equation (2.7) in analogy to equation (2.9) and then differentiating it with respect to t yields

$$\dot{K}(t) = \left[\frac{\pi(t)}{w(t)a_i} - \delta\right] K(t) + \frac{w(t)L}{w(t)a_i} - \frac{E(t)}{w(t)a_i}, \tag{2.12}$$

where we applied the same steps as in the derivation for the aggregate Euler equation shown in appendix A.1.2.[11] Compared to the law of motion for individual capital, there appears no term featuring the mortality rate μ. This captures the fact that $\mu K(t)$ does not represent aggregate capital accumulation but is a transfer - via the life insurance company - from individuals who died to those who survived. As a consequence, aggregate capital accumulates at a rate $\frac{\pi(t)}{w(t)a_i} - \delta$, whereas capital of surviving individuals attracts the actuarial interest rate $\frac{\pi(t)}{w(t)a_i} + \mu - \delta$ for surviving individuals (see Heijdra and van der Ploeg (2002), chapter 16).

Summarizing, the mortality rate μ enters the law of motion for the individual capital stock but disappears in the corresponding aggregate law of motion. This is in sharp contrast to the Euler equation, where μ does not show up at the individual level but is part of the aggregate consumption expenditure growth rate.

2.2.4 Production technology and profit maximization

Profit maximization in the manufacturing and agricultural sector closely follows Baldwin (1999) and yields various intermediate results that simplify the subsequent analysis of the long-run equilibrium. In particular, the way the manufacturing sector is modeled allows us to derive the rental rate of capital as a function of home and foreign capital stocks and expenditures.

Agricultural sector

The homogeneous agricultural good, which can be interpreted as food, is produced according to the following constant returns to scale production

[11] In particular, we substituted for $\dot{k}(t_0, t)$ from equation (2.2).

function

$$Y_z(t) = \frac{1}{a_z} L_z(t), \tag{2.13}$$

where $Y_z(t)$ denotes output of the agricultural sector, $L_z(t)$ represents aggregate labor devoted to agricultural production, and a_z is the unit input coefficient in the production of agricultural goods. Profit maximization under perfect competition results in marginal cost pricing. Moreover, by choice of units for agricultural output, a_z can be set to one implying that the wage rate equals the price of the agricultural good

$$p_z(t) = w_z(t). \tag{2.14}$$

Since labor is perfectly mobile across sectors the wage rate in the economy $w(t)$ satisfies

$$w(t) = w_z(t) = w_m(t) = w_i(t), \tag{2.15}$$

where $w_z(t)$, $w_m(t)$ and $w_i(t)$ denote wages in the agricultural, manufacturing and investment sector. Therefore equation (2.14) pins down the equilibrium wage in the economy. As free trade of the agricultural good between home and foreign equalizes its price, wages are also equalized between the two regions as long as each of them produces some $Y_z(t)$. This can be shown to hold if ξ, the manufacturing share of consumption, is not too large (see Baldwin (1999)) which will be assumed from now on. Finally, choosing the agricultural good as numéraire leads to

$$w(t) = w^*(t) = 1. \tag{2.16}$$

Manufacturing sector

Each firm in the Dixit and Stiglitz (1977) monopolistically competitive manufacturing sector produces a different output variety using labor as variable and one variety-specific machine as fixed input. This machine originates from the investment sector and is equivalent to one unit of capital. Due to the fixed costs, firms face an increasing returns to scale production technology with an associated cost function

$$\pi(t) + w(t)a_m Y_m(i,t), \tag{2.17}$$

where a_m is the unit input coefficient for efficiency units of labor, $Y_m(i,t)$ is total output of one manufacturing good producer and the capital rental rate $\pi(t)$ represents the fixed cost.

Defining[12] $P_m(t) \equiv \int_0^{K(t)} (p_m^H(i,t))^{1-\sigma} di + \int_0^{K^*(t)} (p_{m,\varphi}^F(j,t))^{1-\sigma} dj$ as well as $P_m^*(t) \equiv \int_0^{K^*(t)} (p_m^H(j,t))^{1-\sigma} dj + \int_0^{K(t)} (p_{m,\varphi}^F(i,t))^{1-\sigma} di$ and recognizing that each individual firm has mass zero and hence does not influence these price indexes, leads to the following maximization problem[13] for each firm at time t

$$\begin{aligned}
\max_{p_m^H, p_{m,\varphi}^F} \quad & (p_m^H(i,t) - w(t)a_m) \left(\int_{-\infty}^{t} c_m^H(i,t_0,t) N(t_0,t) dt_0 \right) \\
& + (p_{m,\varphi}^F(i,t) - w(t)\varphi a_m) \left(\int_{-\infty}^{t} c_m^{H*}(i,t_0,t) N^*(t_0,t) dt_0 \right) \\
s.t. \quad & c_m^H(i,t_0,t) = \frac{\xi e(t_0,t)(p_m^H(i,t))^{-\sigma}}{P_m(t)} \\
& c_m^{H*}(i,t_0,t) = \frac{\xi e^*(t_0,t)(p_{m,\varphi}^F(i,t))^{-\sigma}}{P_m^*(t)}.
\end{aligned} \tag{2.18}$$

Carrying out the associated calculations shown in appendix A.1.3 gives optimal prices

$$p_m^H(i,t) = \frac{\sigma}{\sigma - 1} w(t) a_m, \tag{2.19}$$

$$p_{m,\varphi}^F(i,t) = \frac{\sigma}{\sigma - 1} w(t) a_m \varphi. \tag{2.20}$$

The profit maximization problem yields the familiar rule that prices are equal

[12] Note that $p_m^H(j,t) = p_m^{*F}(j,t)$ and $p_{m,\varphi}^F(i,t) = p_{m,\varphi}^{*H}(i,t)$ due to symmetry between the two regions, where $p_{m,\varphi}^{*H}(i,t)$ is the price of a good *produced* in the home economy but *consumed* in the foreign region.

[13] We ignore fixed costs in the derivations here as they do not influence the first order conditions. Therefore we just maximize operating profit defined as revenues from selling the variety to the home and foreign region minus variable production costs (taking into account the effect of transport costs).

to a constant markup over marginal costs which decreases in σ since a higher elasticity of substitution reduces the market power of manufacturing firms. Moreover, mill pricing is optimal, i.e. the only difference between prices in the two regions is due to transport costs (see Baldwin et al. (2003)).

Since we have variety specificity of capital and free entry into the manufacturing sector driving pure profits down to zero, the capital rental rate is equivalent to the Ricardian surplus, i.e. the operating profit of each manufacturing firm. In particular, the insurance companies, which hold all the capital due to the full insurance result (see section 2.2.2), rent their capital holdings to the manufacturing firms and can fully extract all profits. As shown in appendix A.1.3, using optimal prices given in equations (2.19) and (2.20) and redefining global quantities and regional share variables gives operating profits and thus capital rental rates as[14]

$$\pi = \underbrace{\left(\frac{\theta_E}{\theta_K + \phi(1-\theta_K)} + \frac{(1-\theta_E)\phi}{\phi\theta_K + 1 - \theta_K}\right)}_{Bias}\left(\frac{\xi E^W}{\sigma K^W}\right), \tag{2.21}$$

$$\pi^* = \underbrace{\left(\frac{1-\theta_E}{1-\theta_K + \phi\theta_K} + \frac{\theta_E\phi}{\phi(1-\theta_K) + \theta_K}\right)}_{Bias^*}\left(\frac{\xi E^W}{\sigma K^W}\right), \tag{2.22}$$

where $\phi \equiv \varphi^{1-\sigma}$ is a measure of openness between the two regions with $\phi = 0$ indicating prohibitive trade barriers and $\phi = 1$ free trade. World expenditures are defined as $E^W \equiv E + E^*$ and the world capital stock as $K^W \equiv K + K^*$ with θ_K and θ_E being the respective home shares of these quantities, i.e. $\theta_K \equiv \frac{K}{K+K^*}$ and $\theta_E \equiv \frac{E}{E+E^*}$.

As expected, these rental rates are identical to those derived in the constructed capital model of Baldwin (1999), since the introduction of overlapping generations and lifetime uncertainty does not change the production side of the economy. In analogy to Baldwin (1999), the terms labeled *Bias* and *Bias** can be interpreted as the bias in national sales, i.e. *Bias* measures the extent to which a home variety's sales ($\sigma\pi$) differ from the world

[14] We ignore time arguments here. Note, moreover, that $\xi = \xi^*$ and $\sigma = \sigma^*$ due to symmetry between regions.

average sales per variety ($\frac{\xi E^W}{K^W}$). In the symmetric case with $\theta_K = 1/2$ and $\theta_E = 1/2$, $Bias = Bias^* = 1$ implying that operating profits of each manufacturing firm are given by $\frac{\xi E^W}{\sigma K^W}$, a result familiar from the monopolistic competition framework of Dixit and Stiglitz (1977). Additionally, $Bias$ and $Bias^*$ capture the impact of capital and expenditure shifting on profits.[15] At the symmetric equilibrium, shifting expenditure to home ($d\theta_E > 0$) raises π and lowers π^* since it increases the home market size. A higher expenditure share therefore supports agglomeration of capital at home since capital accumulates where the rental rate is higher and decumulates in the other region. Production shifting[16] to home ($d\theta_K > 0$), on the other hand, has the opposite impact as it increases competition in the home market. Both forces are crucial for explaining agglomeration processes in the constructed capital model of Baldwin (1999). In particular, suppose the two regions are in a symmetric equilibrium and capital stocks are slightly perturbed. If this perturbation raises the relative profitability in the region with the increased capital share then the equilibrium is unstable and agglomeration sets in. Whether catastrophic agglomeration of capital occurs is thus determined by the relative strength of the two effects described above. The local competition effect directly decreases the capital rental rate, whereas the higher expenditure share associated with a higher capital share indirectly increases the rate. This last channel is based on the demand-linked circular causality, i.e. a higher capital stock implies a higher income which increases expenditures and thus the capital rental rate. As both the pro-agglomerative expenditure shifting and the anti-agglomerative production shifting effect depend on the level of trade openness ϕ, the link between economic integration and agglomeration can be easily established. In this respect Baldwin (1999) shows that, when starting from a situation of prohibitive trade costs, agglomeration processes set in as soon as economic integration reaches a certain threshold level. The crucial question to be investigated in the following sections is whether and how such

[15] As capital is immobile between regions, the term capital shifting might be misleading. It should, however, only represent an exogenous perturbation of the home capital share (and similarly of the home expenditure share in the case of expenditure shifting).

[16] Recall that the number of varieties in the home region is equal to the capital stock at home. This implies that capital accumulation is tantamount to firm creation.

agglomerative tendencies depend on the regions' demographic structures.

2.3 Long-run equilibrium

The dynamics of this neoclassical growth model with overlapping generations are fully described by the following four dimensional system in the variables E, E^*, K and K^* whose equations were derived in section 2.2.3 and are given by[17]

$$\dot{K} = \left[\frac{\xi}{\sigma a_i}\left(\frac{E}{K+\phi K^*}+\frac{\phi E^*}{\phi K+K^*}\right)-\delta\right]K+\frac{L}{a_i}-\frac{E}{a_i}, \tag{2.23}$$

$$\dot{E} = -\mu(\rho+\mu)a_i K + E\left[\frac{\xi}{\sigma a_i}\left(\frac{E}{K+\phi K^*}+\frac{\phi E^*}{\phi K+K^*}\right)-\rho-\delta\right], \tag{2.24}$$

$$\dot{K^*} = \left[\frac{\xi}{\sigma a_i}\left(\frac{E^*}{K^*+\phi K}+\frac{\phi E}{\phi K^*+K}\right)-\delta\right]K^*+\frac{L}{a_i}-\frac{E^*}{a_i}, \tag{2.25}$$

$$\dot{E^*} = -\mu(\rho+\mu)a_i K^* + E^*\left[\frac{\xi}{\sigma a_i}\left(\frac{E^*}{K^*+\phi K}+\frac{\phi E}{\phi K^*+K}\right)-\rho-\delta\right]. \tag{2.26}$$

Here we used that the equilibrium wage rate is equal to one in both regions and we already substituted for the rental rates from equations (2.21) and (2.22).[18] The mortality rate enters the system only via the turnover correction terms in the aggregate Euler equations. This again emphasizes that the effects of demographic change on the model's dynamics will crucially hinge on the heterogeneity of wealth and therefore expenditure levels with respect to age. Moreover, setting $\mu = 0$, i.e. considering the case of an infinitely lived representative agent and thus ignoring any demographic structures, reduces the law of motions to the ones obtained by Baldwin (1999). Our framework thus nests the constructed capital model as a special case.

A long-run equilibrium characterized by the steady-state values $\bar{E}$, $\bar{K}$, $\bar{E}^*$

[17] We again suppress time arguments here.

[18] Note that we rewrote the rental rates as functions of the variables E, E^*, K and K^* and that, due to the assumption of symmetric regions, we have $L = L^*$ and $\mu = \mu^*$ as well as $a_i = a_i^*$, $\delta = \delta^*$ and $\rho = \rho^*$.

and $\bar{K}^*$ must fulfill the system with the left hand side set equal to zero. It can be verified[19] that the symmetric outcome with $K = K^*$ and $E = E^*$ has this property with the steady-state values given by[20]

$$\bar{E}_{sym} = \frac{L\sigma\left(\sigma\delta^2 + \rho\sigma\delta - 2\mu(\mu+\rho)(\sigma-\xi) + \delta\sqrt{\sigma}\sqrt{\sigma(\delta+\rho)^2 + 4\mu(\mu+\rho)\xi}\right)}{2(\delta\sigma + (\mu+\rho)(\sigma-\xi))(\delta\sigma + \mu(\xi-\sigma))}, \tag{2.27}$$

$$\bar{K}_{sym} = \frac{\delta L\sigma(\sigma+\xi) + L\sqrt{\sigma}(\sigma-\xi)\left(\rho\sqrt{\sigma} - \sqrt{\sigma(\delta+\rho)^2 + 4\mu(\mu+\rho)\xi}\right)}{2a_i(\delta\sigma + (\mu+\rho)(\sigma-\xi))(\delta\sigma + \mu(\xi-\sigma))}. \tag{2.28}$$

Investigating how these steady-state values depend on the mortality rate helps to develop a deeper understanding of the various ways the introduction of overlapping generations and lifetime uncertainty impacts upon the model's behavior. There are two possible channels via which demographic change, as captured by variations in the mortality rate, can influence the steady-state value of aggregate consumption expenditures. On the one hand, a higher mortality rate changes the age structure of the population by increasing the proportion of poor and young to wealthy and old individuals. As the former have lower expenditure levels, this first channel, which basically captures the effects of an increased generational turnover, indicates a negative dependence of equilibrium expenditures upon the mortality rate ("age structure based channel"). On the other hand, a higher mortality rate influences demand patterns of individuals across all generations uniformly by increasing their effective rate of time preference $\rho+\mu$ and thus their marginal propensity to consume out of total wealth (see section 2.2.3). According to this second channel, a higher mortality rate positively affects consumption expenditures since an increased probability of death resulting in a more heavy discounting of the future increases expenditure levels of all individuals ("discount chan-

[19]These and most other results were derived with Mathematica. The corresponding files are available from the authors upon request.

[20]Solving the system for the symmetric equilibrium values in fact yielded two solution pairs. As one of them gives negative equilibrium expenditures for plausible parameter values we restrict our attention to the economically meaningful solution pair.

nel"). We can therefore conclude that the mortality rate's effect on aggregate consumption expenditures is a priori ambiguous since it crucially depends on which of the two effects dominates.

However, as far as the aggregate equilibrium capital stock is concerned, both channels imply a negative dependence. First, a higher mortality rate decreases the proportion of old individuals with high capital stocks which reduces the aggregate capital stock. Second, it increases discounting which depresses savings and thus capital accumulation of all individuals.

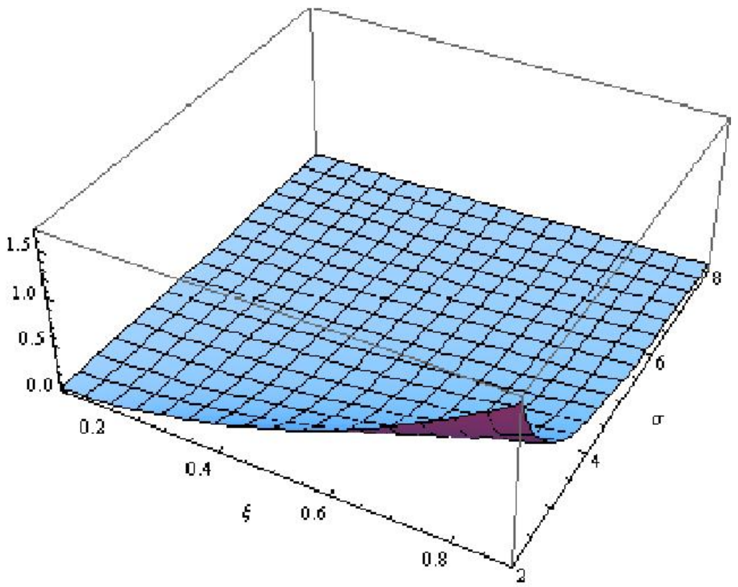
To clarify the above arguing, we investigate the derivatives of $\bar{E}_{sym}$ and $\bar{K}_{sym}$ with respect to μ. As the corresponding signs are analytically ambiguous, we resort to numerical analysis by calibrating our model and evaluating the derivatives at the following plausible parameter values: $\mu = 0.0125$ resulting in a life expectancy of 80 years[21], $\delta = 0.05$ implying that capital fully depreciates on average after 20 years, $\rho = 0.015$ (see Auerbach and Kotlikoff (1987)), $a_i = 2$ by choice of units for the investment good and $L = 1$. Since there is considerable disagreement about the parameter values of σ and ξ, we use a wide range in our numerical calculations. As far as the former is concerned, a plausible lower bound is $\sigma = 2$ as in Baldwin (1999). Most authors, however, use $\sigma \approx 4$ (see Krugman (1991b), Krugman and Venables (1995), Martin and Ottaviano (1999), Puga (1999), Brakman et al. (2005) and Bosker and Garretsen (2007)). In order to consider all reasonable possibilities, we choose as an upper bound $\sigma = 8$. With respect to ξ, which in fact describes the share of consumption expenditures for the good produced under increasing returns to scale (relative to the good produced under constant returns to scale), Puga (1999), Head and Mayer (2003) and Bosker and Garretsen (2007) consider a value of $\xi = 0.1$, Krugman (1991b) and Baldwin (1999) set $\xi = 0.3$, Krugman and Venables (1995) choose $\xi = 0.6$ and Martin and Ottaviano (1999) set $\xi = 0.8$. We therefore consider a possible parameter range of $0.1 \leq \xi \leq 0.9$ to account for this wide spread.[22]

Figure 2.1 and 2.2 reveal that for these parameter ranges the derivative

[21] Since the probability of death during each year equals μ, average life expectancy is $\frac{1}{\mu}$.

[22] Recall, however, that production of the agricultural good in both regions requires ξ to be sufficiently small (see section 2.2.4).

Figure 2.1: Derivative of $\bar{E}_{sym}$ with respect to μ

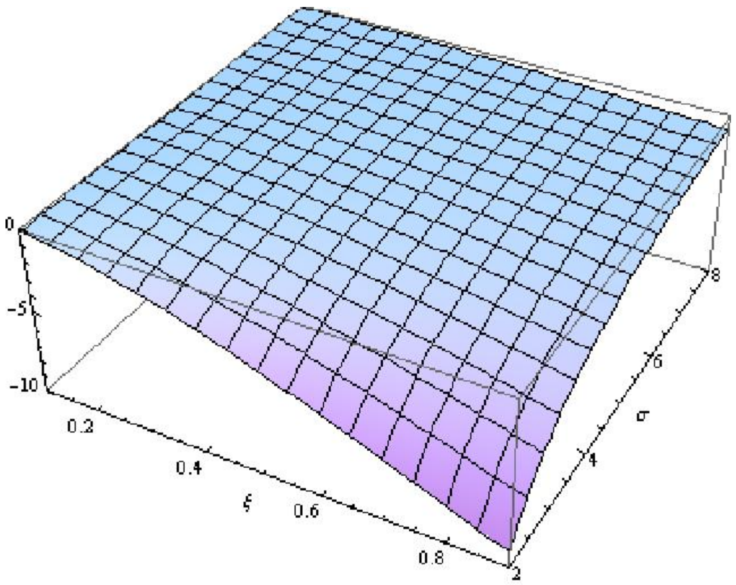


of $\bar{E}_{sym}$ with respect to μ is positive, whereas the derivative of $\bar{K}_{sym}$ is negative.[23] Consequently, a decrease in the mortality rate increases the aggregate equilibrium capital stock and decreases aggregate equilibrium expenditures. We can thus conclude that demographic change influences expenditures primarily via the effect on discounting. The discount channel dominates the age structure channel which captures the effects of variations in μ on the age composition of the population and thus exerts its influence via the heterogeneity of expenditure and wealth levels with respect to age. The positive dependence of aggregate equilibrium expenditures on the mortality rate is fully consistent with the life-cycle savings literature claiming that longer planning horizons, i.e. lower mortality rates, lead to higher individual savings and lower consumption levels (see e.g. Gertler (1999), Futagami and Nakajima (2001) or Zhang et al. (2003)).

As it turns out, when considering the impact of the mortality rate on

[23]We also investigated the derivatives for varying mortality rates. Assuming $0.008 \leq \mu \leq 0.025$ resulting in a life expectancy between 40 and 120 years, and still considering the same values for the other parameters, does not change our findings.

Figure 2.2: Derivative of $\bar{K}_{sym}$ with respect to μ



the steady-state consumption expenditure share[24], $\frac{\bar{E}_{sym}}{\delta \bar{K}_{sym}+\bar{E}_{sym}}$, even analytical results can be derived. This share is obtainable from the ratio of the equilibrium capital stock to equilibrium expenditures[25]

$$\frac{\bar{K}_{sym}}{\bar{E}_{sym}} = \frac{2\xi}{a_i(\delta\sigma + \rho\sigma + \sqrt{\sigma}\sqrt{\sigma(\delta+\rho)^2 + 4\mu(\mu+\rho)\xi})}, \tag{2.29}$$

which obviously depends negatively on the mortality rate. Consistent with our numerical findings, a higher mortality rate thus increases the steady-state consumption expenditure share which again supports the predominant role of the discount channel.

[24]This share is defined as equilibrium consumption expenditures divided by steady-state income, where steady-state income is the sum of replacement investment, δK (equal to savings in steady state), and consumption expenditures.

[25]Simply calculate $\frac{1}{\frac{\delta \bar{K}_{sym}}{\bar{E}_{sym}}+1}$.

2.4 Symmetric equilibrium *stability* - Agglomeration and demographic change

NEG models emphasize that reciprocal liberalization between initially symmetric regions leads to catastrophic agglomeration, i.e. their main focus is on the instability of the symmetric equilibrium. Indeed, if this steady state is unstable, then any slight perturbation will lead us away from an equal distribution of capital and expenditures and thus result in agglomeration processes. In this section we show that the introduction of overlapping generations and lifetime uncertainty considerably reduces the possibility of the symmetric equilibrium to be unstable. As a consequence, agglomeration of economic activity may not set in even if economic integration is promoted up to a high degree.

2.4.1 Formal stability analysis

The stability properties of the symmetric long-run equilibrium for varying levels of trade openness and mortality rates are analyzed by following the classical approach (see e.g. the appendix on mathematical methods in Barro and Sala-i-Martin (2004)) of linearizing the non-linear dynamic system given in equations (2.23), (2.24), (2.25) and (2.26) around the symmetric equilibrium and then by evaluating the eigenvalues of the corresponding 4×4 Jacobian matrix

$$J_{sym} = \begin{pmatrix} J_1 & J_2 \\ J_3 & J_4 \end{pmatrix}, \tag{2.30}$$

where the four symmetric 2×2 sub-matrices J_i for $i = 1, \ldots, 4$ are given in appendix A.1.4. Solving the characteristic equation yields the following four eigenvalues

$$
\begin{aligned}
eig1 &= \frac{1}{2}(r_1 - \sqrt{rad_1}), && (2.31)\\
eig2 &= \frac{1}{2}(r_1 + \sqrt{rad_1}), && (2.32)\\
eig3 &= \frac{1}{2(\phi+1)^2\sqrt{\sigma}}(r_2 - \sqrt{rad_2}), && (2.33)\\
eig4 &= \frac{1}{2(\phi+1)^2\sqrt{\sigma}}(r_2 + \sqrt{rad_2}), && (2.34)
\end{aligned}
$$

where

$$
\begin{aligned}
r_1 &\equiv \frac{A}{\sqrt{\sigma}} - \delta,\\
rad_1 &\equiv \left(\frac{A}{\sqrt{\sigma}} + \delta\right)^2 + \frac{(\sigma-\xi)\left((A+B)^2 + 4\mu(\mu+\rho)\xi\right)}{\sigma\xi},\\
r_2 &\equiv 3\phi A + A - \sqrt{\sigma}\left(\delta\left(2\phi^2+\phi+1\right) + (\phi-1)\phi\rho\right),\\
rad_2 &\equiv \left(A(\phi-1) + (\delta(\phi-1) + \phi(\phi+3)\rho)\sqrt{\sigma}\right)^2 + \\
&\quad \frac{(\phi+1)(\phi\sigma+\sigma+\phi\xi-\xi)\left((A+B)^2(\phi-1)^2 + 4\mu(\phi+1)^2(\mu+\rho)\xi\right)}{\xi},
\end{aligned}
$$

with the parameter clusters $A \equiv \sqrt{\sigma(\delta+\rho)^2 + 4\mu(\mu+\rho)\xi}$ as well as $B \equiv (\delta+\rho)\sqrt{\sigma}$. The signs and nature of these eigenvalues fully characterize the system's local dynamics around the symmetric equilibrium. Analytically investigating them[26] thus results in lemma 2.1.

Lemma 2.1. *Eigenvalue 3 is decisive for the local stability properties of the symmetric equilibrium. A positive eigenvalue 3 implies instability, a negative one saddle path stability.*

Proof. By investigating the expressions for the eigenvalues it is first easily established that all of them are real. This holds since both rad_1 and rad_2 are nonnegative for all possible parameter values (in particular since $\sigma > \xi$).[27]

[26] In order to get a first idea about the signs and nature of the eigenvalues, we also calibrated the model and investigated the eigenvalues numerically. The corresponding findings are presented in appendix A.1.4.

[27] Recall the parameter ranges $\sigma > 1$, $\delta > 0$, $\rho > 0$, $\mu > 0$, $0 < \xi < 1$ and $0 \leq \phi \leq 1$ which imply that $A > 0$ and $B > 0$.

Convergence to or divergence from the symmetric equilibrium is therefore monotonic.

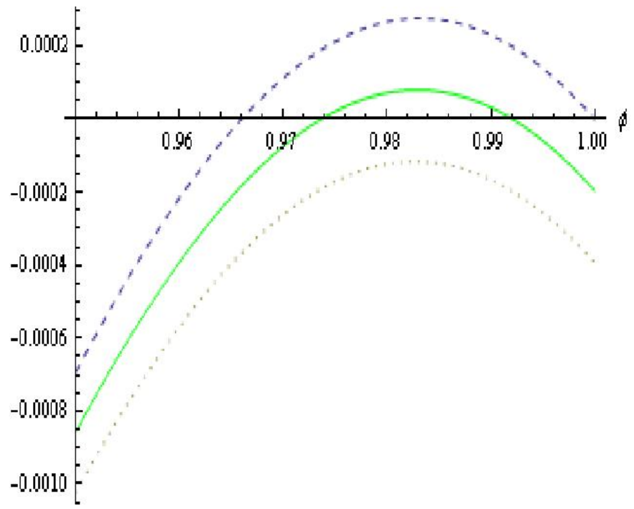
As there are two jump variables E and E^*, saddle path stability prevails if and only if there are two negative eigenvalues. If fewer than two eigenvalues are negative, the system is locally unstable. By inserting the expression for A, it turns out that $r_1 > 0$. We can thus immediately conclude that eigenvalue 2 is positive. In order to find out the sign of eigenvalue 1, we compare r_1 with the corresponding part under the radical, i.e. rad_1. The square of the former is smaller than the latter, implying that eigenvalue 1 is always negative. It remains to investigate the signs of eigenvalues 3 and 4. Again we first check whether r_2 is nonnegative. By inserting the expression for A, r_2 can be rewritten as

$$r_2 = \underbrace{-\sqrt{\sigma}\delta\left(2\phi^2+\phi+1\right)}_{term1} + \underbrace{\sqrt{\sigma}(1-\phi)\phi\rho}_{term2} + \underbrace{(1+3\phi)\sqrt{\sigma(\delta+\rho)^2+4\mu(\mu+\rho)\xi}}_{term3}.$$

All three terms are increasing in ρ, ξ and μ but react differently to changes in ϕ, δ and σ. In order to show that r_2 is nevertheless nonnegative for all parameter values we set ρ, ξ and μ equal to zero resulting in the "worst", i.e. most negative, outcome with respect to these parameters. Since even in this case it is easily established that r_2 is nonnegative for the whole feasible parameter space the fourth eigenvalue is definitely positive. Summarizing, we have shown that eigenvalue 2 and 4 are always positive, whereas eigenvalue 1 is always negative. This proves the crucial role of the third eigenvalue as claimed in lemma 2.1. □

Having demonstrated that changes in the parameter values, and in particular of the mortality rate, can only influence the stability properties of the symmetric equilibrium via eigenvalue 3, it is immediate to investigate this eigenvalue more thoroughly. Figure 2.3 plots eigenvalue 3 as a function of ϕ for three different mortality rates given our choice of the most plausible values of the other parameters ($\rho = 0.015$, $\delta = 0.05$, $\xi = 0.3$ and $\sigma = 4$). The

Figure 2.3: Eigenvalue 3 as a function of ϕ for $\mu = 0$ (dashed line), $\mu = 0.0002$ (solid line) and $\mu = 0.0004$ (dotted line)



graph indicates that, depending on the level of trade openness, eigenvalue 3 switches its sign.[28] Moreover, it is clearly visible that the range of ϕ within which eigenvalue 3 is positive, crucially depends on the mortality rate. This observation is investigated in the following proposition.

Proposition 2.1. *The sign of eigenvalue 3 and hence the stability properties of the symmetric equilibrium depend on the mortality rate.*

Proof. To prove this proposition, we use the concept of the critical level of trade openness ϕ_{break}. This threshold value identifies the degree of economic integration where eigenvalue 3 changes its sign and therefore where the stability properties of the symmetric equilibrium change (i.e. where eigenvalue 3 crosses the horizontal axis in figure 2.3). To analytically obtain ϕ_{break}, we set the expression for the third eigenvalue equal to zero and solve the resulting equation. This yields two solutions for ϕ_{break} as functions of the other parameters.[29] Since these two critical levels in particular also depend on the

[28]The numerical investigation of eigenvalue 3 in appendix A.1.4 also reveals that it is impossible to come up with a definite sign for the whole parameter space.

[29]As the expressions are rather cumbersome they are not presented here but available upon request.

mortality rate (see again figure 2.3 for a graphical illustration), proposition 2.1 holds. □

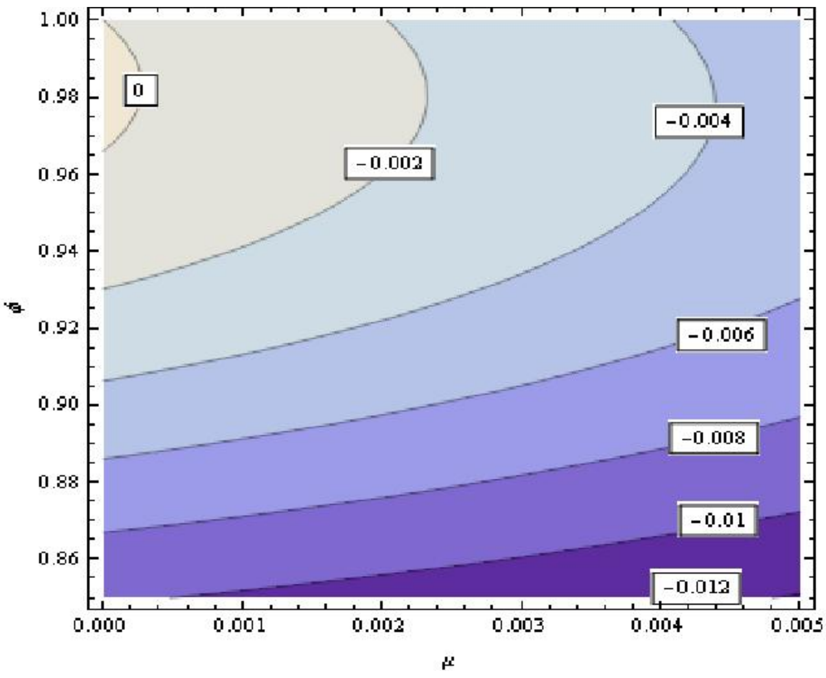
So far, we have shown that changes in the mortality rate influence the stability properties of the symmetric equilibrium. Figure 2.3 moreover already indicates the particular direction of the impact by illustrating that eigenvalue 3 decreases in the mortality rate. The next section is dedicated to investigating this relationship between demographic change and the stability properties of the symmetric equilibrium in more detail.

2.4.2 The impact of demography on agglomeration

The effects of demographic change on agglomeration can be best understood by investigating eigenvalue 3 for varying levels of trade openness and mortality rates. Figure 2.4 plotting the contour lines of eigenvalue 3 for varying μ and ϕ given our choice of the most plausible values of the other parameters ($\rho = 0.015$, $\delta = 0.05$, $\xi = 0.3$ and $\sigma = 4$) illustrates that there only exists a very small range of combinations of μ and ϕ where the sign of the third eigenvalue is positive. This instability region is characterized by parameter combinations inside the zero contour line which yield a nonnegative eigenvalue 3. A higher mortality rate decreases eigenvalue 3 rather quickly for all levels of trade openness. Only in case of a(n) (implausibly) low mortality rate it is possible to find critical values of openness within which the symmetric equilibrium becomes unstable and agglomeration processes can set in. We can therefore conclude that the introduction of overlapping generations and lifetime uncertainty, i.e. increasing μ above zero, profoundly reduces agglomeration tendencies. In particular, a higher mortality rate implies a smaller instability region with respect to the level of economic integration and thus increasingly prevents the two regions from unequal development. The "smallness" of the instability region[30] moreover implies that, in sharp contrast to other NEG frameworks and in particular to the catastrophic agglomeration

[30]Note that we plot this figure only for $\mu \leq 0.005$ and $\phi \geq 0.85$ which indicates how small the instability region relative to the whole parameter range is.

Figure 2.4: Contour plot of eigenvalue 3

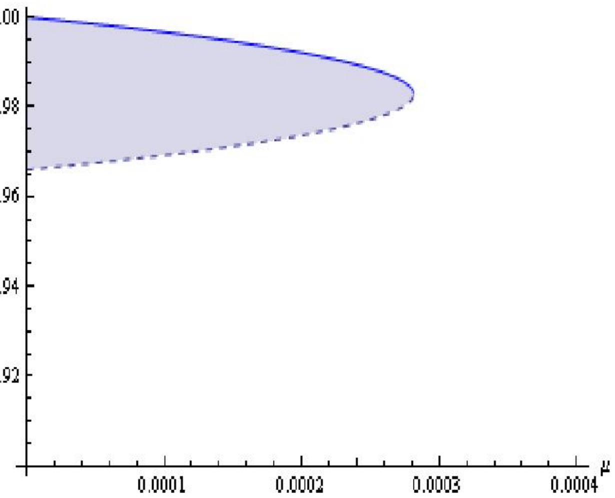


result of Baldwin (1999), our model predicts the symmetric outcome to be predominant even in the presence of high economic integration.

The effects of demographic change on agglomeration are confirmed by investigating how the critical levels of trade openness react to changes in the mortality rate. With an infinitely lived representative individual, i.e. $\mu = 0$, and again applying our choice of the most plausible parameter values, the two critical levels of trade openness are $\phi_{break1} = 0.965974$ and $\phi_{break2} = 1$.[31] In between those values, i.e. for sufficiently low levels of trade costs, the symmetric equilibrium is unstable and agglomeration processes indeed set in. Allowing μ to increase, however, shows that ϕ_{break1} increases, while ϕ_{break2} decreases. The range of trade openness levels within which the symmetric equilibrium is unstable clearly shrinks (in figure 2.3 this shrinkage

[31] When calibrating our model with the parameter values assumed by Baldwin (1999), i.e. $\rho = \delta = 0.1$, $\xi = 0.3$ and $\sigma = 2$, the two critical levels of trade openness exactly coincide with the ones of Baldwin (1999) and are given by $\phi_{break1} = 0.860465$ and $\phi_{break2} = 1$.

Figure 2.5: ϕ_{break1} (dashed) and ϕ_{break2} (solid) as a function of μ



is equivalently represented by the downward shift of eigenvalue 3). Figure 2.5 illustrates this finding by plotting the the two critical levels of trade openness for varying mortality rates as boundaries of the shaded instability region. In particular, we can establish that for $\mu > 0.00028$, corresponding to a life expectancy of less than approximately 3500 years, there exists no level of trade openness such that the symmetric equilibrium is unstable (i.e. the downward shift in figure 2.3 is such that eigenvalue 3 does not cross the horizontal axis anymore where it would become positive).[32] Assuming reasonable values of the mortality rate therefore implies that, again in sharp contrast to other NEG models, deeper economic integration does *not* result in agglomeration processes. We can thus also conclude that the agglomeration induced growth finding of Baldwin (1999) primarily applies in the very special case of infinitely lived individuals.

Figure 2.5 does not only show that the instability region shrinks in the mortality rate but also reveals an interesting feature of the instability set

[32] We also performed these simulations with respect to the critical level of trade openness for other parameter ranges, in particular for the parameter choice $\rho = \delta = 0.1$, $\xi = 0.3$ and $\sigma = 2$ made by Baldwin (1999). In this case the symmetric equilibrium is always stable for $\mu > 0.00395$. This implies that in the setup of Baldwin (1999) a life expectancy of less than approximately 250 years prevents any agglomerative tendencies.

with respect to the level of trade openness: for low but positive mortality rates, the instability set is non-monotone in ϕ implying that agglomeration processes only set in for an intermediate range of trade openness and that the symmetric equilibrium gets stable again for sufficiently high levels of economic integration. This is reflected by ϕ_{break2} being smaller than one which contrasts with the set-up of Baldwin (1999) where ϕ_{break2} is always equal to one and the symmetric equilibrium is thus unstable for all values of ϕ beyond ϕ_{break1} (see figure 2.5 for μ=0).[33] The reason for this distinctive feature of the instability region becomes clear when looking at the forces weakening or fostering agglomeration in our model. As it turns out, these informal stability considerations presented in the next subsection will moreover be useful for developing some economic intuition about the relationship between demographic change and agglomeration, i.e. for understanding the channel through which demography impacts upon the stability properties of the symmetric equilibrium.

2.4.3 Economic intuition - The turnover effect

As shown by Baldwin (1999), the formal stability analysis pursued in section 2.4.1 yields the same results as a more informal way of checking the stability of the symmetric equilibrium. This informal way is based on investigating how an exogenous perturbation of the home share of capital, θ_K, influences the profitability of home-based firms relative to foreign-based firms. A positive impact implies instability as even more firms would locate in the home region, i.e. capital accumulation would set in.

We can isolate three channels via which production shifting influences the relative profitability of home-based firms. First, there is the pro-agglomerative demand-linked circular causality effect. A higher capital share increases capital income in the home region and thus its expenditure share. The associated increased market size positively affects home profitability (see section 2.2.4) and therefore causes further production shifting.[34] The second channel is

[33] Figure 2.3 also illustrates that eigenvalue 3 is positive for all $\phi > \phi_{break1}$ in the case of $\mu = 0$.

[34] This agglomeration force first introduced by Baldwin (1999) is due to the endogeneity

based on the anti-agglomerative local competition effect capturing the negative impact of production shifting upon equilibrium profits due to the more severe competition among home-based firms (see again section 2.2.4). Both these forces are present in the constructed capital model of Baldwin (1999) and explain why agglomeration in this framework sets in for sufficiently high levels of economic integration.

In our model, there appears, however, an additional dispersion force strengthening the stability of the symmetric equilibrium. In particular, the introduction of an overlapping generation structure motivates the anti-agglomerative turnover effect as a third channel via which production shifting changes the profitability of home-based firms. This dispersion force is based on the distributional effects caused by the turnover of generations. An exogenous rise in the home capital share increases wealth and thus expenditure levels of individuals being currently alive in the home region relative to foreign-based individuals. The negative distributional effects on aggregate expenditures resulting from birth and death, i.e. the replacement of these individuals by newborns whose consumption expenditures are lower since they have zero wealth levels (see section 2.2.3), are thus more pronounced in the home region. This, in turn, decreases the home expenditure share and therefore relative profitability. Since a higher mortality rate increases the strength of the anti-agglomerative turnover effect, this third channel intuitively explains the positive impact of μ on the stability of the symmetric equilibrium. We can thus conclude that generalizing the constructed capital model to allow for demographic structures introduces an additional dispersion force that crucially depends on the heterogeneity of individuals with respect to their expenditure and wealth levels: as long as the consumption expenditures of newborns are smaller than average consumption expenditures, the turnover effect is active and the associated distributional effects work against agglomeration.

---

of capital in his model. It hinges critically on the immobility of capital as only in this case capital income cannot be repatriated to its immobile owners and therefore increases the region's own income. In our model with capital immobility it is, however, indeed the case that the equilibrium value of the consumption expenditure share depends, via this income effect, on the capital stock.

Going back to figure 2.5, we can now also explain the non-monotonicity of the instability set with respect to the level of trade openness. From figure 2.3 it is evident that the relative strength of the agglomeration force, as represented by eigenvalue 3, is maximized for $\phi < 1$. This is also the case for the set-up of Baldwin (1999) with a zero mortality rate, i.e. when demographic change is completely ignored. In particular, both the local competition and the agglomeration force present in the constructed capital model decrease in the level of trade openness. The local competition effect diminishes since freer trade makes firms less dependent on the home market, whereas the demand-linked circular causality effect is reduced as local sales increasingly lose importance. Both forces, however, diminish at different speeds with the competition force being reduced more rapidly for high levels of trade costs whereas the decrease of the agglomeration force is stronger for sufficiently low levels of trade barriers. This implies that the relative strength of the agglomeration force is largest for an intermediate level of trade costs. Since eigenvalue 3 remains positive and the symmetric equilibrium thus unstable for all $\phi > \phi_{break1}$ (see eigenvalue 3 for $\mu = 0$ in figure 2.3), this special feature does not have any consequences for the instability set in the case of a zero mortality rate. Positive mortality rates, however, additionally decreases eigenvalue 3 for all levels of trade openness due to the anti-agglomerative turnover effect. For $\mu > 0$, the non-monotonicity of the relative strength of the agglomeration force therefore qualitatively changes the stability properties of the symmetric equilibrium: as shown in figure 2.5, ϕ_{break1} increases *and* ϕ_{break2} *decreases below one* implying that the symmetric equilibrium regains its stability for sufficiently high levels of trade openness. The emergence of an additional dispersion force, when incorporating demographic structures, thus explains the non-monotonicity of the instability set with respect to the level of economic integration in the case of a positive mortality rate.

2.5 Concluding remarks

The model in this paper introduces demography into the NEG literature by generalizing the constructed capital framework of Baldwin (1999) to account for changes in the age structure of the population. Incorporating overlapping generations and lifetime uncertainty allows us to investigate the impact of demographic change on agglomeration tendencies of economic activities. We show that the turnover of generations stabilizes the symmetric equilibrium and thus acts as a force that promotes a more equal distribution of productive factors between two regions.

From the point of view of economic policy, we can conclude that, in sharp contrast to other NEG approaches, our model does not necessarily associate sufficiently deep integration with high interregional inequality. In particular, we have shown that plausible mortality rates are far away from supporting agglomeration processes. Consequently, it might not be necessary to impose any type of trade barriers in order to avoid deindustrialization of one region resulting from decreased transport costs. Especially in the case of the European Union this implies that there is no trade-off between its two most important targets: integration on the one hand and interregional equality on the other hand. Instead, the implementation of appropriate policies to achieve one objective does not interfere with the realization of the other.

However, introducing overlapping generations and lifetime uncertainty was only a first step toward a more comprehensive understanding of the interrelations between demographic change, economic integration and agglomeration. First of all, it is essential to consider a broader view of demographic change that does not only focus on changes in the population age structure but also on varying population growth rates. By relaxing the assumption of equal birth and mortality rates and thus allowing for population growth one could e.g. asses agglomeration processes for various demographic scenarios. In particular, it would be possible to analyze a situation of declining fertility resulting in both population aging and slower population growth. Moreover, the assumption of a constant mortality rate adopted for the sake of analytical tractability is still at odds with reality. Using age-dependent mortality rates is therefore one possible line for future research. Similarly, investigating the effects of age-dependent labor productivity on agglomeration processes might yield important insights, especially when viewing labor productivity as decisive for a region's competitiveness. Last but not least, it would be worthwhile to consider asymmetric regions, in particular with respect to mortality. In such a setting one could investigate how differences in mortality rates are linked to differences in capital accumulation rates, again a question of high relevance for economic policy.

Chapter 3

Agglomeration processes in aging societies[1]

3.1 Introduction

With the ongoing worldwide economic integration during the last decades, economists' and policy makers' interest in the location of economic activity has dramatically risen (see e.g. Fujita and Thisse (2002) and the World Bank (2009) development report on economic geography). Within the European Union for example, regional cohesion policies targeted at a more equal spatial allocation of resources are on top of the political agenda. At the same time, as illustrated in the special report on aging populations of The Economist (2009), demographic change has been creating serious economic challenges for industrialized countries. In particular, declining fertility has caused upward shifts in the mean age of most countries' populations while simultaneously reducing population growth rates (see Eurostat (2004) and United Nations (2007)).

Up to now, these two issues have been analyzed independently of each other, with the New Economic Geography (NEG) addressing the impact of deeper economic integration on the spatial concentration of productive factors and overlapping generation models investigating the effects of demogra-

[1] Joint work with Klaus Prettner.

phy on macroeconomic aggregates. This division is unfortunate since both demographic change and economic integration crucially influence demand patterns which themselves are - via the returns to productive factors - decisive for the spatial distribution of economic activity. Explaining agglomeration processes without accounting for demographic developments thus misses a fundamental point.

The paper in chapter 2 has made a first step toward closing this gap by merging these two research strands and for the first time providing a unified framework within which the linkage between demographic change and agglomeration can be accurately analyzed. In particular, it has shown that introducing an overlapping generation setting where individuals face lifetime uncertainty considerably reduces agglomeration tendencies. By equalizing the birth to the death rate, the analysis was, however, restricted to changes in the population age structure and thus neglected the effects of varying population growth due to demographic change. Since declining fertility leads to decreasing population growth rates while lower mortality rates imply higher population growth, it is essential to reassess the impact of demographic change on the location of industries in a setting with nonconstant population size. Having this purpose in mind, the model presented in this paper extends the approach of chapter 2 by allowing for nonequal birth and mortality rates and thus growing populations. In particular, we generalize the constructed capital model of Baldwin (1999) by incorporating the overlapping generation structure of Buiter (1988) to arrive at a NEG framework featuring both population aging and varying population size.

In the constructed capital model of Baldwin (1999) concentration of economic activity is explained via a demand-linked circular causality whose pro-agglomerative effect is strongest for high levels of economic integration. In particular, with interregionally immobile capital, a higher capital stock raises capital income and thus expenditures which leads to an increase in the capital rental rate and therefore to further capital accumulation. Since higher capital accumulation is typically associated with medium-run growth, this two-region neoclassical growth framework illustrates how economic integration can lead to the emergence of “rust” and “boom belts”. Its intertemporally optimized saving features moreover allow an easy incorporation of overlapping generations with individuals that face a positive probability of death and differ with respect to their age. By using the overlapping generation structure of Buiter (1988), demographic change capturing both changes in the population age structure and/or in the population growth rate can be analyzed via variations in either the birth or the mortality rates. In particular, we can analyze a situation of population aging and declining population growth rates as recently faced by many industrialized countries. Overall, our modeling strategy does not only allow us to accurately analyze agglomeration processes in aging societies but also provides us with a more convincing description of reality compared to the setting with one infinitely lived individual of Baldwin (1999).

Our results show that the possibility of agglomeration crucially hinges on the economies' demographic properties, i.e. on the birth and mortality rate. While declining birth rates strengthen agglomeration processes, declining mortality rates weaken them. These differential effects on the stability properties of the symmetric equilibrium are due to the opposite impact of changes in the birth and mortality rate on the population size. Lower birth rates decrease the population growth rate, while lower mortality rates increase it. Since population growth weakens the wealth and thus expenditure increase due to a higher capital stock, it acts as an important dispersion force. For declining birth rates, the population growth based channel therefore strengthens the pro-agglomerative effects of a lower turnover of generations as identified in chapter 2, while for declining mortality rates the implied increase in the population growth rate counteracts and even dominates the impacts via the turnover channel.

The remainder of the paper is structured as follows. Section 3.2 presents the model framework and derives the equilibrium laws of motion for capital and expenditures. Section 3.3 first verifies the existence of a symmetric long-run equilibrium and then investigates its stability properties. In doing so, we can not only analyze agglomeration processes in aging societies but also isolate the population growth based effect on agglomeration. Finally, section 3.4 contains concluding remarks and sketches possible lines of further research.

3.2 The model

This section describes how we integrate the overlapping generation structure with growing populations of Buiter (1988) into the constructed capital framework of Baldwin (1999) to arrive at a NEG framework allowing for both population aging and varying population size.

There are two symmetric regions or countries, denoted as H (home) and F (foreign)[2], with identical production technologies, trade costs, preferences of individuals, labor endowments and demographic structures. Each region has three economic sectors (agriculture, manufacturing and investment) with two immobile factors (labor L and capital K) at their disposal.

The homogeneous agricultural good, which is also the numéraire good and denoted as z, is produced under constant returns to scale in a perfectly competitive market using labor as the only input with, by choice of units, an input coefficient of one. It can be freely traded between the two regions.

Manufacturing firms behave as in the monopolistic competition framework of Dixit and Stiglitz (1977) and therefore produce horizontally differentiated varieties, m, with one unit of capital as fixed input and a variable per unit requirement of a_m units of labor. Since each variety exactly requires one unit of capital, a continuum of varieties $i \in (0, K]$ is produced at home, whereas a continuum of varieties $j \in (0, K^*]$ is manufactured in the foreign region. In contrast to the agricultural good, trade of manufactures involves iceberg transport costs such that $\varphi \geq 1$ units of a certain good have to be shipped in order to sell one unit abroad (see e.g. Baldwin et al. (2003)). Firms thus face an increasing returns to scale production technology with an associated cost function $\pi + wa_mY_m(i)$, where π is the capital rental rate representing the fixed cost, w is the wage per efficiency unit of labor and

[2]If further distinction is needed, foreign variables are moreover indicated by an asterisk. In particular, the superscript F denotes that a good was *produced* in the foreign region, whereas the asterisk indicates that it is *consumed* in the foreign region. In what follows, emphasis will be on the home region. The corresponding expressions for the foreign region can be derived by symmetry.

$Y_m(i)$ is total output of one manufacturing good producer.

In the Walrasian investment sector, capital, i.e. machines, are produced using labor as the only input with an input coefficient of a_i. Wages of the workers are paid out of individuals' savings. Following Baldwin (1999), a share $\delta > 0$ of the capital stock depreciates at each instant.

Concerning the overlapping generation structure of our model economy, we closely follow Buiter (1988). We assume that at each point in time, $\tau \in [0, \infty)$, a large cohort consisting of new individuals is born. Newborns receive no bequests and thus start their lives without any wealth. Each individual's time of death is stochastic with an exponential probability density function parameterized by the constant instantaneous mortality rate $\mu > 0$. Normalizing initial population size $N(0)$ to one, the size of the cohort born at t_0 at a certain point in time τ is $N(t_0, \tau) = \beta e^{\beta t_0} e^{-\mu\tau}$ (see appendix A.2.1)[3], where $\beta > 0$ is the constant birth rate. Consequently, total population size at time τ is given by

$$\begin{aligned} N(\tau) &= \int_{-\infty}^{\tau} N(t_0, \tau) dt_0 \\ &= \int_{-\infty}^{\tau} \beta e^{\beta t_0} e^{-\mu\tau} dt_0 \\ &= e^{(\beta-\mu)\tau}, \end{aligned} \tag{3.1}$$

where we denote its growth rate as $n \equiv \beta - \mu$.

Since there is no heterogeneity between members of the same cohort, each cohort can be described by one representative individual, who inelastically supplies her efficiency units of labor l on the labor market with perfect mobility across sectors but immobility between regions. Finally, as in Yaari (1965), individuals can insure themselves against the risk of dying with positive assets by buying actuarial notes of a fair life insurance company which are canceled upon their death.

[3] In what follows the first time index of a variable will refer to the birth date, whereas the second will indicate a certain point in time.

3.2.1 Individual consumption behavior

Preferences over the agricultural good and a CES composite of the manufacturing varieties are Cobb-Douglas. The representative individual of the cohort born at t_0 chooses at each instant $\tau > t_0$ consumption of the agricultural good, $c_z(t_0, \tau)$, consumption of varieties produced at home, $c_m^H(i, t_0, \tau)$, and consumption of varieties produced abroad, $c_m^F(j, t_0, \tau)$, to maximize her expected lifetime utility at time t_0[4]

$$U(t_0, t_0) = \int_{t_0}^{\infty} e^{-(\rho+\mu)(\tau - t_0)} \ln \left[(c_z(t_0, \tau))^{1-\xi} (c_m^{agg}(t_0, \tau))^{\xi} \right] d\tau, \tag{3.2}$$

where $\rho > 0$ is the rate of pure time preference, $0 < \xi < 1$ is the manufacturing share of consumption and

$$c_m^{agg}(t_0, \tau) \equiv \left[\int_0^{K(\tau)} \left(c_m^H(i, t_0, \tau) \right)^{\frac{\sigma-1}{\sigma}} di + \int_0^{K^*(\tau)} \left(c_m^F(j, t_0, \tau) \right)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}$$

represents consumption of the CES composite with $\sigma > 1$ being the elasticity of substitution between varieties.

The wealth constraint of a representative individual is given by

$$\dot{k}(t_0, \tau) = \frac{w(\tau)l + \pi(\tau)k(t_0, \tau) - e(t_0, \tau)}{w(\tau)a_i} + \mu k(t_0, \tau) - \delta k(t_0, \tau), \tag{3.3}$$

where $k(t_0, \tau)$ is the individual capital stock and $e(t_0, \tau)$ are individual expenditures for consumption defined as

$$\begin{aligned} e(t_0, \tau) \;\; \equiv \;\; & p_z(\tau) c_z(t_0, \tau) + \int_0^{K(\tau)} p_m^H(i, \tau) c_m^H(i, t_0, \tau) di + \\ & \int_0^{K^*(\tau)} p_{m,\varphi}^F(j, \tau) c_m^F(j, t_0, \tau) dj. \end{aligned}$$

Here $p_z(\tau)$ is the price of the agricultural good, $p_m^H(i, \tau)$ the price of a manu-

[4]Equation (3.2) can be easily derived by calculating expected lifetime utility, where the date of death is a random variable with an exponential probability density function parameterized by a constant instantaneous mortality rate μ.

factured variety produced at home and $p^F_{m,\varphi}(j,\tau)$ the price of a manufactured variety produced abroad with the subscript φ indicating the dependence on transport costs. This wealth constraint displays the structure of the investment sector by showing that individual savings, defined as income minus consumption expenditures, are transformed into capital.

The law of motion for capital given in equation (3.3) is based on the full insurance result of Yaari (1965) implying that individuals save solely in the form of actuarial notes from the life insurance company, whose fair rate exceeds the market rate of return on capital, $\frac{\pi(\tau)}{w(\tau)a_i} - \delta$, by μ (see Yaari (1965)).

The individual's utility optimization problem can be solved by applying a three stage procedure.[5] The first stage analyzes the dynamic savings-expenditure decision and results in the Euler equation for the representative individual of the cohort born at t_0

$$\frac{\dot{e}(t_0,\tau)}{e(t_0,\tau)} = \frac{\pi(\tau)}{a_i w(\tau)} - \delta - \rho. \tag{3.4}$$

Stage two and three finally deal with the static optimal consumption allocation between the CES composite and the agricultural good as well as with the allocation of consumption to each of the varieties. Altogether this leads to the following demand functions for the agricultural good and for each of the manufactured varieties

$$c_z(t_0,\tau) = \frac{(1-\xi)e(t_0,\tau)}{p_z(\tau)}, \tag{3.5}$$

$$c^H_m(i,t_0,\tau) = \frac{\xi e(t_0,\tau)(p^H_m(i,\tau))^{-\sigma}}{\left[\int_0^{K(\tau)}(p^H_m(i,\tau))^{1-\sigma}di + \int_0^{K^*(\tau)}(p^F_{m,\varphi}(j,\tau))^{1-\sigma}dj\right]}, \tag{3.6}$$

$$c^F_m(j,t_0,\tau) = \frac{\xi e(t_0,\tau)(p^F_{m,\varphi}(j,\tau))^{-\sigma}}{\left[\int_0^{K(\tau)}(p^H_m(i,\tau))^{1-\sigma}di + \int_0^{K^*(\tau)}(p^F_{m,\varphi}(j,\tau))^{1-\sigma}dj\right]}. \tag{3.7}$$

[5] For details of the derivations see chapter 2 and its appendix A.1.

3.2.2 Aggregation

Due to the overlapping generation structure, our model setup does not feature *one* single representative individual. Corresponding to any individual variable we thus define population aggregates that follow from aggregating up over all cohorts. For capital and expenditures, these aggregation rules are formally given by

$$
\begin{aligned}
K(t) &\equiv \int_{-\infty}^{t} k(t_0,t)N(t_0,t)dt_0, \\
&= \beta e^{-\mu t}\int_{-\infty}^{t} k(t_0,t)e^{\beta t_0}dt_0, \qquad (3.8)\\
E(t) &\equiv \int_{-\infty}^{t} e(t_0,t)N(t_0,t)dt_0, \\
&= \beta e^{-\mu t}\int_{-\infty}^{t} e(t_0,t)e^{\beta t_0}dt_0, \qquad (3.9)
\end{aligned}
$$

where $K(t)$ is the aggregate capital stock and $E(t)$ denotes aggregate consumption expenditures.[6]

For each population aggregate variable $X(t)$, the corresponding quantity per capita is defined by $\tilde{x}(t) = X(t)e^{-nt}$ (see Buiter (1988)). Using this notational convention, we derive in appendix A.2.1 the following laws of motion for per capita expenditures $\tilde{e}(t)$ and per capita capital $\tilde{k}(t)$

$$
\begin{aligned}
\dot{\tilde{e}}(t) &= \left[\frac{\pi(t)}{w(t)a_i} - \delta - \rho\right]\tilde{e(t)} - \beta(\rho+\mu)a_i w(t)\tilde{k}(t), \qquad (3.10)\\
\dot{\tilde{k}}(t) &= \left(\frac{\pi(t)}{w(t)a_i} - \delta - \beta + \mu\right)\tilde{k}(t) + \frac{\tilde{l}}{a_i} - \frac{\tilde{e}(t)}{w(t)a_i}, \qquad (3.11)
\end{aligned}
$$

where analogous equations hold in the foreign region. In contrast to the setting with a constant population size of chapter 2, where the mortality rate only enters the aggregate Euler equation, demographic parameters appear in both laws of motion. In particular, the law of motion of per capita expenditures differs from the individual Euler equation by $-\beta(\rho+\mu)a_i w(t)\tilde{k}(t)$.

[6] Note that aggregate efficiency units of labor are accordingly given by $L(t) = le^{(\beta-\mu)t}$.

Rewriting equation (3.10) as (see also appendix A.2.1)

$$\frac{\dot{\tilde{e}}(t)}{\tilde{e}(t)} = \frac{\dot{e}(t_0,\tau)}{e(t_0,\tau)} - \beta\frac{\tilde{e}(t) - e(t,t)}{\tilde{e}(t)} \tag{3.12}$$

sheds more light on how to explain the emergence of this additional term. As explained in detail in chapter 2 for the case of a constant population size, the difference between individual and per capita consumption expenditure growth is due to the distributional effects of the turnover of generations. At each instant in time a fraction μ of wealthier individuals with high consumption expenditures dies[7] and is replaced by a fraction β of newborns without capital holdings and thus lower consumption expenditures. This continually ongoing process captured by the difference between average consumption expenditures $\tilde{e}(t)$ and consumption expenditures of newborns $e(t,t)$ slows down per capita consumption expenditure growth (aggregate economy average) relative to individual consumption expenditure growth.

Whereas both the birth and the mortality rate strengthen the turnover correction term and thus decrease per capita expenditure growth[8], their effects on the per capita law of motion of capital are of opposite sign. Obviously, it is in fact the population growth rate $n = \beta - \mu$ that enters the per capita law of motion for capital. A higher n decreases per capita accumulation by decreasing the market rate of return on capital. It is not surprising that this last relationship will become crucial when investigating the effects of population growth on agglomeration tendencies. To do so, we first have to determine the equilibrium factor prices resulting out of profit maximization in order to arrive at a dynamic system for expenditures and capital that fully describes the evolution of our economy.

[7]Due to the law of large numbers, the individual probability of dying is equal to the fraction of individuals who die at each instant.

[8]Recall that by equalizing the birth to the death rate, the framework in chapter 2 was not able to differentiate between the impact of varying birth and mortality rates, i.e. only μ appeared in the turnover correction term.

3.2.3 Profit maximization

Profit maximization and perfect competition in the agricultural sector imply that the price of an agricultural good equals its marginal cost. Since labor is perfectly mobile across sectors, the wage rate in the economy is thus pinned down by the price of the agricultural good. Free trade equalizes this price and thus wages across regions as long as each region produces some agricultural output which will be assumed from now on.[9] We thus have

$$w(t) = w^*(t) = 1, \tag{3.13}$$

since the agricultural good has been chosen as the numéraire.

Profit maximization in the monopolistically competitive manufacturing sector[10] yields

$$\begin{aligned} p_m^H(i,t) &= \frac{\sigma}{\sigma-1} w(t) a_m, & (3.14)\\ p_{m,\varphi}^F(i,t) &= p_m^H(i,t)\varphi. & (3.15) \end{aligned}$$

Prices are equal to a constant markup over marginal costs and mill pricing is optimal, i.e. the only difference between prices in the two regions is due to transport costs (see e.g. Baldwin et al. (2003)).

Since there is free entry into the manufacturing sector, pure profits will be driven down to zero. Consequently, the capital rental rate, which represents the fixed costs of each manufacturing firm, is pinned down by the level of operating profits. Using optimal prices given in equations (3.14) and (3.15) together with equations (3.6) and (3.7) and redefining global quantities and regional share variables gives operating profits and thus capital rental rates

[9] See Baldwin (1999) for details on this assumption.

[10] See again chapter 2 and its appendix A.1 for details of the derivations.

as[11]

$$\pi = \left(\frac{\theta_E}{\theta_K + \phi(1-\theta_K)} + \frac{(1-\theta_E)\phi}{\phi\theta_K + 1 - \theta_K}\right)\left(\frac{\xi E^W}{\sigma K^W}\right), \tag{3.16}$$

$$\pi^* = \left(\frac{1-\theta_E}{1-\theta_K + \phi\theta_K} + \frac{\theta_E\phi}{\phi(1-\theta_K) + \theta_K}\right)\left(\frac{\xi E^W}{\sigma K^W}\right), \tag{3.17}$$

where $\phi \equiv \varphi^{1-\sigma}$ measures trade openness between the two regions with $\phi = 0$ indicating prohibitive trade barriers and $\phi = 1$ free trade. World expenditures are defined as $E^W \equiv E + E^*$ and the world capital stock as $K^W \equiv K + K^*$ with θ_K and θ_E referring to the home shares of these quantities.

At the symmetric equilibrium with $\theta_K = 1/2$ and $\theta_E = 1/2$, shifting expenditure to home ($d\theta_E > 0$) raises π and lowers π^* since it increases the home market size. A higher expenditure share therefore promotes agglomeration of capital at home because capital accumulates where the rental rate is higher and decumulates in the other region. Production shifting to home ($d\theta_K > 0$), on the other hand, has the opposite effects because it increases competition in the home market (local competition effect). The relative strength of both forces determines whether agglomeration processes set in in a framework ignoring demographic change. In particular, Baldwin (1999) shows that the pro-agglomerative force dominates for sufficiently high levels of trade openness.

The paper presented in chapter 2 has already shown that an overlapping generation structure with lifetime uncertainty introduces a crucial third force working via the turnover correction term. Since a higher capital stock in one region implies that dying individuals are on average richer, the distributional effects are more severe relative to the region with the lower capital stock. Consequently, the turnover of generations acts as an anti-agglomerative force by reducing relative aggregate consumption expenditure growth of the region with the higher capital stock. The crucial question to be analyzed in the following sections is how the introduction of a nonconstant population size, i.e. $\mu \neq \beta$, affects this linkage between demographic change and agglom-

[11] We ignore time arguments here. Note, moreover, that $\xi = \xi^*$ and $\sigma = \sigma^*$ due to symmetry between regions. For further details of the derivations see again chapter 2 and its appendix A.1.

eration. In particular, we investigate how population aging as represented by declining fertility rates impacts upon the spatial distribution of economic activity by simultaneously taking into account the associated changes in the population growth rate.

3.3 The impact of demographic change on agglomeration

To assess whether and how demographic change impacts upon agglomeration processes we analyze the stability properties of the symmetric equilibrium more thoroughly. If it turns out that this steady state is unstable, then any slight perturbation will lead us away from an equal distribution of capital and expenditures and thus result in agglomeration processes.

3.3.1 Long-run equilibrium

Using the rental rates from equations (3.16) and (3.17) reformulated as functions of the variables $\tilde{e}$, $\tilde{e}^*$, $\tilde{k}$ and $\tilde{k}^*$ (see appendix A.2.2) as well as the equilibrium wages from equation (3.13) in the per capita laws of motion of capital and consumption expenditures (3.10) and (3.11) yields the following four dimensional system[12]

$$\dot{\tilde{e}} = \left[\left(\frac{\tilde{e}}{\tilde{k}+\phi\tilde{k}^*}+\frac{\tilde{e}^*\phi}{\phi\tilde{k}+\tilde{k}^*}\right)\left(\frac{\xi}{a_i\sigma}\right)-\delta-\rho\right]\tilde{e}-\beta(\rho+\mu)a_i\tilde{k}, \tag{3.18}$$

$$\dot{\tilde{e}}^* = \left[\left(\frac{\tilde{e}^*}{\tilde{k}^*+\phi\tilde{k}}+\frac{\tilde{e}\phi}{\phi\tilde{k}^*+\tilde{k}}\right)\left(\frac{\xi}{a_i\sigma}\right)-\delta-\rho\right]\tilde{e}^*-\beta(\rho+\mu)a_i\tilde{k}^*, \tag{3.19}$$

$$\dot{\tilde{k}} = \left[\left(\frac{\tilde{e}}{\tilde{k}+\phi\tilde{k}^*}+\frac{\tilde{e}^*\phi}{\phi\tilde{k}+\tilde{k}^*}\right)\left(\frac{\xi}{a_i\sigma}\right)-\delta-\beta+\mu\right]\tilde{k}+\frac{\tilde{l}}{a_i}-\frac{\tilde{e}}{a_i}, \tag{3.20}$$

$$\dot{\tilde{k}}^* = \left[\left(\frac{\tilde{e}^*}{\tilde{k}^*+\phi\tilde{k}}+\frac{\tilde{e}\phi}{\phi\tilde{k}^*+\tilde{k}}\right)\left(\frac{\xi}{a_i\sigma}\right)-\delta-\beta+\mu\right]\tilde{k}^*+\frac{\tilde{l}}{a_i}-\frac{\tilde{e}^*}{a_i}. \tag{3.21}$$

These four differential equations in the variables $\tilde{e}$, $\tilde{e}^*$, $\tilde{k}$ and $\tilde{k}^*$ fully describe the dynamics of our NEG model with overlapping generations and a nonconstant population size. Note that they nest both the Baldwin (1999) set-up with $\beta = \mu = 0$ as well as the framework of chapter 2 with $\beta = \mu > 0$ as special cases. The latter will turn out to be particularly useful when we try to isolate the effect of population growth on agglomeration tendencies.

Inserting the symmetric outcome $\tilde{e} = \tilde{e}^*$ and $\tilde{k} = \tilde{k}^*$ into the above system indeed reveals that it is a steady state with the equilibrium values given by[13]

$$\bar{\tilde{e}} = \frac{\tilde{l}\sigma\left((\delta-\mu)\sqrt{\sigma}\,(A+B)+\beta\left((\delta+\rho-2\mu)\sigma+2(\mu+\rho)\xi+A\sqrt{\sigma}\right)\right)}{2((\delta-\mu)\sigma+\beta\xi)((\beta+\delta+\rho)\sigma-(\mu+\rho)\xi)}, \tag{3.22}$$

$$\bar{\tilde{k}} = \frac{\tilde{l}\left((\rho\sigma-\sqrt{\sigma}A)(\sigma-\xi)+2\sigma\xi(\beta-\mu)+\delta\sigma(\sigma+\xi)\right)}{2a_i((\delta-\mu)\sigma+\beta\xi)((\beta+\delta+\rho)\sigma-(\mu+\rho)\xi)}, \tag{3.23}$$

where $A \equiv \sqrt{\sigma(\delta+\rho)^2+4\beta(\mu+\rho)\xi}$ and $B \equiv (\delta+\rho)\sqrt{\sigma}$.

[12] We again suppress time arguments here. Note, moreover, that we have $\tilde{l} = \tilde{l}^*$, $\mu = \mu^*$, $\beta = \beta^*$ $a_i = a_i^*$, $\delta = \delta^*$ and $\rho = \rho^*$ due to symmetry between regions.

[13] These and most of the following results were derived with Mathematica. The corresponding files are available from the authors upon request. Note also that we restrict attention to the economically meaningful solution pair, i.e. where consumption and capital is positive for plausible parameter values.

3.3.2 Formal stability analysis

To analyze the stability properties of the symmetric equilibrium we first linearize the non-linear dynamic system given in equations (3.18), (3.19), (3.20) and (3.21) around the symmetric equilibrium (3.22) and (3.23), and then evaluate the eigenvalues of the corresponding 4×4 Jacobian matrix

$$J_{sym} = \begin{pmatrix} J_1 & J_2 \\ J_3 & J_4 \end{pmatrix}, \tag{3.24}$$

where the four symmetric 2×2 sub-matrices J_i for $i = 1, \ldots, 4$ are given in appendix A.2.3 (for this classical approach of stability analysis see e.g. the appendix on mathematical methods in Barro and Sala-i-Martin (2004)). Solving the characteristic equation yields the following four eigenvalues

$$\begin{aligned}
eig1 &= \frac{1}{2}(r_1 - \sqrt{rad_1}), &(3.25)\\
eig2 &= \frac{1}{2}(r_1 + \sqrt{rad_1}), &(3.26)\\
eig3 &= \frac{1}{2(\phi+1)^2\sqrt{\sigma}}(r_2 - \sqrt{rad_2}), &(3.27)\\
eig4 &= \frac{1}{2(\phi+1)^2\sqrt{\sigma}}(r_2 + \sqrt{rad_2}), &(3.28)
\end{aligned}$$

where

$$\begin{aligned}
r_1 &\equiv \frac{A}{\sqrt{\sigma}} - \beta - \delta + \mu,\\
rad_1 &\equiv \left(\frac{A}{\sqrt{\sigma}} + \beta + \delta - \mu\right)^2 + \frac{(\sigma - \xi)\left((A+B)^2 + 4\beta(\mu+\rho)\xi\right)}{\sigma\xi},\\
r_2 &\equiv \sqrt{\sigma}\left(\mu - \beta(\phi+1)^2 - \delta\left(2\phi^2+\phi+1\right) + \phi(\mu(\phi+2) - \rho(\phi-1))\right)\\
&\quad +A(3\phi+1),\\
rad_2 &\equiv \left(A(\phi-1) + \left(\mu(\phi+1)^2 - \beta(\phi+1)^2 + \delta(\phi-1) + \phi(\phi+3)\rho\right)\sqrt{\sigma}\right)^2 + \\
&\quad \frac{(\phi+1)((\phi+1)\sigma + (\phi-1)\xi)\left(4\beta(\mu+\rho)\xi(\phi+1)^2 + (\phi-1)^2(A+B)^2\right)}{\xi}.
\end{aligned}$$

The nature and signs of these eigenvalues fully characterize the system's local dynamics around the symmetric equilibrium. First it is easily established that all four eigenvalues are real since both rad_1 and rad_2 are nonnegative for all possible parameter values.[14] Turning to the signs of the eigenvalues, the analysis is more involved. Indeed, the above eigenvalues can be used to assess the effects of demographic change on the stability properties of the symmetric equilibrium for all three cases of growing population, i.e. $\beta > \mu$, shrinking population, i.e. $\beta < \mu$, and a constant population size, i.e. $\beta = \mu$. The last scenario has already been investigated in the paper presented in chapter 2 showing that in this case eigenvalue three is decisive for the stability properties of the symmetric equilibrium. In particular, the system is saddle path stable only for parameter ranges that yield a negative eigenvalue three.

For the case of positive population growth, i.e. $\beta > \mu$, eigenvalue three retains its crucial role, which becomes clear when checking the signs of the remaining other eigenvalues. Note first, that the sign of r_1 is ambiguous.[15] As far as eigenvalue 1 is concerned, it is for sure nonpositive as long as $r_1 < 0$. For the case of $r_1 > 0$, we can show that $r_1^2 < rad_1$ implying that eigenvalue 1 never gets positive. The last inequality moreover yields that eigenvalue 2, on the other hand, is always nonnegative.[16] Finally, turning to the sign of eigenvalue 4, note first that r_2 again does not have an unambiguous sign.[17] For $r_2 \geq 0$, eigenvalue 4 is for sure nonnegative. Using Mathematica, it is, however, possible to show that $r_2 + \sqrt{rad_2} > 0$ even for negative r_2 implying that eigenvalue 4 is nonnegative for all possible parameter ranges. Summarizing, we have two positive and one negative eigenvalue. Both for the case of constant population size and population growth the symmetric equilibrium thus becomes unstable for parameter values, and in particular birth and mortality rates, that yield a positive eigenvalue 3.

For the case of population shrinking, i.e. $\mu > \beta$, only eigenvalue 2 and 4 have unambiguous signs. Note first, that $r_1 > 0$ if $\mu > \beta$ which immediately

[14] Recall the parameter ranges $\mu > 0$, $\beta > 0$, $\delta > 0$, $\sigma > 1$, $\rho > 0$, $0 < \xi < 1$ and $0 \leq \phi \leq 1$ which also imply that $A > 0$ and $B > 0$. In particular, note that $\sigma > \xi$.

[15] In particular, r_1 becomes negative for sufficiently high β.

[16] For $r_1 > 0$ this follows trivially, $r_1^2 < rad_1$ also shows it for $r_1 < 0$.

[17] In particular, r_2 becomes negative for sufficiently high β.

proves the nonnegativity of eigenvalue 2. Similarly, it can be shown that $r_2 > 0$ if $\mu > \beta$[18] implying that eigenvalue 4 is always nonnegative as well. Thus, for the case of population shrinking, agglomeration processes will set in for birth or mortality rates for which at least either eigenvalue 1 or eigenvalue 3 is positive.

Since the above findings illustrate that demographic change, i.e. variations in β and μ, can only influence the stability properties of the symmetric equilibrium via eigenvalue 3 for the case of $\beta \geq \mu$ and eigenvalues 1 and 3 for $\beta < \mu$, it is immediate to investigate them more thoroughly. Figures 3.1 and 3.2, which plot the contour lines of eigenvalues 1 and 3 for different birth and mortality rates[19], show that both of them switch their sign depending on the economies' demographic parameters. For the case of population shrinking, i.e. $\beta < \mu$, note moreover that for all combinations of β and μ for which eigenvalue 3 switches sign, eigenvalue 1 is still negative. These observations immediately result in the following proposition.

Proposition 3.1. *The possibility of agglomeration crucially hinges on the economies' demographic properties. In particular, both the birth and the mortality rate are decisive for the stability properties of the symmetric equilibrium.*

Proof. *S*ee figures 3.1 and 3.2 and above arguing on the eigenvalues' signs. □

Proposition 3.1 extends the result presented in chapter 2 that with $\beta = \mu$ agglomeration processes crucially depend on the mortality rate. In a setting with nonconstant population size it is both the birth and the mortality

[18] This follows from rewriting

$$r_2 = \underbrace{A(3\phi+1) + \sqrt{\sigma}\left(-\delta\left(2\phi^2+\phi+1\right) + \phi\rho(1-\phi)\right)}_{term\ 1} + \underbrace{\sqrt{\sigma}\left(-\beta(\phi+1)^2 + \mu + \phi\mu(\phi+2)\right)}_{term\ 2}$$

and noting that term 1 is nonnegative for all parameter ranges (see chapter 2 for details) while term 2 is nonnegative as long as $\mu \geq \beta$.

[19] Figures 3.1 and 3.2 are plotted for $\delta = 0.05$, $\rho = 0.015$, $\xi = 0.3$, $\sigma = 4$ and $\phi = 0.98$. Note the we use different ranges of μ and β to focus on the parameter region where the eigenvalues switch sign.

Figure 3.1: Contour plot of eigenvalue 3

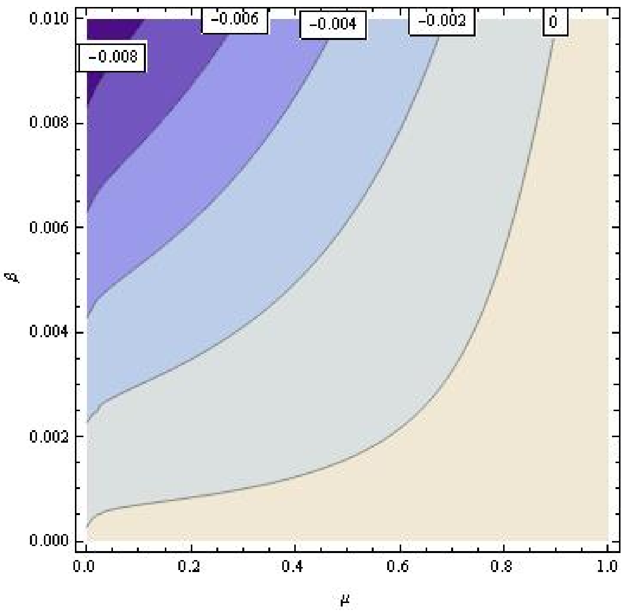
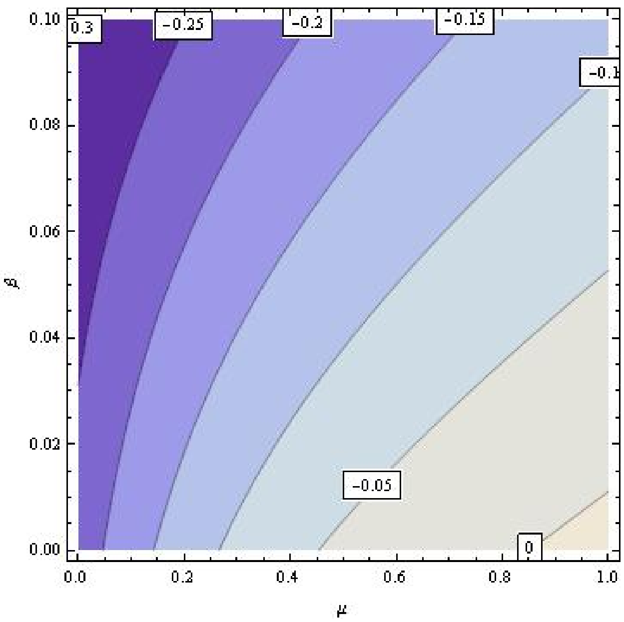


Figure 3.2: Contour plot of eigenvalue 1



rate that determine the stability properties of the symmetric equilibrium. Consequently, in order to fully understand the linkage between demographic change and agglomeration, we must investigate how changes in either of them impact upon agglomeration tendencies. The next subsection precisely deals with this question.

3.3.3 Agglomeration processes in aging societies

In the framework with constant population size of chapter 2, changes in μ and thus β only change the population age structure. With $\beta \neq \mu$, varying either of them, however, also changes the population growth rate. In particular, declines in the birth rate imply both population aging and a lower population growth rate, whereas changes in the mortality rate leave the mean age unchanged[20] and only alter population growth (see chapter 7 in Preston et al. (2001)). In order to assess agglomeration tendencies in aging societies we thus focus on the case of declining birth rates. Moreover, by additionally investigating how the mortality rate impacts upon the stability properties of the symmetric equilibrium, we gain important insights with respect to the population growth based channel on agglomeration.

Figure 3.1 has already indicated the qualitative effect of declining birth and mortality rates on agglomeration processes. Eigenvalue 3 *decreases* in the birth rate and *increases* in the mortality rate. Figures 3.3 and 3.4[21] confirm this observation by plotting eigenvalue 3 as a function of trade openness for different β and μ. Only for sufficiently low β or high μ we can find levels of economic integration for which eigenvalue 3 gets positive and the symmetric equilibrium becomes unstable.[22]

[20]This can be easily shown by noting that the proportion of the population at age $t - t_0$ is given by $\frac{N(t_0,t)}{N(t)} = \beta e^{-\beta(t-t_0)}$ which is independent of μ. Intuitively, a lower mortality rate on the one hand decreases the tendency to die more quickly and thus makes the population older but on the other hand also increases the population growth rate which exactly offsets the individual aging effect by making the population younger.

[21]Figures 3.3 and 3.4 are plotted for $\delta = 0.05$, $\rho = 0.015$, $\xi = 0.3$ and $\sigma = 4$. For figure 3.3 we moreover fix $\mu = 0.001$ and for figure 3.4 we set $\beta = 0.001$.

[22]This also holds for the case of population shrinking, since, as figures 3.1 and 3.2 indicate, the parameter range within which eigenvalue 1 is positive is a subset of the corresponding one of eigenvalue 3. Thus eigenvalue 3 is decisive for the stability properties

Figure 3.3: Eigenvalue 3 as a function of ϕ for varying β

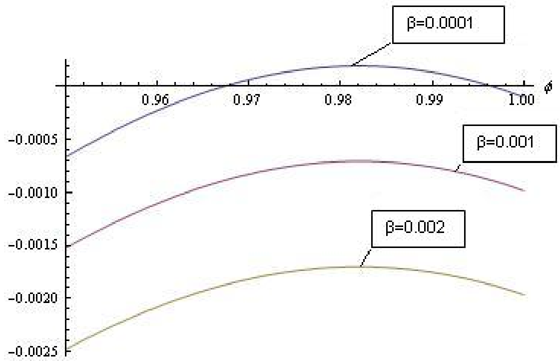
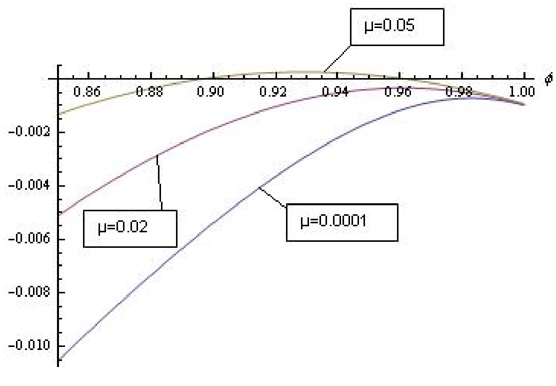


Figure 3.4: Eigenvalue 3 as a function of ϕ for varying μ



Declining fertility rates leading to an older population age structure and lower population growth rates at the same time thus destabilize the symmetric equilibrium, i.e. population aging strengthens agglomeration processes. This is in line with the main finding of chapter 2 that with $\beta = \mu$, increasing μ (and thus also β) above zero (which leads to a younger population structure while leaving the population growth rate unchanged) and thus allowing for a turnover of generations, considerably stabilizes the symmetric equilibrium such that for plausible demographic structures agglomeration processes do not set in. This last observation turns out to also hold in the present setting with nonconstant population size: for a plausible mortality rate of $\mu = 0.0125$ resulting in a life expectancy of 80 years[23] agglomeration processes never take place as long as $\beta > 0.00051$.[24] A fertility rate below such a value would clearly be at odds with reality.[25]

While the birth rate mimics the effects of demographic change on agglomeration found in a setting with constant population size, the mortality rate's impact is completely opposite. In particular, lower mortality rates decrease eigenvalue 3 and thus make the symmetric equilibrium more stable. Since changes in the mortality and birth rate affect population growth differently, it is immediate to suspect a population growth based channel to augment the effects of the turnover of generations, which have been identified in chapter 2 as the only link between demographic change and agglomeration in a setting with constant population size. Indeed, equation (3.10) clearly shows that the turnover correction term increases in both demographic parameters implying that declining μ and β should both strengthen agglomeration processes. Lower birth rates, however, decrease the population growth rate, while lower mortality rates increase it. The above findings thus suggest that population growth rate declines as resulting from lower β act as an additional agglomeration force, while population growth rate increases due to lower μ stabilize the symmetric equilibrium. For declining birth rates, the population growth

of the symmetric equilibrium even for the case of a shrinking population.

[23]Since the probability of death during each year equals μ, average life expectancy is $\frac{1}{\mu}$.

[24]We again set $\delta = 0.05$, $\rho = 0.015$, $\xi = 0.3$, $\sigma = 4$ for this calculation.

[25]A birth rate of $\beta = 0.00051$ would imply 0.00051 children per individual.

based channel simply reinforces the turnover effect, while it counteracts and even dominates the turnover channel for the case of declining mortality rates. Moreover, by recalling from chapter 2 that contemporaneous increases in the birth and mortality rate increase stability, we can also conclude that the birth rate effect dominates the mortality rate effect for the case of constant population size. The next subsection is devoted to verifying this additional population growth based channel on agglomeration processes.

3.3.4 The population growth effect

Since the analysis in chapter 2 has shown that with $\beta = \mu$ demographic change only affects the symmetric equilibrium's stability properties via the turnover channel, it is possible to isolate the effects of changes in the population growth rate on agglomeration tendencies by comparing the instability regions of constant to those of varying population size. This is achieved in figure 3.5 which plots the instability region as a function of the birth rate for the case of zero and the case of positive or negative population growth.[26] In particular, the borders of the instability region are given by the critical levels of economic integration ϕ_{break1} and ϕ_{break2} (see chapter 2 for details) within which eigenvalue 3 is positive and thus agglomeration processes may set in.[27]

Figure 3.5 confirms our above mentioned presumptions about the effects of changes in the population growth rate. We see that for positive population growth, i.e. $\beta > 0.003$, the instability region gets smaller compared to the case of constant population size, whereas it increases for the case of population shrinking, i.e. $\beta < 0.003$. This identifies population growth as an additional dispersion force fostering a more equal distribution of productive factors and explaining the differential impact of β and μ on agglomeration processes.

[26] Note that we now use slightly different parameter values, i.e. $\delta = 0.1$, $\rho = 0.1$, $\xi = 0.4$, $\sigma = 2$, to increase the visibility of the population growth effect. For the varying population size case, we fix μ at 0.003.

[27] Recall that even for the case of population shrinking, eigenvalue 3 is decisive for the stability properties of the symmetric equilibrium since the instability region implicitly defined by eigenvalue 1 is a subset of the instability region defined by eigenvalue 3.

Figure 3.5: Instability region (ϕ_{break1} and ϕ_{break2}) for constant and varying population size

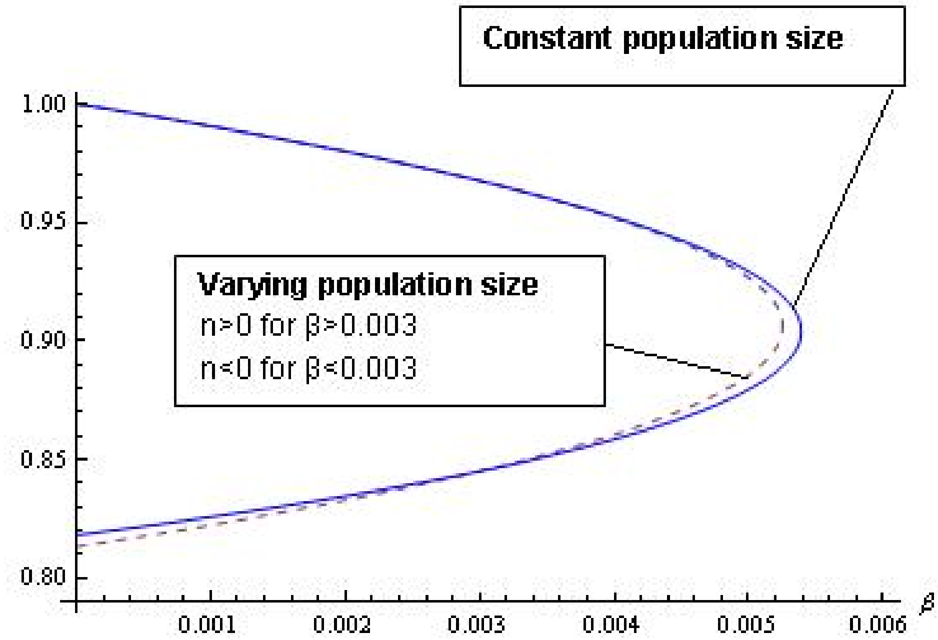


Figure 3.5 also suggests one qualification to the main finding of the paper presented in chapter 2 that agglomeration processes are considerably reduced for positive birth and mortality rates. Indeed, for sufficiently strong population shrinking, i.e. $\mu > \beta > 0$, the instability region is bigger than for the case of $\mu = \beta = 0$ (as represented by the solid line for $\beta = 0$). Only for the case of a nonnegative population growth rate agglomeration tendencies are thus for sure reduced compared to a setting ignoring demographic structures.

To gain some economic intuition about the particular channel via which population growth impacts upon agglomeration processes, it is useful to reconsider the more informal way of checking the stability properties of the symmetric equilibrium followed by Baldwin (1999). This informal way is based on investigating how an exogenous perturbation of the home share of capital, θ_K, influences the home capital rental rate relative to the foreign capital rental rate. A positive impact implies instability as capital accumulation would be fostered in the home relative to the foreign region.

Overall, there are four channels via which capital shifting changes the capital rental rate. The first two are the standard anti-agglomerative local competition effect and the pro-agglomerative demand-linked circular causality first introduced by Baldwin (1999). The latter shows that a higher share of capital in one region increases, via higher wealth levels, expenditures and thus operating profits, i.e. capital rental rates, which speeds up capital accumulation. The former, on the other hand, captures the negative impact of agglomeration of capital, i.e. firms, on capital rental rates due to more severe competition (see section 3.2.3).

In a setting allowing for demographic change in terms of both population aging and varying population size, two additional forces linked to the economies' demographic parameters β and μ appear. First, there is the anti-agglomerative turnover effect (see again section 3.2.3). An exogenous rise in the home capital share increases wealth and thus expenditure levels of individuals being currently alive in the home region relative to foreign-based individuals. The negative distributional effects on per capita expenditures resulting from birth and death, i.e. the replacement of dying individuals by newborns whose consumption expenditures are lower since they have zero wealth levels, are thus more pronounced in the home region. This, in turn, decreases the home expenditure share and therefore relative profitability and capital rental rates.

Both higher birth and mortality rates strengthen this first force between demography and agglomeration, which is in sharp contrast to the population growth based channel that additionally affects the linkage between capital shifting and the capital rental for $\beta \neq \mu$. As already indicated in section 3.2.2, the impact of β and μ in the law of motion of per capita capital is of opposite sign. In particular, equation (3.11) clearly shows that the market rate of return on capital $\frac{\pi}{w(t)a_i} - \delta - \beta + \mu$ depends negatively on $n \equiv \beta - \mu$. The wealth increase due to higher capital income resulting from capital shifting is thus less pronounced the larger is n. This decreased impact upon wealth translates into lower expenditure increases and finally lower increases in the capital rental rate which precisely explains the anti-agglomerative effect of population growth.

3.4 Concluding remarks

The model presented in this paper sheds more light on the linkage between demographic change and agglomeration. In particular, it analyzes agglomeration processes in aging societies and identifies the channels via which demographic developments influence the spatial distribution of economic activity. We extend the framework of chapter 2, that already incorporates an overlapping generation structure and lifetime uncertainty in the constructed capital model of Baldwin (1999), by additionally allowing for a nonconstant population size. Changes in either the birth or mortality rate are thus accompanied by changes in the population growth rate. Since population growth per se acts as an important dispersion force and fertility and mortality have opposite effects on this rate, their impacts with respect to the spatial distribution of economic activity also differ. In particular, declining fertility rates strengthen agglomeration processes while declining mortality rates weaken them.

Our framework is suited to assess the possibility of agglomeration tendencies for various demographic developments. Most relevant for industrialized countries is probably the one of declining fertility rates leading to both population aging and slower population growth. We find that in such a situation agglomeration tendencies are strengthened but still weaker than in a setting that fully ignores demographic structures. In particular, our calibrations suggest that also these countries are currently far away from a situation where catastrophic agglomeration is likely to occur.

Despite the fact that allowing for population growth constitutes one further step toward a more comprehensive understanding of the interrelations between demographic change and agglomeration, many issues still remain open. The above findings e.g. indicate the need for analyzing the combined effect of varying birth and mortality rates. In particular, having the opposite effects of birth and mortality rates on agglomeration processes in mind, it is immediate to ask what happens if an economy faces both declining birth and mortality rates. Our results suggest that the answer to this question will crucially depend on the resulting change in the population growth rate and requires a direct comparison of the quantitative effects of changing birth and mortality rates.

Finally, recognizing the tight link between the spatial distribution of economy activity and economic growth perspectives, it is worth investigating how demographic structures impact upon regional growth rates. Since demography has been shown to be of crucial importance for agglomeration, it is immediate to also investigate its effects on the growth impacts of such concentration tendencies. This requires a NEG framework that allows for both demographic change and endogenous long-run growth, a task being on the top of our research agenda.

Chapter 4

Demographic change, growth and agglomeration

4.1 Introduction

Recently, there has been wide interest in the "economics" of population aging (see e.g. The Economist (2009)). Demographic change has crucial consequences for economic behavior and development along various dimensions, ranging from such diverse aspects as the determination of aggregate labor productivity levels to retirement issues. In particular, it highlights that consumption decisions and incentives to invest in future growth prospects vary over the life-cycle. The latter has important implications for long-run economic growth perspectives. The former, on the other hand, should be decisive for the location of economic activity if one takes into account the mutual dependence between the spatial distribution of production and demand developments as emphasized in the New Economic Geography (NEG) literature (see e.g. Baldwin et al. (2003) for an overview). Both growth and agglomeration processes are, however, themselves interlinked. It is thus necessary not only to investigate the isolated effects of age-dependent heterogeneity on these two issues but to also consider whether and how lifetime uncertainty impacts upon the linkage between growth and the spatial distribution of economic activity.

The relationship between growth and agglomeration processes has been studied extensively in recent years. By claiming that "agglomeration can be thought as the territorial counterpart of economic growth" Fujita and Thisse (2002) emphasize that the emergence of concentration of economic activity is traditionally associated with modern economic growth. The positive link between growth and spatial agglomeration is mainly attributed to the fact that technological spillovers, being the engines of endogenous growth, are localized. Consequently, being close to innovation clusters should have positive effects on productivity and growth perspectives. Such considerations have led to the development of integrated frameworks that combine (endogenous) growth features with NEG models to study the joint process of creation and location of economic activity (see e.g. Martin and Ottaviano (1999), Martin and Ottaviano (2001), Baldwin and Forslid (2000) or Baldwin et al. (2001)).

By introducing endogenous capital in his constructed capital model Baldwin (1999) was the first one allowing for growth features in a NEG framework. The absence of capital mobility in his exogenous growth model implies a demand-linked circular causality fostering agglomeration. Higher capital accumulation increases income and expenditures in the respective region which raises capital rental rates further and thus fosters capital accumulation even more. Baldwin et al. (2001) extended this framework by additionally allowing for learning externalities in the capital creation sector. In their endogenous two-region growth model both the above demand-linked agglomeration force and a growth-linked circular causality strengthen concentration of economic activity. The latter crucially depends on the localized nature of spillovers. As long as spillovers are not fully globalized, spatial concentration of capital in one region implies a lower cost of capital creation and thus speeds up accumulation relative to the other region. Growth itself can thus lead to catastrophic agglomeration of economic activity. Moreover, with localized learning spillovers, agglomeration also affects long-run growth perspectives. In particular, spatial agglomeration is conducive for growth since it decreases costs in the capital accumulation sector.

The traditional line of investigation of all these NEG approaches to growth focuses, however, on the joint consequences of increased economic integration on growth and agglomeration processes while ignoring any potential effect of demographic change. In particular, despite the fact that all these models are intrinsically dynamic, the impact of an economy's demographic structure, and in particular of life-cycle decisions, on consumption and saving patterns, which themselves play a crucial role for agglomeration forces, is completely ignored. The papers presented in chapters 2 and 3 have clearly revealed that this limited perspective misses important mechanism that are fundamental for the location of productive factors. Most importantly, it was shown that incorporating an overlapping generation structure and lifetime uncertainty into the constructed capital of Baldwin (1999) introduces an additional dispersion force that considerably reduces the possibility of agglomeration processes. Prettner (2009), on the other hand, provides evidence for a positive effect of life expectancy on long-run economic growth in an endogenous growth framework in the spirit of Romer (1990).

This paper merges both strands of analysis by generalizing the NEG model with learning spillovers of Baldwin et al. (2001) to allow for an overlapping generation structure with individuals that face a positive probability of death and differ with respect to age. In doing so, the main emphasis is twofold. First, the impact of lifetime uncertainty on long-run growth perspectives is investigated with a view to evaluating the pro-growth effect of spatial concentration in a setting accounting for demographic structures. Second, the stability properties of the symmetric equilibrium with respect to varying mortality rates are analyzed. Here, attention is also paid to the impact of lifetime uncertainty on history-versus-expectations considerations.[1]

What I show is that, consistent with the results presented in chapters 2 and 3, the turnover of generations acts as a dispersion force that dampens the pro-agglomerative growth-linked circular causality being present as long as interregional learning spillovers are not fully perfect. Moreover, the model reveals that lifetime uncertainty has important implications for equilibrium selection. An increase in the mortality rate reduces the possibility that expectations rather than history, represented by initial conditions, are

[1]This debate was initiated by Krugman (1991b) and deals with the question of equilibrium selection in a setting with multiple equilibria (see Baldwin (2001) for a nice overview).

decisive with respect to the question in which region agglomeration might take place. Finally, comparing the negative effect of lifetime uncertainty on equilibrium growth rates in the symmetric and core-periphery outcome, shows that, in sharp contrast to existing NEG growth models with localized knowledge spillovers, spatial agglomeration is not necessarily conducive to growth in a setting accounting for demographic structures.

The remainder of this paper is structured into four sections. The following section 4.2 presents the model framework and derives the dynamic system describing the evolution of the economy. Section 4.3 characterizes the long-run equilibria and investigates the impact of lifetime uncertainty on equilibrium growth rates. The joint effect of demography and spillovers on the stability properties of the symmetric equilibrium is analyzed in section 4.4, which also focuses on the role of mortality for history-versus-expectations considerations. Finally, section 4.5 contains concluding remarks and indicates further lines of research.

4.2 The model

This section describes how a notion of learning as in Baldwin et al. (2001) can be integrated into the generalized constructed capital model of chapter 2 to arrive at a NEG framework featuring both endogenous growth and demographic change.

4.2.1 Basic structure and underlying assumptions

Consider a world economy with two symmetric regions or countries, denoted by H for home and F for foreign[2], with identical production technologies, trade costs, preferences of individuals, labor endowments and demographic structures. Each region has three economic sectors (agriculture, manufactur-

[2]If further distinction is needed, foreign variables are moreover indicated by an asterisk. In particular, the superscript F denotes that a good was *produced* in the foreign region, whereas the asterisk indicates that it is *consumed* in the foreign region. In what follows, emphasis will be on the home region. The corresponding expressions for the foreign region can be derived by symmetry.

ing and investment) with two immobile factors (labor L and capital K) at their disposal.

Technology

The homogeneous agricultural good, z, is produced in a perfectly competitive market under constant returns to scale using labor as the only input with, by choice of units, an input coefficient of one. It can be traded between the two regions without any cost.

Manufacturing firms are modeled as in the monopolistic competition framework of Dixit and Stiglitz (1977) and thus supply horizontally differentiated varieties, m. In contrast to the agricultural good, trade of manufactures involves iceberg transport costs such that $\varphi \geq 1$ units of the differentiated good have to be shipped in order to sell one unit abroad (see e.g. Baldwin et al. (2003)). Each variety is produced with one unit of capital as fixed input and labor as the variable production factor where a_m represents the unit input coefficient for efficiency units of labor. Firms thus face an increasing returns to scale production technology with an associated cost function $\pi(t) + w(t)a_m Y_m(i,t)$, where $\pi(t)$ is the capital rental rate representing the fixed cost, w(t) is the wage per efficiency unit of labor and $Y_m(i,t)$ is total output of one manufacturing good producer. Since each variety exactly requires one unit of capital, a continuum of varieties $i \in (0, K(t)]$ is produced at home, whereas a continuum of varieties $j \in (0, K^*(t)]$ is manufactured in the foreign region.

In the perfectly competitive investment (or innovation) sector, capital is produced using labor as the only input with an input coefficient of $a_i(t)$. Capital is viewed here as new knowledge embedded in an interregionally immobile manufacturing facility. Wages in this sector are being paid out of the individuals' savings. To endogenize long-run growth in this framework, a sector-wide learning curve is modeled by assuming that the marginal cost of producing new capital, $G(t) \equiv a_i(t)w(t)$, declines as the sector's cumulative output rises.[3] Specifically,

$$a_i(t) = \frac{1}{K(t) + \eta K^*(t)}, \tag{4.1}$$

where $0 \leq \eta \leq 1$ determines the degree of internationalization of learning effects[4] with $\eta = 0$ denoting purely localized knowledge spillovers and $\eta = 1$ corresponding to the case of global learning effects. As long as $\eta < 1$, the costs of producing new capital units in each region thus depend on the interregional distribution of capital. The foreign technology is isomorphic with $a_i^*(t) = \frac{1}{K^*(t)+\eta K(t)}$. Following Romer (1990), there is no capital depreciation.

Demographic structure and preferences

As far as the demographic structure of our model economy is concerned, this paper closely follows the framework of chapter 2 by adopting the overlapping generation structure of Blanchard (1985). At each point in time, $\tau \in [0, \infty)$, a large cohort consisting of new individuals is born. These newborns receive no bequests and thus start their lives without any wealth. The size of this cohort is $N(\tau, \tau) = \mu N(\tau)$, where $\mu > 0$ is the constant birth rate and $N(\tau) \equiv \int_{-\infty}^{\tau} N(t_0, \tau) dt_0$ is total population at time τ with $N(t_0, \tau)$ denoting the size of the cohort born at t_0 for any given point in time τ.[5] Consequently, cohorts can be distinguished by the birth date t_0 of their members. Since there is no heterogeneity between members of the same cohort, each cohort can be described by one representative individual, who inelastically supplies her efficiency units of labor on the labor market with perfect mobility across sectors but immobility between regions. Each individual faces

[3] Romer (1990) e.g. rationalizes this assumption by referring to the non-rival nature of knowledge.

[4] New capital units can be thus viewed as having two distinct components. On the one hand, a new capital unit represents private knowledge of how to produce a new variety, which can be sold in the form of a patent to a manufacturing firm. In this sense capital is interregionally immobile. On the other hand, however, it also contains public knowledge since it makes it easier to produce further capital units (imperfectly mobile spillover component).

[5] In what follows the first time index of a variable will refer to the birth date, whereas the second will indicate a certain point in time.

lifetime uncertainty, i.e. her time of death is stochastic with an exponential probability density function. In particular, the instantaneous probability of death of each individual is also given by the age independent parameter μ. This implies that population size is constant and can be normalized to one.[6] Finally, as in Yaari (1965), a perfect life-insurance company offers actuarial notes, which can be bought or sold by each individual and are canceled upon the individual's death.

The overlapping generation structure implies that the overall economy does not feature *one* single representative individual. It is thus necessary to aggregate over the cohorts to arrive at aggregate variables, e.g. aggregate capital stock of the economy at a certain point in time t is defined as

$$K(t) \equiv \int_{-\infty}^{t} k(t_0, t) N(t_0, t) dt_0, \tag{4.2}$$

where $k(t_0, t)$ represents the individual capital stock.

Preferences over the agricultural good and a CES composite of the manufacturing varieties are Cobb-Douglas. In particular, expected lifetime utility of a representative individual of cohort t_0 at time t_0[7] is given by

$$U(t_0, t_0) = \int_{t_0}^{\infty} e^{-(\rho+\mu)(\tau-t_0)} \ln\left[(c_z(t_0, \tau))^{1-\xi}(c_m^{agg}(t_0, \tau))^{\xi}\right] d\tau, \tag{4.3}$$

where $\rho > 0$ is the pure rate of time preference, $0 < \xi < 1$ is the manufac-

[6] From now on, we will refer to μ as the mortality rate. Note, however, that μ equivalently represents the birth rate, e.g. demographic change as captured by variations in μ means that both the mortality and the birth rate change by the same amount such that population size remains constant. This also implies that we restrict attention to changes in the population age structure (lower μ implies population aging) while neglecting variations in the population growth rate due to demographic change. In particular, emphasis is put on comparing a situation that fully ignores demographic structures, i.e. where $\mu = 0$, to one that allows for them by considering nonzero mortality rates.

[7] Equation (4.3) can be easily derived by calculating expected lifetime utility where the date of death is a random variable with an exponential probability density function parameterized by a constant instantaneous mortality rate μ.

turing share of consumption and

$$c_m^{agg}(t_0,\tau) \equiv \left[\int_0^{K(\tau)} \left(c_m^H(i,t_0,\tau)\right)^{\frac{\sigma-1}{\sigma}} di + \int_0^{K^*(\tau)} \left(c_m^F(j,t_0,\tau)\right)^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}}$$

represents consumption of the CES composite with $\sigma > 1$ denoting the elasticity of substitution between varieties.

Capital accumulation

As outlined above, individual savings, defined as income minus consumption expenditures, are converted into capital in the investment sector. The wealth constraint of a representative individual can thus be written as

$$\dot{k}(t_0,\tau) = \frac{w(\tau)l + \pi(\tau)k(t_0,\tau) - e(t_0,\tau)}{G(\tau)} + \mu k(t_0,\tau), \tag{4.4}$$

where l refers to the efficiency units of labor an individual supplies and $e(t_0,\tau)$ are individual total expenditures for consumption defined as

$$\begin{aligned} e(t_0,\tau) \ \equiv \ & p_z(\tau)c_z(t_0,\tau) + \int_0^{K(\tau)} p_m^H(i,\tau)c_m^H(i,t_0,\tau)di + \\ & \int_0^{K^*(\tau)} p_{m,\varphi}^F(j,\tau)c_m^F(j,t_0,\tau)dj. \end{aligned}$$

Here $p_z(\tau)$ is the price of the agricultural good, $p_m^H(i,\tau)$ the price of a manufactured variety produced at home and $p_{m,\varphi}^F(j,\tau)$ the price of a manufactured variety produced abroad with the subscript φ indicating the dependence on transport costs.

The particular law of motion for capital given in equation (4.4) is based on the full insurance result of Yaari (1965) implying that all individuals only hold their wealth in the form of actuarial notes, i.e. each individual itself first converts her savings into capital and then leaves it to the insurance company. Therefore, the market rate of return on capital, $\frac{\pi(\tau)}{G(\tau)}$, has to be augmented by μ to obtain the fair rate on actuarial notes (see Yaari (1965)).

Using the demographic assumptions described above to substitute out

$N(t_0, t)$ in equation (4.2), then differentiating with respect to t and substituting for $\dot{k}(t_0, t)$ from equation (4.4) and for $a_i(t)$ from equation (4.1) yields the aggregate law of motion of capital[8]

$$\dot{K}(t) = [\pi(t)K(t) + w(t)L - E(t)]\,\frac{K(t) + \eta K^*(t)}{w(t)}, \tag{4.5}$$

where aggregate consumption expenditures, $E(t)$, are analogously defined to the aggregate capital stock in equation (4.2).[9] As outlined in chapter 2, the aggregate capital accumulation equation does not feature the mortality rate μ anymore. This is in sharp contrast to the law of motion for individual capital and captures the fact that $\mu K(t)$ does not represent aggregate capital accumulation but is a transfer - via the life insurance company - from individuals who died to those who survived. Similarly, the corresponding law of motion for the foreign capital stock is given by[10]

$$\dot{K^*}(t) = [\pi^*(t)K^*(t) + w^*(t)L - E^*(t)]\,\frac{K^*(t) + \eta K(t)}{w^*(t)}. \tag{4.6}$$

4.2.2 Short-run equilibrium

Analogously to Baldwin et al. (2001), consumers maximize utility, firms maximize profits and all goods and factor markets clear for given levels of $K(t)$ and $K(t)^*$ in the short-run equilibrium.

Utility maximization

The representative individual of cohort t_0 chooses at each instant $\tau > t_0$ consumption of the agricultural good, $c_z(t_0, \tau)$, consumption of varieties produced at home, $c_m^H(i, t_0, \tau)$, and consumption of varieties produced abroad, $c_m^F(j, t_0, \tau)$ to maximize expected lifetime utility given in equation (4.3) subject to the wealth constraint (4.4). This optimization problem can be solved

[8]For details of the aggregation procedure see chapter 2 and its appendix A.1.

[9]The aggregate efficiency units of labor L are equal to the individual supply l since population size is normalized to one.

[10]Note that $L = L^*$ due to symmetry between regions.

by applying a three stage procedure.[11] In the first stage the dynamic saving-consumption decision is analyzed which yields the individual Euler equation

$$\frac{\dot{e}(t_0,\tau)}{e(t_0,\tau)} = \frac{\pi(\tau)}{G(\tau)} - \rho + \frac{\dot{G}(\tau)}{G(\tau)}. \tag{4.7}$$

Note that to each individual the labor input coefficient $a_i(\tau)$ and thus the marginal cost of investment in new capital units, $G(\tau) \equiv a_i(\tau)w(\tau)$, is a parameter. This means that the saving decision's impact on the aggregate capital stock and thus on $a_i(\tau)$ is not taken into account, i.e. no internalization of the knowledge spillover takes place.

To arrive at the law of motion for aggregate consumption expenditures E(t) it is necessary to again "sum" over all cohorts.[12] This yields the "aggregate Euler equation" of the economy

$$\frac{\dot{E}(t)}{E(t)} = -\mu(\rho+\mu)G(t)\frac{K(t)}{E(t)} + \frac{\dot{e}(t_0,t)}{e(t_0,t)}, \tag{4.8}$$

where individual expenditure growth, $\frac{\dot{e}(t_0,\tau)}{e(t_0,\tau)}$, is given in equation (4.7). As described in detail in chapter 2, the difference between individual and aggregate savings behavior is captured by a correction term representing the distributional effects due to the turnover of generations. In particular, aggregate expenditure growth falls short of individual growth as wealthy old individuals with high expenditure levels are continually replaced by newborns with no capital holdings and thus low expenditure levels. Analogously, the corresponding law of motion for the foreign region is given by[13]

$$\frac{\dot{E^*}(t)}{E^*(t)} = -\mu(\rho+\mu)G^*(t)\frac{K^*(t)}{E^*(t)} + \frac{\dot{e^*}(t_0,t)}{e^*(t_0,t)}. \tag{4.9}$$

Stage two and three of the individual optimization problem finally deal with the static consumption allocation between the CES composite and the agricultural good as well as the allocation of consumption to each of the man-

[11] For details see again chapter 2 and its appendix A.1.

[12] For details see again chapter 2 and its appendix A.1.

[13] Note that $\mu = \mu^*$ and $\rho = \rho^*$ due to symmetry between regions.

ufactured varieties. Altogether this leads to the following demand functions for the agricultural good and for each of the manufactured varieties

$$\begin{aligned} c_z(t_0,\tau) &= \frac{(1-\xi)e(t_0,\tau)}{p_z(\tau)}, & (4.10)\\ c_m^H(i,t_0,\tau) &= \frac{\xi e(t_0,\tau)(p_m^H(i,\tau))^{-\sigma}}{\left[\int_0^{K(\tau)}(p_m^H(i,\tau))^{1-\sigma}di+\int_0^{K^*(\tau)}(p_{m,\varphi}^F(j,\tau))^{1-\sigma}dj\right]}, & (4.11)\\ c_m^F(j,t_0,\tau) &= \frac{\xi e(t_0,\tau)(p_{m,\varphi}^F(j,\tau))^{-\sigma}}{\left[\int_0^{K(\tau)}(p_m^H(i,\tau))^{1-\sigma}di+\int_0^{K^*(\tau)}(p_{m,\varphi}^F(j,\tau))^{1-\sigma}dj\right]}. & (4.12)\end{aligned}$$

Profit maximization

Marginal cost pricing in the perfectly competitive agricultural sector and perfect labor mobility across sectors implies that the equilibrium wage rate in the economy is pinned down by the price of the agricultural good. As free trade between home and foreign equalizes this price, wages also equalize between the two regions as long as each of them produces some agricultural output which will be assumed from now on.[14] Finally, choosing the agricultural good as numéraire leads to

$$w(t) = w^*(t) = 1. \tag{4.13}$$

Manufacturing firm's profit maximization yields the familiar rule that prices are equal to a constant markup over marginal costs[15]

$$\begin{aligned} p_m^H(i,t) &= \frac{\sigma}{\sigma-1}w(t)a_m, & (4.14)\\ p_{m,\varphi}^F(i,t) &= p_m^H(i,t)\varphi. & (4.15)\end{aligned}$$

Mill pricing is optimal, i.e. the only difference between prices in the two regions is due to transport costs (see e.g. Baldwin et al. (2003)).

Free entry into the manufacturing sector drives pure profits down to zero which implies that the capital rental rate must equal the operating profit

[14]See Baldwin (1999) for details on this assumption.

[15]For details of the derivations see again chapter 2 and its appendix A.1.

of each manufacturing firm. Using optimal prices given in equations (4.14) and (4.15) together with equations (4.11) and (4.12) and redefining global quantities and regional share variables gives operating profits and thus capital rental rates as[16]

$$\pi = Bias\left(\frac{\xi E^W}{\sigma K^W}\right);\ Bias \equiv \left(\frac{\theta_E}{\theta_K + \phi(1-\theta_K)} + \frac{(1-\theta_E)\phi}{\phi\theta_K + 1 - \theta_K}\right), \tag{4.16}$$

$$\pi^* = Bias^*\left(\frac{\xi E^W}{\sigma K^W}\right);\ Bias^* \equiv \left(\frac{1-\theta_E}{1-\theta_K + \phi\theta_K} + \frac{\theta_E\phi}{\phi(1-\theta_K) + \theta_K}\right), \tag{4.17}$$

where $\phi \equiv \varphi^{1-\sigma}$ is a measure of openness between the two regions with $\phi = 0$ indicating prohibitive trade barriers and $\phi = 1$ free trade. World expenditures are defined as $E^W \equiv E + E^*$ and the world capital stock as $K^W \equiv K + K^*$ with θ_K and θ_E being the respective home shares of these quantities, i.e. $\theta_K \equiv \frac{K}{K^W}$ and $\theta_E \equiv \frac{E}{E^W}$. The terms labeled $Bias$ and $Bias^*$ can be interpreted as the bias in national sales, i.e. $Bias$ measures the extent to which a home variety's sales ($\sigma\pi$) differ from the world average sales per variety ($\frac{\xi E^W}{K^W}$).

The evolution of the economy

Combining the intermediate results of utility and profit maximization yields a system of three differential equations in E, E^* and θ_K that fully describes the evolution of the economy. By substituting for a_i from equation (4.1) and for $\dot{K}$ and $\dot{K}^*$ from equations (4.5) and (4.6) and by imposing the equilibrium wage rate from equation (4.13) as well as the capital rental rates from equations (4.16) and (4.17), the aggregate Euler equations given in (4.8) and

[16]Time arguments are ignored from now on. Note, moreover, that $\xi = \xi^*$ and $\sigma = \sigma^*$ due to symmetry between regions. For further details of the derivations see again chapter 2 and its appendix A.1.

(4.9) can be rewritten as

$$\begin{aligned}
\frac{\dot{E}}{E} &= -\mu(\rho+\mu)\frac{\theta_K}{AE} - (1+\eta\frac{A^*}{A})L + (E+\eta\frac{A^*}{A}E^*) - \rho \\
&\quad +\frac{\xi}{\sigma}(E+E^*)\left[(A-\theta_K)Bias - \eta\frac{A^*}{A}(1-\theta_K)Bias^*\right], \quad (4.18) \\
\frac{\dot{E}^*}{E^*} &= -\mu(\rho+\mu)\frac{1-\theta_K}{A^*E^*} - (1+\eta\frac{A}{A^*})L + (E^*+\eta\frac{A}{A^*}E) - \rho \\
&\quad +\frac{\xi}{\sigma}(E+E^*)\left[(A^*-(1-\theta_K))Bias^* - \eta\frac{A}{A^*}\theta_K Bias\right], \quad (4.19)
\end{aligned}$$

where $A \equiv \theta_K + \eta(1-\theta_K)$ such that $a_i = \frac{1}{K^W A}$ and analogously for A^*.

The law of motion of θ_K is obtained by differentiating the definition of this share variable with respect to time and then substituting for $\dot{K}$ and $\dot{K}^*$ from equations (4.5) and (4.6) which yields

$$\begin{aligned}
\dot{\theta}_K &= \theta_K(1-\theta_K)\left(\frac{\dot{K}}{K} - \frac{\dot{K}^*}{K^*}\right) \\
&= (1-\theta_K)A[L + \frac{\sigma}{\xi}(E+E^*)\theta_K Bias - E] \\
&\quad -\theta_K A^*[L + \frac{\sigma}{\xi}(E+E^*)(1-\theta_K)Bias^* - E^*], \quad (4.20)
\end{aligned}$$

where again the equilibrium wage rate from equation (4.13) as well as the capital rental rate from equation (4.16) is imposed.

The remaining sections 4.3 and 4.4 will analyze this three-dimensional dynamic system (4.18), (4.19) and (4.20) more thoroughly to identify the channels via which the demographic parameter μ impacts upon both the steady-state growth rates and the stability properties of the symmetric equilibrium. Similarly to the framework of chapter 2, the incorporation of overlapping generations and lifetime uncertainty affects the system only via the turnover correction terms in the aggregate Euler equations. This already emphasizes the central role the generational turnover will play for the relationship between demographic change and agglomeration tendencies. Note, moreover, that setting $\mu = 0$, i.e. considering the case of an infinitely lived representative agent, yields the laws of motion obtained by Baldwin et al.

(2001). The framework developed in this paper thus nests their model, which fully ignores demographic structures, as a special case.

4.3 Long-run equilibrium

A long-run equilibrium is characterized by the steady-state values $\bar{E}$, $\bar{E}^*$ and $\bar{\theta}_K$ for which $\dot{E} = \dot{E}^* = \dot{\theta}_K = 0$. Due to the nonlinearities arising from the turnover term in the aggregate Euler equations, one cannot solve for all equilibria analytically. Numerical investigations, however, reveal an equilibrium pattern similar to Baldwin et al. (2001). Before presenting these results, subsection 4.3.1 analytically characterizes the symmetric interior equilibrium as well as the core-periphery outcome.[17]

4.3.1 Lifetime uncertainty in the symmetric and core-periphery equilibrium - Is agglomeration pro-growth?

Inserting the symmetric outcome with $\theta_K = 0.5$ and $E = E^*$ into the three-dimensional system reveals that it is indeed a steady state with the equilibrium level of expenditures given by[18]

$$\bar{E}_{sym} = \bar{E}^*_{sym} = \frac{L(1+\eta) + \rho + \sqrt{(L(1+\eta)^+\rho)^2 + 4\mu(\mu+\rho)}}{2(1+\eta)} \tag{4.21}$$

Equation (4.21) clearly shows that aggregate expenditures in the symmetric equilibrium increase in the mortality rate. This is fully consistent with the findings in the framework without spillovers presented in chapter 2, where the positive dependence has, however, only been shown numerically.[19] The above

[17] These and most other results were derived with Mathematica. The corresponding files are available from the author upon request.

[18] Solving the system for the symmetric equilibrium value of expenditures in fact yielded two solutions. Attention is restricted to the economically meaningful one.

[19] Surprisingly, the introduction of endogenous growth into the benchmark model of chapter 2 substantially simplifies the dynamic system describing the evolution of the economy and thus allows for more analytical results.

result moreover indicates that the mortality rate influences consumption expenditures primarily via its effect on discounting, i.e. a higher mortality rate increases expenditure levels of individuals. This "discount channel" dominates the "age structure based channel". The latter captures the effect of the mortality rate on the age composition of the population and implies a negative dependence since a higher mortality rate increases the proportion of poor and young individuals with low expenditure levels to wealthy and old individuals with higher expenditure levels (see chapter 2 for details).

The growth rate, defined as $g \equiv \frac{\dot{K}}{K}$, in the symmetric equilibrium can be obtained from equation (4.5) by using the equilibrium level of expenditures of equation (4.21) and simplifying. It is given by

$$\begin{aligned} \bar{g}_{sym} &= \bar{g}^*_{sym} = (1+\eta)[L - \frac{\sigma-\xi}{\sigma}\bar{E}_{sym}] \\ &= (1+\eta)L - \frac{(\sigma-\xi)}{2\sigma}[L(1+\eta)+\rho+\sqrt{(L(1+\eta)^+\rho)^2+4\mu(\mu+\rho)}]. \end{aligned} \tag{4.22}$$

Investigating the dependence of this growth rate on the mortality rate immediately yields the following proposition.

Proposition 4.1. *The equilibrium growth rate in the symmetric equilibrium decreases in the mortality rate.*

Proof. The derivative of the growth rate with respect to mortality is

$$\frac{\partial \bar{g}_{sym}}{\partial \mu} = -\frac{(\sigma-\xi)(2\mu+\rho)}{\sigma\sqrt{(L(1+\eta)^+\rho)^2+4\mu(\mu+\rho)}} < 0,$$

since $\sigma > \xi$. □

This finding is fully consistent with Prettner (2009) who investigates the consequences of varying mortality rates for growth perspectives in a one-region endogenous growth model in the spirit of Romer (1990). The negative effect of the mortality rate on the growth rate works via the increased level of equilibrium expenditures (see equation (4.21)). If individuals face lifetime uncertainty, available resources are more heavily used for current consumption purposes than for investment in capital and thus new varieties. This is due to the fact that future is discounted more strongly and thus investment in future growth prospects becomes less important.

Similar findings also apply to the core-periphery outcome.[20] First, for a threshold value of the mortality rate of

$$\mu_{cp} = \frac{-\rho + \sqrt{\left(\frac{(L+\rho)(\sigma-\xi(\phi\eta-1))+2L\xi(1-\frac{\eta}{\phi})}{\sigma+\xi(\phi\eta-1)}\right)^2 - (L^2+2L\rho)}}{2} \tag{4.23}$$

accumulation of capital in only one region, i.e. $\theta_K = 1$, can be shown to be a steady state of three-dimensional system[21] with associated expenditure levels

$$\bar{E}_{cp} = \frac{(L+\rho) + \sqrt{(L+\rho)^2 + 4\mu(\rho+\mu)}}{2}, \qquad \bar{E}^*_{cp} = L. \tag{4.24}$$

Whereas equilibrium expenditures at home again increase in the mortality rate, the foreign expenditure level is of course independent of the mortality rate since, even with infinitely lived individuals, all available resources are immediately used for consumption purposes. The growth rate in the core-periphery equilibrium is finally obtained by combining equation (4.5) with the core-periphery expenditure levels of (4.24) which yields

$$\begin{aligned} \bar{g}_{cp} &= \frac{\sigma+\xi}{\sigma}L - \frac{\sigma-\xi}{\sigma}\bar{E}_{cp} \\ &= \frac{(3\xi+\sigma)L - (\sigma-\xi)(\rho + \sqrt{(L+\rho)^2 + 4\mu(\rho+\mu)})}{2\sigma}. \end{aligned} \tag{4.25}$$

Analogously to the symmetric equilibrium, the impact of lifetime uncertainty on this growth rate can be immediately summarized in the following propo-

[20]Here only the $\theta_K = 1$ case is considered. Using symmetry between the regions, analogous results can be shown to hold for $\theta_K = 0$.

[21]Following Baldwin et al. (2001), who investigate the equilibria of the model for varying trade cost levels instead of mortality rates, one can thus conclude that for all $\mu \leq \mu_{cp}$ the core-periphery outcome $\theta_K = 1$ represents a long-run equilibrium. This follows from taking into account the boundary condition $0 \leq \theta_K \leq 1$.

sition.

Proposition 4.2. *The equilibrium growth rate in the core-periphery equilibrium decreases in the mortality rate.*

Proof. The derivative of the growth rate with respect to mortality is

$$\frac{\partial \bar{g}_{cp}}{\partial \mu} = -\frac{(\sigma - \xi)(2\mu + \rho)}{\sigma\sqrt{(L+\rho)^2 + 4\mu(\rho+\mu)}} < 0,$$

since $\sigma > \xi$. □

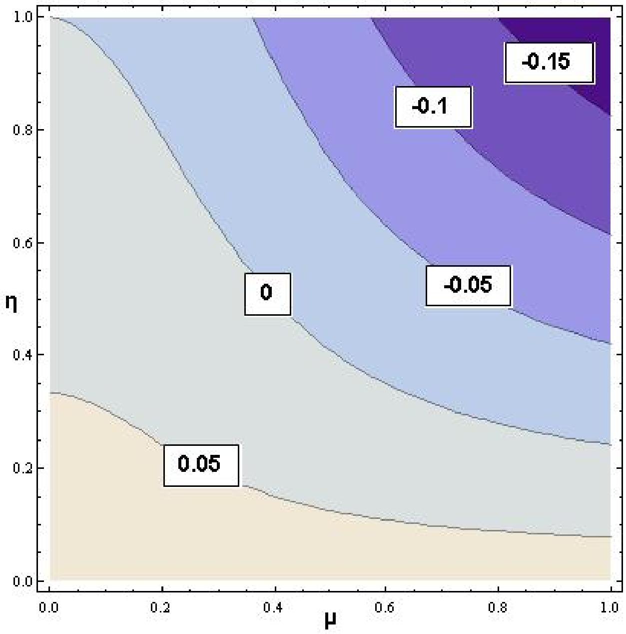
As in the symmetric equilibrium, a positive mortality rate decreases the equilibrium growth rate via its positive impact upon equilibrium *home* expenditures.

Although the mortality rate has the same qualitative effect on the growth rates in the symmetric and core-periphery equilibrium, its quantitative impact differs. This has crucial implications for the growth rate differential between the symmetric and the core-periphery equilibrium.[22] Baldwin et al. (2001) show that in their model with infinitely lived individuals the core-periphery growth rate exceeds the growth rate in the symmetric equilibrium as long as spillovers are localized, i.e. $\eta < 1$. In the case of global spillovers the growth rates equalize which is intuitive since there is no spillover gain from agglomeration of innovative activity. Agglomeration is thus pro-growth unless interregional learning externalities are perfect. Surprisingly, the introduction of overlapping generation structures and lifetime uncertainty qualifies this finding. In particular, for the case of global spillovers, i.e. $\eta = 1$, the growth rate differential is given by

$$\begin{aligned} \bar{g}_{cp} - \bar{g}_{sym}|_{\eta=1} &= \frac{\sigma-\xi}{2\sigma}\sqrt{(2L+\rho)^2 + 4\mu(\rho+\mu)} - L \\ &\quad -\frac{\sigma-\xi}{2\sigma}\sqrt{(L+\rho)^2 + 4\mu(\rho+\mu)}. \end{aligned} \tag{4.26}$$

[22]Note that in both equilibria steady-state growth in real income is equalized across regions. This is due to the fact that real income growth is driven by the steady decrease in the price index along the growth path which itself results from the growing number of worldwide varieties K^W (see Baldwin et al. (2001)).

Figure 4.1: The growth rate differential $\bar{g}_{cp} - \bar{g}_{sym}$ for varying μ and η



It is easily verified that $\bar{g}_{cp} - \bar{g}_{sym}|_{\eta=1}$ for $\mu = 0$ and that this differential decreases in the mortality rate. Consequently, with lifetime uncertainty, the core-periphery outcome even features a smaller growth rate than the symmetric equilibrium for the case of fully globalized spillovers. The growth advantage of the symmetric equilibrium resulting from the possibility of death is, however, dampened the more spillovers are localized. This follows from noting that the growth rate in the symmetric equilibrium is smaller the more spillovers are localized, i.e. the lower is η, whereas the rate in the core-periphery outcome is independent of the spillover parameter. Figure 4.1 illustrates these findings by plotting the level curves of the growth rate differential $\bar{g}_{cp} - \bar{g}_{sym}$ for varying mortality rates and degrees of interregional spillovers given the parameter values $\rho = 0.015$, $\xi = 0.3$, $\sigma = 4$, $L = 1$. In sharp contrast to standard NEG growth models (see e.g. Baldwin and Martin (2004) for an overview), figure 4.1 nicely shows that even in the case of localized spillovers the symmetric equilibrium's growth rate can exceed the core-periphery's one, e.g. for $\eta = 0.9$, mortality rates above 0.127 would imply a negative differential. These observations are summarized in the following proposition.

Proposition 4.3. *The growth rate differential* $\bar{g}_{cp} - \bar{g}_{sym}$ *decreases in the mortality rate with the decrease being dampened the more spillovers are localized.*

Proof.

$$
\begin{aligned}
\frac{\partial(\bar{g}_{cp} - \bar{g}_{sym})}{\partial \mu} &= \frac{(\sigma - \xi)(2\mu + \rho)}{\sigma} \times \\
&\times \left(\frac{1}{\sqrt{(L(1+\eta)+\rho)^2 + 4\mu(\rho+\mu)}} - \frac{1}{\sqrt{(L+\rho)^2 + 4\mu(\rho+\mu)}} \right) \leq 0,
\end{aligned}
$$

since $\eta \geq 0$ makes the term in brackets nonpositive and $\sigma > \xi$. Clearly, the above derivative decreases in η which proves the second part of the proposition. □

In a setting with lifetime uncertainty, spatial agglomeration is thus not necessarily conducive to growth. The intuition for this result becomes clear when looking more thoroughly at the impact of lifetime uncertainty on the growth rates in the symmetric and core-periphery outcome. In general, when spillovers are localized, agglomeration of innovative activity makes innovation cheaper and thus results in higher growth rates. A positive mortality rate, however, countervails this pro-growth effect of spatial agglomeration. In particular, it has a less negative effect on the growth rate in the symmetric equilibrium than in the core-periphery equilibrium if spillovers are not purely local. This can be easily seen by comparing the derivatives of the growth rates with respect to the mortality rate in the proofs of propositions 4.1 and 4.2 which, as the proof of proposition 4.3 also shows, only equalize for $\eta = 0$. Intuitively, in the symmetric equilibrium with some knowledge spillovers from the other region, the decrease in saving incentives due to a higher mortality rate is not as unfavorable for the economy's growth rate as in the core-periphery equilibrium with full concentration of innovative activity. The reason is that spillovers from the other region increase productivity in the innovation sector such that it is ceteris paribus more attractive to use resources for investment purposes than for current consumption. Since this effect is, however, only present in the symmetric equilibrium, a positive mortality rate decreases equilibrium expenditures less in the symmetric than in the core-periphery equilibrium and thus has a lower negative effect on the growth rate.

4.3.2 Interior asymmetric equilibria

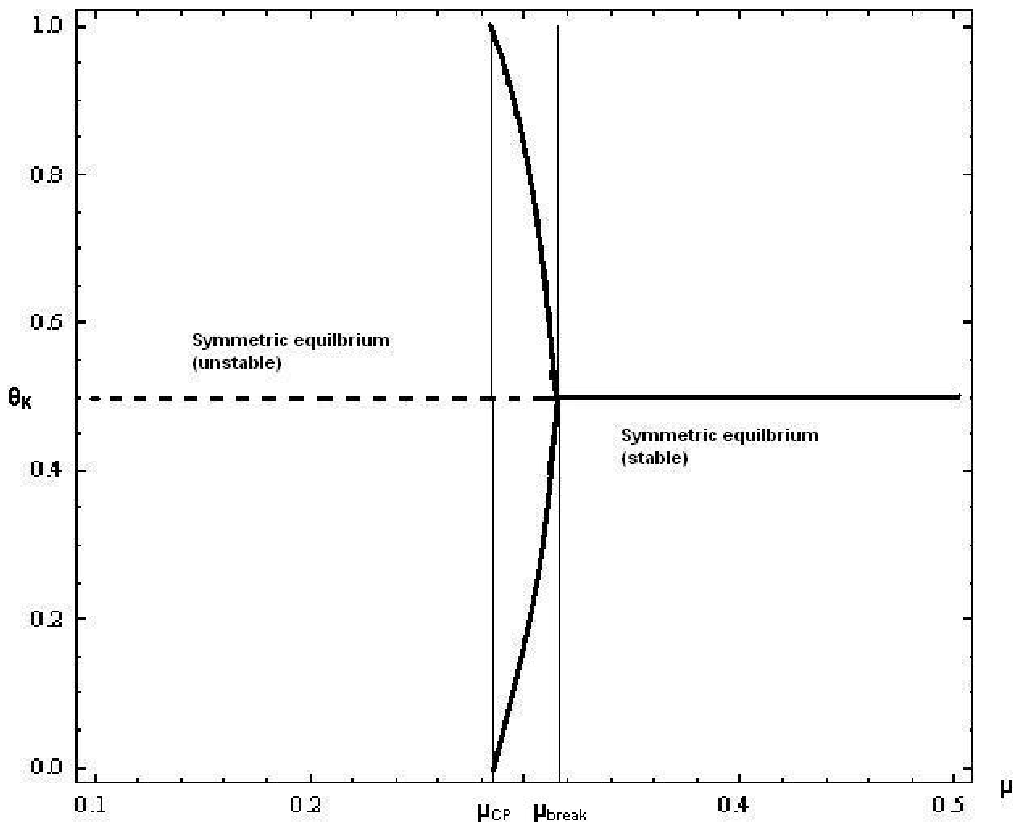
As already mentioned, deriving analytical expressions for all steady states is too cumbersome. It is, however, possible to reduce the system of three equilibrium equations in three variables, i.e. $\dot{E} = \dot{E}^* = \dot{\theta}_K = 0$, to one equation in θ_K, whose roots represent all long-run equilibria. Numerically investigating this equation reveals that for a small range of mortality rates, in particular as soon as the threshold value μ_{cp} is passed, the core-periphery equilibria turn into interior asymmetric equilibria. The following figure 4.2 illustrates these equilibrium characteristics by plotting the steady states of the system as a function of μ.[23] It clearly indicates that there exist, at least for a small range of mortality rates, interior equilibria with an unequal distribution of capital across regions which are symmetric around the persistent steady state $\theta_K = 0.5$. Similar to Baldwin et al. (2001), figure 4.2 is highly reminiscent of a pitchfork bifurcation. The following section will provide additional evidence for the existence of a supercritical pitchfork bifurcation by illustrating that the symmetric equilibrium loses its stability as soon as the two interior asymmetric equilibria appear.

4.4 The joint effect of demographic change and spillovers on agglomeration

Figure 4.2 shows the existence of long-run equilibria that are characterized by an unequal distribution of capital. Whether such an agglomeration of

[23]Note that for the parameter values used, i.e. $\rho = 0.015$, $\xi = 0.3$, $\sigma = 4$, $L = 1$, $\eta = 0.5$ and $\phi = 0.9$, $\mu_{cp} = 0.285455$.

Figure 4.2: Steady-state values of θ_K as a function of μ



economic activity takes place crucially depends on the stability properties of the symmetric equilibrium. If it turns out that this steady state is unstable, any slight perturbation of the symmetric outcome triggers agglomeration processes that might result in either the asymmetric interior or the core-periphery equilibria.

In general, NEG models focus on the role of changing trade costs for the emergence of spatial structures, i.e. they show how economic integration lowering the costs of trading goods leads to concentration of economic activity. Other features of the economy could, however, be similarly decisive for agglomeration processes. Chapter 2 has shown that the adoption of an overlapping generation structure with lifetime uncertainty, i.e. increasing μ above zero, considerably reduces the possibility of the symmetric equilibrium to be unstable in the constructed capital framework of Baldwin (1999). The main question to be investigated in this section is whether an analogous finding also holds in a setting with endogenous growth due to learning spillovers. In particular, it is of interest how demographic structures interact with the additional growth-linked circular causality resulting from the incorporation of knowledge spillovers in the investment sector. Finally, it is worth investigating the role of lifetime uncertainty in the selection among the asymmetric equilibria, i.e. how it influences the importance of initial conditions relative to expectations in choosing the region where agglomeration takes place.

4.4.1 Formal stability analysis

The stability properties of the symmetric long-run equilibrium are analyzed by following the classical approach (see e.g. the appendix on mathematical methods in Barro and Sala-i-Martin (2004)) of linearizing the non-linear dynamic system (4.18), (4.19) and (4.20)[24] around the symmetric equilibrium and then by evaluating the eigenvalues of the corresponding 3×3 Jacobian

[24]Equations (4.18) and (4.19) were multiplied by E or E^* to obtain $\dot{E}$ and $\dot{E}^*$.

matrix

$$\begin{pmatrix} j11 & j12 & j13 \\ j12 & j11 & -j13 \\ j31 & -j31 & j33, \end{pmatrix} \tag{4.27}$$

whose entries are given in appendix A.3.1. Solving the characteristic equation yields the following three eigenvalues

$$\begin{aligned} eig1 &= \sqrt{L^2(\eta+1)^2 + 2L\rho(\eta+1) + (2\mu+\rho)^2}, & (4.28)\\ eig2 &= \frac{1}{2(\eta+1)(\phi+1)^2\sigma}(r - \sqrt{rad}), & (4.29)\\ eig3 &= \frac{1}{2(\eta+1)(\phi+1)^2\sigma}(r + \sqrt{rad}), & (4.30) \end{aligned}$$

where

$$\begin{aligned} r &\equiv (-2L\eta + eig1)((\eta+1)(\phi+1)^2\sigma) - 2\phi(\eta\phi-1)Q\xi, \\ rad &\equiv \big(((\eta-1)eig1 + 2\eta\rho)\sigma(\phi+1)^2 + 2(\phi-\eta)Q\xi\big)^2 \\ &\quad -8\eta(\eta+1)(\phi+1)^3\sigma((\phi+1)\sigma + (\phi-1)\xi) \times \\ &\quad \times \left(\frac{Q\left(-L\eta^2 + L + \frac{1}{2}Q\left(\eta - \frac{2(\eta\phi^2-2\phi+\eta)\xi}{(\phi+1)^2\sigma} - 1\right)\right)}{2(\eta+1)} - \mu(\mu+\rho) \right), \end{aligned}$$

with the parameter cluster $Q \equiv L(\eta+1) + eig1 + \rho$. The signs and nature of these eigenvalues fully characterize the system's local dynamics around the symmetric equilibrium. Since there are two jump variables E and E^*, stability requires at least one eigenvalue to be negative. In particular, saddle path stability prevails if one out of the three eigenvalues is negative.

First it is easily established that eigenvalue 1 is always real and positive for all possible parameter values.[25] As far as the remaining two eigenvalues are concerned things turn out to be more complicated. Checking rad for various parameter specifications shows that it changes sign, i.e. one must differentiate between the case where eigenvalues 2 and 3 are real and the

[25]Recall the parameter ranges $\sigma > 1$, $\rho > 0$, $\mu > 0$, $0 < \xi < 1$ and $0 \le \phi \le 1$, $0 \le \eta \le 1$ and $L > 0$ which also imply that $Q > 0$.

case where they are complex. In both cases stability properties crucially depend on the sign of r. Since $(-2L\eta + eig1) > 0$ and $(\eta\phi - 1) \leq 0$ r is unambiguously positive for all possible parameter ranges.

4.4.2 The case of real eigenvalues - The opposing stability effects of demography and spillovers

With real eigenvalues, $r > 0$ immediately implies that eigenvalue 3 is also positive resulting in lemma 4.1.

Lemma 4.1. *For the case of real eigenvalues, i.e. $rad > 0$, eigenvalue 2 is decisive for the local stability properties of the symmetric equilibrium. A positive eigenvalue 2 implies instability, a negative one saddle path stability.*

Proof. See above arguing. □

Lemma 4.1 implies that changes in the mortality rate can only influence the stability properties of the symmetric equilibrium via eigenvalue 2. Numerically investigating this eigenvalue immediately results in the following proposition.

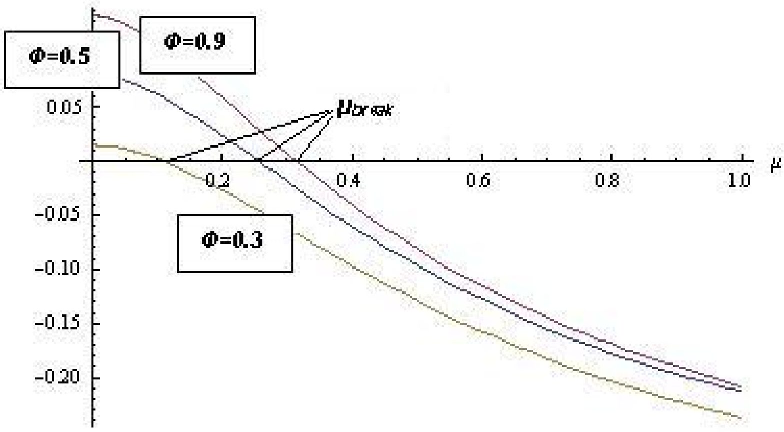
Proposition 4.4. *The possibility of agglomeration crucially hinges on the mortality rate.*

Proof. Figure 4.3[26], which plots eigenvalue 2 as a function of the mortality rate for an intermediate level of interregional spillovers $\eta = 0.5$ and for three different levels of economic integration, clearly reveals that eigenvalue 2 switches sign depending on the mortality rate. □

Proposition 4.4 verifies the crucial importance of demographic structures for agglomeration processes found in chapter 2 also for a setting that additionally allows for endogenous growth due to learning spillovers. Figure 4.3, however, does not only show this decisive role of the mortality rate but also reconfirms the stabilizing effect of introducing overlapping generations and lifetime uncertainty. Consistent with the findings in chapter 2, eigenvalue 2

[26]Figure 4.3 is again plotted for $\rho = 0.015$, $\xi = 0.3$, $\sigma = 4$ and $L = 1$.

Figure 4.3: Eigenvalue 2 as a function of μ



decreases in the mortality rate, i.e. only for sufficiently low mortality rates agglomeration processes may set in. In the framework of chapter 2 without knowledge spillovers, this stabilizing effect even implied that for plausible parameter values instability could never occur. In particular, it was shown that for mortality rates corresponding to life expectancies of less than approximately 3500 years[27] the symmetric equilibrium was stable for all levels of economic integration.[28] This is in sharp contrast to the present setting with endogenous growth due to learning effects. Indeed, figure 4.3 clearly indicates that agglomeration is still a fully possible outcome. It shows that for a rather low level of trade openness $\phi = 0.3$, the critical mortality rate μ_{break} below which eigenvalue 2 is positive and thus the symmetric equilibrium unstable[29] is 0.109. This corresponds to a life expectancy of only 9

[27] Since the probability of death during each year equals μ, average life expectancy is $\frac{1}{\mu}$.

[28] This holds for the most plausible choice of parameter values made in chapter 2, i.e. $\rho = 0.015$, $\delta = 0.05$, $\xi = 0.3$ and $\sigma = 4$. Setting $\delta = 0$ as in this setting only insignificantly reduces this life expectancy threshold value.

[29] The analytical expression for μ_{break} is too unwieldy to report. The Mathematica file deriving it is available upon request.

years, which illustrates that, in contrast to the framework of chapter 2, a plausible choice of parameter values does not eliminate the possibility of agglomeration of economic activity even though we allow for nonzero mortality rates. Summarizing, the findings so far imply that, although the introduction of overlapping generations still acts as a dispersion force in a NEG model with endogenous growth, its impacts are, in general, not strong enough to prevent regions from unequal development.

The intuition for this result is simple. It is based on the existence of two countervailing effects which would not be present in a setting without learning spillovers and ignoring demographic structures. In particular, whether agglomeration takes place in this model economy depends on the relative strength of four distinct agglomeration or dispersion forces. Each of them captures how an exogenous increase of the capital share impacts upon the rate of capital accumulation. If it raises it, agglomeration of capital takes place since a circular causality sets in. Otherwise, the decreased capital accumulation rate acts as a self-correcting force promoting dispersion rather than concentration of economic activity.

First, there are two forces that are neither linked to the demographic structure nor to the spillover specification. These are the standard anti-agglomerative local competition effect and the pro-agglomerative demand-linked circular causality first introduced by Baldwin (1999). The latter shows that a higher share of capital in one region increases expenditures and thus operating profits, i.e. capital rental rates, which speeds up capital accumulation. The former, on the other hand, captures the negative impact of agglomeration of capital, i.e. firms, on capital rental rates due to more severe competition. Both of these forces are, however, not the channels via which the mortality rate on the one hand and learning spillovers on the other hand impact upon the stability properties of the symmetric equilibrium.

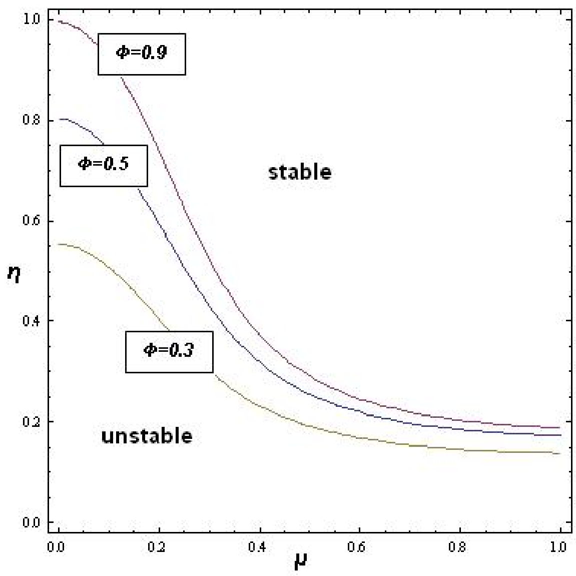
The particular channel via which demography influences agglomeration processes has already been identified in chapter 2 as the anti-agglomerative turnover effect. An exogenous rise in the home capital share increases wealth and thus expenditure levels of individuals being currently alive in the home region relative to foreign-based individuals. The negative distributional effects on aggregate expenditures resulting from death, i.e. the replacement of these individuals by newborns whose consumption expenditures are lower since they have zero wealth levels, are thus more pronounced in the home region. This, in turn, decreases the home expenditure share and therefore relative profitability and the relative capital rental rate.

Finally, the impact of learning spillovers on the stability properties of the symmetric equilibrium are captured by the growth-linked circular causality introduced by Martin and Ottaviano (1999). If learning spillovers are localized, a higher share of capital in the home region lowers the marginal cost of producing new capital relative to the foreign region (see equation 4.1) and thus strengthens capital accumulation. Endogenous growth is thus a powerful destabilizing force that mitigates the stabilizing effect of the turnover of generations. The strength of this growth-linked circular causality crucially depends on the degree of localization of spillovers. As can be seen in figure 4.4[30], which plots the zero level curve of eigenvalue 2, i.e. the dividing line between stability and instability, for three different levels of trade openness, agglomeration is fostered the more learning externalities are localized. In particular, with no learning spillovers across regions, i.e. $\eta = 0$, eigenvalue 2 is positive for all possible parameter ranges and the system is always unstable. Figure 4.4, however, also illustrates the stabilizing effect of increased interregional spillovers. For a given mortality rate of e.g. 0.0125 resulting in a life expectancy of 80 years and intermediate trade openness levels $\phi = 0.5$, fostering interregional knowledge spillovers above a level of about 0.8 could still prevent regions from unequal development. The strength of this stabilizing effect of knowledge spillovers becomes clear if one considers the case of globalized learning effects, i.e. $\eta = 1$. In this case the symmetric equilibrium is stable for all levels of trade costs as long as individuals face a life expectancy of less than approximately 3600 years, i.e. as in the framework presented in chapter 2 agglomeration of economic activity does not occur for plausible parameter values.

Last but not least, the stability findings of this section also provide additional evidence that the dynamic system undergoes a supercritical pitchfork

[30]Figure 4.4 is again plotted for $\rho = 0.015$, $\xi = 0.3$, $\sigma = 4$ and $L = 1$.

Figure 4.4: Symmetric equilibrium stability map



bifurcation as the mortality rate crosses a certain threshold value. In particular, for the parameter values of figure 4.2, the critical mortality rate μ_{break} at which the symmetric equilibrium loses stability is 0.313. As can be seen from figure 4.2, this value exactly coincides with the threshold mortality rate where the two interior asymmetric steady states, that finally turn into the core-periphery equilibria, show up. Following Baldwin (2001), one can thus conclude that as soon as the mortality rate crosses μ_{break} from above, the symmetric steady state loses its stability to the two appearing neighboring asymmetric interior steady states. The question which out of these two equilibria will then be reached, i.e. in which region agglomeration will take place, is briefly addressed in the next subsection.

4.4.3 The case of complex eigenvalues - Lifetime uncertainty and the history-versus-expectations debate

The analysis so far has shown that nonzero mortality rates foster a more equal distribution of economic activity due to the turnover of generations. With complex eigenvalues, i.e. $rad < 0$, the symmetric equilibrium is, however, always unstable since r is unambiguously positive for all possible parameter ranges. As will become clear, the mortality rate nonetheless plays a decisive role for the dynamics of the system by influencing the dividing line between the case of monotone divergence, that occurs for a positive real eigenvalue 2, and the case of diverging oscillations resulting from complex eigenvalues.

Krugman (1991a)[31] shows that in the first situation history, represented by initial conditions, is the crucial factor with respect to equilibrium selection, whereas in the latter self-fulfilling expectations might also be decisive. In particular, as long as all eigenvalues are real, agglomeration of capital will take place in the region with the initially larger share of capital. In the case of complex eigenvalues, on the other hand, equilibrium paths overlap such that there exists a range around the symmetric equilibrium, where a given initial distribution of capital corresponds to paths each leading to agglomeration in a different region and expectations determine which path is chosen. Since the parameters of the model determine whether there are complex or real eigenvalues it is perfectly possible to investigate the role of the mortality rate with respect to such history-versus-expectations considerations.

According to Baldwin (2001), a sufficient condition for there to be some overlap of saddle paths is that the eigenvalues of the Jacobian evaluated at the unstable equilibrium are complex, i.e. $rad < 0$. Checking the dependence of rad on μ reveals that lifetime uncertainty strengthens the role of initial conditions in choosing among the multiple long-run equilibria. This is summarized in proposition 4.5.

[31] Baldwin (2001) gives a nice overview over the history-versus-expectations debate in NEG models initiated by Krugman (1991a).

Proposition 4.5. *The possibility of self-fulfilling expectations rather than initial conditions being decisive for equilibrium selection arises only for sufficiently low mortality rates.*

Proof. Using Mathematica it can be shown that $\frac{\partial rad}{\partial \mu} \geq 0$ for all possible parameter ranges.[32] $\square$

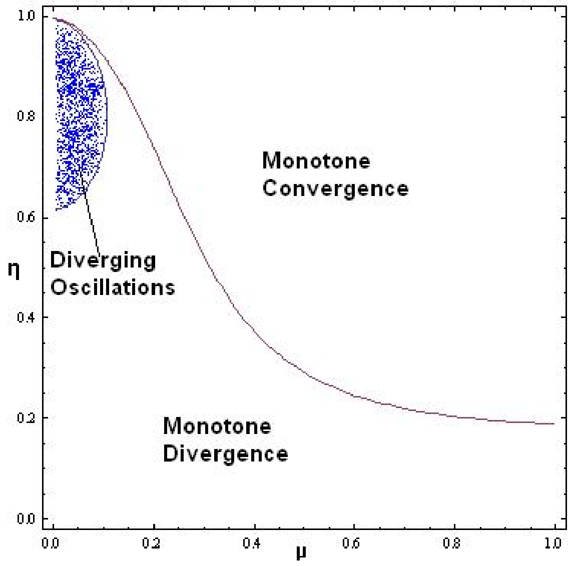
Intuitively, a higher mortality rate implies that individuals discount the future more heavily. This impatience means that they do not care too much about expected profitability of capital accumulation which itself depends on the future investment decisions of other individuals (recall that a region's relative attractiveness with respect to capital accumulation depends on its capital share). But this is exactly one main reason why expectations could be important. If everyone expects that investment will mainly take place in the home region, then this increases the attractiveness of also investing there independently of the current situation, i.e. independent of the relative investment profitability in the initial condition. With low patience such considerations about future investment returns lose, however, importance and it is rather the current relative returns that are decisive.

Finally, numerically investigating the parameter region in which $rad < 0$[33] more thoroughly reveals that the possibility of expectations to be decisive for equilibrium selection in our model is, in general, rather low. For the parameter values of figure 4.2 e.g., the eigenvalues are real for all levels of the mortality rate. Figure 4.5, which replicates, for varying mortality rates and spillovers, the dividing line between stability and instability of figure 4.4 and additionally shows the level curve where $rad = 0$, i.e. the dividing line between monotone divergence and diverging oscillations, indicates a similar conclusion. The area inside the instability region featuring diverging oscillations instead of monotonic divergence is relatively small. This again emphasizes the subordinate role of expectations in answering the question in which region agglomeration will take place.

[32] $\frac{\partial rad}{\partial \mu}$ is too cumbersome to be revealing. The calculations are available upon request.

[33] Note that the threshold value μ_{real}, obtained from setting $rad = 0$, below which there are complex eigenvalues, is lower than μ_{break}, obtained from setting eigenvalue 2 equal to zero, i.e. $r = \sqrt{rad}$, since $r > 0$ and rad has been shown to increase in μ.

Figure 4.5: The subordinate role of expectations relative to initial conditions for equilibrium selection



4.5 Concluding remarks

By incorporating an overlapping generation structure and lifetime uncertainty into a NEG model that features endogenous growth through learning externalities, this paper has shed further light on whether and how demographic structures impact upon the spatial distribution of economic activity. In doing so, it moreover gives deeper insights on how lifetime uncertainty affects the long-run equilibrium growth perspectives of the symmetric relative to the core-periphery outcome.

The main results are twofold. First, consistent with the findings in chapter 2, nonzero mortality rates resulting in age-dependent heterogeneity of individuals support a more equal distribution of productive factors by introducing an additional dispersion force that countervails the agglomeration tendencies resulting from endogenous growth through knowledge spillovers. In particular, if interregional knowledge spillovers are sufficiently encouraged across regions, the turnover of generations can even prevent regions from unequal development. Moreover, the possibility of agglomeration being the result of a self-fulfilling prophecy is substantially reduced when considering individuals who face a positive probability of death. Second, lifetime uncertainty lowers both the symmetric equilibrium's as well as the core-periphery outcome's growth rate. As long as learning spillovers are not purely localized, this decrease is, however, more pronounced for the latter one. Thus, in sharp contrast to existing NEG growth models with localized knowledge spillovers, spatial concentration of economic activity is not necessarily conducive to growth if one takes into account demographic structures. This also implies that there might not be any trade-off between fostering an equal distribution of productive factors and high economic growth which would result from e.g. increased economic integration if agglomeration were unambiguously pro-growth.

Due to analytical tractability, the present framework still draws a very simplified picture of reality. The model could, however, be easily extended to investigate further relevant issues related to the interaction between agglomeration, growth and demography. What happens if regions are asymmetric with respect to mortality rates or the degree of interregional spillovers? What are the exact welfare implications of the results obtained so far? What are the main differences, in particular with respect to growth rates, between partial and full agglomeration equilibria, i.e. between the interior asymmetric and the core-periphery steady states? These are only few questions that could still be investigated more thoroughly within the model framework developed in this paper.

Appendix A

Technicalities

A.1 Chapter 2

A.1.1 The individual's utility optimization problem

The first stage of the individual's utility optimization problem deals with the dynamic savings-expenditure decision. Suppressing time arguments in the optimization procedure, the current value Hamiltonian for the individual's utility optimization problem is

$$H(e,k,\lambda,t) = \ln\left[\frac{e}{P}\right] + \lambda\left(\frac{wl+\pi k-e}{wa_i} + \mu k - \delta k\right) \tag{A.1}$$

where P is the perfect price index translating expenditures into indirect utility.[1] The first order conditions of the problem associated with equation (A.1) are given by

$$\frac{\partial H}{\partial e} \stackrel{!}{=} 0 \quad \Rightarrow \quad \frac{1}{e} = \frac{\lambda}{a_i w}, \tag{A.2}$$

$$\frac{\partial H}{\partial k} \stackrel{!}{=} (\rho+\mu)\lambda - \dot{\lambda} \quad \Rightarrow \quad \frac{\dot{\lambda}}{\lambda} = -\frac{\pi}{a_i w} + \rho + \delta, \tag{A.3}$$

$$\frac{\partial H}{\partial \lambda} \stackrel{!}{=} \dot{k} \quad \Rightarrow \quad \frac{wl+\pi k-e}{wa_i} + \mu k - \delta k = \dot{k} \tag{A.4}$$

[1] This price index can be obtained from the solution to the optimization problem in stage two and three.

and the standard transversality condition. Taking the time derivative of equation (A.2) under the assumption that w is time independent[2] and combining it with equation (A.3) yields the individual consumption Euler equation

$$\frac{\dot{e}}{e} = \frac{\pi}{a_i w} - \delta - \rho.$$

In the second stage, the static problem of dividing consumption between the manufacturing composite and the agricultural good for fixed consumption expenditure e can be formulated as

$$\begin{aligned} &\max_{c_m^{agg}, c_z} (c_z)^{1-\xi} (c_m^{agg})^{\xi} \\ \text{s.t.} \quad &p_z c_z + p_m^{agg} c_m^{agg} = e, \end{aligned} \tag{A.5}$$

where p_m^{agg} is an appropriate price index which can be shown to equal a weighted average of the two Dixit and Stiglitz (1977) price indexes at home and foreign with the foreign price index being augmented by transport costs. Setting up the Lagrangian

$$\ell(c_z, c_m^{agg}, \lambda_a) = (c_z)^{1-\xi} (c_m^{agg})^{\xi} + \lambda_a \left(e - p_z c_z - p_m^{agg} c_m^{agg}\right) \tag{A.6}$$

and solving for the first order conditions yields

$$\frac{\partial \ell}{\partial c_z} \stackrel{!}{=} 0 \quad \Rightarrow \quad (1-\xi)(c_z)^{-\xi} (c_m^{agg})^{\xi} = \lambda_a p_z, \tag{A.7}$$

$$\frac{\partial \ell}{\partial c_m^{agg}} \stackrel{!}{=} 0 \quad \Rightarrow \quad (c_z)^{1-\xi} \xi (c_m^{agg})^{\xi-1} = \lambda_a p_m^{agg}, \tag{A.8}$$

$$\frac{\partial \ell}{\partial \lambda_a} \stackrel{!}{=} 0 \quad \Rightarrow \quad p_z c_z + p_m^{agg} c_m^{agg} = e. \tag{A.9}$$

Manipulating these first order conditions leads to unit elastic demands for the agricultural good and the CES composite of manufactured varieties given

[2]Section 2.2.4 shows that this indeed holds as the wage rate is pinned down by the price of the agricultural good which is chosen to be the numéraire of the economy.

by

$$
\begin{aligned}
c_z &= \frac{(1-\xi)e}{p_z} \\
c_m^{agg} &= \frac{\xi e}{p_m^{agg}}.
\end{aligned}
\tag{A.10}
$$

Due to the Cobb-Douglas specification of utility, a fraction ξ of income used for consumption is spent on manufactures and a fraction $1-\xi$ on the agricultural good.

In the last stage, the static problem of distributing manufacturing consumption among different varieties for fixed manufacturing consumption expenditure ξe can be formulated as

$$
\begin{aligned}
\max_{c_m^H(i), c_m^F(j)} & \left[\int_0^K \left(c_m^H(i)\right)^{\frac{\sigma-1}{\sigma}} di + \int_0^{K^*} \left(c_m^F(j)\right)^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}} \\
\text{s.t.} & \int_0^K p_m^H(i) c_m^H(i) di + \int_0^{K^*} p_{m,\varphi}^F(j) c_m^F(j) dj = \xi e.
\end{aligned}
\tag{A.11}
$$

Setting up the Lagrangian

$$
\begin{aligned}
\ell(c_m^H(i), c_m^F(j), \lambda_m) = & \left[\int_0^K \left(c_m^H(i)\right)^{\frac{\sigma-1}{\sigma}} di + \int_0^{K^*} \left(c_m^F(j)\right)^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}} + \\
& \lambda_m \left[\xi e - \int_0^K p_m^H(i) c_m^H(i) di - \int_0^{K^*} p_{m,\varphi}^F(j) c_m^F(j) dj\right]
\end{aligned}
\tag{A.12}
$$

and solving for the first order conditions yields[3]

$$\frac{\partial \ell}{\partial c_m^H(i)} \stackrel{!}{=} 0 \;\Rightarrow\; \frac{\sigma}{\sigma-1}\left[\int_0^K \left(c_m^H(i)\right)^{\frac{\sigma-1}{\sigma}} di + \int_0^{K^*} \left(c_m^F(j)\right)^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{1}{\sigma-1}} \times \frac{\sigma-1}{\sigma}\left(c_m^H(i)\right)^{-\frac{1}{\sigma}} = \lambda_m p_m^H(i), \tag{A.13}$$

$$\frac{\partial \ell}{\partial c_m^F(j)} \stackrel{!}{=} 0 \;\Rightarrow\; \frac{\sigma}{\sigma-1}\left[\int_0^K \left(c_m^H(i)\right)^{\frac{\sigma-1}{\sigma}} di + \int_0^{K^*} \left(c_m^F(j)\right)^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{1}{\sigma-1}} \times \frac{\sigma-1}{\sigma}\left(c_m^F(j)\right)^{-\frac{1}{\sigma}} = \lambda_m p_{m,\varphi}^F(j), \tag{A.14}$$

$$\frac{\partial \ell}{\partial \lambda_m} \stackrel{!}{=} 0 \;\Rightarrow\; \int_0^K p_m^H(i) c_m^H(i) di + \int_0^{K^*} p_{m,\varphi}^F(j) c_m^F(j) dj = \xi e. \tag{A.15}$$

Recalling the definition of c_m^{agg} given below equation (2.1), these first order conditions can be rewritten as

$$c_m^{agg}\left[\int_0^K \left(c_m^H(i)\right)^{\frac{\sigma-1}{\sigma}} di + \int_0^{K^*} \left(c_m^F(j)\right)^{\frac{\sigma-1}{\sigma}} dj\right]^{-1} \left(c_m^H(i)\right)^{-\frac{1}{\sigma}} = \lambda_m p_m^H(i), \tag{A.16}$$

$$c_m^{agg}\left[\int_0^K \left(c_m^H(i)\right)^{\frac{\sigma-1}{\sigma}} di + \int_0^{K^*} \left(c_m^F(j)\right)^{\frac{\sigma-1}{\sigma}} dj\right]^{-1} \left(c_m^F(j)\right)^{-\frac{1}{\sigma}} = \lambda_m p_{m,\varphi}^F(j). \tag{A.17}$$

Isolating $c_m^H(i)$ and $c_m^F(j)$ on the left hand side, then multiplying both sides

[3] Note that this is in fact a variational problem.

by $p_m^H(i)$ or $p_{m,\varphi}^F(j)$ and finally integrating over all varieties yields

$$\int_0^K p_m^H(i)c_m^H(i)di =$$
$$\frac{\lambda_m^{-\sigma}\int_0^K(p_m^H(i))^{1-\sigma}di\left[\int_0^K(c_m^H(i))^{\frac{\sigma-1}{\sigma}}di+\int_0^{K^*}(c_m^F(j))^{\frac{\sigma-1}{\sigma}}dj\right]^{-\sigma}}{(c_m^{agg})^{-\sigma}},$$
$$\int_0^{K^*} p_{m,\varphi}^F(j)c_m^F(j)dj =$$
$$\frac{\lambda_m^{-\sigma}\int_0^{K^*}(p_{m,\varphi}^F(j))^{1-\sigma}dj\left[\int_0^K(c_m^H(i))^{\frac{\sigma-1}{\sigma}}di+\int_0^{K^*}(c_m^F(j))^{\frac{\sigma-1}{\sigma}}dj\right]^{-\sigma}}{(c_m^{agg})^{-\sigma}}.$$

Adding these two expressions, using the budget constraint from above and isolating λ_m gives the Lagrange multiplier

$$\lambda_m = \frac{(\xi e)^{-\frac{1}{\sigma}}c_m^{agg}\left[\int_0^K(p_m^H(i))^{1-\sigma}di+\int_0^{K^*}(p_{m,\varphi}^F(j))^{1-\sigma}dj\right]^{\frac{1}{\sigma}}}{\left[\int_0^K(c_m^H(i))^{\frac{\sigma-1}{\sigma}}di+\int_0^{K^*}(c_m^F(j))^{\frac{\sigma-1}{\sigma}}dj\right]}, \qquad \text{(A.18)}$$

i.e. the shadow price of manufacturing consumption. Plugging this expression back into equations (A.16) and (A.17) finally leads to the demands for all varieties

$$c_m^H(i) = \frac{\xi e(p_m^H(i))^{-\sigma}}{\left[\int_0^K(p_m^H(i))^{1-\sigma}di+\int_0^{K^*}(p_{m,\varphi}^F(j))^{1-\sigma}dj\right]},$$
$$c_m^F(j) = \frac{\xi e(p_{m,\varphi}^F(j))^{-\sigma}}{\left[\int_0^K(p_m^H(i))^{1-\sigma}di+\int_0^{K^*}(p_{m,\varphi}^F(j))^{1-\sigma}dj\right]}.$$

A.1.2 Aggregation over individuals

Following chapter 16 in Heijdra and van der Ploeg (2002), the aggregate Euler equation can be derived as follows. Taking the time derivative of aggregate consumption expenditures defined in equation (2.9) yields

$$\begin{aligned}\dot{E}(t) &= \mu e(t,t) + \mu \int_{-\infty}^{t} \dot{e}(t_0,t)e^{-\mu(t-t_0)} + e(t_0,t)(-\mu)e^{-\mu(t-t_0)}dt_0 \\ &= \mu e(t,t) - \mu E(t) + \mu \int_{-\infty}^{t} \dot{e}(t_0,t)e^{-\mu(t-t_0)}dt_0, \qquad \text{(A.19)}\end{aligned}$$

where we again used the definition of aggregate consumption expenditures in going from the first to the second line. Before arriving at the final aggregate Euler equation, it is necessary to derive optimal consumption expenditures $e(t,t)$ of newborns in the planning period t and the aggregate consumption expenditure rule $E(t)$. To achieve this, we reformulate the individual's optimization problem as follows. In line with equation (2.1), expected utility $U(t_0,t)$ at an arbitrary point in time t of a consumer born at time $t_0 \leq t$ is given by

$$U(t_0,t) \equiv \int_{t}^{\infty} e^{-(\rho+\mu)(\tau-t)} \ln\left(\frac{e(t_0,\tau)}{P(\tau)}\right) d\tau, \qquad \text{(A.20)}$$

where we again used the perfect price index P translating expenditures in indirect utility (see appendix A.1.1). The law of motion of capital given in equation (2.2) can be rewritten as

$$\begin{aligned}\dot{k}(t_0,\tau) &= \frac{w(\tau)l + \pi(\tau)k(t_0,\tau) - e(t_0,\tau)}{w(\tau)a_i} + \mu k(t_0,\tau) - \delta k(t_0,\tau) \\ &= \left(\frac{\pi(\tau)}{w(\tau)a_i} + \mu - \delta\right) k(t_0,\tau) + \frac{l}{a_i} - \frac{e(t_0,\tau)}{w(\tau)a_i}. \qquad \text{(A.21)}\end{aligned}$$

From equation (A.21) the individual's lifetime budget can be derived. First, both sides of the equation are multiplied by $e^{-R^A(t,\tau)} \equiv e^{-\int_t^\tau \left(\frac{\pi(s)}{w(s)a_i}+\mu-\delta\right)ds}$ and rearranged to

$$\left[\dot{k}(t_0,\tau) - \left(\frac{\pi(\tau)}{w(\tau)a_i} + \mu - \delta\right) k(t_0,\tau)\right] e^{-R^A(t,\tau)} = \left[\frac{l}{a_i} - \frac{e(t_0,\tau)}{w(\tau)a_i}\right] e^{-R^A(t,\tau)}. \qquad \text{(A.22)}$$

Observing that the left hand side of equation (A.22) is $d\left[k(t_0,\tau)e^{-R^A(t,\tau)}\right]/d\tau$ by applying the Leibniz rule to recognize that $dR^A(t,\tau)/d\tau = \frac{\pi(\tau)}{w(\tau)a_i} + \mu - \delta$

and integrating over the interval $[t, \infty)$ yields

$$\int_t^\infty d\left[k(t_0,\tau)e^{-R^A(t,\tau)}\right] = \int_t^\infty \left[\frac{l}{a_i} - \frac{e(t_0,\tau)}{w(\tau)a_i}\right] e^{-R^A(t,\tau)} d\tau.$$

This expression can be solved to

$$\lim_{\tau\to\infty} k(t_0,\tau)e^{-R^A(t,\tau)} - k(t_0,t)e^{-R^A(t,t)} = h(t) - \int_t^\infty \frac{e(t_0,\tau)}{w(\tau)a_i} e^{-R^A(t,\tau)} d\tau, \tag{A.23}$$

where $h(t) \equiv \int_t^\infty \frac{w(\tau)l}{w(\tau)a_i} e^{-R^A(t,\tau)} d\tau$ denotes human wealth of individuals in capital units consisting of the present value of lifetime wage income using the annuity factor $R^{A(t,\tau)}$ for discounting. Note that $e^{-R^A(t,t)} = 1$ and that the first term on the left hand side represents "terminal capital holdings". These holdings must be equal to zero because first, the insurance company will ensure their nonnegativity, and second, it is suboptimal for an individual to have positive terminal assets as there is neither a bequest motive nor satiation from consumption. Taking this into account, yields the following solvency condition

$$\lim_{\tau\to\infty} e^{-R^A(t,\tau)} k(t_0,\tau) = 0, \tag{A.24}$$

which prevents an individual from running a Ponzi game against the life-insurance company. The No-Ponzi-Game condition can be inserted in equation (A.23) to obtain the individual's lifetime budget restriction

$$k(t_0,t) + h(t) = \int_t^\infty \frac{e(t_0,\tau)}{w(\tau)a_i} e^{-R^A(t,\tau)} d\tau. \tag{A.25}$$

The present value of an individual's consumption expenditure plan in capital units must be equal to the sum of human wealth in capital units and capital holdings (=total wealth). Evaluating the lifetime budget constraint at $t = t_0$ shows that the discounted sum of lifetime labor earnings must equal discounted consumption expenditures.[4] This implies, from investigating the law of motion of capital, that discounted savings are equal to discounted accumulated profits, i.e. savings are only used for reallocating consumption

[4]Recall that capital holdings of newborns $k(t_0, t_0)$ are zero by assumption (no bequests).

across lifetime.

Maximizing expected utility given in equation (A.20) subject to the budget constraint in equation (A.25) yields the following first order condition

$$\frac{1}{e(t_0,\tau)}e^{-(\rho+\mu)(\tau-t)} = \lambda(t)\frac{1}{w(\tau)a_i}e^{-R^A(t,\tau)}, \quad \tau \in [t,\infty), \tag{A.26}$$

where $\lambda(t)$ represents the marginal expected lifetime utility of wealth.[5] Individuals should therefore plan consumption expenditures in a way such that the appropriately discounted marginal utility of expenditures and wealth are equated.

Applying equation (A.26) to the planning period ($\tau = t$) yields $e(t_0,t) = \frac{w(t)a_i}{\lambda(t)}$. Using this result and then substituting for $\lambda(t)$ also from the first order condition in equation (A.26) helps to establish the following equality

$$\begin{aligned}
\int_t^\infty e(t_0,t)e^{-(\rho+\mu)(\tau-t)}d\tau &= \int_t^\infty \frac{w(t)a_i}{\lambda(t)}e^{-(\rho+\mu)(\tau-t)}d\tau \\
&= a_i w(t)\int_t^\infty \frac{e(t_0,\tau)}{a_i w(\tau)}e^{-R^A(t,\tau)}d\tau.
\end{aligned}$$

Integrating out and using the lifetime budget constraint of equation (A.25) finally yields consumption expenditures $e(t_0,t)$ in the planning period t

$$\begin{aligned}
\frac{e(t_0,t)}{\rho+\mu}\left[-e^{-(\rho+\mu)(\tau-t)}\right]_t^\infty &= a_i w(t)[k(t_0,t)+h(t)] \\
e(t_0,t) &= (\rho+\mu)a_i w(t)[k(t_0,t)+h(t)]. \qquad \text{(A.28)}
\end{aligned}$$

The above equation clearly shows that optimal consumption expenditures in the planning period t in capital units, $\frac{e(t_0,t)}{a_i w(t)}$, are proportional to total wealth with the marginal propensity to consume out of total wealth being constant

[5]Differentiating this first order condition with respect to τ, inserting the expression for $\lambda(t)$ also obtainable from this first order condition and simplifying yields the following Euler equation

$$\frac{\dot{e}(t_0,\tau)}{e(t_0,\tau)} = \frac{\pi(\tau)}{w(\tau)a_i} - \rho - \delta + \frac{\dot{w}(\tau)}{w(\tau)}. \tag{A.27}$$

With time-invariant wages (see section 2.2.4), this Euler equation is exactly the same as the one obtained in equation (2.6).

and equal to the effective rate of time preference $\rho + \mu$.

Using this expression for optimal consumption expenditures in the definition of aggregate consumption expenditures in equation (2.9) yields the following very simple aggregate consumption expenditure rule

$$\begin{aligned} E(t) &\equiv \mu \int_{-\infty}^{t} e^{-\mu(t-t_0)}(\rho+\mu)a_i w(t)[k(t_0,t)+h(t)]dt_0 \\ &= (\rho+\mu)a_i w(t)\mu \left[\int_{-\infty}^{t} e^{-\mu(t-t_0)}k(t_0,t)dt_0 + \int_{-\infty}^{t} e^{-\mu(t-t_0)}h(t)dt_0\right] \\ &= (\rho+\mu)a_i w(t)\left[K(t)+h(t)\right], \end{aligned} \tag{A.29}$$

where the aggregate capital stock is defined in equation (2.7) and can be rewritten in analogy to aggregate consumption expenditures in equation (2.9). Moreover it is easily established that $\mu h(t)\left[\frac{e^{-\mu(t-t_0)}}{\mu}\right]_{-\infty}^{t} = h(t)$.[6]

Finally, we modify equation (A.19) by substituting for $e(t,t)$ and $E(t)$ from the derived expressions of equation (A.28) evaluated at birth date t[7] and equation (A.29) as well as for $\dot{e}(t_0,t)$ from the individual Euler equation given in expression (A.27). Dividing by $E(t)$ then gives the aggregate Euler equation

$$\begin{aligned} \frac{\dot{E}(t)}{E(t)} &= -\mu(\rho+\mu)a_i w(t)\frac{K(t)}{E(t)} + \\ &\quad \frac{\mu}{E(t)}\int_{-\infty}^{t} e(t_0,t)\left[\frac{\pi(t)}{w(t)a_i} - \rho - \delta + \frac{\dot{w}(t)}{w(t)}\right]e^{-\mu(t-t_0)}dt_0 \\ &= -\mu(\rho+\mu)a_i w(t)\frac{K(t)}{E(t)} + \frac{\pi(t)}{w(t)a_i} - \rho - \delta + \frac{\dot{w}(t)}{w(t)} \\ &= -\mu\frac{E(t)-e(t,t)}{E(t)} + \frac{\dot{e}(t_0,t)}{e(t_0,t)}, \end{aligned}$$

where in the third line we used again the definition of aggregate consumption expenditures from equation (2.9) and the term $\dot{w}(t)/w(t)$ disappears in the

[6] This aggregation property of consumption expenditures is due to the fact that we assume a constant probability of death implying an age independent marginal propensity to consume out of total wealth (see equation (A.28)).

[7] Note again that $k(t,t) = 0$ and newborns therefore consume a fraction of their human wealth at birth, i.e. $e(t,t) = (\rho+\mu)a_i w(t)h(t)$.

case of time-invariant wages (see section 2.2.4).

A.1.3 The manufacturing firm's profit maximization problem - Derivation of rental rates

By substituting for optimal demands for varieties from the constraints of the maximization problem as stated in equation (2.18), operating profit can be rewritten as

$$\begin{aligned}&(p_m^H(i,t) - w(t)a_m)\left(\int_{-\infty}^{t} \frac{\xi e(t_0,t)(p_m^H(i,t))^{-\sigma}}{P_m(t)} N(t_0,t)dt_0\right) + \\ &(p_{m,\varphi}^F(i,t) - w(t)\varphi a_m)\left(\int_{-\infty}^{t} \frac{\xi e^*(t_0,t)(p_{m,\varphi}^F(i,t))^{-\sigma}}{P_m^*(t)} N^*(t_0,t)dt_0\right),\end{aligned} \tag{A.30}$$

whose derivatives with respect to $p_m^H(i,t)$ and $p_{m,\varphi}^F(i,t)$ are set equal to zero to yield the first order conditions

$$\frac{\int_{-\infty}^{t} \xi e(t_0,t)N(t_0,t)dt_0}{P_m(t)}\left[(1-\sigma)(p_m^H(i,t))^{-\sigma} + \sigma w(t)a_m(p_m^H(i,t))^{-\sigma-1}\right] = 0,$$

$$\frac{\int_{-\infty}^{t} \xi e^*(t_0,t)N^*(t_0,t)dt_0}{P_m^*(t)}\left[(1-\sigma)(p_{m,\varphi}^F(i,t))^{-\sigma} + \sigma w(t)a_m\varphi(p_{m,\varphi}^F(i,t))^{-\sigma-1}\right] = 0.$$

Rearranging and simplifying gives optimal prices

$$\begin{aligned}p_m^H(i,t) &= \frac{\sigma}{\sigma-1}w(t)a_m, \\ p_{m,\varphi}^F(i,t) &= \frac{\sigma}{\sigma-1}w(t)a_m\varphi.\end{aligned}$$

Using these pricing rules and the definition of aggregate expenditures given in equation (2.8) in equation (A.30) and simplifying yields operating profits as

$$\pi(t) = \frac{\xi E(t)}{\sigma(K(t)+\varphi^{1-\sigma}K^*(t))} + \frac{\xi\varphi^{1-\sigma}E^*(t)}{\sigma(\varphi^{1-\sigma}K(t)+K^*(t))}, \tag{A.31}$$

where an equivalent equation holds in the foreign region. Note that the variety index i can be dropped since prices and therefore profits are equal for all firms. Applying the definitions of regional share variables and global quantities as well as the definition of trade openness yields the final expressions for regional rental rates[8]

$$\begin{aligned}
\pi &= \left(\frac{\theta_E}{\theta_K + \phi(1-\theta_K)} + \frac{(1-\theta_E)\phi}{\phi\theta_K + 1 - \theta_K}\right)\left(\frac{\xi E^W}{\sigma K^W}\right), \\
\pi^* &= \left(\frac{1-\theta_E}{1-\theta_K + \phi\theta_K} + \frac{\theta_E\phi}{\phi(1-\theta_K) + \theta_K}\right)\left(\frac{\xi E^W}{\sigma K^W}\right).
\end{aligned}$$

A.1.4 Intermediate results for the stability analysis

The Jacobian matrix J_{sym}, which is evaluated at the symmetric equilibrium and given in equation (2.30), has the following entries J_i for $i = 1, \ldots, 4$:

$$\begin{aligned}
J_1 &= \frac{1}{2(\phi+1)\sqrt{\sigma}}\begin{pmatrix} A(\phi+2) - B\phi & (A+B)\phi \\ (A+B)\phi & A(\phi+2) - B\phi \end{pmatrix}, \\
J_2 &= \begin{pmatrix} \frac{-a_i(A+B)^2(\phi^2+1)}{4(\phi+1)^2\xi} - a_i\mu(\mu+\rho) & -\frac{(A+B)^2 a_i\phi}{2(\phi+1)^2\xi} \\ -\frac{(A+B)^2 a_i\phi}{2(\phi+1)^2\xi} & \frac{-a_i(A+B)^2(\phi^2+1)}{4(\phi+1)^2\xi} - a_i\mu(\mu+\rho) \end{pmatrix}, \\
J_3 &= \frac{1}{a_i(\phi+1)\sigma}\begin{pmatrix} \xi - (\phi+1)\sigma & \phi\xi \\ \phi\xi & \xi - (\phi+1)\sigma \end{pmatrix}, \\
J_4 &= \begin{pmatrix} \frac{\phi(A+\rho\sqrt{\sigma}) - \delta(\phi^2+\phi+1)\sqrt{\sigma}}{(\phi+1)^2\sqrt{\sigma}} & -\frac{(A+B)\phi}{(\phi+1)^2\sqrt{\sigma}} \\ -\frac{(A+B)\phi}{(\phi+1)^2\sqrt{\sigma}} & \frac{\phi(A+\rho\sqrt{\sigma}) - \delta(\phi^2+\phi+1)\sqrt{\sigma}}{(\phi+1)^2\sqrt{\sigma}} \end{pmatrix}
\end{aligned}$$

with the parameter clusters $A \equiv \sqrt{\sigma(\delta+\rho)^2 + 4\mu(\mu+\rho)\xi}$ as well as $B \equiv (\delta+\rho)\sqrt{\sigma}$.

In order to get a first insight into the nature and signs of the eigenvalues of J_{sym}, we calibrated the model using the parameter values $\rho = 0.015$ and $\delta = 0.05$ and allowing the elasticity of substitution and the manufacturing share of consumption to vary within the ranges $2 \leq \sigma \leq 8$ and $0.1 \leq \xi \leq 0.9$. Figures A.1, A.2, A.3 and A.4 illustrate the numerical investigation of the signs of the eigenvalues for $\sigma = 4$, $\xi = 0.3$ and varying μ and ϕ.[9]

[8] We ignore time arguments here.

[9] We also conducted the same simulations for other values of σ and ξ within the considered range. Overall, our findings with respect to the signs of the eigenvalues are insensitive to changes in those parameters.

Figure A.1: Eigenvalue 1 for varying μ and ϕ

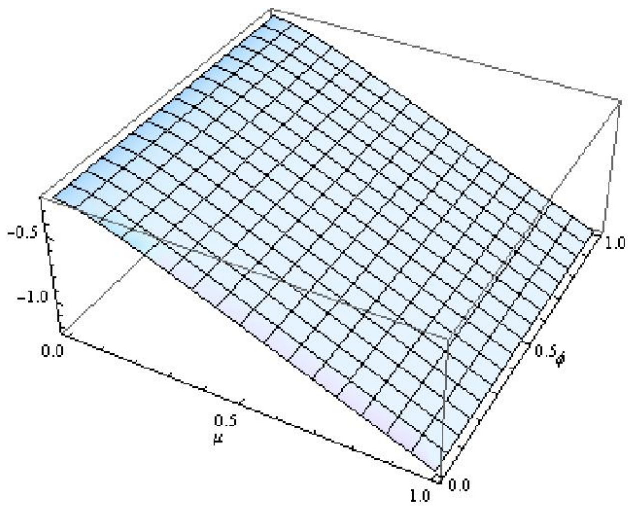


Figure A.2: Eigenvalue 2 for varying μ and ϕ

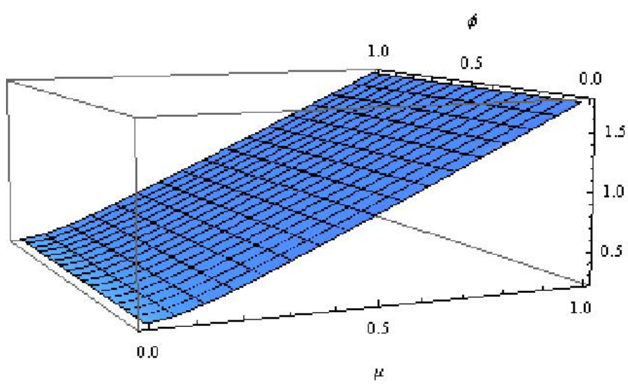


Figure A.3: Eigenvalue 3 for varying μ and ϕ

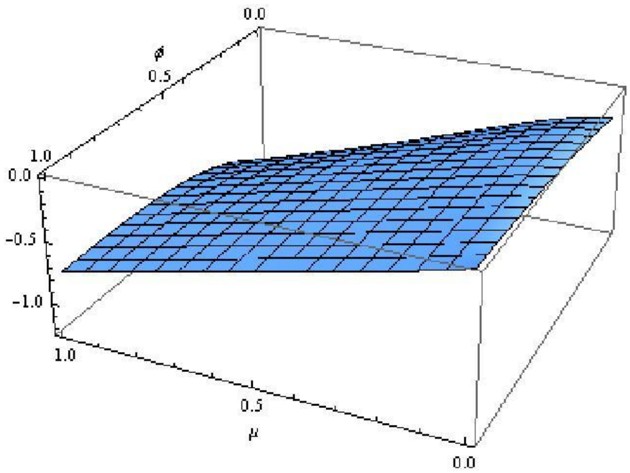
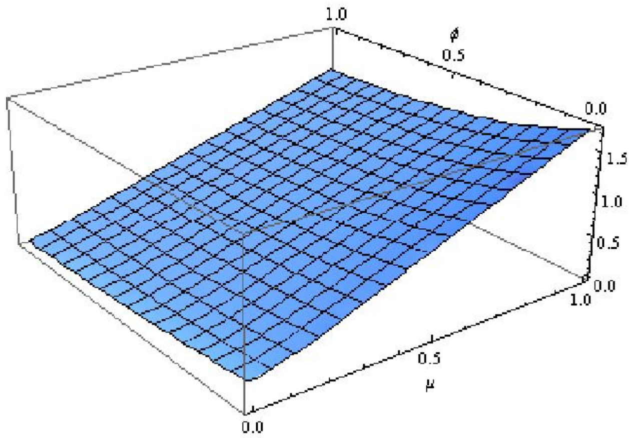


Figure A.4: Eigenvalue 4 for varying μ and ϕ



First, the figures suggest that all eigenvalues are real for the chosen parameter space. Moreover, figures A.1, A.2 and A.4 show that the first eigenvalue is always negative, whereas the second and fourth are always positive. This result is independent of the level of transport costs and the mortality rate. Saddle path stability of the symmetric equilibrium therefore seems to crucially depend on the third eigenvalue by requiring it to be negative. As can be seen from the 3D plot in figure A.3 there only exists a very small range of combinations of low μ and high ϕ where the sign of the third eigenvalue is positive. One is therefore tempted to conclude that with a sufficiently high mortality rate, the symmetric equilibrium is stable for all levels of trade openness.

A.2 Chapter 3

A.2.1 Aggregation over individuals

Cohort size

Since $e^{-\mu(t-t_0)}$ is the probability of an individual of a cohort born at t_0 to survive to time t and the cohort size of newborns is given by $N(t_0, t_0) = \beta N(t_0)$, the size of a cohort born at t_0 at time t can be rewritten as

$$\begin{aligned} N(t_0, t) &= \beta N(t_0) e^{-\mu(t-t_0)} \\ &= \beta N(0) e^{n t_0} e^{-\mu(t-t_0)} \\ &= \beta N(0) e^{(\beta-\mu) t_0} e^{-\mu(t-t_0)} \\ &= \beta N(0) e^{\beta t_0} e^{-\mu t} \\ &= \beta e^{\beta t_0} e^{-\mu t}, \end{aligned} \tag{A.32}$$

where we use that population size N(t) grows with $n = \beta - \mu$ and we normalize $N(0) = 1$.

Aggregate expenditures and capital

To obtain the laws of motion of per capita expenditures and capital, we must first derive the aggregate consumption rule and the aggregate law of motion of capital. First, note that individual utility maximization yields the individual consumption expenditures rule[10]

$$e(t_0, t) = (\rho + \mu) a_i w(t) [k(t_0, t) + h(t)], \tag{A.33}$$

where

$$h(t) \equiv \int_t^{\infty} \frac{l}{a_i} e^{-R^A(t,\tau)} d\tau \tag{A.34}$$

is human wealth of individuals in capital units, i.e. the present value of lifetime wage income with the annuity factor $e^{-R^A(t,\tau)} \equiv e^{-\int_t^{\tau} \left(\frac{\pi(s)}{w(s) a_i} + \mu - \delta \right) ds}$ used for discounting. Optimal individual consumption expenditures in the

[10]For details of the derivations see chapter 2 and its appendix A.1.

planning period t in capital units, $\frac{e(t_0,t)}{a_i w(t)}$, are proportional to total wealth with the marginal propensity to consume out of total wealth being constant and equal to the effective rate of time preference $\rho+\mu$. As will become clear soon, it is also necessary to derive an expression for the law of motion of individual human wealth being equal to per capita human wealth.[11] Applying twice the Leibniz rule to equation (A.34) yields

$$\begin{aligned}\dot{\tilde{h}}(t) = \dot{h}(t) &= -\frac{l}{a_i} + \int_t^\infty \frac{l}{a_i} e^{-R^A(t,\tau)}(-1)\left[-\left(\frac{\pi(t)}{w(t)a_i} + \mu - \delta\right)\right] d\tau \\ &= \left(\frac{\pi(t)}{w(t)a_i} + \mu - \delta\right) h(t) - \frac{l}{a_i}. \qquad \text{(A.35)}\end{aligned}$$

Using equation (A.33) in equation (3.9) yields the aggregate consumption expenditure rule

$$\begin{aligned}E(t) &= \beta \int_{-\infty}^{t} (\rho+\mu) a_i w(t)[k(t_0,t) + h(t)] e^{-\mu t + \beta t_0} dt_0 \\ &= \beta(\rho+\mu) a_i w(t) e^{-\mu t} \int_{-\infty}^{t} [k(t_0,t) + h(t)] e^{\beta t_0} dt_0 \\ &= (\rho+\mu) a_i w(t) \left[K(t) + H(t)\right], \qquad \text{(A.36)}\end{aligned}$$

where

$$H(t) = \beta e^{-\mu t} \int_{-\infty}^{t} h(t) e^{\beta t_0} dt_0 = h(t) e^{(\beta-\mu)t} \qquad \text{(A.37)}$$

represents aggregate human wealth.

The law of motion of aggregate capital can be obtained from equation

[11] With age independency, individual variables of course equal per capita variables, e.g. $\tilde{h}(t) = h(t)$ and $\tilde{l} = l$.

(3.8) by again applying the Leibniz rule as

$$\begin{aligned}
\dot{K}(t) &= \beta\left(\underbrace{k(t,t)e^{(\beta-\mu)t}}_{0}-0\right)+\\
&\quad \beta\left[\int_{-\infty}^{t}\dot{k}(t_0,t)e^{\beta t_0}e^{-\mu t}dt_0+\int_{-\infty}^{t}k(t_0,t)e^{\beta t_0}e^{-\mu}(-\mu)dt_0\right]\\
&= -\mu K(t)+\\
&\quad \beta\int_{-\infty}^{t}\left(\frac{w(t)l+\pi(t)k(t_0,t)-e(t_0,t)}{w(t)a_i}+(\mu-\delta)k(t_0,t)\right)e^{\beta t_0}e^{-\mu t}dt_0\\
&= -\mu K(t)+\frac{L(t)}{a_i}-\frac{E(t)}{w(t)a_i}+\frac{\pi(t)}{w(t)a_i}K(t)+\mu K(t)-\delta K(t)\\
&= \left(\frac{\pi(t)}{w(t)a_i}-\delta\right)K(t)+\frac{L(t)}{a_i}-\frac{E(t)}{w(t)a_i}, \qquad \text{(A.38)}
\end{aligned}$$

where we used the individual law of motion of capital given in equation (3.3) to go from the first to the second line.[12] In contrast to the individual law of motion of capital (3.3), the aggregate law of motion for capital does not feature the mortality rate, since $\mu K(t)$ just captures the transfer of capital of dying to surviving individuals by the life insurance companies which does not change the rate of return on aggregate capital.

Based on this aggregate law of motion of capital we are now ready to obtain the law of motion of per capita capital as

$$\begin{aligned}
\dot{\tilde{k}}(t) &= \dot{K}(t)e^{-nt}+K(t)e^{-nt}(-n)\\
&= \left[\left(\frac{\pi(t)}{w(t)a_i}-\delta\right)K(t)+\frac{L(t)}{a_i}-\frac{E(t)}{w(t)a_i})\right]e^{-nt}+K(t)e^{-nt}(-n)\\
&= \frac{\tilde{l}}{a_i}-\frac{\tilde{e}(t)}{w(t)a_i}+\left(\frac{\pi(t)}{w(t)a_i}-\delta\right)\tilde{k}(t)-n\tilde{k}(t)\\
&= \left(\frac{\pi(t)}{w(t)a_i}-\delta-\beta+\mu\right)\tilde{k}(t)+\frac{\tilde{l}}{a_i}-\frac{\tilde{e}(t)}{w(t)a_i}.
\end{aligned}$$

Finally, the per capita Euler equation is obtained from differentiating the per capita version of the aggregate consumption rule given in equation

[12] Recall also that capital holdings of newborns k(t,t) are zero by assumption (no bequests).

(A.36) with respect to time and substituting in the per capita law of motions of capital and human wealth. This yields

$$
\begin{aligned}
\tilde{e}(t) &= (\rho+\mu)a_i w(t)\left[\tilde{k}(t)+\tilde{h}(t)\right] \qquad \text{(A.39)}\\
\dot{\tilde{e}}(t) &= (\rho+\mu)a_i w(t)\left[\dot{\tilde{k}}(t)+\dot{\tilde{h}}(t)\right]\\
&= (\rho+\mu)a_i w(t)\left[\left(\frac{\pi(t)}{w(t)a_i}-\delta-n\right)\tilde{k}(t)+\frac{\tilde{l}}{a_i}-\frac{\tilde{e}(t)}{w(t)a_i}\right]+\\
&\quad (\rho+\mu)a_i w(t)\left[\left(\frac{\pi(t)}{w(t)a_i}-\delta+\mu\right)\tilde{h}(t)-\frac{\tilde{l}}{a_i}\right]\\
&= (\rho+\mu)a_i w(t)\left(-\frac{\tilde{e}(t)}{w(t)a_i}\right)+(\rho+\mu)a_i w(t)\left(\frac{\pi(t)}{w(t)a_i}-\delta\right)\left[\tilde{k}(t)+\tilde{h}(t)\right]+\\
&\quad (\rho+\mu)a_i w(t)\left[-n\tilde{k}(t)+\mu\tilde{h}(t)\right].
\end{aligned}
$$

Substituting in $\tilde{e}(t)$ from equation (A.39), we can rewrite $\dot{\tilde{e}}(t)$ as

$$
\begin{aligned}
\dot{\tilde{e}}(t) &= (\rho+\mu)\left(-\tilde{e}(t)\right)+\tilde{e(t)}\left(\frac{\pi(t)}{w(t)a_i}-\delta\right)+(\rho+\mu)a_i w(t)\left[-n\tilde{k}(t)+\mu\tilde{h}(t)\right]\\
&= \left[\left(\frac{\pi(t)}{w(t)a_i}-\delta\right)-(\rho+\mu)\right]\tilde{e}(t)+(\rho+\mu)a_i w(t)\mu\tilde{h}(t)-n(\rho+\mu)a_i w(t)\tilde{k}(t)\\
&= \left[\left(\frac{\pi(t)}{w(t)a_i}-\delta\right)-(\rho+\mu)\right]\tilde{e}(t)+(\rho+\mu)a_i w(t)\mu\left[\frac{\tilde{e}(t)}{(\rho+\mu)a_i w(t)}-\tilde{k}(t)\right]\\
&\quad -n(\rho+\mu)a_i w(t)\tilde{k}(t)\\
&= \left[\frac{\pi(t)}{w(t)a_i}-\delta-\rho\right]\tilde{e}(t)+\left[-\mu(\rho+\mu)a_i w(t)(\tilde{k}(t))-n(\rho+\mu)a_i w(t)\tilde{k}(t)\right]\\
&= \left[\frac{\pi(t)}{w(t)a_i}-\delta-\rho\right]\tilde{e}(t)-\beta(\rho+\mu)a_i w(t)\tilde{k}(t),
\end{aligned}
$$

where we used that $\tilde{h}(t)=\frac{\tilde{e}(t)}{(\rho+\mu)a_i w(t)}-\tilde{k}(t)$.

Therefore we have a two dimensional dynamic system of the following form

$$\begin{aligned}
\dot{\tilde{e}}(t) &= \left[\frac{\pi(t)}{w(t)a_i} - \delta - \rho\right]\tilde{e}(t) - \beta(\rho+\mu)a_i w(t)\tilde{k}(t) \\
\dot{\tilde{k}}(t) &= \left(\frac{\pi(t)}{w(t)a_i} - \delta - \beta + \mu\right)\tilde{k}(t) + \frac{\tilde{l}}{a_i} - \frac{\tilde{e}(t)}{w(t)a_i}
\end{aligned}$$

with analogous equations holding in the foreign region.

Note, finally, that by using equation (A.39) and equation (A.33) for $t_0 = t$ the law of motion of per capita consumption expenditures can be rewritten as

$$\begin{aligned}
\frac{\dot{\tilde{e}}(t)}{\tilde{e}(t)} &= \left[\frac{\pi(t)}{w(t)a_i} - \delta - \rho\right] - \beta\frac{\tilde{e}(t) - e(t,t)}{\tilde{e}(t)} \\
&= \frac{\dot{e}(t_0,\tau)}{e(t_0,\tau)} - \beta\frac{\tilde{e}(t) - e(t,t)}{\tilde{e}(t)}.
\end{aligned}$$

A.2.2 Derivation of rental rates

Rental rates given in equations (3.16) and (3.17) can be rewritten as[13]

$$\begin{aligned}
\pi &= \left(\frac{E}{K+\phi K^*} + \frac{E^*\phi}{\phi K + K^*}\right)\left(\frac{\xi}{\sigma}\right), \\
\pi^* &= \left(\frac{E^*}{K^*+\phi K} + \frac{E\phi}{\phi K^* + K}\right)\left(\frac{\xi}{\sigma}\right).
\end{aligned}$$

By multiplying the nominator as well as the denominator by e^{-nt}, we arrive at

$$\pi = \left(\frac{\tilde{e}}{\tilde{k}+\phi\tilde{k}^*} + \frac{\tilde{e}^*\phi}{\phi\tilde{k} + \tilde{k}^*}\right)\left(\frac{\xi}{\sigma}\right), \tag{A.40}$$

$$\pi^* = \left(\frac{\tilde{e}^*}{\tilde{k}^*+\phi\tilde{k}} + \frac{\tilde{e}\phi}{\phi\tilde{k}^* + \tilde{k}}\right)\left(\frac{\xi}{\sigma}\right). \tag{A.41}$$

[13]Note that we suppress time arguments here.

A.2.3 Intermediate results for the stability analysis

The Jacobian matrix J_{sym}, which is evaluated at the symmetric equilibrium and given in equation (3.24), has the following entries J_i for $i = 1, \ldots, 4$

$$
\begin{aligned}
J_1 &= \frac{1}{2(\phi+1)\sqrt{\sigma}} \begin{pmatrix} A(\phi+2) - B\phi & \phi(A+B) \\ \phi(A+B) & A(\phi+2) - B\phi \end{pmatrix}, \\
J_2 &= \begin{pmatrix} -\frac{a_i\left(\phi^2+1\right)(A+B)^2}{4(\phi+1)^2\xi} - \beta a_i(\mu+\rho) & -\frac{a_i\phi(A+B)^2}{2(\phi+1)^2\xi} \\ -\frac{a_i\phi(A+B)^2}{2(\phi+1)^2\xi} & -\frac{a_i\left(\phi^2+1\right)(A+B)^2}{4(\phi+1)^2\xi} - \beta a_i(\mu+\rho) \end{pmatrix}, \\
J_3 &= \frac{1}{a_i(\phi+1)\sigma} \begin{pmatrix} \xi - (\phi+1)\sigma & \phi\xi \\ \phi\xi & \xi - (\phi+1)\sigma \end{pmatrix}, \\
J_4 &= \begin{pmatrix} \frac{A\phi+\sqrt{\sigma}\left(-\beta(\phi+1)^2-\delta\left(\phi^2+\phi+1\right)+\mu(\phi+1)^2+\phi\rho\right)}{(\phi+1)^2\sqrt{\sigma}} & -\frac{\phi(A+B)}{(\phi+1)^2\sqrt{\sigma}} \\ -\frac{\phi(A+B)}{(\phi+1)^2\sqrt{\sigma}} & \frac{A\phi+\sqrt{\sigma}\left(-\beta(\phi+1)^2-\delta\left(\phi^2+\phi+1\right)+\mu(\phi+1)^2+\phi\rho\right)}{(\phi+1)^2\sqrt{\sigma}} \end{pmatrix}
\end{aligned}
$$

with the parameter clusters $A \equiv \sqrt{\sigma(\delta+\rho)^2 + 4\beta(\mu+\rho)\xi}$ as well as $B \equiv (\delta+\rho)\sqrt{\sigma}$.

A.3 Chapter 4

A.3.1 Intermediate results for the stability analysis

The Jacobian matrix given in equation (4.27) has the following entries

$$
\begin{aligned}
J_{11} &= -\eta L - L - \rho + \frac{(\eta L + L + \rho + eig1)\left(\eta + \frac{\eta\xi - \eta\phi\xi}{\phi\sigma + \sigma} + 2\right)}{2(\eta+1)}, \\
J_{12} &= \frac{\eta\,(\eta L + L + \rho + eig1)\left(\frac{(\phi-1)\xi}{(\phi+1)\sigma} + 1\right)}{2(\eta+1)}, \\
J_{13} &= \frac{4\eta\left(\frac{(\eta L+L+\rho+eig1)\left(-L\eta^2+L+\frac{1}{2}(\eta L+L+\rho+eig1)\left(\eta-\frac{2\left(\eta\phi^2-2\phi+\eta\right)\xi}{(\phi+1)^2\sigma}-1\right)\right)}{2(\eta+1)} - \mu(\mu+\rho)\right)}{(\eta+1)^2}, \\
J_{31} &= -\frac{(\eta+1)((\phi+1)\sigma + (\phi-1)\xi)}{4(\phi+1)\sigma}, \\
J_{33} &= \frac{(\eta L + L + \rho + eig1)\left(\eta - \frac{(\eta\phi^2 - 2\phi + \eta)\xi}{(\phi+1)^2\sigma}\right)}{\eta+1} - 2L\eta,
\end{aligned}
$$

where $eig1 = \sqrt{L^2(\eta+1)^2 + 2L\rho(\eta+1) + (2\mu+\rho)^2}$.

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Zusammenfassung

Wie beeinflussen demografische Strukturen Agglomerationsprozesse? Die drei Artikel dieser Dissertation widmen sich der Beantwortung dieser Frage, indem sie die Auswirkungen von demografischem Wandel sowohl auf die räumliche Verteilung von ökonomischer Aktivität als auch auf den Zusammenhang von Wachstum und Agglomeration untersuchen.

Der erste Aufsatz entwickelt ein neuartiges theoretisches Basismodell, das einen New Economic Geography Ansatz um überlappende Generationen erweitert. Dies ermöglicht es, den grundlegenden Zusammenhang zwischen demografischen Strukturen und Agglomeration zu analysieren. Durch die Berücksichtigung von Lebenszyklen, im Besonderen durch die Verteilungseffekte infolge des Generationenwechsels, werden Agglomerationsprozesse deutlich abgeschwächt. Für plausible demografische Strukturen findet Agglomeration somit auch bei zunehmender ökonomischer Integration nicht statt.

Der zweite Artikel erweitert das Basismodell um eine detaillierte Modellierung von demografischen Wandel, indem er unterschiedliche Geburten- und Sterberaten und somit variierende Bevölkerungsgrößen zulässt. Dadurch können sowohl die Auswirkungen von Änderungen in der Altersstruktur als auch in der Bevölkerungswachstumsrate analysiert werden. Während Bevölkerungsalterung Agglomerationstendenzen verstärkt, erweist sich Bevölkerungswachstum als Dispersionskraft.

Erweitert man, wie im dritten Artikel, das Modell schließlich um endogenes Wachstum aufgrund von Lerneffekten, kann auch die Rolle von demografischen Strukturen für den Zusammenhang zwischen Wachstum und räumlicher Verteilung von ökonomischer Aktivität analysiert werden. Generell senkt die Berücksichtigung von begrenzter Lebensdauer die langfristigen Wachstumsraten. Gleichzeitig wird dadurch der wachstumsfördernde Effekt von Agglomeration infolge von räumlich begrenzten Lerneffekten abgeschwächt.

Abstract

How do demographic structures impact upon agglomeration processes? The three articles comprising this thesis are dedicated toward answering this question by focusing on the effects of demographic change on both the spatial distribution of economic activity and the linkage between growth and agglomeration.

The first essay develops a novel theoretical baseline framework that merges the two research strands of New Economic Geography and overlapping generation models. This allows to establish and analyze the main linkage between demography and agglomeration processes. We find that the introduction of an overlapping generation structure considerably reduces agglomeration tendencies due to the distributional effects from the turnover of generations. For plausible demographic structures, agglomeration processes thus do not set in even if economic integration is promoted up to a high degree.

The second article extends the above benchmark model with a view to focusing on the effects of changes in both the population age structure and in the population growth rate on agglomeration processes. To do so, it incorporates a more detailed analysis of demographic change by allowing for unequal birth and death rates and thus varying population size. While population aging strengthens agglomeration tendencies, population growth acts as a dispersion force.

Finally, the third essay additionally allows for endogenous growth due to learning spillovers in order to also investigate the impact of demography on the linkage between growth and the spatial distribution of economic activity. Lifetime uncertainty is shown to decrease long-run growth perspectives. In doing so, it mitigates the pro-growth effects of agglomeration resulting from the localized nature of learning spillovers.

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