# MASTERARBEIT 

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# „Transfer Pricing and Strategic Delegation" 

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## Foreword

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A major force that led to the successful completion of this thesis has been the sum of supportive actions undertaken by my friends and family who believed in me, my abilities and my willingness to complete it, even if I had doubts sometimes. Thank you!

## Declaration on Oath

Herewith I affirm that I have written the master's thesis "Transfer Pricing and Strategic Delegation" entirely on my own and have not used outside sources without declaration in the text. Any concepts or quotations applicable to these sources are clearly attributed to them.

This diploma thesis has not been submitted in the same or substantially similar version, not even in part, to any other authority for grading and has not been published elsewhere.

Signature, Date:

(Patrick-Philipp Valda, Bakk. rer. soc. eec.)

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## Abbreviations

1A Division 1 of company AB
1B Division 2 of company AB
$\Pi_{1 A} \quad$ Profit of division 1 of company AB consisting of two divisions, A and B
$\Pi_{i A} \quad$ Profit of division A of company $i$
$\Pi_{1 B} \quad$ Profit of division 2 of company AB consisting of two divisions, A and B
$\Pi_{A B} \quad$ Company AB's consolidated profit
$\gamma \quad$ Level of substitution
$\delta \quad$ Subsidy or markup set by the central office
$a_{F} \quad$ Buyers' willingness to pay for the good on the final market
$a_{Z} \quad$ Buyers' willingness to pay for the good on the intermediate market
c Unit costs for production of goods on the intermediate market
$P_{F} \quad$ Price of the good sold on the final market
$P_{Z} \quad$ Price of the good on the intermediary market
$q_{F} \quad$ Quantity offered on the final market
$q_{Z, i} \quad$ Quantity offered from company $i$ on the intermediary market
$t \quad$ Transfer price
FOC First order condition
Ibid. Latin, short form for Ibidem: "the same place"
MNE Multinational Enterprise
OECD Organization for Economic Cooperation and Development

## 1 Introduction and Motivation

As transfer pricing has become a topic of increasing importance due to numerable reasons, the topic has also received more attention from scientific sources over the years. Transfer pricing theory is an essential part of making a partitioned company with more than one independently acting division an effective construction. After all, if there are no synergies and ways to efficiently use these synergies, then there are no reasons for companies' divisions to be economically related in the first place. Instead, they probably would be better off being independent units competing with each other on the open market, relying on the popular "invisible hand" by Adam Smith, to maximize efficiency. The effects and objectives of transfer pricing strategies can be manifold. The topics of tax avoidance and the maximization of consolidated profits through strategically thought-through transfer pricing probably are the most familiar applications coming to one's mind. Another very important purpose of transfer pricing strategies is mitigating coordination problems which arise in segregated companies.

The core of this thesis will be the focus on extending and modifying the model described in a working paper published by Anil Arya and Brian Mittendorf in 2008 titled "Pricing Internal Trade to get a Leg up on External Rivals" which deals with internal transaction pricing and its strategic effects. Arya/Mittendorf focus on a model composed of a parent firm that offers an "intermediate" good on an intermediate market where it faces Cournot competition from a second firm and a subsidiary that uses this "intermediate" good to produce a final good for a final good market where it enjoys monopoly power. The parent division in the model of Arya/Mittendorf tries to maximize consolidated profit whereas the subsidiary maximizes its own profit. Therefore, the parent can influence the transfer price either indirectly by adjusting its output to the intermediate market since the transfer price is derived from the price on the market which is affected by supply or directly via an internal subsidy. Then, the authors examine among others the effects of the transfer price strategy on the competitor as well as the internal effect under market based transfer pricing as well as cost based transfer pricing.

In contrast to the model of Arya/Mittendorf, in this thesis the model has been modified in a way that the "parent", which then will be regarded as a simple division of the company, no longer tries to maximize consolidated profit but rather pays attention to its own gains. The subsidiary, as in the model of Arya/Mittendorf, still maximizes its own profit on the final good market. Another difference is the introduction of a third party in the organization, namely the "central office", which then tries to maximize consolidated company profits using
its power of being able to set the value of a subsidization/markup factor. This introduction of the central office is the main difference between the models of Arya/Mittendorf and the one proposed in this thesis. Cost based transfer prices will be excluded from the analysis in the modified model since the focus will be laid on the effect of a market based transfer price. Differences between the relevant results of Arya/Mittendorf and the modified model in this thesis will be highlighted.

After this first introduction and analysis of the modified model's results the model will be further altered to move away from the proposition of a duopoly on the intermediate market. The modified model will be standardized to portrait the competition of $N$ competitors on the intermediate market to examine the influence of increasing competition on the actions of the decision making units within the company. Interestingly, the model does not react as planned under the postulated assumptions.

Besides focusing on these models and their results, the thesis will give an overview of the importance of transfer pricing in today's business world.

## 2 Theory of Transfer Prices

Simply spoken, transfer prices are needed for intra-company accounting purposes to evaluate intra-company goods and services, which are transferred from one division to another division.

Transfer pricing theory is a field of study with a history that reaches back more than half a century, see, for example, publications by Stone (1956) or Paul W. Cook (1955). According to these publications, the upcoming interest on transfer pricing theory was strongly related to the increasing decentralization of companies. That is consequential, as decentralization of companies into several divisions leaves the central offices' managers with the question of how to make the divisions' executives manage their division in the shareholders' best interest (i.e. to maximize consolidated profit) and still be able to enjoy the opportunities provided by having several internal divisions working independently in conjunction with the arm's length principle.

Although the topic has already been picked up decades ago, the initial problem of setting internal transfer prices between the divisions of a company is still subject to intense research. Since transfer prices are also a subject to conflicts of objectives, the perfect solution might very well remain a theoretical concept and in the real world decision-makers could be forced
to be content with the approach of converging to the theoretically best possible result. The topic of transfer pricing is also closely related to the problem of setting suitable incentives to maximize profits. Since multidivisional companies seek to evaluate their divisions, they will want to enforce certain managerial structures on them, organizing them for instance as profit centers, which then are evaluated according to their profits. These profits however are influenced by the transfer pricing strategies enforced by the central office. This setup thereby bares risks of leaving one of the parties unsatisfied and could even have a share in proving compensation-schemes useless if divisional managers feel they are restricted in optimizing the outcomes of their performance figures by what they feel are unjustly set transfer prices. (Stone, 1956)

In any case, divisional performance measurement is an important topic of its own. On the one hand, separately measuring divisional performances increases incentives for these divisions to perform well. This decreases the problematic issue of divisional managements that profit largely from consolidated companies results without contributing as positively as they could, also called the "free rider problem". On the other hand, divisional performance measuring can lead to the exact opposite, namely increasing divisional profit by sacrificing the whole companies' profits. (Zimmerman, 1997)

Zimmerman (1997) does also give a very neat and intuitive example of how performance measurement and transfer pricing act in concert. The example gives an idea about the depth of the topic. Zimmerman describes a scenario of a Casino divided in three divisions. Two of them are achieving a negative performance according to Economic Value Added (EVA). However, the third division, the "Gaming" division, is highly profitable. Regarded separately, the two unprofitable divisions should be disposed of or restructured, but that wouldn't pay any respect to the synergies which in this case exist. The two divisions' services are highly relevant to the success of the third division. There is obviously a need for internal compensation, otherwise the managers of the unprofitable division will most likely choose to become profitable on their own and thereby undermine the great results of the consolidated company. In this case however, there's not even an open market for their services. So, what kind of transfer price should be introduced? There is no definitive answer to that question. Zimmerman points out that accounting for synergies is often unprofitable and this example gives a first impression of the difficulties of transfer pricing. (Zimmerman, 1997, p. 99ff)

When it comes to intercompany transactions, the transfer pricing strategy has to be fine-tuned with the divisional performance measures. Assume, for instance, the application of a market
based transfer pricing strategy on a cost center. In this theoretical example the division would then be judged by its costs which would depend on the market price, on which it would in most cases not have any influence at all. Divisional management should only be judged by measures they can influence. Transfer pricing choices for cost centers have to be related to cost. But again, the situation is not so easy to be assessed since it's not obvious what kind of cost should be contemplated for evaluation. Actual cost would lead to an incentive of the producing division's management to act careless since the buying division would be the one to suffer from exuberating costs. The appliance of standard cost however solves the problem by leaving any variances, positive or negative, in the selling division. That leads to a fair transfer price from which the consolidated company profits as the selling division tries to control its cost. (Fabozzi, Drake, \& Polimeni, 2008, p. 406f)

There are several types of transfer prices known in literature and practice. Ewert and Wagenhofer (2005, p. 585) roughly separate them into market based transfer prices, cost based transfer prices and negotiated transfer prices. Coenenberg (2003, p. 526) refers to Riebel/Paudtke/Zscherlich (1973, p. 29ff) who are more exact and divide transfer prices into 9 categories, which include differentiations according to the manner of occurence, material and time-wise orientation, cost-figures, length of validity, consistency, diversity, multi-part features and complementary transfer price types. Since this itemization may be exact it is rather cryptical which supposedly leads Coenenberg (2003, p. 527ff) to focus on highlighting the most important transfer prices which are closely related to the description summarized in Ewert and Wagenhofer (2005, p. 585) mentioned before, namely the market-based transfer prices, cost-based transfer prices and miscellaneous transfer prices, which include for instance negotiated transfer prices.

The importance of transfer pricing can also be conceived by having a glance at the latest global transfer pricing tax authority survey published by Ernst \& Young. It tells a story of just how much effort governments around the world have started to put into monitoring companies‘ internal transfer prices and their effects on tax burdens. (Ernst \& Young, 2012) However, any of the big four accounting companies has transfer pricing on their agenda and does their own copious studies in this field.

The OECD also publishes „Transfer Pricing Guidelines for Multinational Enterprises and Tax Administrations" which affects multinational companies addresses not only governmental interests like taxes, but also taxpayers interests like avoiding double taxation.

According to the global transfer pricing survey of Ernst \& Young (2010), the awareness of managers to the importance of transfer pricing has been increasing during the last years. Although levels have been decreasing since the peak of awareness in 2005, only $5 \%$ of managers take the transfer pricing topic as "not very important" or "not at all important". Ernst \& Young account the decrease in concern about transfer pricing to the fact that managers nowadays feel more in control about the topic. This is comprehensible since in 2010 a striking $86 \%$ of parent company respondents have indicated their transfer pricing policies have been examined by tax authorities, up from $52 \%$ in 2007, so dealing with transfer pricing policies has become a vital part of the overall managerial responsibility. Another interesting finding of the study is that there's an ongoing shift from transactional methods of transfer pricing to profit-based methods (Comparable profit method/Transactional net margin method), which are preferred by tax authorities. (Ernst \& Young, 2010, p. 6ff)

At the time of writing this thesis the most recent case of transfer pricing violation involves the finish company Nokia, which may have violated transfer pricing norms in India. Nokia’s local subsidiary allegedly has transferred profits to its headquarters and there have been intracompany transactions of software without meeting all legal requirements. Beside from RS 3.000 crore for tax violations Nokia will probably have to pay RS 10.000 crore for transfer pricing issues. This would equal about 1.4 billion Euros for disobeying transfer pricing rules. Even if allegations would be alleviated, this is just the latest example of how much effect transfer pricing rules could possibly have on multinational companies. (The Economic Times, 2013) ${ }^{1}$

Another hint concerning the importance of transfer pricing decisions can be deduced from a legal point of view. Since about 60 to 70 percent of worldwide trade happens within companies and tax-regulations vary from country to country, multinational companies tend to use transfer pricing strategies to minimize tax burden. (Sheppard, 2012)

Reducing tax burden is however not the only strategic effect of transfer prices. As described in the paper of Arya/Mittendorf and also in this thesis, transfer pricing strategy affects divisions' as well as competitors' behavior and therefore has to be set with care to prevent unwanted signaling.

[^0]Generally speaking, the functions of internal transfer prices are income calculation for assessment of divisional profit-contribution, divisional coordination and steering, calculation for pricing decisions, calculation for financial assessment of goods that run through several divisions and simplification through application of standardized figures for planning (e.g. master budget). Especially worth mentioning is the strategic function of transfer prices, which comes with the commitment to a specific action and helps to get a leg up on the external competitor. (Ewert \& Wagenhofer, 2005, p. 579ff)

Regarding adjustments in the form of intra company discounts of the market based transfer pricing, for instance Baldenius and Reichelstein (2004) have found that under certain circumstances these adjustments can improve consolidated company profits. They also mention the possibility to use to the prevailing market based transfer price for tax purposes whilst using an adjusted one for internal profit measurement, since this is allowed by most tax authorities.

Regarding the types of transfer prices, a pressing question is most certainly: "What kind of transfer price would be theoretically the best one?" The answer is probably simpler than expected: "The costs which arise from giving the good or service away for less than could be achieved elsewhere!" That kind of costs are also called opportunity costs. However, these opportunity costs are hard to determine because they usually are only known to the specific division. Moreover, the opportunities of a division are hard to determine as well. For instance, if a division has excess capacity its opportunity costs are variable costs of production since there are no further costs related to additional production than the costs related to keeping the machines running. But if there are capacity constraints, the opportunity costs are likely higher than variable costs. Consider for instance the case where an additional machine has to be installed in order to increase production - simply charging variable costs would not account for the initial costs of purchasing another machine. Summarized, it's the mentioned difficulty of determining accurate opportunity costs to a division's product or service that makes companies adopt transfer prices which are more objective. Those transfer prices are determined with the intention to be in proximity of the actual opportunity costs. (Zimmerman, 1997, p. 99ff)

From said perspective, the market based transfer price is an opportunity based transfer price as well, since a delivering upstream division would receive compensation on an open market at a level with the market price for its goods. So if there is a market for the good, it has either the opportunity to sell inside the company or outside, making the market price its opportunity
costs. On the topic of transfer prices' effectiveness for instance Loeffler and Pfeiffer (2011) have written a paper which gives insight about the specifications transfer prices should have in various settings and how market conditions affect the suitability of certain types of transfer prices. Hirshleifer (1956) has famously shown that in theory without capacity constraints marginal costs of the selling division are the optimal transfer price under a specific set of assumptions. This means that the consolidated company's optimum is maximized as long as the transfer price equals marginal costs (i.e. costs which arise from producing one additional unit). Under these circumstances, a firm would increase its output until marginal revenue would equal marginal costs and the production of one additional unit would result in a decreasing profit. Although this seems a relatively easy transfer pricing rule to solving the coordination problem which results from decentralized decision-making, it's not applicable to reality, as will be explained in chapter 2.2.

Hirshleifer has also pointed out that in a competitive market, the transfer price should be equal to the market price. The firm then would be a price taker and would not be able to influence the price. If the price on the market is higher than marginal costs of production, this would lead to a higher output of the company since it would increase production until marginal revenue (which would be determined by a predetermined market price) meets marginal costs of production. Moreover, the central office would be indifferent about a separation of the companies' divisions since the selling division would be indifferent between selling to the market or the purchasing division as the purchasing division would be indifferent between buying from the market or the selling division.

Multinational enterprises (MNE) often are constraint by regulators to use transfer prices tied to the "arm's length principle". This is a terminus one will certainly stumble across when dealing with the topic of transfer pricing. Governments need to ensure that multinational enterprises with several divisions don't evade tax burdens by artificially shifting profits out of their jurisdiction. However, taxpaying multinationals need to make sure to avoid falling under double taxation which can arise when divisions operate in countries which are not at odds concerning arm's lengths pricing method. Therefore, the Organization for Economic Cooperation and Development (OECD) publishes "Transfer Pricing Guidelines for Multinational Enterprises and Tax Administrations" which can be found on the OECD website and be read for free or downloaded by subscribers. Therein the arm's length transfer principle is referred to as:
"...the international transfer pricing standard that OECD member countries have agreed should be used for tax purposes by MNE groups and tax administrations." (Organisation for Economic Co-operation and Development (OECD), 2010, p. 33)
"Arm's length transfer pricing" relates to the requirement that intra-company compensation is based on levels of value conform to those that would have been applied if the transaction would have been conducted between unrelated parties. This sounds reasonably simple. However, there is a set of rules defining not only the amount of compensation but also the manner of transaction (e.g. one-time payment versus stream of payments). PricewaterhouseCoopers (2012) for instance provide an international transfer pricing report that gives an overview of this set of rules. Even though the OECD currently states the arm'slength principle as a fair and reliable basis for choosing transfer pricing, they are well aware of criticism concerning the arm's length principle.
"The arm's length principle is viewed by some as inherently flawed because the separate entity approach may not always account for the economies of scale and interrelation of diverse activities created by integrated businesses." (Organisation for Economic Co-operation and Development (OECD), 2010, p. 34)

They then even discuss an alternative approach called "Global formulary apportionment" that would use a predetermined formula to allocate a segregated multinational company's profits amongst its divisions. The main concern appears to be problems arising from double taxation as international coordination and consensus regarding the formula in question would be needed. (Organisation for Economic Co-operation and Development (OECD), 2010, p. 37f)

Picturing all the regulations regarding the excessively detailed reports from OECD, PricewaterhouseCoopers, Ernst \& Young, etc., it is interesting that Arya and Mittendorf note that these regulatory constraints can even be advantageous instead of harming for multinationals since they provide strategic opportunities on intermediate good markets. These advantages result from credible competitive posturing, i.e. making the competitors believe the company is more committed to achieving a target than its competition. (Arya \& Mittendorf, 2008, p. 711)

To provide executives with an overview of the transfer pricing jumble, accounting companies like Ernst \& Young or the formerly mentioned PricewaterhouseCoopers on their part publish guides, for instance the "Transfer pricing global reference guide" by Ernst \& Young. "The guide outlines basic information for the covered jurisdictions regarding their transfer pricing
tax laws, regulations and rulings, Organization for Economic Co-operation and Development (OECD) guidelines treatment, priorities and pricing methods, penalties, the potential for relief from penalties, documentation requirements and deadlines, statute of limitations, required disclosures, audit risk and opportunities for advance pricing agreements (APAs)." (Ernst \& Young, 2010)

Transfer pricing strategies can also be a means to mitigate intra-company coordination problems. In the modified model this coordination problem will play a role since the sales decision is not made from a centralized perspective.

### 2.1 Types of Transfer Prices

According to the OECD guidelines, the appropriate transfer price is chosen with respect to the availability of reliable information and the degree of comparability between controlled and uncontrolled transactions as there is no transfer price suitable for all circumstances. (Organisation for Economic Co-operation and Development (OECD), 2010, p. 59)

Correspondent to OECD (2010), to apply the arm's length principle, traditional transaction methods compile the following types:

- Comparable uncontrolled price method, which compares the prices of goods and services of a controlled transaction to those of an uncontrolled transaction happening on the market.
- The resale price method, that takes the price for which a product has been effectively sold (resale price) by an associated division to an outside company and then determines the transfer price by calculating backwards, deducting a profit margin and selling costs from that resale price.
- Cost plus method, that begins with the costs that actually occur at the selling division and then adds a markup. The determination of the costs thereby raises questions since there are several types of costs suitable for internal cost allocation.

The transactional profit methods may be used under certain conditions to approximate arm's length conditions. The idea is that profits arising from controlled transactions are examined and regarded as indicators whether the transactions differ from those made by independent companies. They comprise two methods:

- Transactional net margin method, which examines the net margin in a controlled environment by comparing it to an appropriate base (e.g. cost, sales, assets...). This
net margin is compared to the net margin achieved by unrelated companies operating in the same field of work.
- Transactional profit split method that divides the combined intra-company profits of related divisions in a way that unassociated, independent companies would agree on. This method is especially interesting, if the individual divisions are closely interrelated and the contribution to the profit of the transaction is not easily determinable. This method suggests examining unrelated companies on the market to get an idea about the profit allocation. (Organisation for Economic Co-operation and Development (OECD), 2010, p. 63ff)

A more general means of classification is the division of transfer prices into market-based transfer prices, cost-based transfer prices and negotiated transfer prices.

### 2.2 The Coordination Problem

Successfully guided companies require attention on various aspects of coordination. According to Ewert and Wagenhofer (2005), factual reasons for coordination are a result of problems arising from:

- Limited resources, which have to be allocated efficiently across companies' divisions.
- Interdependencies across divisions - in order to achieve the best possible result, divisions have to coordinate their actions. Think about two divisions in a company, one producing printer and the other printer cartridges.
- Stochastic correlation of measures executed by intra-company divisions. Think about several, yet to be executed, measures depending on rising/falling market prices of a specific commodity.
- Possible interrelations concerning assessments of performances if the subjective assessment method is dependent on the characteristic of other variables. This problem arises from the characteristics of the utility function applied. For instance, if due to a utility function the outcomes of projects with 2 stages (year 1, year 2) have to be assessed and the outcome at stage 1 does not interfere with the choices for stage 2, there are no interdependencies. However, if outcome 2 depends on outcome 1, a need for coordination arises.

Beside the factual reasons for coordination are also personnel reasons for coordination. The need for coordination scales among other factors with the size of the company and the
quantity of decision-making-units or persons involved in the process. Information usually is not distributed evenly across all of these decision-makers. More likely there will be asymmetries in the distribution of information which arise due to the delegation of tasks in a company and are hardly avoidable. Theoretical solutions like simply requesting divisional managers to pass their superior information on to the top management is unrealistic as conflicts of interest hinder divisional managers to do so. (Ewert \& Wagenhofer, 2005, p. 402ff)

For the sake of achieving a greater good, all the players on a football field must work together. They combine forces and it's not unusual that the youth, the impulsive manner and the arrogance of some players are hard to control by the teams coach. Especially as some of these players, usually the ones, who score the most goals, shine brighter than others.

The football team example mirrors an integrated company quite well in some respect. Take for instance a vertically integrated company with two profit centers, which are accountable for both costs and revenues, as profits equal revenues minus costs. Even if one of the two divisions had the possibility to assist its intra-company team member in a way that overall profitability could be increased to a higher level than could be achieved by simply adding up the results of both divisions, it would still seem as if the supported division performed better and the supporting division performed worse.

Hirshleifer (1956) graphically presents a model where division A produces an intermediate product which is processed by division B to be sold to a final monopolistic market. There is no market for the intermediate product. The divisions decide on their own upon their output and the question is how the transfer price has to be set to maximize consolidated profit. Hirshleifer concludes that the optimal transfer price is the producing division's marginal costs but remarks the following:
"The full solution involves one of the divisions presenting to the other its supply schedule (or demand schedule, as the case may be) as a function of the transfer price. The second division then establishes its output and the transfer price by a rule which leads to the optimum solution specified above for the firm as a whole." (Hirshleifer, 1956, p. 183)

Without this information about the supply schedule, the result is not optimal.
Let's look at an example of the coordination problem based on the model proposed by Hirshleifer (1956) and portrayed by Ewert and Wagenhofer (2005, p. 598ff).

Consider a company with two divisions, whereas the cost functions of division A and B are:
$c_{A}=10+q^{2}$ and $c_{B}=5+q$

The price sales function for the product on the end market - there is no intermediate market is:
$p(q)=25-q$

The consolidated company would determine its profit $\left(\Pi_{A B}\right)$ by maximizing its profit function with respect to the quantity:
$\Pi_{A B}=p(q) q-c_{A}-c_{B}=(25-q) q-\left(10+q^{2}\right)-(5+q)$
$\max _{q}\left[(25-q) q-\left(10+q^{2}\right)-(5+q)\right]$

This yields a consolidated profit $\left(\Pi_{A B}\right)$ of 57 and an optimal output $\left(q_{o p t}\right)$ of 6 units. Now consider a decentralization of decision-making regarding the output of the divisions. What transfer price should the central office enforce to make the divisional managers choose their output in line with the superior target of maximizing consolidated profit? There is exactly one transfer price that suits this objective, namely the marginal costs of the selling division in the optimum. The divisional profits now are dependent on an intra-company transfer price $(t)$ :
$\Pi_{A}=t q-c_{A}(q)$ and $\Pi_{B}=p(q) q-t q-c_{B}(q)$

If the central office enforces a transfer price $(t)$ of the marginal costs in the optimum ( $q_{o p t}=$ $6)$ of the selling division, that is:
$\left.\max _{q}\left[c_{A}\right]\right|_{q_{\text {opt }}}=\left.\max \left[10+q^{2}\right]\right|_{q_{o p t}}=\left.2 q\right|_{q_{\text {opt }}}=12$

If $t=12$, then division A wants to sell
$\max _{q}\left[t q-c_{A}(q)\right]=\max _{q}\left[t q-\left(10+q^{2}\right)\right]=t-2 q=12-2 q$ and again $q=6$

And division B wants to sell:
$\max _{q}\left[p(q) q-t q-c_{B}(q)\right]=\max _{q}[(25-q) q-t q-(5+q)]=24-2 q-t=$ $24-2 q-12$ and again $q=6$

Therefore, both companies want to produce the quantity that maximizes consolidated profit ( $\Pi_{A B}$ is again 57). The imposition of any other transfer price has the effect that the allocation of profits would be either advantageous for the buying division, if the transfer price would be smaller, or advantageous for the selling division, if the transfer price would be higher. However, if the transfer price would be any higher than 12, the buying division would not want to produce the optimal quantity of 6 in the first place, since its profit would be maximized at a lower quantity as its marginal costs would intersect with its marginal revenue earlier. The same principle applies for the selling division, with the difference that it would want to produce a lower quantity than 6 if the transfer price would be lower than 12 as its profit would be maximized already at that lower quantity.

Stated the above example, it just looks like the coordination problem has been solved although it really hasn't. Remember that the central office has to fix the transfer price at exactly $t=12$ to maximize the consolidated profit in a decentralized setup. This bears the question how the central office is able to determine this optimal transfer price in the first place. In order to know the optimal transfer price, the central office would have to solve the decision problem and could thereby decide the optimal output on its own, rendering the divisional managers useless. This thereby is a problem of circularity as decentralized decision-making under a predetermined transfer price by the central office solves a pseudo problem. (Ewert \& Wagenhofer, 2005, p. 600)

### 2.3 Strategic Function of Transfer Pricing

Under observable transfer pricing and price competition Robert F. Göx (2000) shows the effects of strategic transfer pricing in a model. The model depicts two companies in price competition. He portrays a case where an outperformance can be achieved if not the central offices decide upon the price directly but the companies employ divisional managers and profit from decentralization. The outcome for both companies in competition is higher than under centralized management. Göx (2000) shows, that the competitors' transfer prices affect the pricing decision of each company, whereby the marginal costs curves shift upward, resulting in a lower output to the market with higher prices. Thereby, the outcome drifts closer to a cartel solution. Both competitors benefit as the consolidated profits of both companies exceed the profit achieved by price decisions undertaken directly by the central offices. The higher output can only be achieved in the decentralized setting, since the divisional managers can credible commit to setting a transfer price above marginal costs as
the divisions are structured as profit centers. Considering a centralized solution, the companies could not credible commit to set transfer prices above marginal cost as the intermediate product costs would be an exogenous parameter for the consolidated companies profit function. In a centralized setting, the Nash equilibrium would require setting the price of the intermediate good to marginal cost. (Göx, 2000, p. 332ff)

As it is more intuitive to reproduce, the effect of strategic transfer prices shall be explained in an example on the basis of Ewert and Wagenhofer (2005) who took the work of Robert F. Göx, "Strategische Transferpreispolitik im Dyopol" (1999) as foundation. Therefore, consider two companies in price competition. Unit costs are equal in both companies with $c, c>0$ and the level of product substitution is $\gamma, 1>\gamma>0$. The customers' willingness to pay is $a, a>c$. The prices they achieve on the market are denoted $P_{1}, P_{2}$. Their inverse price-sales functions are:
$q_{1}=a-P_{1}+\gamma P_{2}$ and $q_{2}=a-P_{2}+\gamma P_{1}$

Their profits functions of company 1 and $2\left(\Pi_{1}, \Pi_{2}\right)$ are:
$\Pi_{1}=\left(P_{1}-c\right) q_{1}=\left(P_{1}-c\right)\left(a-P_{1}+\gamma P_{2}\right)$
$\Pi_{2}=\left(P_{2}-c\right) q_{2}=\left(P_{2}-c\right)\left(a-P_{2}+\gamma P_{1}\right)$

Maximizing these profit functions with respect to $P_{1}$ respectively $P_{2}$ yields the optimal price for each company, since the solution fulfills the profit maximizing condition that marginal revenue equals marginal cost.

$$
\begin{aligned}
& \max \left[\left(P_{1}-c\right) q_{1}\right]=\max \left[P_{1} q_{1}-c q_{1}\right]=\max \left[P_{1} q_{1}\right]-\max \left[c q_{1}\right]= \\
& \max \left[P_{1} q_{1}\right]=\max \left[c q_{1}\right] \hat{=} \text { Marginal Revenue }=\text { Marginal Cost }
\end{aligned}
$$

The first order condition of the profit functions with respect to the prices yields the following reaction curves:

$$
P_{1}=\frac{1}{2}\left(a+c+\gamma P_{2}\right) \text { and } P_{2}=\frac{1}{2}\left(a+c+\gamma P_{1}\right)
$$

Inserting one curve in the other and solving for the prices results in:
$P_{1}=P_{2}=\frac{a+c}{2-\gamma}$

This is the equilibrium price and the Bertrand-Nash equilibrium. Inserting these prices into the price-sales function results in the optimal profit for both companies under centralized management:
$\Pi_{1}=\Pi_{2}=\frac{(a+c(\gamma-1))^{2}}{(\gamma-2)^{2}}$

Now let's assume that each company decentralizes its pricing decision to a manger of a profit center. The central office decides upon the transfer price $(t)$ which is publicly observable.

Profit of the profit centers are now:
$\Pi_{1}=\left(P_{1}-t\right) q_{1}=\left(P_{1}-t\right)\left(a-P_{1}+\gamma P_{2}\right)$
$\Pi_{2}=\left(P_{2}-t\right) q_{2}=\left(P_{2}-t\right)\left(a-P_{2}+\gamma P_{1}\right)$
Again, after maximizing profits with respect to the prices and intersecting the reaction curves, both managers decide to choose the following prices:
$P_{1}(t)=P_{2}(t)=\frac{a+t}{2-\gamma}$

Profits of the consolidated companies differ from the profits of the profit centers, since the transfer price does not express actual cost but is in this case merely an instrument for steering:
$\Pi_{1}=\left(P_{1}-c\right) q_{1}=\left(\frac{a+t}{2-\gamma}-c\right)\left(a-P_{1}+\gamma P_{2}\right)=\left(\frac{a+t}{2-\gamma}-c\right)\left(\frac{a-t+t \gamma}{2-\gamma}\right)$
$\Pi_{2}=\left(P_{2}-c\right) q_{2}=\left(\frac{a+t}{2-\gamma}-c\right)\left(a-P_{2}+\gamma P_{1}\right)=\left(\frac{a+t}{2-\gamma}-c\right)\left(\frac{a-t+t \gamma}{2-\gamma}\right)$
To determine the optimal transfer price from the consolidated companies' perspectives, that are the perspectives of the central offices, the above portrayed profits of the companies have to be differentiated with respect to the transfer price.
$\frac{\partial \Pi_{1,2}}{\partial \mathrm{t}}=c+\frac{2(a+t)}{(-2+\gamma)^{2}}+\frac{a+c+2 t}{-2+\gamma}$

Optimal transfer price $\left(t_{\text {opt }}\right)$ is:
$t_{o p t}=\frac{c(-2+\gamma)(-1+\gamma)+a \gamma}{2-2 \gamma}=c+\frac{\gamma(a-(1-y) c)}{2-2 \gamma}>c$

As the transfer price $\left(t_{\text {opt }}\right)$ is actually higher than unit costs $(c)$, the introduction of a transfer price results in a strategic effect. Now, the profit is:
$\Pi_{1,2}\left(t_{o p t}\right)=\frac{(a-(1-\gamma) c)^{2}}{4-4 \gamma}$
Comparing the consolidated profit under a transfer pricing strategy with the consolidated profit without strategic transfer pricing, it's evident that the profit under strategic transfer pricing is higher for every level of product substitution, as long as there is even the slightest difference in products. $(\gamma>0)$ :
$\Pi_{1,2}\left(t_{\text {opt }}\right)>\Pi_{1,2} \widehat{=} \frac{(a-(1-\gamma) c)^{2}}{4-4 \gamma}>\frac{(a+(\gamma-1) c)^{2}}{\gamma^{2}-4 \gamma+4}$
The reason for this advantageous effect for both companies on the market is due to the increase of selling prices by the managers in the new equilibrium. The higher the level of similarity of the products the two companies produce $(\gamma)$, the more pronounced this effect becomes. Price-competition with two competitors would already lead to a scenario of perfect competition, where no profit above economic cost of production could be achieved. This favorable solution is only applicable under decentralized decision-making because if the central offices would directly decide upon the market prices, the two companies would not have the possibility to credible choose a higher market price in the equilibrium. Only if the central offices of the companies enforce a publicly observable transfer price which cannot be altered after being made publicly observable, whereas the optimal transfer price is higher than the actual unit costs, a higher market price is credible. Both managers enforce a higher market price by their own will, as, from their perspective, this higher market price yields the optimal result with respect to the competing manager's market price (i.e. they take each other's actions in consideration by regarding the reaction curves). If it were not for the managers, the company choosing a higher market price than the equilibrium market price would offer its competitor the possibility to increase profits at its own expense. (Ewert \& Wagenhofer, 2005, p. 634f) (Pfähler \& Wiese, 2006, p. 125ff)

## 3 Game-Theoretical Background

The necessary mathematical skills to understand and follow the model, which will soon be presented, are not on a breathtaking level. However, it is of use to have a basic knowledge about game theory, since multilateral interests are in the focus of the analysis. The few concepts that will be necessary to understand the model shall be highlighted briefly.

The following model and its modification will be based on competition in quantities and not competition in prices. This is not imperative; the model could also be transformed and interpreted in an environment of price competition. The decision, if one assumes competition in quantities or prices, depends essentially on the expectation about the long-term action parameter, if it's rather the price or quantity. An oil producing company cannot set prices since it takes quite some time to extract and deliver oil via freighter, thereby its action parameter will be the quantity produced. The action parameter for an operator of a gas station on the other hand is the price. This separation is important since it leads to different models which are applicable: The Bertrand model in the case of price competition or the Cournot model in the case of competition in quantities. Which one of these models is more appropriate depends largely on the market position a company finds itself in (e.g. industry, retailer), as stated in the above example. One important difference is that in the Bertrand competition, even a market with just two competitors, a duopoly, is enough to make both competitors offer their goods for just marginal costs which is the result of a market with perfect competition. This means, the companies would not make any profits on their goods and are just about able to cover their economic costs (Bertrand paradox). This is not the case in a market in Cournot competition. (Pfähler \& Wiese, 2006, pp. 67ff, 125ff)

Consider a company that operates on a market and faces one competitor. It has to decide upon the quantity it sells on this market, taking its competitor's output into equation. Its competitor deals with the exact same thoughts. Mathematically speaking, this is the reaction curve, which shows the possible action alternatives whilst taking the behavior of the competitors into consideration.
$q_{1}^{R}\left(q_{2}\right)=\operatorname{argmax} \Pi_{1_{q 1}}\left(q_{1}, q_{2}\right)$
(Pfähler \& Wiese, 2006, p. 128)

The argument of the maximum determines all the possible quantities on the quantity-curve from company $1\left(q_{1}^{R}\right)$, where company 1 's profit $\left(\Pi_{1}\right)$ is maximized with respect to the competitor's output $\left(q_{2}\right)$. These quantities are deduced by maximizing the profit of company

1 with respect to its own quantity $\left(q_{1}\right)$. At the same time, company 2 determines its own reaction function.
$q_{2}^{R}\left(q_{1}\right)=\operatorname{argmax} \Pi_{2 q 2}\left(q_{2}, q_{1}\right)$

These reaction functions likewise yield the best possible outcome for both companies, taking each other's actions into account. The reaction curves have a negative slope since the increase of output of one company makes the market price shrink. Thus it's profitable for the other company to decrease its output as marginal revenue still needs to equal marginal cost to maximize profits and marginal revenue just decreased in said situation. The Cournot model assumes that both actors simultaneously determine their outputs. Thus, strictly speaking, they have to know about each other's organization and costs. Stackelberg (1934) for instance dealt critically with the issue that Cournot-competitors choose their quantity simultaneously and proposed sequential competition of quantities. Thereby he showed that the Stackelberg leader, that is the market participant who determines his output first, has a strategically advantageous position and can thereby achieve a higher output compared to the Stackelberg follower and the actors in a Cournot competition. However, this requires that the Stackelberg follower is aware of the quantity that the leader proposes. Availability of information thereby is critical. (Pfähler \& Wiese, 2006, pp. 125ff, 140ff)

Another aspect to successfully understanding the following model and its modification is the process of backward induction, or backward solving. By using this process, it is possible to examine if a solution to a multiparty and multilevel problem is in a subgame perfect equilibrium. This is the case if the whole problem or game is in a Nash-Equilibrium and every subgame of the multilevel game is in a Nash-Equilibrium, too. A Nash-Equilibrium is achieved in a game if all parties have decided upon a strategy, which they unilaterally don't want to change or adjust. If no party sees an advantage by changing its strategy, the game is in a Nash-Equilibrium. Keep in mind though that this doesn't necessarily mean the solution is Pareto efficient. Pareto efficiency goes one step further and would express a situation where no player could be made better off without making another player worse off. (Pfähler \& Wiese, 2006, p. 28ff)

Related to the reaction curves, the Nash-Equilibrium is determined by intersecting both reaction curves. The thereby determined equilibrium tells the involved market participants about the quantity they need to produce in order to maximize their profit while accounting for the actions of each other. In this situation, the market participants share the interest to increase
the customers' willingness to pay on their mutual market. This could for instance be achieved by jointly realized marketing campaigns. Another mutual interest is to decrease cost, which could be achieved by amplified efforts of lobbying or agreements related to the negotiations with labor unions. (Ibid, p. 129ff)

## 4 Basic Setup of Arya and Mittendorf

Arya and Mittendorf focus in their paper titled "Pricing Internal Trade to Get a Leg up on External Rivals" (2008) on the influence of transfer pricing on competitors behavior. They highlight cost based transfer pricing as well as market based transfer pricing in multiple model configurations. For this thesis, one configuration under market based transfer pricing is of particular interest. According to the results of Arya/Mittendorf, the competitor can be forced to cede market share through a credible, aggressive signal via market based transfer pricing. A vital function of this kind of transfer pricing next to tax compliance and shifting tax burdens thereby can be competitive posturing. Considering this competitive role, a market based transfer price might even be considered when the market in question is thin. Usually, internal coordination is problematic due to the subsidiaries interest to procure a less than optimal quantity of goods from the upstream division unless the transfer price is equal to marginal costs. This proves costly in the final market but useful in the intermediate market since the parent, that oversees the profit of the consolidated company, has an incentive to reduce costs for the subsidiary's procurement and thereby also reduce the intermediate product's market price. This of course puts Cournot competitors under pressure as the parent aggressively pushes to decrease the market price and, having the subsidiary's procurement at the back of its mind, has credible incentive for its course of action. (Arya \& Mittendorf, 2008, p. 709ff)

In order to show the strategic effect a transfer price can have on competitors' behavior, Arya/Mittendorf introduce a simple model as follows: A parent and a subsidiary form the components of a vertically integrated company. The parent produces an input good for the subsidiary, the intermediate good, and additionally competes on an intermediate market with one competitor in Cournot-competition. The subsidiary enjoys monopoly power on the final good market. Figure 1 illustrates the setup:

Figure 1: Visualization of the Setup by Arya/Mittendorf

(Source: Own figure)

In this setting, with market-based transfer prices, the parent has incentives to drive down the market price for the intermediate good to increase internal procurement of the subsidiary. This is because the subsidiary will increase its procurement when it has to pay a smaller transfer price to the parent for the input good. This transfer price depends on the market price, which is called market based transfer price for that reason. The result is an aggressive behavior of the parent resulting in a softened response from its competitor. This means that the parent has not only a primary incentive to maximize its own profits but also a secondary incentive to increase the willingness of the subsidiary to procure more in order to sell more goods on the monopoly market, which is highly profitable. Thereby the parent maximizes its own profits as well as the profits of the subsidiary and thus the consolidated company. To manage distortions - which in this case are purposefully introduced for competitive advantages - and balance the effects of market-based transfer pricing on the profits of both divisions, the firm uses intra company discounts which are set by the parent. These intra company discounts are basically an additional mechanism for the parent to steer the value of the transfer price and thereby the amount the subsidiary procures. (Ibid.)

To make the setup of Arya/Mittendorf comparable to the modified model, which will be proposed later on in this thesis, we shall have a closer look at the basic conditions and the main results of Arya/Mittendorf. The following terms and equations are taken directly from their above mentioned paper, with the only difference being the denotation of the variables in order to simplify comparison later on as nomenclature will already be known and similar.

## Important Assumptions:

$$
c>0 \quad \gamma \in(0,1) \quad a_{Z}, a_{F}>c \quad \text { Conversion costs }=0
$$

Arya/Mittendorf make the assumptions that marginal unit costs $c$ exceed 0 and the customers' willingness to pay both on the intermediate market $\left(a_{Z}\right)$ and on the final market $\left(a_{F}\right)$ exceed costs $c$. Moreover, they introduce a substitution factor $\gamma$ which can have any value between 0 , inferring total dissimilarity of the product the company in focus produces compared to the products of its competitor, and 1, inferring total similarity. Hereby it has to be clarified that this substitution factor will later on, in the modified model, be neglected for simplicity. More specifically, it will be set to the value of 1 . This simplification results formulas which depict the case that the products of the consolidated company and its competitors are equal in every manner.

Table 1: Timeline of actions set by the actors of the company 1

| T=0 | $\mathbf{T}$ T=1 | $\mathbf{T}=\mathbf{2}$ |
| :--- | :--- | :--- |
| Company 1 specifies publicly <br> observable transfer pricing <br> strategy | Company 1 and 2 compete in <br> the intermediate market | The subsidiary of company 1 <br> $(=1 B)$ <br> quantity on the final good <br> market |

Source: own figure based on "Figure 1. Timeline of Events in the Base Model" (Arya \& Mittendorf, 2008, p. 714)

To have a strategic effect (i.e. express credible commitment on the intermediate market through transfer pricing strategy), the transfer price has to be publicly observable by competitors and unchangeable after adoption. (Arya \& Mittendorf, 2008, p. 719)

### 4.1 Input

$P_{F}=\left(a_{F}-q_{F}\right)$

The price on the final market $\left(P_{F}\right)$ is dependent on the willingness of the customers to pay for a final good $\left(a_{F}\right)$ and only the quantity which the subsidiary sells on the final good market $\left(q_{F}\right)$, since it operates in a monopoly in Arya/Mittendorf's base setting.
$P_{Z, i}=\left(a_{Z}-q_{Z, i}-\gamma q_{Z, j}\right)$ whereas $i, j=1,2$ and $i \neq j$

In the model of Arya/Mittendorf, the price on the intermediate market is different for company 1 and company 2 , since their products are not equal except for the case where the level product substitution is $1 \quad(\gamma=1)$. If the customers' willingness to pay on the intermediate market $\left(a_{Z}\right)$ increases, the price does so as well. It decreases simply if aggregated supply is increased by the competing companies. However, the substitution-factor $\gamma$ determines the magnitude of the effect this increase in the competitor's supply has on the intermediate market price that the company in focus can achieve.
$t=\left(P_{Z, i}-\delta\right)$

The transfer price $(t)$ links the two divisions of the multi-divisional company in focus. It is simply the price that the upstream division of the company receives for delivering its goods to the downstream division, which is responsible for the further steps in the manufacturing process. In this case the transfer price is a market-based transfer price. This means the opinion of the market $\left(P_{Z, i}\right)$ is consulted when it comes to the question about the value of the good on the intermediate market and then adjusted by the parent by applying an intra-company discount or markup $(\delta)$ to optimize consolidated profit.
$\Pi_{1 B}=P_{F} q_{F}-t q_{F}=\left(a_{F}-q_{F}\right) q_{F}-t q_{F}$

The subsidiary's revenue $\left(\Pi_{1 B}\right)$ depends on the price on the final market $\left(P_{F}\right)$ times the quantity $\left(q_{F}\right)$ sold on the final market. According to the changes in demand and supply the price reacts to the subsidiaries output. This can be seen by looking at the equation depicting the price on the final market $\left(P_{F}\right)$. Then the costs, in this case the costs for the subsidiary are equal to the transfer price times the quantity it procures, are subtracted. This is due to a simplification introduced by Arya/Mittendorf: They set conversion costs to zero. Thereby any additional costs of production arising from further process done by the subsidiary to finish the product are neglected. One input good procured by the subsidiary from the parent is turned directly into one final good, which the subsidiary then sells on the final market for the price achievable there $\left(P_{F}\right)$.

$$
\begin{align*}
\Pi_{1 A} & =P_{z, 1} q_{z, 1}+t q_{F}-c\left(q_{z, 1}+q_{F}\right) \\
& =\left(a_{Z}-q_{z, 1}-\gamma q_{z, 2}\right) q_{z, 1}+t q_{F}-c\left(q_{z, 1}+q_{F}\right) \tag{5}
\end{align*}
$$

The parent's revenues $\left(\Pi_{1 A}\right)$ are increased by either a higher price on the intermediate market $\left(P_{Z, 1}\right)$ or a higher quantity sold on the intermediate market $\left(q_{z, 1}\right)$. That however affects the price on the intermediate market negatively since that price is dependent on the output of the parent and its single competitor on the intermediate market. Additionally, the parent receives revenues as a result of the procurement by the subsidiary. The amount of these revenues depends on the level of the transfer price $(t)$ which depends on the intermediate market price $\left(P_{Z, i}\right)$ and the level of subsidization/markup ( $\delta$ ) by the parent. Finally, deducting the cost of production for the quantity sold on the intermediate market by the parent and those sold to the subsidiary, leads to the profit of the parent $\left(\Pi_{1 A}\right)$.
$\Pi_{A B}=\left(a_{F}-q_{F}\right) q_{F}+\left(a_{Z}-q_{Z, 1}-\gamma q_{Z, 2}\right) q_{Z, 1}-c\left(q_{Z, 1}+q_{F}\right)$

By substituting for the intermediate market price $\left(P_{Z, 1}\right)$ and the price on the final good market $\left(P_{F}\right)$, the sum of the two divisional profits $\left(\Pi_{A}+\Pi_{B}\right)$ results in consolidated profit $\left(\Pi_{A B}\right)$. This eliminates the transfer price $(t)$ since the profit and costs of the quantity transferred stays within the company. In the consolidated company, one division's receivables are the others division's liabilities.

At last, the counterpart of the parent is the standalone division $2 A$ of the competing company. The competitor only consists of this division and thus doesn't have to manage any intracompany transactions. Of course, it thereby has no possibility to profit from the adaption of a transfer pricing policy. Its profit is:
$\Pi_{2 A}=P_{Z, 2} q_{Z, 2}-c q_{Z, 2}=\left(a_{Z}-q_{Z, 2}-\gamma q_{Z, 1}\right) q_{Z, 2}-c q_{z, 2}$

### 4.2 Cost-Based Transfer Price

First, Arya and Mittendorf introduce their model with a cost-based transfer price in order to later on provide a comparison between the efficiency of cost-based transfer prices versus market-based transfer prices in their proposed model.

Using backward induction, transfer pricing policy shall now be considered under the objective to maximize consolidated firm profit.

Irrespective of the transfer price $(t)$, the subsidiary's objective is to maximize its profit $\left(\Pi_{1 B}\right)$ on the final good market. Inserting the term for the price on the final good market $\left(P_{F}\right)(1)$ into the equation for division 2 's profit $\left(\Pi_{1 B}\right)(4)$ yields:

$$
\begin{equation*}
\max _{q_{F}}\left[\left(a_{F}-q_{F}\right) q_{F}-t q_{F}\right] \tag{8}
\end{equation*}
$$

The first order condition (FOC) yields the optimal quantity in dependency of the transfer price ( $t$ ):
$\tilde{q}_{F}=\frac{a_{F}-t}{2}$

The optimal quantity for the subsidiary therefore increases with the willingness to pay $\left(a_{F}\right)$ on the final good market and decreases (increases) with an increasing (decreasing) value of the transfer price ( $t$ ).

The next step in the backward induction concerns the parent division that maximizes consolidated profit $\left(\Pi_{A B}\right)$ (6) with respect to the intermediate quantity $\left(q_{Z, 1}\right)$.
$\max _{q_{Z, 1}}\left[\left(a_{F}-q_{F}\right) q_{F}+\left(a_{Z}-q_{Z, 1}-\gamma q_{Z, 2}\right) q_{Z, 1}-c\left(q_{Z, 1}+q_{F}\right)\right]$
FOC then yields:
$q_{Z, 1}=\frac{1}{2}\left(a_{Z}-c-\gamma q_{Z, 2}\right)$
This is the optimal quantity the parent should sell on the intermediate market $\left(q_{z, 1}\right)$ in dependency of the quantity which the competing company decides to sell $\left(q_{z, 2}\right)$. The other variables are all exogenous.

Meanwhile, the second company, the competitor to our company in focus, also seeks to maximize its profits on the intermediate market according to its quantity $\left(q_{z, 2}\right)$ in consideration of the quantity its competitor unloads $\left(q_{z, 1}\right)$. According to its profit function $\left(\Pi_{2 A}\right)(7)$ the following function has to be differentiated:
$\max _{q_{Z, 2}}\left[\left(a_{Z}-\gamma q_{z, 1}-q_{z, 2}\right) q_{z, 2}-c q_{z, 2}\right]$
FOC yields:
$q_{Z, 2}=\frac{1}{2}\left(-c+a_{z}-\gamma q_{z, 1}\right)$

This, in contrast to (11), is the situation from the competitor's point of view. It will adjust its output $\left(q_{z, 2}\right)$ in dependency of the quantity company $1\left(q_{z, 1}\right)$ puts out on the intermediate market.

As explained in chapter 3 (Game-Theoretical Background), both companies will decide at the same point in time what quantity to sell and therefore need to anticipate each other's action. Equations (11) and (13) therefore are called reaction curves. Since both companies decide simultaneously upon their output, they don't know about the competitors output at the decision's point in time. They rather decide upon the expected quantity of the competitor. After inserting (13) in (11), and the other way round, and solving for the quantity $q_{z, 1}$, we end up with the Cournot quantities which define the Cournot equilibrium. Unilateral improvements of the profits are not possible in this environment. This equilibrium therefore describes a situation in which both companies have maximized their profits in consideration of the competitor's output. (Pfähler \& Wiese, 2006, p. 128f)

Cournot quantities:
$\tilde{q}_{Z, 1}=\tilde{q}_{Z, 2}=\frac{a_{Z}-c}{2+\gamma}$
To reach optimal profit under cost-based transfer pricing, optimal quantities $\tilde{q}_{Z, 1}(14)$ and $q_{F}$ (9) are inserted into the profit function of the consolidate company $\Pi_{A B}(6)$.

Optimal profit is:
$\Pi_{A B}=\left[\frac{a_{Z}-c}{2+\gamma}\right]+\left[\frac{\left(a_{F}-c\right)}{2}-\frac{(t-c)^{2}}{2}\right]$

The two terms stand for the intermediate market and the final good market and show that there is no interconnection between the two. The left term in the squared brackets stands for the intermediate market, the right one in the second pair of squared brackets for the final good market. As long as the transfer price ( $t$ ) exceeds marginal costs ( $c$ ), there is still room for profit-optimization until the transfer price equals marginal cost $(t=c)$. This yields to equation (16). (Arya \& Mittendorf, 2008, p. 714f)

$$
\begin{equation*}
\widetilde{\Pi}_{A B}=\left[\frac{a_{Z}-c}{2+\gamma}\right]+\left[\frac{\left(a_{F}-c\right)}{2}\right] \tag{16}
\end{equation*}
$$

### 4.3 Market-Based Transfer Price

Next, the same model will be regarded under market-based transfer pricing. Remember, the transfer price is defined in equation (3) as $t=\left(P_{Z, i}-\delta\right)$. As in equation (9), the subsidiary maximizes its profit on the final good market, taking the newly defined transfer price as well the price on the intermediate market $\left(P_{Z, i}\right)$ into account. The optimal output for the subsidiary under market-based transfer price is:
$q_{F}=\frac{a_{F}-t}{2}=\frac{a_{F}-\left(P_{Z, 1}-\delta\right)}{2}=\frac{a_{F}-\left(a_{Z}-q_{Z, 1}-\gamma q_{Z, 2}\right)+\delta}{2}$
Larger internal discounts $(\delta)$ increase the output to the final good market and thus also the demand of the subsidiary, since conversion costs are assumed to be zero. Now, in contrast to the cost-based transfer pricing method, the intermediate good market and the final good market are linked together! The market price on the intermediate good market $\left(P_{Z, 1}\right)$ is dependent on the quantity of goods $\left(q_{z, 1}\right.$ and $\left.\gamma q_{Z, 2}\right)$ supplied to the market. $P_{Z, 1}$ then influences the quantity which the subsidiary sells on the final market $\left(q_{F}\right)$ via the transfer price $(t)$.

The parent takes this into account as it solves the exact same equation (10) as in the costbased transfer pricing case, but with a different quantity on the final market $\left(q_{F}\right)$, as in equation (17):
$\max _{q_{Z, 1}}\left[\left(a_{F}-q_{F}\right) q_{F}+\left(a_{Z}-q_{Z, 1}-\gamma q_{Z, 2}\right) q_{Z, 1}-c\left(q_{Z, 1}+q_{F}\right)\right]$
(10) from chapter 4.2

Inserting (17) for $q_{F}$ the FOC yields:
$q_{z, 1}=\frac{1}{5}\left(3\left(a_{z}-c-\gamma q_{z, 2}\right)-\delta\right)$
The optimum quantity on the intermediate market $\left(q_{z, 1}\right)$ now looks similar to the one depicted in (11) from chapter 4.2, but is not only dependent on the output of the competitor $\left(q_{z, 2}\right)$ but also the subsidy or markup ( $\delta$ ). If the markup increases (decreases) it becomes less (more) attractive for the parent to sell to the intermediate market.

The competitor solves again for (12), the differences also being hidden within the quantities:
$\max _{q_{Z, 2}}\left[\left(a_{Z}-\gamma q_{z, 1}-q_{z, 2}\right) q_{Z, 2}-c q_{z, 2}\right]$
(12) from chapter 4.2

Again, FOC is:
$q_{z, 2}=\frac{1}{2}\left(-c+a_{z}-\gamma q_{z, 1}\right)$
(13) from chapter 4.2

Inserting (13) in (18), and the other way round, plus solving for both quantities ( $q_{Z, 1}$ and $q_{z, 2}$ ) results again in the - compared to the cost-based ones quite different - Cournot quantities:
$q_{Z, 1}=\frac{3\left[a_{z}-c\right][2-\gamma]-2 \delta}{10-3 \gamma^{2}}$
$q_{z, 2}=\frac{\left[a_{z}-c\right][5-3 \gamma]+\delta \gamma}{10-3 \gamma^{2}}$

Now, both quantities offered on the intermediate market $\left(q_{z, 1}, q_{z, 2}\right)$ depend on the subsidy or markup ( $\delta$ ) which the parent of company 1 introduces! Interestingly, company 1 decides to offer less if the subsidy/markup increases and company 2 offers more. Since the quantity of company 1 in (19) exceeds its cost-based cousin in (14) in the case that that there's no subsidy provided by the central office ( $\delta=0$, i.e. transfer price is similar to the market price), the parent can act more aggressively using a market-based transfer price. This comes as with the Cournot quantities in the cost-based scenario, the subsidiary doesn't procure enough to operate in the monopoly optimum on the final market since the costs for procurement would be above marginal cost. Thanks to the link between the markets through the transfer price, which is now market based, the parent tries to actively lower the transfer price to increase procurement of its subsidiary. The first way to do so is increasing its output to the intermediate market, which according to (2) lowers the market price $\left(P_{Z, i}\right)$ and thereby the transfer price $(t)$ seen in (3). This credible commitment to increase output on the intermediate market decreases the output of the competitor. The second way to improve procurement of the subsidiary is to increase the subsidy ( $\delta$ ). However, this action comes with the undesired drawback that company 2 increases and company 1 decreases its output, as can be seen in (19). Therefore it remains to be seen which level of subsidy is optimal to achieve the wanted strategic effect without compromising the output of company 1 to the intermediate market and thereby increasing the transfer price again! By plugging the optimal quantities $q_{F}$ and $q_{Z, 1}$
from (17) and (19) into the equation of consolidated profit (6) and differentiating it with respect to the subsidy $(\delta)$, we end up with the optimal subsidy, i.e. the difference between the market price and the internal transfer price. (Arya \& Mittendorf, 2008, p. 716)

Arya and Mittendorf use caret (e.g. $\hat{q}_{Z, 1}$ ) to display market-based equilibriums and tilde (e.g. $\left.\tilde{q}_{Z, 1}\right)$ to display cost-based equilibriums.

The optimal subsidy $(\hat{\delta})$ is:
$\hat{\delta}=\frac{5\left[a_{Z}-c\right]\left[4-2 \gamma-2 \gamma^{2}+\gamma^{3}\right]}{2\left[20-10 \gamma^{2}+\gamma^{4}\right]}$

This optimal subsidy is now plugged into the Cournot-quantities from division $\mathrm{A}\left(q_{z, 1}\right)$ and division $\mathrm{B}\left(q_{z, 2}\right)$ depicted in (19) to determine the optimal quantities from the central offices point of view under market-based transfer pricing:
$\hat{q}_{F}=\tilde{q}_{F}-\frac{\left[a_{Z}-c\right][2-\gamma] \gamma^{2}}{2\left[20-10 \gamma^{2}+\gamma^{4}\right]}$
This is the optimal quantity on the final market under market-based transfer pricing $\left(\hat{q}_{F}\right)$ in relation to the optimal quantity on the final market under cost-based transfer pricing ( $\tilde{q}_{F}$ ). Under the assumptions of Arya/Mittendorf, the quantity on the final market is always higher with cost-based transfer prices, since transfer prices exceeding marginal cost lead to restrictions in procurement by the purchasing division (1B). If the products are absolutely diverse (i.e. $\gamma=0$ ), the quantities are equal. If products would be absolutely unrelated in every aspect, the companies would not be competing in the same market and there would not be effects of cannibalization. This effect is expressed in the equation for the market price (2) in this model.
$\hat{q}_{Z, 1}=\tilde{q}_{Z, 1}+\frac{\left[a_{Z}-c\right] \gamma^{2}}{[2+\gamma]\left[20-10 \gamma^{2}+\gamma^{4}\right]}$
Contrarian to the quantity on the final market $\left(\hat{q}_{F}\right)$ in (21), the company sells more units on the intermediate market ( $\hat{q}_{Z, 1}$ ) under market-based transfer pricing, again with the exceptional case when products are absolutely diverse.
$\hat{q}_{Z, 2}=\tilde{q}_{Z, 2}-\frac{\left[a_{Z}-c\right] \gamma^{3}}{2[2+\gamma]\left[20-10 \gamma^{2}+\gamma^{4}\right]}$

Company 2 on the other hand loses share on the intermediate market compared to cost-based transfer pricing.

To reach the term for the optimal transfer price under market-based transfer pricing, the optimal quantities ( $\hat{q}_{Z, 1}$ and $\hat{q}_{Z, 2}$ ) need to be plugged into the equation of the market price in (2), which then has to be inserted into the equation of the transfer price (3). Finally, the optimal subsidy $(\hat{\delta})$ from (20) needs to be applied to equation (3) as well, which results in:
$\hat{t}=c+\frac{\left(a_{z}-c\right)(2-\gamma) \gamma^{2}}{20-10 y^{2}+\gamma^{4}}$

From (20) Arya and Mittendorf deduce that regardless of the level of substitution ( $\gamma$ ) between the companies' products, the subsidy $(\hat{\delta})$ is positive. Thus, the subsidiary can procure its input goods cheaper inside the company than on the open market but still has to pay more than marginal cost, which they deduce from (24). Arya/Mittendorf conclude that intra-company subsidy is used to provide the subsidiary an incentive to procure more than it would under unadjusted market-based transfer pricing, but still less than with marginal cost. The upside is that the parent can send a credible signal of commitment to its competitor which results in competitive posturing on the intermediate market. When the products become more similar (i.e. $\gamma$ increases), intra company subsidy decreases. One reason for this development is a general decrease in the market price which goes along with increased competition on the intermediate market. From (2) and common sense it's inferable that products converging in similarity decrease the price on the market. Interestingly, the reasoning doesn't stop there. From (24) we know that with increasing $\gamma$ the transfer price $(\hat{t})$ diverges from marginal cost (c). As long as the competitors' products are absolutely different (i.e. $\gamma=0$ ) transfer price should be the same as in the cost-based model $(\hat{t}=c)$ where markets are not linked. With a convergence of the products the transfer price $(\hat{t})$ rises and the market price $\left(P_{Z, i}\right)$ falls. Besides however, the quantity on the intermediate market offered by company $1\left(\hat{q}_{Z, 1}\right)$ rises according to (22). In favor of showing its teeth to its competitor, company 1 sacrifices profit on the final good market for profit and an aggressive signal on the intermediate market. Arya/Mittendorf note, that with the fitting subsidization ( $\delta$ ) there is a benefit irrespective of the level of similarity $(\gamma)$.

At last, they compare the profit under market-based transfer pricing ( $\widehat{\Pi}_{A B}$ ) relative to profit under cost-based transfer pricing $\left(\widetilde{\Pi}_{A B}\right)$ :
$\widehat{\Pi}_{A B}=\widetilde{\Pi}_{A B}+\frac{\left[a_{Z}-c\right]^{2} \gamma^{4}}{4[2+\gamma]^{2}\left[20-10 y^{2}+\gamma^{4}\right]}$
It is apparent that regardless of the level of product difference $(\gamma)$ the profit with market-based transfer pricing exceeds profit with cost-based transfer pricing. As $\gamma$ increases, the advantage of market-based transfer pricing advances. Summed up, via market-based transfer pricing the company enhances its output on the intermediate market by credibly committing to mitigate the problem of undersupply in the final good market. In this model, the market price, which is the basis for the transfer price, is dependent on the output of two companies. The results show that the company with an upstream and a downstream division can convince its competitor on the intermediate market that it needs to put out more units as this will boost procurement by the downstream division. The subsidy $(\delta)$ balances the effect to maximize consolidated company profit under any level of substitution ( $\gamma$ ). (Arya \& Mittendorf, 2008, p. 718)

Subsequently, Arya and Mittendorf expand their model to represent symmetric competition, i.e. two companies competing on both markets, intermediate good market and final good market. They find that in that changed setting both companies would introduce market-based transfer pricing as a disadvantage in profits would occur for the company which adopts costbased transfer pricing. Moreover, from their model, it seems that both firms would want to use the credible commitment pictured above to increase market share. This seems like disadvantageous for both firms as they just increase competition. However, Arya/Mittendorf point out even more harmful results can be deduced from their model, if the companies could not link markets via market-based transfer pricing. (Arya \& Mittendorf, 2008, p. 725)

## 5 Modified Setup

In this chapter we will look at a modified model that is related to the one of Arya/Mittendorf. This will lead to a configuration that is in a way more related to the actual circumstances of reality, in other ways however it is simplified to keep it neat and focused. Now the question is how things change when the supplying division (1A) doesn't have intrinsic motivation, like the parent in the model of Arya/Mittendorf, to support the purchasing division (1B). The simple fact in the matter is that an intra-company support is desirable to make the 1 B procure more from 1A since 1B will restrict procurement the more the transfer price $(t)$ diverges from
marginal cost (c). In the basic model of Arya/Mittendorf the strategic effect of transfer pricing is examined. Now, next to the strategic effect a coordination problem needs to be overcome.

In this modified model the focus now lies on a company with two rather self-involved decision-making units and one unit which tries to interconnect these units and their productive efficiency: These units shall be tagged as follows: Central office, upstream division 1A and downstream division 1B. Division 1A and 1B are related by internal transactions. 1A produces a good which it delivers to the intermediate market on the one hand and to 1 A on the other. Doing this, 1A faces Cournot-competition from NA competitors, which themselves are not subject to in-depth analysis and do not operate within a segmented company. $N=1$ resembles the special case where only one competitor offers its goods on the intermediate market. Thus, this is the most interesting case for comparison with the results of Arya/Mittendorf and will be dealt with before the second case, which then resembles a multicompetitor situation on the intermediate market where $N=x$ and $x \in \mathbb{N}$. Division 1B internally receives a good from 1A and it has, as in the Paper of Arya/Mittendorf, no access to the external intermediate market. This assumption makes more sense in the model of Arya/Mittendorf since they account for differences between the products of 1A and 2A (see the substitution factor $\gamma$ in their model). Those differences in products will be neglected in the comparison as well as in the forthcoming calculations for simplicity.

The intermediate good is produced at unit costs $c, c>0$. However, this assumption will later on be alleviated to $c=0$ for the comparison of results. Otherwise the terms would be unnecessarily hard to interpret, without much added value. Like in the basic model of Arya/Mittendorf it will be assumed that without loss of generality conversion costs of one intermediary product to one final product are zero. Customers' willingness to pay is likewise assumed to be larger than the costs on both intermediate and final market $a_{Z}, a_{F}>c$.

To calculate the results, backwards induction is used. Starting point is the profit maximization of 1 B , following by the profit of 1 A in conjunction with 2 A respectively $N \mathrm{~A}$. In the last step of the calculation, when the profits of all other parties are maximized, the optimal subsidy/markup factor $(\delta)$ is determined. Of course in reality the time line of actions is as follows:

Table 2: Timeline of actions set by the actors of the company AB

| $\mathbf{T}=\mathbf{0}$ | $\mathbf{T}=\mathbf{1}$ | $\mathbf{T}=\mathbf{2}$ |
| :--- | :--- | :--- |
| Central office determines | 1 A maximizes profit | 1 B maximizes profit with |
| subsidy/markup ( $\delta)$, the | according to its quantity | respect to its quantity $\left(q_{F}\right)$ |
| transfer pricing policy is | $\left(q_{Z, 1}\right)$ under consideration of |  |
| publicly observable | 2 A's actions |  |

(Source: own table)
It's important to mention that the central office knows about the goals of 1 A and 1 B as does 1A know about 1B's objective, which is the maximization of their profit according to their profit functions. The main difference to the Model of Arya/Mittendorf is the observation of a divergence in interests of party 1 A . It can either choose to make more profits on the intermediary market by pushing its sales there or it tries to use its market power to increase the transfer price which it receives from 1B by decreasing its sales on the intermediary market. The central office on the other hand wants to maximize consolidated profit by introducing a subsidization/markup. In the setting of Arya/Mittendorf the power of influencing the market price $\left(P_{Z, i}\right)$ by increasing or decreasing output and setting the subsidy/markup ( $\delta$ ) was combined in one division, which they called the "parent". This division is now segregated into the central office and 1 A , which makes the subsidy/markup $(\delta)$ a much more important tool for maximizing consolidated profit.

### 5.1 Case of One Competitor on the Intermediate Market

The modified model's visualization for the case of one competitor is depicted in Figure 2: Visualization of the Modified Model with one Competitor. The model's calculations will be reviewed on the following pages. Then, the outcome will be subject to an in-depth analysis with the outcomes of Arya/Mittendorf under some simplifications.

Figure 2: Visualization of the Modified Model with one Competitor

(Source: Own figure)

### 5.1.1 Input

The demand function for the intermediate product is
$P_{Z}=\left(a_{Z}-q_{Z, 1}-q_{Z, 2}\right)$,
where $P_{Z}$ denotes the price on the intermediate market for the good 1A produces, $a_{Z}$ denotes the customers' willingness to pay on the intermediate market and $q_{Z, 1}$ respectively $q_{Z, 2}$ denotes the quantity which 1 A and 2 A intend to sell on the intermediate market. Compared to Arya/Mittendorf, there is no price for each of the competing companies on the intermediary market $\left(P_{Z, i}\right)$ but one price for both of them $\left(P_{Z}\right)$. This is due to the assumed lack of product differentiation.

The demand function on the final good market is exactly similar to the one in chapter 4.1:
$P_{F}=\left(a_{F}-q_{F}\right)$,
(1) from chapter 4.1
$P_{F}$ denotes the price on the final market, $a_{F}$ denotes the customers willingness to pay on the final market and $q_{F}$ denotes the quantity that 1 B chooses to sell on the final market.

Unlike in the setting of Arya/Mittendorf pictured in chapter 4, division 1A no longer tries to maximize consolidated profit, taking into account both intermediate and final good markets. Now, in this model, 1A is as egocentric as 1B. Hence, both of them try to maximize their own profits without bearing any shortcomings on the consolidated level in mind. There is however a third player in the game that tries to maximize consolidated profits. This is the central office, which overlooks the overall business activity and tries to maximize both divisions
aggregated profits. In order to calculate the optimal profit from the central office's point of view, three steps can be distinguished through backwards-induction. Again, backwards induction stands for the process of calculating the optimal outcome by starting at the level of the calculation, where no interdependencies exist, then working your way backwards until the level is reached, where all the other optimal outcomes have to be considered. At first, division 1B's maximal profit $\Pi_{1 B}$
$\Pi_{1 B}=P_{F} q_{F}-t q_{F}$
(4) from chapter 4.1
is determined with respect to the quantity on the final market $q_{F}$. The equation stays the same as the one already introduced in chapter 4.1.

The equation of the transfer $(t)$ price slightly changes due to the simplification of the now neglected level of product substitution, now accounting for just one price on the intermediate market since there's the same unique product offered by two companies $\left(P_{Z}\right)$ :
$t=\left(P_{Z}-\delta\right)=\left(a_{Z}-q_{Z, 1}-q_{Z, 2}\right)-\delta$

The factor $\delta$ therein determines the subsidy/markup set by the central office to steer the transfer price and thereby influence the behavior of division 1A and 1B. Secondly, 1A's maximal profit, taking (26) into consideration,
$\Pi_{1 A}=P_{Z} q_{Z, 1}+t q_{F}-c\left(q_{Z, 1}+q_{F}\right)=\left(a_{Z}-q_{Z, 1}-q_{Z, 2}\right) q_{Z, 1}+t q_{F}-c\left(q_{Z, 1}+q_{F}\right)$
is determined according to the quantity it sells on the intermediary market $\left(q_{z, 1}\right)$. Simultaneously the output of its competitor 2 A on the intermediate market has to be considered ( $q_{Z, 2}$ ). This is achieved by determining the intersection of the reaction curves of both 1A's and 2A's demand curves. It is of special interest to mention that 1A not only influences its profit directly by determining the optimal amount of goods $\left(q_{z, 1}\right)$ it is going to sell on the intermediate market, but also through the not so obvious effect of its decision of quantity of sales $\left(q_{Z, 1}\right)$ on the intermediate market price $\left(P_{Z}\right)$. Thereby it has an opportunity to influence the transfer price $(t)$ it receives from 1B. In the model of Arya/Mittendorf, the parent $(=1 \mathrm{~A}+\mathrm{CO})$ didn't abuse the opportunity to increase the transfer price to achieve a higher profit itself since it maximized for consolidated profit. Now however, 1A doesn't care about the sake of 1B and the central office is left with the opportunistic behavior of 1A and
needs to make sure that 1 B serves the monopoly market well enough since it's highly profitable, even if 1 A increases the transfer price far above marginal costs.

Profits of the consolidated company are the sum of (4) and (28):

$$
\begin{align*}
\Pi_{A B} & =P_{Z} q_{Z, 1}+P_{F} q_{F}-c\left(q_{Z, 1}+q_{F}\right) \\
& =\left(a_{Z}-q_{Z, 1}-q_{Z, 2}\right) q_{Z, 1}+\left(a_{F}-q_{F}\right) q_{F}-c\left(q_{Z, 1}+q_{F}\right) \tag{29}
\end{align*}
$$

The profits achieved on the intermediate market (market price times quantity sold: $P_{z} q_{z, 1}$ ), plus the profits achieved on the final good market $\left(P_{F} q_{F}\right)$ minus total cost of production $\left(c\left(q_{Z, 1}+q_{F}\right)\right)$ of these goods equal the consolidated company's profits. The transfer price $(t)$ does not directly appear in this equation. However, it influences consolidated profits by influencing its constituent variables.

The competitor on the intermediate market (2A) has the following profit function that resembles equation (7) in chapter 4.1 without accounting for product differentiation:
$\Pi_{2 A}=P_{Z} q_{Z, 2}-c q_{z, 2}=\left(a_{Z}-q_{Z, 2}-q_{z, 1}\right) q_{Z, 2}-c q_{z, 2}$

### 5.1.2 Calculation and On the Fly Interpretation

At first, according to backward induction we have to consider the optimal result from the downstream division $(1 B)$ which like in chapter 4 has to decide upon its optimal output to the final market $\left(q_{F, o p t}\right)$. Therefore we reach the same equation, however not accounting for product substitution ( $\gamma$ ) anymore:
$q_{F}=\max _{q_{F}}\left[\left(a_{F}-q_{F}\right) q_{F}-t q_{F}\right]=\frac{a_{F}-\left(a_{Z}-q_{Z, 1}-q_{Z, 2}\right)+\delta}{2}$

Next, we have to construct the two reaction curves for the optimal quantities of division 1A and 2 A and deduce the Cournot-Nash equilibrium:
$\max _{q_{Z, 1}}\left[\left(a_{Z}-q_{Z, 1}-q_{Z, 2}\right) q_{Z, 1}+t q_{F}-c\left(q_{Z, 1}+q_{F}\right)\right]$
Substituting transfer price $(t)$ for (27) and the quantity on the final market $\left(q_{F}\right)$ for (17), FOC yields to the reaction curve that depicts the quantity that 1 A sells on the intermediate market with considering the output of 2 A :
$q_{Z, 1}=\frac{1}{6}\left(4 a_{Z}-a_{F}-3 c-2 \delta-4 q_{Z, 2}\right)$

2A simultaneously maximizes its output:
$\max _{q_{Z, 2}}\left[\left(a_{Z}-\gamma q_{z, 1}-q_{z, 2}\right) q_{Z, 2}-c q_{z, 2}\right]$

FOC yields the second reaction curve that similarly shows the quantity that $2 A$ sells on the intermediate market with considering the output of 1 A :
$q_{Z, 2}=\frac{1}{2}\left(a_{Z}-c-q_{Z, 1}\right)$

Intersecting these curves (i.e. plugging (34) in (32) and solving for $q_{Z, 1}$ as well as plugging (32) in (34) and solving for $q_{Z, 2}$ ) we get the optimal quantities in a Cournot-equilibrium of 1 A and 2A:
$q_{Z, 1}=\frac{1}{4}\left(2 a_{Z}-a_{F}-c-2 \delta\right)$
$q_{Z, 2}=\frac{1}{8}\left(2 a_{Z}+a_{F}-3 c+2 \delta\right)$

Again, as in chapter 4.3 the quantities offered on the intermediate market by 1 A and 2 A depend on the subsidy or markup $(\delta)$ introduced by the central office (not the parent!). If the central office decides to increase subsidy so that division 1B can procure closer to marginal costs and thereby skim the monopoly market more efficiently, division 1A will decrease its output on the intermediate market. As can be seen in the equations in (19), an increase in subsidy by the central office results in $-\frac{2 \delta}{4}$ in output on the intermediate market $\left(q_{z, 1}\right)$ and is not completely balanced by the increase of the output on the intermediate market by the competitor 2A which increases its output by just $\frac{2 \delta}{8}$. According to (26) that results in a higher market price $\left(P_{Z}\right)$ and thereby a higher market-based transfer price $(t)$. Thus, the central office has to decide upon the subsidy/markup $(\delta)$ with care to improve overall company results. In this matter the willingness to pay-ratio $\left(a_{Z} / a_{F}\right)$ plays huge role since the profit from the final market has to make up for the missed share on the intermediate market. The question is how profitable the monopoly is compared to the intermediate market. Moreover, the quantities on the intermediate market in comparison to the results from Arya/Mittendorf in (19) not only
depend on the willingness to pay on the intermediate market $\left(a_{z}\right)$ but also the willingness to pay on the final market $\left(a_{F}\right)$. Mathematically the reason for this change is in the differentiation of division 1A's profit with respect to the quantity it sells on the intermediate market $\left(q_{z, 1}\right)$ in equation (31). The logical reasoning is that 1 A decides to sell less on the intermediate market as $a_{Z}$ increases, since an increase of $a_{Z}$ would naturally lead to an increase of $q_{F}$ from 1B (and thereby increased procurement from 1A) and as a result 1A could achieve a higher profit by simply pushing higher the market price and thereby the transfer price, and vice versa.

Now, as both individual divisions of company 1 have maximized their profits according to their quantities, the central office adds the profit functions of the two parties up and maximizes consolidated profits with respect to the subsidy/markup $(\delta)$ which it is empowered to determine for strategic steering. The FOC of (29) in conjunction with optimal $q_{F}(17), q_{Z, 1}$ and $q_{Z, 2}$ (35) results in (for a more voluminous presentation of the equation please refer to formula (A18) in the appendix and account for $N=1$ ):

$$
\begin{align*}
\delta_{o p t} & =\left.\max _{\delta}\left[\left(a_{Z}-q_{Z, 1}-q_{Z, 2}\right) q_{Z, 1}+\left(a_{F}-q_{F}\right) q_{F}-c\left(q_{Z, 1}+q_{F}\right)\right]\right|_{\left\{q_{F}, q_{Z, 1}, q_{Z, 2}\right\}} \\
& =\frac{1}{34}\left(6 a_{Z}-5 a_{F}-c\right) \tag{36}
\end{align*}
$$

The optimal subsidy/markup ( $\delta_{o p t}$ ) increases more with the willingness to pay on the intermediate market $\left(a_{Z}\right)$ than it decreases with the willingness to pay on the final market $\left(a_{F}\right)$. For instance consider the case when costs $c=0$, then as long as $a_{F}<\frac{6}{5} a_{Z}, \delta_{o p t}$ will be a subsidy. If $a_{F}=\frac{6}{5} a_{Z}$, the transfer price will be exactly the market price. For $a_{F}>\frac{6}{5} a_{Z}$, we would have a markup over the market price on the transfer price. Thus, the central office will decide upon the profitability of the markets and the costs whether it introduces a markup or subsidy and thus steers the transfer price so that either 1A or 1B will profit.

The optimal subsidy/markup ( $\delta_{o p t}$ ) is then taken and plugged into optimal quantities from the divisions' 1A and 1B point of view, displayed in equations (17) and (35). The results are the optimal quantities with respect to the central offices influence on the transfer price.

Based on the basic equation for the quantity on the final market (17) the optimal quantity on the final market ( $q_{F, o p t}$ ) from the central offices point of view is expressed. The basic
equation needs to be updated with the Cournot quantities $q_{Z, 1}$ and $q_{Z, 2}$ (35). Then, optimal subsidy/markup $\delta_{o p t}$ in (36) is inserted and the result is:

$$
\begin{align*}
q_{F, o p t} & =\left.\frac{a_{F}-\left(a_{Z}-q_{Z, 1}-q_{Z, 2}\right)+\delta}{2}\right|_{\left\{q_{Z, 1}, q_{Z, 2}, \delta_{o p t}\right\}} \\
& =\frac{1}{34}\left(13 a_{F}-2 a_{Z}-11 c\right) \tag{37}
\end{align*}
$$

Since the value of the transfer price $(t)$ influences procurement of division 2 A (see equation (17)), it depends on both the willingness to pay on the intermediate as the final market ( $a_{Z}$, $a_{F}$,). All things being equal, if $a_{Z}$ rises, the intermediate market price $\left(P_{Z}\right)$ rises. Thus, transfer price $(t)$ rises and that makes procurement for 2A less interesting. On the other hand, if the $a_{F}$ rises, the price on the final market $\left(P_{F}\right)$ increases and 2 A will increase its output.
$q_{Z, 1, o p t}=\frac{1}{17}\left(7 a_{Z}-3 a_{F}-4 c\right)$
$q_{Z, 2, \text { opt }}=\frac{1}{34}\left(10 a_{Z}+3 a_{F}-13 c\right)$
1A increases distribution to the intermediate market with $a_{Z}$ and decreases it with $a_{F}, 2 \mathrm{~A}$ increases its output to the final market with both, $a_{Z}$ and $a_{F}$. The reasoning is already outlined above, next to equation (35). By subtracting $q_{Z, 2, o p t}$ from $q_{Z, 1, o p t}$ we can interpret whether a strategic effect of the transfer pricing policy is achieved, like in the model of Arya/Mittendorf.
$q_{Z, 1, \mathrm{opt}}-q_{Z, 2, o p t}=\frac{1}{34}\left(5 c-9 a_{F}+4 a_{Z}\right)$
Assuming for simplicity that $c=0$ then if $a_{Z}>\frac{9 a_{F}}{4}$ (but $a_{Z}<\frac{13 a_{F}}{2}$ since that would render $q_{F, o p t}$ in (37) negative) there is still a strategic effect of transfer pricing as in the equilibrium, division 1 A of company AB decides to sell more units than its competitor 2 A on the intermediate market. This helps again to increase the procurement of 1B. The willingness to pay on the intermediate market $\left(a_{Z}\right)$ has to be relatively high to the willingness to pay on the final market $\left(a_{F}\right)$ in order to make it attractive for division 1A to increase its output to the intermediate market, to stimulate procurement.

By plugging the optimal quantities $\left(q_{z, 1, o p t}, q_{z, 2, o p t}\right)$ and optimal subsidy/markup ( $\delta_{o p t}$ ) into the transfer price equation in (27) we arrive at the optimal transfer price:
$t_{o p t}=\frac{1}{17}\left(4 a_{F}+2 a_{Z}+11 c\right)$

The optimal transfer price $\left(t_{o p t}\right)$ increases with the willingness to pay on the final market ( $a_{F}$ ) and half as much for every increase on the intermediate market ( $a_{z}$ ). In the Arya/Mittendorf model, without a coordination problem, there isn't even an $a_{F}$ involved in the related equation, see (24). Here however, the $a_{F}$ does act a part in the transfer price, since it already influences $q_{z, 1, o p t}, q_{z, 2, o p t}$ and $\delta_{o p t}$.

Finally, by applying all the optimal outcomes so far to the equation the consolidated company's profit (29) we arrive at optimal consolidated profit $\left(\Pi_{A B, o p t}\right)$ :

$$
\begin{equation*}
\Pi_{A B, o p t}=\frac{1}{68}\left(15 a_{F}^{2}+8 a_{Z}^{2}+21 c^{2}-2\left(7 c+a_{F}\right) a_{Z}-28 c a_{F}\right) \tag{41}
\end{equation*}
$$

### 5.1.3 Comparison with Results of Arya/Mittendorf

Finally we reach the second most interesting part of this thesis: The results of Arya/Mittendorf from chapter 4.3 will be compared with those of the modified model of chapter 5.1.2.

Let's have a look at a brief summary concerning the differences of the two models:

- Arya/Mittendorf present a "two-step model" where the parent has to balance the effects of achieving a higher transfer price by producing less on the one hand and producing more for the intermediate market to lower transfer price and to put pressure on its competitor on the other hand. This is a problem of strategic delegation. The parent does so by maximizing its profit function according to amount of goods sold on the intermediate market, then inserting this quantity and the optimal output of the subsidiary on the final market into consolidated profits and maximizing this to subsidy/markup factor $(\delta)$. In contrast to the modified model's division 1A (which produces the same good for the intermediate market and the subsidiary as the parent), the parent does take the subsidiaries profit into account when maximizing its profits with respect to the intermediate good! The subsidiary will procure less the more the transfer price is above marginal cost, which lessens its profits.
- The modified model introduces a "third step" since divisions 1B and 1A maximize their profit without taking each other's interests into account. 1B still maximizes its profit with respect to its output, but in contrast to the basic model Arya/Mittendorf
portrayed, 1A does so as well. The central office then tries to get the optimum out of the situation by imposing a subsidy/markup. This is the third step in this model that separates the results compared to those of Arya/Mittendorf.

To compare the results, they need to be adjusted to reach a mutual foundation. Arya/Mittendorf have established the assumptions that costs $c>0$ and product substitution parameter $\gamma \in(0,1)$. For simplicity, these assumptions will be loosened to $c=0$ and $\gamma=1$ and thereby facilitate the formulas of Arya/Mittendorf accordingly. This step simplifies comparison without compromising the core message; otherwise the terms would be too excessive. This means that we are comparing the models for the special case that the production of one marginal unit costs nothing (imaginable would be a setting affected by a high ratio of fixed costs, e.g. software development if distribution costs are neglected) and total substitution, which is assumed for simplicity and seldom encountered in reality (thinkable could be the case of Coca-Cola and Pepsi, which mostly differentiate their products by means of brand image). Of course, the comparison is drawn for the case of one competitor on the intermediate market ( $N=1$ ), since this is the case which Arya/Mittendorf computed. This gives us the following table of results to analyze:

Table 3: Arya/Mittendorf's simplified results versus the modified model's results

| Figure | Arya/Mittendorf (X) | Modified Model (Y) | Divergence (X-Y) |
| :--- | :---: | :---: | :---: |
| $\boldsymbol{q}_{\boldsymbol{F}}$ | $\frac{1}{2}\left(a_{F}-\frac{a_{Z}}{11}\right)$ | $\frac{1}{34}\left(13 a_{F}-2 a_{Z}\right)$ | $\frac{1}{374}\left(44 a_{F}+5 a_{Z}\right)$ |
| $\boldsymbol{q}_{\boldsymbol{Z}, \mathbf{1}}$ | $\frac{4 a_{Z}}{11}$ | $\frac{1}{17}\left(-3 a_{F}+7 a_{Z}\right)$ | $\frac{3}{187}\left(11 a_{F}-3 a_{Z}\right)$ |
| $\boldsymbol{q}_{Z, \mathbf{2}}$ | $\frac{7 a_{Z}}{22}$ | $\frac{1}{34}\left(3 a_{F}+10 a_{Z}\right)$ | $-\frac{3 a_{F}}{34}+\frac{9 a_{Z}}{374}$ |
| $\boldsymbol{\delta}$ | $\frac{5 a_{Z}}{22}$ | $\frac{1}{34}\left(-5 a_{F}+6 a_{Z}\right)$ | $\frac{1}{374}\left(55 a_{F}+19 a_{Z}\right)$ |
| $\boldsymbol{t}$ | $\frac{a_{Z}}{11}$ | $\frac{2}{17}\left(2 a_{F}+a_{Z}\right)$ | $\frac{1}{187}\left(-44 a_{F}-5 a_{Z}\right)$ |
| $\boldsymbol{\Pi}_{\boldsymbol{B}}$ | $\frac{1}{484}\left(-11 a_{F}+a_{Z}\right)^{2}$ | $\frac{\left(13 a_{F}-2 a_{Z}\right)^{2}}{1156}$ | $\frac{3\left(110 a_{F}-13 a_{Z}\right)\left(44 a_{F}+5 a_{Z}\right)}{139876}$ |


| $\boldsymbol{\Pi}_{\boldsymbol{A}}$ | $\frac{1}{242} a_{Z}\left(11 a_{F}+27 a_{Z}\right)$ | $\frac{1}{578}\left(43 a_{F}^{2}+9 a_{F} a_{Z}\right.$ <br> $\left.+66 a_{Z}^{2}\right)$ | $-\frac{\left(473 a_{F}-61 a_{Z}\right)\left(11 a_{F}-3 a_{Z}\right)}{69938}$ |
| :--- | :---: | :---: | :---: |
| $\boldsymbol{\Pi}_{\mathbf{A B}}$ | $\frac{a_{F}^{2}}{4}+\frac{5 a_{Z}^{2}}{44}$ | $\frac{1}{68}\left(15 a_{F}^{2}-2 a_{F} a_{Z}+8 a_{Z}^{2}\right)$ | $\frac{1}{748}\left(22 a_{F}^{2}+22 a_{F} a_{Z}-3 a_{Z}^{2}\right)$ |

(Own figure)
To avoid negative quantities it's a necessary condition to mind the restrictions $\frac{13}{2} a_{F}>a_{Z}>$ $\frac{3}{7} a_{F}$ derived from the equations depicted in the column of the modified model $(\mathrm{Y})$ in the rows $q_{F}$ and $q_{Z, 1}$ !

At first glance it's eye-catching that in the Arya/Mittendorf model, where the parent aggregates the modified models central office plus the first division, the quantities on the final and intermediate market ( $q_{F}$ respectively $q_{Z, 1}$ ) are bigger which can be derived from the positive divergence. In the case of the quantity on the intermediate market $\left(q_{Z, 1}\right)$, this depends however on the ratio of the willingness to pay on the final market to the willingness to pay on the intermediate market ( $a_{F}$ respectively $a_{Z}$ ). Should the willingness to pay on the intermediate market $\left(a_{Z}\right)$ be more than $\frac{11}{3} a_{F}$, the quantity sold on the intermediate market $\left(q_{z, 1}\right)$ in the model of Arya/Mittendorf becomes less than in the modified one. On the other hand, the quantity sold on the intermediate market by the competitor $\left(q_{z, 2}\right)$ is less in the Arya/Mittendorf configuration unless the willingness to pay on the intermediate market ( $a_{Z}$ ) is again $\frac{11}{3} a_{F}$. It seems that this, as expected, means that the company dealing with the coordination problem in the modified setup is in worse condition of putting pressure on its competitor than the company not dealing with the coordination problem. This result coincides with the intuition, that the better result in a segregated company is achieved when the divisions acknowledge that they are members of the same team.

There's another urging question: Why don't the quantity on the intermediate and final market $\left(q_{z, 1}, q_{z, 2}\right)$ and the subsidy/markup ( $\delta$ ) in the Arya/Mittendorf configuration depend on both, willingness to pay on the intermediate and final market $\left(a_{Z}, a_{F}\right)$, like in the modified model? Thinking through the interrelations of the modified model, the result comes intuitively: Division 1A will increase its output $\left(q_{Z, 1}\right)$ with the willingness to pay on the intermediate market $\left(a_{Z}\right)$ since, all other things kept equal, a higher willingness to pay on the intermediate
market $\left(a_{Z}\right)$ increases the intermediate market price $\left(P_{Z}\right)$ and 1 A will yield a higher profit $\left(\Pi_{1 A}\right) .1 \mathrm{~A}$ will however decrease its output $\left(q_{Z, 1}\right)$ the more profitable the final market becomes in order to increase its profits by increasing the market price $\left(P_{Z}\right)$ through the mechanics of supply and demand. In these models, as there are only two companies sharing the intermediate market, each of the companies has substantial market power to influence the market price. 1A seeks to increase market price $\left(P_{Z}\right)$ if the final market is reasonably profitable relative to the intermediate market in order to increase the market-based transfer price $(t)$. Thereby, 1A maximizes its profits through division 2A's increased willingness to procure, since willingness to pay on the monopoly $\left(a_{F}\right)$ has driven the price on the final good market $\left(P_{F}\right)$ and thereby the quantity 2 A wants to sell to that market $\left(q_{F}\right)$ increases. 1A's profit function depends on the transfer price $(t)$ times the quantity 2 A procures $\left(q_{F}\right)$, which gives 1 A the said incentive to restrict sales to the intermediate market if the final market is relatively profitable.

The mathematical explanation is that in the Arya/Mittendorf configuration, the parent maximizes consolidated profit (see equation (6), there is no transfer price in the equation!) and in the modified model, 1A maximizes its own profit (see equation (28), there is a transfer price!). The outcome is that the parent takes the subsidiaries maximized profit as given and doesn't try to increase its own profit by pushing the price on the intermediary market. In contrast, that's exactly what 1A does, since it accounts for the transfer price it receives from 1B. To be more specific, the parent does also influence the output on the intermediary market, but for a different ulterior motive: It wants to decrease the market price to stimulate procurement of the subsidiary. Therefore this results in an aggressive, competitive posturing. Arya/Mittendorf show in their paper that, compared to cost-based transfer pricing, the parent does in fact increase its amount to the intermediary market in a setting with market-based transfer pricing! They conclude that if the usual Cournot quantities were chosen, the subsidiary would restrict procurement below the monopoly optimum, because in a Cournotequilibrium, prices are well above marginal costs. The parent does react to this undesirable circumstance and increases its output on the intermediate market $\left(q_{Z, 1}\right)$ to bring the market price nearer to marginal cost. This additional supply is also the reason for a softened response by the rival - its output decreases compared to cost based transfer pricing! (Arya \& Mittendorf, 2008, p. 716)

In contrast, division 1A adjusts its output directly to boost its own profit from the transfer price. The higher the willingness to pay on the final market, the less it will produce since it
becomes more and more lucrative to decrease the market price in order to increase the transfer price. 1 A simply does not see the big picture and therefore does not care that the monopoly situation would make it very profitable for 1B to produce at the monopoly optimum. Thus, there is no competitive posturing without team spirit.

Let's have a look at subsidy/markup $(\delta)$ and the transfer price $(t)$, which is the intermediate market price $\left(P_{Z}\right)$ less the subsidy/markup $(\delta)$. The divergence in the subsidy/markup-terms turns out to be always positive since the willingness to pay on the markets is assumed to be positive. This goes hand in hand with the suspicion that there might be a higher incentive for the parent, which combines the central office and the first division, compared to a separated central office/first division to support the second division with its dealings on the monopoly market. The divergence in transfer price terms is always negative, meaning the modified model's transfer price is always higher. This can be interpreted as the results of the division 1A's interest to increase the transfer price in order to achieve a higher profit. In the modified model the central office has to interfere more cautiously, effectively setting a lower subsidy which then yields a higher transfer price. This results in decreases procurement from 2A and thus diverges from an optimal situation.

In the Arya/Mittendorf configuration, subsidy ( $\delta$ ) is positively dependent only on the willingness to pay on the intermediate market $\left(a_{Z}\right)$. This is because an increase in $a_{Z}$ also increases the intermediate market price $\left(P_{Z}\right)(42)$ and therefore reduces division 1B's procurement. To counterbalance this effect, the parent increases the subsidy ( $\delta$ ). In the modified model, the subsidy/markup ( $\delta$ ) increases with $\mathrm{a}_{\mathrm{Z}}$ and decreases with willingness to pay on the final market $\left(a_{F}\right)$. If the willingness to pay on the intermediary market ( $a_{Z}$ ) increases, the price on the intermediate market $\left(P_{Z}\right)(42)$ will increase and the transfer price ( $t$ ) (A4) will increase as a result. Thus, 1B will restrict procurement and produce far below the monopoly optimum. The central office therefore increases the subsidy ( $\delta$ ) to increase 1B's procurement again.

Subsequently, the subsidiaries profit $\left(\Pi_{B}\right)$ in the Arya/Mittendorf model is considerably larger than in the modified model, unless, according to the equations, willingness to pay on the intermediate market ( $a_{Z}$ ) would more than around 8.46 times willingness to pay on the final market $\left(a_{F}\right)$ - which is restricted as we would see a negative quantity on the final market $\left(q_{F}\right)$. Hence, the subsidiaries profit $(=1 \mathrm{~B})$ always performs better in a setting without a coordination problem.

According to the equations, the parents profit $\left(\Pi_{A}\right)$ is less in the Arya/Mittendorf configuration if not willingness to pay on the intermediate market ( $a_{Z}$ ) would be about 7.75 bigger times than willingness to pay on the final market $\left(a_{F}\right)$. This is also impossible due to restrictions, as we would again see a negative quantity on the final market $\left(q_{F}\right)$. Thus, division 1 A outperforms in a setting with the coordination problem, of course at the expense of division 1B.

In the end, the consolidated profit of the Arya/Mittendorf model exceeds the one of the modified configuration, unless $a_{Z}=\frac{1}{3}\left(11 a_{F}+\sqrt{187} a_{F}\right)$, which is about 8.22 times $a_{F}$ and again impossible according to the restrictions. The conclusion is that the profit in a setting without the coordination problem exceeds the profit in a setting that suffers from that problem for all possible multiples of the willingness to pay-factors $\left(a_{Z}, a_{F}\right)$.

### 5.2 Standardized Case: Multiple Competitors on the Intermediate Market

This alteration of the modified model is visualized in Figure 3: Visualization of the Modified Model with $N(N \in \mathbb{N})$ Competitors. The model's outputs will be reviewed on the following pages. The full calculation of the outcomes can be found in the appendix and is left out here since the process is a more complicated iteration of the one already explained in chapter 5.1.2. After explaining the alterations concerning the input, the outcomes interpretation then will highlight an issue that puts even the theoretical applicability of the model in a multicompetitor environment in question.

Figure 3: Visualization of the Modified Model with $N(N \in \mathbb{N})$ Competitors

(Source: Own figure)

### 5.2.1 Input

The demand function for the intermediate product is
$P_{Z}=\left(a_{Z}-q_{Z, 1}-N q_{Z, i}\right)$,
where $P_{Z}$ denotes the price on the intermediate market for all competitors 1 A to $N \mathrm{~A}$, since there is no product differentiation, $a_{Z}$ denotes the customers' willingness to pay on the intermediate market and $q_{Z, 1}$ respectively $q_{Z, 2}, \ldots, q_{Z, i}$ denote the quantities which 1 A and $2 \mathrm{~A}, \ldots, N \mathrm{~A}$ intend to sell on the intermediate market.

The demand function on the final good market is the same as ever:
$P_{F}=\left(a_{F}-q_{F}\right)$
(1) from chapter 4.1

Like before, $P_{F}$ denotes the price on the final market, $a_{F}$ denotes the customers' willingness to pay on the final market and $q_{F}$ denotes the quantity that $1 B$ chooses to sell on the final market.

The transfer price $(t)$ does change slightly with the newly introduced change in the equation of the market price $\left(P_{Z}\right)$ that account for multiple competitors:
$t=\left(P_{Z}-\delta\right)=\left(a_{Z}-q_{Z, 1}-N q_{Z, i}\right)-\delta$

Division $1 B$ 's profit function still is the same as in (4) from chapter 4.1, but also varies with the number of competitors on the intermediate market $(N)$ through the transfer price $(t)$ :
$\Pi_{1 B}=P_{F} q_{F}-t q_{F}=\left(a_{F}-q_{F}\right) q_{F}-\left(\left(a_{Z}-q_{Z, 1}-N q_{Z, i}\right)-\delta\right) q_{F}$

Division 1A's maximal profit is also still the same. However, keep in mind that all equations which depend either directly on the market price for the intermediate $\operatorname{good}\left(P_{Z}\right)$ or indirectly through transfer price $(t)$ change in a way so they are now influenced by the number of competitors on the intermediate market ( $N$ ). Therefore the profit of division 1 A is:

$$
\begin{align*}
\Pi_{1 A} & =P_{Z} q_{Z, 1}+t q_{F}-c\left(q_{Z, 1}+q_{F}\right) \\
& =\left(a_{Z}-q_{Z, 1}-N q_{Z, i}\right) q_{Z, 1}+\left(\left(a_{Z}-q_{Z, 1}-N q_{Z, i}\right)-\delta\right) q_{F}-c\left(q_{Z, 1}+q_{F}\right) \tag{45}
\end{align*}
$$

By having a glance at the number of competitors in the equation ( $N$ ), it's already easily deducible that division 1A's profit will decrease with increasing competition both directly through the part of the profit achieved on the intermediate market and indirectly by a decrease in the market-based transfer price $(t)$. Also keep in mind that the coordination problem should start to evaporate as competition on the intermediate market becomes fiercer as every competitor increasingly becomes a price taker. In reality this would be the case for homogenous goods which suits the model as we assumed goods are indeed homogenous. In an environment characterized by perfect competition, it would not make a difference if the company operates as one unit or is divided into subdivisions under market-based transfer pricing.

$$
\begin{align*}
\Pi_{A B} & =P_{Z} q_{Z, 1}+P_{F} q_{F}-c\left(q_{Z, 1}+q_{F}\right) \\
& =\left(a_{Z}-q_{Z, 1}-N q_{Z, i}\right) q_{Z, 1}+\left(a_{F}-q_{F}\right) q_{F}-c\left(q_{Z, 1}+q_{F}\right) \tag{46}
\end{align*}
$$

The consolidated profit $\left(\Pi_{A B}\right)$ also decreases with the number of competitors on the intermediate market.
$\Pi_{N A}=\left(a_{Z}-q_{Z, 1}-N q_{Z, i}\right) q_{Z, i}-c q_{Z, i}$
Under the premise that every single competitor $N$ produces the exactly same good in the exactly same environment, this is the equation of every competitor's profit. Every competitor chooses to sell the exact same quantity of goods on the intermediate market $\left(q_{Z, i}\right)$. Due to its unique organization compared to its competitors, company 1 or AB produces a different quantity $\left(q_{z, 1}\right)$.

### 5.2.2 Output and Interpretation

For simplicity, the outcomes will again, like in chapter 5.1.3, be analyzed under the assumption that cost $=0$. Moreover, willingness to pay on the intermediate and final market will be assumed to equal $1\left(a_{Z}, a_{F}=1\right)$ to focus wholly on the effects arising from various levels of competition.

At first, let us have a look at the optimal price on the intermediate market $\left(P_{Z, o p t}\right)$. This is the price on the intermediate market that is established after divisions 1B and 1A have fixed their outputs and the central office has determined the optimal subsidy or markup to maximize consolidated profit. The only factor feasible for adjustment is $N$, which determines the number of competitors on the intermediate market. For the derivation of $P_{Z, o p t}$ before simplifications, please see (A23) in the appendix. To examine the effects a change in the number of competitors has on the price on the intermediate market, differentiate $P_{Z, o p t}$ with respect to $N$ :
$\frac{\partial P_{Z, o p t}}{\partial N}=-\frac{56+N(20+3 N)}{2(8+N(8+N))^{2}}$

Figure 4: $P_{Z, o p t}$ change to $N$ according to (48)

(Source: Own figure)

With rising competition, the price on the intermediate market shrinks and converges towards zero. The more competitors enter the intermediate market, the slower the pace of the decline in price. Even though theory and model comply at this point, keep the marker at $N=4$ in mind.

In a duopoly-setting (one competitor: $N=1$ ) on the intermediate market, the slope of $P_{Z, o p t}$ is $-\frac{79}{578}$, which tells us that a marginal increase (decrease) in $N$ decreases (increases) the intermediate market price by $\frac{79}{578}$. As can be seen from the graphic, the step from 1 competitor to 5 has a more pronounced effect than from 5 to 10 . The numerator of equation (48) increases slower with the number of competitors ( $N$ ) than the denominator and thereby the FOC converges against zero with $N$. The consequent and intuitive assertion is that eventually a higher number of $N$ does not anymore decrease the market price since it will equal marginal cost in the end. We'd end up in a market with perfect competition. So far, intuition and model comply.

Next, it is of interest how the central office acts under increasing competition. Does the management enforce a subsidy to facilitate procurement for division 1B with a higher number of competitors, or will we see a markup on the transfer price instead?
$\frac{\partial \delta_{o p t}}{\partial N}=\frac{-88+N(-12+5 N)}{2(8+N(8+N))^{2}}$
The differentiation of the optimal subsidy/markup ( $\delta_{o p t}$ ) from (A19) tells us how the subsidy/markup reacts to a change in the number of competitors on the intermediate market. When we apply numbers from 1 until 10 to $N$, we will see that the slope is negative until $N=6$. That means for every number of competitors smaller than 6 , an additional competitor will push the central office to reduce the subsidy. In fact, the equation also tells us that at
$N=10$ the slope is maximized and then the curve runs towards to zero. The central office seemingly decides to reduce the subsidy with an increase in $N$ until $N=6$ and then increases subsidy again. How does that make sense?

To find an answer to this unsettling outcome, we should have a glance at the original equation of the subsidiary factor, $\delta_{o p t}$ (A19). After inserting $c=0$ and $a_{Z}, a_{F}=1$ the equation is:

$$
\begin{equation*}
\delta_{o p t}=\frac{6-5 N}{2(8+N(8+N))} \tag{50}
\end{equation*}
$$

Plotting this we see the following, rather unexpected outcome:

Figure 5: $\delta_{o p t}$ change to $N$


The graph shows very straightforward that competition from a second competitor on the intermediate market ( $N=2$ ) drives the central office to impose a markup instead of a subsidy.
(Source: Own figure)

This result gives us a first reasonably strong suspicion to question the model's validity, since the interpretation of that outcome goes as follows: Ceteris paribus, the central office introduces a markup when 1A faces competition from at least 2 competitors. The central offices motive is to support 1A with its distribution of goods on the intermediate market. The reason why the central office considers this supportive action a necessity lies in the actions of 1 A . As we shall see in a moment, 1 A decides to quit offering its products on the intermediate market up to a level where, according to a model, its output even is negative (i.e. it would start "neutralizing" goods on the intermediate market) in order to increase the transfer price by pushing the market price to increase its profits. This is of course utter nonsense and a result of missing restrictions.

This hypothetical outcome would of course not be in the interest of consolidated profit maximization since the consolidated profit increases when 1 A also sells on the intermediate market. Therefore the central office would try to prevent this trend by imposing a negative subsidy, which is a markup.

To prove this point and to show where exactly things start to go wrong and restrictions should be introduced, a look at the optimal output of division 1A (A22) is helpful. Simplified according to the assumptions mentioned above it is:

$$
\begin{equation*}
q_{z, 1}=\frac{6-(-3+N) N}{2(8+N(8+N))} \tag{51}
\end{equation*}
$$

Figure 6: $q_{Z, 1}$ responds unexpectedly to $\Delta N$


At the central offices optimal subsidy $(\delta), \quad 1 \mathrm{~A}$ relatively quickly decides that already with $5(N=5)$ competitors it doesn't want to compete on the intermediate market at all. According to the model, 1 A then would offer "negative quantities".
(Source: Own figure)

Mathematically, the reason for this outcome is the definition of the intermediate market price equation in (1) respectively (42). Without restrictions it enables 1A to counter a decreasing market price by "neutralizing enemy production".

Summed up we have to enforce restrictions for the model before the number of competitors on the intermediate market hits $5(N=5)$. The easiest way to do so is to cut the model at $N=4$. By double-checking if any other quantity or optimal outcome yields unreasonable results it is revealed that indeed division 1 A is the weak point in the design of the model, given the assumptions made.

By reason of the interrelations, the unexpected outcomes continue: Since 1 A is increasingly reluctant to compete on the intermediate market due to the possibility to maximize its profits via internal trade, it pushes the central office towards a policy involving a markup, instead of
a subsidy. As seen in Figure 5, with $N=2$ the subsidy already is not a subsidy anymore but a markup. 1B therefore has to pay a higher price than the market price and consequently 1 A is rewarded by the central office with a subsidy for selling its goods to the intermediate market. Under the assumptions, a generalization could be that the division, which is exposed to a contested market, should be the one to be subsidize, not the division that enjoys monopoly power. On the other hand this is a problem of the model since goods are supposed to be equal according to the simplification. Technically speaking, with absolute substitutability, 1B has no reason to procure from 1 A , as it also could procure from any other competitor on the intermediate market. Thus, 1A would not have the power to increase its profits on at 1B's expense via the transfer price in the first place, since 1 B would not be forced to procure from 1 A .

At $N=4.37$ rounded up $N=5$ competitors 1A "offers" a negative quantity. This means $N=4$ is the definite point where the model doesn't make sense anymore and a restriction has to be introduced!

To get an aggregated perspective of the effect that the number of competitors has on the transfer price, let's have a look at the plot. According to optimal transfer price (A25):

$$
\begin{equation*}
\frac{\partial t}{\partial N}=-\frac{4\left(-4+N+N^{2}\right)}{(8+N(8+N))^{2}} \tag{52}
\end{equation*}
$$

Figure 7: Slope of transfer price

(Source: Own figure)

According to the graph, the transfer price increases until $N=1.5$ and falls thereafter. This result is merely interesting to understand the model, since per definition $N \in \mathbb{N}$. It's obvious that even before a second competitor enters the market, the model proposes a decreasing transfer price.

We now know that the transfer price decreases as the second competitor enters the intermediate market.

At last we want to have a look at the profit functions of 1B (A26), 1A (A27) and the consolidated company AB (A28) (again, simplified for the assumptions mentioned above) and how they react to changes in competition. They are plotted in one diagram which shows results fitting the interpretation above. According to the findings related to intermediate market competition, $N$ is already cut before $N=5$.

Figure 8: Profit functions' responses to $N$


1B, 1A and the consolidated profit AB in one plot. 1B's profit actually rises slightly with $N$ whereas the profit of $\mathbf{1 A}$ declines with $N$. AB simply is the sum of both curves and shows that increasing competition decreases overall profit.
(Source: Own figure)

The obvious question to be raised here is: What's the reason for 1B's increase in profit with rising competition on the intermediate market? Remember that increased competition drives down the market price for the intermediate good. The market price is linked to the company's internal transfer price. 1B competes in a monopoly, making its profit dependent on the price it has to pay 1A for the input good, which is lessened with higher competition since this, as mentioned, drives down intermediate market price. The development of 1A's curve on the other hand is self-explanatory. Rising competition on the intermediate market drives down its profits as a logical conclusion of supply and demand, which is expressed equation (42). The consolidated company's profit curve is simply the sum of the two divisional profits curves as the consolidated profit equals the profit of division 1A plus the profit of division 1B. It shows a decrease in overall profits with increasing competition on the intermediate market. The difference in profits arising from one additional competitor is more pronounced from the first to the second competitor compared to the third to the forth since 1B profits from increased competition on the intermediate market and 1A profits from the markup imposed by the central office as soon as $N$ equals an artificial " 1.2 competitors".

To determine whether there is still a strategic effect on the intermediate market, the quantities of company AB's division 1A ( $q_{Z, 1, o p t}$, see (A22)) and the quantity of its competitors, NA ( $q_{z, i, o p t}$, see (A21)), shall be compared.

$$
\begin{equation*}
q_{Z, 1, o p t}-q_{Z, i, o p t}=\frac{c\left(4+N^{2}\right)-(2+N)^{2} a_{F}+4 N a_{Z}}{2(8+N(8+N))} \tag{53}
\end{equation*}
$$

For simplicity, assume cost $c=0$. Then, a strategic effect exists if $\frac{(6+N(6+N)) a_{F}}{2 N}>a_{Z}>$ $\frac{(2+N)^{2} a_{F}}{4 N}$ what means the intermediate market, i.e. the customers' willingness to pay $\left(a_{Z}\right)$, has to be more attractive as more competitors ( $N$ ) enter that intermediate market. ${ }^{2}$ Also, if the final market $\left(a_{F}\right)$ becomes more attractive in relation to the intermediate market, 1A's willingness to produce more units for the intermediate market and thereby affect 1B's level of procurement positively, decreases. This is because 1A tries to maximize its profits from 1B's gains on the final market.

## 6 Conclusion

The modified model proves not to be perfectly extendable to depict multi-competition on the intermediate market under the assumptions made. Attention is raised from the analysis of the effect that an increasing number of competitors on the intermediate market leads to supportive actions from the central office to help division 1A instead of division 1B. This alone is interesting but doesn't make the models results invalid. Especially illuminative in showing that the model needs to be restricted is the insight that division 1A's output on the intermediate market eventually becomes negative (51). To make any sense of the proposed model, strict restrictions have to be applied. The modified model has to be cut at 4 competitors as division 1 A of the vertically integrated company AB otherwise shows illegitimate behavior. Another means of restricting the model in order to achieve sensible outcomes would be to introduce the restriction $q_{Z, 1} \geq 0$ which however would lead to a division 1A that's not producing anything as soon as the fifth competitor enters the market under the assumptions made.

The most interesting difference between the outcome of Arya/Mittendorf and the alterations in the modified model is that the parent in the model of Arya/Mittendorf, which basically

[^1]resembles a combination of division 1A and the central office in the modified model, seeks to drive down market price $\left(P_{Z}\right)$ to enable its subsidiary $(=1 \mathrm{~B})$, which is the same division in both configurations, to procure more. The reason for this is to be found in the absence of the coordination problem as the parent does account for consolidated profits. The parent is in competition with another competitor on the intermediate market but the subsidiary enjoys monopoly power including the price advantage. Therefore the parent intelligently uses the transfer price for competitive posturing since it has an incentive to make a profit on the intermediate market but another incentive to make the subsidiary procure more. Thus the parent chooses to sell more units on the intermediate market than it would have usually done without a promisingly profitable subsidiary that's enjoying monopoly power and is contributing to consolidating profit.

Splitting the parent into two profit centers (1A and 1B) and a central office, that seeks to mitigate the arising coordination problem inside the vertically integrated company, bears a wholly different outcome. The division 1A has an incentive to drive up the market price to make transfer price $(t)$ move up. Therefore it uses its market power in a duopoly by selling less of its goods on the intermediate market. Even with just one competitor on the intermediate market, the strategic effect vanishes due to a lack of credible commitment since division 1A acts egocentric. In an attempt to optimize the consolidated result, the central office tries to mitigate the coordination problem.

In the case of multiple ( $N$ ) competitors on the intermediate market, the model's outcomes demand scrutiny. The expectation with higher competition on the intermediate market (i.e. increasing $N$ ) would be that by converging to perfect competition, the coordination problem should disappear without any intervention at all. After all, division 1A does incrementally lose market power with rising competition on the intermediate market. Without restricting the model however, 1A restricts production of the intermediate product and relies on a markup introduced by the central office to stimulate production of 1A again.

Another interesting question for further research would definitely be, if it's that easy to get a leg on upon external rivals if you're not acting in a duopoly and you have no market power in a more appropriate setting under more appropriate assumptions. Then, the model could be incrementally extended to depict variable levels of product diversity and eventually competition on the final market.

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## 8 Appendix

### 8.1 Proof of Calculation for a variable Number of Competitors

The demand function for the intermediate product is:
$P_{Z}=\left(a_{z}-q_{Z, 1}-N q_{Z, i}\right)$

The demand function for the final good market is:
$P_{F}=\left(a_{F}-q_{F}\right)$,

Division 1B's profit function is:
$\Pi_{1 B}=P_{F} q_{F}-t q_{F}$

The transfer price is defined as:
$t=\left(P_{Z}-\delta\right)$

Division 1A's profit function is:
$\Pi_{1 A}=P_{Z} q_{Z, 1}+t q_{F}-c\left(q_{Z, 1}+q_{F}\right)$

Inserting (A1), (A2), (A4) in (A3), we end up with:
$\Pi_{1 B}=\left(a_{F}-q_{F}\right) q_{F}-\left(\left(a_{Z}-q_{Z, 1}-N q_{Z, i}\right)-\delta\right) q_{F}$

1B's profit depends on the price of the good sold on the intermediate market $\left(P_{Z}\right)$. This price is influenced by the quantity $1 \mathrm{~A}, 2 \mathrm{~A} \ldots N \mathrm{~A}, N \in \mathbb{N}$, determine to offer. The denotation of the competitor's amount on the intermediary market shall be $q_{Z, i}$, whereas the index $i=(N+1)$. $q_{Z, i}$ denotes every competitor's quantity on the intermediary market. More specific, if $N$ equals 1 , there is just one competitor on the intermediate market (see the case in chapter 5.1) which makes $i$ equal to 2 , denoting the quantity that the first competitor of the company with divisions 1 A and 1 B , let's call it "company AB ", offers on the intermediary market $\left(q_{z, 2}\right)$. If $N$ equals 2, then $i$ is 3 and $q_{Z, 3}$ denotes the amount both the first and the second competitor of company AB simultaneously choose to offer on the intermediate market. Their amounts are the same due to the assumption of symmetry in organization, unit cost $c$ and product
characteristics - see (A9) for further details. The subsidy/markup ( $\delta$ ) increases 1B's profit if the central office sets an internal discount ( $\delta$ is positive) and decreases it, if it sets an internal markup ( $\delta$ is negative) to the price obtainable on the intermediate market.

According to microeconomic theory, the optimal output is reached when marginal revenue (MR) equals marginal cost (MC). In other words, until one additional marginal unit's revenue exceeds its costs, it shall be produced.
$\frac{d \Pi_{1 B}}{d q_{F}}=M R\left[\left(a_{F}-q_{F}\right) q_{F}\right]-M C\left[\left(\left(a_{Z}-q_{Z, 1}-N q_{Z, i}\right)-\delta\right) q_{F}\right]$
Thereby maximizing 1B's profit (A6) with respect to $q_{F}$ yields the FOC:
$q_{F}=\frac{1}{2}\left(a_{F}-a_{Z}+q_{Z, 1}+N q_{Z, i}+\delta\right)=\frac{1}{2}\left(a_{F}-P_{Z}+\delta\right)$
This is the optimal quantity according to 1B. It increases with an internal discount ( $\delta$ ) and decreases with the price of the good on the intermediate market $\left(P_{Z}\right)$ which is influenced by the number of competitors $(N)$ and the quantities $\left(q_{z, i}\right)$ they offer to the market.

On the second level, the optimal quantities for division 1 A and every competitor $N \mathrm{~A}$ on the intermediary market are determined. All the participants on the market take each other's intentions into account and thereby form a Nash-Cournot-Equilibrium.

Generally speaking, without taking the symmetry/equality assumption into account yet, the profit of every competitor on the market is depicted as:
$\Pi_{i A}=\left(a_{Z}-q_{Z, i}-\sum_{j=1}^{n} q_{j} \gamma_{j}\right) q_{Z, i}-c q_{Z, i}$
FOC yields:
$q_{Z, i}=\frac{1}{2}\left(a_{Z}-c-\sum_{j=1}^{n} q_{j} \gamma_{j}\right)$
At this point, the symmetry assumption is introduced: Assuming that all the competitors choose to offer exactly the same quantity under equal circumstances, mainly offering identical products ( $\gamma=1$ ), having equal costs per unit (c) and equal internal organization, this formula can be adjusted for simplification. Mark that company AB does not offer the same quantity as
the others on the internal market. This comes due to internal strategic and coordination processes related to the transfer price that affects the segregated company with two divisions and decentralized decision-making. Thereby the quantity it offers $\left(q_{z, 1}\right)$ differs from all the others companies.
$q_{Z, i}=\frac{1}{2}\left(a_{Z}-(N-1) q_{Z, i}-q_{Z, 1}-c\right)$

The quantity of competitor $i$ equals the willingness to pay on the intermediate market ( $a_{z}$ ) less the quantity all other competitors $\left(q_{z, i}\right)$ with one division (A) that effectively choose to offer the same quantity $\left(q_{z, i}\right)$ less the quantity that division A of the vertically integrated company AB decides to offer $\left(q_{z, 1}\right)$.

Simplified this equates to:
$q_{Z, i}=\frac{a_{Z}-c-q_{Z, 1}}{1+N}$
At the same time, 1A maximizes its profit according to its output on the intermediary market $\left(q_{z, 1}\right)$, taking the optimal quantity on the final market $\left(q_{F}\right)$ into equation. Inserting (A8), (A4), (A1) in (A5) results in:

$$
\begin{align*}
\Pi_{1 A} & =\left(a_{Z}-q_{Z, 1}-N q_{Z, i}\right) q_{Z, 1} \\
& +\left(\left(a_{Z}-q_{Z, 1}-N q_{Z, i}\right)-\delta\right)\left(\frac{1}{2}\left(a_{F}-a_{Z}+q_{Z, 1}+N q_{Z, i}+\delta\right)\right)  \tag{A13}\\
& -c\left(q_{Z, 1}+\left(\frac{1}{2}\left(a_{F}-a_{Z}+q_{Z, 1}+N q_{Z, i}+\delta\right)\right)\right)
\end{align*}
$$

The FOC then yields the optimal quantity for 1A under consideration of 1B's optimal quantity on the final market:
$q_{Z, 1}=\frac{1}{6}\left(-3 c-2 \delta-a_{F}+4 a_{Z}-4 N q_{Z, i}\right)$

Now, that we have calculated the optimal quantities of both 1 A and the individual optimum of each and every one of its competitors on the intermediary market, a Nash-CournotEquilibrium can be achieved by intersecting the two relation curves (A12) and (A14).

Nash-Cournot-Equilibrium determined according to these equations then is:
$q_{Z, i}=\frac{a_{F}+2 a_{Z}-3 c+2 \delta}{2(3+N)}$
$q_{Z, 1}=\frac{4 a_{Z}-(1+N) a_{F}+c(-3+N)-2(1+N) \delta}{2(3+N)}$

It is of interest that $q_{Z, 1}$ not only reacts positively on the willingness to pay on the intermediary market $\left(a_{Z}\right)$ but decreases with the willingness to pay on the final market $\left(a_{F}\right)$. When the willingness to pay on the final market shrinks, 1 A wants to decrease its output on the intermediary market, too. This is due to the decreased unit sales to 1 B when willingness to pay on the final market shrinks. 1A tries to counter this development with an increased transfer price. 1A itself can influence the transfer price by reducing its amount sold on the intermediary market, thus decreasing the intermediate market price $\left(P_{Z}\right)$. Of course, from the central office's point of view that might not be a preferred intervention.

Finally, on the third level of backward induction, the optimal quantities of 1 A and 1 B have to be taken into account to determinate the optimal subsidy/markup ( $\delta$ ), fixed by the central office. Thereafter, the optimal $\delta$ can be utilized to compute the optimal profit of the consolidated company from the central office's point of view.

In order to calculate optimal subsidy/markup ( $\delta$ ), the consolidated profit has to be depicted and all optimal quantities have to be inserted into the equation. Consolidated profit simply is 1A's profit depicted in (A5) added up with 1B's profit, depicted in (A3), thereby eliminating transfer prices in the formula.
$\Pi_{A B}=P_{Z} q_{Z, 1}+P_{F} q_{F}-c\left(q_{Z, 1}+q_{F}\right)$

Replacing $P_{Z}$ (A1) and $P_{F}$ (A2) and inserting optimized terms of $q_{F}$ (A8), $q_{z, i}$ (A15) and $q_{z, 1}$ (A16) into the equation of consolidated profit yields (A18)

$$
\begin{align*}
\Pi_{A B} & =\left(a_{Z}-\left(\frac{c(-3+N)-2(1+N) \delta-(1+N) a_{F}+4 a_{Z}}{2(3+N)}\right)-N\right. \\
& \left.*\left(\frac{-3 c+2 \delta+a_{F}+2 a_{Z}}{2(3+N)}\right)\right) *\left(\frac{c(-3+N)-2(1+N) \delta-(1+N) a_{F}+4 a_{Z}}{2(3+N)}\right) \\
& -c\left(\left(\frac{c(-3+N)-2(1+N) \delta-(1+N) a_{F}+4 a_{Z}}{2(3+N)}\right)+\left(\frac { 1 } { 2 } \left(\delta+a_{F}-\left(a_{Z}\right.\right.\right.\right. \\
& -\left(\frac{c(-3+N)-2(1+N) \delta-(1+N) a_{F}+4 a_{Z}}{2(3+N)}\right)-N \\
& \left.\left.\left.*\left(\frac{-3 c+2 \delta+a_{F}+2 a_{Z}}{2(3+N)}\right)\right)\right)\right)+\left(a_{F}-\left(\frac { 1 } { 2 } \left(\delta+a_{F}-\left(a_{Z}\right.\right.\right.\right. \\
& -\left(\frac{c(-3+N)-2(1+N) \delta-(1+N) a_{F}+4 a_{Z}}{2(3+N)}\right)-N  \tag{A18}\\
& \left.\left.\left.*\left(\frac{-3 c+2 \delta+a_{F}+2 a_{Z}}{2(3+N)}\right)\right)\right)\right) *\left(\frac { 1 } { 2 } \left(\delta+a_{F}-\left(a_{Z}\right.\right.\right. \\
& -\left(\frac{c(-3+N)-2(1+N) \delta-(1+N) a_{F}+4 a_{Z}}{2(3+N)}\right)-N \\
& \left.\left.*\left(\frac{-3 c+2 \delta+a_{F}+2 a_{Z}}{2(3+N)}\right)\right)\right)
\end{align*}
$$

FOC with respect to $\delta$ and then solved for $\delta$ determines the optimal subsidy/markup $\left(\delta_{o p t}\right)$ :

$$
\begin{equation*}
\delta_{o p t}=\frac{c(-6+5 N)-(2+3 N) a_{F}-2(-4+N) a_{Z}}{2(8+N(8+N))} \tag{A19}
\end{equation*}
$$

In order to calculate the optimal consolidated profit it's necessary to calculate the optimal quantities from the central office's point of view and insert those into the consolidated profit equation.

First, take 1B's optimal $q_{F}$ (A8). Inserting $P_{Z, 1}$ (A1) in the first step, continuing with the Nash-Cournot-Equilibrium optima's $q_{z, i}$ (A15) and $q_{z, 1}$ (A16) and finally inserting the optimal $\delta$ (A19) results in the optimal quantity on the final market from the central office's point of view in dependency on the number of competitors on the intermediary market ( $N$ ):
$q_{F, o p t}=\frac{(6+N(6+N)) a_{F}-2 N a_{Z}-c(6+N(4+N))}{2(8+N(8+N))}$

Optimal values for the quantities, which divisions $N \mathrm{~A}$ and 1 A offer on the intermediary market, are calculated by inserting optimal subsidy/markup $\delta_{\text {opt }}$ (A19) into the respective terms (A15) and (A16):
$q_{Z, i, o p t}=\frac{-c(10+3 N)+(2+N) a_{F}+2(4+N) a_{Z}}{2(8+N(8+N))}$
$q_{Z, 1, o p t}=\frac{c(-6+(-3+N) N)-(1+N)(2+N) a_{F}+(8+6 N) a_{Z}}{2(8+N(8+N))}$
The optimal price on the intermediary market $\left(\mathrm{P}_{Z, \text { opt }}\right)$ is calculated by taking the original intermediate market price equation (A1) and replacing the quantities with the terms in (A21) and (A22). The optimal price on the final market is expressed similarly by taking expression (A2) and replacing the quantity with the optimum depicted in (A20):
$\mathrm{P}_{Z, o p t}=\frac{c(6+N)(1+2 N)+(2+N) a_{F}+2(4+N) a_{Z}}{2(8+N(8+N))}$
$P_{F, \text { opt }}=\frac{c(6+N(4+N))+(10+N(10+N)) a_{F}+2 N a_{Z}}{2(8+N(8+N))}$
Optimum transfer price results from inserting (A21) and (A22) into the original transfer price formula (A4):
$t=\frac{c(6+N(4+N))+2(1+N) a_{F}+2 N a_{Z}}{8+N(8+N)}$

Finally, the optimal profits of 1 A and 1 B as well as consolidated profit can be depicted. Therefore the original formula for 1B's profit (A3) is merged with the optimal $P_{F, o p t}$ (A24), $q_{F, o p t}$ (A20) and $t$ (A25). 1A's optimal profit equation is expressed by taking the original equation (A5) and applying $\mathrm{P}_{Z, \text { opt }}$ (A23), $q_{Z, 1, \text { opt }}$ (A22), $t$ (A25) as well as $q_{F, \text { opt }}$ (A20) to it.
$\Pi_{B}=\frac{\left(c(6+N(4+N))-(6+N(6+N)) a_{F}+2 N a_{Z}\right)^{2}}{4(8+N(8+N))^{2}}$

$$
\begin{align*}
\Pi_{A} & =\frac{1}{4(8+N(8+N))^{2}}\left(c^{2}(84+N(112+5 N(7+N)))+(1+N)(20+N(20\right. \\
& +3 N)) a_{F}^{2}-2 c(64+N(68+N(8+N))) a_{Z}+4(16+N(16+N)) a_{Z}^{2}  \tag{A27}\\
& \left.+2 a_{F}\left(-c(20+N(4+N)(11+4 N))+N(2+N)^{2} a_{Z}\right)\right)
\end{align*}
$$

Profit 1A (A27) and 1B (A26) added up equals consolidated profit:

$$
\begin{align*}
\Pi_{\mathrm{AB}} & =\frac{1}{4(8+N(8+N))}\left(c^{2}(15+N(5+N))+(7+N(7+N)) a_{F}^{2}+2 c(-8\right.  \tag{A28}\\
& \left.+N) a_{Z}+8 a_{Z}^{2}-2 a_{F}\left(c(7+N(6+N))+N a_{Z}\right)\right)
\end{align*}
$$

The competitors' profits after the central office sets the subsidy/markup is expressed by applying $q_{z, i, o p t}(\mathrm{~A} 21)$ and $\mathrm{P}_{Z, o p t}$ (A23) to the following general profit function of the competitors:
$\Pi_{i A}=P_{Z} * q_{Z, i}-c q_{Z, i}$

Simplified, this yields:
$\Pi_{i A}=\frac{\left(-c(10+3 N)+(2+N) a_{F}+2(4+N) a_{z}\right)^{2}}{4(8+N(8+N))^{2}}$
$\Pi_{2 A}(N=1)=\frac{\left(-13 c+3 a_{F}+10 a_{Z}\right)^{2}}{1156}$

### 8.2 Split Up Profit Function

To examine which terms of the model's outcomes are of particular interest for a differential and graphical analysis, a closer look at the profit function is informative. A full breakdown of the profit function looks like (A31).

$$
\begin{align*}
\frac{d \Pi_{\mathrm{AB}}}{d \delta} & =\frac{\partial \Pi_{\mathrm{AB}}}{\partial \delta}+\frac{\partial \Pi_{\mathrm{AB}}}{\partial q_{Z, 1}} \frac{\partial q_{Z, 1 \mathrm{opt}}}{\partial \delta}+\frac{\partial \Pi_{\mathrm{AB}}}{\partial q_{Z, i}} \frac{\partial q_{Z, \text { iopt }}}{\partial \delta}+\frac{\partial \Pi_{\mathrm{AB}}}{\partial q_{F}} \frac{\partial q_{\mathrm{Fopt}}}{\partial \delta} \\
& +\frac{\partial \Pi_{\mathrm{AB}}}{\partial q_{F}} \frac{\partial q_{\mathrm{Fopt}}}{\partial q_{Z, 1}} \frac{\partial q_{Z, \text { opt }}}{\partial \delta}+\frac{\partial \Pi_{\mathrm{AB}}}{\partial q_{F}} \frac{\partial q_{\mathrm{Fopt}}}{\partial q_{Z, i}} \frac{\partial q_{Z, \text { iopt }}}{\partial \delta} \tag{A31}
\end{align*}
$$

In this massive term all the (inter-)dependencies are depicted and the general steps of calculation for $N$ competitors basically recapitulated. However, some of them are unnecessary regarding the analysis since $\Pi_{A B}$ includes $\Pi_{1 B}$ and $\Pi_{1 A}$. The terms in green can be itemized to:
$\frac{\partial \Pi}{\partial q_{Z, 1}} \frac{\partial q_{Z, 1 \text { opt }}}{\partial \delta}+\frac{\partial \Pi}{\partial q_{F}} \frac{\partial q_{F o p t}}{\partial q_{Z, 1}} \frac{\partial q_{Z, 1 o p t}}{\partial \delta}$
$\hat{=} \frac{\partial \Pi_{1 A}}{\partial q_{Z, 1}} \frac{\partial q_{Z, 1 \mathrm{opt}}}{\partial \delta}+\frac{\partial \Pi_{1 A}}{\partial q_{F}} \frac{\partial q_{F o p t}}{\partial q_{Z, 1}} \frac{\partial q_{Z, 1 o p t}}{\partial \delta}+\frac{\partial \Pi_{1 B}}{\partial q_{Z, 1}} \frac{\partial q_{Z, 1 o p t}}{\partial \delta}+\frac{\partial \Pi_{1 B}}{\partial q_{F}} \frac{\partial q_{F o p t}}{\partial q_{Z, 1}} \frac{\partial q_{Z, 1 o p t}}{\partial \delta}$
Now let's have a closer look on how to further simplify this term. At first, with the terms (A6) and (A8) we have defined that:

$$
\begin{equation*}
\frac{\partial \Pi_{1 B}}{\partial q_{F}}=0 \tag{A33}
\end{equation*}
$$

In order to reach outcome (A14) it must be true that:
$\frac{d \Pi_{1 A}}{d q_{Z, 1}}=\frac{\partial \Pi_{1 A}}{\partial q_{Z, 1}}+\frac{\partial \Pi_{1 A}}{\partial q_{F}} \frac{\partial q_{F o p t}}{\partial q_{Z, 1}}=0$
Given the knowledge about (A32), (A33) and (A34), the terms of interest for further analysis, as the profit function of $A B$ is concerned, can be reduced to the following ones:

$$
\begin{equation*}
\frac{d \Pi_{A B}}{d \delta}=\frac{\partial \Pi_{1 B}}{\partial q_{Z, 1}} \frac{\partial q_{Z, 1 o p t}}{\partial \delta}+\frac{\partial \Pi_{A B}}{\partial q_{Z, i}} \frac{\partial q_{Z, \text { iopt }}}{\partial \delta}+\frac{\partial \Pi_{1 A}}{\partial q_{F}} \frac{\partial q_{\text {Fopt }}}{\partial \delta}+\frac{\partial \Pi_{1 A}}{\partial q_{F}} \frac{\partial q_{F o p t}}{\partial q_{Z, i}} \frac{\partial q_{Z, \text { iopt }}}{\partial \delta} \tag{A35}
\end{equation*}
$$

By looking closely at (A35) it appears there are essentially four major terms, which are separated by pluses. This fragmentation will serve to distinct the terms during the analysis. Another point worth mentioning is that the parts of terms marked in orange resemble each other.

### 8.3 Differential Analysis for One- and Multiple Competitors

To analyze the outcome of the model and to get a grip on the interrelations it is helpful to look at the meaningful differentials. Therefore, the outcomes will be analyzed for the case that 1 A is in competition with one competitor and $N$ competitors on the intermediate market. For that reason $N=1$ will be applied to the general formulas expressed at the beginning of the appendix.

## Effect of a change in 1A's quantity on the intermediate market ( $\boldsymbol{q}_{Z, 1}$ ) on 1B's profit ( $\boldsymbol{\Pi}_{1 B}$ )

According to (A35), $\Pi_{1 B}$ (A6) is differentiated with respect to $q_{z, 1}$ :
$\frac{\partial \Pi_{1 B}}{\partial q_{Z, 1}}=q_{F}(N)=\left(\frac{1}{2}\left(\delta+a_{F}-\left(a_{Z}-q_{Z, 1}-N q_{Z, i}\right)\right)\right)$
$\frac{\partial \Pi_{1 B}}{\partial q_{Z, 1}}=\quad q_{F}(N=1)=\left(\frac{1}{2}\left(\delta+a_{F}-\left(a_{Z}-q_{Z, 1}-q_{Z, 2}\right)\right)\right)$

1B's profit on the final market grows (shrinks) as the amount $q_{Z, 1}$ which 1 A chooses to sell on the intermediate market increases (decreases). The reason for this development is to be found in the inverse influence of the units sold on the intermediate market ( $q_{z, 1}$ and $q_{z, i}$ ) on the price on the intermediate market $\left(P_{Z}\right)$ as can be seen in (A1). Disregarding the subsidy/markup ( $\delta$ ), the price on the intermediate market $\left(P_{Z}\right)$ eventually affects the transfer price $(t)$ (A4) unidirectional: If 1A increases its output $\left(q_{z, 1}\right)$ on the intermediate market, the price on this market $\left(P_{Z}\right)$ sinks and the transfer price $(t)$ is reduced as well. As the transfer price $(t)$ is regarded as cost-factor by 1 B , its profit $\left(\Pi_{1 B}\right)$ increases as the transfer price decreases.

The difference for multiple competitors lies in the equation of the amount that 1 B offer on the final market $\left(q_{F}\right)$, which now is also dependent on the number of competitors ( $N$ ). Quantity on the final market $\left(q_{F}\right)$ is increased with a higher subsidy $(\delta)$ from the central office, a higher willingness of the customers on the final market to pay $a_{F}$ and is decreased with the cost of the input good on the intermediate market $P_{Z}(\mathrm{~A} 1)$, which is exactly the part of the equation that now depends on the number of competitors on the intermediate market. It is observable that with more competitors on the intermediate market, 1A's effect via $q_{z, 1}$ is decreased and the slope $\frac{\partial \Pi_{1 B}}{\partial q_{Z, 1}}$ decreases. For every additional unit sold on the intermediate market by 1 A , the increase in 1B's profit then is $q_{F}(5>N>1)<q_{F}(N=1)$. In other words, 1B's marginal revenue decreases with $N$ because the influence of 1A's output on the price on the intermediate market decreases as there are more opportunities for buyers to choose from.

## Effect of the change in subsidy/markup ( $\delta$ ) on 1 A's quantity ( $q_{z, 1}$ ) on the intermediate market

$q_{Z, 1}$ (A16) is differentiated with respect to $\delta$ :

$$
\begin{align*}
& \frac{\partial q_{Z, 1}}{\partial \delta}=-\frac{1+N}{3+N}  \tag{A38}\\
& \frac{\partial q_{Z, 1}(N=1)}{\partial \delta}=-\frac{1}{2} \tag{A39}
\end{align*}
$$

For one competitor on the intermediate market (A38), this outcome tells us that in the case of an increase (decrease) of the subsidy/markup by 1 , the quantity sold on the intermediate market by 1A decreases (increases) by half of that. A higher $\delta$ decreases the transfer price $(t)$ (A4). That is not in the favor of division 1 A as it is striving for the highest possible profit, which, not accounting for changes in 1B's output $\left(q_{F}\right)$, comes with the highest possible transfer price that it receives from 1B. At this point, 1A itself has a possibility to influence the transfer price indirectly. This connection becomes more tangible by closely examining the definition of the market price $\left(P_{Z}\right)$ in equation (A1) in conjunction with the equation of the transfer price (A4). The price on the intermediate good market is dependent on the change of the quantity 1A chooses to sell, as well as on the quantity its competitor chooses to sell or choose to sell, as more competitors enter the market. The intermediate market price ( $P_{Z}$ ) increases as 1A decreases the quantity it offers on the intermediate market $\left(q_{Z, 1}\right)$. This decline in consequence influences $P_{Z}$ to rise, which then raises the transfer price which is to 1A's liking. A higher subsidy $\delta$ thus has two mentionable effects on the transfer price. Firstly, it decreases the transfer price directly. Secondly however, it increases the transfer price through the implicitly consequential action of 1 A . According to this result, the central office has to cautiously balance these effects to maximize consolidated profit because the indirect effect backfires on the company's performance.

With an increasing number of players ( $N$ ) on the intermediate market 1A's influence decreases. With a sufficiently high $N$, no party would be able to influence the market price and we would be in perfect competition. Since the model is restricted at $N=4$ as it is, this is not depicted in the following plot.

Figure 9: Plot of (A38)


This plot shows the effect of the markup/subsidy ( $\delta$ ) on 1A's quantity $\left(q_{z, 1}\right)$. With increasing competition ( $N$ ) the effect of the subsidy becomes smaller, the central office's action have decreasing effects.
(Source: Own figure)

Effect of a change in the competitors' $\left(q_{z, 2}, q_{z, i}\right)$ quantities on $A B ' s$ $\operatorname{profit}\left(\Pi_{\mathrm{AB}}\right)$
$\frac{\partial \Pi_{A B}}{\partial q_{Z, 2}}=\quad-q_{Z, 1}=\frac{1}{6}\left(-3 c-2 \delta-a_{F}+4 a_{Z}-4 N q_{Z, i}\right)$
$\frac{\partial \Pi_{A B}}{\partial q_{Z, i}}=\quad-N q_{Z, 1}=-N * \frac{1}{6}\left(-3 c-2 \delta-a_{F}+4 a_{Z}-4 N q_{Z, i}\right)$

The result in a duopoly ( $-q_{Z, 1}$ ) tells, that an increase (decrease) of the competitor's quantity $\left(q_{z, 2}\right)$ on the intermediate good market by 1 decreases (increases) consolidated profit of AB by $q_{Z, 1}$. This is logical since an increase (decrease) of the competitors quantity decreases (increases) market price, which decreases (increases) AB's profit.

Figure 10: Plot of (A41)


The plot shows increasing effect of the competitors' quantities on the intermediate market as the number of competitors increase. This is comprehensible since each additional competitor takes another share in the market. The effect is also dependent on division 1A's interest in the intermediate market. If its output is relatively high, sharing with additional competition hurts more than if its output is relatively low.

Effect of a change in the subsidy/markup factor on the quantity of 2 A
$\frac{\partial q_{z, 2 o p t}}{\partial \delta}=\frac{1}{4}$
$\frac{\partial q_{Z, \text { iopt }}}{\partial \delta}=\frac{1}{3+N}$
If the subsidy factor increases by one unit, the quantity that competitor 2 A offers on the intermediary market increases by $\frac{1}{4}$ (A42). This is because 1 A reduces its quantity on the intermediate market with increasing subsidy $\delta$ in order to push the market price $P_{Z}$ and thereby indirectly raise the transfer price $t$ it receives from 1B for the intermediate good. Using the subsidy factor, the central office is able to slightly influence the degree of competition on the intermediate market.

With an increasing number of competitors on the intermediate market, the effect of a change in the subsidy factor $\delta$ on $N A$ 's quantity $q_{Z, i}$ becomes less pronounced. The central office
influences the quantity of 1A's competitors in way that each of them increases (decreases) their $q_{Z, \mathrm{i}}$ by $\frac{1}{3+N}$ with every unit of increased (decreased) subsidy ( $\delta$ ).

## Effect of a change in 1B's quantity on 1A's profit

$$
\begin{align*}
& \frac{\partial \Pi_{1 A}}{\partial q_{F}}=-c+t=-c-\delta+a_{Z}-q_{Z, 1}-q_{Z, 2}  \tag{A44}\\
& \frac{\partial \Pi_{1 A}}{\partial q_{F}}=-c+t=-c-\delta+a_{Z}-q_{Z, 1}-N q_{Z, i} \tag{A45}
\end{align*}
$$

The effect is exactly the transfer price $t$ (A4) minus cost $c$, see (A44). If 1B's quantity on the final market $\left(q_{F}\right)$ increases, the profit of $1 \mathrm{~A}\left(\Pi_{1 A}\right)$ increases as well. When 1B increases its output by one unit, the profit of 1 A increases exactly by the amount of money that 1 B initially paid 1A to receive one input unit to produce that unit for the final market. This is due to the conversion rate being set at 1 . Transfer price $t$ (A4) is always positive with the single exception if market price $P_{Z}$ is equal to marginal cost $c$ which we defined as zero in a sufficiently competitive market. Then the transfer price would also be zero.

With increasing competition the effect of a change in 1B's quantity on 1A's profit lessens, as to be seen in (A45). It converges to zero with an evolving perfect market. This is because eventually market price $P_{Z}$ will equal marginal cost.

## Effect of a change in subsidy/markup on 1 B 's quantity

$\frac{\partial q_{\text {Fopt }}}{\partial \delta}=\frac{1}{2}$

The optimal quantity on the final good market deduced with respect to subsidy factor is $\frac{1}{2}$, see (A46). If the subsidy increases (decreases) by one unit, the amount on the final good market increases (decreases) by half that. The relation between the two figures is the transfer price which decreases (increases) when the subsidy/markup factor increases (decreases). 1B will want to procure more input goods when prices are low to maximize its profit. In this case, the effect doesn't scale with the number of competitors on the intermediate market $(N)$.

## Effect of a change in 2A's quantity on 1B's quantity

$\frac{\partial q_{\text {Fopt }}}{\partial q_{Z, 2}}=\frac{1}{2}$
$\frac{\partial q_{\text {Fopt }}}{\partial q_{Z, i}}=\frac{N}{2}$
According to (A48) the result is $\frac{1}{2}$ which tells that an increasing (decreasing) output of 2 A on the intermediate market $q_{Z, 2}$ increases the output of 1 B on the final good market $q_{\mathrm{F}}$. The two amounts are related by the market price $P_{Z}$ which in turn affects the transfer price $t$ (A4). If the intermediate market price decreases (increases) because of an increased (decreased) output of 2 A , the transfer price ceteris paribus (i.e. given an unchanged subsidy/markup ( $\delta$ )) will decrease (increase). A decreased (increased) transfer price then will boost (reduce) 1B's procurement.

With increasing competition on the intermediate market the increase in $q_{\mathrm{F}}$ on the final market picks up momentum. Again this is closely related to the diminishing effect of increased competition on the intermediate market price $P_{Z}$ which inversely affects transfer price $t$ (A4). Basically, consolidated AB can shift its focus from intermediate market to the final good market with monopoly-prices.

## 9 Summary

This thesis gives an introduction to (market-based) transfer prices and highlights interrelations which are linked to transfer pricing policies. A strategic effect of transfer pricing and the delegation problem are highlighted. On the basis of a paper published by Arya and Mittendorf (2008), a basic model of a vertically integrated company is introduced. After examining the results of Arya and Mittendorf (2008) a modified model is portrayed to show the changes in the effect of the transfer pricing policy in comparison with the basic model. Further on, the modified model is extended to portray an intermediate market that converges to full competition. The rather unexpected outcome is again presented and the problems arising from the assumptions are examined. It is observed that strategic effects of transfer pricing requires credible commitment and an additional coordination problem in the modified model compromises corporate efficiency.

## 10 Zusammenfassung

Diese Arbeit gibt eine Einführung zu (marktbasierten) Transferpreisen und zeigt in Folge unternehmensinterne Zusammenhänge auf, die mit der Transferpreis-Thematik in Verbindung stehen. Auf Basis eines Arbeitspapiers, das von Arya und Mittendorf (2008) publiziert wurde, wird das Basismodell von Arya und Mittendorf vorgestellt. Nach der Analyse der Ergebnisse von Arya und Mittendorf (2008) wird ein modifiziertes Modell präsentiert und es werden im Vergleich zum Basismodell Veränderungen beleuchtet, die die Transferpreispolitik betreffen. Darauffolgend wird das modifizierte Modell so angepasst, dass es einen Zwischenmarkt abbildet, der in Richtung eines perfekten Wettbewerbs konvergiert. Daraus und aus den gewählten Annahmen resultieren allerdings unerwartete Probleme, die dann beleuchtet werden. Im Ergebnis scheint auf, dass durch ein zusätzliches Koordinationsproblem im modifizierten Modell der Gewinn im Vergleich zum Basismodell vermindert wird.

## 11 Curriculum Vitae

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[^0]:    ${ }^{1}$ Exchange rate from the website of the ECB on February $6{ }^{\text {th }} 2013$, EUR $1=$ INR 71.8490 ; 1 crore equals $10^{7}$

[^1]:    ${ }^{2} a_{Z}<\frac{(6+N(6+N)) a_{F}}{2 N}$ according to non-negativity restriction resulting from $q_{F, \text { opt }}$ (A20)

