Diplomarbeit

Titel der Diplomarbeit

"Modeling Oligopolistic Competition in Electricity Markets with Forward Contracts"

Verfasser

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Abstract

Since the liberalization of the electricity markets in Europe at the beginning of the 1990s several models were developed to strengthen the abilities of electric companies to compete. Many of these models are also used by researchers in their analysis of market data. My diploma thesis provides an introduction to the various models that are used in this analysis. Part 1 of my thesis focuses on how the usage of game theoretical developments of John Nash's different equilibrium strategies impacts companies in the spot market. In part 2, I incorporate the supply function equilibrium and the Cournot equilibrium without capacity constraints to the forward market. In my final part, I discuss the effects of capacity constraints on the Cournot equilibrium model for companies with limited resources.

Zusammenfassung

Seit Beginn der Liberalisierung der Elektrizitätsmärkte in Europa Anfang der neunziger Jahre wurden verschiedene Modelle entwickelt, die die Konzerne verwenden um ihre Wettbewerbsfähigkeit zu stärken, die auch unter anderem für die Marktanalyse des jeweiligen Marktes dienen. Diese Diplomarbeit gibt eine Einführung in die verschiedenen Modelle, die zu diesem Zweck verwendet werden. Der erste Teil behandelt unter Einsatz der spieltheoretischen Entwicklungen von John Nash die verschiedenen Strategien der Konzerne auf dem Spotmarkt. Im zweiten Teil stelle ich das Gleichgewichtsmodell fr Angebotsfunktionen und das Cournot-Gleichgewichtsmodell für den Terminmarkt ohne Nebenbedingungen vor. Zum Schluss befasse ich mich mit dem Terminmarkt unter Einbeziehung von Kapazitätsnebenbedingungen und deren Auswirkungen auf das Marktmodell.

Contents

Chapter 1

Introduction

Electricity markets are really complex. The non-storability[1] of electricity, the high variation of supply and demand, and various environmental and energy political interventions lead to a remarkable price- and quantity-dynamic. This holds for both short and long term markets. That the demand and supply are inelastic is well-known if the capacity reserves are getting tight. For example This leads to an increased market power potential [29].

Market power is defined as the ability to raise the market price up to a profitable amount. Market power on the electricity market can practiced by reserving some of the production capacity.[2] This diploma thesis provides an introduction to the electricity market, how companies compete in the spot and forward markets by using various methods for maximizing their profit.

The seminal development of John Nash's equilibrium in a strategic game from game theory in 1950 is in fact the main instrument to solve the problem of competing companies.

This diploma thesis provides an overview of the Austrian electricity market

[1] Exception storager plants like pump storage plants which are used for load balancing. These plants consist of a high-level reservoir, a low-level reservoir and a pump turbine. The pump-turbine is able to store electricity in off-peak hours and sell it in the peak hours by pumping the water upwards or downwards, respectively [17].

[2] See OCKENFELS [29]

and of several models for competitors try to maximize their profit leading to equilibria in an electricity market. There are several of these equilibrium models. The most common models are based on Cournot- and supply function equilibrium (SFE) competition.

The supply function equilibrium approach was introduced by KLEMPERER & MEYER [26] and applied by GREEN & NEWBERY [22] and GREEN [20] and VON DER FEHR & HARBORD [35], as a contrary to the SFE which is based on discrete supply auctions, to the electricity spot market in England and Wales.

NEWBERY [28] generalized the SFE model considering contracts in the market which was applied to the electricity market in England and Wales in GREEN [21].

The second model which will be discussed in more detail in this diploma thesis is the cournot model with existing contracts. This model was developed first by ALLAZ & VILA [7]. GANS et al. [16] adopted the result of ALLAZ & VILA [7] and approved it. MURPHY & SMEERS [27] showed that the ALLAZ & VILA [7] results do not hold if there exist capacity constraints in the market. But models will be discussed in more detail later.

Chapter 2

Background

In the last 30 years the European electricity markets have evolved from a monopoly to an oligopoly. The liberalization of the European electricity market has started in the beginning of the 1990s with the collaboration of Great Britain and the Scandinavian countries. The European Union directive on the internal electricity market is in force since February 1997. The member states had two years granted to transpose the directive into domestic legislation. The aim of that directive was to strengthen the European industry against competition from the USA and Japan. In Amsterdam the Amsterdam Power Exchange (APX) was established in 1999, in 2000 the energy market European Energy Exchange (EEX) in Frankfurt and the Leipzig Power Exchange (LPX) fusioned to become the EEX located in Leipzig in 2002. The spot market of the EEX was transformed in 2009 into the EPEX SPOT SE [3].

2.1 Energy Market Liberalization in Austria

Austria put the European Union directive into practice with the 'Elektrizitätswirtschafts- und Organisationsgesetz 1998 (ElWOG)'. 'Energie-Control' (e-control) as the regulatory authority was installed to regulate these legislative frameworks. E-control has published in 2011 a resume of the liberalization in Austria for the tenth anniversary ”10 years energy market liberalization”. This will be the main source of this chapter.

The ElWOG aims of the liberalization were [2]:

- To harmonize the Austrian electricity regulations with the EU-directive.
- To create a legal framework and increase the competitiveness of the domestic industry in an international environment as a consequence.

The principles that have been defined for this internal energy market of the European Union are sustainability, security of supply, and competitiveness. At the same time the objectives of the Austrian energy policy (environmental and social sustainability, security of supply and cost minimization) were to be continued with the new EIWOG.

The main issues of EIWOG are as follows [31]:

1. Unbundling the businesses of the electricity companies in generation, transmission, distribution and other to ensure transparency.
2. Price- and tariff politics
    - Price determination: This is implemented by the corresponding electricity price regulation system. This system has to ensure the competitiveness of the Austrian energy market and at the same time to maintain the interests of the consumers by setting price limits.
    - No pass through of loss of revenues to small customers.
    - Fixing a tariff for a system use of the electricity network.
3. Business and industrial parks will also be regarded as points of consumption to gain a better position for a larger group of companies at the electricity market.
4. Gradual opening of the electricity market from February 1999 to 2009 for end users and distribution companies.

- from February 1999 with annual consumption > 40GWh
- from February 2000 with annual consumption > 20GWh
- from February 2003 with annual consumption > 9GWh

The reason for this gradual liberalization is to give participating electricity utilities, consumers, and public authorities time to adjust.

Finally, the Austrian electricity market is fully liberalized and subject to the rules of free competition since October 2001.

2.1.1 Effects of the liberalization

Liberalization has positive economic consequences. The gross domestic product of Austria would be approximately 1% lower if the liberalization had not happened ($\widehat{=}$ 3bn €). The consumer expenditure would be nearly 500m € lower [2].

The situation of energy companies has improved as new strategies have been developed and expansion has started.

Production companies shared in the increases in earnings. After some Austrian companies got rid of their old debts at the beginning of the liberalization, they could make revenues already at relatively low prices. Furthermore, the increasing electricity prices since 2003/2004 have increased their benefits by 126 % between 2001 and 2010 [2].

Not forgetting, the benefits for the consumers. Between 2001 and 2009 the consumers saved overall approximately 10bn € due to the lower electricity prices and 1.3bn € due to the lower gas prices as compared to a situation where liberalization had not taken place due to Kratena's (2011) calculations as shown in Table 2.1 [2].

EFFECTS OF LIBERALISATION ON CONSUMERS (2001-2009, bn €)		
	Electricity	Gas
Business consumers	8.90	1.20
Households	1.30	0.08
Total	**10.20**	**1.28**

Table 2.1: Savings of the Consumers [2]

2.1.2 Market Structure for Electricity

In the following I will introduce the structure of the three main components of the electricity market.

(i) **Wholesale Market**

The wholesale market for electricity creates a link between production and subordinated markets. Since the implementation of liberalization is completed trading of power does not only have the role of exchange of physical power, moreover it serves as hedging, investing and arbitrage [2].

The wholesale electricity market has the following trading forms[2]:

- OTC (Over the counter) trade, which is based on bilateral contracts usually settled outside of an exchange trade market.
- Stock exchange trade, which can be partitioned in spot and forward markets.
- Financial derivative trade

(ii) **Generating Electricity**

Because of the absence of bottlenecks the Austrian and German wholesale markets generally are a single price area. The four leading suppliers in the Austrian and German market area are [2]:

- EnBW 'Energie Baden-Wurttemberg',
- E.ON,

- RWE,
- Vattenfall.

The above mentioned four German companies have a total market share of more than 75 % and so Austrian electricity producers do not play a price making role on the market because of their small market share. If or how they practice their market power is the theme of several analyses [2].

Concentration numbers like the HHI[1] on Table 2.2 show that the electricity market is absolutely highly concentrated. Through different measures the market share of the above mentioned companies has been reduced from year to year [2].

Year	Capacity(HHI)	Generated Amount (HHI)
2003-2005	1914	2143
2007	2093	2183
2008	2045	2145

Table 2.2: HHI by installed capacity and generated amount [2]

The electricity generation mix in Austria and Germany has changed in the last decade. Not only because of the catastrophy of Fukushima in 2011, but also the price support for renewable generation by means of injection tariffs and the introduction of the European Union Trading Scheme have made investment in coal and nuclear power plants less attractive and have greatly accelerated the development of wind power in Germany (Figure 2.2). In Austria electricity generation is based on a hydro-thermal system. The most important energy source for electricity generation is hydro power. But as we can see in Figure 2.1 there is a decrease of hydrodynamic power which is replaced by an increase of wind, biomass and gas facilities, which are distributed mainly by the Austrian Power Grid (APG). APG owns more than 92% of the 380 kV (total 1145 km) and 220kV (total 1902km) of the Austrian grid [19].

[1] The Herfindahl-Hirschman-Index (HHI) is a degree of concentration of a market. A market with a HHI over 1800 is called highly concentrated.

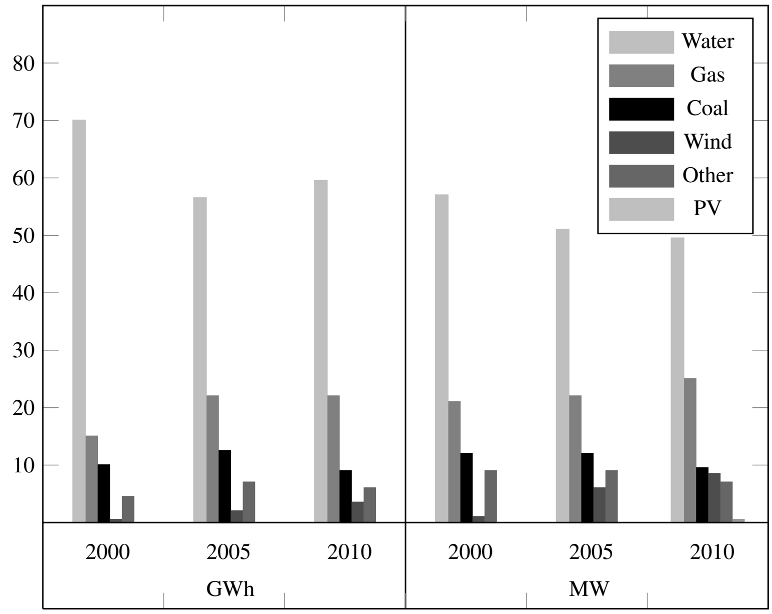

Figure 2.1: Production-Mix in Austria in % source: [2]

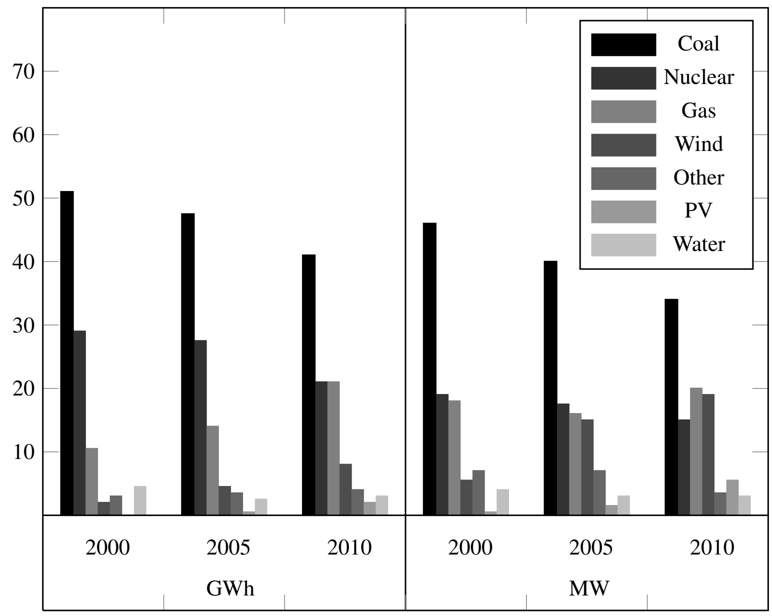

Figure 2.2: Production-Mix in Germany in % source:[2]

(iii) **Trading Electricity**

Since the electricity market has been liberalized the trading with electricity went through essential changes. For example companies like banks, financial institutions, or industrial companies now also trade electricity. These arbitragers try to make profit from price fluctuations [2].

The major electricity stock exchanges in Europe are EPEX Spot/EEX, APX-ENDEX, and Nordpool. EXAA is the Austrian electricity spot market.

In addition to trading electricity there are system services which are also very important for the electricity sale and are necessary for the maintenance of the system. Table 2.3 shows the temporal structure of different trading forms [2].

System Services	control energy procurement			procurement costs for grid losses
OTC Market	Spot maket		Forwards/Options and structured products	
Financial Market	Intraday	Day-Ahead	Weakly-/Monthly-contracts	futures and options
	today	tomorrow	1 month	1 year 2 years 3 years

Table 2.3: Temporal structure of different trading forms[2]

2.1.3 EXAA[2]

"With the aim of increasing competition in production and support the liberalization of the Austrian electricity market began in 1999. The Energy Exchange Austria (EXAA) was founded in June 2001 as a spot market for power and carbon dioxide. But trading of carbon dioxide electricity has began in March 2002. Since the start of trading the number of market participants, which are not only energy companies but also banks and financial service companies, have increased from 12 to 90. There are traders from 16 different countries."[3]

[2] See also http://www.exaa.at

”In addition to the classical tasks related to exchange trading, EXAA is also responsible for the settlement of financial transactions (clearing) and assumes the counter party risk for all trades executed. Over the years, the trading territory and tasks at EXAA have been enlarged.” [3]

One of the advantages for a market participant of trading at the stock exchange compared to OTC is minimizing the counterpart risk, i.e., the risk that the trade partner can not fulfill the contract conditions. Exchange participants have to provide a security at any conclusion of contracts to ensure also the longterm forward contracts. These securities are called margins. Table 2.4 shows how much electricity was traded in the last five years at EEX and EXAA [2].

Spot Market Volumes, TWh					
	2007	2008	2009	2010	2011
EEX, Germany	124	146	203	279	314
EXAA, Austria	2	3	4.6	6.4	7.6
Forward Market Volume, TWh					
EEX, Germany	1150	1165	1025	1208	1075
At EXAA there does not exists a futures market					

Table 2.4: Traded Spot and Future Market Volumes since 2007 at EEX and EXAA /source: [1, 3]

To generators and retailers the greatest risk posed by electricity pools is the financial consequences of fluctuating pool prices. Pool prices will vary each hour (in Austria) and will be determined by the balance of supply and demand. Whilst the level of demand can be estimated, the availability of generation capacity in the market is less predictable. Generators themselves will choose how much electricity they will offer to produce. The power station with the highest marginal bid that is operating at any point in time (and the price they require to operate) determines the pool price [16].

2.2 Mathematical Structure

From a mathematical point of view, research development of electricity market modeling follows three trends which are presented in VENTOSA et al. [34] in more detail:

(i) Optimization models: are maximization problems for one company.

(ii) Equilibrium models: represent the overall market behavior considering competition between all companies.

(iii) Simulation models: are used if consideration of all the market participants in the equilibrium model is too complex to be addressed within a formal framework.

(i) and (ii) are schematically represented in Figure 2.3. In the simulation models the market is synthesized in the representation of the price clearing process, which can be modeled as exogenous to the optimization program or as depending on the quantity supplied by the company of interest [34].

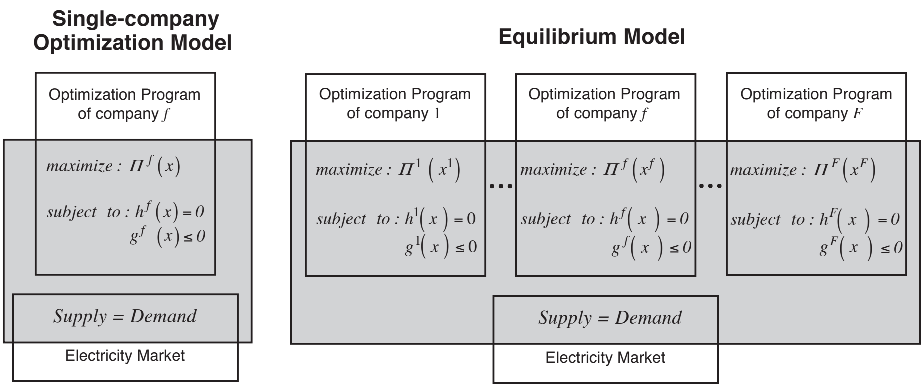

Figure 2.3: Mathematical structure of single-company optimization and equilibrium models [34]

Single-company optimization models can be classified into two types [34].

- price modeled as exogenous variable
- price modeled as a function of the demand supplied by the company of study.

The equilibrium models will be the main part of this diploma thesis and will be discussed in the following chapters.

Chapter 3

Nash Equilibrium

3.1 Nash Equilibrium

The following chapter will give an introduction to the fundamental game theory concepts, Nash equilibrium as a strategic game and will summarize the main applications and definitions. For this I will closely follow the books 'A course in game theory' by OSBORNE & RUBINSTEIN [30] and 'A Primer in Game Theory' by GIBBONS [18].

In order to explain the Nash equilibrium model and its conclusion the following definitions have to be made:

Definition 3.1.1. *(Preference Relation):*
Let A be a non-empty set. A relation $\succeq$ is called ***preference relation****, if the following properties are satisfied:*

- *Reflexivity:*
  For all $a \in A : a \succeq a$
- *Completeness:*
  For $a, b \in A$ with $a \neq b : a \succeq b$ or $b \succeq a$

- *Transivity:*
  For all $a, b, c \in A$ if $a \succeq b$ and $b \succeq c$ then $a \succeq c$

Definition 3.1.2. *(Strategic Game):*
*A **strategic game** Γ is defined as a triple $\langle N, (A_i)_{i \in N}, (\succeq_i)_{i \in N} \rangle$ with:*

(i) *N is a finite set of companies.*

(ii) *Each company i is assigned the non-empty set A_i; each element $a_i \in A_i$ is called decision or strategy of the company i and A_i its set of strategies.*

(iii) *$\succeq_i$ is a preference relation for the company i.*

The Play Γ is called finite if the set A_i is finite for all i.

Important

Decisions of the companies are done independent from each other, i.e., each company makes its own choice without being informed about the selection of other companies.

Notation

We write $A := \prod_{i \in N} A_i$.

Let $a = (a_i)_{i \in N}$ be a strategy profile $(a \in A = \prod_{i \in N} A_i)$. Then $a_{-i} := (a_j)_{j \in N \setminus i}$ and $(a_{-i}, a_i) := (a_j)_{j \in N} = a$.

Definition 3.1.3. *(Nash Equilibrium):*
Let $\Gamma = \langle N, A, \succeq \rangle$ be a strategic game. A strategy profile (strategy-combinations of the n companies) $a^* \in A$ such that for all $i \in N$:
*$a^* = (a^*_{-i}, a^*_i) \succeq_i (a^*_{-i}, a_i)$ for all $a_i \in A_i$, is called **Nash Equilibrium**.*

Remark

In a Nash equilibrium no company changes its strategy onesided, because the impact of this decision will not lead to an improvement.

Definition 3.1.4. *(Best-Response Function):*
For all $a_{-i} \in A_{-i}$ define $B_i(a_{-i})$ to be the set of the best actions of company i's given a_{-i}: $B(a_{-i}) = \{a_i \in A_i : (a_{-i}, a_i) \succeq_i (a_{-i}, a_i') \quad \forall a_i' \in A_i\}$. We call the function B_i the ***best response function*** *of company i.*

Remark
A Nash equilibrium is a profile a of actions for which $a \in B_i(a_i)$ for all $i \in N$. Based on this consequence Nash equilibria can be determined by the following procedure.

1. Calculate the best-response function of each company i.

2. Find a profile $a^*$of actions for which $a^* \in B_i(a^*_{-i}) \quad$ for all $i \in N$.

3. Solve the n equations in the n unknowns.

A strategic game can also be represented by its **normal form**. For this, we replace the preference relation $\succeq_i$ by a payoff function $u_i : A \to \mathbb{R}$, in the sense if $a \succeq_i b$ then $u_i(a) \geq u_i(b)$ and define $\langle N, A, u_i \rangle$ as follows:

Definition 3.1.5. *(Nash Equilibrium)*
In the n-company normal form game $\langle N, A, u_i \rangle$ the strategies $(a_1^*, \ldots, a_n^*)$ are a ***Nash equilibrium*** *if a_i^* is player i's best response to the strategies of the other $n-1$ companies:*

$$u_i(a_1^*, \ldots, a_i^*, \ldots, a_n^*) \geq u_i(a_1^*, \ldots, a_i^*, \ldots, a_n^*)$$

for all i.

3.2 Applications

The development and the results of the Nash equilibrium have been used in several types of strategy games. In the financial market the main usage of the Nash equilibrium is in models concerning competition. Thus, it is the main game plan to compete with other rivals.

3.2.1 Cournot Model

Cournot (1838) anticipated the Nash equilibrium model one century ago by focusing on duopoly markets. Subject to the best response function companies regulate their quantities simultaneously by choosing their best response against the other companies' previous outputs. It is easy to see that this dynamic converges to a Nash equilibrium whenever it converges.

Below a simple derivation of the Cournot game in a duopoly market is presented. The case of n companies can be obtained simply by replacing j by $q_{-i} = \sum\limits_{j \neq i} q_j$ and will be represented in Section 4.

Let q_i be the quantities produced by company i and $P(Q) = a - Q$ be the inverse demand function when the total quantity on the market is the sum of the quantities which are sold by the market participants. In our case $Q = q_i + q_j$. Assume that the total cost for company i to produce quantity q_i is linear, $C_i(q_i) = cq_i$.

To find the Nash equilibrium of the Cournot game where the players choose their quantities simultaneously, we first express the problem in its normal form. To define and solve the equilibrium we assume that the company's payoff is its profit. Hence, the payoff $u_i(a_i, a_j)$ in a duopoly game can be denoted as

$$\pi_i(q_i, q_j) = q_i[P(q_i + q_j) - c] = q_i[a - (q_i + q_j) - c]$$

For every feasible strategy $a_i \in A_i$ the strategy pair (a_i^*, a_j^*) is a Nash equilibrium if

$$u_i(a_i^*, a_j^*) \geq u_i(a_i, a_j^*)$$

for each company i. This is equivalent to the fact that for each company i, a_i^* must solve the optimization problem

$$\max_{a_i \in A_i} u_i(a_i, a_j^*).$$

The quantity pair (q_i^*, q_j^*) in the Cournot model is a Nash equilibrium if q_i^* solves

$$\max_{0 \le q_i < \infty} \pi_i(q_i, q_j^*) = \max_{0 \le q_i < \infty} q_i[a - (q_i + q_j^*) - c]$$

for all i.

Now assume $q_j^* < a - c$, the first order condition for company i's optimization problem is necessary and sufficient, which leads to

$$q_i = \frac{1}{2}(a - q_j^* - c).$$

Therefore, if (q_1^*, q_2^*) is a Nash equilibrium, the companies' quantity choices have to satisfy

$$q_1^* = \frac{1}{2}(a - q_2^* - c)$$

and analogously for q_2^*. Solving these two equations we get

$$q_1^* = q_2^* = \frac{a - c}{3},$$

which is really less than $a - c$, as assumed.

One can intuitively expect that each company would like to be a monopolist in the market. In that case the company would choose q_i to maximize $\pi_i(q_i, 0)$ with a corresponding quantity of $q_m = \frac{a-c}{4}$ which yields the monopoly profit $\pi_i(q_m, 0) = \frac{(a-c)^2}{4}$. In a market that consists of two companies, the total quantity of the duopoly profit would be maximized if the sum $q_1 + q_2$ would be equal to the monopoly quantity q_m. But the monopoly quantity is low and the corresponding price $P(q_m)$ is high. The companies would like to increase their quantity at this price. However the increase leads to a decrease of the market-clearing price.

A second way to solve the Nash equilibrium is graphically. Figure 3.1 shows the Nash equilibrium for a Cournot duopoly with the best response functions. Indeed

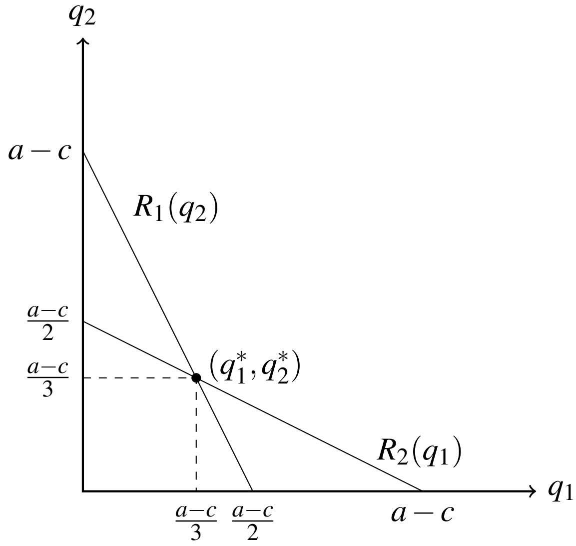

Figure 3.1: Cournot Equilibrium

assuming that company 1 satisfies $q_1 < a-c$, company 2's best response is

$$R_2(q_1) = \frac{a - q_1 - c}{2};$$

analogous for company 2

$$R_1(q_2) = \frac{a - q_2 - c}{2}.$$

As seen in Figure 3.1 these two best response functions intersect only once, at (q_1^*, q_2^*) which is our equilibrium.

Example 3.2.1. *Let the inverse demand function be*

$$P(Q) = 100 - Q$$

and the cost function

$$c_i = 10q_i.$$

Let q_i be the output of company i and q_j that of company j. Therefore $Q = q_i + q_j$ and the profit functions are:

$$\pi_i(q_i, q_j) = q_i[100 - (q_i + q_j)] - 10q_i$$
$$\pi_j(q_i, q_j) = q_j[100 - (q_i + q_j)] - 10q_j.$$

Remembering the Nash equilibrium; an equilibrium is a pair of outputs (q_i^*, q_j^*) *such that*

$$\pi_i(q_i^*, q_j^*) \geq \pi_i(q_i, q_j^*) \quad \forall q_i \geq 0$$
$$\pi_i(q_i^*, q_j^*) \geq \pi_j(q_i^*, q_j) \quad \forall q_j \geq 0.$$

Therefore, if we fix q_j *at the value* q_j^* *and consider* π_i *as a function of* q_i *on its own, this function is maximal at* $q_i = q_i^*$ *if it satisfies the first order condition*

$$\frac{\partial \pi_i}{\partial q_i}(q_i^*, q_j^*) = 0.$$

Analogous for q_j.

Now we can obtain the Nash equilibrium by solving the following two equations:

$$\frac{\partial \pi_i}{\partial q_i}(q_i^*, q_j^*) = 90 - 2q_i - q_j = 0$$
$$\frac{\partial \pi_j}{\partial q_j}(q_i^*, q_j^*) = 90 - q_i - 2q_j = 0.$$

The equilibrium pair is:

$$(q_i^*, q_j^*) = (30, 30)$$

Therefore,

$$P = 40 \qquad \textit{and} \qquad \pi_i = \pi_j = 900.$$

If we compare this solution with the monopoly outcome where $q = Q$ *and hence* *the profit function is as follows:*

$$\pi(Q) = (100 - Q - 10)Q = (90 - Q)Q.$$

The first order condition for maximization yields Q=45 which implies

$$P = 65 \quad and \quad \pi = 2025.$$

Remark

- Example 3.2.1 illustrates that every competitor would like to act like a monopolist.
- Of course, companies can have different cost functions. But it is not difficult to compute the equilibrium in this case. The approach is the same.

3.2.2 Bertrand Model

Bertrand's (1883) idea is based on the assumption that companies actually choose prices instead of quantities like in Cournot's model. Therefore, Bertrand's model differs in strategy spaces, pay off functions, and the behavior in the Nash equilibria from Cournot's model. But there is **no** difference in the equilibrium concept used in both games. *In both games Nash equilibria are used.*

Consider a market with homogeneous goods and two companies (duopoly). As in the previous model, the Nash equilibrium was determined by first translating the problem into a normal form game. By assuming that negative prices are not feasible, each company's strategy space can represented as $A_i = [0, \infty)$, and a typical strategy a_i is to choose a price $p_i \geq 0$.

Let the profit of company i when choosing price p_i and the competitor chooses p_j be

$$\pi_i(p_i, p_j) = q_i(p_i, p_j)[p_i - c] = [a - p_i + p_j][p_i - c].$$

Hence, the pair (p_1^*, p_2^*) is a Nash equilibrium if, for each company i, p_i^* solves

$$\max_{0 \leq p_i < \infty} \pi_i(p_i, p_j^*) = \max_{0 \leq p_i < \infty} [a - p_i + p_j^*][p_i - c].$$

The solution of the optimization problem of company i is

$$p_i^* = \frac{1}{2}(a + p_j^* + c).$$

(p_1^*, p_2^*) is a Nash equilibrium if

$$p_1^* = p_2^* = \frac{a+c}{2}.$$

That means that two competing companies can sell their products if they set the same market prices. If for example $p_i < p_j$ then company i will sell its total output and company two can only sell the residual demand and vice versa.

Suppose the price which is chosen by the companies is $p_i = p_j > c$. This case cannot be an equilibrium, since company i or j can decrease its price and so the consumers would buy from i or j, respectively.

The only price which would prevent mutual underbidding is equal to the marginal cost[1].

3.2.3 Stackelberg Model

The Stackelberg model (1934) concentrates on an oligopoly market in which a leader company moves first and then the follower companies move successively. We will discuss the case where the companies choose their quantities. Basically, companies can either choose quantities or prices.

Assuming two companies in the market, the schedule of the game is:

[1] Marginal cost is the derivative of the total production costs with respect to the level of quantity, i.e. let $c_i(q)$ be the cost function then $\frac{\partial}{\partial q} c_i(q)$ denotes the marginal cost.

1. company 1 chooses a quantity $q_1 \geq 0$,
2. company 2 observes q_1 and then makes the choice of quantity q_2,
3. The payoff function of company i is given by

$$\pi_i(q_i, q_j) = q_i[P(Q) - c].$$

In order to solve the backward induction[2] result of this game, the reaction of company 2 to an arbitrary quantity of company 1 is computed.

$R_2(q_1)$ is a solution of

$$\max_{q_2 \geq 0} \pi_2(q_1, q_2) = \max_{q_2 \geq 0} q_2[a - q_1 - q_2 - c].$$

This implies for $q_1 < a - c$

$$R_2(q_1) = \frac{a - q_1 - c}{2}.$$

Note that in Section 3.2.1 the equation for $R_2(q_1)$ was the same. The difference is that here $R_2(q_1)$ is company 2's reaction, compared to Cournot's model where $R_2(q_1)$ is company 2's best answer to an unknown choice of company 1.

Now, company 1's problem in the first stage of the game is

$$\begin{aligned} \max_{q_1 \geq 0} \pi_1(q_1, R_2(q_1)) &= \max_{q_1 \geq 0} q_1[a - q_1 - R_2(q_1) - c] \\ &= \max_{q_1 \geq 0} q_1 \frac{a - q_1 - c}{2}, \end{aligned}$$

and by backward induction we obtain

$$q_1^* = \frac{a - c}{2} \quad \text{and} \quad R_2(q_1^*) = \frac{a - c}{4}$$

as the result of the Stackelberg game for a duopoly market.

[2]That means we take the best response for company 2 at first and substitute it into the profit function of company 1. For $n > 2$ we start with the company which moves last and so on.

Example 3.2.2. *We take the same inverse demand and cost function as in Example 3.2.1*

$$P(Q) = 100 - Q \quad \text{and} \quad c_i = 10q_i.$$

Therefore the follower $-i$ has to solve

$$\max_{q_{-i}}[(100 - (q_i + q_j) - 4]q_j = 90q_j - q_i q_j - q_j^2.$$

The first order conditions yield

$$q_{-i} = \frac{90 - q_i}{2},$$

which is the best response function like in the Cournot game.

If the leader company i would put q_i into the market it knows that company $-i$'s best response would be $\frac{90-q_i}{2}$. Therefore, company i must take $\frac{90-q_i}{2}$ into account in its profit maximization:

$$\max_{q_1} [100 - (q_i + \frac{90 - q_i}{2} - 4)]q_i = 45q_i - \tfrac{3}{2}q_i^2.$$

After differentiating the first order conditions yield

$$q_i = 45.$$

Thus, the amount which is sold by company j is

$$q_j = 22.5,$$

which yields us the market price $p = 32.5$. And therefore, the profits of the companies are

$$\pi_i = 1012.5 \quad \text{and} \quad \pi_j = 506.25, \text{ respectively.}$$

Table 3.1 summarizes the results of every single model described above. As you can see, in a monopoly market the highest profit can be reached although the quantity which is sold is the smallest.

Game	q_i	q_j	P	π_i	π_j	$\pi_i + \pi_j$
Monopoly	45		65	2025		2025
Cournot	30	30	40	900	900	1800
Stackelberg	45	22.5	32.5	1012.5	506.25	1519.75

Table 3.1: Table of Outcomes

Chapter 4

Spot Market

Spot market is a market in which the commodities are traded for immediate delivery. The electricity spot market is not like any other market, not only because electricity is difficult to store but also the prices are determined for each degree of demand expected during the whole day for every half hour in England and every hour in Austria. In order to model such a market it has to be considered whether generators sell to a central auction (POOLCO) or bilaterally to customers. The POOLCO model includes also independent arbitragers that can influence the market price [23].
In the following the different strategies which companies can use in their models will be summarized. Some of them are already discussed in Chapter 3.

4.1 Strategies

4.1.1 Perfect Competition

Pure Competition is a market strategy in which companies sell products without having any influence on the product price. Therefore, the companies are price-takers not price-makers. q_i is a decision variable of the company i's revenue pq_i. p is fixed. The Karush-Kuhn-Tucker conditions (KKTs) for profit maximization yield [13]:

$$\partial(pq)/\partial q = p. \qquad \text{(Marginal Revenue(MR))}$$

The KKT conditions are equal to the marginal cost because the price does not change with the amount of quantity sold ($p(q) = 0$).

4.1.2 Bertrand Strategy (”Game in Prices”)

HOBBS [24] used Bertrand strategy in electricity market for studying the restructuring of the electricity industry in the US. Because of the difficulty to store electricity, it leads to a short term price competition, thus to a Bertrand model [33].

As shown in the previous chapter Bertrand follows a duopoly market form strategy in which companies set the prices p_i. He describes interactions among companies and their customers that choose quantities at price p_i. In this strategy if p_i is not greater than the lowest price of the rivals then i can sell as much as it wants, otherwise $q_i = 0$.

4.1.3 Cournot Strategy[1] (”Game in Quantities”)

The Cournot model is one of the most used schemes in industrial organizations. As already showed in the previous chapter, the Cournot model is a game in which each company chooses its quantity that is going to be sold in the market. As there are more than two competing companies in the electricity market the case of n competing companies in an oligopoly market should be analyzed.

Let q_i, $i = 1, \ldots, n$ be the quantities produced by n companies. Hence, the market clearing price is

[1] See FRIEDMAN [15]

$$P(Q) = \begin{cases} a - Q, & Q < a \\ 0, & Q \geq a \end{cases}$$

with $P(Q) = a - Q$ and $Q = \sum_{i=1}^{n} q_i$. Assume that the cost function of company i is linear $c(q_i) = cq_i$. So we choose as the marginal cost constant c with $c < a$. Suppose again that the companies choose their quantities simultaneously. The profit function for company i can be written as:

$$\pi_i(q_1, \ldots, q_n) = q_i P(Q) - c(q_i).$$

then the Cournot oligopoly has one equilibrium that can be determined to be

$$q_i = \frac{a-c}{n+1}, \quad \forall i \in 1, \ldots, n.$$

For $n = 1$ and $n = 2$ we get the solution for monopoly and duopoly as already shown in the previous chapter.

4.1.4 Supply Function Equilibria (SFE)

KLEMPERER & MEYER [26] developed the supply function equilibrium concept. They modeled an oligopoly market whose participants are confronted with uncertain demand. Each company is creating its own strategy by determining a supply function $q(p)$. The supply functions map for every price p the quantity q of goods that the company is willing to sell at this price. GREEN & NEWBERY [22] applied the supply function model of Klemperer and Meyer to the British electricity spot market. They pointed out that the uncertainty of the demand is equivalent to a time dependent demand and they used the SFE model in the electricity market in England and Wales to generate optimal supply functions. Since then the SFE model has been widely used to analyze the bidding behavior in electricity spot markets. BALDICK et al. [11] analyzed the existence of supply function equilibria by considering price caps. BALDICK [9] showed that the equilibrium also depends on the parameterization of the supply functions [8].

Table 4.1 shows several developments of the SFE model under different assumptions.

Author	Marginal Costs	Demand Curve	Supply Functions	Solution Method
KLEMPERER & MEYER [26]	Convex	Concave	C^2	Necessary conditions
GREEN & NEWBERY [22]	Quadratic	Linear	C^2	Numerical Integration
GREEN [21]	Linear	Linear	Affine	Closed-form expression
FERRERO et al. [14]	Affine	Inelastic	Affine	Exhaustive enumeration
RUDKEVICH et al. [32]	Stepwise	Inelastic	Differentiable	Closed-form expression
BALDICK et al. [10]	Affine	Linear	Piecewise linear	Heuristics
BERRY et al. [12]	Affine	Linear	Affine	Heuristics
HOBBS et al. [25]	Affine	Linear	Affine	MPEC

Table 4.1: Characterization of SFE models [34]

First we will observe the case of a duopoly, which can be generalized to an n-company oligopoly. I will closely follow GREEN & NEWBERY [22] and assume that the load duration curve of supply at any moment is predictable with certainty and is given by $D(p,t)$, where t denotes time (number of hours of demand higher than D), and p is the spot price. Klemperer and Meyer discuss also the case under certainty. Now assume for all (p,t) that $-\infty < D_p < 0$, $D_{pp} \leq 0$, and $D_{pt} = 0$.

The company i is confronted with the net demand $D(p,t) - S^j(p)$ at time t where $S^j(p)$ is the supply schedule of the other company j. Let the generating costs of supplying the quantity q be $C(q)$ with marginal cost $C'(q)$. Now, the aim of company i is to develop a function which maps price to a level of output independent of time, $t : S^i : [0,\infty) \rightarrow (-\infty,\infty)$. Each company presents the supply function simultaneously to the dispatcher, and the dispatcher establishes the spot price and the supply of company i by solving the price output pair that equates supply to demand at each time t. I.e., the dispatcher assigns the lowest price $p(t)$ such that

$D(p(t),t) = S^i(p(t)) + S^j(p(t))$, if it exists. The companies earn nothing if there is no such a price. Assume that the profit maximizing price-output can be described by a supply function $q_i = S^i(p)$ for all t. Thus, the profit-maximizing solution can be determined by maximizing $\pi_i(p) = pq_i - C(q_i)$ with respect to p:

$$\pi_i(p) = p[D(p,t) - q_j(p)] - C\left(D(p,t) - q_j(p)\right).$$

The first order condition for this problem is:

$$0 = D - q_j(p) + p\left(D_p - \tfrac{\partial q_j(p)}{\partial p}\right) - C'(D - q_j(p))\left(D_p - \tfrac{\partial q_j(p)}{\partial p}\right)$$

which is equal to:

$$\frac{\partial q_j(p)}{\partial p} = \frac{q_i}{p - C'(q)} + D_p.$$

The assumption of symmetry, $q_i = q_j = q$, leads to

$$\frac{\partial q(p)}{\partial p} = \frac{q}{p - C'(q)} + D_p.$$

Let us observe the points (q,p) such that

$$C'(q) < p < C'(q) - \frac{q}{D_p}.$$

For these points $0 < \frac{\partial q}{\partial p} < \infty$ holds, and the corresponding trajectory through this points exhibits a positive directional slope which is well-defined. Now, consider the stationaries $C'(q)$ and $C'(q) - \frac{q}{D_p}$. For $p = C'(q)$ we have $\frac{\partial q}{\partial p} = \infty$ and so $\frac{\partial p}{\partial q} = 0$. This curve describes a perfectly competitive company's supply schedule. Every trajectory that meets $C'(q)$ has a horizontal slope at the intersection, see Figure 4.1, and after the intersection the slope of the trajectory will be smaller then zero.

If the trajectory meets the monopoly solution, its slope will be $\frac{\partial q}{\partial p} = 0$ respectively $\frac{\partial p}{\partial q} = \infty$ at that point. It will intersect the monopoly solution, which is also called the Cournot supply schedule, and then bend back. Thus, the profit-maximizing choice of p satisfies

$$q_i + [p - C*(q_i)]D_p = 0,$$

and

$$C'(q) - \frac{q}{D_p},$$

respectively.

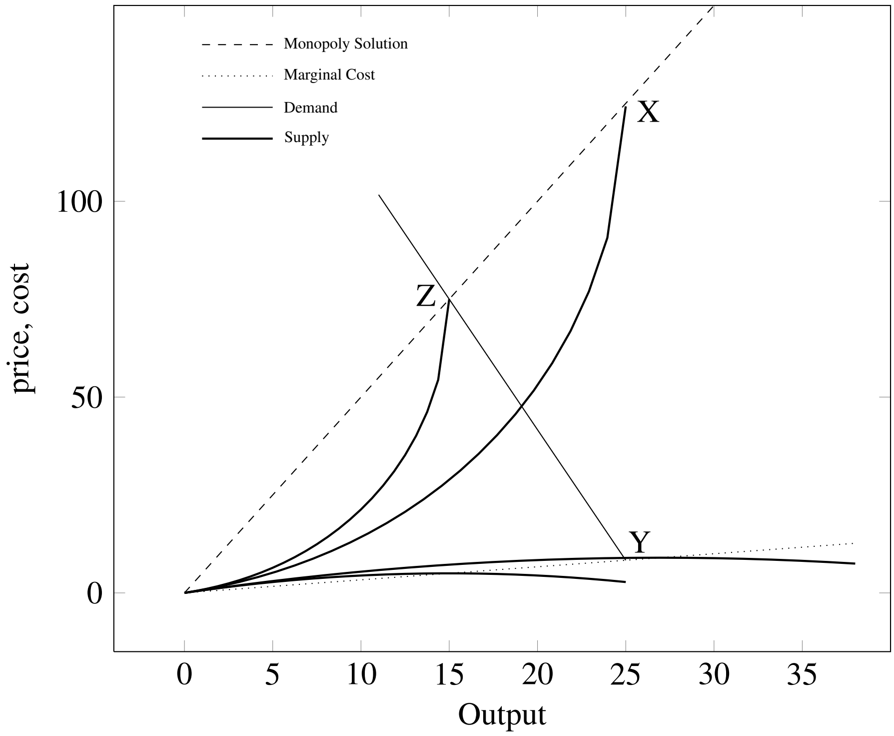

Figure 4.1: Feasible supply function equilibria [22]

Normally, the duopoly schedules are between the competitive and Cournot schedules along a curve such as $0X$ in Figure 4.1. Possible equilibria supply schedules need not intersect either stationary over the range of possible price-output pairs. KLEMPERER & MEYER [26] prove that if the demand schedule can be arbitrarily high, then there is a unique solution; if there is not such a unique point there may be a set of equilibria which is bounded above and below by the supply schedules. Thus in Figure 4.1, if YZ is the maximum demand $D(p,0)$, then all solutions of the differential equation above which are between $0Y$ and $0Z$ are possible. Now, we know that if company i is known to choose its schedule $q_i(p)$ and if there exist no supply constraints, then the solution $q_j(p)$ of the differential equation is the profit-maximizing response of company j.

Example 4.1.1.

(i) Duopoly
By adopting the linear demand and cost function from the previous chapter and make the latter also dependent on time $t = [0,1]$ we get:

$$Q = D(p,t) = 100 + 5(1-t) - p, \quad c_i = 10q_i.$$

For $t = 1$, peak time, we had our solutions of the Nash Cournot equilibrium:

$$q_i = q_{-i} = 30, \quad p = 40.$$

By solving the differential equation

$$\frac{\partial q}{\partial p} = \frac{q}{p-10} - 1$$

we obtain

$$q = A(p-10) - (p-10)\log(p-10),$$

where A is a constant of integration which we can compute by determining the boundary condition, for example where supply intersects the Cournot solution. Therefore,

$$q = (1 + \log(30))(p-10)\log(p-10).$$

The solution of our example is shown in Figure 4.2.

(ii) Oligopoly
Assume there are $n > 2$ symmetric companies that are companies competing with a homogenous product and equal cost functions in the market. Then

$$a - bp - (n-1)q(p)$$

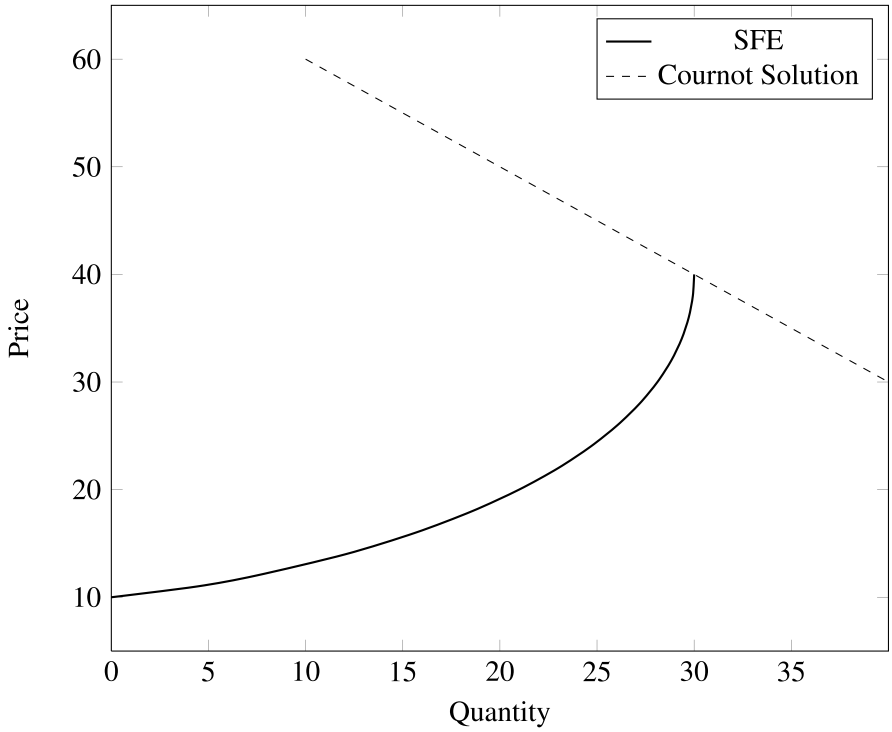

Figure 4.2: Supply Function Equilibrium

is the net demand which each company is concerned with. The first order condition for profit maximizing yields

$$(n-1)\frac{\partial q}{\partial p} - \frac{q}{p-c'} = -b.$$

By applying this to the numerical example $p = 100 - Q$ *and* $c_i = 10q_i$ *for* $n = 5$ *then*

$$4 * \frac{\partial q}{\partial p} - \frac{q}{p} = -1,$$

since the constant marginal cost c' *is normalized to zero. By solving this ordinary differential equation the result is*

$$q = Ap^{1/4} - \frac{p}{3},$$

for a constant of integration A which can be determined by boundary conditions.

4.1.5 Conjectured Supply Function[2] (CSF)

The conjectured supply function model was developed by DAY et al. [13]. The production output of the competitors in this model are anticipated. The company i has to react to corresponding function $q_{-i}(p)$. The CSF model is like the SFE model superficially and has four components for the bilateral market [13].

(i) Generation Companies Model

(ii) Independent System Operator (ISO) Model

(iii) Arbitrager Model

(iv) Market Clearing Conditions

Notation

In the following we use the index $i = 1,\ldots,n$ for companies, $f = 1,\ldots,m$ for nodes at which electricity is traded, $h = 1,\ldots,l$ for generators, and $t = 1,\ldots,k$ for flowgates between nodes.

C'_{ifh} ...[€/MWh] Marginal cost of generator h at node f owned by i

G_{ifh} ...[MW] Upper bound of generation

w_f ...[€/MWh] Price of transmission

c_{if} ...[€/MWh] Generation cost

α_f ...Assumed price intercept of the CSF for i at f

g_{ifh} ...[MW] Production of generator h at node f owned by i

s_{if} ...[MW] Sales of i to consumers at f

$s_{-if} \ldots \sum_{j \neq i} s_{fj}$

a_f ...[MW] is the net amount of power sold by arbitragers at f

γ_{if} ...dual variable of generation limit

[2] See DAY et al. [13]

φ_f ...dual variable of sales

η_{if} ...dual variable of generation

$PDTF_{fk}$ ...Power Distribution Factor for flowgate k at node f

y_f ...[MW]transmission service provided from the hub[3] to f

λ_t ...dual variable upon the flow constraint for flowgate t

(i) Generation Companies Model

Generation companies have to solve the following problem

$$
\begin{aligned}
\max\ \ & \pi_f = \sum_f (p_{if} - w_f) s_{if} - \sum_{f,h} (C_{ifh} - w_f) g_{ifh} & & \\
\text{s.t.:}\ & s_{if} + s_{-if} + a_f = q_f(p_{if}) & \forall f \qquad & \text{(Demand Functions)} \\
& s_{-if} = s_{-if}(p_{if}) & \forall f \qquad & \text{(CSFs)} \\
& g_{ifh} \le G_{ifh} & \forall f,h \qquad & \text{(Generation Limits)} \\
& \sum_f s_{if} = \sum_{f,h} g_{ifh} & & \text{(Energy Balance)} \\
& \forall s_{if}, g_{if} \ge 0 & &
\end{aligned}
$$

In region f the electricity price p_{if} is an affine function of the total sales S_f:

$$p_{if} = P_f - \frac{P_f}{Q_f}(S_f + a_{if}),$$

where

$$S_f = \sum_f s_{fi}.$$

The model assumes that the s_{-if} are linear functions of p_{if}:

$$s_{if} = s^*_{-if} + \beta_{if}(p^*_f, s_{-if})(p_{if} - p^*_f),$$

where (p^*_f, s^*_{-if}) is an equilibrium (price,sales) pair, and the function $\beta_{if}(x,y)$

has one of the two forms:

(a) a positive constant: β_{if}, (fixed slope)

(b) a rational function: $\frac{y}{x-\alpha_{if}}$. (fixed intercept)

The first case leads to the equation

$$p_{if} = (p_f^* - \frac{s_{-if}^*}{\beta_{if}}) + \frac{1}{\beta_{if}} s_{-if} \Longrightarrow \begin{cases} \beta_{if} = 0, & \text{(Cournot)} \\ \beta_{if} = \infty, & \text{(Bertrand)} \end{cases}$$

and in the second case

$$p_{if} = \alpha_{if} + \frac{p_f^* - \alpha_{if}}{s_{-if}^*} s_{-if} \Longrightarrow \alpha_{if} = -\infty. \qquad \text{(Cournot)}$$

By substituting s_{-if} into p_{if} in case of fixed intercept

$$p_{if} = \frac{Q_f - s_{if} - a_{if} + \frac{\alpha_{if}}{p_i^* - \alpha_{if}} s_{-if}^*}{\frac{Q_f}{P_f} + \frac{s_{-if}^*}{p_f^* - \alpha_{if}}}.$$

Thus, the problem is reduced to

$$\begin{aligned} \max & \sum_i (p_{if} - w_f) s_{if} - \sum_{f,h} (C_{ifh} - w_f) g_{ifh} \\ \text{s.t.: } & g_{ifh} \leq G_{ifh} \\ & \sum_f s_{if} = \sum_{f,h} g_{ifh} \\ & \forall s_{if}, g_{if} \geq 0. \end{aligned}$$

The KKT conditions for this model yield a Mixed Complementarity Problem that is either linear (in case (i)) or nonlinear (in case (ii)).

The KKTs are

$$
\begin{aligned}
&0 \leq s_{if} \perp -p_i + \frac{\sum_{j\in N} s_{ij}}{\sum_{j\in N}(\frac{Q_j}{P_j} + \frac{s_{ij}}{p_j^*-\alpha_j})} + \varphi_i \geq 0,\\
&0 \leq g_{if} \perp c_{if} - w_f + \gamma_{if} - \varphi_i \geq 0,\\
&0 \leq \gamma_{if} \perp C_{if} - g_{if} \geq 0,\\
&0 = \sum_{f\in N} (s_{if} - g_{if}).
\end{aligned}
$$

(ii) The ISO model

The independent system operator coordinates, controls, and observes the operation of the electrical power system. Its model represents the efficient rationing of transmission capacity. There are two types of variables:

- $y_f \ldots$ the amount of transmission service provided from the hub which is the main distribution node to f
- $\lambda_t \ldots$ the dual variable for the flow constraint for flowgate t

The model maximizes the value of services $\sum_f w_f^* y_f$ subject to the DC load[4] flow, yielding KKTs:

- for y_f, $\quad \forall f : w_f - \sum_k PTDF_{fk\lambda_k}$
- for λ_t, $\quad \forall k : 0 \leq \lambda_t \perp (\sum_f PTDF_{ft} y_f - T_t)$

(iii) The arbitrgers' model

The arbitrager can buy power in one location and sell it in another. The only cost the arbitrager incurs is the ISO's transmission fees between the

[4]The DC load flow is an approximation for the real AC flow, called the DC approximation which is quite precise if the network parameters are well-known. It uses Kirchhoff's laws to compute the so called power transfer distribution factors (PTDF) which describe the rate of flow over all transmission links if all inputs and outputs at the nodes are known.

two locations. In equilibrium, arbitrage will eliminate any price differences between nodes that are not based on cost, implying that:

$$p^*_f = p^*_{hub} + w^*_f \qquad \forall f \neq hub.$$

(iv) Market clearing conditions

A market-clearing condition is an equation (or other representation) stating that supply equals demand. A market-clearing price is a price that causes supply and demand to be equal.
For all i, f:

$$\begin{aligned} y_f &= \sum_j s_{jf} + a^*_f - \sum_{j,h} g_{jfh} \\ p_{if} &= p^*_f. \end{aligned}$$

Together with the conditions of the ISO model, arbitrager model, and the market clearing conditions the KKT conditions for each generator i yield a mixed complementarity problem.

The advantages of a CSF model are [13]:

- $q_{-i}(p)$ can be modeled as a smooth function.
- CSF gives modelers the flexibility to consider more realistic supply responses unlike Cournot.
- It is feasible for large problems unlike SFE models.

Chapter 5

Forward Market

5.1 Preliminaries

The aim of this diploma thesis is to model an electricity market where also a forward market exists. First a two stage game using Cournot equilibria according to ALLAZ & VILA [7] and then using the supply function equilibria according to GREEN [21] will be described. In the next chapter we will discuss the case of a model with three stages where also capacity constraints exist according to MURPHY & SMEERS [27].

In the first case we denote two periods:

- **Contract Market**
- **Spot Market**

Allaz and Vila generalized this also to the case of $m > 2$ periods in the forward market.

5.1.1 Types of Contracts

We differ two types of contracts [16]:

(i) one-way contracts,

(ii) two-way contracts.

ad (i)

One-way contracts, as shown in Figure 5.1, are sold at a strike price to a purchaser. If the spot price is less than the strike price purchaser pays the spot price. But if the price is bigger than the strike price the production company pays the difference to the purchaser. These types of contracts are in principle the same as a call option. If the strike price and the pool price coincide the parties are risk-neutral. Hence the aim of the production companies is to sell the contracts at the future spot price to minimize the contract trading costs [16].

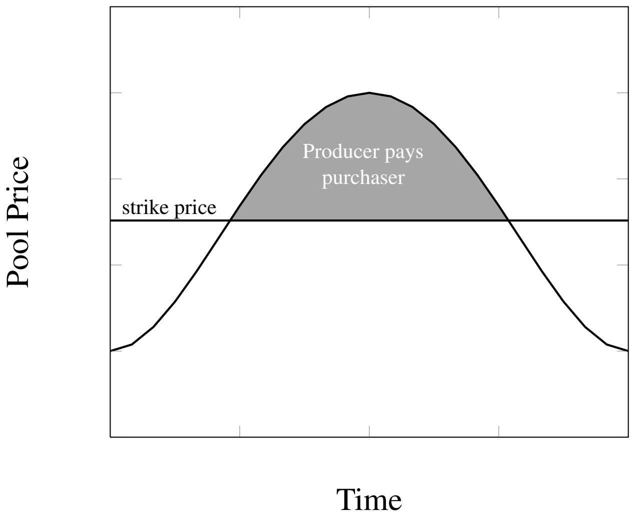

Figure 5.1: One-Way Contracts [16]

ad (ii)

The difference between one-way contracts and two-way contracts is that the price of the two way contract is fix for the purchaser and the producer as shown in Figure 5.2. Two way contracts are equivalent to forward contracts which are sold at the strike price. From now on the latter ones will be discussed [16].

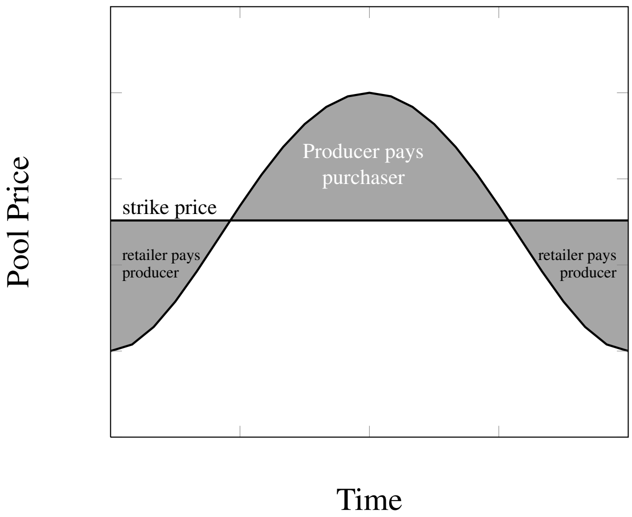

Figure 5.2: Two-Way Contracts [16]

This chapter shows the emmergence of trading forward contracts and the effect on the profit and behavior of the producers.

5.2 Market Model as Cournot Game

ALLAZ & VILA [7] first present the market model as a Cournot game. They documented the equilibrium solutions of a duopoly at first with two and then with m periods. Following ALLAZ & VILA [7] I will also show the equivalence of an existing forward market to the *Prisoner's Dilemma*.

5.2.1 Spot Market Equilibrium

Two production companies with a homogenous good and equal marginal cost c' in a two period model are considered. In the first period, both companies simultaneously choose the quantity of forward sales that they want to sell in the forward market for delivery in the second period. The amount of the forward sales is denoted by x_i and the forward price by p' [7].

Given the producers' forward choice x_1 and x_2, the production game in the spot market is defined by the two payoff functions

$$\pi_1(x_1,x_2) = p(q_1+q_2)(q_1-x_1) - c_1(q_1)$$
$$\pi_2(x_1,x_2) = p(q_1+q_2)(q_2-x_2) - c_2(q_2)$$

Obviously, if one company has already sold x_i in the first period, it can only sell the amount $q_i - x_i$ in the spot market. If x_i is more than the amount of the production capacity the difference has to be bought in order to compensate the residual. In this case the producer i has to buy the residual amount from its competitor j (at the spot price) or the contract can be redeemed [7].

Following ALLAZ & VILA [7] assume that the inverse demand and the cost functions are linear:

$$\begin{aligned} c_1(q_1) &= cq_1; \\ c_2(q_2) &= cq_2; \\ p(Q) &= a - Q; \\ 0 < c &< a. \end{aligned}$$

Thus,

$$\pi_1(x_1,x_2) = (a-q_1-q_2)(q_1-x_1) - cq_1,$$
$$\pi_2(x_1,x_2) = (a-q_1-q_2)(q_2-x_2) - cq_2.$$

The best response function is then given by

$$R_1(q_2) = \frac{a - c + x_1 - q_2}{2}.$$

Proposition 5.2.1. *There exists only one Nash equilibrium with*

$$\begin{aligned} q_i &= \frac{a - c + 2x_i - x_j}{3} \quad i \neq j, \\ p &= \frac{a + 2c - x_i - x_j}{3} \quad i \neq j. \end{aligned}$$

Proof. Follows directly from Section 3.2.1. □

If only one of the production companies is able to trade forward contracts then that producer faces the following game: Find an x such that the Nash equilibrium output in the second spot market is optimal [7].

Proposition 5.2.2. *The equilibrium output is the solution of the Stackelberg output of the Cournot duopoly game without a forward market when company 1 is the leader:*

$$x_1 = \frac{a - c}{4}; \quad q_1 = \frac{a - c}{2}; \quad q_2 = \frac{a - c}{4}; \quad p = \frac{a + 3c}{4}.$$

Proof. Follows directly from Section 3.2.3. □

Only the company which can trade forward contracts is able to increase profits. Comparing Proposition 5.2.1 and Proposition 5.2.2 shows that the total output increases from $\frac{2(a-c)}{3}$ to $\frac{3(a-c)}{4}$.

If producer 1 can trade forward contracts, he gains great profit. Thus, producer 2 wants to trade forward contracts too in order to improve its profit. This proven fact is the reason of emmergence of the forward market [7].

5.2.2 Forward Market Equilibrium

In deciding how many contracts one company to offer in the forward market, company i has the following total profit function:

$$\pi_i' = p'(q_i, q_j)x_i + \pi_i(x_i, x_j)$$

by substituting

$$\pi_i'(x_i, x_j) = (p(x_i, x_j)q_i(x_i, x_j) - cq_i(x_i, x_j)) + (p'(x_i, x_j) - p(x_i, x_j))x_i$$

q_i and p are already given from Proposition 5.2.1 The first term in this equation is the standard Cournot profit and the second one is the arbitrage profit. For the arbitrager the following condition holds:

$$cq_i(x_i, x_j) + (p'(x_i, x_j) - p(x_i, x_j))x_i = 0.$$

Proposition 5.2.3. *There exists only one forward market equilibrium such that*

$$q_i = q_j = \frac{2(a-c)}{5}, \tag{5.1}$$

$$x_i = x_j = \frac{a-c}{5}, \tag{5.2}$$

$$p = c + \frac{a-c}{5}. \tag{5.3}$$

Proof. The payoff function of the first producer depending on x_i and x_j is:

$$\pi_i(x_i, x_j) = (p-c)q_i = \tfrac{1}{9}(a - c - x_i - x_j)(a - c + 2x_i - x_j).$$

The first order condition of maximizing π_i' ($\partial \pi_i' / \partial x_i = 0$) yields the best response function

$$R_i(q_j) = \frac{1}{4}(a - c - x_j).$$

Analogous

$$R_j(q_i) = \frac{1}{4}(a - c - x_i).$$

Thus, (5.1)-(5.3) hold in equilibrium. □

Remark:

(i) Obviously, when one of the producers is able to trade forward contracts, he benefits a lot from doing so. But if both trade forward contracts, their profit eventually become worse compared to the case where they do not enter into the forward market. Hence, trading forward contracts is equivalent to the Prisoner's Dilemma.

(ii) We can also point out that the solution of Proposition 5.2.3 is more competitive than the Cournot game where only a spot market exists. Allaz and Vila proved moreover in their Proposition 3:
If there exists a forward market with N periods, in equilibrium for $N \to \infty$ the residual production of the spot market tends to 0 and the price p tends to the marginal cost c.

Example 5.2.4.
Let

$$\begin{aligned} P &= 100 - Q, \\ c_i &= 10q_i, \end{aligned}$$

for $i = 1,2$ be the inverse demand function and the cost functions of the companies.

We will solve the problem backwards. First we will determine the Nash equilibrium for the spot market as a function of the amount of the contracts which are sold in the contract market. By Proposition 5.2.1

$$\begin{aligned} q_1 &= \tfrac{1}{3}(90 + 2x_1 - x_2), \\ q_2 &= \tfrac{1}{3}(90 + 2x_2 - x_1), \end{aligned}$$

$$and\ p = \frac{1}{3}(90 - x_1 - x_2).$$

The payoff functions of the first producer depending on x_1 and x_2 are:

$$\pi_1(x_1,x_2) = (p-c)q_1 = \frac{1}{9}(90 - x_1 - x_2)(90 + 2x_1 - x_2)$$
$$\pi_2(x_1,x_2) = (p-c)q_2 = \frac{1}{9}(90 - x_1 - x_2)(90 + 2x_2 - x_1).$$

The first order conditions for both π_1 and π_2 are

$$0 = \frac{1}{9}(90 - 4x_1 - x_2) \qquad and$$
$$0 = \frac{1}{9}(90 - x_1 - 4x_2).$$

Therefore, we have an equilibrium by Proposition 5.2.3 at

$$q_1 = q_2 = 36$$
$$x_1 = x_2 = 18$$
$$p = 28.$$

Comparing the results of the market equilibrium with and without forwards (Table 5.1) we see that the total profit of the companies decrease by trading forward contracts.

Game	q_i	q_{-i}	x_i, x_j	P	π_i	π_{-i}	$\pi_i + \pi_{-i}$
Monopoly	45			65	2025		2025
Cournot	30	30		40	900	900	1800
Stackelberg	45	22.5		32.5	1012.5	506.25	1519.75
Cournot with contracts	36	36	18	28	648	648	1296

Table 5.1: Result of trading forward contracts

5.3 Market Model with Supply Function Equilibria

NEWBERY [28] and GREEN [21] represent models with forward markets using supply functions in the spot market. For modeling the electricity market with SFE I will follow GREEN [21].

5.3.1 Spot Market

Goods which are sold in the spot market have to be delivered immediately at the spot price. The spot price is determined by the market clearing condition where demand equals supply, i.e. $a - bp^* = Q$. Every production company wants to maximize its profit, given by the net forward contract sales minus the production costs

$$\pi_i = pq_i(p) + (p' - p)x_i - c_i(q_i(p)).$$

In order to rewrite this terminus as a function of the market price, the residual demand has to be inserted instead of the companys output:

$$\pi_i(p) = p(D(p,t) - q_j(p) - x_i) + p'x_i - c_i(D(p,t) - q_j(p))^2$$

Maximizing this function with respect to p the first order conditions yield:

$$0 = D(p,t) - q_j(p) - x_i + (p - c_i(D(p,t) - q_j(p)))\left(-b - \frac{dq_j}{dp}\right). \quad (\star)$$

This leads to a differential equation as can be seen from Chapter 4,

$$q_i(p) = x_i + (p - c_i q_i(p))\left(b + \frac{dq_j}{dp}\right).$$

Analogous we can apply this method to $q_j(p)$. These two equations can be solved simultaneously to obtain the two supply function equilibria.

GREEN [21] proved in his Proposition 1 that for any equilibrium supply function, a company will offer a quantity equal to the amount covered by its contracts at a price equal to its marginal cost at that output level. This result is a generalization of Allaz and Vila's Proposition 3 for any positive sloping supply function. Green assumed that his supply function is linear because of the uniqueness of the equilibrium for each pair of contract sales. In the non linear case many solutions for equation ($\star$) can be found. GREEN [20] showed that the linear supply function can be determined straightforward and that supply function equilibrium of company i is independent of the contract sales of company j. In the non-linear case, supply functions depend on each company's contract sales. Considering the linear case $q_i = \alpha_i + \beta_i p$ leads to

$$\alpha_i + \beta_i p = x_i + (p - c_i(\alpha_i + \beta_i p))(b + \beta_j).$$

This equation holds for

$$\alpha_i = \frac{x_i}{1 + c_i(b + \beta_j)}$$

and

$$\beta_i = \frac{b + \beta_j}{1 + c_i(b + \beta_j)}.$$

The second derivative of the profit function with respect to the price is

$$\frac{d^2\pi_i}{dp^2} = -(b + \beta)(2 + c_i(b + \beta)),$$

Which is negative. Hence, the supply function gives a maximum. If company i and j both have linear supply functions, the equilibrium price will be

$$p = \frac{1}{b + \beta_i + \beta_j}\left(a - x_i\frac{\beta_i}{b + \beta_j} - x_j\frac{\beta_j}{b + \beta_i}\right).$$

The output of company i can then be obtained from its supply function

$$q_i = \frac{\beta_i}{b + \beta_i + \beta_j}\left(a - x_j\frac{\beta_j}{b + \beta_i} + x_i\right).$$

Of course x_i and x_j can reach a value smaller than zero. In this case in some markets, e.g. in the EXAA the market rules would set these to zero.

5.3.2 Contract Market

Compared to the spot contracts where the goods are delivered immediately by selling them, in the futures contracts the payment and the delivery of the goods are at a fixed date. In electricity markets, generators often sell contracts in order to reduce their risks.

The contract market is actually the first stage of this model. Again I will closely follow GREEN [21] and assume that the production companies have the same marginal costs, i.e., the slopes of the supply functions in the spot market are equal. With the offering of contracts by the production companies the purchasers can establish the market clearing price. Green also assumed that a sufficiently large proportion of the purchasers in the contract market are risk-neutral with rational expectations. For this reason the determined contract price p' should be equal to the expected spot price p. Hence, the price of the contract depends on the amount of contracts that are going to be sold

$$p' = \frac{1}{2\beta + b}\left(a - \tfrac{\beta}{\beta + b}\right)$$

Now we can define the company's profit function in terms of the futures price

$$\pi_i = p'(x_i, x_j)q_i(x_i, x_j) - \tfrac{1}{2}cq_i(x_i, x_j)^2.$$

The first order condition for profit maximizing yields after some simplifications

$$x_i = -q_i \frac{(2\beta + b)\frac{dx_j}{dx_i}}{\beta + b - \beta\frac{dx_j}{dx_i}}. \tag{5.4}$$

An equilibrium in the forward market consists of a pair of forward contract sales that solves (5.4) for $i = 1, 2$, given the conjectural variations of the companies.

Remark
The actions that happen in the market can be summarized as follows:

- The producers choose their forward contract amounts x_i simultaneously (first period).
- The producers choose the quantity q_i of the goods that they are going to put into the market supply functions $q(p)$ −again simultaneously (second period).
- The spot price can be determined by the inverse demand function.
- The payoffs of the production companies can be denoted as

$$\pi_i = p' x_i + p(q_i) - c_i(x_i).$$

Example 5.3.1.[1]
Again for the case $p = 100 - Q$ and the linear cost function $c_i = 10q_i$ with marginal cost $c' = 10$ the first order condition yields

$$\frac{\partial q_j}{\partial p} = \frac{q_i - x_i}{p} - 1, \quad i \neq j,$$

where the marginal cost is normalized to zero and it can be solved by

$$\begin{array}{llll} q_i = x_i + Ap - p\log(p), & i = 1,2, & q_i > x_i, & p > 0, \\ q_i = x_i, & i = 1,2, & & p = 0, \\ q_i = x_i + Bp + p\log(-p), & i = 1,2, & q_i < x_i, & p < 0. \end{array}$$

The last line is the case where companies are overcontracted. In this case they are willing to drive the spot price down.

The first case holds if the minimum demand crosses the supply function at a price greater then zero. That happens if the amount of all contracts X is less than the competitive baseload demand $a_1 - bp(1)$, i.e., $X := \sum x_i \leq a_1$, where $p(1)$ is zero if $X = a_1$ and $t = 1$ is the time of minimal demand. Therefore, we have the aggregate Cournot schedule

$$Q^c = \sum q_i^c = \sum (x_i + p) = X + 2p.$$

[1] See NEWBERY [28]

By summing up $q_i = x_i + Ap - p\log(p)$ we can determine the total supply Q which equals the demand D

$$\begin{aligned}
&Q \equiv \sum_i q_i = X + 2Ap - 2p\log p,\\
&Q \leq Q^c = X + 2p, \quad X \leq a_1,\\
&Q = D = a(t) - p.
\end{aligned}$$

If A is given $p(t)$ can be obtained by

$$2Ap(t) + p(t) - 2p(t)\log p(t) = a(t) - X.$$

Obviously, the total demand and price depends only on X, the aggregate contract supply, not on the individual forward positions of the companies.

The next step is to find the value of A and the choice of the contracts x_i. Therefore, $p^* = p(0)$ is denoted as the market clearing price for maximal demand. Let $a_o - p^*$ at $t = 0$, where $a_o \equiv a(0)$. Solving

$$A(X, p^*) = \frac{a_0 - X}{2p^*} + \left(\log p^* - \frac{1}{2}\right), \quad X \equiv x_i + x_{-i} \leq a_1.$$

p^Δ is the maximum value of p^*, which is given by intersecting the Cournot solution with the maximal demand, i.e.,

$$\sum q_i^c =: Q^c = X + 2p^\Delta,$$

or

$$p^\Delta = \frac{a_0 - X}{3}.$$

Therefore, the value of A is equal to

$$1 + \log\left(\frac{a_0 - X}{3}\right),$$

and the corresponding supply function is

$$q_i = x_i + p\left[1 + \log\left(\frac{a_0 - X}{3p}\right)\right], \quad p \le \frac{a_0 - X}{3}, \quad X \le a_1 < a_0.$$

The variation of aggregate forward positions on price $\frac{\partial p(t)}{\partial X}$ *depends on how companies coordinate on their choice of supply positions. The three choices are as follows*

$$\begin{aligned}
\frac{\partial p}{\partial X} &= \frac{-p}{a(t) - 2p - X}, && \textit{holding A constant,}\\
\frac{\partial p}{\partial X} &= \frac{-p\left(1 - \frac{p}{p^*}\right)}{a(t) - 2p - X}, && \textit{holding } p^* \textit{ constant,}\\
\frac{\partial p}{\partial X} &= -\left(\frac{a_0 - 2p - X}{a(t) - 2p - X}\right)\left(\frac{p}{a_0 - X}\right), && \textit{highest price-bid decision.}
\end{aligned}$$

For $X \le a_1$ *we have* $\frac{\partial p(t)}{\partial X} < 0$ *in all three cases. The higher the forward position, the more competitively the companies will act in the spot market, and the less the prices will be at each* t. *The time-weighted average price in the spot market is*

$$P = \int_0^1 p(t)dt = P(X, A).$$

For arisk-neutral contract retailer the contract price p' *has to be equal to the time-weighted spot price*

$$p' = P(X, A),$$

where A is again fixed by p^* *and* p^Δ. *The decision of forward position is taken by each company as in a Cournot game. Therefore,*

$$\frac{\partial p}{\partial x_i} = \frac{\partial p}{\partial X},$$

and the average daily operating profit for company i is

$$\pi_i = \int_0^1 pq_i(p)dt + x_i \int_0^1 (p' - p)dt = \int_0^1 pq_i(p)dt.$$

Therefore the companies make their forward decisions as follows:

- *Each company will have zero forward positions, if they suppose that their positions do not change the forward positions of their competitors.*
- *Each company will offer the same forward position, if they adjust supply to gain a given highest spot price.*

Chapter 6

Capacity Constraints

Obviously, the problems of Chapter 5 do not contain capacity constraints. That means that the generators can deliver infinite quantities. What happens to the results if there exist such constraints? To solve this problem I will follow MURPHY & SMEERS [27] and their appendix [6].

MURPHY & SMEERS [27] and in their appendix [6] proved that the result of ALLAZ & VILA [7] does not hold when capacities are endogenously limited. This means that if there are capacity constraints the forward contracts do not necessarily mitigates market power. In this regard they replaced the two-stage model by a three-stage model:

- investment / capacity game
- forward market
- spot market

Including a forward market in a model, the spot market equilibrium is derived given the capacities and forward contracts. The equilibrium in the forward market can be obtained by given capacities, taking into account the subsequent equilibrium of the spot market.

Some notations:

- $P = a - Q$[1] (Inverse demand function)
- d_i (Investment cost parameter, measured in €/MW)
- c_i (Operating cost parameter, measured in €/MWh)
- The quantity variables for $i = 1,2$
  - y_i (The quantity invested by company i)
  - x_i (The quantity of company i's forward) contracts
  - $q_i(a)$ (Output of company i after the demand realization a)

6.1 Stochastic Demand

What MURPHY & SMEERS [27] obtain in the spot market is a model leading to an equilibrium problem subject to equilibrium constraints (EPEC):

6.1.1 The Spot Market

Under uncertainty of demand the inverse demand function can be obtained after the companies' decisions. For each a, the companies act as Cournot competitor.

We will denote $q_i(a)$ as the energy delivered by company i when a is realized.

Now, each company i takes the production q_{-i} of the rivals as given and solves

$$\begin{aligned}
\max_{q_i} \ & \pi_i = (a - q_i - q_{-i})(q_i - x_i) - c_i q_i \\
s.t. \ & 0 \leq q_i \\
& 0 \leq y_i - q_i
\end{aligned}$$

[1] a is a random intercept with density $f(a)$ defined over (l,u). $l = u$ yields the deterministic case and a is used as the intercept.

I.e., after the decision of the forward amount at the forward price the motivation to influence the market by restricting q_i is bounded by the residual amount $q_i - x_i$.

The first order conditions for this problem which the companies solve simultaneously are

$$
\begin{aligned}
&0 \le a - 2q_i - q_{-i} - c_i + x_i - \lambda_i + \mu_i \perp q_i \ge 0 \quad i = 1,2 \\
&0 \le y_i - q_i \perp \lambda_i \ge 0 \quad i = 1,2 \\
&0 \le q_i \perp \mu \ge 0 \quad i = 1,2.
\end{aligned}
$$

Obviously, a parametric complementarity problem is obtained, where λ and μ are the dual variables of the constraints.

Hence we have a spot market profit after solving the EPEC for q_i and q_{-i} of

$$(a - q_i - q_{-i} - c_i)(q_i - x_i).$$

Example 6.1.1.
Let

$$p(Q) = 100 - Q \quad \textit{and} \quad c_i = 10q_i$$

and denote the investment cost as $d_i = 4$, i=1,2 €/Mw.

This example is modeled in AMPL [4] and solved with the MINOS [5] solver. The input is as follows.

Listing 6.1: Numerical solution of spot market with investments

```
param a := 100;
param c := 10;                   #operation cost
param d := 4;                    #investment cost
param y := (a-c-d)/3;            #capacity equilibrium
var q1 >=0;
var q2 >=0;
var w;                           #dual vairable
var lambda=a-2*q1-q2-c+w;        #dual variable
```

```
var p=a-q1-q2;                    #price
var profit1=p*(q1)-q1*(c-d);      #profit
subj to comp1:
y-q1 >= 0 complements l >= 0;
subj to comp2:
q1 >= 0 complements w >= 0;
subj to equi:
q1=x;
```

The results are as follows

	q	x	p	profit
company 1	28.667	28.667	42.667	1051.11
company 2	28.667	28.667	42.667	1051.11

6.1.2 The Forward Market

Let us assume that x_i is sold at the expected value of the spot price, such that the distribution $f(a)$ of the parameter a as a risk neutral probability is obtained from the forward sales. The forward price that the companies can expect is

$$\int_l^u (a - q_i - q_{-i}) f(a) da.$$

The profit of company i where x_{-i} is the forward position of company $-i$ is

$$x_i \int_l^u (a - q_i - q_{-i}) f(a) da + \int_l^u (a - q_i - q_{-i})(q_i - x_i) f(a) da = \int_l^u (a - q_i - q_{-i}) q_i f(a) da.$$

So the companies have to solve the following problem.

$$\max_{x_i} \int_l^u (a - q_i - q_{-i}) q_i f(a) da,$$

where q_i and q_{-i} are the solutions of the complementarity problem of the spot market.[2]

From the uniqueness of the solution of the EPEC follows that there exists unique functions

$$q_i(x_i, x_{-i}; a) \quad \text{and} \quad q_{-i}(x_i, x_{-i}; a)$$

that solve the EPEC. Which leads by substitution to

$$\max_{x_i} \; g(y_i, y_{-i}) = \int_l^u (a - q_i(x_i, x_{-i}; a) - q_{-i}(x_i, x_{-i}; a)) q_i(x_i, x_{-i}; a) f(a) da$$

Obviously, the result is a standard Nash equilibrium problem and not an EPEC.

6.1.3 Capacity Game

Let the net profit of the company *i* be:

$$\pi_i(y_i, y_{-i}) = g(y_i, y_{-i}) - d_i y_i$$

Thus, the companies simultaneously solve

$$\max_{x_i \geq 0} \pi_i(y_i, y_{-i}).$$

6.1.4 Solutions of the Three Stage Game

(i) *Spot Market*

Under the assumptions the equilibrium of the spot market always exists and is unique. The equilibrium is distinguished by the constraints. We distinguish the following cases:

[2] Striking out the integrals, gives the deterministic case.

- None of the companies is constrained:

$$0 < q_i(a) < y_i \quad i = 1,2.$$

- One of the companies is constrained:

$$0 < q_i(a) < y_i \quad 0 < q_{-i}(a) = y_{-i}.$$

- All are constrained:

$$0 < q_i(a) = y_i \quad i = 1,2.$$

- One does not produce anything:

$$0 < q_i(a) \leq y_i \quad 0 < q_{-i}(a) = y_{-i}.$$

- None produces anything:

$$0 < q_i(a) \leq y_i \quad i = 1,2.$$

These cases appear in the deterministic and stochastic demand schemes. Sufficiently, its enough to consider only equilibria for which $q_i > 0$ holds. Therefore, only the first three cases will be discussed. The EPEC problem from Subsection 6.1.1 can be simplified to

$$\begin{aligned} a - 2q_i - q_{-i} - c_i + x_i + \lambda_i = 0 \quad i = 1,2, \\ 0 \leq y_i - q_i \perp \lambda_i \geq 0 \quad i = 1,2, \end{aligned}$$

respectively.

Which of the three cases holds depends on the value of a. Two definitions are introduced: Let

$$\alpha_i(y,x) \quad \text{and} \quad \alpha_{-i}(y,x)$$

be the smallest values of a such that

$$q_{-i}(a) = y_{-i} \quad \text{and} \quad q_i(a) < y_i \quad \text{for} \quad a = \alpha_{-i}(y,x),$$
$$q_{-i}(a) = y_{-i} \quad \text{and} \quad q_i(a) = y_i \quad \text{for} \quad a = \alpha_i(y,x).$$

Setting $x = 0$ yields a model without forward markets and obviously we get

$$\alpha_{-i}(y,x) < \alpha_i(y,x).$$

Comparing our three cases in the spot market with forward contracts:

- *Case 1* (no capacity constrains):
  By solving
  $$0 = a - 2q_i - q_{-i} - c_i + x_i \quad i = 1,2$$
  the following result is obtained.
  $$q_i^* = \tfrac{1}{3}[a - 2(c_i - x_i) + (c_{-i} - x_{-i})],$$
  which yields a spot market profit of
  $$\tfrac{1}{9}(a - x_i - x_{-i} - 2c_i + c_{-i})(a - 2c_i + 2x_i + c_{-i} - x_{-i})$$
  and the market clearing price
  $$p(a) = \tfrac{1}{3}[a + (c_i - x_i) + (c_{-i} - x_{-i})].$$
  The same result was derived by Allaz and Vila (1993) although different cost functions were used. Murphy and Smeers (2010) presented an adapted version and showed that the forward market positions of the companies are
  $$x_i = \tfrac{1}{5}[\int_l^u af(a)da - 3c_i + 2c_{-i}],$$
  $$x_{-i} = \tfrac{1}{5}[\int_l^u af(a)da - 3c_{-i} + 2c_i].$$

- *Case 2* (one is constrained):
  q_i is obtained by solving

$$a - 2q_i - q_{-i} - c_i + x_i + \lambda_i = 0 \quad i = 1,2$$

  for company i such that $q_{-i} = y_{-i}$ and $q_i < y_i$. This yield

$$a - 2q_i - y_{-i} - c_i + x_i = 0$$

  or

$$q_i = \frac{a - y_{-i} - c_i + x_i}{2}.$$

  Therefore, the profit of the company i together with the profit from the forward contracts is

$$\tfrac{1}{4}(a - y_{-i} - c_i - x_i)(a - y_{-i} - c_i + x_i) = \tfrac{1}{4}[(a - y_{-i} - c_i)^2 - x_i^2],$$

  and for company $-i$

$$\tfrac{1}{2}(a - y_{-i} - 2c_{-i} + c_i - x_i)y_i,$$

  respectively.

- *Case 3* (both are constrained): The profit is

$$(a - y_i - y_{-i} - c_i)y_i,$$

  since $q_i = y_i \quad$ for $i = 1,2$.

In the stochastic case α_{-i} is the value where the profit function turns from *Case 1* into *Case 2* and α_i from *Case 2* into *Case 3*. We have

$$y_{-i} = \tfrac{1}{3}[a - 2(c_{-i} - x_{-i}) + (c_i - x_i)]$$

or

$$\alpha_{-i}(x,y) = 3y_{-i} + 2(c_{-i} - x_{-i}) - (c_i - x_i).$$

This follows from the fact that $\alpha_{-i}(x,y)$ is the value where the spot market equilibrium of *Case 1* equals capacity y_{-i}.

Analogous

$$\alpha_i(x,y) = 2y_i + y_{-i} + c_i - x_i.$$

(ii) *Forward Market*

Assume there is a forward market under uncertain demand. Together with the relation $\alpha_{-i}(y,x) < \alpha_i(y,x)$ the profit functions π_i and π_{-i} of the companies i and $-i$ are defined as

$$\begin{aligned}\pi_i(y,x) =& \tfrac{1}{9}\int_l^{\alpha_{-i}(y,x)} (a - x_i - x_{-i} - 2c_i + c_{-i})\\ &(a + 2x_i - x_{-i} - 2c_i + c_{-i}) f(a)da\\ &+\tfrac{1}{4}\int_{\alpha_{-i}(,x)}^{\alpha_i(y,x)} [(a - y_{-i} - c_i)^2 x_i^2] f(a)da\\ &+\int_{\alpha_i(y,x)}^{u} (a - y_i - y_{-i} - c_i) y_i f(a)da - d_i y_i\end{aligned}$$

and

$$\begin{aligned}\pi_i(y,x) =& \tfrac{1}{9}\int_l^{\alpha_{-i}(y,x)} (a - x_i - x_{-i} + c_i - 2c_{-i})\\ &(a - x_i + 2x_{-i} + c_i - 2c_{-i}) f(a)da\\ &+\tfrac{1}{2}\int_{\alpha_{-i}(,x)}^{\alpha_i(y,x)} (a - y_{-i} + c_i - 2c_{-i} - x_i) x_{-i} f(a)da\\ &+\int_{\alpha_i(y,x)}^{u} (a - y_i - y_{-i} - c_{-i}) y_{-i} f(a)da - d_{-i} y_{-i}.\end{aligned}$$

This follows by adding the profits of the three cases. If $\alpha_i < u$ and $\alpha_{-i} > l$ then π_i and π_{-i} are differentiable. So the equilibrium is calculated by solving the first order conditions

$$\frac{\partial \pi_i(y,x)}{\partial x_i} = \frac{\partial \pi_{-i}(y,x)}{\partial x_{-i}} = 0,$$

given that such an equilibrium exists. For the existence and uniqueness the second order conditions are

$$\frac{\partial^2 \pi_i(y,x)}{\partial x_i^2} < 0 \quad \text{and} \quad \frac{\partial^2 \pi_{-i}(y,x)}{\partial x_{-i^2}} < 0,$$

which are discussed in the e-companion appendix of MURPHY & SMEERS [27]. They show in their work that if capacity constraints exist in the forward market than there is no certainty that the market has an equilibrium.

(iii) *Capacity Game*
The Capacity game profit function with a forward market can be obtained after substituting x_i by the equilibrium solution $x(y)$ of the forward market:

$$\pi_i(y) = \pi_i[y, x(y)] \quad i = 1, 2.$$

The capacity game profit functions without a forward market of the company i and $-i$ are obtained by setting x_i and x_{-i} zero ($\pi_i(y,0)$ and $\pi_{-i}(y,0)$).

Example 6.1.2.
The first order conditions of the forward market as already mentioned are

$$\frac{\partial p_i}{\partial x_i} = \frac{\partial p_{-i}}{\partial x_{-i}} = 0.$$

This implies

$$\begin{aligned}\frac{\partial p_i}{\partial x_i} =& \frac{1}{9}\int_l^{\alpha_{-i}(x,y)} (a - 4x_i - x_{-i} + c_i - 2c_{-i}) f(a) da - \\ & \frac{x_i}{2}\int_{\alpha_{-i}(x,y)}^{\alpha_i(x,y)} f(a) da = 0\end{aligned}$$

and

$$\frac{\partial p_{-i}}{\partial x_{-i}} = -\frac{1}{9}\int_l^{\alpha_{-i}(x,y)} (a - x_i - 4x_{-i} - c_i - 2c_{-i}) f(a) da = 0.$$

The solution of these equations when

$$\alpha_{-i}(x,y) = 3y_{-i} + 2(c_{-i} - x_{-i}) - (c_i - y_i)$$
$$\alpha_i(x,y) = 2y_i + y_{-i} + c_i - x_i$$

is applied is a possible equilibrium on the forward market.

For example it is obvious that for

$$x_i = 0 \qquad \text{and} \qquad \alpha_{-i}(x,y) = l$$

the previous conditions hold. Thus the constraint $\alpha_{-i}(x,y) > l$ *has to be added.*

6.2 Deterministic Demand

The result of ALLAZ & VILA [7] that trading forwards mitigate market power do not hold in a model with capacity constraints. This fact will be shown next and can also be found in the appendix [6] of MURPHY & SMEERS [27].

6.2.1 Single Stage Game

Assume an open-loop model, where companies choose their strategy simultaneously. This game can be interpreted as a game where both companies generate capacity and immediately sell the whole amount on the forward market. In this case no spot market exists.

A solution of the Cournot equilibrium (y_i^*, y_{-i}^*) can be reached when y_i^* solves

$$\max_{y_i \geq 0} \; [a - (y_i + y_{-i}^*)]y_i - (c_i + d_i)y_i, \quad i = 1,2.$$

There exists only one solution in this game. To make the comparison of the three games efficient, the case with a strictly positive equilibrium is observed. The first order conditions of the above optimization problem yield:

$$a - 2y_i - y_{-i} - (c_i + k_i) = 0$$
$$a - y_i - 2y_{-i} - (c_{-i} + k_{-i}) = 0.$$

Therefore,

$$\begin{aligned} y_i &= \tfrac{1}{3}[a - 2(c_i + d_i) + (c_{-i} + d_{-i})] \qquad\qquad i = 1,2, \\ p &= a - y_i - y_{-i} = \tfrac{1}{3}[a + (c_i + d_i) + (c_{-i} + d_{-i})], \\ \pi_i^\circ &= \tfrac{1}{3}[a - 2(c_i + d_i) + (c_{-i} + d_{-i})], \\ \pi_i &= \tfrac{1}{9}[a - 2(c_i + d_i) + (c_{-i} + d_{-i})]^2. \end{aligned}$$

where p is the electricity price, π_i° the unit profit and π_i is the total profit. Obviously, y_i is strictly positive if and only if

$$a - 2(c_i + d_i) + (c_{-i} + d_{-i}) > 0 \quad i = 1,2.$$

Proposition 6.2.1. *$q_i = y_i$, $i = 1,2$ in the open-loop game.*

Proof. Applying the solution of Section 3.2.1 the Cournot solution is

$$q_i = \tfrac{1}{3}[a - 2(c_i + d_i) + (c_{-i} + d_{-i})],$$

which is equal to that of the capacity output. □

6.2.2 Two Stage Game (Investment/Spot Model)

In this game companies invest in capacities and trade on the spot market. This case can be observed in the Austrian[3] and Spanish markets as there is no forward market. Working backward from spot to the capacity market will be the approach to obtain the equilibrium of this model.

[3] Austrian companies trade forward contracts in the EEX forward market.

Let y_i be the amount of capacity adopted from the investment stage. Then each company has to solve the following problem

$$\max_{0 \le q_i \le y_i} [a - (q_i + q_{-i})]q_i - c_i q_i.$$

The first order conditions yield a complementarity problem whose solution is the unique equilibrium.

$$\begin{aligned} &a - 2q_i - q_{-i} - c_i + \mu_i = \lambda_i, \\ &x_i - q_i \ge 0 \qquad \lambda_i \ge 0 \qquad (y_i - q_i)\lambda_i = 0, \\ &q_i \ge 0 \qquad\quad\; \mu_i \ge 0 \qquad\qquad q_i\mu_i = 0, \end{aligned} \tag{CP1}$$

for $i = 1, 2$.

The solution of these equilibrium conditions results in a function $q(y)$ of capacities y adopted from the capacity stage. This function $q(y)$ is continuous and linear and in addition continuously differentiable in y.

Again three cases will be observed in which the equilibrium satisfies $0 < q_i \le y_i$. For $i = 1, 2$

- $0 < q_i(a) < y_i$
- $0 < q_i(a) < y_i \quad 0 < q_{-i}(a) = y_{-i}$
- $0 < q_i(a) = y_i$

The next step is to derive the equilibrium in the capacity market that affects the companies' attitude in the spot market.

Definition 6.2.2. *(Closed-loop[4] equilibrium)*
*A **closed-loop equilibrium** of the two-stage game y^*, $q^*(y)$ satisfies the following conditions.*

(i) $q^*(y)$ is a Nash equilibrium of the spot market game for every feasible y

[4]A closed loop game is a game where all past datas are known.

(ii) *y^* is a Nash equilibrium of the capacity market game where the payoffs of the agents are*

$$u_i(y_i; y_{-i}) = u_i[y_i, q_i^*(y); y_{-i}, q_{-i}(x)], \quad i = 1,2.$$

If there exists a closed-loop equilibrium y^*, $q^*(y)$, then there exists a feasible neighborhood $N(y^*)$ of y^* such that

- $q^*(y)$ is a Nash equilibrium in the spot market for all points y, $y \in N(y^*)$.
- y^* is a Nash equilibrium of the capacity market with payoffs $u_i(y_i, y_{-i})$ for $i = 1,2$ in $N(y^*)$.

y^*, $q^*(y)$ is a local equilibrium if y^*, $q^*(y)$ is located in a feasible neighborhood around y^*. This fact can be redefined as

Definition 6.2.3. *(Local closed-loop equilibrium)*
*A **local closed-loop equilibrium** of the two-stage game is a closed-loop equilibrium of the game where y is restricted to a non-empty full dimensional subset of the capacity space.*

For extending Proposition 6.2.6 into a two-stage game the case $0 < q_i < y_i$ and the case $0 < q_i < y_i$; $0 < q_{-i} = y_i$ for $i = 1,2$ cannot hold in an equilibrium.

Lemma 6.2.4. *Suppose there is a closed-loop equilibrium of the two stage game. Then the case where both are constrained ($0 < q_i < y_i$, $i = 1,2$) cannot hold at this equilibrium.*

Proof. Assume

$$0 < q_i^* < y_i^* \qquad i = 1,2.$$

Then the complementarity system becomes

$$a - 2q_i^* - q_{-i}^* - c_i = 0 \qquad i = 1,2$$

or

$$q_i^* = \tfrac{1}{3}[a - (2c_i - c_{-i})].$$

There exists a B_{y^*}, a ball with center y^*, such that for all $y \in B_{y^*}$ $q^*(y) = y^*$ is the best response. It follows that $(y^*, q^*(y^*))$ is a local optimum of the capacity market. For this equilibrium the payoff of i before paying its investments is

$$\tfrac{1}{9}[a - (2c_i - c_{-i})]^2.$$

Thus, the profit is

$$\tfrac{1}{9}[a - (2c_i - c_{-i})]^2 - d_i y_i^*.$$

But this cannot be a local maximum of the profit of i with respect to y_i because we can decrease y_i in order to improve the profit. □

Lemma 6.2.5. *Assume there is a closed-loop equilibrium of the two stage game. Then the case where one of the companies is constrained ($0 < q_i < y_i$; $0 < q_{-i} = y_-$) cannot hold at this equilibrium.*

Proof. Suppose

$$0 < q_i^* < y_i^* \quad \textit{and} \quad q_{-i}^* = y_{-i}^*.$$

Again this assumption with our complementarity problem (CP1) yields:

$$\begin{aligned} q_i^* &= \tfrac{1}{2}(a - y_{-i}^* - c_i) \\ q_{-i} &= y_{-i}^* \end{aligned}$$

So

$$\begin{aligned} q_i(x) &= \tfrac{1}{2}(a - y_{-i} - c_i) \quad \textit{and} \\ q_{-i}(x) &= y_{-i}. \end{aligned}$$

Again as in Lemma 6.2.1 we can reduce y_i by a small amount and receive a higher profit. □

Proposition 6.2.6. *A closed-loop equilibrium of the two stage game satisfies*

$$q_i^* = y_i^*, \quad i = 1,2,$$

if it exists.

Proof. This follows from the process of elimination by Lemma 6.2.1 and Lemma 6.2.2. □

If the closed-loop equilibrium exists Proposition 6.2.2 allows to relate the open-loop to the closed-looped equilibrium

Theorem 6.2.7. *The open-loop equilibrium of the single stage game is equivalent to the closed-loop equilibrium of the two stage game, if it exists.*

Proof. Assume y_i^c and q_i^c, $i = 1,2$ are the closed-loop solution of the two stage game such that $q_i^c = y_i^c$, $i = 1,2$ (Proposition 6.2.2), if it exists. Therefore,

$$\alpha - 2y_i^c - y_{-i}^c - c_i = \lambda_i^c \geq 0, \quad i = 1,2.$$

If y_i would decrease while keeping $y_{-i} = y_{-i}^c$ then $q_i = y_i$, $i = 1,2$ satisfies the first order conditions of the two-stage game. Therefore, the result for the first-stage objective function of i is

$$u_i(y_i; y_{-i}^c) := (\alpha - y_i - y_{-i}^c - c_i)y_i - d_i y_i,$$

where y_i is decreased with $y_{-i} = y_{-i}^c$. u_i reaches a maximum at y_i^c for $y_{-i} = y_{-i}^c$ because of the closed-loop equilibrium. We have

$$\alpha - 2y_i^c - x_{-i}^c - c_i - k_i \geq 0$$

thus

$$\lambda_i^c \geq d_i > 0.$$

This result ($\lambda_i^c > 0$) implies that there exists a neighborhood U of y^c such that for $y \in U$ setting $q_i = y_i$ satisfies the complementarity system (CP1) of our first order conditions in that neighborhood. Adapting the above reasoning to variations of y_i in excess of y_i^c one finds $\lambda_i^c = k_i$. Therefore, the closed-loop equilibrium of the two-stage game y^c, if it exists, satisfies the same conditions as the open loop equilibrium [27]. $\square$

6.2.3 Three Stage Game

Now I want to analyze the three-stage game where the market participants can sell their invested goods by forward contracts and the residual in the spot market. Therefore, the definitions of the closed-loop equilibrium are extended by introducing additional notation.

Let q be the vector of total production in the spot market,
x the amount sold by forward contracts,
y the invested capacity.

The three stage game can be again solved backwards. A spot market equilibrium q is a vector-valued function $q(y,x)$ where q_i solves

$$\max_{0 \le q_i \le x_i} \{u_i^s(y,x;q_i,q_{-i}^*) = [a-(q_i+q_{-i}^*)](q_i-x_i)-c_iq_i\}.$$

If such an equilibrium exists write

$$u_i^f(y;x) := u_i^s[y,x;q(y,x)].$$

Thus, the forward equilibrium defines a set valued map $x : \mathbb{R}^n \to \prod_{i=1}^m \mathbb{PR}^{k_i}$ with

$x_i(y)$ being the solution of

$$\max_{x_i} t_i^f(y;x_i;x_{-i}^*) \quad \text{for } i = 1,2 \tag{6.1}$$

If such a solution exists, we define using

$$\max_{y_i \geq 0} [a - (y_i + y_{-i}^*)]y_i - (c_i + d_i)y_i \qquad i = 1,2. \qquad \text{(Single stage game)}$$

we define

$$\mathbf{q_i}(y) = q_i[y;x(y)]. \quad i = 1,2.$$

The fact that $\mathbf{q_i}(y)$ is a unique point although the solutions of (6.1) are not unique will be shown below. Therefore we can define

$$u_i(y_i,y_{-i}) = \{a - [\mathbf{q_i}(y_i,y_{-i}) + \mathbf{q_{-i}}(y_i,y_{-i})] - c_i\}\mathbf{q_i}(_i,y_{-i}) - d_i y_i \quad \textit{for } i = 1,2.$$

The equilibrium solution of the investment stage is a vector y^* where y_i^* is a solution of

$$\max_{0 \leq y_i} u_i(y_i,y_{-i}^*) \quad i = 1,2.$$

Now we can extend Definition 6.2.1 to

Definition 6.2.8. *A closed loop equilibrium* $(y^*,x^*(y),q^*(y,x))$ *of the three stage game satisfies the following three conditions.*

(i) $q^*(y,x)$ *is a Nash equilibrium of the spot market for every feasible y, x*

(ii) $x^*(y)$ *is a Nash equilibrium of the forward market for every feasible y*

(iii) y^* *is a Nash equilibrium of the capacity market.*

The next approach is to calculate the different stages of this equilibrium.

Spot market equilibrium with forwards

The equilibrium conditions of the spot market for given forward positions of the companies are

$$
\begin{aligned}
& a - 2q_i - q_{-i} - c_i + x_i + \mu_i = \lambda_i, \\
& x_i - q_i \geq 0 \qquad \lambda_i \geq 0 \qquad (y_i - q_i)\lambda_i = 0, \\
& q_i \geq 0 \qquad \mu_i \geq 0 \qquad q_i \mu_i = 0,
\end{aligned}
\tag{CP2}
$$

for $i = 1, 2$.

Since companies can sell and buy forwards x_i can have positive and negative consequences. Suppose there exists an equilibrium $(y^*, x(y^*), q[y^*; x(y^*)])$ and that both companies have positive production. Then the equilibrium of the spot market $q^* = q[y^*, x(y^*)]$ satisfies one of the following conditions

(i) $\quad 0 < q_i^* < y_i^* \quad i = 1, 2$

(ii) $\quad 0 < q_i^* < y_i^* \quad 0 < q_{-i}^* = y_{-i}^*$

(iii) $\quad 0 = q_i^* < y_i^* \quad 0 < q_{-i}^* < y_{-i}^*$.

The following lemmata will show like in Section 6.2.2 that the first two cases do not hold in equilibrium.

Lemma 6.2.9. *If an equilibrium exists, then case (i) does not hold.*

Proof. Assume $y^*, x^* = x(y^*), q^* = q[y^*, x(y^*)]$ is the equilibrium which satisfies condition (i). The equilibrium conditions are for $i = 1, 2$

$$
\begin{aligned}
a - 2q_i^* - q_{-i}^* - c_i + x_i^* &= 0 \\
a - q_i^* - 2q_{-i}^* - c_{-i} + x_{-i}^* &= 0 \\
0 < q_i^* &< y_i^*.
\end{aligned}
$$

Substituting $c_i + d_i$ by $c_i - x_i^*$ in the solution of the single-stage (open-loop) game yields

$$q_i^* = q_i^*(x^*) = \tfrac{1}{3}[a - 2(c_i - x_i^*) + (c_{-i} - x_{-i}^*)].$$

$0 < q_i(y^*, x) < y_i^*$ holds in a neighborhood $N(x^*)$ of x^* thus an equilibrium of the spot market is obtained. Therefore

$$u_i^f[y^*, x] = \tfrac{1}{9}[a - 2(c_i - x_i) + (c_{-i} - x_{-i}^*)]^2.$$

The first order condition yields

$$x_i^* = \tfrac{1}{5}[a - (3c_i - 2c_{-i})]$$

and

$$q_i^* = \tfrac{2}{5}[a - (3c_i - 2c_{-i})].$$

Therefore, there exists a neighborhood $N(y^*)$ of y^* such that

$$x^*(y) = x^*$$

and

$$\mathbf{q}^*(y) = q^*[y, x^*(y)] = q^*$$

are the best responses to any $y \in N(y^*)$. For all $y \in N(y^*)$

$$u_i(y) = \tfrac{2}{25}[a - (3c_i - 3c_{-i})]^2 - d_i y_i.$$

Obviously one can increase the payoff by decreasing y like in Lemma 6.2.4. □

Lemma 6.2.10. *If an equilibrium exists, then case (ii) does not hold.*

Proof. Assume $y^*, x(y^*), q[y^*, x(y^*)]$ is an equilibrium satisfying case (ii). Thus

$$\begin{aligned} &a - 2q_i^* - q_{-i}^* - c_i + x_i^* = 0 && 0 < q_i^* < y_i^* \\ &a - q_i^* - 2q_{-i}^* - c_{-i} + x_{-i}^* = \lambda_{-i}^* && 0 < q_{-i}^* = y_{-i}^*. \end{aligned}$$

If it is an equilibrium, it is also a local equilibrium. Keeping y fixed at y^* and letting x move around $x(y^*)$, leads to the following solution

$$\begin{aligned}
q_i &= \tfrac{1}{2}(a - y^*_{-i} - c_i + x_i)\\
\lambda_{-i} &= a - 2y^*_{-i} - c_{-i} + y_{-i} - \tfrac{1}{2}1(a - y^*_{-i} - c_i + x_i)\\
&= \tfrac{a}{2} - \tfrac{3}{2}y_{-i^*} - \tfrac{1}{2}(2c_{-i} - c_i) + \tfrac{1}{2}(2x_{-i} - x_i).
\end{aligned}$$

What happens to the payoff of i if x_i varies for fix x^*_{-i} in the forward market? For the spot price the result is

$$a - q_i - y^*_{-i} = a - \tfrac{1}{2}(a - y^*_{-i} - c_i + x_i) - y^*_{-i} = \tfrac{(}{a} - y^*_{-i} + c_i - x_i).$$

The associated profit of company i in the forward market is

$$\begin{aligned}
(a - q_i - y^*_{-i} - c_i)q_i &= \tfrac{1}{2}(a - y^*_{-i} - c_i - x_i)\tfrac{1}{2}(a - y^*_{-i} - c_i + x_i)\\
&= \tfrac{1}{4}[(a - y^*_{-i} - c_i)^2 - x_i^2].
\end{aligned}$$

Whereas x^*_i maximizes the payoff of i, i.e. x^*_i must be zero. Therefore, the medium term payoff of i on the forward market is $\frac{1}{4}[(a - y^*_{-i} - c_i)]^2$. In this case the profit is

$$\tfrac{1}{4}[(a - y^*_{-i} - c_i)]^2 - d_i y^*_i.$$

A suficiently small decrease of y^*_i to $y_i < y^*_i$, keeps $x_i = 0$ as the optimal strategy on the futures market and q_i stays as before and strictly less than y_i. This procedure improves the payoff of company i in the capacity market. Thus, the profit was not an optimum. □

These two lemmata imply

Proposition 6.2.11. *A closed loop equilibrium of the three-stage game satisfies $q_i = y_i$, $i = 1, 2$, if one exists.*

Proof. By Lemma 6.2.9 and Lemma 6.2.10. □

The lemmata and the proposition from the two stage game are now applied for the three stage game. The next aim is to adapt Theorem 6.2.7 to the three stage game.

Therefore, the space of investment variables are partitioned into various subsets and their equilibrium properties can be characterized.

In the following consider the case $q_i < y_i$. As already shown in the previous lemmata this cannot hold at an equilibrium. But it can be a characteristic of a disequilibrium point if one wants to show that there does not exist an equilibrium. Therefore the characteristic of the forward and spot market equilibria for all possible $y_i > 0$ is considered.

Assume the case where the investment variables satisfy $a - 2y_i - y_{-i} - c_i > 0$, $i = 1,2$ (both companies use all of their investment capacity in the spot market). The next lemma shows the property of the equilibrium in the forward market for that case.

Lemma 6.2.12. *Let* (y_i, y_{-i}) *satisfy*

$$a - 2y_i - y_{-i} - c_i > 0 \quad i = 1,2$$

then

$$x_i \geq \tilde{x}_i(x) = -(a - 2y_i - y_{-i} - c_i) < 0, \quad i = 1,2$$

is a closed-loop equilibrium of the forward market.

Proof. Let the invested capacity y be given and $\tilde{x}_i = \tilde{x}_i(y)$ be the optimal reaction of i to a forward position $x_{-i} \geq \tilde{x}_{-i}$ of $-i$. Assume $x_i > \tilde{x}_i$, then

$$a - 2y_i - y_{-i} - c_i + x_i = \lambda_i > 0$$
$$a - y_i - 2y_{-i} - c_{-i} + x_{-i} = \lambda_{-i} \geq 0$$

and $q_i = x_i$ stays an equilibrium on the spot market. Taking $x_i > \tilde{x}_i$ therefore keeps the profit of company i unchanged independent of $-i$'s decision as long as $x_{-i} > \tilde{x}_{-i}$. Let $x_i < \tilde{x}_i$, $x_{-i} \leq \tilde{x}_{-i}$. The total output gets smaller than the forward position and the equilibrium conditions of the spot market are

$$a - 2q_i - y_{-i} - c_i + x_i = 0$$
$$a - q_i - 2y_{-i} - c_i + x_{-i} = \lambda_{-i} > 0.$$

Thus

$$q_i = \tfrac{1}{2}(a - y_{-i} - c_i + x_i)$$

and

$$u_i^f(y; x_i, x_{-i}) = \tfrac{1}{4}[(a - y_{-i} - c_i)^2 - x_i^2].$$

The maximum profit of company i is achieved for $x_i = 0$ with a payoff equal to $\frac{1}{4}(a - y_{-i} - c_i)^2$ which is the global optimum of i if and only if

$$0 = x_i < \tilde{x}_i = -(a - 2y_i - y_{-i} - c_i) < 0$$

which cannot hold.

Thus, to $x_{-i} \geq \tilde{x}_{-i}$ the optimal reaction of company i cannot be $x_i < \tilde{x}_i$. That implies that $\tilde{x}_i(y)$, $i = 1,2$ is a closed-loop equilibrium of the forward market and any $x_i \geq \tilde{x}_i$, $i = 1,2$ is also a closed-loop equilibrium of the forward market. □

The next lemma shows that $-i$ can always force i out of the forward market by selecting x_{-i} large enough.

Lemma 6.2.13. *Let (y_i, y_{-i}) be given. $x_i = 0$ is the optimal response of i to any $x_{-i} \geq \tilde{x}_{-i}(y)$ if $a - 2y_i - y_{-i} - c_i < 0$ and $a - y_i - 2y_{-i} - c_{-i} > 0$.*

Proof. Assume $-i$ has a forward position $\bar{x}_{-i} \geq \tilde{x}_{-i}$. Then we suppose that the equilibrium in the spot market is

$$a - 2q_i - y_{-i} - c_i = 0$$
$$a - q_i - 2y_{-i} - c_{-i} + \bar{x}_{-i} = \lambda_i \geq 0.$$

That holds because

- there exists some $q_i < y_i$ since $a - 2y_i - y_{-i} - c_i < 0$ such that q_i solves $a - 2q_i - y_{-i} - c_i = 0$.
- $a - y_i - 2y_{-i} - c_{-i} + \tilde{x}_{-i}(y) = 0$ by the definition of $\tilde{x}_{-i}(y)$. Thus, any $q_i < y_i$ and $x_{-i} > \tilde{x}_{-i}(y)$ satisfies $a - q_i - 2y_{-i} - c_{-i} + x_{-i} = \lambda_{-i} \geq 0$, which yields the equilibrium at $q_i < y_i$ and $q_{-i} = y_{-i}$.

Consider the reaction of $-i$ to $x_{-i} > 0$. $a - y_i - 2y_{-i} - c_{-i} + x_{-i} > 0$ and $a - q_i - 2y_{-i} - c_{-i} > 0$ for all $q_i < y_i$ since $x_{-i} \geq \tilde{x}_{-i}(y)$. Thus, $q_{-i} = y_{-i}$ whenever $x_{-i} \geq \tilde{x}_{-i}(y)$, whatever the position of i on the forward market is.

Consider the subsequent strategies of i on the forward market. Since the objective function depends on the value of x_i, we analyze two cases:

(i) $x_i \geq \tilde{x}_i(y) = -(a - 2y_i - y_{-i} - c_i) > 0$

(ii) $x_i \geq \tilde{x}_i(y) = -(a - 2y_i - y_{-i} - c_i) < 0$

The payoff of company i in case (i) stays constant at $(a - y_i - y_{-i} - c_i)y_i$ for all $x_i \geq \tilde{x}_i(y)$. Thus, company i cannot improve its payoff by choosing $x_i \geq \tilde{x}(y)$ and obtain a global optimum for the case (ii).

The payoff of company i in case (ii) can be obtained as follows. Since $x_i \leq \tilde{x}_i(y)$, $q_i \leq y_i$ and q_i solves

$$
\begin{gathered}
a - 2q_i - y_{-i} - c_i + x_i = 0 \\
a - q_i - 2y_{-i} - c_{-i} + x_{-i} = \lambda_{-i} > 0.
\end{gathered}
$$

The best response of company i by Lemma 6.2.12 is

$$q_i = \tfrac{1}{2}(a - y_{-i} - c_i + x_i) < y_i)$$

and

$$u_i^f(y; x_i, x_{-i}) = \tfrac{1}{4}[(a - y_{-i} - c_i)^2 - x_i^2].$$

The maximum profit can be reached for $x_i = 0$ if the company's payoff is equal to $\frac{1}{4}(a - y_{-i} - c_i)^2$. This is the global solution of the payoff of company i if it has both

$$0 = x_i < \tilde{x}_(y) = -(a - 2y_i - y_{-i} - c_i) > 0$$

and

$$\tfrac{1}{4}(a - y_{-i} - c_i)^2 > (a - y_i - y_{-i} - c_i)y_i. \qquad (\star)$$

By assumption, the first condition holds. In order to check the second order condition, $(\star)$ must be rewritten as

$$(a - y_{-i} - c_i)^2 - 4(a - y_{-i} - c_i)y_i + 4y_i^2 > 0$$

or

$$(a - 2y_i - y_{-i} - c_i)^2 > 0.$$

Therefore, if company $-i$ chooses $x_{-i} \geq \bar{x}_{-i}$ and $a - 2y_i - y_{-i} - c_i < 0$ the best response of company i is $x_i = 0$. The result is unique because of the strict concavity of the objective function. □

Company $-i$ (which did not exhaust his capacity, i.e. $a - y_i - 2y_{-i} - c_- - i > 0$) can always force to sell its capacity in the spot market, does not matter what company i (which overinvestet, $a - 2y_i - y_{-i} - c_i < 0$) decides, by taking any forward position $x_{-i} \geq \tilde{x}_{-i}(y)$.

Since $q_{-i} = y_{-i}$ and is invariant with x_i, the profit function of company i in the forward market is

$$\pi_i(y,x) = \tfrac{1}{4}[(a - y_{-i} - c_i)^2 - x_i^2].$$

This profit is maximized if x_i is equal to zero as proved in the next Lemma.

Lemma 6.2.14. *Assume $a - y_i - 2y - i - c_{-i} > 0$ and $a - 2y_i - y_{-i} - c_i < 0$. The best response of company $-i$ to $x_i = 0$ is $x_{-i} = \tilde{x}_{-i}$.*

Proof. Let x_i be zero and define $\tilde{q}_i$ such that $a - 2\tilde{q}_i - y_{-i} - c_i = 0$. Therefore, $\tilde{q}_i$ is less than the maximal capacity of company i, because $a - 2y_i - y_{-i} - c_i < 0$. The following three strategies of company $-i$ on the forward market will be assigned, due to the form of the objective function of company $-i$. It depends on how the spot positions are at capacity.

(i) The forward positions are chosen to ensure $q_{-i} = y_{-i}$.

(ii) The forward positions are chosen to maximize the payoff in the region where $q_i < y_i$, $q_{-i} < y_{-i}$.

(iii) The forward positions are chosen to maximize the payoff in the region where $q_i = y_i$, $q_{-i} < y_{-i}$.

The payoff of company $-i$ for the three cases is:

(i) Company $-i$ ensures the full recovery of its capacity in the forward market and it takes $x_{-i} \geq \hat{x}(y)$ where $\hat{x}_{-i}(y)$ is defined by

$$a - \tilde{q} - 2y_{-i} - c_{-i} + \hat{x}(y) = 0.$$

For x_i and $x_{-i} \geq \hat{x}_{-i}$ the equilibrium in the spot market is $q_i = \tilde{q}_i$ and $q_{-i} = \bar{q}_{-i}$. Thus, the payoff of company $-i$ is

$$(a - \tilde{q}_i - y_{-i} - c_i)y_{-i} = \tfrac{1}{2}(a - y_{-i} - c_i)y_i.$$

(ii) Assume that $x_{-i} = \hat{x}_{-i}(y) - \varepsilon_{-i}$ where ε_{-i} is small enough to ensure that q_i hits the capacity and that q_{-i} does not reach zero. Then the system

$$\begin{aligned} a - 2q_i - q_{-i} - c_i &= 0, \\ a - q_i - 2q_{-i} - c_{-i} + x_{-i} &= 0 \end{aligned}$$

can be solved with q_i and q_{-i} as a function of x_{-i}. The payoff function of $-i$ is obtain by setting $y_i = 0$ in the payoff function $u_i^f[y^*, x]$ in Lemma 6.2.9.

$$u_{-i^f}[y; 0, x_{-i}] = \tfrac{1}{9}[a - 2(c_{-i} - x_{-i}) + c_i]^2.$$

Deriving u^f_{-i} with respect to y_{-i} yields

$$\tfrac{4}{9}[a-2(c_{-i}-x_{-i})+c_i].$$

This evaluates to

$$\tfrac{4}{9}[a-(2c_{-i}+c_i)+\hat{x}_{-i}] = \tfrac{4}{9}y_{-i} > 0$$

at $\hat{x}_{-i}$, when q_{-i} reaches the capacity amount y_{-i}. Since the derivate of u^f_{-i} is positive at $\hat{x}_{-i}$ and is concave in x_{-i} it is still increasing at that point. Therefore, the maximum of u^f_{-i} cannot be $x_{-i} < \hat{x}_{-i}(y)$, which means that $x_{-i} = \hat{x}_{-i} - \varepsilon_{-i}$ cannot be the optimal reaction of company $-i$.

(iii) Again we will use the same concavity argument to show that $u^f_{-i}[y;0,x_{-i}]$ cannot be maximized by decreasing x_{-i} to a level where the total output of company i is equal to the capacity x_i or that the total output of company $-i$ reaches zero. Therefore, there is no benefit for company $-i$ to decrease its forward positions if its total output reaches zero before $q_i = y_i$. It would imply that its payoff would be zero. One other case is that q_i reaches y_i and $q_{-i} > 0$. This happens for some $\bar{q}_{-i}$ such that

$$a-2y_i-\bar{q}_{-i}-c_i = 0$$

Now we show that decreasing of the forward position of company i cannot increase the profit by the resulting price.

Let $q_{-i} = \bar{q}_{-i}+\varepsilon$. Therefore the profit of company $-i$ is

$$(a-y_i-\bar{q}_{-i}-\varepsilon-c_{-i})(\bar{q}_{-i}+\varepsilon).$$

If this term is derived at $\varepsilon = 0$ then

$$3y_i+(2c_i-c_{-i})-a,$$

which is greater than zero. Since

$$2(-a+2y_i+y_{-i}+c_i)+(a-y_i-2y_{-i}-c_{-i})>0$$

by assumption, this means that company i cannot reduce the forward position x_i beyond the point where $q_i = y_i$. Therefore $x_{-i} \geq \tilde{x}_{-i}$ ensures the optimal profit of company $-i$ if the forward position of company i is zero.

□

If company i has excess capacity the following result will emerge.

Lemma 6.2.15. *Assume that* (y_i, y_{-i}) *is such that*

$$\begin{aligned}
&a-2y_i-y_{-i}-c_i<0\\
&a-x_i-2y_{-i}-c_{-i}>0\\
&\tilde{x}_{-i}(y)=-(a-y_i-2y_{-i}-c_{-i}).
\end{aligned}$$

Then

$$\begin{aligned}
x_i &= 0,\\
x_{-i} &\geq \tilde{x}_{-i}(y)
\end{aligned}$$

is a closed loop equilibrium of the forward market. At the equilibrium $q_i < y_i$ *holds.*

Proof. This follows from the combination of the Lemmata 6.2.13 and 6.2.14. □

Lemma 6.2.16. *There does not exists an equilibrium of the capacity game with a forward market such that* $a-2y_i-y_{-i}-c_i<0$ *and* $a-y_i-2y_{-i}-c_{-i}>0$.

Proof. Suppose such an equilibrium exists. Then $x_i = 0$ and $x_{-i} \geq \bar{x}_{-i}$ is the equilibrium on the forward market with the corresponding spot market equilibrium $q_i = \frac{1}{2}(a-y_{-i}-c_i)$, $q_{-i} = y_{-i}$. This spot market equilibrium yields $q_i < y_i$ which is a contradiction to Proposition 6.2.6. □

Lemma 6.2.17. *A capacity game equilibrium with a forward market cannot satisfy* $a - 2y_i - y_{-i} - c_i < 0$*,* $i = 1,2$*.*

Proof. By Proposition 6.2.6 the equilibrium, if it exists, satisfies $q_i = y_i$ for $i = 1,2$. Since the marginal revenue of both companies is negative at that point, it cannot be an optimal choice for both of them. □

The conclusion of the above is that a capacity equilibrium with a forward market satisfies $a - 2y_i - y_{-i} - c - i \geq 0$, for $i = 1,2$, if it exists. The result is summarized in the following proposition.

Proposition 6.2.18. *A Capacity game equilibrium with a forward market satisfies* $a_i - 2y_i^* - y_{-i}^* - c_i \geq 0$*,* $i = 1,2$*, if it exists.*

Proof. That is proved in Lemmata 6.2.16 and 6.2.17. □

Theorem 6.2.19. *A capacity game equilibrium, is the open-loop equilibrium, if it exists.*

Proof. Suppose an equilibrium of the three-stage game exists. $a - 2y_i - y_{-i} - c_i \geq 0$, $i = 1,2$ holds by Proposition 6.2.18. $a - 2y_i - y_{-i} - c_i$ is also the marginal revenue of company i from its actings in the spot and forward market. The marginal revenue is equal to d_i because of the optimality of the action in the capacity game of company i. Therefore

$$a - 2y_i - y_{-i} - c_i - d_i = 0, \quad i = 1,2,$$

which are the open-loop equilibrium conditions. □

Remark

If an equilibrium in the three-stage game exists, it is the open-loop equilibrium. Therefore, one cannot enlarge the production in the spot market by using a forward market. As a consequence of the capacity game, the companies are faced with the destructive competition as a result of the existence of a forward market. They block this possibility by setting capacity levels. If we compare the two equilibrium conditions the same solutions will be reached. Thus, the capacity stage produce a contradiction to the ALLAZ & VILA [7] result that the the initiation of a forward game mitigates market power.

Conclusion

In my diploma thesis I have discussed several equilibrium models that are used by companies in the electricity market. The complexity of the electricity market creates many risks for companies, forcing them to trade forward contracts to reduce those risks. However, trading forward contracts induces other risks such as a potential decrease of a company's profits therefore reducing the company's market power. This proven fact is equivalent to the *Prisoner's Dilemma*, however, if we insert capacity constraints into the model this favorably impacts the results especially, since we have seen that forward contracts do not change the equilibrium, if it exists, in a model with deterministic demand.

As further developments of my thesis, it would be interesting to extend this model by pump storage plants. This however would make the model much more complex since these plants can effectively store electricity.

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Deutsch	Sehr gute Kenntnisse in Wort und Schrift
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