DISSERTATION

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„Topics in Dual Sourcing under Disruption Risk"

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Abstract

Suppliers as a central factor of successful supply chain management face the risk of several disruptions, e.g. natural disasters, bankruptcy, labor strikes, machine breakdowns, and business failures. Buyers that rely on a single supplier, therefore, expose themselves to great risk of disruptions. Dual sourcing, i.e. sourcing from two suppliers, is a potential countermeasure.

This thesis studies dual sourcing models in the presence of supply disruption risks. Thereby, the optimal sourcing and allocation policy, i.e. how much to order from which supplier, depends on various factors, such as diversity in reliabilities, prices, and geographical location of the suppliers. To examine the value of the buyer's optimal dual sourcing policy it is compared to single sourcing policies and simple (heuristic) dual sourcing policies.

We study different model variants that include supply disruptions from the perspective of the buying firm. First, we analyze the optimal inventory and allocation policy under the risk of supply disruptions, stochastic lead times, and stochastic demand. Second, we study the optimal allocation policy under supply disruptions and learning, i.e. suppliers reduce production cost over time through learning. By considering supply disruption risk and learning, we analyze the trade-off between risk reduction via dual sourcing and learning benefits on supplier sourcing costs induced by long-term relationships with a single supplier. Finally, the sensitivity of this model to unknown supplier reliabilities, a stochastic demand, and risk averse decision makers is investigated. Our results illustrate the advantage of a flexible dual sourcing strategy and provide important insights into effective supply disruption risk management.

Zusammenfassung

Lieferanten spielen im erfolgreichen Supply Chain Management eine zentrale Rolle. Sie sind durch Naturkatastrophen, Insolvenzen, Arbeiterstreiks oder Maschinenausfälle u.ä. dem Risiko von Produktionsstörungen oder -ausfällen ausgesetzt. Verlassen sich Unternehmen lediglich auf einen Zulieferer (Single Sourcing) setzen sie sich dadurch einem sehr hohen Ausfallrisiko aus. Eine potentielle Gegenmaßnahme ist es, den Bedarf auf zwei Lieferanten aufzuteilen (Dual Sourcing). Die vorliegende Dissertation befasst sich mit Dual Sourcing Modellen zur Ermittlung der optimalen Beschaffungspolitik. Hierbei werden das Ausfallrisiko, die geographischen Lagen sowie die Preise der Lieferanten berücksichtigt. Die optimale Beschaffungspolitik wird anschließend mit Single- und nicht optimalen (heuristischen) Dual-Sourcing Politiken verglichen.

Anhand des ersten Modells wird die optimale Beschaffungs- und Bestandspolitik unter Berücksichtigung temporärer Lieferantenausfälle, stochastischer Lieferzeiten und stochastischer Nachfrage untersucht. Anhand des zweiten Modells wird die optimale Beschaffungspolitik unter Berücksichtigung von Ausfallrisiken und Lieferanten-Lernkurven (Produktionskosten sinken durch Lerneffekte) betrachtet. Aufgrund der Lernkurven kann die Single-Sourcing Politik der Dual-Sourcing Politik, trotz erhöhtem Ausfallrisiko, überlegen sein. Diesen Effekt gilt es zu berücksichtigen. Abschließend wird die Sensitivität dieses Modells auf unbekannte Lieferantenzuverlässigkeiten, einer stochastischer Nachfrage und risikoaversen Entscheidungsträgern untersucht. Die entwickelten Modelle verdeutlichen den Vorteil einer flexiblen Dual Sourcing Strategie und liefern wichtige Einsichten in das risikobehaftete Lieferantenmanagement.

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Chapter 1

Introduction

1.1 Motivation and research objectives

Today's trends of globalization and outsourcing increases the interdependence among buyers and its suppliers. As buyers become more supplier dependent, they also become more vulnerable to supply disruptions. Disruptions in supply may be caused by different forces from inside and outside the organizations. These disruptions can have a significant impact on the supply chain performance in the short-run and the long-run. In particular, supply disruption can increase the sourcing costs considerably. Therefore, buying firms need to prepare for such events.

For example, in 1997 a fire at an Aisin Seiki Co. plant disrupted Toyota's supply chain. Aisin was the sole source for P-valves at low cost. P-valves where crucial for all Toyota vehicles and manufactured exclusively at this plant. As a result, Toyota had to restructure the supply chain of P-valves. Within some days, suppliers with little experience started to manufacture the required components to minimize the impact of the disruption caused by the fire. However, the financial damage to Toyota was an overall loss of about $160 billion in revenues (Nishiguchi and Beautdet (1997)). Another example where a fire incident caused a supply disruption with serious implications for purchasing organizations is the Nokia-Ericsson case. In 2000, a fire shut down Philips' semiconductor plant which supplied both buyers Nokia and Ericsson. Nokia managed to source from alternative suppliers immediately and hence could minimize the negative impact of the disruption. Ericsson, however, had no other source of chips and failed to manage the shortages. In the end Ericsson lost $400 million (Latour (2001)). Major catastrophes or terrorist incidents can also cause disruptions. In April 2010, the car manufacturer BMW had to stop production in three German plants because electronic components, normally air-shipped, could not be delivered due to the ash cloud over Europe (Friese et al. (2010)). In March 2011, the earthquake in Japan caused companies around the world to rebuild their supply chains to cope with supply disruption and search for new suppliers to avoid running out of components that had been previously obtained from Japan (Hookway and Poon (2011)). After the 9/11 terrorist attack, the U.S. border and air traffic were closed, resulting in high losses for many supply lines (Sheffi and Rice (2005)). A prominent example of a supplier bankruptcy, another reason for supply disruption, is the automotive parts manufacturer Delphi Corporation who filed for Chapter 11 bankruptcy protection in 2005. The original equipment manufacturer experienced substantial problems when dealing with the resulted loss of Delphi's production.

This recent incidents show that risks of disruptive events exist. Further, different reactions from the buyers to supply disruptions illustrate the importance of effective supply risk management. Risk diversification, i.e. sourcing from two or multiple suppliers, is an effective strategy to protect against disruptions in supply. Although, single sourcing allows for an improvement of the buyer-supplier relationships that makes the supply chain very efficient in terms of quality and costs, it is also very risky. Therefore, having the flexibility to source from multiple suppliers can reduce the risk substantially.

This research is inspired by the challenges in dealing with supply disruptions. We focus on a two-stage supply chain where the buyer can source from two suppliers that face the risk of supply disruptions. These suppliers may differ in their characteristics, e.g. differ in their geographically regions, prices and reliabilities. As disruptions influence the present and the future systems performances we focus on multi-period models. The objective is to minimize the buyer's total cost. Further, we deal with different forms of supply uncertainties.

In particular, we consider stochastic dynamic models that monitor the supply chain's changing factors which allows the buyer to manage his sourcing and order allocation strategy over time in the presence of supply disruptions. The models under investigation differ in the ways how supply disruptions are defined (temporary or permanent), how decisions are made (continuous or discrete) and in the considered time horizon (finite or infinite). First, we study the impact of temporary supply disruptions on the buyer's optimal inventory and sourcing strategy under lead time risk and demand risk. We consider an infinite horizon model formulating a semi-Markov decision process where decisions are made in continuous time. Second, we study the impact of permanent disruptions on the buyers optimal sourcing strategy. Thereby, we assume that suppliers reduce future supply cost with cumulative productions, i.e. due to learning effects. We consider a finite horizon discrete time model applying Bellman's principle of recursion without considering lead time risk under deterministic and stochastic demand. An overview on the particular modeling elements that are incorporated in this thesis is shown in Table 1.1.

The objective of this work is to provide managerial insights by characterizing the buyer's optimal sourcing and order allocation policy and evaluating the benefits of risk diversification due to the dual sourcing strategy. In this context, the central research questions are:

- What is the optimal sourcing policy (single or dual sourcing) over time that minimizes the buyers' total cost? Is it beneficial to use dual sourcing to diversify the risk or should a buyer accept the risk of potential disruptions and source from a single supplier?

- How should the demand be allocated among the supply base in a dynamic environment with regards to the system states?

- How do supplier characteristics such as purchase cost, cost improvements, lead times, and reliabilities influence the optimal sourcing and allocation strategy?

- Effective sourcing strategies, further, depend on the nature of supply uncertainty. What is the effect of permanent or temporary disruption on the optimal policy? If disruptions are temporary, how does the frequency impact the optimal policy?

- If supplier cost improve through learning, i.e. suppliers learn based on production experience: What is the trade-off between the disruption risk which favors dual sourcing and the learning effects which favors single sourcing?

In regards of the buyers' benefit of the optimal diversification strategy we address the following research questions:

- What is the benefit of the optimal dual sourcing strategy compared to an acceptance strategy in which the buyer sources from a single supplier and does not mitigate the supply risk through diversification?

- What is the benefit of the optimal sourcing policy compared to simple dual sourcing policies or heuristic dual sourcing policies that are common in practice?

	Supply disruptions	Lead time risk	Demand risk	Learning effects
Model 1 (Chapter 3)	temporary	✓	✓	x
Model 2a (Chapter 4)	permanent	x	x	✓
Model 2b (Chapter 5)	permanent	x	✓	✓

Table 1.1: Overview of model elements included in this thesis.

1.2 Structure of the thesis

This thesis is organized as follows.

Chapter 2 provides an overview on fundamentals in supply related risks, and sourcing strategies. This chapter also reviews relevant literature on sourcing strategies under supply uncertainty, and buyer-supplier relationships and the learning curves.

Chapter 3 deals with the problem of a single item inventory system with stochastic demand, stochastic lead times, and supplier disruptions. A supply chain with one buyer facing Poisson demand who can procure from a set of potential suppliers who are not perfectly reliable, is studied. Each supplier is fully available for a certain amount of time (ON periods) and then breaks down for a certain amount of time during which it can supply nothing at all (OFF periods). The problem is modeled by a semi-Markov decision process (SMDP). The objective is to minimize the buyer's long run average cost, including purchasing, holding and penalty costs. In a numerical study focusing on the dual sourcing case, the trade-off between single and dual sourcing, as well as keeping inventory and having a back-up supplier is investigated. Further, the value of full information about the supplier status switching events is analyzed. The performance of the optimal policy is compared to an order-up-to-S policy and a simple heuristic is developed where the order-up-to-levels are heuristically derived. Simulations with more general distributions are performed and a more dramatic break-down scenario is discussed. The Chapter is based on Silbermayr and Minner (2014).

Chapter 4 analyzes the trade-off between risk reduction via dual sourcing under disruption risk and learning benefits on sourcing costs induced by long-term relationships with a single supplier. In particular, the suppliers realize reductions in their future production cost due to volume-based learning, but face the risk of permanent disruptions. The buyer's optimal sourcing and volume allocation strategy over a finite dynamic planning horizon is identified to obtain insights on how reliability, cost and learning ability of potential suppliers influence the buyer's sourcing decision. Further, the benefit from dual sourcing compared to single sourcing and heuristic dual sourcing policies is quantified. The Chapter is based on Silbermayr and Minner (2013).

Chapter 5 extends the basic model analyzed in Chapter 4 to study the sensitivity of the model with respect to limiting model assumptions. Thereby, the impact of the extended assumptions on the optimal sourcing and volume allocation strategy and the value of dual sourcing are discussed. First, the model assumptions are extended by assuming that supplier reliabilities are uncertain. In addition to the volume-based learning, a second type of learning, Bayesian learning about supplier reliability, is introduced. Then, the assumption of deterministic demand is relaxed to study the influence of uncertain demand on the optimal policy and the value of dual sourcing. Finally, a risk averse framework is analyzed to study the impact of risk aversion. The Chapter is based on Silbermayr and Minner (2013), where the main results are briefly discussed in the final Section.

Chapter 6 concludes this thesis by summarizing the findings and discussing possible extensions for future research.

Chapter 2

Foundations and Literature

In this Chapter an overview of the topics of interests and the related literature is given.

2.1 Foundations

This Section discusses the different types of uncertainties that can occur at the supply side and deals with effective sourcing strategies in supplier management.

2.1.1 Types of supply uncertainty

A disruption in the supply chain can be defined as a (random) event which has negative consequences for the supply chain performance causing operational and financial risks. The focus in this Section is to provide an overview of different classifications of supply-related disruptions that affect or disrupt the supply chain.

Any uncertainties arising in supply can be caused by internal or external factors of the supply chain. They may be due to accidents (e.g. fires, explosions), acts of nature (e.g. earthquakes, extreme weather events), acts of humans (e.g. terrorist attacks, war), or intra-firms interactions (e.g. labor strikes, machine failures, quality failures, complex production processes). The uncertainties can either be discrete or continuous and the consequences can be either temporary or permanently.

The different forms of supply uncertainty can be specified as follows (Snyder et al. (2012)):

ON and OFF Disruptions: In this case, suppliers are either available (ON) or completely unavailable (OFF) for a certain amount of time, where the ON and OFF times are random variables. As long as a supplier is available (ON) he is able to deliver goods, but when his status changes to OFF he gets unable to provide any goods. Existing models differ in if an order is placed with an available supplier that switches from ON to OFF will still be processed, will be continued to process when the supplier is ON again or will be lost as soon as a supplier status switches from ON to OFF.

Yield uncertainty: Here supply uncertainty is treated as randomness in yield, where the suppliers are either reliable delivering the desired amount or unreliable delivering less than the demanded amount with some probability. Hence, a buyer ordering q^o units from an unreliable supplier receives $q = yq^o$ units with y being a random variable. A special case of random yield is the all-or-nothing delivery, where y follows a Bernoulli distribution, i.e. a supplier either delivers the total required quantity $q = q^o$, with some probability p or nothing at all $q = 0$, with probability $1 - p$ (Bernoulli yield).

Capacity uncertainty: Unlike random yield, random capacity modeling treats the capacity as an uncertain variable that is typically independent of the order quantity. The amount the buyer receives is $q = \min\{q^o, yCap\}$, with Cap being the regular capacity of the supplier and y the random variable.

Lead time uncertainty: Another form of supply uncertainty is characterized by stochastic lead times of orders placed with the suppliers, a common problem in industry.

Supply cost uncertainty: In this case, uncertain supply is reflected by stochastic procurement prices of the suppliers.

At this point it should be mentioned that the boundaries among the above forms of supply uncertainty are often blurry (Snyder et al. (2012)). Bernoulli yield, for example, can be viewed as a special case of ON and OFF disruptions. Similarly, capacity uncertainty can be seen as lead time uncertainty.

2.1.2 Strategic sourcing

Creating effective sourcing strategies is crucial for successful supply chain management in consideration of supply disruption risks. The general sourcing problem includes the following decisions: How many suppliers should be included in the procurement process, which suppliers should receive an order and how should the total order quantity be allocated among these suppliers? The buyer may choose between sourcing from one supplier (single sourcing) and multiple suppliers (multiple sourcing). These suppliers may be local suppliers (local sourcing) or global suppliers (global sourcing). In the following we discuss these different concepts and their advantages and disadvantages.

The strategy of single or multiple sourcing concerns the decision on the number of suppliers a buying firms wants to have a relationship with, for a certain product:

Single sourcing: In this case, the buyer uses a single supplier to procure the desired product. The main advantages of this strategy are that the buyer can achieve cost reductions trough higher volumes, develop a long-term relationship, build up high trust with the supplier and achieve high quality standards. Disadvantages of this strategy are the high dependency on a single supplier, the lack of competition, and the low flexibility for the buying firm in the supply chain. This means that single sourcing requires a high reliability, quality and flexibility of the supplier. If disruption risk is low or the cost of mitigating the risk are high single sourcing is an appropriate strategy (Schmitt and Tomlin (2012)).

Dual sourcing: If, in contrast, the buyer's strategy is to procure from two suppliers a close supplier relationship still remains, but with a higher supply guarantee than in the single sourcing strategy in consideration of disruption risk. In dual sourcing the lower cost supplier often receives a higher volume than the other supplier, but the second supplier is kept to diminish the supply risk. This sourcing policy is a special case of the multiple sourcing policy that still provides cost reductions through high volumes, however, with medium flexibility of the buyer.

Multiple sourcing: The major benefit from multiple sourcing is a reduced risk in case of supplier disruptions and the maintenance of supplier competition. Moreover, the probability of meeting the customer's volume requirement increases with the number of suppliers as this lowers the probability that any of these sources become unavailable. However, a multiple sourcing strategy implies higher cost for the buyer due to lower volumes assigned to the individual suppliers, higher administrative and management cost and less loyalty in the buyer-supplier relationship. Using a sourcing strategy with two or three suppliers, often reflects a reasonable balance between risk mitigation and supply chain rationalization (Schmitt and Tomlin (2012)).

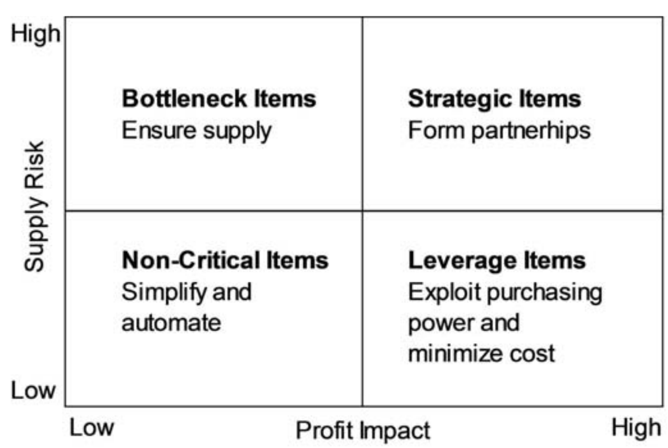
The strategy of local or global sourcing concerns the decision on the geographic location of the suppliers:

Local sourcing: In this case, the engaged supplier is located in the immediate neighborhood of the buyer, where logistic disturbances are minimized. The buying firm can benefit, e.g. from short lead times, low transportation costs, high product quality standards, and the same currency. A major disadvantage of local sourcing is that the high reliability in the procurement process often implies high prices.

Global sourcing: This refers to an internationalization of sourcing opportunities and a world wide selection of suppliers. Low prices from global suppliers often come along with low reliabilities, quality risks, exchange rate risks, and logistic problems. In addition, troubles resulting from cultural differences, political instabilities and different languages should be kept in mind.

General trends in supplier management show increasing global sourcing, a supply base reduction, and long term-relationships with suppliers (Minner (2003)). As a consequence, more and more cost effective single sourcing relationships occur, that can be very risky. Firms that accept the risk of disruptions and do not allow for the flexibility of diversifying the risk due to multiple sourcing need to be clear about the negative consequences of potential supply disruptions. Therefore, in terms of supply-side related risk the objective of a buyer is to find the optimal number of suppliers to balance the cost of managing the suppliers and the cost resulting from supply disruptions (Meena et al. (2011)). Of the above explained sourcing strategies, which will be the most effective one, depends on the individual situation of the buying firm, the nature of the suppliers considered in the supply base, and the cost structure. In order to choose the right strategy, the buyer has to weigh out the advantages and disadvantages of each strategy in his particular context to balance the disruption risk and the impact on the cost.

Kraljic (1983) presents a simple framework for developing individual supply strategies. He argues that the appropriate procurement strategy depends on two dimensions: The profit impact and the supply risk. Based on these dimensions Kraljic's supply matrix defines four types of items that require different supply strategies (see Figure 2.1).



Source: "Designing and managing the supply chain: concepts, strategies and case studies", Semchi-Levi, D., Kaminsky, P., Semchi-Levi, E., McGraw-Hill (2008)

Figure 2.1: Supply matrix.

Strategic items (high supply risk, high profit impact), typically high-value components that are crucial for the buyer, require a supply strategy focusing on cost reduction and on risk minimization. Hence, developing long-term relationships with suppliers is appropriate. Leverage items (low supply risk, high profit impact) are items that have many suppliers. Forcing competition to reduce cost which have a large impact should be adapted in this case. Useful approaches for bottleneck items (high supply risk, low profit impact) include over-ordering when the item is available to ensure supply, and look for potential suppliers. For non-critical items (low supply risk, low profit impact) that are easy to buy and produced in standard configuration, the objective is to simplify and automate the procurement process.

2.2 Literature

This Section discusses literature related to this thesis. We classify the literature into two categories. The first category focuses on literature dealing with random supply and sourcing strategies and the second category focuses on the buyer-supplier relationships and the learning curve.

2.2.1 Random supply and sourcing strategies

Existing models in the literature dealing with supply risk management, sourcing strategies and decision making in presence of unreliable supply differ with respect to several characteristics: (i) the type of supply uncertainty, (ii) the type of demand (deterministic versus stochastic), (iii) the type of supply lead time (zero versus positive deterministic versus stochastic), (iv) the type of the sourcing strategy (single versus dual versus multiple sourcing), and (v) the type of the planning horizon (single versus multiple periods and finite versus infinite horizon). Our discussion first will give an overview on papers dealing with supply disruption management strategies. Then, literature will further be classified into different forms of supply risk and will mainly focus on the multiple supplier option models. For further information the interested reader is referred to the following reviews: Comprehensive literature reviews on sourcing strategies and the optimal number of suppliers are provided by Elmaghraby (2000) and Minner (2003). A review on supply chain risk management is given in Tang (2006). Reviews covering the random yield literature is given in Yano and Lee (1995) and Grosfeld-Nir and Gerchak (2004). A review on pooling lead time risk by splitting the order among multiple suppliers can be found in Thomas and Tyworth (2006). A very recent review on quantitative models for supply chain disruptions is found in Snyder et al. (2012).

Supply disruption management strategies

Kleindorfer and Saad (2005) state that for successful supply disruption management, first the nature and the impact of the supply risk have to be evaluated, and finally the risk mitigation strategy needs to be defined. They present a conceptual framework to mitigate risks. Tang and Tomlin (2009) provide a framework for examining the benefits of flexible supply via multiple suppliers and via flexible supply contracts to mitigate the negative impact of supply risk. They point out that already low levels of flexibility can have a major reduction in the negative consequences resulting from disruptions.

Tomlin (2006) discusses supply disruption management strategies for an infinite-horizon model with two suppliers, one reliable supplier with flexible capacity and one unreliable but less expensive supplier. The discussed strategies to cope with the disruptions are inventory control, sourcing and acceptance of disruption risk. The author shows that the optimal strategy depends on the percentage uptime and disruption length and that inventory is preferred over supplier diversification if the supply disruptions are relatively short and frequent, but sourcing is preferred if disruptions are long and rare. Tomlin (2009a) discusses supply disruption management strategies for short life-cycle products where keeping inventory is no possible strategy. He considers a risk neutral and risk averse decision maker and finds that, as supplier reliability increases, dual sourcing becomes advantageous unless demand uncertainty is very low. Saghafian and Van Oyen (2012) discuss the value of secondary flexible backup supplier which is capable to respond to the requests of a firm in the case of a disruption and the value of information concerning the disruption risk. They consider a two-stage setting with a primary risky supplier and the recourse option of ordering from a flexible backup supplier. They conclude that sourcing from a flexible backup supplier is more valuable when the perception errors about the unreliability supplier status are low and disruption risk information is more valuable when perception errors are high. Schmitt and Tomlin (2012) study infinite horizon models focusing on sourcing strategies (diversification and backup strategies) to manage disruption risk. They assume that suppliers are either fully operational or temporary completely unavailable and examine the preferred strategy depending on the firm's particular situation. They conclude that the average disruption length and the frequency of disruptions have a major impact on the preferred strategy, but there is no one-size-fits-all solution to the disruption risk problem. Consequently, the best strategy needs to be evaluated for each particular context.

Optimal number of supplier under supply failure risk

Several single-period decision models compare single and multiple sourcing in the presence of supply failure risk using a decision tree approach focusing on determining the optimal number of suppliers. Berger et al. (2004) investigate the optimal number of suppliers subject to failure, including super-events which affect all suppliers, as well as unique events which affect only a single supplier. The cost considered are the supplier operating cost and the supply failure cost. Ruiz-Torres and Mahmoodi (2007) extend this model by considering the independent risks of individual supplier failure. They conduct a sensitivity analysis to study the impact of the input parameters on the optimal number of suppliers and conclude, like Berger et al. (2004), that having a large number of suppliers is an effective strategy only in extreme cases where suppliers are very unreliable and the loss to operational cost are high. Berger and Zeng (2005) also study the optimal size of the supply base relaxing the assumption of identical supply failure probabilities and the assumption of linear costs. A decision tree model considering super-events affecting all suppliers, unique events affecting one supplier and semi-super events affecting a set of suppliers, exhibiting regional effects, is discussed in Sarkar and Mohapatra (2009). They find that a strategy having suppliers from as many locations as possible is always preferable.

Supply modelled as ON and OFF process

Early disruption models where supply is modelled as an ON-OFF process are restricted to the single supplier framework where no alternative source is available (e.g. Meyer et al. (1979), Moinzadeh and Aggarwal (1997), Parlar (1997)) and Mohebbi and Hao (2006)). Meyer et al. (1979) discuss a production-storage system with constant demand subject to stochastic failure and repair processes and give an expression for the average inventory level. An unreliable bottleneck production-storage system with random disruptions and positive set-up cost is studied in Moinzadeh and Aggarwal (1997), where properties of the policy parameters minimizing expected total cost are developed. Further, Parlar (1997) considers a continuous-review inventory problem with random demand and random lead time where supplier availability is modelled as a semi-Markov process. Using renewal reward theory he constructs the average cost function of the underlying problem. Mohebbi and Hao (2006) consider a continuous-review inventory system with Erlang-distributed lead times and lost sales.

Parlar and Perry (1996) and Gürler and Parlar (1997) extend the research on supply disruption to the multiple supplier case to study the benefits of multiple sourcing. The buying firm faces a deterministic demand rate and sources from cost-identical, infinite-capacity suppliers. In Parlar and Perry (1996) the ON (available) and OFF (unavailable) periods are exponentially distributed. They propose a suboptimal ordering policy in the two supplier case which is solved numerically and show that as the number of suppliers becomes large, the objective function of the multiple supplier problem reduces to that of the classical EOQ model. More general repair and failure processes are considered in Gürler and Parlar (1997). They numerically evaluate a cost expression considering the case of Erlang failure times and general repair times.

Random yield

Single- and multi-period discrete time models on the optimal order quantity in the presence of two uncertain suppliers are examined in Anupindi and Akella (1993)) combining stochastic demand and supply uncertainty under various stochastic yield assumptions. They show that the optimal policy depending on the current inventory has three regions: (i) order from both suppliers when inventory is low, (ii) order only from the cheaper supplier when inventory is moderate and (iii) order nothing when inventory is high. Agrawal and Nahmias (1997) examine a one-period multiple supplier framework including supply risk and fixed supplier costs. They determine the optimal number of suppliers and optimal lot sizes in the presence of yield uncertainty for identical as well as nonidentical suppliers concluding that small orders from multiple sources can reduce yield uncertainty and fixed cost provide a penalty for having too many suppliers. Babich et al. (2012) extend the analysis of the identical-supplier case of Agrawal and Nahmias (1997) by adding financial decisions and financial constraints of the manufacturer affecting the optimal number of suppliers. They consider the joint procurement and financing decisions with either an uncertain demand or an uncertain supply. They observe that the optimal number of sources may increase when supplier costs increase. Federgruen and Yang (2008) investigate a newsvendor framework procuring from multiple sources, where each source faces a random yield factor and some fixed cost. They consider a service constraint model and a total cost model. In the service constraint model the random demand has to be covered by the available supply with a given probability and in the total cost model, orders are allocated by minimizing procurement, inventory and shortage costs. Federgruen and Yang (2009) also consider multiple unreliable suppliers analyzing two planning models, a service constraint model and a total cost model.

Chopra et al. (2007) investigate a system with both disruption risk and random yield showing the importance of recognizing and decoupling disruption and yield risk in terms of mitigation strategies, in a single-period model. They consider the backup strategy with a primary unreliable supplier with yield variability and probability of complete failure of an order under deterministic demand. Schmitt and Tomlin (2012) extend their setting and investigate the infinite-horizon case. Giri (2011) analyses the model settings of Chopra et al. (2007) from a risk-averse retailer point of view. He considers an inventory model with a primary, cheap but unreliable supplier and a perfectly reliable but more expensive supplier. He shows that the order quantities from the primary supplier are lower in the context of a risk-averse retailer than in the risk-neutral case.

The impact of supplier reliability forecasting using Bayesian updating and the decision whether to use dual sourcing or single sourcing is discussed in Tomlin (2009b). The suppliers face Bernoulli yield and it is assumed that supplier learning (updating) is only used for one supplier and that the reliability of the other supplier is known. The author finds that increasing reliability forecast uncertainty increases the attractiveness of the supplier but reduces the buyer's willingness to protect against future supply disruptions.

Random capacities

Ciarallo et al. (1994) discuss optimal ordering policies with random capacities in a single product production model in the single-period, multi-period and infinite-horizon setting. An EOQ model including multiple suppliers having random capacities leading to uncertain yield in orders is analyzed in Erdem et al. (2006). They discuss the effect of diversification under the assumption of identical suppliers. The optimal control of an assembly system when component suppliers have random capacities is studied in Bollapragada et al. (2004).

Dada et al. (2007) consider the problem of a newsvendor that is served by multiple suppliers differing in cost and reliability. The framework features both supply and demand uncertainty for a single demand season. The suppliers are either unreliable and supply strictly less than the required amount with some probability or perfectly reliable. They conclude that from the buyer's perspective the quantity ordered is higher than in the standard newsvendor setting where supply is certain and that the size of the selected supplier's order is dependent on whether he is reliable or not.

In Wang et al. (2010) the authors propose a model of process improvement in which improvement efforts increase the supplier reliability in the case of single and dual sourcing. They consider both random capacity and random yield types of uncertain supply and show that for random capacity, improvement is preferred over dual sourcing when the cost difference of the suppliers increases.

Stochastic lead times

There are various works that focus on models dealing with uncertain lead times. In general, however, the exact analysis of multiple suppliers with stochastic lead times is intractable and exact analysis can only be obtained for special cases (Tang (2006)).

The advantage that order splitting reduces the variability of the item arrivals was first presented in Sculli and Wu (1981). They provide tables to determine the minimum size of replenishment orders sourcing from two suppliers in the presence of normally distributed demand. Ryu and Lee (2003) consider a dual sourcing model with exponentially distributed lead times and constant demand to investigate the value of lead time reductions.

Abginehchi and Larsen (2012) study a lost sales inventory system with two non-identical suppliers assuming Poisson demand, Erlang distributed replenishment lead times and no more than one outstanding order for each supplier. Their problem is modeled as a semi-Markov decision process where the decision maker decides to use dual sourcing with order splitting, dual sourcing with emergency order, or single sourcing.

Song et al. (2014) consider a manufacturing supply chain with multiple suppliers in the presence of uncertain supplies, stochastic production lead times and random demand subject to supply and production capacity constraints focusing on the supplier management such as supply base reduction and supplier differentiation. They conclude that increasing supply base, increasing supplier capacity, shortening material delivery time and improving reliability benefits the manufacturer.

For a detailed analysis of models that deal with lead time uncertainty see Zipkin (2000).

Financial default/Supplier cost uncertainty

Financial instability of suppliers and the consequences of supplier defaults, insolvencies, or bankruptcies in a one-period model is discussed in Babich et al. (2007). They focus on the effects of supplier default risk including correlation with competing risky suppliers being the leaders in a Stackelberg game with the retailer and predict an advantage of competition and diversification for the retailer as the number of suppliers increases. Wan and Beil (2009) analyze a supply base diversification problem faced by a buyer who periodically holds auctions in the presence of supply cost shocks and show how supplier competition can be affected by correlations across suppliers' cost shocks. A study based on empirical data from the automotive industry of supplier default dependency as consequences of supplier bankruptcies is given in Wagner et al. (2009) demonstrating that default dependencies among suppliers often exit.

Babich (2010) discusses supply disruptions caused by supplier bankruptcy which can be controlled by financial investments by the manufacturer using a dynamic, stochastic, periodic-review model analyzing the optimal subsidy and ordering decisions. Swinney and Netessine (2009) analyze a single supplier and multiple supplier Stackelberg game with stochastic production cost where the bargaining power lies with the buyer demonstrating the value of long-term contracts in buyer-supplier relationships under supplier default. Li (2009) studies the supply base design of a buyer sourcing from two potential suppliers that can invest in supplier capacities under supply cost and demand uncertainties concluding that the buyer should engage suppliers in close competition if production cost show high uncertainties.

2.2.2 Buyer-supplier relationship and learning curve

The management of buyer-supplier relationships improves supply chain performance. Maloni and Benton (1997) provide an extensive review on supply chain partnerships. A conceptual framework for analyzing and managing supplier relationships can be found in Tang (1999). Cannon and Homburg (2001) develop a marketing study on the importance of a supplier-buyer relationship finding that collaboration reduces purchasing cost. Humphreys et al. (2004) analyze the role of supplier development from the perspective of the buying firm. Their empirical results show that a successful supplier development, trust, and communication improve the buyer-supplier performance.

One possibility to model the dynamic cost impact of supplier relationships is the use of the learning curve. Most organizations learn and improve over time as complex production processes leave room for the suppliers to optimize their production processes. As they perform a task over and over, they learn how to perform the task more efficiently resulting a cost reduction in relation to cumulative production or experience. The phenomenon was first reported by Wright (1936) who observed that the number of direct labor hours required to produce one unit of output decreases uniformly as the produced unit of output doubles and hence, gives a simple formulation of the learning curve, the power form. The learning curve describes the relationship between the cost of producing an item and the firm's experience in producing that item (Zangwill and Kantor (1998)). Many functional forms of the learning curve have been proposed (e.g., Adler and Clark (1991), Zangwill and Kantor (1998), Zangwill and Kantor (2000)). For a comparative analysis of different learning curves, see Plaza et al. (2010). A review of approaches dealing with the effect of learning in the EMQ/EOQ setting is given in Jaber and Bonney (1999). A discussion on the learning curve assuming imperfect production processes is presented in Jaber and Guiffrida (2004) developing a learning curve which is the sum of two learning curves described by the production of non-defective and defective units respectively.

Empirical analysis of the learning phenomenon are examined e.g. in Hirsch (1952) who analyzed data of a large US machine tool manufacturer, Baloff (1971) discussing data from the apparel and from the automobile industry and Plaza et al. (2010) with data from three manufacturing companies producing vegetable oils and meals, providing standards and certifications and producing spice ingredients, respectively.

Zangwill and Kantor (1998) focus not only on learning-by-doing but also on learning through continuous improvement efforts. In Zangwill and Kantor (2000) they extend their work by a framework that gives information on whether improvement efforts are successful or not.

The relationship between process improvement and the learning curve focusing on a single sourcing strategy is discussed in Carrillo and Gaimon (2000). Their dynamic model considers short-term loss due to a disruption when a new process is implemented and the long-term gain in effective capacity due to improvement because of the process change. They consider two types of knowledge - one resulting from investment and the other as a by-product of the process change - and study the firm's optimal process change strategy. Elmaghraby and Oh (2004) study the optimal design of a contract in a two period setting where suppliers experience learning by doing with a cost reduction in the second period. They find that a buyer is often better off to open competition running sequential auctions than to contract with a single supplier. Li and Debo (2009) study the benefit of second sourcing (option to source from a new supplier in future) relative to sole sourcing (single sourcing over the entire horizon with a supplier that may change future cost due to learning) in the presence of demand uncertainty. Gray et al. (2009) discuss in a two-period game how a manufacturer's production outsourcing decision is affected by the contract type and production cost reduction through learning-by-doing. They find that simultaneous outsourcing and in-house production can be optimal, as well as that dynamic outsourcing, where the strategy is changed from one period to the next, can be an optimal strategy. Xiao and Gaimon (2013) analyze the effect of volume-based learning for both the buyer and the supplier in a two-period Stackelberg game with a supplier and buyer. They show that even if marginal cost of outsourcing is less than that of in-house production, partial outsourcing is optimal when its future value is sufficient. Also, they provide insights into how this future value is affecting the buyer's outsourcing level.

In summary, existing research dealing with supply disruptions differ from each other in terms of the definition of uncertain supply and the individual management strategies to cope with these uncertainties. However, they all have in common that the question how to deal with the disruption risk and, hence, how to design effective strategies and profitable relationships is an important question for supply chain management.

Our modeling approaches that characterize the optimal sourcing strategies (single sourcing versus dual sourcing) position this research among the existing literature reviewed in this Chapter. First, we study the impact of ON/OFF supply disruptions on the optimal dual sourcing ordering policy under the complicating assumptions of stochastic demand and stochastic lead times. Afterwards, we characterize the buyer-supplier relationship when supply cost follow the learning curve, hence, integrating supplier learning to supply disruption risk management.

Chapter 3

A Multiple Sourcing Inventory Model under Disruption Risk

In this Chapter, a buyer that operates an infinite horizon, continuous review inventory system is considered. The buyer can source from two suppliers in the presence of uncertain demand, uncertain lead times, and temporary supply disruptions affecting the status of the suppliers.

3.1 Introduction

For successful supply chain management, a buyer has to consider that suppliers may not always be available. Temporary supply interruption can occur due to machine breakdowns, material shortages, unexpected incidents, and labor strikes. As these disruptions can have a severe impact on the supply process, a buyer may source from more than one supplier to protect against supply risk. There are numerous practical examples of supply disruption where a dual sourcing strategy resulted in high cost savings. E.g. the Nokia-Ericsson case in 2000, where Nokia managed to source form alternative suppliers right after the fire incident disrupted the supplying plant and minimized the negative impact of that disruption while Ericsson, without alternative suppliers, lost $400 million. This real life example shows that buyers can reduce the risk of supply shortfalls by sourcing from multiple suppliers when the supply process is subject to disruptions.

Our approach incorporates a multiple sourcing inventory system with stochastic demand, stochastic lead times and suppliers subject to disruptions, varying in cost, speed, and reliability. In order to compute the optimal decisions regarding supplier selection and reorder quantities for each state of the system, we formulate a semi-Markov decision process (SMDP) under Poisson demand, exponentially distributed lead times, and exponentially distributed periods where a supplier is available (ON) and unavailable (OFF). A state consists of the buyer's inventory level, the outstanding orders, and the availability of the suppliers. The mathematical problem is formulated for a general multiple supplier case with K potential suppliers. As suppliers typically differ in service and cost, we investigate the optimal sourcing strategies depending on supplier characteristics like cheap/expensive, fast/slow, and reliable/unreliable. In an inventory model, unfilled demands due to stock-outs are typically considered either backordered or lost, hence, we discuss both options. In the case of backorders, the customers arriving at the system where no inventory is on hand accept this situation and wait for the fulfillment of the demand. In the case of lost sales, the customers arriving at the system when out of stock are lost.

The remainder of this Chapter is organized as follows. In Section 3.2 we give a detailed description of the model assumptions. Section 3.3 deals with the semi-Markov decision process (SMDP) formulation. In Section 3.4 we focus on using the model to discuss the optimal sourcing strategy dependent on the states of the inventory system for the dual sourcing case and to study the behavior of the model. Specifically, in that Section, we focus on the following research questions:

- What is the structure of the optimal sourcing and order policy under Poisson demand, exponentially distributed lead times, and exponentially distributed ON and OFF times of the suppliers?

- What is the influence of supplier characteristics cost, speed, and availability on the optimal sourcing and order policy?

- What is the benefit of dual sourcing compared to single sourcing?

- What is the value of having full information about the supplier status?

- How do simple policies and a simple heuristic perform compared to the optimized dual souring policy?

- What is the impact on the model performance of assuming more general distributed lead times and ON and OFF times?

- What is the impact of the optimal policy when considering a more dramatic supplier breakdown scenario?

In Section 3.5 we summarize the results and give concluding remarks.

3.2 Problem description

We consider a single item inventory system where a buyer facing stochastic demand can source from $k \in K$ potential suppliers which are subject to temporary disruptions. The availability status of a supplier is subject to changes and the respective times are called ON and OFF periods. The lengths of these periods are exponentially distributed with mean $1/\mu_{ON}^k$ and $1/\mu_{OFF}^k$, respectively (see Figure 3.1). The set of suppliers is the union of the set of available and unavailable suppliers, $K^{ON} \cup K^{OFF} = K$. The availability of a supplier k the fraction of time where the supplier is operating without any disruptions, A_k is defined as $A_k = \mu_{OFF}^k/(\mu_{OFF}^k + \mu_{ON}^k)$.

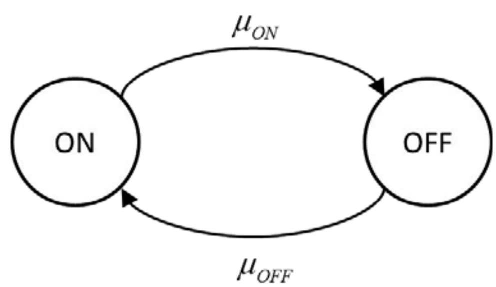


Figure 3.1: Supplier status.

If a supplier is ON and fully available, the buyer can order any desired quantity from the supplier without capacity restrictions. Whenever a supplier switches to OFF, this supplier is interrupted and no orders can be placed. Nevertheless, the orders on the way to the buyer, placed prior to a disruption, are not affected and will arrive after the corresponding lead time of that supplier. The replenishment lead times are exponentially distributed with a mean $1/\mu_L^k$, $k \in K$. The lead times can be interpreted as the time required to process an item. Each item is handled separately and thus the lead times are independently and identically random variables which also implies order crossovers.

Customers arrive at the buyer according to a Poisson-process with rate λ. Demands are satisfied immediately if the buyer has physical inventory on hand. We consider a lost sales model and a backorder model. In the lost sales case, unsatisfied demand is lost and subject to a penalty cost p. In the backorder case, unsatisfied demand is backordered and subject to a backorder cost b per unit per time unit until the backorder is satisfied. Further, inventories are subject to holding cost h (independent of past procurement cost) per unit per time unit. The unit procurement costs are c_k, $k \in K$. The problem framework under investigation is illustrated in Figure 3.2 and a complete list of the notation is summarized in Table 3.1.

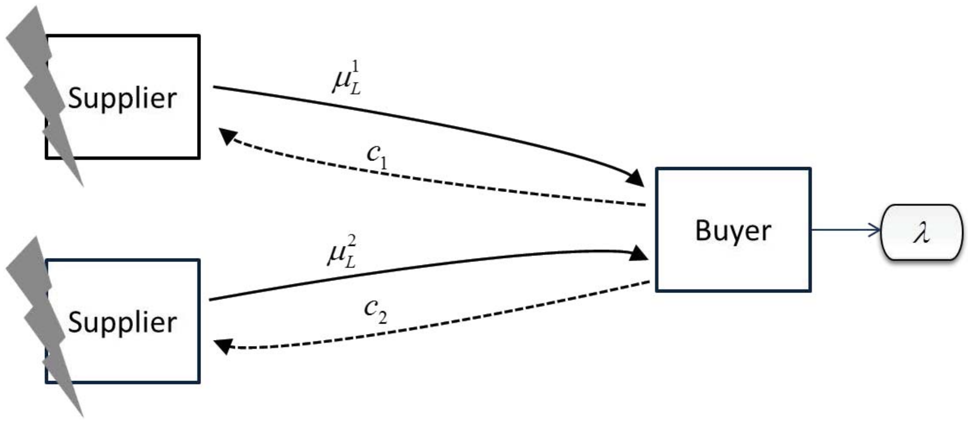


Figure 3.2: Framework for two potential suppliers $K = 2$.

K	Set of potential suppliers
K^{ON}	Set of suppliers currently ON
K^{OFF}	Set of suppliers currently OFF
$1/\mu_{ON}^k$	Expected ON period of supplier k
$1/\mu_{OFF}^k$	Expected OFF period of supplier k
$1/\mu_L^k$	Expected lead time of supplier k
λ	Customer arrival rate
p	Penalty cost per unit of lost customer
b	Backorder cost per unit of customer and per time unit
h	Holding cost
c_k	Unit procurement cost of supplier k
s	Buyer's net inventory = units on hand - backorders
n_k	Number of outstanding orders with supplier k
v_k	Status of supplier k, $v_k = 1$ if supplier $k \in K^{ON}$, $v_k = 0$ otherwise
$A(i)$	Set of all possible actions in state i
a	Action, set of order quantities assigned to suppliers
$p_{ij}(a)$	Transition probability from state i to j under action a
$\tau_{ij}(a)$	Expected time until next decision epoch under action a in state i
$c_i(a)$	Expected cost until next decision epoch under action a in state i

Table 3.1: List of notation.

3.3 Semi-Markov decision process formulation

To compute the optimal policy, in terms of the supplier selection and order allocation, the problem is formulated as a semi-Markov decision process. We will first describe the concept and the solution methods for solving these problems in general and apply it to the formulated problem thereafter. For further details on semi-Markov decision processes see e.g. Tijms (2003).

3.3.1 General semi-Markov decision process formulation

Following Puterman (2009) a Markov decision process can be described by the following five elements: Decision epochs, states, actions, transition probabilities, and rewards/cost. The term Markov property means that the set of actions, the transition probability and the cost functions depend only on the actual state of the system and the action selected in that state. If the number of points in time at which decisions are made is finite the problem is called finite horizon problem, otherwise it is called infinite horizon problem. When decision epochs occur at discrete time points the underlying problem is called discrete-time Markov decision process, which is discussed in the next Chapters for a finite horizon. When decision epochs occur randomly then we call the problem semi-Markov decision process.

This Chapter considers an infinite horizon dynamic system whose state is reviewed at random epochs. At each decision epoch a decision has to be made and cost result from a decision taken. For each state $i \in I$, where I denotes the set of all possible states there exit a set of possible actions $A(i)$. This controlled dynamic system is a semi-Markov decision process for $i \in I$ and $A(i)$ finite. The objective is to solve the long-run average cost per time unit which can be determined by $p_{ij}(a)$, the transition probability from state i to j if action a is chosen in state i, $\tau_i(a)$, the expected time until the next decision epoch if action a is chosen in state i and $c_i(a)$, the expected cost resulting from action a in state i until the next decision epoch. The optimal policy which minimizes the long-run average cost specifies the optimal decision to be taken in each state. Three methods can be applied to derive an optimal solution and the resulting minimal average cost per time unit C^*: Value-iteration, policy-iteration and linear programming.

The linear program to be solved for the semi-Markov decision model is:

$$C^* = \min \sum_{i \in I} \sum_{a \in A(i)} c_i(a) u_{ia} \tag{3.1}$$

$$\text{s.t} \quad \sum_{a \in A(j)} u_{ja} = \sum_{i \in I} \sum_{a \in A(i)} p_{ij}(a) u_{ia}, \tag{3.2}$$

$$\sum_{i \in I} \sum_{a \in A(i)} \tau_i(a) u_{ia} = 1, \quad u_{ia} \geq 0, a \in A(i), i \in I \tag{3.3}$$

The minimal average cost C^* per time unit results from the sum of the cost associated to the states $c_i(a)$ multiplied with u_{ia}, the long-run fractions of decision epochs at which the system is in a state i and action a is chosen. Constraint (3.2) states that the transitions from state j have to be equal to the transitions into state j for all $j \in J$. Additionally, the sum of all fractions has to be equal to one, which is reflected in the normalization constraint (3.3).

The value iteration algorithm for the semi-Markov decision process is as follows:

1. Choose a function $V_0(i)$ with $0 \leq V_0(i) \leq \min_a \{\frac{c_i(a)}{\tau_i(a)}\} \forall i \in I$. Choose a number τ with $0 \leq \tau \leq \min_{i,a} \tau_i(a)$ and a stopping condition $\epsilon > 0$. Let $n := 1$.

2. Compute the value function $V_n(i), i \in I$ from
$$V_n(i) = \min_{a \in A(i)} \{\frac{c_i(a)}{\tau_i(a)} + \frac{\tau}{\tau_i(a)} \sum_{j \in J} p_{ij}(a) V_{n-1}(j) + (1 - \frac{\tau}{\tau_i(a)}) V_{n-1}(i)\}. \tag{3.4}$$
Let $R(n)$ be a stationary policy whose actions minimize the right-hand side of (3.4).

3. Compute the bounds $m_n = \min_{j \in J}\{V_n(j) - V_{n-1}(j)\}$ and $M_n = \max_{j \in J}\{V_n(j) - V_{n-1}(j)\}$ and stop with policy $R(n)$ when $0 \leq (M_n - m_n) \leq \epsilon m_n$. Otherwise go to step 4.

4. $n := n + 1$ and go to step 2.

The value iteration algorithm stops after finitely many iterations where the average cost function $C(R(n))$ of the policy $R(n)$ satisfies $0 \leq (C(R(n)) - C^*)/C^* \leq \epsilon$ and $C^* = \lim_{n \to \infty} M_n = \lim_{n \to \infty} m_n$.

The third method, the policy-iteration, is not discussed here as we did not apply it to the formulated problem. A detailed description of this method is given in Tijms (2003).

3.3.2 Application to the formulated problem

State of the system and possible decisions

The state of the system is described by the buyer's net inventory s the number n_k of outstanding orders with each supplier, and the status v_k of the respective supplier. Thus, the state space of the semi-Markov decision process (SMDP) is defined as
$$I = \{(s, n_1, ..., n_K, v_1, ..., v_K)\}, \tag{3.5}$$
where $v_k = 1$ indicates that supplier k is ON and $v_k = 0$ that supplier k is OFF.

The states of the system are reviewed at random epochs when either a demand arrives or the availability status of a supplier changes. At these random epochs, decisions have to be made.

The buyer decides whether to place a new order, the order quantity, and which suppliers to assign the order to. Thus, the action a at a certain decision epoch is defined as $a = (a_1, ..., a_K)$, where a_k is the order quantity assigned to supplier k. Depending on the status of each supplier, only certain actions are feasible in a state. For each state $i = (s^i, n_1^i, ..., n_K^i, v_1^i, ..., v_K^i)$ the set of all possible actions is defined as $A(i) = \{a_k, k \in K : a_k = 0, k \in K^{OFF}; a_k \geq 0, k \in K^{ON}\}$. For states $i = (s^i, n_1^i, ..., n_K^i, 0, ..., 0)$ where all suppliers are OFF, the buyer cannot place a new order, and $A(i)$ is a K-dimensional vector of zeros.

State-transition probability

The state dynamics of the inventory system are specified by the state transition probabilities from state $i = (s^i, n_1^i, ..., n_K^i, v_1^i, ..., v_K^i)$ to state $j = (s^j, n_1^j, ..., n_K^j, v_1^j, ..., v_K^j)$, $p_{ij}(a)$ choosing action a in state i and the expected time until the next decision epoch in a state i, $\tau_i(a)$ choosing action a in state i for each state-action combination. Thus, when in state i action $a \in A(i)$ is chosen, the expected time until a new state is entered is

$$\tau_i(a) = \frac{1}{\lambda + \sum_{k \in K}(a_k + n_k)\mu_L^k + \sum_{k \in K}(1 - v_k)\mu_{OFF}^k + \sum_{k \in K} v_k \mu_{ON}^k}. \tag{3.6}$$

In the above expression the denominator represents the probability of a demand arrival, of a delivery of an item including new ordered items and of a status change in the current state, thus $\tau_i(a)$ stands for the mean time that the system remains unchanged. The probability that the state of the system switches from i to j at the next decision epoch if action a is chosen in the present state i is

$$p_{ij}(a) = \begin{cases} \lambda\tau_i(a), & s^i - 1 = s^j, n_k^i + a_k = n_k^j, k \in K^{ON} & \text{(a customer arrives)} \\ (n_k^i + a_k)\mu_L^k \tau_i(a), & s^i + 1 = s^j, (n_k^i + a_k) - 1 = n_k^j, k \in K & \text{(item from } k \text{ arrives)} \\ (1 - v_k^i)\mu_{OFF}^k \tau_i(a), & v_k^i + 1 = v_k^j & (k\text{: OFF} \to \text{ON}) \\ v_k^i \mu_{ON}^k \tau_i(a), & v_k^i - 1 = v_k^j & (k\text{: ON} \to \text{OFF}) \\ 0, & \text{otherwise} & \end{cases}$$

Cost

The total cost of the buyer consists of the holding cost per time unit, the procurement cost per unit, and backorder cost per unit per time unit or lost sale penalty costs per unit, respectively. We define $s^+ = \max\{s, 0\}$, $s^- = \max\{-s, 0\}$ and $s^0 = \delta_{s,0}$, where the function $\delta_{i,j} = 1$ if i is equal to j and $\delta_{i,j} = 0$ otherwise. For the backorder model the cost incurred in each state i until the next decision epoch if action a is chosen in state i is

$$c_i(a) = \sum_{k \in K} a_k c_k + h\tau_i(a)s^+ + b\tau_i(a)s^-. \tag{3.7}$$

Inventory levels do not change between decision epochs. Therefore, the holding cost until the next decision epoch is $h\tau_i(a)s^+$ and backorder cost is $b\tau_i(a)s^-$.

In the lost sales model, cost in state i for decision a is

$$c_i(a) = \sum_{k\in K} a_k c_k + h\tau_i(a)s^+ + \lambda p\tau_i(a)s^0, \tag{3.8}$$

where $\lambda\tau_i(a)s^0$ is the probability that the next event is a demand, in all states where the buyer is out of stock ($s = 0$).

Method for the optimal policy

We applied linear programming and value iteration to derive the optimal long-run average cost of the semi-Markov decision process. However, as we found that linear programming is a very efficient method for small problems in terms of computation time we kept on linear programming for our numerical study.

3.4 Numerical results and discussion

We illustrate various effects on the optimal policy for the special case of a dual sourcing inventory model under supply disruptions. First, we give an example and analyze the optimal policy in detail when the suppliers differ in cost, lead time and availability and show the benefit of dual sourcing over single sourcing. Furthermore, we study the effects of incomplete information, where the decision maker has no information about a supplier becoming available or unavailable. We compare the performance of the optimal policy with a simpler policy, the order-up-to-S policy and develop a simple heuristic. Finally, we discuss the sensitivity of our model to more generally distributed lead times and ON and OFF times using simulation and compare our model to a more dramatic supply breakdown scenario.

To keep the state space finite, we assume that the inventory position cannot exceed a storage capacity S and that backorders are bounded by B. Customers arriving after the backorder capacity B is reached are lost and subject to a penalty cost p. However, if the bounds on the state space are chosen sufficiently large, there are no limitations on the optimal decisions. Under these assumptions, the state space is given by

$$I = \{(s, n_1, ..., n_K, v_1, ..., v_K) \mid -B \le s + \sum_{k\in K} n_k \le S\}.$$

The semi-Markov decision model is solved according to the linear programming formulation using the solver XPRESS-IVE. For computational tractability, the maximum bound on the state space is $B = S = 30$.

3.4.1 Example

Consider the following example with a set of potential suppliers $K = 2$, a demand rate $\lambda = 2$, and holding cost $h = 0.6$. Penalty cost p and backorder cost b, respectively are either low $p = 4$ and $b = 2$ or high $p = 8$ and $b = 4$. The buyer's supply base consists of one fast, reliable, and expensive supplier S1 and one slow, unreliable, but cheaper supplier S2. The cost of S1 is $c_1 = 2$ and the expected lead time $1/\mu_L^1 = 0.5$. Supplier 2 is 15% cheaper than S1 but also 50% slower than S1 on average, thus the cost of S2 are $c_2 = 1.7$ and the expected lead time $1/\mu_L^2 = 1$. Mean ON and OFF times of the reliable supplier S1 are $1/\mu_{OFF}^1 = 0.3$ and $1/\mu_{ON}^1 = 3$. Mean ON and OFF times of the unreliable supplier S2 are $1/\mu_{OFF}^2 = 1/\mu_{ON}^2 = 1$. The overall availabilities of the suppliers are $A_1 = 90\%$, $A_2 = 50\%$.

Table 3.2 summarizes the cost of the optimal dual sourcing policy for each option. Further, it shows the savings of the optimal dual sourcing policy when compared to the optimal single sourcing policy defined by

$$\text{Sav. } (\%) = \frac{\text{Avg.cost}_{\text{single}} - \text{Avg.cost}_{\text{dual}}}{\text{Avg.cost}_{\text{dual}}} \times 100$$

and the split of total demands that are lost, or assigned to S1 and S2, respectively. The savings of dual sourcing over single sourcing with S1 decrease when penalty costs are high ($p = 8$) as S1 has a high reliability of 90%. Savings over single sourcing with S2, however, increase for high penalty cost because S2 has a low reliability of only 50%. The results for the split of demands show that the percentage of demands triggering an order with the expensive but reliable supplier S1 is higher when penalty costs are high ($p = 8$) compared to the case when penalty cost are low ($p = 4$). To understand the behavior of the optimal policy, Figure 3.4.1 depicts the partition of the total cost as penalty costs and backorder cost, respectively change. The optimal policy in this case allocates more than 20% of the cost to holding cost in the lost sales model and less than 20% in the backorder model. The allocation of total procurement cost, in contrast, is less in the lost sales model than in the backorder model. This also explains the difference in the savings of dual sourcing compared to single sourcing. Due to the relatively long durations of the OFF periods of S2 ($1/\mu_{OFF}^2 = 1$ compared to $1/\mu_{OFF}^1 = 0.3$), more emergency orders are placed with the expensive supplier S1.

To analyze the structure of the optimal dual sourcing policy, Table 3.3 illustrates the optimal order quantities assigned to respective suppliers for the lost sales option and the backorder option ($p = 4$ and $b = 2$). For a given inventory level s outstanding orders with S1 and S2 n_1, n_2, and the status of the suppliers v_1, v_2, where v_k is either ON or OFF for $k = 1, 2$, the optimal order quantities assigned to supplier 1 ($a_1^* > 0$) and supplier 2 ($a_2^* > 0$) are summarized. For reasons of clarity and readability, only order decisions for $s > -2$ are presented in the backorder case. From the results we observe that the optimal policy is rather complex and not of a base-stock-type, even in states where there is only a single supplier available.

			Savings of dual sourcing (%)		Split of demands (%)		
		Avg. Cost	Supplier 1	Supplier 2	Lost	Supplier 1	Supplier 2
Lost sales	p=4	5.2	4.6	3.9	9.6	26.3	64.1
	p=8	5.6	3.9	11.7	3.3	32.6	64.1
Backorder	b=2	4.5	5.8	12.1	-	30.5	69.5
	b=4	4.8	5.3	19.3	-	34.5	65.5

Table 3.2: Avg. cost, savings of dual sourcing and split of demands of base case.

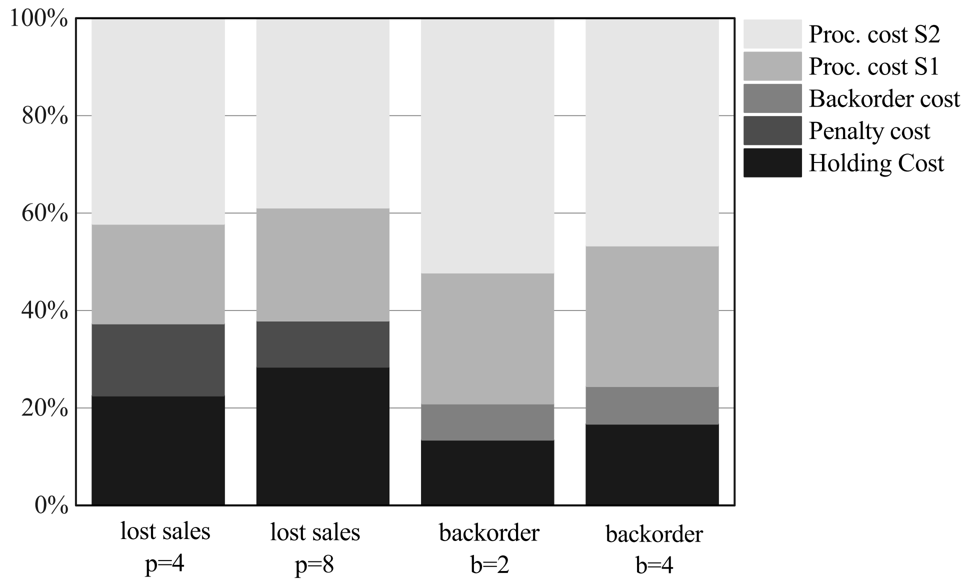


Figure 3.3: Cost partition base case $p = 4, 8$ (lost sales) and $b = 2, 4$ (backorder).

In general, three events trigger an order: i) a demand occurs, where a single unit is replenished, ii) if a supplier becomes unavailable, then it will be attractive to immediately replenish multiple units with the other remaining supplier to increase inventory and secure availability, iii) as soon as a supplier becomes available again, it will be wise to use this opportunity and improve the supply state (e.g. order cheap items or refill the pipeline for fast delivery). Further, the optimal order quantities depend not only on the current inventory level but also on the orders outstanding from the fast supplier and those outstanding from the slow supplier.

Lost sales $a^* > 0$							Backorder $a^* > 0$ and $s > -2$						
s	n_1	n_2	v_1	v_2	a_1^*	a_2^*	s	n_1	n_2	v_1	v_2	a_1^*	a_2^*
0	0	4	ON/OFF	ON	-	1	-1	0	4	ON/OFF	ON	-	1
0	1	3	ON/OFF	ON	-	1	-1	1	2	ON/OFF	ON	-	1
0	2	1	ON/OFF	ON	-	1	-1	2	1	ON/OFF	ON	-	1
0	3	0	ON/OFF	ON	-	1	-1	3	0	ON/OFF	ON	-	1
1	0	2	ON/OFF	ON	-	1	0	0	2	ON/OFF	ON	-	1
1	1	0	ON/OFF	ON	-	1	0	1	0	ON/OFF	ON	-	1
2	0	0	ON/OFF	ON	-	1	1	0	0	OFF	ON	-	1
0	0	3	ON/OFF	ON	-	2	-1	0	3	ON/OFF	ON	-	2
0	1	2	ON/OFF	ON	-	2	-1	1	1	ON/OFF	ON	-	2
0	2	0	ON/OFF	ON	-	2	-1	2	0	ON/OFF	ON	-	2
1	0	1	ON/OFF	ON	-	2	0	0	1	ON/OFF	ON	-	2
0	0	2	OFF	ON	-	3	-1	0	2	OFF	ON	-	3
0	1	1	ON/OFF	ON	-	3	-1	1	0	OFF	ON	-	3
1	0	0	OFF	ON	-	3	0	0	0	ON/OFF	ON	-	3
0	0	1	OFF	ON	-	4	-1	0	1	OFF	ON	-	4
0	1	0	OFF	ON	-	4	-1	0	0	OFF	ON	-	5
0	0	0	OFF	ON	-	5	-1	0	2	ON	OFF	1	-
0	0	3	ON	OFF	1	-	-1	0	3	ON	OFF	1	-
0	1	1	ON	OFF	1	-	-1	1	1	ON	OFF	1	-
0	1	2	ON	OFF	1	-	-1	2	0	ON	OFF	1	-
0	2	0	ON	OFF	1	-	-1	0	1	ON	OFF	2	-
1	0	0	ON	OFF	1	-	-1	1	0	ON	OFF	2	-
0	0	1	ON	OFF	2	-	-1	0	0	ON	OFF	3	-
0	0	2	ON	OFF	2	-							
0	1	0	ON	OFF	2	-							
0	0	0	ON	OFF	3	-							

Table 3.3: Optimal order policy base case (lost sales case $p = 4$ and backorder case b =2).

3.4.2 Numerical design

The following experiment is designed to observe the effects of the impact of supplier characteristics (cost, speed and availability). We fix the unit procurement cost and the expected lead time of supplier 1 and vary supplier 2's unit procurement cost $c_2 = c_1 - \Delta c$ and its expected lead time with $1/\mu_L^2 = 1/\mu_L^1 + \Delta\mu_L$ to have a cheaper but slower second supplier. For supplier 1 $c_1 = 2$ and $1/\mu_L^1 = 0.5$, and for supplier 2 $\Delta c \in \{0\%, 10\%, 25\%\}$ and $\Delta\mu_L \in \{0\%, 50\%\}$. The availability for supplier 1 is either high or low $A_1 \in \{90\%, 50\%\}$ and $A_2 \in 50\%$. The mean OFF periods are either both short, short and long, or both long $(1/\mu_{OFF}^1, 1/\mu_{OFF}^2) \in (1/3, 1/3), (1/3, 1), (1, 1)\}$.

The inventory holding cost are fixed at $h = 0.6$, penalty costs are either low $p = 4$ or high $p = 8$, and backorder cost are $b = 2$ or $b = 4$. Demand is either low $\lambda = 4$ or high $\lambda = 10$.

The results of all parameter combinations are presented in Tables 3.7-3.10 at the end of this Chapter. The average cost of dual sourcing is compared to the average cost of single sourcing, the cost of dual sourcing under incomplete information and to the cost of an order-up-to-S policy and expressed in the percentage savings of dual sourcing compared to the other policies. The average results of the lost sales model and the backorder model are summarized in Table 3.4.

		$p = 4, b = 2$							$p = 8, b = 4$						
Model	λ	*Avg. cost*	*Sav. S1*	*Sav. S2*	*Sav. Info*	*Sav. S fix*	*Sav. S var*	*Sav. Heur*	*Avg. cost*	*Sav. S1*	*Sav. S2*	*Sav. Info*	*Sav. S fix*	*Sav. S var*	*Sav. Heur*
Lost sales	4	9.44	9.0	4.9	1.2	3.5	1.9	1.0	10.04	11.8	12.8	2.1	4.2	2.8	2.8
	10	21.66	11.1	7.0	0.7	3.9	1.9	1.7	22.64	15.2	17.1	1.1	4.4	2.7	2.0
Backorder	4	8.53	12.3	10.	1.7	4.9	2.7	1.4	8.89	14.3	16.5	2.2	5.1	3.3	1.5
	10	20.19	13.7	9.9	0.8	4.5	2.5	1.8	20.78	16.4	18.7	1.1	5.0	2.8	1.9

Table 3.4: Average results of lost sales model and backorder model.

3.4.3 Dual sourcing vs. single sourcing

In Tables 3.7-3.10 we observe that there are significant savings of dual sourcing compared to single sourcing. The savings of a dual sourcing policy increase as supplier availability decreases because penalty cost and backorder cost increase. The results indicate that, apart from a difference in speed and cost of the suppliers, the availability and the length of the disruption of the supplier also influence the optimal dual sourcing policy. Thus when the disruption periods of supplier 2 are short and thus the mean ON periods are long ($1/\mu^2_{OFF} = 1/3$), almost all demands trigger orders with supplier 2 if he has a cost advantage. The average savings of dual sourcing compared to single sourcing are higher in the backorder case than in the lost sales case.

To investigate the trade-off between dual sourcing and single sourcing when suppliers differ in cost and availability, we analyze the extreme case where supplier 1 is fully available ($A_1 = 100\%$) and the costs of supplier 2 are varied with and its availability ranges from 50% to 100%. We look at the percentage of demands that trigger an order with supplier 2 and are not lost. Figure 3.4 shows the share of S2 for various cost differences and availabilities. The results indicate that the share of S2 rises fast, even with a small cost difference and a low availability of S2. Comparing the result to the case where S2 has a long mean OFF time (left) with the results where S2 has short mean OFF times (right), we see that even with a low availability of S2 (e.g.: $A_2 = 50\%$) the share of S2 is 100% when the cost difference is 50% and disruption periods are short, but only around 80% when disruption periods are long.

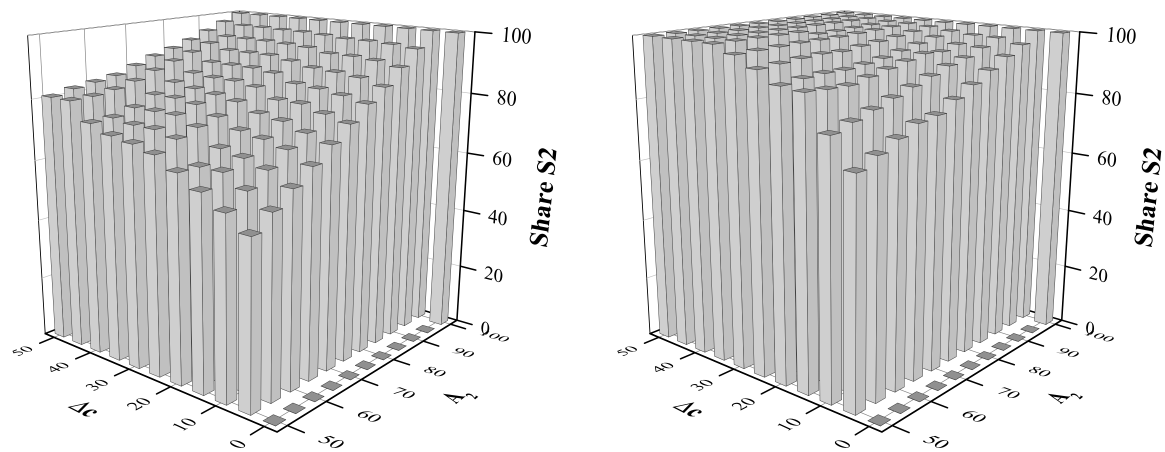


Figure 3.4: Share of supplier 2 ($\Delta\mu = 0\%$) for cost differences and availabilities of supplier 2 $c_1 = 2, 1/\mu_L^1 = 0.5, p = 04, \lambda = 4, A_1 = 100\%$, where $(1/\mu_{OFF}^1, 1/\mu_{OFF}^2) = (1/3, 1)$(left) and $(1/\mu_{OFF}^1, 1/\mu_{OFF}^2) = (1/3, 1/3)$ (right).

3.4.4 The value of full information

So far we assumed that the buyer has full information about the supplier status. The buyer is informed when a supplier becomes unavailable or again available. He is also able to reorder from an available supplier when the other supplier becomes unavailable to build up inventory before this supplier becomes unavailable as well or to reorder from a cheap supplier if he switches from the OFF to the ON status again. If the buyer has no information about a supplier becoming available or unavailable the model has to be modified as no orders can be placed when there is a change in the status of a supplier. As a consequence, the buyer's order will only be triggered by an incoming demand. In terms of the model formulation, we have to reduce the action space. Therefore, in the modified SMDP, no decision can be made whenever there is a change in the status of a supplier and decisions are only made whenever a demand arrives. Tables 3.7-3.10 show the savings of full information compared to incomplete information. The savings increase with penalty cost and average disruption lengths and decrease as supplier availability increases. The value of full information is high if supplier availabilities are low and ON and OFF periods change frequently.

3.4.5 Comparison with simple policies and a heuristic

The optimal policy as shown in Section 3.4.1 has a complex form. Therefore, we compare the optimal policy with an optimized order-up-to-S policy. This means that the buyer reorders a single unit with every incoming demand with the optimal decision rule to minimize the cost, given that a supplier is available and orders up to S whenever both suppliers have been unavailable, and one supplier became available again. First, the optimal policy is compared to an order-up-to-S policy where the order-up-to level S is identical for all supplier status combinations meaning that $S = S_{ON,ON} = S_{ON,OFF} = S_{OFF,ON}$, where $S_{i,j}$ is the order-up-to level where supplier 1 has status i and supplier 2 status j. Secondly, as a fixed S may not be favorable when the suppliers differ in speed, cost and availability, the optimal policy is compared to an order-up-to-S policy with an individual S for each supply base for each supplier status combination - meaning that $S_{ON,ON}, S_{ON,OFF}$ and $S_{OFF,ON}$ are optimized individually. The results in Tables 3.7-3.10 indicate that the difference between an order-up-to-S policy with a fixed S and an order-up-to-S policy with individual S is noticeable especially when the suppliers are not identical.

For practical purposes, a simple heuristic to easily calculate the base-dependent order-up-to-levels is desired. We follow an idea in Kiesmüller and Minner (2003) and compute the order-up-to levels S for each supply base according to the critical ratio of the newsvendor model. S satisfies

$$F_{crt}(S) = \frac{c_u}{c_u + c_o}, \tag{3.9}$$

where $F_{crt}(S)$ in (3.9) is the demand distribution over the critical (uncertain) replenishment lead time crt. To calculate the base-dependent order-up-to levels $S_{ON,ON}$, $S_{ON,OFF}$ and $S_{OFF,ON}$ we first discuss how we set overage and underage cost, and secondly, how we set the critical replenishment time, depending on the actual supply base.

If only the fast but expensive supplier S1 is available, the underage cost are the penalty cost and backorder cost $c_u = p$ and b, respectively and the overage cost consist of two parts, the inventory holding cost and the extra cost for using the fast supplier $c_o = h + (c_1 - c_2)$. For the order-up-to level of the cheap but slow supplier S2, the underage cost are again $c_u = p$ and b, respectively and the overage cost are the per unit holding cost $c_o = h$. We present two options for setting the critical replenishment time, one for the lost sales case and the other for the backorder case. In the lost sales case, we set the critical replenishment time equal to the sum of the mean lead time of the respective supplier and the mean OFF time if he is unavailable. This is for S1: $crt^1 = 1/\mu_L^1 + (1 - A_1) \times 1/\mu_{OFF}^1$, and for S2: $crt^2 = 1/\mu_L^2 + (1 - A_2) \times 1/\mu_{OFF}^2$. When OFF periods of S2 are long, or S2 has a lead time disadvantage, the critical replenishment time of S2, crt^2 gets very long according to our formula, as the probability that S2 is unavailable is 50%. Hence, the resulting order-up-to level of the heuristic is high and indicates a small overestimation compared to the order-up-to level of the optimal order-up-to policy. Our numerical results show that an overestimation is more favorable than an underestimation in the lost sales case, especially when penalty costs are high and thus we choose this option. In the backorder case, however, the order-up-to levels generally are lower and this overestimation becomes unfavorable. To counteract this overestimation, we present a second option which we apply in the backorder case, where we extend the formulation of the critical replenishment time of S2 to $crt^2 = 1/\mu_L^2 \times A_2 + (1 - A_2) \times \min\{1/\mu_L^1 + 1/\mu_{OFF}^1, 1/\mu_L^2 + 1/\mu_{OFF}^2\}$. Hence, if S2 is not available, we consider the time until S1 gets available again and supplies and the time until S2 gets available again and supplies, and take the minimum of these times. The intuition is that if S2 is not available, there exist situations in our numerical design where the time until S1 returns and supplies is shorter than the time until S2 returns and supplies. Therefore, considering the minimum of the respective times reduces the critical replenishment time for that supply base, which improves the overall performance of the heuristic in the backorder case. Substituting overage and underage cost, and the critical replenishment times obtained above into the newsvendor formula leads to the following determination of the order up-to-levels for the lost sales case: $S_{ON,OFF} = F_{crt^1}^{-1}(\frac{p}{p+h+c_1-c_2})$ and $S_{OFF,ON} = F_{crt^2}^{-1}(\frac{p}{p+h})$, and for the backorder case: $S_{ON,OFF} = F_{crt^1}^{-1}(\frac{b}{b+h+c_1-c_2})$ and $S_{OFF,ON} = F_{crt^2}^{-1}(\frac{b}{b+h})$. If both suppliers are available, we use a simple structure: the order-up-to level is calculated according to the rule where only S2 is available, given that S2 is cheaper than S1 ($\Delta c > 0$) and according to the rule where only S1 is available, if S2 has no cost advantage ($\Delta c = 0$). As we can choose between two suppliers, the decision rule in this situation is to order from the cheap but slow supplier when the inventory position ($IP = s + n_1 + n_2$) is at least $S_{ON,OFF}$ and to order from the expensive but fast supplier if the inventory position is below $S_{ON,OFF}$. The performance of the heuristic is compared to the order-up-to-S policy with base-dependent levels and expressed in percentage cost difference

$$\text{Sav. Heur} = \frac{\text{Avg.cost}_{\text{heuristic}} - \text{Avg.cost}_{\text{uptoSvar}}}{\text{Avg.cost}_{\text{uptoSvar}}} \times 100$$

(see Table 3.7-3.10). The results show moderate cost increases and therefore a reasonable performance. The average deviation in the lost sales case is less than 1.9% and less than 1.7% in the backorder case.

3.4.6 More general distributions

As the assumptions of exponential ON and OFF times and exponential lead times are limiting, we test the performance of the model by simulating deterministic and gamma distributed lead times and ON and OFF times, respectively. In our analysis, we keep the mean lead times and ON and

OFF times fixed while changing the value of the shape parameter k of the gamma distribution. The values of the coefficient of variation considered are $cv = 1/\sqrt{k} = \{0, 0.2, 0.5, 2\}$. The results presented were obtained by simulating the inventory system with the different shape parameters k using the optimal policy obtained from the model with exponential ON and OFF and lead times with a relative precision of at least 0.05 at the 0.95 confidence interval (see (Law 2006)). The simulation results are summarized in Table 3.5 showing the cost difference in percentage of our model compared to the simulation result of more general distributions

$$\Delta = \frac{\text{Avg.cost}^{\text{sim}}_{\text{gamma}} - \text{Avg.cost}_{\text{dual,exp}}}{\text{Avg.cost}_{\text{dual,exp}}} \times 100,$$

where Δ^{ONOFF} (cost difference of the model compared to simulation result with gamma distributed ON and OFF times), Δ^{leadtime} (cost difference of our model compared to simulation result with gamma distributed lead times) and Δ^{both} (cost difference of our model compared to simulation result with both, gamma distributed lead times and ON and OFF times). The

	cv	Δ^{ONOFF}	Δ^{leadtime}	Δ^{both}
LS	0	-0.7	15.6	15.2
	0.2	-1.5	9.7	7.9
	0.5	-1.2	3.8	2.4
	2	3.5	-1.6	2.0
BO	0	-0.9	10.5	9.9
	0.2	-1.7	8.4	6.3
	0.5	-1.3	3.6	2.0
	2	15.0	-1.7	12.0

Table 3.5: Average cost difference (%) of our model compared to simulation results with gamma distributed lead times, and ON and OFF times for different coefficient of variations.

results indicate a significant sensitivity with respect to changes in the variability of the lead times and ON and OFF times. Changing the shape of the lead time and the ON and OFF time distributions has mixed impacts on the average cost. While the performance of the policy is not very sensitive with regard to the distribution of the ON and OFF times when these are deterministic ($cv = 0$) or the coefficient of variation is low, it is sensitive when lead times are deterministic or the coefficient of variations is low. A high variability in ON and OFF times leads to situations where the time during which both suppliers are unavailable are very long, especially when the mean OFF times are long. This leads to higher total backorder cost and penalty cost, respectively. A high variability in lead times, on the other hand, reduces backorder cost and penalty cost, respectively (see Thomas and Tyworth (2006)).

3.4.7 Comparison to a more dramatic supplier breakdown scenario

Finally, we analyze a model where a supply breakdown is not only affecting the buyer's order decision (he only can order from an available supplier), but also the orders outstanding. Therefore, if a supplier status changes from ON to OFF, the outstanding orders are lost. In terms of the model formulation, for states $i = (s^i, n_1^i, n_2^i, v_1^i, v_2^i)$ where the status v_k is OFF it implies that $n_k = 0$, $k = 1, 2$, and the probability that a supplier k switches from ON to OFF in state i and taking decision a is $p_{ij}(a)$ affecting the status of supplier k in state j, v_k^j and reducing the outstanding orders of supplier k in state j to zero. Thus, in a more dramatic supplier breakdown scenario the probability that the state of the system switches from i to j at the next decision epoch if action a is chosen in the present state i is

$$p_{ij}(a) = \begin{cases} \lambda\tau_i(a), & s^i - 1 = s^j, n_k^i + a_k = n_k^j, k \in K^{ON} \quad \text{(a customer arrives)} \\ (n_k^i + a_k)\mu_L^k\tau_i(a), & s^i + 1 = s^j, (n_k^i + a_k) - 1 = n_k^j, k \in K^{ON} \quad \text{(item from } k \text{ arrives)} \\ (1 - v_k^i)\mu_{OFF}^k\tau_i(a), & v_k^i + 1 = v_k^j \quad (k\text{: OFF} \to \text{ON}) \\ v_k^i\mu_{ON}^k\tau_i(a), & v_k^i - 1 = v_k^j, n_k^j + a_k^j = 0 \quad (k\text{: ON} \to \text{OFF}) \\ 0, & \text{otherwise} \end{cases}$$

The performance of this dramatic supplier breakdown scenario for the parameter values of the base example in Section 3.4.1 is shown in Table 3.6 and Figure 3.5. As the availability of supplier

			Savings of dual sourcing (%)		Split of demands (%)		
		Avg. Cost	Supplier 1	Supplier 2	Lost	Supplier 1	Supplier 2
Lost sales	p=4	6.04	0.0	29.9	12.9	87.1	0.0
	p=8	6.6	0.0	52.7	3.1	95.6	1.3
Backorder	b=2	5.53	0.0	70.0	-	100.0	0.0
	b=4	5.8	0.0	78.2	-	100.0	0.0

Table 3.6: Avg. cost, savings of dual sourcing and split of demands of a dramatic breakdown scenario, parameter values of base case.

2 is only 50%, the OFF times are rather long, and the lead time is short, it is too risky to place an order with the unreliable and slow supplier. In the lost sales case with high penalty cost ($p = 8$), it is optimal in a single state to reorder a single unit with S2 when S1 is unavailable, the buyer is out of stock and no orders are outstanding.

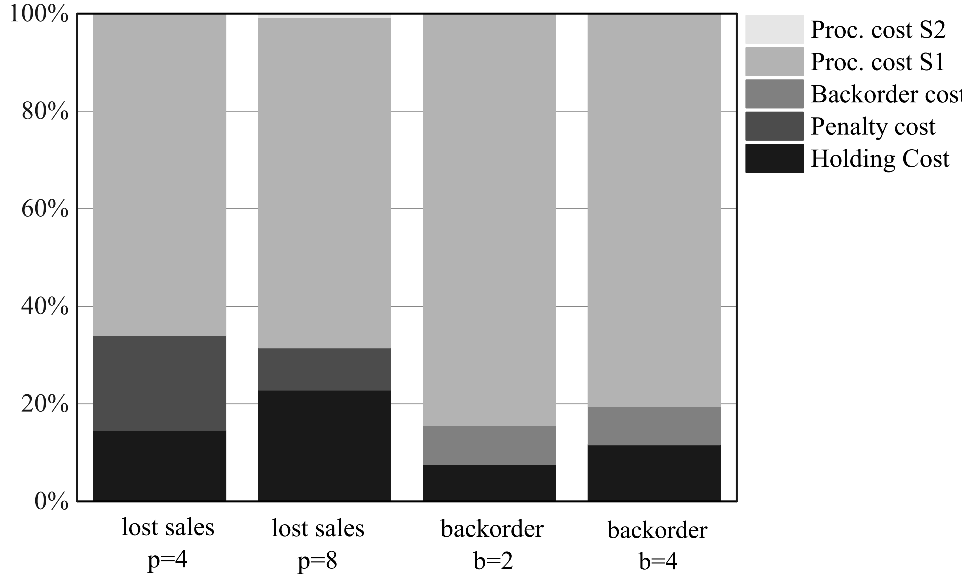


Figure 3.5: Cost partition of a dramatic breakdown scenario, parameter values of base case.

3.5 Conclusion

This Chapter analyzed an inventory system where a buyer orders from multiple suppliers with different cost and reliabilities which are subject to temporary failures. A semi-Markov decision process was formulated to optimize the order decisions of the buyer depending on the supplier's availabilities for a lost sales model and a backorder model, assuming Poisson demand, exponentially distributed lead times, and exponentially distributed ON and OFF periods. The solution of the semi-Markov decision process provides the optimal sourcing strategy depending on the actual inventory, the outstanding orders and the supplier status for both models. The numerical examples show that the optimal policy is rather complex and illustrate the benefit of dual sourcing compared to single sourcing when supply is subject to failure. In an illustrative example we analyzed the optimal policy for the lost sales and backorder setting. Computational experiments show that the benefit of dual sourcing over single sourcing is high especially when penalty costs are high and disruption periods are long. We analyze the value of having full information about the suppliers becoming available and unavailable, which is very important when the supplier availabilities are low and disruption periods are frequent. Further, we compared the optimal policy with a simple order-up-to-S policy where S may not be equal for all supply bases and depends on the supplier characteristics of the actual supply base. A simple heuristic is developed providing good results compared to the optimal order-up-to-S policy with individual S. Simulation results indicate that our model is sensitive with respect to more general ON and OFF and lead time distributions. Further, our model is compared to a more dramatic supply breakdown scenario.

Possible extensions to the current model would be to generalize to more general distributions and to relax the assumption of independent lead times and allow for batch ordering. Future research should include the investigation of the impact of different shapes of distribution for the demand, lead time and ON and OFF times.

3.6 Appendix: Tables

						$p = 4, b = 2$							$p = 8, b = 4$						
$\Delta\mu$	Δc	A_1	A_2	$1/\mu^1_{OFF}$	$1/\mu^2_{OFF}$	*Avg. cost*	*Sav. S1*	*Sav. S2*	*Sav. Info*	*Sav. S fix*	*Sav. S var*	*Sav. Heur*	*Avg. cost*	*Sav. S1*	*Sav. S2*	*Sav. Info*	*Sav. S fix*	*Sav. S var*	*Sav. Heur*
0	0	0.9	0.5	1/3	1/3	9.91	0.7	3.3	0.3	1.8	1.7	0.1	10.39	1.5	6.2	0.5	3.1	2.8	8.3
				1	1	9.96	1.8	10.1	0.4	1.8	1.7	0.1	10.52	5.4	23.3	0.9	2.8	2.6	0.6
				1/3	1	9.93	0.5	10.5	0.3	1.8	1.7	0.1	10.42	1.3	24.4	0.6	2.9	2.7	7.6
		0.5	0.5	1/3	1/3	9.98	2.6	2.6	1.4	1.7	1.5	0.2	10.52	5.0	5.0	2.6	2.2	2.2	1.6
				1	1	10.19	7.6	7.6	1.9	1.2	1.1	2.3	11.06	17.2	17.2	40	1.7	1.7	0.8
				1/3	1	10.03	2.1	9.4	1.8	1.6	1.3	1.1	10.62	3.9	22	3.6	2.0	2.0	1.5
	10	0.9	0.5	1/3	1/3	9.40	6.2	1.2	0.8	3.4	1.8	0.0	9.87	6.9	4.0	1.0	4.3	2.9	4.1
				1	1	9.54	6.4	8.1	0.4	2.4	1.5	0.9	10.09	9.9	21.3	0.9	3.1	2.5	0.7
				1/3	1	9.51	4.9	8.4	0.4	2.4	1.5	1.0	10.01	5.5	22.3	0.6	3.2	2.6	2.1
		0.5	0.5	1/3	1/3	9.42	8.6	0.9	1.8	2.7	1.4	0.0	9.95	11.0	3.2	3.1	3.1	2.3	2.6
				1	1	9.73	12.7	6.0	1.9	1.3	1.1	2.3	10.58	22.5	15.6	4.2	1.8	1.4	1.0
				1/3	1	9.60	6.6	7.4	1.7	1.6	1.2	0.9	10.20	8.3	20.0	3.6	1.9	1.9	0.9
	25	0.9	0.5	1/3	1/3	8.38	19.2	0.2	2.5	9.0	3.2	0.7	8.89	18.7	2.3	2.2	8.8	3.6	6.4
				1	1	8.79	15.4	5.9	0.8	4.3	2.0	0.1	9.35	18.5	19.1	1.2	4.6	2.8	0.7
				1/3	1	8.78	13.7	6.2	0.7	4.4	2.0	0.0	9.28	13.7	20.0	0.9	4.7	2.9	0.7
		0.5	0.5	1/3	1/3	8.38	22.2	0.1	3.4	6.9	2.4	1.3	8.94	23.6	1.8	4.3	6.6	2.8	5.9
				1	1	8.92	22.9	4.4	2.4	2.7	1.4	1.0	9.79	32.4	13.8	4.6	2.7	1.6	0.1
				1/3	1	8.84	15.8	5.4	2.1	3.1	1.5	0.1	9.46	16.7	17.7	4.0	2.9	2.0	0.0
50	0	0.9	0.5	1/3	1/3	9.94	0.4	4.5	0.2	1.9	1.8	0.7	10.43	1.1	7.2	0.4	3.0	3.0	10.1
				1	1	9.98	1.6	9.1	0.3	2.0	1.8	0.4	10.54	5.2	19.1	0.8	3.3	2.8	1.1
				1/3	1	9.95	0.3	9.5	0.2	1.9	1.9	0.8	10.45	0.9	20.1	0.5	2.9	2.9	9.1
		0.5	0.5	1/3	1/3	10.08	1.6	3.0	1.1	2.1	1.9	0.8	10.66	3.6	5.0	2.1	3.2	3.2	4.1
				1	1	10.26	6.8	6.1	1.4	1.8	1.6	2.6	11.10	16.8	13.1	3.1	2.9	2.3	1.0
				1/3	1	10.11	1.3	7.8	1.5	1.9	1.7	2.8	10.73	2.9	17.0	2.9	2.8	2.8	3.6
	10	0.9	0.5	1/3	1/3	9.55	4.5	1.1	0.4	3.7	2.3	0.1	10.08	4.7	3.3	0.5	4.6	3.4	4.4
				1	1	9.64	5.3	6.0	0.3	3.4	2.2	2.2	10.22	8.5	15.6	0.7	4.4	3.1	2.2
				1/3	1	9.62	3.8	6.2	0.3	3.3	2.3	2.2	10.15	3.9	16.3	0.5	4.3	3.2	3.9
		0.5	0.5	1/3	1/3	9.57	6.9	0.8	1.1	3.3	2.1	0.1	10.16	8.7	2.5	1.9	4.4	3.4	1.8
				1	1	9.78	12.1	4.4	1.4	2.7	1.7	2.9	10.60	22.3	11.4	3.1	3.5	2.3	1.3
				1/3	1	9.69	5.7	5.5	1.3	2.8	1.9	2.6	10.31	7.1	14.6	2.6	3.6	3.0	2.2
	25	0.9	0.5	1/3	1/3	8.53	17.0	0.2	1.5	9.5	3.1	0.9	9.08	16.2	1.9	1.3	10.0	4.6	3.8
				1	1	8.81	15.1	4.2	0.6	6.9	2.3	1.1	9.39	18.1	14.0	1.0	7.9	3.9	1.8
				1/3	1	8.80	13.4	4.4	0.6	6.9	2.4	1.4	9.33	13.1	14.6	0.7	7.8	4.0	1.9
		0.5	0.5	1/3	1/3	8.54	19.9	0.1	2.1	7.8	2.6	0.3	9.12	21.1	1.5	2.7	8.9	4.0	1.7
				1	1	8.91	23.1	3.1	1.8	5.4	1.9	1.4	9.71	33.5	10.2	3.5	5.7	2.7	0.2
				1/3	1	8.85	15.7	3.8	1.6	5.6	2.0	1.2	9.47	16.6	12.9	3.0	6.6	3.1	1.0

Table 3.7: Results of optimal dual sourcing policy and the savings compared to single sourcing, incomplete information, and order-up-to-S policy lost sales case $\lambda = 4$.

						$p=4,b=2$							$p=8,b=4$						
$\Delta\mu$	Δc	A_1	A_2	$1/\mu^1_{\text{OFF}}$	$1/\mu^2_{\text{OFF}}$	*Avg. cost*	*Sav. S1*	*Sav. S2*	*Sav. Info*	*Sav. S fix*	*Sav. S var*	*Sav. Heur*	*Avg. cost*	*Sav. S1*	*Sav. S2*	*Sav. Info*	*Sav. S fix*	*Sav. S var*	*Sav. Heur*
0	0	0.9	0.5	1/3	1/3	22.88	1.0	4.4	0.1	1.8	1.8	0.6	23.50	2.4	8.5	0.3	2.9	2.8	2.6
				1	1	23.03	2.6	14.1	0.2	1.8	1.8	0.3	23.92	7.7	30.8	0.5	2.8	2.6	0.4
				1/3	1	22.92	0.8	14.7	0.2	1.8	1.8	0.9	23.61	1.9	32.5	0.4	2.7	2.7	2.7
		0.5	0.5	1/3	1/3	23.06	3.6	3.6	0.8	1.6	1.6	0.2	23.85	6.9	6.9	1.5	2.2	2.2	0.0
				1	1	23.81	10.4	10.4	1.0	1.3	1.2	5.0	25.69	21.8	21.8	2.3	1.8	1.2	2.2
				1/3	1	23.23	2.8	13.1	1.0	1.6	1.4	2.6	24.21	5.3	29.2	2.1	2.1	1.8	1.8
	10	0.9	0.5	1/3	1/3	21.56	7.1	2.1	0.4	3.4	1.9	0.2	22.19	8.4	6.1	0.5	4.2	2.8	0.6
				1	1	21.97	7.5	12.2	0.2	2.2	1.8	2.9	22.85	12.7	28.9	0.6	3.1	2.6	2.3
				1/3	1	21.87	5.6	12.6	0.2	2.2	1.9	3.0	22.57	6.6	30.5	0.4	2.9	2.7	2.5
		0.5	0.5	1/3	1/3	21.65	10.3	1.7	1.0	2.8	1.5	0.5	22.46	13.6	4.9	1.7	3.0	2.1	0.6
				1	1	22.64	16.1	8.8	1.1	1.4	1.2	4.1	24.54	27.5	20.0	2.4	1.6	1.1	1.5
				1/3	1	22.16	7.8	11.2	1.0	1.5	1.4	2.9	23.18	10.0	27.1	2.1	1.9	1.8	2.3
	25	0.9	0.5	1/3	1/3	19.05	21.2	0.7	1.3	9.0	2.5	1.4	19.77	21.8	4.4	1.2	9.1	3.2	2.4
				1	1	20.17	17.1	9.8	0.4	3.9	1.8	1.7	21.07	22.2	26.7	0.7	4.4	2.7	0.9
				1/3	1	20.10	14.9	10.2	0.3	3.9	1.9	1.5	20.83	15.5	28.2	0.5	4.2	2.8	1.6
		0.5	0.5	1/3	1/3	19.08	25.2	0.5	1.8	7.1	1.9	3.1	19.94	27.9	3.5	2.4	6.8	2.3	3.0
				1	1	20.69	27.0	7.0	1.3	2.5	1.2	3.6	22.63	38.2	18.0	2.6	2.1	1.0	1.7
				1/3	1	20.33	17.4	8.9	1.2	2.6	1.4	1.8	21.44	19.0	24.6	2.3	2.3	1.7	0.8
50	0	0.9	0.5	1/3	1/3	22.92	0.8	4.3	0.1	2.1	2.1	1.5	23.56	2.2	7.1	0.3	3.3	3.2	4.9
				1	1	23.05	2.5	11.0	0.2	2.3	2.0	0.3	23.93	7.6	23.6	0.4	3.8	2.8	0.7
				1/3	1	22.95	0.6	11.5	0.1	2.0	2.0	1.5	23.66	1.7	25.0	0.3	3.0	3.0	4.2
		0.5	0.5	1/3	1/3	23.21	2.9	2.9	0.6	2.1	2.1	0.9	24.03	6.1	5.1	1.1	3.2	3.1	2.5
				1	1	23.80	10.4	7.5	0.8	2.1	1.7	3.6	25.46	22.9	16.2	1.7	2.7	2.1	1.5
				1/3	1	23.33	2.4	9.7	0.8	2.0	1.9	2.3	24.31	4.9	21.7	1.6	2.7	2.6	2.8
	10	0.9	0.5	1/3	1/3	21.70	6.4	1.4	0.3	4.7	2.5	0.2	22.39	7.5	4.0	0.3	5.5	3.6	2.4
				1	1	22.00	7.4	8.5	0.2	4.3	2.5	2.1	22.90	12.4	21.1	0.3	5.3	3.4	2.9
				1/3	1	21.93	5.3	8.8	0.1	4.2	2.5	2.2	22.71	6.0	22.1	0.1	4.7	3.3	3.3
		0.5	0.5	1/3	1/3	21.77	9.7	1.1	0.6	4.1	2.2	0.1	22.58	13.0	3.1	1.0	5.0	3.3	1.5
				1	1	22.49	16.8	6.1	0.8	3.4	1.8	2.6	24.12	29.7	14.9	1.8	3.6	2.2	1.1
				1/3	1	22.15	7.8	7.8	0.7	3.4	2.1	2.1	23.14	10.2	19.9	1.5	3.8	2.8	2.8
	25	0.9	0.5	1/3	1/3	19.08	21.1	0.4	0.8	11.4	2.9	0.2	19.81	21.5	2.7	0.8	12.1	4.4	2.9
				1	1	19.92	18.6	6.8	0.3	8.2	2.5	0.9	20.86	23.4	19.5	0.2	9.0	3.4	1.4
				1/3	1	19.88	16.2	7.0	0.3	8.0	2.5	1.2	20.63	16.6	20.9	0.3	8.9	3.8	2.6
		0.5	0.5	1/3	1/3	19.10	25.1	0.3	1.1	9.4	2.5	0.1	19.92	28.0	2.1	1.5	10.1	3.7	1.4
				1	1	20.28	29.6	4.8	1.0	6.1	1.9	1.8	21.90	42.8	13.9	2.0	5.9	2.2	0.8
				1/3	1	20.04	19.2	6.1	0.9	6.4	2.1	1.0	21.05	21.2	18.5	1.7	6.6	2.8	1.4

Table 3.8: Results of optimal dual sourcing policy and the savings compared to single sourcing, incomplete information, and order-up-to-S policy lost sales case $\lambda = 10$.

						$p=4, b=2$							$p=8, b=4$						
$\Delta\mu$	Δc	A_1	A_2	$1/\mu^1_{\text{OFF}}$	$1/\mu^2_{\text{OFF}}$	*Avg. cost*	*Sav. S1*	*Sav. S2*	*Sav. Info*	*Sav. S fix*	*Sav. S var*	*Sav. Heur*	*Avg. cost*	*Sav. S1*	*Sav. S2*	*Sav. Info*	*Sav. S fix*	*Sav. S var*	*Sav. Heur*
0	0	0.9	0.5	1/3	1/3	9.01	0.9	4.1	0.3	2.6	2.1	0.0	9.29	1.4	5.8	0.4	3.1	3.0	0.2
				1	1	9.08	4.3	19.4	0.6	2.7	2.0	0.0	9.40	7.7	29.6	1.0	3.3	2.9	0.1
				1/3	1	9.03	0.7	20.1	0.4	2.6	2.1	0.0	9.31	1.1	30.9	0.5	3.1	2.9	0.1
		0.5	0.5	1/3	1/3	9.08	3.3	3.3	1.8	2.7	2.2	1.2	9.38	4.8	4.8	2.5	2.7	2.7	2.5
				1	1	9.39	15.4	15.5	3.1	2.4	2.0	3.2	9.88	23.4	23.3	4.7	3.4	2.7	3.4
				1/3	1	9.13	2.7	18.7	2.5	3.0	2.1	0.9	9.47	3.8	28.7	3.6	2.8	2.8	1.7
	10	0.9	0.5	1/3	1/3	8.46	7.4	1.4	0.9	4.4	2.4	0.8	8.75	7.6	3.1	1.0	4.6	3.0	1.8
				1	1	8.63	9.8	16.4	0.7	3.3	2.4	1.9	8.95	13.1	27.1	1.1	3.7	2.8	3.4
				1/3	1	8.59	5.9	17.0	0.4	3.1	2.5	0.2	8.88	6.1	28.3	0.6	3.5	2.8	1.4
		0.5	0.5	1/3	1/3	8.48	10.6	1.1	2.3	4.1	2.3	0.5	8.80	11.6	2.5	3.0	3.7	2.6	1.4
				1	1	8.89	21.9	13.1	3.2	2.7	2.2	3.3	9.39	29.9	21.3	4.9	3.5	2.8	3.6
				1/3	1	8.68	8.1	15.7	2.5	3.2	2.2	0.1	9.03	8.8	26.2	3.7	2.8	2.7	0.5
	25	0.9	0.5	1/3	1/3	7.38	23.2	0.0	3.3	11.3	4.4	2.9	7.74	21.7	1.1	2.5	10.2	4.1	0.9
				1	1	7.87	20.4	12.5	1.1	5.4	2.2	2.0	8.21	23.3	24.1	1.3	5.5	3.0	3.1
				1/3	1	7.84	16.0	13.0	0.8	5.3	2.2	1.2	8.15	15.5	25.1	0.9	5.2	2.9	0.0
		0.5	0.5	1/3	1/3	7.38	27.1	0.0	4.3	9.7	3.4	2.3	7.75	26.8	0.9	4.5	8.1	3.3	3.0
				1	1	8.05	34.6	10.1	3.7	4.2	2.4	4.8	8.56	42.4	19.0	5.4	4.8	3.0	2.1
				1/3	1	7.91	18.6	12.0	2.8	4.9	2.3	0.2	8.27	18.8	23.2	4.0	4.0	2.6	0.6
50	0	0.9	0.5	1/3	1/3	9.04	0.6	5.9	0.3	3.0	2.6	0.0	9.32	1.0	7.6	0.4	3.4	3.2	0.2
				1	1	9.11	4.0	17.4	0.5	3.2	2.2	0.1	9.43	7.4	25.4	0.9	4.4	2.9	0.2
				1/3	1	9.05	0.5	18.1	0.3	3.0	2.4	0.0	9.34	0.8	26.6	0.5	3.3	3.1	0.1
		0.5	0.5	1/3	1/3	9.19	2.1	4.2	1.5	3.2	1.8	1.5	9.52	3.2	5.4	2.0	3.8	3.6	0.6
				1	1	9.47	14.4	12.9	2.6	3.0	2.6	1.8	9.95	22.5	18.8	3.8	3.8	3.1	2.0
				1/3	1	9.22	1.7	15.9	2.2	3.0	2.3	0.7	9.57	2.6	23.5	3.1	3.5	3.3	0.4
	10	0.9	0.5	1/3	1/3	8.63	5.3	1.7	0.5	4.6	3.0	1.6	8.95	5.2	3.1	0.6	5.0	3.6	1.6
				1	1	8.74	8.4	13.2	0.5	4.5	2.9	1.2	9.08	11.5	21.4	0.8	5.1	3.4	2.1
				1/3	1	8.70	4.5	13.7	0.3	4.2	2.9	1.0	9.02	4.4	22.3	0.5	4.7	3.4	1.1
		0.5	0.5	1/3	1/3	8.66	8.3	1.4	1.5	4.8	2.1	2.7	9.00	9.1	2.5	2.0	5.3	4.0	1.4
				1	1	8.95	21.0	10.5	2.6	4.0	2.8	1.6	9.44	29.2	16.8	3.8	4.6	3.3	1.8
				1/3	1	8.78	6.8	12.7	2.0	4.2	2.9	1.4	9.14	7.5	20.6	2.8	4.6	3.6	1.2
	25	0.9	0.5	1/3	1/3	7.56	20.2	0.2	1.8	12.2	4.7	2.5	7.92	18.9	1.4	1.6	11.6	5.3	3.1
				1	1	7.88	20.2	10.4	0.9	9.1	3.4	0.3	8.23	23.0	19.4	1.2	9.3	4.3	0.8
				1/3	1	7.86	15.7	10.7	0.7	8.8	3.4	1.6	8.18	15.0	20.1	0.8	8.9	4.4	2.4
		0.5	0.5	1/3	1/3	7.56	24.0	0.2	2.7	10.2	4.1	2.5	7.93	23.8	1.2	2.9	10.7	4.8	1.9
				1	1	8.03	34.9	8.3	3.0	7.0	3.0	2.2	8.52	43.1	15.4	4.3	7.4	3.6	0.7
				1/3	1	7.91	18.5	9.9	2.3	7.4	3.1	2.0	8.29	18.6	18.6	3.2	8.1	3.8	1.6

Table 3.9: Results of optimal dual sourcing policy and the savings compared to single sourcing, incomplete information, and order-up-to-S policy backorder case $\lambda = 4$.

						$p=4, b=2$							$p=8, b=4$						
$\Delta\mu$	Δc	A_1	A_2	$1/\mu^1_{\text{OFF}}$	$1/\mu^2_{\text{OFF}}$	*Avg. cost*	*Sav. S1*	*Sav. S2*	*Sav. Info*	*Sav. S fix*	*Sav. S var*	*Sav. Heur*	*Avg. cost*	*Sav. S1*	*Sav. S2*	*Sav. Info*	*Sav. S fix*	*Sav. S var*	*Sav. Heur*
0	0	0.9	0.5	1/3	1/3	21.47	1.0	4.2	0.1	2.0	1.9	0.1	21.85	1.6	6.3	0.2	2.5	2.4	0.2
				1	1	21.65	4.7	21.6	0.3	2.1	1.9	1.2	22.17	9.3	33.8	0.5	3.0	2.5	1.3
				1/3	1	21.51	0.8	4.0	0.2	2.0	1.9	0.0	21.91	1.3	35.3	0.2	2.5	2.4	0.1
		0.5	0.5	1/3	1/3	21.62	3.5	3.5	0.7	1.8	1.8	0.5	22.06	5.3	5.3	1.1	2.2	2.1	2.0
				1	1	22.46	17.2	17.2	1.4	2.1	1.8	5.2	23.48	26.3	26.3	2.3	3.2	1.6	6.3
				1/3	1	21.76	2.8	2.8	1.0	2.0	1.7	0.4	22.30	4.1	33.0	1.6	2.5	1.9	1.6
	10	0.9	0.5	1/3	1/3	20.10	7.9	1.4	0.5	3.7	2.4	0.9	20.51	8.2	3.5	0.5	4.1	2.4	1.6
				1	1	20.55	10.3	18.4	0.3	2.5	2.0	4.1	21.08	14.9	31.1	0.5	3.3	2.6	5.5
				1/3	1	20.44	6.1	19.1	0.2	2.3	1.9	0.4	20.86	6.4	32.6	0.3	2.8	2.4	1.0
		0.5	0.5	1/3	1/3	20.14	11.1	1.2	1.0	3.1	2.0	1.6	20.62	12.6	2.9	1.3	3.3	2.1	1.9
				1	1	21.22	24.0	14.7	1.5	2.3	2.0	5.4	22.27	33.1	24.1	2.4	3.2	2.6	4.7
				1/3	1	20.66	8.3	17.8	1.0	2.0	1.8	0.3	21.22	9.4	30.3	1.6	2.4	2.3	0.6
	25	0.9	0.5	1/3	1/3	17.37	24.8	0.0	2.2	10.9	5.9	0.6	17.99	23.4	1.3	1.4	9.9	3.8	1.3
				1	1	18.70	21.2	14.4	0.5	4.2	2.4	3.1	19.26	25.8	28.0	0.7	4.9	2.6	4.5
				1/3	1	18.62	16.4	14.9	0.3	4.0	2.3	0.0	19.08	16.4	29.2	0.4	4.3	2.5	0.3
		0.5	0.5	1/3	1/3	17.37	28.8	0.0	2.4	8.7	4.6	2.1	18.02	28.8	1.1	2.1	7.8	3.0	4.3
				1	1	19.18	37.3	11.5	1.7	3.5	2.0	5.5	20.27	46.3	21.6	2.6	4.2	2.6	4.2
				1/3	1	18.79	19.1	13.9	1.1	3.2	1.8	0.6	19.41	19.7	27.0	1.8	3.3	2.2	1.0
50	0	0.9	0.5	1/3	1/3	21.51	0.8	4.6	0.1	2.2	2.2	0.2	21.90	1.3	6.1	0.2	2.9	2.8	0.3
				1	1	21.68	4.5	17.9	0.3	2.8	2.1	0.8	22.19	9.2	27.6	0.5	4.0	2.7	0.8
				1/3	1	21.54	0.6	18.7	0.1	2.2	2.1	0.2	21.95	1.1	28.9	0.2	2.8	2.8	0.2
		0.5	0.5	1/3	1/3	21.77	2.8	3.3	0.6	2.6	2.5	1.2	22.24	4.4	4.5	0.8	3.3	3.1	0.6
				1	1	22.49	17.0	13.6	1.2	3.0	2.4	2.6	23.42	26.6	20.8	1.8	3.7	2.8	3.2
				1/3	1	21.88	2.3	16.8	0.9	2.5	2.3	1.0	22.42	3.6	26.2	1.3	3.0	2.9	0.5
	10	0.9	0.5	1/3	1/3	20.23	7.2	1.3	0.3	5.2	2.8	2.1	20.68	7.3	2.7	0.3	5.5	3.3	2.0
				1	1	20.57	10.2	14.5	0.3	4.9	2.8	0.7	21.10	14.8	24.7	0.4	5.8	3.4	1.5
				1/3	1	20.48	5.9	15.0	0.2	4.5	2.8	1.2	20.92	6.1	25.7	0.2	4.9	3.4	1.1
		0.5	0.5	1/3	1/3	20.27	10.4	1.1	0.7	4.9	2.8	1.9	20.77	11.8	2.3	0.9	5.3	3.4	1.5
				1	1	21.11	24.7	11.6	1.2	4.3	2.6	1.8	22.05	34.5	19.3	1.9	4.8	3.0	1.5
				1/3	1	20.66	8.3	14.0	0.8	4.0	2.7	1.2	21.22	9.4	24.0	1.3	4.4	3.1	1.2
	25	0.9	0.5	1/3	1/3	17.48	24.0	0.1	1.2	13.2	3.7	4.5	18.02	23.2	1.3	0.9	13.1	4.3	3.3
				1	1	18.44	22.9	11.6	0.4	9.1	2.8	0.3	18.99	27.6	22.7	0.6	10.0	3.6	0.5
				1/3	1	18.37	18.0	12.0	0.3	8.8	2.8	3.0	18.84	17.8	23.7	0.4	9.4	3.6	2.3
		0.5	0.5	1/3	1/3	17.48	28.0	0.1	1.4	11.1	3.4	4.2	18.05	28.6	1.1	1.4	11.1	4.0	2.5
				1	1	18.81	39.9	9.4	1.4	7.2	2.7	1.9	19.77	50.0	17.9	2.1	7.6	3.1	1.3
				1/3	1	18.50	20.9	11.2	1.0	7.3	2.5	3.1	19.10	21.6	22.0	1.5	7.5	3.2	1.7

Table 3.10: Results of optimal dual sourcing policy and the savings compared to single sourcing, incomplete information, and order-up-to-S policy backorder case $\lambda = 10$.

Chapter 4

Dual Sourcing under Disruption Risk and Cost Improvements through Learning

This Chapter investigates a finite time dynamic supplier selection and allocation problem of a buyer that can source from two suppliers subject to supply disruptions.

4.1 Introduction

When analyzing an optimal sourcing strategy, a buyer has to consider ineffectiveness of the supply chain resulting from supply disruptions. Disruptions may be caused by different forces from inside and outside the organization, e.g. supplier bankruptcy or natural disasters, and can have a strong impact on the supply performance and, in particular, increase sourcing costs considerably.

To operate efficiently despite potential disruptions, dual sourcing is a prevailing strategy for mitigating against the supply risk. However, dual sourcing forfeits some potential economies of scale from single sourcing. The challenge for the buying firm is to deal with supply disruptions and integrate long-term learning effects of the suppliers based on production experience into the optimal sourcing decision. Existing literature mostly focuses on ways how to manage supply disruptions, assuming constant costs of the suppliers over the entire planning horizon. In long-term planning, however, the phenomenon that suppliers learn how to reduce cost over time through production has to be included into the buyer's procurement decision (see Figure 4.1).

Paying attention to both managing supply disruptions and quantifying supplier relationships

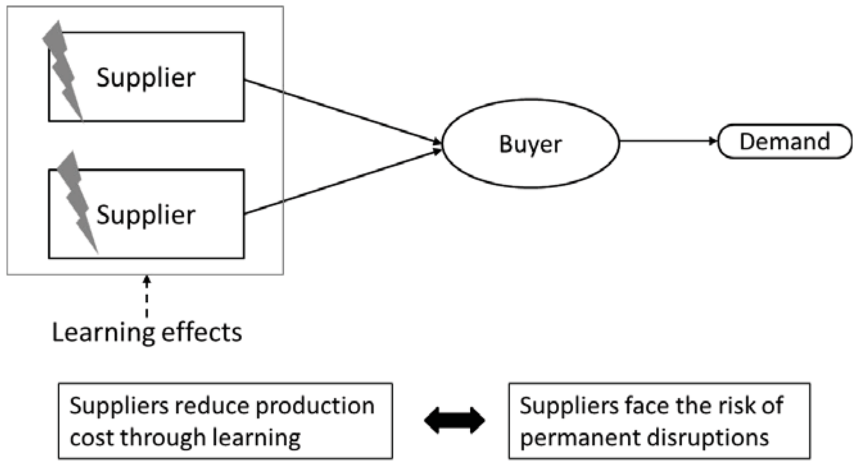


Figure 4.1: Framework for dual sourcing and illustration of key trade-off.

when supply cost follows a learning curve requires a general supply cost function, where costs decrease with cumulative production. When accounting for supplier cost improvements through learning alone, the buyer will clearly favor a single sourcing strategy to benefit the most from future cost reductions. However, the risk of a disruption is generally higher when ordering from a single supplier than when ordering from two suppliers. Under dual sourcing, in case of a disruption at least the other supplier survives and accumulates experience which decreases future supply cost. Therefore, a diversification between two suppliers to reduce the risk of higher future purchasing cost can result in overall lower total cost for the buyer.

The objective of this Chapter is to provide managerial insights into how a buyer should optimally allocate demand volume between two suppliers under given supplier reliability, procurement cost and cost improvement potentials through learning. We develop a discrete time stochastic dynamic program with a single buyer purchasing from two potential suppliers who face the risk of permanent supply disruptions. We analyze the optimal policy for the buyer sourcing from two potential suppliers, that either are identical or differ in their reliability, learning ability, or cost structure (e.g. the supply base may consist of one unreliable but low cost supplier and one reliable but high cost supplier) and compare it to a single sourcing policy. The central research questions in this Chapter are:

- What is the interaction of supplier characteristics, reliability, cost, and learning ability on the optimal policy?
- What is the influence of the relevant parameters on the sourcing strategy and on the optimal volume allocation between the suppliers?

- What is the benefit from dual sourcing compared to single sourcing, equal split or 75:25 split dual sourcing?

The model under investigation is mostly related to Tomlin (2009b) who also studies a discrete time finite horizon problem with two unreliable suppliers. However, our work differs as follows: Tomlin (2009b) assumes that supply costs do not change over time, whereas we assume that costs decline with experience. Further, Tomlin (2009b) focuses on supply disruption where an order placed with a supplier might succeed or fail. Our work assumes that a supplier might survive and gain experience, which reduces future supply cost or else fails and gets permanently disrupted and needs to be replaced by a supplier without experience. In Tomlin (2009b), total expected cost depends on past events only through the reliability forecast and the optimal decision only influences the current period's expected cost. The optimal decision in our model depends on past allocation decisions and influences both, the current period's and expected future cost.

The remainder of this Chapter is organized as follows. In Sections 4.2, we present the problem description, in Section 4.3 the dynamic model formulation. Section 4.4 and 4.5 derive structural properties of the buyer's optimal policy analytically and through numerical examples. In Section 4.6, we give concluding remarks. The proofs of the results are provided in 4.7.

4.2 Problem description

Consider a buyer with a T-period planning horizon with $T \geq 2$. In every period t, the buyer can use dual sourcing to purchase from two potential suppliers to satisfy a constant deterministic demand d. The buyer decides on the order allocation $\mathbf{q_t} = \{q_{1,t}, q_{2,t}\}$ between suppliers 1 and 2 to minimize the total expected cost over the planning horizon and thereby implicitly on the sourcing strategy (single sourcing or dual sourcing). For reasons of simplicity, we assume that the buyer does not store inventory strategically and shortages are not allowed, therefore $q_{1,t} + q_{2,t} = d$ for $t = 1, ..., T$. Further, the lead times of the suppliers are negligible compared to the strategic time bucket of the sourcing allocation problem and both potential suppliers have ample capacities.

The suppliers' per unit costs change dynamically based on past cumulative production (learning effect), i.e., the suppliers realize a quantity-based learning benefit. Hence, the per unit purchase costs associated with each supplier $i = 1, 2$ in period t, $c_i(x_{i,t})$ are functions of experience defined as cumulative production $x_{i,t} = \sum_{\tau=1}^{t} q_{i,\tau}$. The cost function of supplier i, $c_i(.)$, is decreasing and convex in experience, $c_i'(x) < 0$ and $c_i''(x) > 0$. An example of such a cost function is the power learning function $c(x) = c_0 x^{-b}$, where c_0 is the initial cost, x the experience and b the learning factor. To obtain further insights when necessary we will use the power learning function to specify the cost function. Supplier experience at the beginning of the first period is normalized to zero, $x_{i,1} = 0$ and initial supply costs are given by $c_i(0) = c_{i,0}$. We define supplier i's reliability by the survival probability p_i, and assume that the supplier survival probabilities are known and independent between the suppliers and across periods. A supplier i who does not survive to the next period $t+1$ loses gained experience and faces a permanent disruption, but a new supplier can start production at initial cost $c_{i,0}$, i.e. the replacement is by a supplier of identical type. In case of a supplier disruption, however, a new supplier with the same qualification gets available and replaces the disrupted supplier. The preselected suppliers are of certain types $i = 1, 2$, i.e. a type one supplier may differ e.g. geographically, politically or in the infrastructure installations from a type 2 supplier. Then, supplier i's experience in period $t+1$ is given by

$$x_{i,t+1} = \begin{cases} x_{i,t} + q_{i,t} & \text{with probability} \quad p_i, \\ 0 & \text{with probability} \quad 1 - p_i \end{cases}.$$

The suppliers may have different characteristics: initial cost, learning ability and reliability and will not necessarily have the same ability to learn, i.e. one supplier may learn faster than the other.

The sequence of events is as follows: (1) at the beginning of each period t, the buyer knows the supplier experiences $\mathbf{x_t} = (x_{1,t}, x_{2,t})$ before production, hence the actual within period supply cost, (2) the buyer decides on order quantities $\mathbf{q_t} = (q_{1,t}, q_{2,t})$ to satisfy demand $d = q_{1,t} + q_{2,t}$. The production decision affects the future supply cost with probability p_i, where the experience of supplier $i = 1, 2$ at the beginning of the next period accumulates to $x_{i,t+1} = x_{i,t} + q_{i,t}$. With probability $1 - p_i$, the supplier gets disrupted and a new inexperienced supplier with identical initial characteristics can start production (see Figure 4.2).

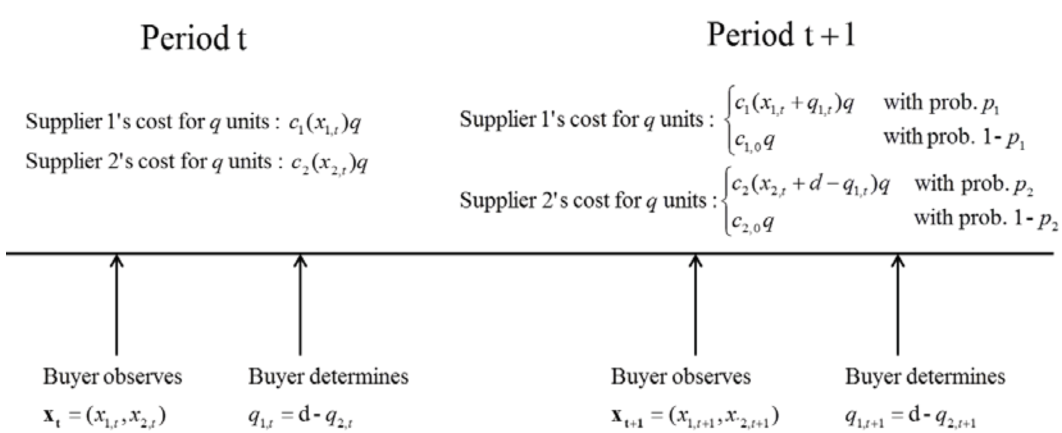


Figure 4.2: Sequence of events.

4.3 Dynamic programming formulation

In order to analyze this sequential decision problem with a probabilistic law of motion and a finite planning horizon, we first discuss the method for solving these types of problems to apply it to the formulated problem afterwards. Further details on stochastic dynamic programming models and its applications can be found in e.g. Heyman and Sobel (2003), Bertsekas (2005), Bertsekas (2007), and Puterman (2009).

4.3.1 General stochastic dynamic programming formulation - The finite horizon problem

Dynamic programming is a computational approach for analyzing sequential decision problems over a finite time horizon T based on states, the principle of optimality, and functional equations. It is a recursive procedure for calculating optimal value functions from functional equations. These functional equations follow the principle of optimality which states that an optimal policy behaves optimally for every state at each stage of the system. Hence, the optimal immediate decision depends only on the current state and not on how you got there. The goal is to select the sequence of decisions (policy) which minimizes the systems expected total cost.

The state of the system in period t that summarizes past information that is relevant for future optimization is denoted by x_t. The decision to be selected at time t out of the set of possible decisions $A_t(x_t)$ is denoted by q_t. As a result of choosing decision $q_t \in A_t(x_t)$ in state x_t at time t immediate cost $c_t(x_t, q_t)$ occur. The system state of the stochastic dynamic problem evolves according to

$$x_{t+1} = f_t(x_t, q_t, \xi_t), \quad t = 0, 1, ..., T-1$$

where ξ_t is a stochastic variable. For a given state and decision at the actual stage the future state of the system is uncertain and the expected value is used to deal with the uncertainty. The value function $V_t(x_t)$ defines the optimal expected cost for all remaining periods if the system is in state x_t at time t. The functional equations, or Bellman equations, which relate $V_t(x_t)$ to $V_{t+1}(x_{t+1})$ are given by

$$V_t(x_t) = \min_{q_t \in A(x_t)} \{c_t(x_t, q_t) + EV_{t+1}(x_{t+1})\}, \quad t = 1, ..., T \tag{4.1}$$

with the boundary condition $V_{T+1}(x_{T+1}) = 0$.

$V_t(x_t)$ is the value of an optimal balance of short-run and long-run cost. Equation (4.1) is the basis for the backward induction algorithm. This algorithm decomposes the problem into a sequence of minimization problems that proceeds as follows:

1. Start in the last stage $t = T$, where the problem is static as $V_{T+1}(x_{T+1}) = 0$ and compute $V_T(x_T) = \min_{q_T \in A(x_T)} c_T(x_T, q_T)$ to obtain the optimal policy $q_T^*(x_T)$ for the last stage that minimizes the right-hand side of the preceding equation for each of the possible states x_T in that stage.

2. Go one step back, set $t = t-1$ and compute $V_t(x_t) = \min_{q_t \in A(x_t)}\{c_t(x_t, q_t) + EV_{t+1}(x_{t+1})\}$ inserting the optimal value functions for stage $t+1$ to obtain $q_t^*(x_t)$.

3. Stop if $t = 1$, otherwise go to step 2.

Returning to the problem formulated in Section 4.2 and applying this recursive procedure is straight forward and will be discussed in the following.

4.3.2 Application to the formulated problem

Variables and parameters used for the stochastic dynamic model are summarized in Table 4.1. The decision variable is $q_{1,t}$, which is the order quantity assigned to supplier 1 and $q_{2,t} = d - q_{1,t}$. For $t = 1, ..., T$, $V_t(\mathbf{x_t})$ defines the optimal undiscounted cost for the remaining periods, given that the actual supplier experience at the beginning of period t is $\mathbf{x_t} = (x_{1,t}, x_{2,t})$. The stochastic dynamic programming recursion is formulated as

$$\begin{aligned} V_t(\mathbf{x_t}) \quad = \quad \min_{0 \le q_{1,t} \le d} &\{c_1(x_{1,t})q_{1,t} + c_2(x_{2,t})(d - q_{1,t}) + p_1 p_2 V_{t+1}(x_{1,t} + q_{1,t}, x_{2,t} + d - q_{1,t}) + \\ &p_1(1-p_2)V_{t+1}(x_{1,t} + q_{1,t}, 0) + (1-p_1)p_2 V_{t+1}(0, x_{2,t} + d - q_{1,t}) + \\ &(1-p_1)(1-p_2)V_{t+1}(0,0)\}, \end{aligned} \tag{4.2}$$

with terminal condition $V_{T+1}(\mathbf{x_{T+1}}) = 0$.

T	Planning horizon
t	Index for periods in time horizon $t = 1, ..., T$
d	Per period demand
$q_{i,t}$	Order quantity assigned to supplier $i = 1, 2$ in period t
$x_{i,t}$	Experience (cumulative production) of supplier $i = 1, 2$ at the beginning of period t
$c_i(x_{i,t})$	Per unit purchase cost of supplier $i = 1, 2$ in period t
$c_{i,0}$	Initial per unit purchase cost of supplier $i = 1, 2$ if $x_{i,t} = 0$
p_i	Probability that supplier $i = 1, 2$ will survive to the next period
$V_t(.)$	Optimal cost for the t-period problem

Table 4.1: List of notation.

4.4 Model analysis

4.4.1 Structural properties

In the last period T, the objective is to minimize total procurement cost for realized supplier experiences $\mathbf{x_T}$

$$V_T(\mathbf{x_T}) = \min_{0 \le q_{1,T} \le d} \{c_1(x_{1,T})q_{1,T} + c_2(x_{2,T})(d - q_{1,T})\}.$$

Clearly, in the absence of future supply disruption risk the buyer will only procure from the lower cost supplier in the last period.

$$q^*_{1,T} = \begin{cases} d, & \text{if} \quad c_1(x_{1,T}) \le c_2(x_{2,T}) \\ 0, & \text{otherwise} \end{cases}, \quad V_T(\mathbf{x_T}) = \begin{cases} c_1(x_{1,T})d, & \text{if} \quad c_1(x_{1,T}) \le c_2(x_{2,T}) \\ c_2(x_{2,T})d, & \text{otherwise} \end{cases}. \tag{4.3}$$

This implies that the last period decision only depends on the realization of actual supplier experiences (cumulative production) and as the buyer does not face the risk of losing experience for a future cost reduction, the optimal decision does not depend on the reliabilities p_i. Hence, there is no benefit from dual sourcing. For periods $t < T$, the optimal procurement decisions $\mathbf{q}^* = (q^*_{1,1}, q^*_{1,2}, ..., q^*_{1,T-1})$ are as follows. Unless otherwise stated, $V'(.)$ and $c'(.)$ denote $\frac{\partial V(.)}{\partial q_{1,t}}$ and $\frac{\partial c(.)}{\partial q_{1,t}}$, respectively.

Theorem 4.4.1. *There are three possible candidate decisions for an optimal allocation in period $t < T$: single sourcing with supplier 1 ($q^*_{1,t} = d$), single sourcing with supplier 2 ($q^*_{1,t} = 0$) and dual sourcing ($0 < q^*_{1,t} < d$). The optimal dual sourcing procurement quantity ($0 < q^*_{1,t} < d$) solves*

$$\begin{aligned} c_2(x_{2,t}) - c_1(x_{1,t}) \quad &= \quad p_1 p_2 V'_{t+1}(x_{1,t} + q^*_{1,t}, x_{2,t} + d - q^*_{1,t}) + p_1(1-p_2)V'_{t+1}(x_{1,t} + q^*_{1,t}, 0) + \\ &\quad\; (1-p_1)p_2 V'_{t+1}(0, x_{2,t} + d - q^*_{1,t}). \end{aligned} \tag{4.4}$$

The optimal order quantity from dual sourcing $0 < q^*_{1,t} < d$ trades off the actual marginal supply cost difference $c_2(x_{2,t}) - c_1(x_{1,t})$ of assigning an additional unit to supplier 2 rather than to supplier 1 and the expected change of total cost (derivative of the future value function) from all future periods with the currently obtained experiences $x_{i,t} + q_{i,t}$ and weighted with the probabilities of different supply bases containing all or one (of the two) current suppliers.

In the two-period case $T = 2$, the optimality condition for sourcing volume allocation reduces to

$$c_{2,0} - c_{1,0} \quad = \quad \begin{cases} p_1 d c'_1(q^*_{1,1}) - p_2(1-p_1) d c'_2(d - q^*_{1,1}), & \text{if} \quad c_1(q^*_{1,1}) \le c_2(d - q^*_{1,1}) \\ p_1(1-p_2) d c'_1(q^*_{1,1}) - p_2 d c'_2(d - q^*_{1,1}), & \text{otherwise} \end{cases}, \tag{4.5}$$

which offers a clearer economic interpretation. The optimal first-period allocation is chosen such that the per unit extra cost of using supplier 2 becomes equal to the expected total loss if supplier 1 survives minus the savings if supplier 1 fails and supplier 2 survives. Equation (4.5) implies the following.

Lemma 4.4.2. *In a two-period problem for non-identical suppliers, the optimal first period procurement quantity $q^*_{1,1}$ is increasing in p_1 and decreasing in p_2. For $c_i(x_i) = c_{i,0}x_i^{-b_i}, (i = 1, 2)$, the optimal first-period procurement quantity $q^*_{1,1}$ is increasing in d, decreasing (increasing) in b_1 if $b_1 \ln(q^*_{1,1}) > (<)1$ and increasing (decreasing) in b_2 if $b_2 \ln(d - q^*_{1,1}) > (<)1$.*

In the special case of two identical suppliers assuming $q_{1,1} \geq d/2$ (supplier 1 is awarded the larger quantity of first period demand, however as the suppliers are symmetric this assumption is just a technical one and will hold throughout this paper), (4.5) reduces to

$$c'(q^*_{1,1}) = (1 - p)c'(d - q^*_{1,1}). \tag{4.6}$$

This has the following implications:

Lemma 4.4.3. *In a two-period problem for identical suppliers, (i) the optimal first-period order quantity $q^*_{1,1}$ is increasing in p and d. For $c(x_i) = c_0x_i^{-b}$, the optimal first-period procurement quantity $q^*_{1,1}$ is decreasing in b. (iii) The optimal first-period order quantity $q^*_{1,1}$ is independent of the initial cost c_0. (iv) $q^*_{1,1} \neq q^*_{2,1}$, i.e equal order splitting is not optimal.*

The relationship between the optimal procurement quantities and survival probabilities are intuitive: when the survival probability of supplier 1 increases, the risk of a disruption decreases and more volume will be allocated to supplier 1. The last implication might appear counterintuitive at first sight, as one would expect a symmetric allocation when sourcing from two symmetric suppliers. But due to the cost improvements in the second period, one supplier will always be awarded a higher amount of demand than the other supplier as long as they are unreliable ($p < 1$).

4.4.2 Comparison of dual and single sourcing

Having characterized the optimal sourcing strategy, we compare the optimal sourcing strategy with single sourcing from supplier 1 or 2 and analyze under which conditions dual sourcing is preferred over single sourcing.

If the buyer chooses single sourcing with supplier i, he will procure the required quantity d from this supplier in each period t, resulting in the decision $q_{i,t}^{\text{single}} = d$ for all t, where the expected total cost is given by

$$V_t^{\text{single i}}(x_{i,t}) = c_i(x_{i,t})d + p_i V_{t+1}(x_{i,t}+d) + (1-p_i)V_{t+1}(0) \tag{4.7}$$

Thus, dual sourcing is preferred over single sourcing when $V_1^{\text{single*}} - V_1^{\text{dual}} > 0$, where $V_1^{\text{single*}} = \min\{V_1^{\text{single 1}}, V_1^{\text{single 2}}\}$. With supplier 1 being the optimal supplier choice for single sourcing, for notational convenience, dual sourcing is preferred over single sourcing if

$$\begin{aligned} &p_1V_2(d) + (1-p_1)V_2(0) - ((c_{1,0}-c_{2,0})(q_{1,1}^*-d) + (1-p_1)(1-p_2)V_2(0,0) + \\ &\quad p_1p_2V_2(q_{1,1}^*, d-q_{1,1}^*) + p_1(1-p_2)V_2(q_{1,1}^*,0) + (1-p_1)p_2V_2(0,d-q_{1,1}^*)) > 0 \end{aligned} \tag{4.8}$$

Therefore, there is a benefit from dual sourcing if the actual purchasing cost difference between single and dual sourcing together with the future purchasing cost difference is positive. The economic interpretation is that the expected future cost from single sourcing needs to compensate the sum of the actual extra cost from dual sourcing and expected future cost from dual sourcing. In the two-period model, (4.8) reduces to

$$p_2(1-p_1)dc_{1,0} - ((c_{1,0}-c_{2,0})(q_{1,1}^*-d) + p_1d(c_1(q_{1,1}^*) - c_1(d)) + (1-p_1)p_2dc_2(d-q_{1,1}^*)) > 0 \tag{4.9}$$

The first term of (4.9) is positive and the last three terms are negative. Thus, when the loss resulting from a disruption of supplier 1 ($p_2(1-p_1)dc_{1,0}$) is sufficient, dual sourcing is preferred over single sourcing with supplier 1. Note that for the case of identical suppliers, (4.9) can be simplified to

$$(1-p)c_0 - (c(q_{1,1}^*) - c(d) + (1-p)c(d-q_{1,1}^*)) < 0 \tag{4.10}$$

With (4.9) and (4.10) the sensitivity of the advantage of dual sourcing can be discussed. The results are summarized below.

Lemma 4.4.4. *In a two period problem for non-identical suppliers and $V_1^{single*} = V_1^{single\ 1}$, the advantage of dual sourcing $V_1^{single*} - V_1^{dual}$ decreases (increases) in p_1 if $p_2(c_{1,0} - c_2(d-q_{1,1}^*)) >$ $(<)$ $c_1(d) - c_1(q_{1,1}^*)$ and increases (decreases) in p_2 if $c_2(d-q_{1,1}^*) < (>)$ $c_{1,0}$.*

The advantage of dual sourcing decreases (increases) in p_1 if the extra cost of supplier 1 using dual sourcing $c_1(d) - c_1(q_{1,1}^*)$ is smaller (greater) than the extra benefit/loss if supplier 2 survives $p_2(c_{1,0} - c_2(d-q_{1,1}^*))$. Further, the advantage of dual sourcing increases (decreases) in p_2, if there is a benefit (loss) of dual sourcing in case that supplier 1 does not survive. For the other parameters, there is a complex interaction and we discuss the resulting effects in the numerical Section.

For identical suppliers, the sensitivity of the advantage of dual sourcing decreases is as follows.

Lemma 4.4.5. *In a two period problem for identical suppliers, the advantage of dual sourcing* $V_1^{single} - V_1^{dual}$ *decreases in* p.

For further illustration, we formulate indifference curves for $t = T-1$ between single and dual sourcing of $q^*_{1,T-1}$ by setting $q^*_{1,T-1}$ to 0 or d, respectively in the first-order condition (4.4). In the case of identical suppliers where $p_1 = p_2 = p$, $c_{1,0} = c_{2,0} = c_0$ and $c_1(x) = c_2(x) = c(x)$, the indifference curves are

$$
\begin{aligned}
L_1 &= c(x_{1,T-1}) - c(x_{2,T-1}) - p(1-p)dc'(x_{2,T-1}) + pdc'(x_{1,T-1} + d) \\
L_2 &= c(x_{1,T-1}) - c(x_{2,T-1}) - pdc'(x_{2,T-1} + d) + p(1-p)dc'(x_{1,T-1})
\end{aligned}
$$

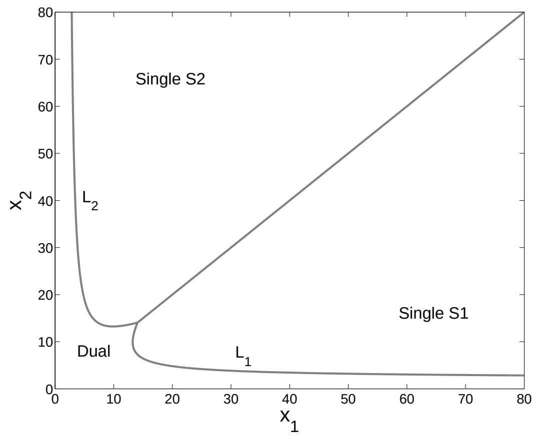
OBSERVATION 4.1. *The dual sourcing region increases with learning slope and decreases with survival probability (see Figure 4.3).*

The indifference curves L_1 and L_2 that state the regions for dual and single sourcing with supplier 1 and 2 are shown in Figure 4.3 for $c(x) = c_0 x^{-b}$. As the suppliers are identical, the buyer is indifferent between supplier 1 and 2 on the diagonal line.

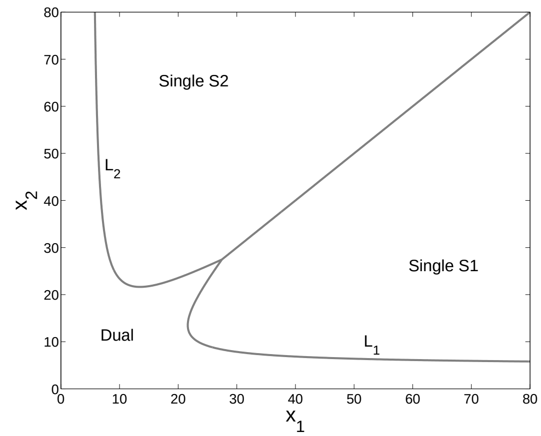
4.5 Numerical results and discussion

The following numerical studies complement our analytical findings. We analyze the impact of supplier reliability, cost difference, and learning ability on the optimal volume allocation and quantify the benefit of dual over single sourcing, equal split dual sourcing, and 75:25-split dual sourcing. Buyer demand d is 100. Supplier 1 initial unit costs are defined by $c_{1,0} = c_{2,0} - \Delta c$ with $c_{2,0} = 10$. For example, supplier 1 is a foreign supplier offering a low initial per unit cost, but is less reliable facing a higher risk of disruption due to less experienced workforce and supplier 2 is a domestic supplier with high per unit cost but a high reliability. For the supplier reliabilities, we use three scenarios: low, medium and high reliability with $p \in \{0.7, 0.8, 0.9\}$. We use the power learning function to specify the cost function. The learning rate expressed as percentage is l, where $b = \frac{-log(l)}{log(2)}$ and $0 < b < 1$. A learning slope of $b = 0.3$ implies a 81% learning rate which means that for every doubling of cumulative production, costs go down by 29%. Depending on the industry, the learning rate typically ranges from 70% to 90% in most industrial situations (see e.g. Yelle (1979), Jaber (2013)). Thus, we consider learning slopes $b \in \{0.1, 0.3, 0.5\}$, which implies a learning rate of $l \in \{90\%, 81\%, 71\%\}$ with corresponding improvement rates of $\{10\%, 19\%, 29\%\}$. We compare the optimal policy to other policies. Define Δ^{DS} as the percentage savings of optimal compared to single sourcing

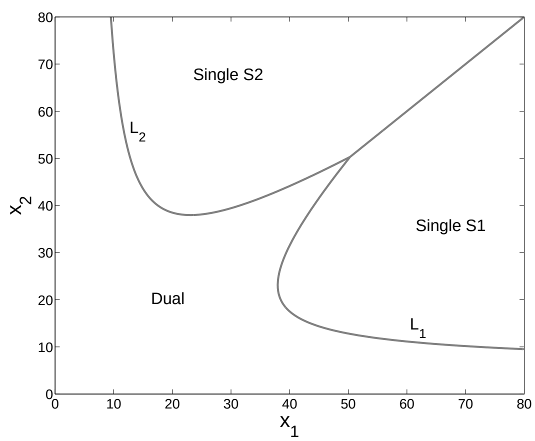
$$
\Delta^{DS} = \frac{V_1^{\text{single}} - V_1}{V_1} \times 100. \tag{4.11}
$$



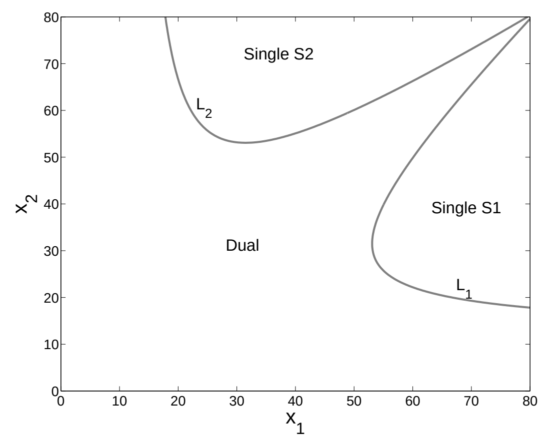
(a) $b = 0.1, p = 0.9$



(b) $b = 0.1, p = 0.9$



(c) $b = 0.1, p = 0.7$



(d) $b = 0.5, p = 0.7$

Figure 4.3: Optimal sourcing strategy $t = T - 1$ in the (x_1, x_2)-plane with $c_0 = 10$ and $d = 100$.

and Δ^{50} as the percentage savings of optimal sourcing compared to dual sourcing with equal split. Further, as proposed in Allon and Van Mieghem (2010), we also compare the optimal policy to a policy with a 75%:25% allocation, Δ^{75*} where 75^* indicates the best 75:25 split ($75^* = \min\{75 : 25 \text{ policy}, 25 : 75 \text{ policy}\}$), in the case of non-identical suppliers. In Allon and Van Mieghem (2010) they consider a firm that can source from one low-cost offshore supplier and one nearshore supplier. Simulations results indicated that, for the majority of parameter values, total cost where minimal when around 75% (i.e., more than 50% but less than 100% given that only five allocations were investigated) was sourced from the low-cost supplier.

4.5.1 Two periods

For the case of two identical suppliers and $T = 2$, we obtain a closed form solution of the optimal first-period order allocation for the power learning function $c(x) = c_0 x^{-b}$.

$$q^*_{1,1} = \frac{d}{1+\sqrt[b+1]{1-p}}, \tag{4.12}$$

i.e., the optimal order quantity $q^*_{1,1}$ in the first period is smaller than d for $p < 1$ and increases in reliability p, as long as supplier 1 gets the larger part of the demand, and decreases with learning ability b. The optimal order quantity (4.12) is shown in Figure 4.4.

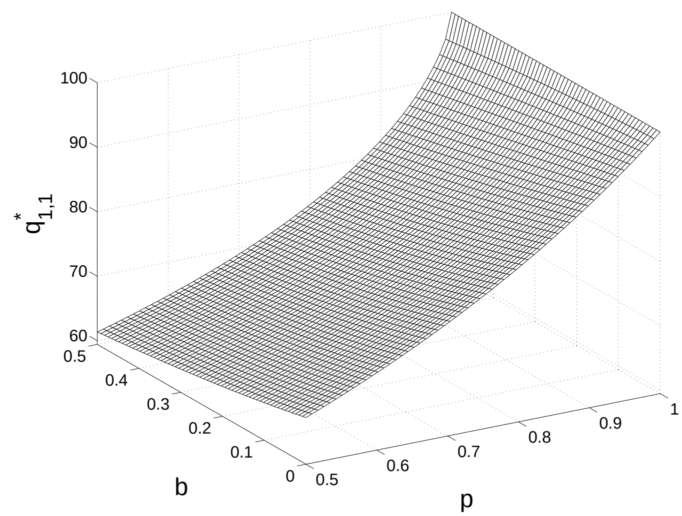
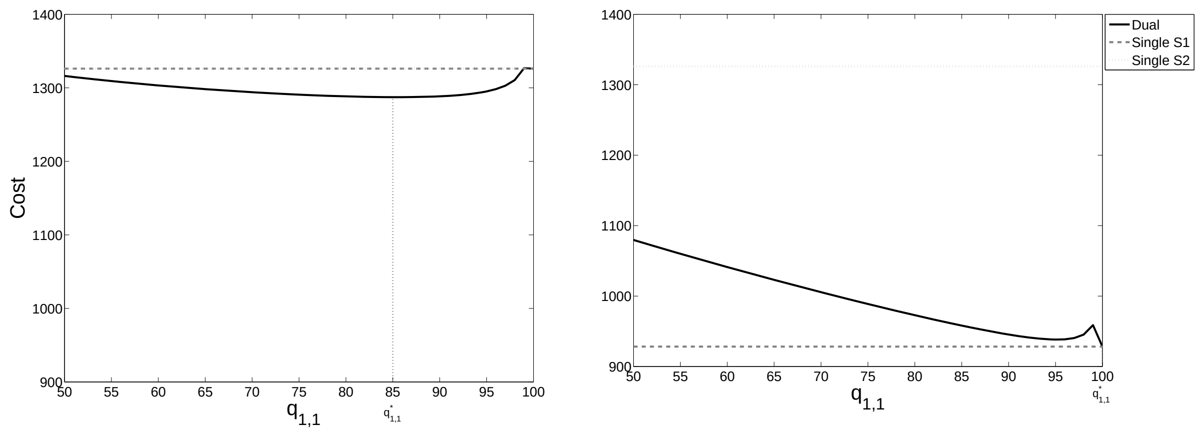


Figure 4.4: Optimal order allocation $q^*_{1,1}$ for various b and p (identical suppliers $d = 100$).

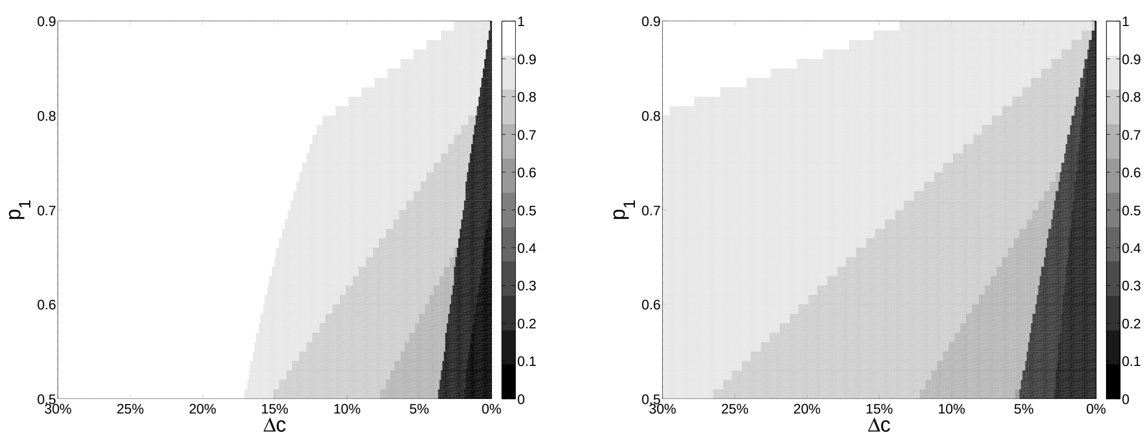
For a two-period problem, Figure 4.5 plots the total expected cost as a function of the first-period order allocation of supplier 1, q_1 (black curve) for $\Delta c = 0\%$ and 30%, and the expected total cost of single sourcing, where $q_{1,1} = d$ and $q_{2,1} = d$, respectively (dashed lines). The minimal expected costs for dual sourcing increase and the optimal order quantity assigned to supplier 1 increases with the cost advantage of supplier 1 ($\Delta c \uparrow$). Dual sourcing is preferred over single sourcing for $\Delta c = 0\%$, but as the cost advantage reaches a certain level, which is the case in Figure 3 when $\Delta c = 30\%$, the optimal dual sourcing strategy becomes single sourcing with supplier 1.

In Figure 4.6, we illustrate the optimal sourcing strategy as a function of initial cost difference Δc and supplier reliability p_1 when p_2 is fixed to 0.9. The white area is the region where supplier 1 gets more than 90% of demand and the black area is the region where supplier 2 gets more than 90% of demand. The split-up of total demand are presented by their respective grey shades.



(a) $\Delta c = 0\%$ (b) $\Delta c = 30\%$

Figure 4.5: Expected cost of dual sourcing as a function of $q_{1,1}$ compared to single sourcing from supplier 1 and 2 varying Δc ($c_{2,0} = 10,\ d = 100, p_1 = p_2 = 0.9, b_1 = b_2 = 0.3$).



(a) Learning factor $b = 0.1$ (b) Learning factor $b = 0.5$

Figure 4.6: Buyer's first period optimal order allocation to supplier 1 expressed as percentage of demand varying Δc and p with $c_{2,0} = 10, p_2 = 0.9$.

From Figure 4.6 we make the following observations: When the cheaper supplier 1 has a high cost advantage and a moderate survival probability the buyer will never choose to work with the expensive supplier and will always order from the cheap supplier. A high cost advantage of the cheap supplier favors single souring when the reliability is moderate and the learning factor is low.

Learning rates differ significantly across industry, but can even differ between firms in the same industry (see e.g. Li and Rajagopalan (1998), Hayes and Clark (1986)). The results of a two-period problem where we assume that the second supplier may have an initial cost disadvantage ($\Delta c > 0$) but a higher learning rate $b_1 < b_2$ are summarized in Table 4.2. An initial cost disadvantage accompanied by a learning ability advantage can lead to a volume advantage for supplier 2 in the first period ($q^*_{1,1} < d/2$), e.g. in the case $\Delta c = 10\%$ and $b_1 = 0.1, b_2 = 0.5$. However, when the initial cost advantage of supplier 1 is large, the higher cost improvements of supplier 2 do not pay off and supplier 1 will get the larger share of demand. The buyer can significantly reduce total expected cost through the optimal dual sourcing policy in the first period. The cost reduction through optimal dual sourcing is high, especially when learning is high, even under a cost advantage of supplier 1 of 20%. Intuitively, the savings of the optimal policy increase when learning increases and decrease when reliability decreases. Further, a 75:25 policy performs better than a 50:50 policy, especially when the suppliers are significantly heterogeneous.

			$\Delta c = 0$					$\Delta c = 10\%$					$\Delta c = 20\%$				
(p_1, p_2)	b_1	b_2	$q^*_{1,1}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}	$q^*_{1,1}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}	$q^*_{1,1}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}
(0.9,0.9)	0.1	0.1	89	0.8	0.8	3.2	2.8	100	0.0	11.1	8.1	4.9	100	0.0	25.0	15.2	8.6
		0.5	13	41.8	1.2	24.8	14.0	46	31.6	4.3	21.2	13.7	64	23.8	10.4	20.4	16.3
	0.3	0.3	85	3.0	3.0	7.1	6.5	92	1.7	13.0	11.5	8.2	94	0.4	25.5	17.4	10.6
		0.5	18	15.6	3.7	13.6	10.1	47	7.4	7.1	11.0	10.5	93	1.9	14.3	11.9	8.4
	0.5	0.5	82	5.3	5.3	8.6	8.2	90	3.7	15.2	12.9	9.7	93	2.3	27.9	18.7	12.0
(0.8,0.9)	0.1	0.1	10	2.9	0.6	4.1	3.1	90	0.3	9.0	7.0	4.5	100	0.0	22.3	13.7	7.8
		0.5	12	44.7	1.0	26.0	14.4	45	34.2	4.1	22.2	14.0	64	26.3	10.2	21.3	16.5
	0.3	0.3	13	8.4	2.6	9.3	7.2	85	4.8	10.3	11.5	9.6	90	2.6	21.4	16.1	11.0
		0.5	17	21.5	3.2	16.0	10.9	46	12.8	6.5	13.2	11.1	64	6.1	12.6	12.9	10.9
	0.5	0.5	17	12.5	4.6	11.6	9.2	84	8.9	12.5	13.9	12.7	89	6.7	23.9	18.6	14.1
(0.7,0.8)	0.1	0.1	17	3.7	1.5	4.6	3.7	87	0.8	9.6	7.2	4.7	100	0.0	22.4	13.4	7.5
		0.5	19	39.0	2.1	23.1	13.4	51	30.4	6.5	21.0	14.4	67	23.7	13.6	21.2	18.1
	0.3	0.3	21	10.8	5.2	11.3	9.3	81	7.0	12.9	13.3	11.5	87	4.3	23.8	17.5	12.3
		0.5	26	22.4	6.2	17.3	12.9	52	14.9	10.7	15.8	14.3	67	8.8	18.0	16.4	13.8
	0.5	0.5	24	16.1	8.4	14.9	12.6	81	12.3	16.6	17.1	15.8	86	9.5	27.9	21.6	16.8
(0.7,0.7)	0.1	0.1	75	2.6	2.6	4.5	4.2	89	0.5	11.7	8.0	4.9	100	0.0	25.0	14.5	8.0
		0.5	27	31.6	3.6	19.9	12.4	57	24.7	9.0	19.1	14.7	70	19.1	17.1	20.5	19.4
	0.3	0.3	72	8.4	8.4	11.4	10.9	83	5.8	17.6	14.8	11.4	88	3.4	29.3	19.5	12.7
		0.5	34	18.2	9.7	16.7	14.3	58	11.9	15.4	16.4	15.2	88	7.1	24.2	18.4	13.8
	0.5	0.5	69	13.0	13.0	15.4	15.2	82	10.3	22.5	18.9	15.6	87	7.8	34.8	23.9	16.9

Table 4.2: Optimal first period decision and savings (%) from dual sourcing, $T = 2$.

4.5.2 Multiple periods

Table 4.3 shows how the optimal allocation decision and the savings from dual sourcing depend on survival probability, learning rate, and the time horizon $T \geq 2$ for the case of identical suppliers, where $c_{1,0} = c_{2,0} = c_0, b_1 = b_2 = b$ and $p_1 = p_2 = p$ and, as supplier are identical, ordered experiences $x_{1,t} \geq x_{2,t}, \forall t$. Table 4.3 shows $q^*_{1,1}$ for a zero initial experience level and omits $q^*_{1,t}$ for $1 < t < T$, which depend on the supplier experience realizations in each stage. The initial order allocation $q^*_{1,1}$ decreases in T and as supplier costs decrease with experience, $q^*_{1,t}$ increases with time t if $x_{1,t} > x_{2,t}$ and supplier 1 accumulates more experience than supplier 2. The total savings from dual sourcing clearly increase with T. Thus, we state the per period saving from dual sourcing in Table 4.3 for a fair comparison, which decreases in T for a low learning rate and increases or decreases in T for a high learning rate.

The optimal order allocation in period t depends on whether both suppliers survived, one and which of the two suppliers survived, or no one survived. Thus, the optimal policy changes with the realization of the supplier experience levels (see Table 4.4, where the complete policy for $T = 3$ and the savings from dual sourcing conditioned on $\mathbf{x_2} = (x_{1,2}, x_{2,2})$ for the remaining two periods is shown). Further, it can be optimal for a buyer to return to dual after single sourcing in previous periods. This case can be explained as follows: Assume the system has reached a state where one of the two suppliers has accumulated enough experience so that it is optimal for the buyer to single source in that period. If then both suppliers disrupt ($\mathbf{x_{t+1}} = (0, 0)$) and there are still $k \geq 1$ periods to go, dual sourcing can be optimal and the optimal order quantity coincides with $q^*_{1,1}$ of a $k-$period problem. Note that also reaching a state where only one of the two suppliers survived can lead to dual after single sourcing, if the experience level of the survived supplier is not sufficient for single sourcing in that stage (see Figure 4.3).

		$b = 0.1$				$b = 0.3$				$b = 0.5$			
p	T	$q^*_{1,1}$	$\frac{\Delta^{DS}}{T}$	$\frac{\Delta^{50}}{T}$	$\frac{\Delta^{75}}{T}$	$q^*_{1,1}$	$\frac{\Delta^{DS}}{T}$	$\frac{\Delta^{50}}{T}$	$\frac{\Delta^{75}}{T}$	$q^*_{1,1}$	$\frac{\Delta^{DS}}{T}$	$\frac{\Delta^{50}}{T}$	$\frac{\Delta^{75}}{T}$
0.9	2	89	0.4	1.6	1.0	85	1.5	3.5	2.1	82	2.6	4.3	2.5
	3	86	0.4	1.5	0.9	82	1.7	3.7	2.2	78	3.1	4.8	2.7
	4	84	0.3	1.4	0.8	79	1.6	3.6	2.0	75	3.2	4.8	2.6
	5	81	0.3	1.2	0.7	77	1.5	3.3	1.8	73	3.1	4.7	2.4
0.7	2	75	1.3	2.3	1.2	72	4.2	5.7	3.1	69	6.5	7.7	4.1
	3	71	1.1	2.0	1.0	67	4.0	5.5	2.7	65	6.6	7.8	3.7
	4	68	0.9	1.7	0.8	65	3.5	4.9	2.2	63	6.1	7.2	3.2
	5	67	0.8	1.4	0.7	64	3.1	4.3	1.9	61	5.6	6.6	2.8

Table 4.3: Optimal order quantity allocation to supplier 1 depending on time horizon.

	$b = 0.1$					$b = 0.3$					$b = 0.5$				
p	$\mathbf{x_2}$	$q^*_{1,2}$	Δ^{DS}	Δ^{50}	Δ^{75}	$\mathbf{x_2}$	$q^*_{1,2}$	Δ^{DS}	Δ^{50}	Δ^{75}	$\mathbf{x_2}$	$q^*_{1,2}$	Δ^{DS}	Δ^{50}	Δ^{75}
0.9	(86,14)	99	1.6	9.7	5.3	(82,18)	99	10.1	33.6	18.0	(78,22)	99	33.3	69.0	36.3
	(86,0)	100	0.0	17.7	8.9	(82,0)	97	0.4	76.5	36.6	(78,0)	97	4.0	181.7	84.7
	(14,0)	97	0.1	11.0	5.6	(18,0)	97	0.9	47.6	22.9	(22,0)	97	3.5	115.9	54.1
	(0,0)	89	0.8	3.2	2.0	(0,0)	85	3.0	7.1	4.3	(0,0)	82	5.3	8.6	5.0
0.7	(71,29)	99	4.5	8.7	4.4	(67,33)	99	22.8	32.7	16.5	(65,35)	99	57.3	70.0	35.1
	(71,0)	95	0.9	16.3	7.2	(67,0)	95	6.3	64.1	27.8	(65,0)	95	17.5	131.9	56.8
	(29,0)	94	1.1	13.3	5.9	(33,0)	94	6.0	53.6	23.2	(35,0)	95	16.1	113.2	48.7
	(0,0)	75	2.6	4.5	2.5	(0,0)	72	8.4	11.4	6.2	(0,0)	69	13.0	15.4	8.2

Table 4.4: Optimal order quantity allocation depending on realized experience in period 2, $T = 3$.

The main results of the numerical experiments are summarized in the following observations.

OBSERVATION 4.2. *Dual sourcing can lead to significant savings that increase with learning ability and decrease with supplier reliability and the per period savings of dual sourcing decrease with the time horizon for low learning rates and first increase and then decrease with time horizon for high learning rates.*

OBSERVATION 4.3. *The optimal order allocation $q^*_{1,t}$ increases with time as supplier experience increases due to learning. But $q^*_{1,t}$ changes in the time horizon as it depends on the actual supplier experience, if no, one or both suppliers survived from the previous period. Thus, it can be optimal for the buyer to return to dual sourcing in later periods, even if single sourcing has been optimal in previous periods.*

4.6 Conclusion

This Chapter analyzed the optimal allocation decisions that depend on the suppliers' characteristics - initial cost difference, learning slope and reliability.

We show that, if the future value gained by allocating the demand to both sources is sufficient, dual sourcing is optimal. We find that, even though supplier cost declines with experience and one would expect the decision maker to source the total required quantity from a single supplier, the savings from dual sourcing can be significant if demand is allocated optimally between the two suppliers. Interestingly, this optimal allocation is not symmetric even in the case of symmetric suppliers. Moreover, we show cases where, even when a supplier has higher cost than the other supplier, it is optimal for the buyer to use dual sourcing to compensate for the risk of supplier disruptions. In addition, a supplier with a relatively fair initial cost disadvantage but a learning ability advantage can get larger parts of demand in the actual period. Further, we find that the savings of dual sourcing compared to single sourcing or other policies can be significant and are sensitive to the problem parameters.

As our model is limited to a supply base of two suppliers at most, future research should analyze a model with more than two suppliers to study the effect of the size of the supply base on the buyer's decision. Further, we only focus on the learning phenomenon and do not consider forgetting when e.g. disruption is only temporary, rather than permanent. Possible extensions could also allow for order quantities that are greater than the per period demand, i.e. an inventory carryover.

4.7 Appendix: Mathematical proofs

Proof. of Theorem 4.4.1. First we show that the objective function is convex in its decision variable for all t. The proof is by induction. Clearly, for $t = T$ the function

$$C_T(\mathbf{x_T}, q_{1,T}) = c(x_{1,T})q_{1,T} + c(x_{2,T})(d - q_{1,T})$$

is convex in $q_{1,T}$ as it is linear in $q_{1,T}$. For $t = T - 1$, the cost function is

$$\begin{aligned} C_{T-1}(\mathbf{x_{T-1}}, q_{1,T-1}) &= c_1(x_{1,T-1})q_{1,T-1} + c_2(x_{2,T-1})(d - q_{1,T-1}) + (1-p_1)(1-p_2)V_T(0,0) + \\ &\quad p_1(1-p_2)V_T(x_{1,T-1} + q_{1,T-1}, 0) + (1-p_1)p_2V_T(0, x_{2,T-1} + d - q_{1,T-1}) + \\ &\quad p_1p_2V_T(x_{1,T-1} + q_{1,T-1}, x_{2,T-1} + d - q_{1,T-1}) \end{aligned}$$

Recalling that

$$V_T(x_{1,T}, x_{2,T}) = \begin{cases} c_1(x_{1,T})d & \text{if} \quad c_1(x_{1,T}) \leq c_2(x_{2,T}) \\ c(x_{2,T})d & \textit{otherwise} \end{cases}$$

the first and the second derivatives are

$$\begin{aligned} &\frac{\partial C_{T-1}(\mathbf{x_{T-1}}, q_{1,T-1})}{\partial q_{1,T-1}} = \\ &\quad = c_1(x_{1,T-1}) - c_2(x_{2,T-1}) + p_1(1-p_2)dc_1'(x_{1,T-1} + q_{1,T-1}) - p_2(1-p_1)dc_1'(x_{2,T-1} + d - q_{1,T-1}) \\ &\quad +p_1p_2 \begin{cases} dc_1'(x_{1,T-1} + q_{1,T-1}), & \text{if} \quad c_1(x_{1,T}) \leq c_2(x_{2,T}) \\ -dc_2'(x_{2,T-1} + d - q_{1,T-1}), & \text{otherwise} \end{cases} \end{aligned}$$

$$\begin{aligned} &\frac{\partial^2 C_{T-1}(\mathbf{x_{T-1}}, q_{1,T-1})}{\partial q_{1,T-1}^2} = \\ &\quad = \begin{cases} p_1dc_1''(x_{1,T-1} + q_{1,T-1}) + (1-p_1)p_2dc_2''(x_{1,T-1} + d - q_{1,T-1}) > 0 & \text{if} \quad c_1(x_{1,T}) \leq c_2(x_{2,T}) \\ p_1(1-p_2)dc_1''(x_{1,T-1} + q_{1,T-1}) + p_2dc_2''(x_{1,T-1} + d - q_{1,T-1}) > 0 & \text{otherwise} \end{cases} \end{aligned}$$

For $t = T - 1$, the cost function $C_{T-1}(\mathbf{x_{T-1}}, q_{1,T-1})$ is convex in $q_{1,T-1}$ due to the convexity of the supplier cost functions. Assume, that for a general $t + 1$ that $C_{t+1}(\mathbf{x_{t+1}}, q_{1,t+1})$ is convex in $q_{1,t+1}$. To complete the proof, it has to be shown that $C_t(\mathbf{x_t}, q_{1,t})$ is convex in $q_{1,t}$. As $C_{t+1}(\mathbf{x_{t+1}}, q_{1,t+1})$ is convex (sum of convex cost function) it follows that $\partial^2 C_t(\mathbf{x_t}, q_{1,t})/\partial q_{1,t}^2 = p_1p_2V_{t+1}''(x_{1,t}+q_{1,t}, x_{2,t}+d-q_{1,t}) + p_1(1-p_2)V_{t+1}''(x_{1,t}+q_{1,t}, 0) + (1-p_1)p_2V_{t+1}''(0, x_{2,t}+d-q_{1,t}) > 0$ and $C_t(\mathbf{x_t}, q_{1,t})$ is convex in $q_{1,t}$ which completes the proof.

The Lagrangian function for general $t < T$ subject to the constraint $0 \leq q_{1,t} \leq d$ is

$$L = c_1(x_{1,t})q_{1,t} + c_2(x_{2,t})(d - q_{1,t}) + E(V_{t+1}(x_{1,t} + q_{1,t}, x_{2,t} + d - q_{1,t})) - \lambda_1 q_{1,t} + \lambda_2(q_{1,t} - d)$$

where λ_1 and λ_2 are the Lagrangian multipliers. The optimality condition is

$$\frac{\partial L}{\partial q_{1,t}} = c_1(x_{1,t}) - c_2(x_{2,t}) + E(V'_{t+1}(x_{1,t} + q_{1,t}, x_{2,t} + d - q_{1,t})) - \lambda_1 + \lambda_2 \overset{!}{=} 0$$

with the complementary slackness conditions

$$\lambda_1 \geq 0 \quad \text{and} \quad \lambda_1(-q_{1,t}) = 0,$$
$$\lambda_2 \geq 0 \quad \text{and} \quad \lambda_2(q_{1,t} - d) = 0$$

leading to three feasible solutions:
(i) $\lambda_1^* > 0, \lambda_2^* = 0$ and $q_{1,t}^* = 0$
(ii) $\lambda_1^* = 0, \lambda_2^* > 0$ and $q_{1,t}^* = d$
(iii) $\lambda_1^* = 0$ and $\lambda_2^* = 0$ and $0 < q_{1,t}^* < d$ the solution to

$$c_1(x_{1,t}) - c_2(x_{2,t}) + E(V'_{t+1}(x_{1,t} + q_{1,t}, x_{2,t} + d - q_{1,t})) = 0 \Rightarrow$$

$$\begin{aligned} c_2(x_{1,t}) - c_1(x_{2,t}) &= p_1 p_2 V'_{t+1}(x_{1,t} + q_{1,t}, x_{2,t} + d - q_{1,t})) + p_1(1-p_2)V'_{t+1}(x_{1,t} + q_{1,t}, 0)) + \\ &\quad (1-p_1)p_2 V'_{t+1}(x_{1,t} + q_{1,t}, 0)) \end{aligned}$$

□

Proof. of Lemma 4.4.2. We use the implicit function theorem on equation (4.5): $c_{2,0} - c_{1,0} = p_1 d c_1'(q_{1,1}^*) - p_2(1-p_1) d c_2'(d - q_{1,1}^*)$ (i.e. for the case $c_1(q_{1,1}^*) \leq c_2(d - q_{1,1}^*)$, but for $c_1(q_{1,1}^*) > c_2(d - q_{1,1}^*)$ it is analogous).

$$\begin{aligned} \frac{\partial q_{1,1}^*}{\partial p_1} &= -\frac{d c_1'(q_{1,1}^*) + p_2 d c_2'(d - q_{1,1}^*)}{p_1 d c_1''(q_{1,1}^*) + p_2(1-p_1) d c_2''(d - q_{1,1}^*)} > 0 \\ \frac{\partial q_{1,1}^*}{\partial p_2} &= -\frac{(p_1 - 1) d c_2'(d - q_{1,1}^*)}{p_1 d c_1''(q_{1,1}^*) + p_2(1-p_1) d c_2''(d - q_{1,1}^*)} < 0 \end{aligned}$$

Using the power learning function $c(x) = c_0 x^{-b}$

$$\begin{aligned} \frac{\partial q_{1,1}^*}{\partial d} &= -\frac{p_1 c_1'(q_{1,1}^*) - p_2(1-p_1)(c_2'(d - q_{1,1}^*) + \frac{\partial^2 c_2(d - q_{1,1}^*)}{\partial q_{1,1}^* \partial d})}{p_1 d c_1''(q_{1,1}^*) + p_2(1-p_1) d c_2''(d - q_{1,1})} \\ &= -\frac{-p_1 c_{1,0} b_1 (q_{1,1}^*)^{-b_1 - 1} - p_2(1-p_1) c_{2,0} b_2 (d - q_{1,1}^*)^{-b_2 - 2}(q_{1,1}^* + b_2 d)}{p_1 d c_1''(q_{1,1}^*) + p_2(1-p_1) d c_2''(d - q_{1,1})} > 0 \end{aligned}$$

$$\begin{aligned}
\frac{\partial q^*_{1,1}}{\partial b_1} &= -\frac{c_{1,0} d p_1 q^{*-b_1-1}_{1,1}(b_1 \ln(q^*_{1,1}) - 1)}{p_1 d c''_1(q^*_{1,1}) + p_2(1-p_1) d c''_2(d - q^*_{1,1})} \\
&\Rightarrow \frac{\partial q^*_{1,1}}{\partial b_1} < (>) 0 \Leftrightarrow b_1 \ln(q^*_{1,1}) > (<) 1 \\
\frac{\partial q^*_{1,1}}{\partial b_2} &= -\frac{-c_{2,0} d p_2 (1-p_1)(d - q^*_{1,1})^{-b_2-1}(b_2 \ln(d - q^*_{1,1}) - 1)}{p_1 d c''_1(q^*_{1,1}) + p_2(1-p_1) d c''_2(d - q^*_{1,1})} \\
&\Rightarrow \frac{\partial q^*_{1,1}}{\partial b_2} > (<) 0 \Leftrightarrow b_2 \ln(d - q^*_{1,1}) > (<) 1
\end{aligned}$$

□

Proof. of Lemma 4.4.3. We use the implicit function theorem on equation (4.6): $c'(q^*_{1,1}) = (1-p)c'(d - q^*_{1,1})$

$$\begin{aligned}
\frac{\partial q^*_{1,1}}{\partial p} &= -\frac{p d c'(d - q^*_{1,1})}{c''(q^*_{1,1}) + (1-p)c''(d - q^*_{1,1})} > 0 \\
\frac{\partial q^*_{1,1}}{\partial d} &= -\frac{-(1-p)\frac{\partial^2 c(d - q^*_{1,1})}{\partial q^*_{1,1} \partial d}}{c''(q^*_{1,1}) + (1-p)c''(d - q^*_{1,1})} > 0
\end{aligned}$$

$\frac{\partial q^*_{1,1}}{\partial b} < 0$ can be directly seen from the explicit solution (4.12). (iii) and (iv) are straightforward.

□

Proof. of Lemma 4.4.4. We use equations (4.9) and (4.10) to show (i) and (ii), taking the partial derivatives and using the results from Lemma 4.4.2.

(i) From (4.9) we define $V_1^{\text{single } 1} - V_1 = \Delta$ by

$$\Delta = p_2(1-p_1) d c_{1,0} - ((c_{1,0} - c_{2,0})(q^*_{1,1} - d) + p_1 d (c_1(q^*_{1,1}) - c_1(d)) + (1-p_1) p_2 d c_2(d - q^*_{1,1}))$$

$$\begin{aligned}
\frac{\partial \Delta}{\partial p_1} &= -d p_2 c_{1,0} - ((c_{1,0} - c_{2,0})\frac{\partial q^*_{1,1}}{\partial p_1} + d(c_1(q^*_{1,1}) - c_1(d)) + p_1 d c'(q^*_{1,1})\frac{\partial q^*_{1,1}}{\partial p_1} \\
&\quad - p_2 d c_2(d - q^*_{1,1})) - (1-p_1) p_2 d c'_2(d - q^*_{1,1})\frac{\partial q^*_{1,1}}{\partial p_1}) \\
&= -d(p_2((c_{1,0} - c_2(d - q^*_{1,1})) + (c_1(q^*_{1,1}) - c_1(d)) \\
&\quad - \frac{\partial q^*_{1,1}}{\partial p_1}((c_{1,0} - c_{2,0}) + p_1 d c'(q^*_{1,1}) - (1-p_1) p_2 d c'_2(d - q^*_{1,1})) \\
&= -d(p_2((c_{1,0} - c_2(d - q^*_{1,1})) + (c_1(q^*_{1,1}) - c_1(d)) - 0 \\
&\quad \Rightarrow \frac{\partial \Delta}{\partial p_1} < (>) 0 \text{ if } p_2(c_{1,0} - c_2(d - q^*_{1,1})) > (<) c_1(d) - c_1(q^*_{1,1})
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Delta}{\partial p_2} &= (1-p_1)dc_{1,0} - ((c_{1,0}-c_{2,0})\frac{\partial q^*_{1,1}}{\partial p_2} + p_1 dc'(q^*_{1,1})\frac{\partial q^*_{1,1}}{\partial p_2} \\
&\quad +(1-p_1)dc_2(d-q^*_{1,1})) - (1-p_1)p_2 dc'_2(d-q^*_{1,1})\frac{\partial q^*_{1,1}}{\partial p_2}) \\
&= (1-p_1)d(c_{1,0} - c_2(d-q^*_{1,1})) \\
&\quad -\frac{\partial q^*_{1,1}}{\partial p_2}((c_{1,0}-c_{2,0}) + p_1 dc'(q^*_{1,1}) - (1-p_1)p_2 dc'_2(d-q^*_{1,1})) \\
&= (1-p_1)d(c_{1,0} - c_2(d-q^*_{1,1})) - 0 \\
&\quad \Rightarrow \frac{\partial \Delta}{\partial p_2} > (<) 0 \text{ if } c_2(d-q^*_{1,1}) < (>) c_{1,0}
\end{aligned}$$

Analogous, when $V_1^{\text{single}*} = V_1^{\text{single } 2}$ then the advantage of dual sourcing increases (decreases) in p_1 if $c_1(q^*_{1,1}) < (>) c_{2,0}$ and decreases (increases) in p_2 if $p_1(c_{2,0} - c_1(q^*_{1,1})) > (<) c_2(d) - c2(d - q^*_{1,1})$. □

Proof. of Lemma 4.4.5. From (4.10) define $V_1^{single} - V_1 = \Delta$ by

$$\Delta = (1-p)c_0 - (c(q^*_{1,1}) - c(d) + (1-p)c(d-q^*_{1,1}))$$

$$\begin{aligned}
\frac{\partial \Delta}{\partial p} &= c(d-q^*_{1,1}) - c_0 - \frac{\partial q^*_{1,1}}{\partial p}(c'(q^*_{1,1}) - (1-p)c'(d-q^*_{1,1})) \\
&= c(d-q^*_{1,1}) - c_0 < 0
\end{aligned}$$

□

Chapter 5

Disruption Risk and Cost Improvements through Learning: Sensitivity Analysis and Implications

In this Chapter, the sensitivity of the model presented in Chapter 4 with regard to three limiting assumptions is discussed: known reliability of suppliers, known demand, and a risk neutral decision maker.

5.1 Introduction

We extend the model dealing with supply disruption risk and cost improvements trough learning of Chapter 4 to the following assumptions.

First, we relax the assumption that the buyer knows the true reliability of the two suppliers. When the buying firms do not know the true distributions of the supplier reliabilities they have to make their sourcing decision based on the beliefs they have about the supply uncertainty. However, they can learn about the true supplier reliabilities based on the experience gained with the suppliers.

Second, we study the impact of demand uncertainty assuming that the buyer does not know the exact demand at the point of time when the sourcing and volume allocation decision has to be done.

Finally, we relax the assumption of a risk neutral decision maker by allowing for risk aversion in the sourcing and volume allocation decision.

The central research questions in this Chapter are:

- What are the implications of these model extensions on the optimal sourcing and volume allocation policy?
- What are the implications of these model extensions on the benefit from dual sourcing compared to single sourcing, equal split or 75:25 split dual sourcing?

In order to demonstrate the main effects, we limit the following analysis of the dynamic decision problem to the minimum number of periods, i.e. two or three periods planning horizons, respectively.

This Chapter is organized as follows: In Section 5.2 we discuss the impact of unknown survival probability on the optimal policy for a three period problem. Section 5.3 we incorporate stochastic demand into the basic model assumptions for a two period problem. In Section 5.4 the impact of a risk averse decision maker on the optimal two period policy is analyzed. In 5.5 we summarize the results. The proofs of the results are provided in 5.6.

5.2 Unknown survival probability

This Section considers the case where the true supplier reliabilities are unknown for the buyer. Thereby, we present a Bayesian analysis by describing the uncertainty about the unknown supplier survival probability probabilistically.

5.2.1 Model formulation

We assume that the buyer has imperfect information of the supplier reliability using Bayesian updating of the unknown parameter. For reasons of simplicity, we consider the case of identical suppliers and $T = 3$. The buyer has prior information about the supply disruption probability, i.e., at the beginning of the planning horizon, the buyer has a certain belief on the unknown suppliers' survival probability p represented by the probability density function $f(p) = \text{prior}(p)$ - which will be updated after each period's supply base observation. Specifically, assume that the prior distribution for p in period one is $Beta(\alpha_1, \beta_1)$ with $\alpha_1 > 0$, $\beta_1 > 0$ and density

$$f(p \mid \alpha_1, \beta_1) = \frac{1}{B(\alpha_1, \beta_1)} p^{\alpha_1 - 1}(1-p)^{\beta_1 - 1}, \quad 0 \leq p \leq 1$$

where $1/B(\alpha_1, \beta_1) = \Gamma(\alpha_1 + \beta_1)/\Gamma(\alpha_1)\Gamma(\beta_1)$ and $\Gamma(.)$ is the gamma function. The expected value $E(p)$ and variance $Var(p)$ of the prior are

$$E(p) = \frac{\alpha_1}{\alpha_1 + \beta_1} \quad \text{and} \quad Var(p) = \frac{\alpha_1 \beta_1}{(\alpha_1 + \beta_1)^2(\alpha_1 + \beta_1 + 1)}.$$

Based on this initial belief in the first period, the buyer makes an allocation decision for meeting current demand. Thereby, the right choice of the prior is relevant: When $\alpha_1 < \beta_1$ the density has more weight on the upper half, the opposite is true when $\alpha_1 > \beta_1$. When $\alpha_1 = \beta_1$ the density is symmetric. Note that when $\alpha_1 = \beta_1 = 1$ then the prior becomes uniform, which would be a non-informative prior (see Berger (2004)).

At the beginning of the next period the buyer observes the number of suppliers that survived to the next period $n = \{0, 1, 2\}$ and updates the belief about p according to the observation n. The prior distribution, and the observed n lead to the posterior distribution at the beginning of period 2

$$f(p \mid \alpha_1, \beta_1, n) = \frac{1}{B(\alpha_1 + n, 2 - n + \beta_1)} p^{\alpha_1 + n - 1}(1-p)^{2-n+\beta_1-1}, \quad 0 \leq p \leq 1,$$

which is $Beta(\alpha_1 + n, \beta_1 + 2 - n)$ and has a mean of $(\alpha_1 + n)/(\alpha_1 + \beta_1 + 2)$. Thus, the resulting posterior distribution with an update on the supplier reliability is again a beta distribution with new parameters obtained by adding the number of survived suppliers to α_1 to get $\alpha_2 = \alpha_1 + n$ and the number of disrupted suppliers to β_1 to get $\beta_2 = \beta_1 + 2 - n$ at the beginning of period 2.

The dynamic recursion for $t = 1, ..., 3$ with V_t denoting the optimal cost for the remaining periods is

$$\begin{aligned}
&V_t(\mathbf{x_t}, \alpha_t, \beta_t) = \\
&\quad \min_{0 \leq q_{1,t} \leq d} \{c(x_{1,t})q_{1,t} + c(x_{2,t})(d - q_{1,t}) + \int_0^1 f(p \mid \alpha_t, \beta_t)[(1-p)^2 V_{t+1}(0, 0, \alpha_t, \beta_t + 2) \\
&\quad + p(1-p)(V_{t+1}(x_{1,t} + q_{1,t}, 0, \alpha_t + 1, \beta_t + 1) + V_{t+1}(0, x_{2,t} + d - q_{1,t}, \alpha_t + 1, \beta_t + 1)) \\
&\quad + p^2 V_{t+1}(x_{1,t} + q_{1,t}, x_{2,t} + d - q_{1,t}, \alpha_t + 2, \beta_t)]\mathrm{d}p\}, \qquad (5.1)
\end{aligned}$$

with terminal condition $V_4(\mathbf{x_4}, \alpha_4, \beta_4) = 0$.

The right-hand side of (5.1) can be simplified (see Appendix) to

$$\begin{aligned}
&V_t(\mathbf{x_t}, \alpha_t, \beta_t) = \\
&\quad \min_{0 \leq q_{1,t} \leq d} \{c(x_{1,t})q_{1,t} + c(x_{2,t})(d - q_{1,t}) + \frac{(\beta_t + 1)\beta_t}{(\alpha_t + \beta_t + 1)(\alpha_t + \beta_t)} V_{t+1}(0, 0, \alpha_t, \beta_t + 2) \\
&\quad + \frac{\alpha_t \beta_t}{(\alpha_t + \beta_t + 1)(\alpha_1 + \beta_1)} V_{t+1}(x_{1,t} + q_{1,t}, 0, \alpha_t + 1, \beta_t + 1) \\
&\quad + \frac{\alpha_t \beta_t}{(\alpha_t + \beta_t + 1)(\alpha_1 + \beta_1)} V_{t+1}(0, x_{2,t} + d - q_{1,t}, \alpha_t + 1, \beta_t + 1) \\
&\quad + \frac{(\alpha_t + 1)\alpha_t}{(\alpha_t + \beta_t + 1)(\alpha_t + \beta_t)} V_{t+1}(x_{1,t} + q_{1,t}, x_{2,t} + d - q_{1,t}, \alpha_t + 2, \beta_t)\}, \qquad (5.2)
\end{aligned}$$

Note that if the buyer chooses a single sourcing strategy with supplier i in period t, then supplier i will survive to the next period with probability $\frac{\alpha_t}{\alpha_t + \beta_t}$, simply the expected value, and disrupt with probability $\frac{\beta_t}{\alpha_t+\beta_t}$. The value function based on that fact can be reduced to

$$V_t(\mathbf{x_t}, \alpha_t, \beta_t) = c(x_{i,t})d + \frac{\beta_t}{\alpha_t + \beta_t} V_{t+1}(0, \alpha_t, \beta_t + 1) + \frac{\alpha_t}{\alpha_t + \beta_t} V_{t+1}(x_{i,t} + d, \alpha_t + 1, \beta_t).$$

5.2.2 Structural properties

Starting at $T = 3$, there is no risk of a future disruption. Thus, $q^*_{1,3}$ is independent from the probability distribution of the unknown parameter p and single sourcing with the cheaper supplier is optimal in accordance with the results in the previous Chapter and $V_3(\mathbf{x_3})$ coincides with (4.3). For the second and first period, the optimal (interior) procurement quantity for dual sourcing ($0 < q^*_{1,t} < d$) solves

$$\begin{aligned} c(x_{2,t}) - c(x_{1,t}) = & \frac{(\alpha_t + 1)\alpha_t}{(\alpha_t + \beta_t + 1)(\alpha_t + \beta_t)} V'_{t+1}(x_{1,t} + q^*_{1,t}, x_{2,t} + d - q^*_{1,t}, \alpha_t + 2, \beta_t) + \\ & \frac{\alpha_t \beta_t}{(\alpha_t + \beta_t + 1)(\alpha_t + \beta_t)} V'_{t+1}(x_{1,t} + q^*_{1,t}, 0, \alpha_t + 1, \beta_t + 1) + \\ & \frac{\alpha_t \beta_t}{(\alpha_t + \beta_t + 1)(\alpha_t + \beta_t)} V'_{t+1}(0, x_{2,t} + d - q^*_{1,t}, \alpha_t + 1, \beta_t + 1). \end{aligned} \tag{5.3}$$

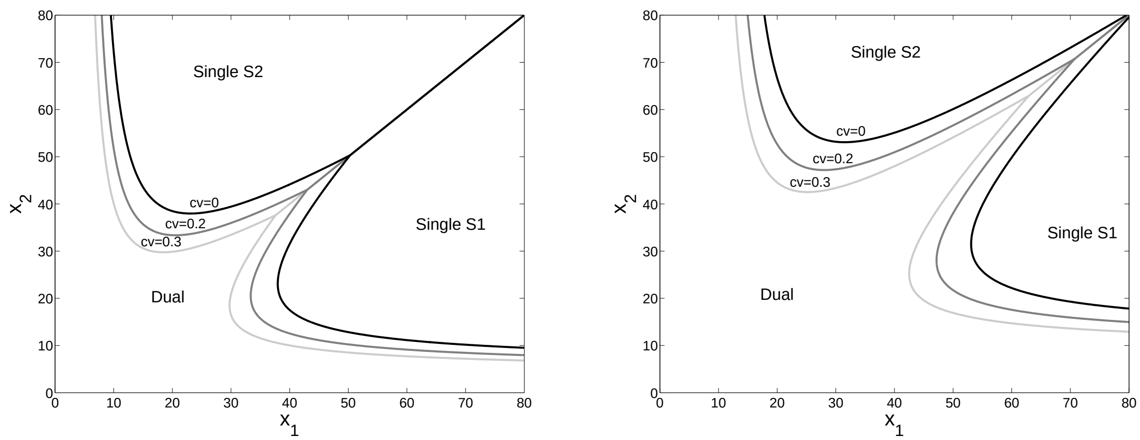
Interpreting (5.3), the marginal current supply cost difference has to equal the future value from dual sourcing in consideration of the probability distribution of the unknown parameter p. At period 2, $q^*_{1,2}$ is obtained by solving (5.3) and is affected by the observation after the first period, since the posterior distribution of the survival probability p depends on n.

For period $T-1$, we plot the indifference curves L_1 and L_2 stating the regions for single and dual sourcing for different values of α and β in the second period fixing the mean. Figure 5.1 indicates the following.

OBSERVATION 5.1. *The dual sourcing region is increasing with increasing information about the supplier survival probability p.*

5.2.3 Numerical example and discussion

To answer our research questions regarding the optimal order allocation and the value of dual sourcing under unknown survival probability, we complement the above results numerically. We consider the same parameter values as in Chapter 4 with identical suppliers ($c_0 = 10, d = 100$) and different parameters of the prior (α_1, β_1), with an expected value of the prior $E(p) \in \{0.9, 0.7\}$. To indicate the value of Bayesian updating, we fix the mean and let the variance decrease as e.g. $(\alpha_1, \beta_1) = (3, 1/3), (6, 2/3)$ and also state the results under perfect information, assuming that p is known and equal to the mean of the prior $\alpha_1/(\alpha_1 + \beta_1)$. Moreover, we state the results of a non-informative prior, where $\alpha_1 = \beta_1 = 1$ and compare it to $\alpha_1 = \beta_1 = 2$ and the perfect



(a) $b = 0.1$ (b) $b = 0.5$

Figure 5.1: Optimal sourcing strategy $t = T - 1$ in the (x_1, x_2)-plane with $E(p) = 0.7, c_0 = 10, d = 100$ varying cv.

information case with $p = 0.5$. Table 5.1 shows the optimal decision under Bayesian updating, total expected cost and savings of the optimal policy compared to single sourcing, an equal, and a 75:25 split of a three period problem. The complete policy is listed: the optimal first period allocation and the optimal second period allocations for the given supplier experience realizations (both, one or no one survived: $(\mathbf{x_2} = (q^*_{1,1}, d - q^*_{1,1}), \mathbf{x_2} = (q^*_{1,1}, 0), \mathbf{x_2} = (d - q^*_{1,1}, 0), \mathbf{x_2} = (0, 0))$.

OBSERVATION 5.2. *Knowledge about the supplier's reliability balances the initial order quantity allocation.*

We observe decreasing initial allocations to the supplier with the largest volume when the coefficient of variation of the survival probability decreases. The second period allocation depends on the prior distribution and the observation after the first period and we observe both, slightly increasing or decreasing allocations to the main supplier.

OBSERVATION 6. *The savings of the optimal policy compared to single sourcing, equal split, and 75:25 split increase with reliability information.*

Consequently, the savings of dual sourcing compared to single sourcing and an equal split increase with information on p. Put the other way, with increasing uncertainty regarding supplier reliability, the heuristic policies perform better.

			$b = 0.1$					$b = 0.5$				
$E(p)$	$Var(p)$	(α_1, β_1)	$q^*_{1,1}$	$q^*_{1,2}(\mathbf{x_2})$	Δ^{DS}	Δ^{50}	Δ^{75}	$q^*_{1,1}$	$q^*_{1,2}(\mathbf{x_2})$	Δ^{DS}	Δ^{50}	Δ^{75}
0.9	0.02	(3,1/3)	90	(99,96,97,95)	0.7	4.1	2.7	82	(99,96,95,66)	6.5	11.6	6.5
0.9	0.01	(6,2/3)	88	(99,97,95,76)	0.9	4.3	2.7	80	(99,96,96,70)	7.6	12.7	7.1
0.9	0	perfect inf.	86	(99,100,97,89)	1.1	4.6	2.8	78	(99,97,97,82)	9.3	14.5	8.0
0.7	0.04	(2.8,1.2)	76	(99,95,94,67)	2.6	5.1	2.7	69	(99,95,95,63)	15.0	18.5	9.0
0.7	0.02	(5.6,2.4)	74	(99,95,94,70)	2.9	5.5	2.8	67	(99,95,95,65)	17.1	20.7	10.0
0.7	0	perfect inf.	71	(99,95,94,75)	3.4	6.0	3.0	65	(99,95,95,69)	20.0	23.5	11.2
0.5	0.08	(1,1)	71	(99,95,95,61)	2.6	4.3	2.2	65	(99,95,95,58)	12.3	14.3	6.8
0.5	0.05	(2,2)	67	(99,95,94,62)	3.2	5.0	2.4	62	(99,95,95,59)	15.2	17.3	8.2
0.5	0	perfect inf.	62	(99,94,94,65)	4.2	5.9	2.8	58	(93,94,94,61)	19.8	22.0	10.2

Table 5.1: Results Bayesian updating vs. perfect information and savings of optimal policy.

5.3 Stochastic demand

This Section extends the basic model assuming that demand is a random variable defined by a demand distribution.

5.3.1 Model formulation

We now assume stochastic demand for a two-period model. At the beginning of each period - when the orders $\mathbf{q_t} = (q_{1,t}, q_{2,t}), t = 1, 2$ need to be placed - the actual demand is unknown. However, the buyer knows the distribution of the demands $D_t, t = 1, 2$, which are assumed to be non-stationary and independent over successive periods. $f_t(d_t)$ denotes the probability density function and $F_t(d_t)$ the cumulative density function of the random demand in period t, μ_t its mean, σ_t its standard deviation and d_1 and d_2 are the realizations of the first and second period demands. To simplify our analysis, we assume that the buyer does not carry strategic inventory into future periods. Therefore, supplier experiences $\mathbf{x_t}$ are still the only required state variables. Unsatisfied demand is lost and subject to a penalty cost π and any leftover units are discarded.

Let $L(q)$ denote the expected penalty cost with q units ordered. This can be written as

$$L(q) = \pi \int_q^\infty (d - q) f(d) \mathrm{d}d = \pi(\mu - q) + \pi \int_0^q F(d) \mathrm{d}d$$

The dynamic recursion for $t = 1, 2$ is

$$\begin{aligned} V_t(\mathbf{x_t}) = \min_{q_{1,t}, q_{2,t} \geq 0} \{ & c_1(x_{1,t})q_{1,t} + c_2(x_{2,t})q_{2,t} + L(q_{1,t} + q_{2,t}) + p_1 p_2 V_{t+1}(x_{1,t} + q_{1,t}, x_{2,t} + q_{2,t}) + \\ & p_1(1 - p_2)V_{t+1}(x_{1,t} + q_{1,t}, 0) + (1 - p_1)p_2 V_{t+1}(0, x_{2,t} + q_{2,t}) + (1 - p_1)(1 - p_2)V_{t+1}(0, 0) \}, \end{aligned} \tag{5.4}$$

with terminal condition $V_3(\mathbf{x_3}) = 0$.

5.3.2 Structural properties

In the second period, the problem reduces to the standard newsvendor problem selecting the order quantity that minimizes total cost. The buyer's optimization problem in period two for realized supplier experiences $\mathbf{x_2}$ is given by

$$V_2(\mathbf{x_2}) = \min_{q_{1,2},q_{2,2}\geq 0}\{c_1(x_{1,2})q_{1,2} + c_2(x_{2,2})q_{2,2} + L(q_{1,2}+q_{2,2})\}.$$

The optimal decision and optimal cost in the second period are

$$(q^*_{1,2}, q^*_{2,2}) = \begin{cases} (F^{-1}(\frac{\pi - c_1(x_{1,2})}{\pi}), 0), & \text{if} \quad c_1(x_{1,2}) \leq c_2(x_{2,2}) \\ (0, F^{-1}(\frac{\pi - c_2(x_{2,2})}{\pi})), & \text{otherwise,} \end{cases} \tag{5.5}$$

$$V_2(\mathbf{x_2}) = \begin{cases} c_1(x_{1,2})q^*_{1,2} + L(q^*_{1,2}), & \text{if} \quad c_1(x_{1,2}) \leq c_2(x_{2,2}) \\ c_2(x_{2,2})q^*_{2,2} + L(q^*_{2,2}), & \text{otherwise.} \end{cases} \tag{5.6}$$

In the first period, the buyer needs to solve (5.4) for $t = 1$. The Karush-Kuhn-Tucker conditions are $q^*_{1,1} \geq 0$, $q^*_{2,1} \geq 0$, and

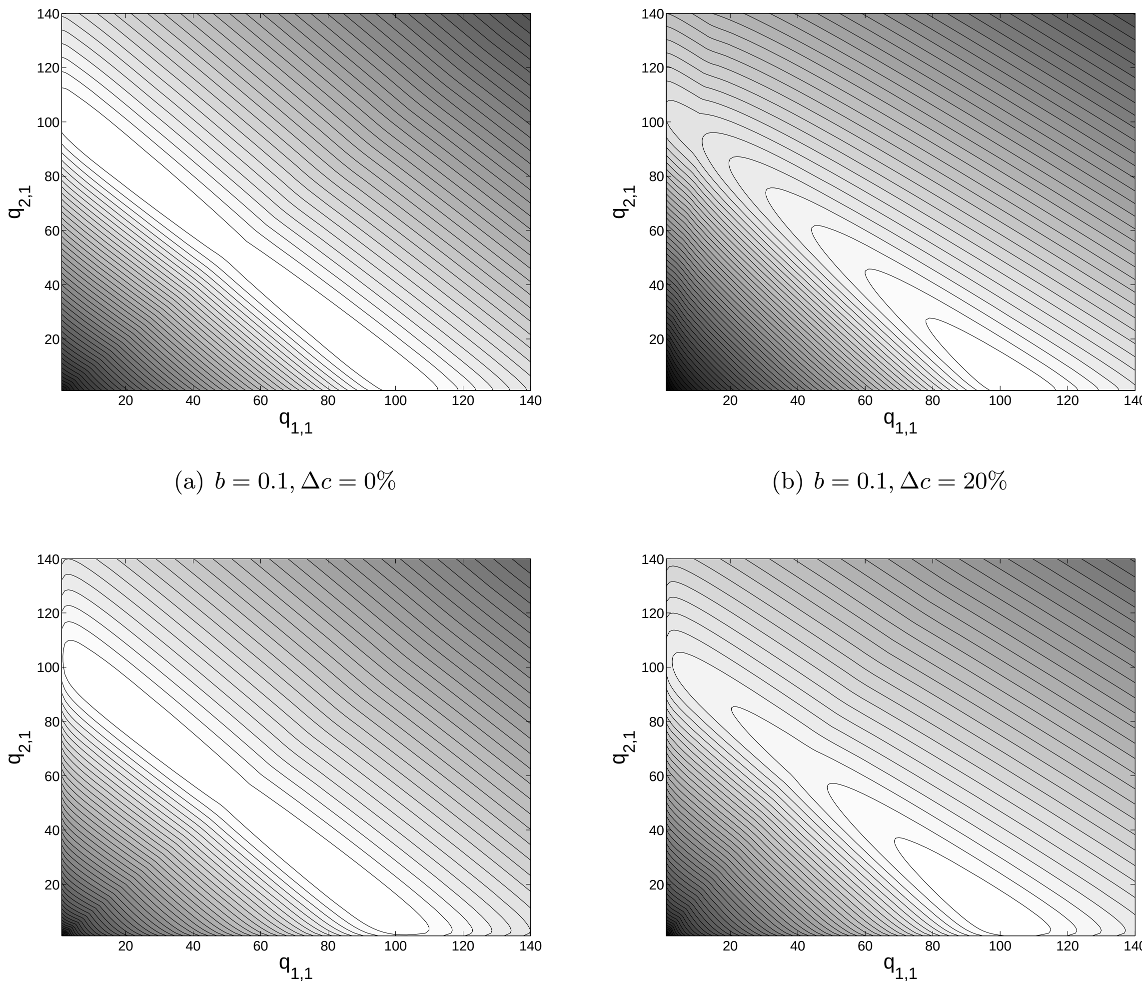
$$\begin{aligned} & q^*_{1,1}[c_{1,0} - \pi(1 - F(q^*_{1,1} + q^*_{2,1})) + \\ & \quad p_1 \begin{cases} (c'_1(q^*_{1,1})q^*_{1,2} + c_1(q^*_{1,1})\frac{\partial q^*_{1,2}}{\partial q^*_{1,1}} + \frac{\partial L(q^*_{1,2})}{\partial q^*_{1,2}}\frac{\partial q^*_{1,2}}{\partial q^*_{1,1}})] = 0, \text{if } c_1(q^*_{1,1}) \leq c_2(q^*_{2,1}) \\ (1-p_2)(c'_1(q^*_{1,1})q^*_{1,2} + c_1(q^*_{1,1})\frac{\partial q^*_{1,2}}{\partial q^*_{1,1}} + \frac{\partial L(q^*_{1,2})}{\partial q^*_{1,2}}\frac{\partial q^*_{1,2}}{\partial q^*_{1,1}})] = 0, \text{otherwise} \end{cases}, \end{aligned} \tag{5.7}$$

$$\begin{aligned} & q^*_{2,1}[c_{2,0} - \pi(1 - F(q^*_{1,1} + q^*_{2,1})) + \\ & \quad p_2 \begin{cases} (c'_2(q^*_{2,1})q^*_{2,2} + c_2(q^*_{2,1})\frac{\partial q^*_{2,2}}{\partial q^*_{2,1}} + \frac{\partial L(q^*_{2,2})}{\partial q^*_{2,2}}\frac{\partial q^*_{2,2}}{\partial q^*_{2,1}})] = 0, \text{if } c_1(q^*_{1,1}) > c_2(q^*_{2,1}) \\ (1-p_1)(c'_2(q^*_{2,1})q_{2,2} + c_2(q^*_{2,1})\frac{\partial q^*_{2,2}}{\partial q^*_{2,1}} + \frac{\partial L(q^*_{2,2})}{\partial q^*_{2,2}}\frac{\partial q^*_{2,2}}{\partial q^*_{2,1}})] = 0, \text{otherwise} \end{cases}. \end{aligned} \tag{5.8}$$

This leads to three possible solutions in period 1: i) single sourcing with supplier 1: $q^*_{1,1} > 0, q^*_{2,1} = 0$, ii) single sourcing with supplier 2: $q^*_{1,1} = 0, q^*_{2,1} > 0$ or iii) dual sourcing: $q^*_{1,1} > 0, q^*_{2,1} > 0$. Thus, the optimal decisions in the first period can be obtained by solving equations (5.7) and (5.8), which need to be evaluated numerically.

5.3.3 Numerical example and discussion

We start with the following illustrative example assuming normally distributed demand with $\mu_1 = \mu_2 = 100$ and $\sigma_1 = \sigma_2 = 10$ to investigate the value of the total expected cost by simultaneously changing the order quantities assigned to the suppliers in the first period $q_{1,1}$ and $q_{2,1}$ with $\pi = 30$, $c_{2,0} = 10$, $c_{1,0} = c_{2,0} - \Delta c$ and $p_1 = p_2 = 0.9$. The results are shown in Figure 5.2 changing the learning slope b and the initial cost advantage of supplier 1 Δc while keeping all other parameters fixed. When supplier 1 has a cost advantage, it is optimal to use manly supplier 1 to achieve low total cost. When there is no initial cost difference, the expected cost is very flat in terms of splitting the order or single sourcing. Further, a high learning rate increases the allocation balances.



(a) $b = 0.1, \Delta c = 0\%$ (b) $b = 0.1, \Delta c = 20\%$

(c) $b = 0.5, \Delta c = 0\%$ (d) $b = 0.5, \Delta c = 20\%$

Figure 5.2: Expected total cost as a function of $q_{1,1}$ and $q_{2,1}$.

To study the impact of demand uncertainty on the optimal order allocation and the value of dual sourcing, we fix the mean demand in period one and two (μ_1, μ_2) and vary the standard deviation with a coefficient of variation $cv = \sigma/\mu \in \{0.1, 0.3\}$. Further, we discuss the impact of different second period mean demands. We consider three cases: the normal product case where $\mu_1 = \mu_2 = 100$, a new product case, where the mean demand increases in the second period

$\mu_1 = 100 < \mu_2 = 200$, and a final phase product case, where the mean demand decreases in the second period $\mu_1 = 100 > \mu_2 = 50$. We fix the coefficient of variation cv in both periods to 0.1 and 0.3, respectively. The results for $p_1 = p_2 = 0.9$ and $\pi = 30$ are summarized in Table 5.2.

			$cv = 0.1$					$cv = 0.3$				
Δc	b_1	b_2	$(q^*_{1,1}, q^*_{2,1})$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}	$(q^*_{1,1}, q^*_{2,1})$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}
$\mu_1 = 100, \mu_2 = 100$												
0	0.1	0.1	(94, 11)	0.7	0.7	3.1	2.7	(103, 12)	0.7	0.7	2.9	2.5
		0.5	(12, 93)	41.9	1.1	27.2	14.4	(12, 103)	42.0	1.0	27.7	15.0
	0.3	0.3	(90, 15)	2.9	2.9	7.0	6.4	(100, 16)	2.7	2.7	7.0	6.3
		0.5	(19, 86)	15.8	3.6	14.4	10.3	(20, 95)	16.1	3.4	14.8	10.5
	0.5	0.5	(87, 18)	5.1	5.1	8.7	8.2	(95, 20)	4.9	4.9	8.8	8.3
20	0.1	0.1	(107, 0)	0.0	23.6	14.7	8.2	(120, 0)	0.0	21.6	13.6	7.6
		0.5	(66, 40)	24.3	9.4	22.8	16.0	(72, 47)	25.1	8.1	23.0	15.7
	0.3	0.3	(101, 6)	0.4	24.2	17.4	10.4	(114, 7)	0.6	22.4	16.7	10.1
		0.5	(100, 7)	2.0	12.8	12.7	8.1	(72, 47)	2.3	10.8	12.1	8.0
	0.5	0.5	(99, 8)	2.3	26.4	19.1	11.9	(112, 8)	2.4	24.5	18.5	11.8
$\mu_1 = 100, \mu_2 = 200$												
0	0.1	0.1	(95, 11)	1.0	1.0	4.4	3.8	(105, 12)	1.0	1.0	4.2	3.6
		0.5	(12, 93)	72.4	1.9	47.1	24.9	(13, 104)	72.3	1.7	47.7	25.8
	0.3	0.3	(91, 15)	4.7	4.7	11.4	10.4	(102, 16)	4.4	4.4	11.2	10.2
		0.5	(19, 87)	27.8	6.3	25.4	18.1	(20, 97)	28.1	5.9	25.8	18.4
	0.5	0.5	(88, 18)	9.1	9.1	15.4	14.7	(97, 20)	8.7	8.7	15.7	14.8
20	0.1	0.1	(107, 0)	0.0	23.7	15.9	9.1	(122, 0)	0.0	21.7	14.8	8.5
		0.5	(45, 61)	47.9	8.1	38.1	22.9	(47, 72)	48.9	7.0	38.6	23.1
	0.3	0.3	(99, 9)	1.8	25.9	22.1	14.2	(113, 10)	1.8	24.3	21.4	13.9
		0.5	(48, 59)	9.5	12.7	20.1	16.2	(51, 69)	10.5	11.5	20.4	16.9
	0.5	0.5	(96, 11)	5.6	30.6	26.4	18.1	(110, 12)	5.7	28.7	26.0	18.2
$\mu_1 = 100, \mu_2 = 50$												
0	0.1	0.1	(94, 11)	0.5	0.5	1.9	1.7	(102, 12)	0.4	0.4	1.8	1.6
		0.5	(12, 93)	22.7	0.6	14.7	7.8	(12, 102)	22.9	0.5	15.1	8.2
	0.3	0.3	(90, 15)	1.6	1.6	4.0	3.6	(98, 16)	1.5	1.5	4.0	3.6
		0.5	(19, 86)	8.5	1.9	7.7	5.5	(19, 95)	8.7	1.8	8.0	5.7
	0.5	0.5	(87, 18)	2.7	2.7	4.6	4.4	(95, 19)	2.6	2.6	4.7	4.5
20	0.1	0.1	(106, 0)	0.0	23.5	13.6	7.3	(119, 0)	0.0	21.4	12.5	6.8
		0.5	(81, 25)	10.7	12.1	14.7	12.8	(89, 30)	11.2	10.5	14.5	12.6
	0.3	0.3	(107, 0)	0.0	23.5	14.7	8.2	(120, 0)	0.0	21.5	13.8	7.7
		0.5	(102, 5)	0.5	16.7	11.6	6.6	(115, 5)	0.6	14.5	10.8	6.2
	0.5	0.5	(101, 5)	0.6	24.3	15.0	8.5	(113, 6)	0.7	22.2	14.3	8.2

Table 5.2: Optimal first period decision, stochastic demand $(p_1, p_2) = (0.9, 0.9)$, $\pi = 30$ and $cv_1 = cv_2 = cv$.

OBSERVATION 5.3. *The total order quantity increases with uncertainty when penalty costs are high (volume effect). The savings gained by the optimal policy compared to the heuristics of single sourcing and equal or 75:25-split decreases with uncertainty of demand (see Table 5.2).*

No general conclusion about the balance of the optimal order allocation can be found, as the optimal order quantities depend on the interaction of initial cost, learning slope, reliability and demand uncertainty. Thus, the balance of the first-period order allocations may increase or decrease with demand uncertainty. As single sourcing is optimal in the second period, the equal split and 75:25 split policies perform poorly because both suppliers are used in both periods.

OBSERVATION 5.4. *The total order quantity increases when the mean demand of the second period increases. The value of dual sourcing increases in the new product scenario and decreases in the final phase product scenario (see Table 5.2).*

An increase in second period demand affects the first-period order quantity and increases the savings gained from dual sourcing. Ordering more in the first period decreases the second period cost, hence the total expected cost as μ_2 increases.

As $\pi = 30$ accounts for a newsvendor-ratio higher than 0.5, we also investigated the case of $\pi = 15$, for a newsvendor-ratio lower than 0.5. The structure of the results do not change when $\pi = 15$, the only difference that was found is that the total order quantity decreases with demand uncertainty (see Tables 5.4 to 5.9 in the Appendix Section 5.7).

5.4 Risk aversion

So far we have focused on characterizing the optimal policy so as to minimize the expected total cost, which is appropriate for a risk neutral decision maker. This Section extends the two period model discussed in Chapter 4 by assuming a risk averse decision maker.

5.4.1 Model formulation

Consider a risk-averse buyer who minimizes cost and sources from two identical suppliers over a two period planning horizon. $u(.)$ denotes the disutility function which is assumed to be non-decreasing and convex $u'(.) > 0$ and $u''(.) < 0$, e.g. higher cost implies a higher disutility level. In the multi-period context, we assume additive utility, i.e., the objective is the summation of the disutility from the cost in each period (see e.g. Chen et al. (2007), Raiffa and Keeney (1993)) and use an exponential disutility function of the form

$$u(x) = \frac{1}{r}e^{rx},$$

with $r > 0$ denoting the degree of risk-aversion. Period cost C_t is a random variable with certainty equivalent

$$R(C) = \frac{1}{r} \ln (Ee^{rC}),$$

i.e. the inverse of the expected disutility $R(C) = u^{-1}(Eu(C))$ (see Eisenführ et al. (2010)).

In the two-period problem, $C = C_1 + C_2$, where the second-period cost C_2 is a random variable as the realization of supplier experience levels and hence supplier cost is uncertain. Hence the problem is to solve

$$\begin{aligned} V_1 \quad = \quad & \min_{0 \le q_{1,1} \le d} \{c_0 q_{1,1} + c_0(d - q_{1,1}) + \frac{1}{r} \ln (p^2 e^{rV_2(q_{1,1}, d-q_{1,1})} + \\ & p(1-p)(e^{rV_2(q_{1,1}, d-q_{1,1})} + e^{rV_2(0, d-q_{1,1})}) + (1-p)^2 e^{rV_2(0,0)})\}, \end{aligned} \tag{5.9}$$

where the value function V_1 is the sum of first period cost plus the certainty equivalent of the second period. The order allocation $q_{1,1} = d - q_{1,2}$ has to be chosen to minimize the first-period cost and the uncertain second period cost with risk aversion parameter r. Note that a risk-averse policy yields larger expected total cost but a smaller risk than the risk-neutral policy.

5.4.2 Structural properties

Starting in the last period, there is no risk for a future disruption and the total demand is allocated to the cheapest supplier with $q_i^* = d$ and $V_2(\mathbf{x_2}) = dc(x_{i,2})$ for $x_{i,2} \le x_{j,2}, i \ne j$.

In the first period the first-order condition from problem (5.9) is

$$e^{rc(q_{1,1}^*)d} c'(q_{1,1}^*) = (1-p) e^{rc(d-q_{1,1}^*)d} c'(d - q_{1,1}^*). \tag{5.10}$$

The optimal order quantity in the risk-averse case solves the trade-off of the derivative of the future value function if supplier 1 survives or if only supplier 2 survives weighted with the exponential disutility under risk-aversion parameter r. Equation (5.10) has the following implication.

Lemma 5.4.1. *The optimal order quantity $q_{1,1}^*$ decreases with risk-aversion r and increases with reliability p.*

This implies that the balance of the order in the first period increases with increasing risk aversion.

5.4.3 Numerical example and discussion

Table 5.3 shows the influence of risk-aversion on the optimal order allocation and the value of dual sourcing with $r \in \{0.005, 0.01, 0.025\}$ for the case of identical suppliers and $c_0 = 10, d = 100$.

For small degrees of risk aversion, the initial order quantity $q^*_{1,1}$ decreases in the learning rate whereas for larger risk-aversion, it first decreases and then slightly increases in the learning rate as the cost function gets very flat for a dual sourcing order allocation ($d/2 \leq q_{1,1} < d$).

OBSERVATION 5.5. *The savings of dual sourcing compared to single sourcing and equal volume split first increase with r and then decrease. The savings of dual sourcing compared to a 75:25 split decrease with r (see Table 5.3).*

Slight risk-aversion therefore leads to more balanced supply allocations and increasing benefits of dual sourcing. The savings of dual sourcing compared to single sourcing peak first and then decrease due to the exponential function of the disutility function putting a non-linear weight on high costs with increasing r.

		$b = 0.1$				$b = 0.3$				$b = 0.5$			
p	r	$q^*_{1,1}$	Δ^{DS}	Δ^{50}	Δ^{75}	$q^*_{1,1}$	Δ^{DS}	Δ^{50}	Δ^{75}	$q^*_{1,1}$	Δ^{DS}	Δ^{50}	Δ^{75}
0.9	0	89	0.8	3.2	2.0	85	3.0	7.1	4.3	82	5.3	8.6	5.0
	0.005	83	3.1	3.5	2.0	78	17.3	8.6	3.9	77	25.6	10.1	3.9
	0.01	78	6.3	3.5	1.8	73	14.4	2.7	0.7	73	14.8	1.4	0.2
	0.025	69	5.0	0.6	0.1	65	5.1	0.0	0.0	67	5.1	0.0	0.0
0.7	0	75	2.6	4.5	2.5	72	8.4	11.4	6.2	69	13.0	15.4	8.2
	0.005	70	5.1	4.0	2.0	66	12.6	5.4	2.1	66	14.3	4.6	1.5
	0.01	66	5.3	2.5	1.1	63	6.8	0.8	0.2	63	6.8	0.3	0.0
	0.025	61	2.5	0.2	0.0	58	2.5	0.0	0.0	59	2.5	0.0	0.0

Table 5.3: Optimal first period decision for different values of the risk aversion parameter r, $T = 2$.

5.5 Conclusion

This Chapter investigated the sensitivity and its implications of the analysis presented in Chapter 4 with respect to limiting model assumptions. The basic analysis of Chapter 4 is extended by studying the impact of incomplete information of supplier reliability, stochastic demand and a risk-averse buyer on the optimal sourcing and order policy and the benefit from the optimal policy compared to single sourcing and heuristic dual sourcing policies.

Using Bayesian updating, when the true survival probability is not known for the buyer, in a three period problem we find that increasing uncertainty about reliability leads to a less balanced initial volume allocation. The savings of the optimal policy decrease with uncertainty about reliability. We find that demand uncertainty affects the total order quantity and the balance between the supplier and the savings of dual sourcing decreases with uncertainty in a two period problem. Moreover, higher risk-aversion leads to a more balanced allocation decision between the suppliers.

This analysis is an important first step in understanding the implication of the basic model extensions discussed. Future work should further explore the multi-period problem and general number of suppliers.

5.6 Appendix: Mathematical proofs

Deviation of equation (5.2).
As $\Gamma(\alpha+n)=(\alpha+n-1)(\alpha+n-2)...(\alpha+1)\alpha\Gamma(\alpha), \alpha>0, n\in\mathbb{N}$ and $\int_0^1 p^{\alpha-1}(1-p)^{\beta-1}\mathrm{d}p = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ we have

$$\begin{aligned}\int f(p)p^2\mathrm{d}p &= \int \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}p^{\alpha-1}(1-p)^{\beta-1}p^2\mathrm{d}p = \int \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}p^{\alpha+1}(1-p)^{\beta-1}\mathrm{d}p\\ &= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\frac{\Gamma(\alpha+2)\Gamma(\beta)}{\Gamma(\alpha+\beta+2)} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)}\frac{(\alpha+1)\alpha\Gamma(\alpha)}{(\alpha+\beta+1)(\alpha+\beta)\Gamma(\alpha+\beta)}\\ &= \frac{(\alpha+1)\alpha}{(\alpha+\beta+1)(\alpha+\beta)}\end{aligned}$$

$$\begin{aligned}\int f(p)p(1-p)\mathrm{d}p &= \int \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}p^{\alpha-1}(1-p)^{\beta-1}p(1-p)\mathrm{d}p = \int \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}p^{\alpha}(1-p)^{\beta}\mathrm{d}p\\ &= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{\Gamma(\alpha+\beta+2)} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\frac{\alpha\Gamma(\alpha)\beta\Gamma(\beta)}{(\alpha+\beta+1)(\alpha+\beta)\Gamma(\alpha+\beta)}\\ &= \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)}\end{aligned}$$

$$\begin{aligned}\int f(p)(1-p)^2\mathrm{d}p &= \int \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}p^{\alpha-1}(1-p)^{\beta-1}(1-p)^2\mathrm{d}p = \int \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}p^{\alpha-1}(1-p)^{\beta+1}\mathrm{d}p\\ &= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\frac{\Gamma(\alpha)\Gamma(\beta+2)}{\Gamma(\alpha+\beta+2)} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\frac{\Gamma(\alpha)(\beta+1)\beta\Gamma(\beta)}{(\alpha+\beta+1)(\alpha+\beta)\Gamma(\alpha+\beta)}\\ &= \frac{(\beta+1)\beta}{(\alpha+\beta+1)(\alpha+\beta)}\end{aligned}$$

Proof. of Lemma 5.4.1. The optimal order quantity $q^*_{1,1}$ is the solution to

$$e^{rc(q^*_{1,1})d}c'(q^*_{1,1}) - (1-p)e^{rc(d-q^*_{1,1})d}c'(d-q^*_{1,1}) = 0$$

assuming $\frac{d}{2}\le q_{1,1}\le d$. Using implicit function theorem we get

$$\frac{\partial q^*_{1,1}}{\partial r} = -\frac{e^{rc(q^*_{1,1})d}c(q_{1,1})dc'(q^*_{1,1}) - (1-p)e^{rc(d-q^*_{1,1})d}c(d-q^*_{1,1})dc'(d-q^*_{1,1})}{e^{rc(q^*_{1,1})d}rd(c'(q^*_{1,1}))^2 + e^{rc(q^*_{1,1})d}c''(q^*_{1,1}) + (1-p)(e^{rc(d-q^*_{1,1})d}rd(c'(d-q^*_{1,1}))^2 + e^{rc(q^*_{1,1})d}c''(d-q^*_{1,1}))}$$

The denominator of $\partial q^*_{1,1}/\partial r$ clearly is positive. The numerator is the first-order condition multiplied with $c(q^*_{1,1})d$ and $c(d-q^*_{1,1})d$, respectively. As the first term of the first order condition is < 0 and the second term positive and by assumption $q^*_{1,1} > \frac{d}{2} \Rightarrow c(q^*_{1,1}) < c(d - q^*_{1,1})$, the numerator is positive and it follows that $\frac{\partial q^*_{1,1}}{\partial r} < 0$.

$$\frac{\partial q^*_{1,1}}{\partial p} =$$

$$-\frac{e^{rc(d-q^*_{1,1})d}c'(d-q^*_{1,1})}{e^{rc(q^*_{1,1})d}rd(c'(q^*_{1,1}))^2 + e^{rc(q^*_{1,1})d}c''(q^*_{1,1}) + (1-p)(e^{rc(d-q^*_{1,1})d}rd(c'(d-q^*_{1,1}))^2 + e^{rc(q^*_{1,1})d}c''(d-q^*_{1,1}))}$$

As $e^{rc(d-q^*_{1,1})d}c'(d-q^*_{1,1}) < 0$ and the denominator of $\partial q^*_{1,1}/\partial r$ is positive it follows that $\partial q^*_{1,1}/\partial p > 0$ $\square$

5.7 Appendix: Tables

The complete results of the numerical study from Section 5.3 (stochastic demand) for low penalty cost ($\pi = 15$) and high penalty cost ($\pi = 30$) are summarized in Tables 5.4 to 5.9.

				$\pi = 30$										$\pi = 15$									
				$cv = 0.1$					$cv = 0.3$					$cv = 0.1$					$cv = 0.3$				
(p_1, p_2)	Δc	b_1	b_2	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}
(0.9,0.9)	0	0.1	0.1	(94, 11)	0.7	0.7	3.1	2.7	(103, 12)	0.7	0.7	2.9	2.5	(87, 10)	0.7	0.7	3.0	2.6	(82, 9)	0.5	0.5	2.7	2.3
			0.5	(12, 93)	41.9	1.1	27.2	14.4	(12, 103)	42.0	1.0	27.7	15.0	(11, 86)	41.8	1.0	27.3	14.5	(8, 84)	41.7	0.6	28.0	15.2
		0.3	0.3	(90, 15)	2.9	2.9	7.0	6.4	(100, 16)	2.7	2.7	7.0	6.3	(83, 14)	2.7	2.7	7.0	6.3	(80, 13)	2.2	2.2	6.8	6.1
			0.5	(19, 86)	15.8	3.6	14.4	10.3	(20, 95)	16.1	3.4	14.8	10.5	(17, 80)	15.8	3.4	14.6	10.3	(15, 77)	16.2	2.7	15.2	10.6
		0.5	0.5	(87, 18)	5.1	5.1	8.7	8.2	(95, 20)	4.9	4.9	8.8	8.3	(80, 17)	4.9	4.9	8.7	8.2	(77, 15)	4.3	4.3	8.9	8.3
	10	0.1	0.1	(106, 0)	0.0	10.5	7.8	4.7	(117, 0)	0.0	9.6	7.3	4.4	(98, 0)	0.0	10.2	7.7	4.6	(95, 0)	0.0	8.8	6.8	4.2
			0.5	(45, 60)	31.9	3.9	23.7	13.8	(45, 71)	32.5	3.3	24.1	14.1	(40, 58)	31.9	3.5	23.6	13.7	(29, 65)	32.4	2.4	24.1	13.9
		0.3	0.3	(97, 9)	1.6	12.3	11.6	8.0	(108, 10)	1.6	11.5	11.2	7.9	(91, 8)	1.5	11.9	11.4	7.9	(89, 8)	1.2	10.3	10.8	7.6
			0.5	(47, 58)	7.8	6.6	12.1	10.4	(48, 68)	8.5	5.9	12.4	10.4	(42, 56)	7.9	6.2	12.2	10.3	(33, 61)	8.8	4.8	12.6	10.1
		0.5	0.5	(96, 10)	3.6	14.5	13.1	9.7	(105, 12)	3.6	13.6	13.0	9.7	(88, 10)	3.5	14.0	13.1	9.6	(86, 10)	3.1	12.2	12.8	9.6
	20	0.1	0.1	(107, 0)	0.0	23.6	14.7	8.2	(120, 0)	0.0	21.6	13.6	7.6	(100, 0)	0.0	23.0	14.4	8.0	(100, 0)	0.0	19.9	12.7	7.2
			0.5	(66, 40)	24.3	9.4	22.8	16.0	(72, 47)	25.1	8.1	23.0	15.7	(60, 39)	24.1	8.7	22.5	15.6	(54, 44)	24.7	6.2	22.2	14.6
		0.3	0.3	(101, 6)	0.4	24.2	17.4	10.4	(114, 7)	0.6	22.4	16.7	10.1	(94, 6)	0.3	23.5	17.2	10.2	(95, 6)	0.3	20.5	16.0	9.7
			0.5	(100, 7)	2.0	12.8	12.7	8.1	(72, 47)	2.3	10.8	12.1	8.0	(93, 7)	1.8	11.8	12.3	8.0	(54, 44)	2.3	8.7	11.6	8.0
		0.5	0.5	(99, 8)	2.3	26.4	19.1	11.9	(112, 8)	2.4	24.5	18.5	11.8	(93, 7)	2.2	25.6	18.9	11.8	(92, 8)	2.0	22.2	17.9	11.5
(0.8,0.9)	0	0.1	0.1	(10, 95)	2.7	0.6	3.9	3.0	(11, 104)	2.6	0.6	3.7	2.8	(9, 88)	2.6	0.5	3.8	2.9	(8, 83)	2.2	0.4	3.4	2.6
			0.5	(11, 94)	44.6	0.9	30.3	14.8	(11, 104)	44.6	0.8	30.7	15.4	(10, 87)	44.5	0.8	30.4	14.9	(8, 84)	44.0	0.5	30.8	15.6
		0.3	0.3	(14, 91)	8.1	2.5	9.2	7.2	(15, 101)	7.7	2.3	9.1	7.1	(13, 84)	7.8	2.3	9.1	7.1	(12, 81)	6.9	1.9	8.9	6.9
			0.5	(17, 88)	21.5	3.1	17.5	11.1	(18, 97)	21.7	2.9	17.8	11.4	(16, 81)	21.5	2.9	17.6	11.2	(14, 78)	21.4	2.3	18.1	11.5
		0.5	0.5	(17, 88)	12.2	4.4	11.7	9.3	(18, 97)	11.9	4.2	12.0	9.5	(16, 81)	11.9	4.2	11.8	9.3	(14, 78)	10.9	3.7	12.0	9.5
	10	0.1	0.1	(96, 10)	0.3	8.5	6.9	4.3	(106, 11)	0.3	7.8	6.5	4.1	(89, 9)	0.2	8.3	6.7	4.2	(86, 9)	0.1	7.1	5.9	3.7
			0.5	(44, 61)	34.4	3.6	26.8	14.1	(45, 71)	34.9	3.1	27.1	14.4	(39, 59)	34.3	3.3	26.7	14.0	(27, 66)	34.6	2.2	26.9	14.2
		0.3	0.3	(91, 15)	4.7	9.7	11.7	9.5	(101, 17)	4.6	8.9	11.4	9.3	(84, 15)	4.5	9.3	11.5	9.3	(82, 15)	3.9	7.8	10.8	8.8
			0.5	(46, 59)	13.1	6.0	15.2	11.1	(47, 69)	13.7	5.4	15.5	11.1	(41, 57)	13.1	5.6	15.2	11.0	(32, 62)	13.7	4.3	15.6	10.9
		0.5	0.5	(89, 17)	8.8	11.9	14.5	12.7	(98, 19)	8.7	11.0	14.5	12.7	(82, 16)	8.5	11.4	14.4	12.6	(79, 17)	7.8	9.6	14.0	12.4
	20	0.1	0.1	(107, 0)	0.0	21.0	13.4	7.4	(120, 0)	0.0	19.1	12.4	6.9	(100, 0)	0.0	20.5	13.2	7.3	(100, 0)	0.0	17.7	11.6	6.5
			0.5	(66, 40)	26.6	9.2	26.1	16.3	(72, 47)	27.3	7.9	26.1	16.0	(60, 39)	26.4	8.4	25.8	15.8	(53, 45)	26.7	5.9	25.2	14.9
		0.3	0.3	(96, 11)	2.7	20.3	16.8	10.8	(108, 13)	2.7	18.7	16.1	10.6	(89, 11)	2.4	19.6	16.5	10.6	(90, 11)	2.2	16.9	15.2	10.0
			0.5	(66, 40)	6.5	11.6	15.4	11.1	(72, 47)	7.2	10.3	15.3	11.5	(60, 39)	6.4	10.8	15.1	11.0	(54, 44)	7.0	8.2	14.7	11.3
		0.5	0.5	(95, 12)	6.7	22.6	19.9	14.0	(106, 14)	6.7	20.8	19.4	14.0	(88, 12)	6.4	21.8	19.6	13.8	(87, 13)	6.0	18.5	18.6	13.4

Table 5.4: Optimal first period decision, stochastic demand $\mu_1 = 100, \mu_2 = 100$ and $\sigma_1 = \sigma_2 = \sigma$.

				$\pi = 30$										$\pi = 15$									
				$cv = 0.1$					$cv = 0.3$					$cv = 0.1$					$cv = 0.3$				
(p_1, p_2)	Δc	b_1	b_2	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}
(0.7,0.8)	0	0.1	0.1	(18, 87)	3.5	1.4	4.5	3.5	(19, 96)	3.3	1.3	4.2	3.4	(16, 81)	3.3	1.3	4.3	3.4	(15, 76)	2.8	1.0	3.8	3.0
			0.5	(19, 86)	38.8	2.0	28.4	13.7	(19, 96)	38.7	1.8	28.6	14.2	(17, 80)	38.6	1.8	28.3	13.7	(13, 78)	38.0	1.3	28.4	14.1
		0.3	0.3	(21, 84)	10.5	5.0	11.2	9.2	(23, 93)	10.0	4.7	11.1	9.1	(20, 77)	10.1	4.7	11.0	9.1	(18, 75)	8.8	4.0	10.6	8.6
			0.5	(26, 79)	22.3	5.9	19.2	13.0	(28, 87)	22.2	5.6	19.4	13.2	(24, 73)	22.1	5.6	19.2	12.9	(21, 71)	21.6	4.7	19.4	13.0
		0.5	0.5	(25, 80)	15.7	8.2	15.0	12.7	(27, 88)	15.3	7.9	15.3	12.9	(23, 74)	15.3	7.9	15.0	12.7	(21, 71)	14.1	7.1	15.2	12.7
	10	0.1	0.1	(93, 13)	0.8	9.1	7.2	4.6	(102, 15)	0.8	8.3	6.8	4.3	(86, 12)	0.7	8.8	7.0	4.4	(83, 12)	0.5	7.4	6.1	3.9
			0.5	(51, 54)	30.5	5.9	26.6	14.4	(53, 63)	30.8	5.2	26.7	14.5	(46, 52)	30.3	5.5	26.4	14.2	(36, 58)	30.2	4.0	26.1	13.9
		0.3	0.3	(86, 20)	6.9	12.3	13.7	11.3	(96, 22)	6.7	11.4	13.4	11.1	(80, 19)	6.6	11.8	13.5	11.1	(78, 19)	5.8	10.0	12.6	10.4
			0.5	(53, 52)	15.0	10.1	18.3	14.2	(56, 61)	15.4	9.3	18.4	14.1	(48, 50)	14.9	9.6	18.2	14.0	(40, 54)	15.0	7.8	18.1	13.4
		0.5	0.5	(85, 21)	12.1	15.8	18.0	15.8	(94, 23)	11.9	14.8	18.0	15.9	(78, 20)	11.8	15.2	17.8	15.7	(75, 21)	10.9	13.1	17.4	15.3
	20	0.1	0.1	(107, 0)	0.0	21.0	13.2	7.1	(120, 0)	0.0	19.1	12.2	6.6	(100, 0)	0.0	20.5	13.0	7.0	(99, 0)	0.0	17.6	11.4	6.3
			0.5	(70, 36)	23.9	12.4	27.3	17.6	(76, 43)	24.4	10.9	27.0	17.1	(63, 36)	23.6	11.6	26.8	17.1	(58, 40)	23.7	8.7	25.7	15.7
		0.3	0.3	(93, 14)	4.3	22.5	18.5	12.1	(104, 16)	4.4	20.8	17.9	11.9	(86, 14)	4.0	21.7	18.1	11.9	(87, 14)	3.6	18.6	16.8	11.2
			0.5	(70, 36)	9.1	16.8	19.6	13.9	(77, 42)	9.6	15.1	19.4	14.3	(64, 35)	8.9	15.8	19.2	13.8	(60, 39)	9.1	12.7	18.4	13.8
		0.5	0.5	(92, 15)	9.5	26.5	23.3	16.7	(103, 17)	9.6	24.5	22.9	16.7	(86, 14)	9.2	25.6	22.9	16.5	(84, 16)	8.6	21.9	21.8	16.0
(0.7,0.7)	0	0.1	0.1	(79, 26)	2.5	2.5	4.4	4.0	(87, 28)	2.4	2.4	4.1	3.8	(73, 24)	2.4	2.4	4.2	3.9	(69, 22)	1.9	1.9	3.7	3.4
			0.5	(27, 78)	31.5	3.3	24.2	12.6	(28, 87)	31.5	3.1	24.3	12.9	(24, 73)	31.4	3.1	24.1	12.5	(19, 72)	30.8	2.3	24.0	12.7
		0.3	0.3	(75, 30)	8.1	8.1	11.3	10.8	(84, 32)	7.7	7.7	11.2	10.6	(70, 27)	7.8	7.8	11.1	10.6	(67, 26)	6.7	6.7	10.6	9.9
			0.5	(35, 70)	18.1	9.3	18.2	14.3	(37, 78)	18.0	8.9	18.4	14.4	(32, 65)	17.8	8.9	18.1	14.1	(28, 64)	17.3	7.7	18.1	13.9
		0.5	0.5	(73, 32)	12.7	12.7	15.6	15.2	(80, 35)	12.3	12.3	15.9	15.3	(67, 30)	12.4	12.4	15.5	15.1	(64, 28)	11.2	11.2	15.6	14.9
	10	0.1	0.1	(94, 12)	0.5	11.1	7.9	4.8	(104, 13)	0.6	10.2	7.4	4.5	(87, 11)	0.4	10.7	7.7	4.6	(84, 11)	0.3	9.1	6.7	4.1
			0.5	(57, 48)	24.8	8.3	23.7	14.6	(60, 56)	25.1	7.4	23.7	14.4	(52, 46)	24.6	7.8	23.5	14.3	(43, 51)	24.5	5.9	22.9	13.6
		0.3	0.3	(88, 18)	5.7	16.8	15.1	11.3	(98, 20)	5.6	15.7	14.7	11.1	(82, 17)	5.4	16.2	14.8	11.0	(80, 17)	4.8	14.0	13.9	10.4
			0.5	(60, 46)	12.0	14.6	18.5	15.3	(64, 53)	12.3	13.6	18.5	15.6	(54, 44)	11.9	14.0	18.3	15.2	(47, 48)	11.8	11.7	17.9	15.3
		0.5	0.5	(87, 19)	10.1	21.7	19.6	15.6	(96, 21)	10.0	20.5	19.6	15.7	(80, 18)	9.8	21.0	19.4	15.4	(77, 19)	9.1	18.6	19.0	15.2
	20	0.1	0.1	(107, 0)	0.0	23.5	14.3	7.6	(120, 0)	0.0	21.5	13.2	7.1	(100, 0)	0.0	22.9	14.0	7.5	(99, 0)	0.0	19.6	12.3	6.7
			0.5	(73, 33)	19.3	15.8	25.4	18.8	(80, 39)	19.8	14.0	25.0	18.1	(67, 32)	19.1	14.9	24.9	18.2	(62, 36)	19.1	11.5	23.6	16.4
		0.3	0.3	(94, 13)	3.4	27.8	20.3	12.5	(106, 14)	3.5	25.8	19.6	12.3	(88, 12)	3.2	26.9	19.9	12.2	(88, 13)	2.9	23.2	18.4	11.6
			0.5	(94, 13)	7.0	22.5	20.7	13.6	(81, 38)	7.4	20.5	20.3	13.7	(87, 13)	6.8	21.4	20.2	13.3	(64, 35)	6.9	17.5	19.1	13.2
		0.5	0.5	(93, 14)	7.8	33.2	25.3	16.9	(104, 16)	7.9	31.0	24.9	17.0	(87, 13)	7.5	32.1	24.9	16.7	(86, 14)	7.1	28.0	23.8	16.3

Table 5.5: Optimal first period decision, stochastic demand $\mu_1 = 100, \mu_2 = 100$ and $\sigma_1 = \sigma_2 = \sigma$.

				$\pi = 30$										$\pi = 15$									
				$cv = 0.1$					$cv = 0.3$					$cv = 0.1$					$cv = 0.3$				
(p_1, p_2)	Δc	b_1	b_2	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}
(0.9,0.9)	0	0.1	0.1	(95, 11)	1.0	1.0	4.4	3.8	(105, 12)	1.0	1.0	4.2	3.6	(87, 11)	0.9	0.9	4.3	3.7	(85, 10)	0.7	0.7	3.8	3.3
			0.5	(12, 93)	72.4	1.9	47.1	24.9	(13, 104)	72.3	1.7	47.7	25.8	(11, 87)	72.0	1.7	47.0	24.9	(9, 87)	70.8	1.1	47.5	25.8
		0.3	0.3	(91, 15)	4.7	4.7	11.4	10.4	(102, 16)	4.4	4.4	11.2	10.2	(85, 14)	4.4	4.4	11.3	10.2	(85, 13)	3.6	3.6	10.8	9.7
			0.5	(19, 87)	27.8	6.3	25.4	18.1	(20, 97)	28.1	5.9	25.8	18.4	(17, 81)	27.7	5.9	25.6	18.1	(15, 81)	27.9	4.8	26.0	18.3
		0.5	0.5	(88, 18)	9.1	9.1	15.4	14.7	(97, 20)	8.7	8.7	15.7	14.7	(81, 17)	8.7	8.7	15.4	14.6	(80, 16)	7.5	7.5	15.6	14.5
	10	0.1	0.1	(99, 7)	0.0	10.5	8.9	5.6	(111, 8)	0.1	9.8	8.4	5.3	(99, 0)	0.0	10.3	8.8	5.5	(98, 0)	0.0	9.0	7.9	5.0
			0.5	(26, 80)	59.7	4.3	41.9	23.2	(25, 92)	60.3	3.8	42.6	23.9	(22, 76)	59.4	3.9	41.8	23.1	(16, 80)	59.3	2.8	42.3	23.7
		0.3	0.3	(95, 12)	3.3	14.2	16.1	12.0	(108, 13)	3.2	13.3	15.7	11.8	(89, 11)	3.1	13.7	15.9	11.9	(91, 11)	2.5	12.1	15.1	11.3
			0.5	(31, 75)	18.3	8.8	22.2	17.4	(32, 86)	19.1	8.2	22.7	17.5	(28, 71)	18.4	8.3	22.3	17.2	(23, 74)	19.2	6.8	22.8	17.2
		0.5	0.5	(93, 13)	7.4	18.7	20.3	16.1	(104, 15)	7.2	17.7	20.2	16.2	(86, 13)	7.1	18.1	20.2	16.0	(87, 13)	6.3	15.9	19.9	15.9
	20	0.1	0.1	(107, 0)	0.0	23.7	15.9	9.1	(122, 0)	0.0	21.7	14.8	8.5	(101, 0)	0.0	23.1	15.7	9.0	(102, 0)	0.0	20.3	14.0	8.1
			0.5	(45, 61)	47.9	8.1	38.1	22.9	(47, 72)	48.9	7.0	38.6	23.1	(41, 59)	47.6	7.4	37.8	22.5	(32, 67)	48.0	5.4	37.9	22.3
		0.3	0.3	(99, 9)	1.8	25.9	22.1	14.2	(113, 10)	1.8	24.3	21.4	13.9	(92, 9)	1.6	25.2	21.8	14.0	(96, 9)	1.3	22.4	20.5	13.4
			0.5	(48, 59)	9.5	12.7	20.1	16.2	(51, 69)	10.5	11.5	20.4	16.9	(43, 57)	9.4	11.9	20.0	16.2	(36, 64)	10.5	9.5	20.1	16.7
		0.5	0.5	(96, 11)	5.6	30.6	26.4	18.1	(110, 12)	5.7	28.7	26.0	18.2	(91, 10)	5.3	29.7	26.2	18.0	(92, 11)	4.8	26.3	25.4	17.8
(0.8,0.9)	0	0.1	0.1	(10, 96)	3.9	0.9	5.6	4.2	(11, 106)	3.7	0.8	5.3	4.0	(10, 88)	3.7	0.8	5.4	4.1	(9, 86)	3.1	0.6	4.8	3.6
			0.5	(11, 94)	77.1	1.6	52.4	25.6	(12, 105)	76.7	1.4	52.8	26.5	(10, 88)	76.5	1.4	52.2	25.6	(8, 87)	74.7	0.9	52.3	26.6
		0.3	0.3	(14, 92)	13.2	4.0	14.9	11.6	(15, 103)	12.5	3.8	14.7	11.4	(13, 86)	12.6	3.7	14.7	11.4	(12, 86)	11.0	3.0	14.1	10.9
			0.5	(18, 88)	37.8	5.4	30.7	19.5	(19, 98)	37.8	5.1	31.1	20.0	(16, 82)	37.5	5.1	30.8	19.5	(14, 82)	36.8	4.1	31.1	19.8
		0.5	0.5	(17, 89)	21.6	7.8	20.8	16.5	(19, 98)	20.9	7.5	21.1	16.8	(16, 82)	21.0	7.5	20.8	16.5	(15, 81)	19.0	6.5	20.9	16.5
	10	0.1	0.1	(92, 14)	0.9	8.3	8.1	5.5	(103, 16)	0.9	7.7	7.7	5.3	(86, 13)	0.8	8.0	7.9	5.4	(85, 13)	0.6	7.0	7.0	4.8
			0.5	(25, 81)	63.9	3.9	47.3	23.7	(24, 93)	64.3	3.4	47.8	24.5	(21, 77)	63.5	3.6	47.1	23.6	(15, 81)	63.0	2.6	47.2	24.3
		0.3	0.3	(87, 20)	8.6	10.4	16.7	14.7	(98, 22)	8.3	9.7	16.4	14.4	(81, 19)	8.2	9.9	16.5	14.4	(83, 19)	7.0	8.5	15.5	13.5
			0.5	(30, 76)	27.6	7.9	27.6	18.6	(31, 87)	28.2	7.3	28.1	18.8	(26, 73)	27.5	7.4	27.6	18.4	(22, 75)	27.6	6.0	28.0	18.5
		0.5	0.5	(85, 21)	16.9	14.5	23.1	20.3	(95, 24)	16.5	13.5	23.1	20.0	(79, 20)	16.3	13.9	22.8	20.0	(79, 21)	14.7	11.8	22.4	19.2
	20	0.1	0.1	(107, 0)	0.0	20.0	14.1	8.0	(121, 0)	0.0	18.3	13.1	7.5	(101, 0)	0.0	19.6	13.9	7.9	(102, 0)	0.0	17.2	12.4	7.2
			0.5	(45, 61)	51.8	7.7	43.7	23.3	(47, 72)	52.7	6.6	44.0	23.6	(40, 60)	51.4	7.0	43.3	22.9	(32, 67)	51.4	5.1	42.9	22.8
		0.3	0.3	(92, 16)	6.0	20.5	21.7	15.6	(105, 18)	6.0	19.1	21.1	15.3	(86, 15)	5.6	19.9	21.3	15.3	(89, 16)	4.9	17.4	20.0	14.4
			0.5	(47, 60)	18.1	11.7	25.7	18.8	(50, 70)	19.0	10.6	26.0	18.7	(42, 58)	17.9	11.0	25.5	18.4	(36, 64)	18.5	8.7	25.4	17.9
		0.5	0.5	(90, 17)	13.9	24.9	28.5	22.5	(102, 20)	13.9	23.1	28.2	22.5	(85, 16)	13.4	24.0	28.1	22.2	(85, 18)	12.4	20.7	27.2	21.7

Table 5.6: Optimal first period decision, stochastic demand $\mu_1 = 100, \mu_2 = 200$ and $\sigma_1 = \sigma_2 = \sigma$.

				$\pi = 30$										$\pi = 15$									
					$cv = 0.1$					$cv = 0.3$					$cv = 0.1$					$cv = 0.3$			
(p_1, p_2)	Δc	b_1	b_2	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}
(0.7,0.8)	0	0.1	0.1	(18, 87)	5.0	2.0	6.3	5.0	(19, 98)	4.7	1.9	6.0	4.8	(16, 82)	4.7	1.9	6.1	4.9	(15, 79)	3.9	1.5	5.4	4.2
			0.5	(19, 86)	64.4	3.3	47.0	22.7	(19, 98)	63.9	3.0	47.2	23.4	(17, 81)	63.8	3.0	46.8	22.6	(14, 81)	62.0	2.2	46.4	23.1
		0.3	0.3	(22, 84)	16.7	8.0	17.8	14.7	(24, 94)	15.9	7.5	17.5	14.4	(20, 79)	16.0	7.5	17.5	14.4	(19, 79)	13.9	6.3	16.6	13.5
			0.5	(26, 80)	37.9	10.1	32.6	22.1	(28, 89)	37.6	9.5	32.8	22.4	(24, 74)	37.4	9.6	32.4	21.9	(22, 74)	36.0	8.0	32.3	21.7
		0.5	0.5	(25, 81)	27.2	14.2	26.0	22.0	(27, 90)	26.3	13.7	26.3	22.2	(23, 75)	26.4	13.7	25.8	21.7	(22, 74)	24.0	12.1	25.7	21.5
	10	0.1	0.1	(87, 19)	1.7	9.2	8.7	6.0	(98, 21)	1.7	8.5	8.2	5.7	(81, 18)	1.6	8.9	8.4	5.8	(81, 17)	1.2	7.6	7.4	5.1
			0.5	(34, 72)	54.2	7.1	44.6	22.8	(35, 83)	54.4	6.4	44.7	23.2	(30, 69)	53.8	6.6	44.2	22.6	(24, 73)	52.9	5.0	43.6	22.6
		0.3	0.3	(82, 25)	12.2	14.7	20.0	17.6	(92, 28)	11.8	13.8	19.6	17.3	(76, 24)	11.6	14.1	19.6	17.2	(77, 24)	10.1	12.0	18.4	16.1
			0.5	(39, 67)	29.4	14.1	31.5	22.8	(41, 77)	29.5	13.1	31.6	22.8	(35, 64)	29.0	13.4	31.2	22.5	(31, 67)	28.4	11.2	30.9	21.8
		0.5	0.5	(80, 26)	22.4	21.5	28.8	26.2	(89, 30)	21.9	20.3	28.9	25.9	(74, 25)	21.7	20.7	28.5	25.7	(74, 26)	19.9	18.0	27.9	24.7
	20	0.1	0.1	(107, 0)	0.0	20.1	13.8	7.6	(121, 0)	0.0	18.3	12.8	7.1	(100, 0)	0.0	19.7	13.6	7.5	(101, 0)	0.0	17.1	12.1	6.8
			0.5	(52, 55)	44.8	12.5	43.5	24.3	(55, 65)	45.3	11.0	43.3	24.2	(47, 53)	44.3	11.6	42.9	23.8	(40, 60)	43.9	8.9	41.7	22.9
		0.3	0.3	(87, 20)	8.9	24.6	24.7	17.9	(100, 23)	8.8	22.9	24.1	17.7	(82, 19)	8.4	23.7	24.2	17.5	(84, 20)	7.5	20.6	22.6	16.5
			0.5	(54, 53)	21.1	19.5	31.4	24.6	(58, 62)	21.7	18.0	31.4	24.2	(49, 51)	20.8	18.6	31.0	24.1	(44, 57)	20.7	15.3	30.3	22.7
		0.5	0.5	(86, 21)	18.9	32.0	34.4	27.2	(98, 24)	18.8	29.9	34.1	27.3	(81, 20)	18.3	30.8	33.9	26.8	(81, 22)	17.0	26.8	32.7	26.2
(0.7,0.7)	0	0.1	0.1	(79, 26)	3.6	3.6	6.2	5.7	(88, 29)	3.4	3.4	5.8	5.4	(74, 24)	3.4	3.4	5.9	5.5	(71, 23)	2.7	2.7	5.2	4.7
			0.5	(27, 78)	50.5	5.4	38.8	20.2	(28, 89)	50.2	4.9	38.8	20.6	(24, 74)	50.1	4.9	38.5	20.0	(20, 75)	48.7	3.8	38.0	20.0
		0.3	0.3	(76, 30)	12.8	12.8	17.8	17.0	(33, 85)	12.1	12.1	17.5	16.6	(71, 28)	12.2	12.2	17.4	16.6	(71, 27)	10.5	10.5	16.5	15.4
			0.5	(35, 71)	30.0	15.5	30.2	23.7	(38, 79)	29.7	14.7	30.4	23.7	(32, 66)	29.5	14.8	30.0	23.4	(30, 67)	28.3	12.7	29.6	22.7
		0.5	0.5	(73, 33)	21.6	21.6	26.4	25.7	(81, 36)	20.8	20.8	26.7	25.8	(68, 30)	20.9	20.9	26.2	25.4	(67, 29)	18.8	18.8	26.0	24.8
	10	0.1	0.1	(89, 17)	1.3	12.0	9.6	6.2	(100, 19)	1.3	11.1	9.1	5.9	(83, 16)	1.2	11.6	9.3	6.0	(83, 15)	0.9	9.9	8.2	5.3
			0.5	(43, 63)	42.8	10.5	38.2	22.0	(45, 73)	43.0	9.4	38.1	22.0	(39, 60)	42.4	9.8	37.8	21.6	(32, 65)	41.8	7.6	36.9	20.9
		0.3	0.3	(84, 23)	10.1	21.7	21.9	17.4	(94, 26)	9.8	20.4	21.4	17.1	(78, 22)	9.6	20.9	21.5	17.0	(79, 22)	8.4	18.1	20.2	16.0
			0.5	(48, 58)	23.1	21.0	30.5	26.0	(52, 67)	23.3	19.6	30.6	25.6	(43, 56)	22.8	20.0	30.2	25.4	(39, 59)	22.2	17.0	29.6	24.2
		0.5	0.5	(82, 24)	18.6	31.1	31.1	26.1	(91, 28)	18.2	29.6	31.2	26.2	(76, 23)	18.0	30.1	30.7	25.7	(75, 24)	16.5	26.8	30.1	25.2
	20	0.1	0.1	(107, 0)	0.0	23.6	15.3	8.3	(121, 0)	0.0	21.6	14.2	7.7	(100, 0)	0.0	23.0	15.0	8.2	(101, 0)	0.0	19.9	13.3	7.4
			0.5	(58, 49)	35.5	17.2	38.8	25.1	(63, 57)	36.0	15.4	38.4	24.6	(53, 47)	35.1	16.2	38.2	24.5	(47, 53)	34.8	12.8	36.6	22.8
		0.3	0.3	(88, 19)	7.2	32.6	27.1	18.2	(101, 21)	7.2	30.5	26.4	18.0	(83, 18)	6.8	31.5	26.6	17.8	(86, 18)	6.1	27.5	24.8	16.9
			0.5	(60, 47)	16.4	27.9	31.9	23.7	(66, 55)	16.9	25.9	31.7	24.1	(55, 45)	16.1	26.7	31.4	23.4	(51, 50)	16.0	22.4	30.3	23.3
		0.5	0.5	(88, 19)	15.6	42.8	37.2	27.0	(99, 22)	15.5	40.4	37.0	27.1	(82, 19)	15.0	41.5	36.7	26.6	(83, 20)	14.0	36.6	35.5	26.1

Table 5.7: Optimal first period decision, stochastic demand $\mu_1 = 100, \mu_2 = 200$ and $\sigma_1 = \sigma_2 = \sigma$.

				$\pi = 30$										$\pi = 15$									
				$cv = 0.1$					$cv = 0.3$					$cv = 0.1$					$cv = 0.3$				
(p_1, p_2)	Δc	b_1	b_2	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}
(0.9,0.9)	0	0.1	0.1	(94, 11)	0.5	0.5	1.9	1.7	(102, 12)	0.4	0.4	1.8	1.6	(86, 10)	0.4	0.4	1.9	1.6	(80, 9)	0.3	0.3	1.7	1.5
			0.5	(12, 93)	22.7	0.6	14.7	7.8	(12, 102)	22.9	0.5	15.1	8.2	(11, 85)	22.7	0.5	14.9	7.9	(8, 81)	22.9	0.3	15.4	8.4
		0.3	0.3	(90, 15)	1.6	1.6	4.0	3.6	(98, 16)	1.5	1.5	4.0	3.6	(83, 14)	1.5	1.5	4.0	3.6	(78, 12)	1.3	1.3	3.9	3.5
			0.5	(19, 86)	8.5	1.9	7.7	5.5	(19, 95)	8.7	1.8	8.0	5.7	(17, 79)	8.5	1.8	7.9	5.6	(14, 76)	8.8	1.5	8.3	5.8
		0.5	0.5	(87, 18)	2.7	2.7	4.6	4.4	(95, 19)	2.6	2.6	4.7	4.5	(79, 17)	2.6	2.6	4.7	4.4	(75, 15)	2.3	2.3	4.8	4.5
	10	0.1	0.1	(103, 0)	0.0	10.5	6.9	3.9	(113, 0)	0.0	9.5	6.3	3.6	(95, 0)	0.0	10.2	6.7	3.9	(91, 0)	0.0	8.7	5.9	3.4
			0.5	(65, 40)	15.4	4.6	13.2	8.9	(69, 47)	15.9	3.8	13.3	8.8	(59, 39)	15.4	4.1	13.1	8.7	(49, 44)	15.8	2.8	13.0	8.3
		0.3	0.3	(100, 6)	0.6	11.2	8.5	5.4	(110, 7)	0.6	10.3	8.1	5.2	(92, 6)	0.5	10.8	8.4	5.3	(89, 6)	0.4	9.2	7.7	5.0
			0.5	(66, 39)	2.2	6.1	6.8	5.0	(70, 46)	2.6	5.3	6.8	5.3	(60, 38)	2.2	5.6	6.7	5.0	(50, 43)	2.8	4.2	6.7	5.4
		0.5	0.5	(98, 7)	1.5	12.2	9.0	6.0	(109, 8)	1.6	11.2	8.8	5.9	(91, 7)	1.5	11.8	8.9	5.9	(87, 7)	1.3	10.0	8.5	5.8
	20	0.1	0.1	(104, 0)	0.0	23.5	13.6	7.3	(117, 0)	0.0	21.4	12.5	6.8	(98, 0)	0.0	22.9	13.3	7.2	(97, 0)	0.0	19.5	11.6	6.4
			0.5	(81, 25)	10.7	12.1	14.7	12.8	(89, 30)	11.2	10.5	14.5	12.6	(74, 25)	10.6	11.3	14.4	12.6	(70, 28)	11.0	8.3	13.6	11.2
		0.3	0.3	(103, 0)	0.0	23.5	14.7	8.2	(115, 0)	0.0	21.5	13.8	7.7	(96, 0)	0.0	22.9	14.5	8.1	(95, 0)	0.0	19.6	13.2	7.6
			0.5	(102, 5)	0.5	16.7	11.6	6.6	(115, 5)	0.6	14.5	10.8	6.2	(95, 5)	0.5	15.8	11.3	6.4	(94, 5)	0.4	12.1	9.8	5.8
		0.5	0.5	(101, 5)	0.6	24.3	15.0	8.5	(113, 6)	0.7	22.2	14.3	8.2	(95, 5)	0.5	23.5	14.7	8.3	(94, 5)	0.5	20.0	13.5	7.9
(0.8,0.9)	0	0.1	0.1	(10, 95)	1.7	0.4	2.5	1.9	(11, 103)	1.6	0.4	2.3	1.8	(9, 87)	1.6	0.3	2.4	1.8	(8, 81)	1.4	0.3	2.1	1.6
			0.5	(11, 94)	24.2	0.5	16.4	8.0	(11, 103)	24.3	0.4	16.7	8.4	(10, 86)	24.2	0.4	16.5	8.1	(7, 82)	24.2	0.3	16.9	8.6
		0.3	0.3	(14, 91)	4.6	1.4	5.2	4.1	(15, 99)	4.4	1.3	5.2	4.0	(13, 84)	4.4	1.3	5.2	4.0	(11, 79)	3.9	1.1	5.1	4.0
			0.5	(17, 88)	11.6	1.6	9.4	6.0	(18, 96)	11.7	1.6	9.6	6.2	(16, 80)	11.6	1.5	9.5	6.0	(13, 77)	11.7	1.3	9.9	6.3
		0.5	0.5	(17, 88)	6.5	2.4	6.3	5.0	(18, 96)	6.4	2.3	6.4	5.1	(16, 80)	6.4	2.3	6.3	5.0	(14, 76)	5.9	2.0	6.5	5.2
	10	0.1	0.1	(99, 0)	0.0	9.0	6.1	3.5	(109, 0)	0.0	8.2	5.6	3.2	(92, 0)	0.0	8.8	6.0	3.4	(88, 0)	0.0	7.4	5.2	3.1
			0.5	(65, 40)	16.8	4.4	14.9	9.0	(69, 47)	17.2	3.7	14.9	9.0	(59, 39)	16.7	4.0	14.7	8.8	(49, 44)	17.0	2.7	14.6	8.5
		0.3	0.3	(95, 11)	2.2	9.4	8.4	6.0	(105, 12)	2.1	8.6	8.0	5.8	(88, 10)	2.0	9.0	8.2	5.9	(84, 11)	1.8	7.5	7.6	5.5
			0.5	(66, 39)	5.1	5.8	8.5	7.0	(69, 47)	5.4	5.0	8.5	7.1	(59, 39)	5.0	5.3	8.4	7.0	(50, 43)	5.5	3.9	8.3	6.6
		0.5	0.5	(93, 12)	4.1	10.5	9.6	7.4	(103, 14)	4.1	9.6	9.4	7.4	(87, 11)	4.0	10.1	9.4	7.3	(82, 12)	3.7	8.4	9.0	7.1
	20	0.1	0.1	(102, 0)	0.0	21.9	12.8	6.8	(115, 0)	0.0	19.9	11.8	6.3	(96, 0)	0.0	21.3	12.5	6.7	(96, 0)	0.0	18.1	10.9	5.9
			0.5	(81, 25)	12.0	11.9	16.6	13.5	(89, 30)	12.5	10.3	16.3	12.8	(74, 25)	11.8	11.1	16.2	13.0	(70, 28)	12.1	8.2	15.2	11.3
		0.3	0.3	(100, 7)	0.7	20.6	13.7	7.8	(112, 8)	0.8	18.8	13.0	7.6	(93, 7)	0.6	19.9	13.4	7.7	(92, 7)	0.5	16.9	12.1	7.1
			0.5	(99, 8)	2.4	15.3	12.4	7.5	(111, 9)	2.5	13.2	11.6	7.2	(92, 8)	2.3	14.4	12.0	7.3	(91, 8)	2.1	10.7	10.6	6.7
		0.5	0.5	(98, 8)	2.6	21.8	15.1	9.2	(110, 9)	2.7	19.9	14.4	9.1	(92, 8)	2.5	21.0	14.7	9.1	(91, 8)	2.3	17.6	13.6	8.6

Table 5.8: Optimal first period decision, stochastic demand $\mu_1 = 100, \mu_2 = 50$ and $\sigma_1 = \sigma_2 = \sigma$.

				$\pi = 30$										$\pi = 15$									
				$cv = 0.1$					$cv = 0.3$					$cv = 0.1$					$cv = 0.3$				
(p_1, p_2)	Δc	b_1	b_2	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}	$\mathbf{q_1^*}$	Δ^{DS1}	Δ^{DS2}	Δ^{50}	Δ^{75*}
(0.7,0.8)	0	0.1	0.1	(18, 87)	2.2	0.9	2.8	2.2	(19, 95)	2.1	0.8	2.7	2.1	(16, 80)	2.1	0.8	2.7	2.2	(15, 74)	1.8	0.7	2.4	1.9
			0.5	(19, 86)	21.7	1.1	15.8	7.6	(19, 95)	21.7	1.0	16.0	7.9	(16, 80)	21.6	1.0	15.8	7.7	(13, 76)	21.4	0.7	16.0	8.0
		0.3	0.3	(21, 84)	6.0	2.9	6.4	5.3	(23, 91)	5.8	2.7	6.4	5.2	(20, 77)	5.8	2.7	6.3	5.2	(18, 72)	5.1	2.3	6.2	5.0
			0.5	(26, 79)	12.2	3.3	10.5	7.1	(27, 87)	12.2	3.1	10.7	7.3	(23, 73)	12.2	3.1	10.5	7.1	(20, 70)	12.0	2.6	10.8	7.2
		0.5	0.5	(25, 80)	8.5	4.5	8.2	6.9	(26, 88)	8.3	4.3	8.3	7.0	(23, 73)	8.3	4.3	8.2	6.9	(21, 69)	7.7	3.9	8.3	7.0
	10	0.1	0.1	(97, 8)	0.1	9.1	6.1	3.5	(107, 9)	0.2	8.4	5.7	3.3	(90, 8)	0.0	8.8	5.9	3.3	(86, 8)	0.0	7.4	5.1	2.9
			0.5	(69, 36)	15.3	5.9	15.3	9.6	(73, 43)	15.6	5.1	15.2	9.4	(63, 35)	15.2	5.4	15.1	9.3	(53, 40)	15.3	3.9	14.7	8.7
		0.3	0.3	(92, 14)	3.3	10.7	9.4	6.9	(101, 16)	3.3	9.9	9.1	6.8	(85, 13)	3.1	10.3	9.2	6.8	(80, 14)	2.8	8.6	8.5	6.3
			0.5	(69, 36)	6.5	8.2	10.5	8.5	(74, 42)	6.8	7.4	10.5	8.7	(63, 35)	6.4	7.8	10.4	8.5	(54, 39)	6.6	6.1	10.1	8.4
		0.5	0.5	(90, 15)	5.9	12.5	11.4	9.1	(99, 17)	5.8	11.6	11.3	9.1	(84, 14)	5.7	12.1	11.2	9.0	(79, 15)	5.3	10.2	10.8	8.7
	20	0.1	0.1	(101, 0)	0.0	21.9	12.7	6.7	(114, 0)	0.0	19.9	11.7	6.2	(94, 0)	0.0	21.3	12.4	6.6	(95, 0)	0.0	18.1	10.8	5.8
			0.5	(83, 23)	11.1	14.0	17.8	13.3	(92, 27)	11.5	12.3	17.3	13.4	(76, 23)	10.9	13.1	17.3	13.1	(72, 26)	11.0	10.0	16.1	12.2
		0.3	0.3	(97, 9)	1.5	21.6	14.6	8.4	(110, 10)	1.6	19.8	13.8	8.2	(91, 9)	1.3	20.9	14.2	8.2	(90, 9)	1.2	17.7	12.9	7.6
			0.5	(96, 10)	3.8	18.0	14.7	9.0	(109, 11)	3.9	15.8	13.9	8.8	(91, 9)	3.6	17.0	14.3	8.8	(89, 10)	3.4	13.2	12.7	8.1
		0.5	0.5	(96, 10)	4.1	23.7	16.8	10.6	(108, 11)	4.2	21.7	16.2	10.5	(89, 10)	3.9	22.8	16.4	10.4	(89, 10)	3.7	19.2	15.2	9.9
(0.7,0.7)	0	0.1	0.1	(79, 26)	1.6	1.6	2.8	2.6	(86, 28)	1.5	1.5	2.6	2.4	(72, 24)	1.5	1.5	2.7	2.5	(67, 22)	1.2	1.2	2.3	2.1
			0.5	(27, 78)	18.0	1.9	13.8	7.2	(27, 87)	18.0	1.7	13.9	7.4	(24, 72)	17.9	1.8	13.8	7.2	(19, 70)	17.8	1.3	13.9	7.3
		0.3	0.3	(75, 30)	4.7	4.7	6.5	6.2	(82, 32)	4.5	4.5	6.5	6.2	(70, 27)	4.5	4.5	6.4	6.1	(65, 25)	3.9	3.9	6.2	5.8
			0.5	(35, 70)	10.1	5.2	10.1	8.0	(37, 77)	10.0	5.0	10.3	8.0	(31, 65)	10.0	5.0	10.1	7.9	(28, 62)	9.8	4.3	10.2	7.8
		0.5	0.5	(73, 32)	7.0	7.0	8.6	8.4	(79, 35)	6.8	6.8	8.7	8.4	(67, 29)	6.8	6.8	8.5	8.3	(63, 27)	6.2	6.2	8.7	8.3
	10	0.1	0.1	(98, 7)	0.0	10.5	6.6	3.6	(108, 8)	0.1	9.6	6.1	3.4	(91, 0)	0.0	10.2	6.4	3.6	(87, 0)	0.0	8.6	5.6	3.2
			0.5	(72, 33)	12.7	7.5	14.2	10.0	(78, 38)	12.9	6.6	14.0	9.7	(66, 32)	12.5	7.0	14.0	9.7	(57, 36)	12.6	5.2	13.4	8.8
		0.3	0.3	(93, 13)	2.7	13.4	10.3	7.0	(103, 14)	2.7	12.5	10.0	6.9	(86, 12)	2.5	13.0	10.1	6.9	(82, 12)	2.2	11.1	9.3	6.5
			0.5	(73, 32)	5.1	11.0	11.0	8.1	(79, 37)	5.4	10.1	10.9	8.2	(67, 31)	5.1	10.5	10.8	8.0	(59, 34)	5.2	8.6	10.4	8.0
		0.5	0.5	(92, 13)	4.9	15.9	12.4	9.1	(101, 15)	4.9	14.8	12.3	9.1	(85, 13)	4.7	15.4	12.3	9.0	(80, 14)	4.4	13.3	11.8	8.8
	20	0.1	0.1	(102, 0)	0.0	23.5	13.4	7.0	(114, 0)	0.0	21.3	12.3	6.5	(95, 0)	0.0	22.8	13.1	6.9	(95, 0)	0.0	19.4	11.4	6.1
			0.5	(85, 21)	9.0	16.2	17.2	12.1	(94, 25)	9.3	14.4	16.6	12.2	(78, 21)	8.8	15.3	16.7	11.9	(75, 23)	8.9	11.9	15.3	11.5
		0.3	0.3	(98, 8)	1.1	24.8	15.7	8.8	(111, 9)	1.2	22.8	15.0	8.5	(92, 8)	1.0	24.0	15.4	8.6	(91, 8)	0.9	20.5	13.9	8.0
			0.5	(97, 9)	3.1	21.7	15.8	9.3	(110, 10)	3.2	19.4	15.0	9.0	(91, 9)	2.9	20.7	15.4	9.1	(90, 9)	2.8	16.6	13.8	8.4
		0.5	0.5	(97, 9)	3.3	27.5	18.1	10.9	(109, 10)	3.4	25.4	17.5	10.8	(90, 9)	3.1	26.6	17.7	10.7	(90, 9)	3.0	22.8	16.5	10.2

Table 5.9: Optimal first period decision, stochastic demand $\mu_1 = 100, \mu_2 = 50$ and $\sigma_1 = \sigma_2 = \sigma$.

Chapter 6

Summary and Outlook

This dissertation provides dual sourcing models under supply disruption risk. The key contributions of this work can be summarized as follows:

- We build a semi-Markov decision model that we use to find the optimal sourcing and ordering policy of a buyer that sources from two (or multiple) suppliers under Poisson demand, exponentially distributed lead times, and exponentially distributed ON and OFF times of the suppliers (Chapter 3).
- We develop a finite horizon dynamic programming model to find the optimal dual sourcing allocation policy with disruption risk and supply cost improvements through learning (Chapter 4).
- We discuss the sensitivity of the model of Chapter 4 to the following limiting model assumptions: complete information about supplier reliability, known demand and risk neutral decision maker (Chapter 5).

Chapter 3 focused on an inventory problem of a buyer facing stochastic demand who can source from multiple potential suppliers subject to disruption risk and with uncertain lead times. Each supplier is fully available for a certain amount of time (ON periods) and then breaks down for a certain amount of time during which it can supply nothing at all (OFF periods). A semi-Markov decision process formulation is presented assuming Poisson demand and exponentially distributed lead times and ON and OFF times. Solution methods are discussed to compute the buyer's optimal sourcing policy that minimizes the buyer's long run average cost, including purchasing, holding and penalty costs. In an illustrative example the optimal policy is found to be rather complex and not of a base stock type. An extensive numerical study that focuses on the dual sourcing case is conducted. The results illustrate the benefit from dual souring compared to single sourcing and show the influence of the suppliers' characteristics cost, speed, and availability on the optimal policy. The value of full information about the supplier status switching events is analyzed and found to be very important especially when supplier availabilities are low and disruption periods are long. The performance of the optimal policy is compared to an order-up-to-S policy with fixed S and variable S for each supply base combination. The later outperforms the fixed S case in consideration of nonidentical suppliers. A simple heuristic providing good results compared to the optimal solution is developed. Finally, due to simulation results we find that the optimal policy is very sensitive with respect to more general ON and OFF and lead time distributions. A comparison to a more dramatic supplier break down scenario shows that the optimal policy is also sensitive to the definition of the impact of a disruption.

Chapter 4 focused on a supplier selection and demand allocation problem of a buyer that can source from two potential suppliers. To optimize the buyer's allocation problem two facts are considered. First, the suppliers reduce future cost through learning based on past production which favors single sourcing to benefit the most from learning. Second, the suppliers face the risk of a permanent disruption where dual sourcing is a prevailing strategy to diversify the supply risk. Hence, we analyze the trade-off between risk reduction via dual sourcing under disruption risk and learning benefits on sourcing costs induced by long-term relationships with a single supplier. The buyer's optimal volume allocation strategy over a finite dynamic planning horizon is identified and we find that a symmetric demand allocation is not optimal even if suppliers are symmetric. We obtain insights on how reliability, cost and learning ability of potential suppliers impact the buyer's sourcing decision and find that the allocation balance increases with learning rate and decreases with both reliability and demand level. Moreover, we show cases where, even if a supplier has higher cost than the other supplier, it is optimal for the buyer to use dual sourcing to compensate the risk of supply disruptions when the future value gained by dual sourcing is sufficient. In addition, a supplier with a relatively fair actual cost disadvantage but a future learning ability advantage can get larger parts of demand in the actual period. Further, we quantify the benefit of dual sourcing compared to single sourcing, which increases with the learning rate and decreases with the reliability. We find that the benefit of the optimal dual sourcing policy can be significant, especially compared to single sourcing.

Chapter 5 extended the basic model assumption of Chapter 4 to analyze the sensitivity of the model to limited assumptions. The basic analysis is extended by the assumptions of incomplete information about supplier reliabilities, stochastic demand and a risk-averse decision maker. We find that increasing uncertainty about reliability leads to a less balanced initial volume allocation. Assuming stochastic demand we find that demand uncertainty affects both, the total order quantity and the balance between the suppliers. The savings of dual sourcing decrease with demand uncertainty. Moreover, the assumption of a risk-averse buyer leads to a more balanced order allocation between the suppliers.

Our results provide important implications for purchasing organizations that have to deal with supply disruption risk. We show that relying on a single supplier, in most of the cases, has serious impacts for the supply chain in the presence of supply disruption risk and that dual sourcing is an effective countermeasure. Considering supply uncertainty in supply chain models in order to derive optimal policies is essential for choosing the right purchase strategy and for improving a firm's performance.

Based on the results of this thesis, future work could extend the dual sourcing models by allowing an arbitrary number of suppliers. This could answer the following questions: "How many suppliers are best when dealing with the supply disruption risk?" and "What is the value added by extending the supply base from two to multiple suppliers?". The answer to the latter question would provide insights on the efficiency of the dual sourcing policy compared to the multiple (more than two) sourcing policy.

Another interesting extension would allow for correlation of supply disruptions between the suppliers. The assumption of supply risk dependency may be reasonable for certain supplier portfolios and could further shed light on successful disruption risk management.

A further possible direction for future research would be to establish a framework for supplier qualification before choosing effective sourcing strategies. We assume that suppliers in the supply base are already qualified. Integrating the qualification process, associated with cost, into the decision process could provide further insights into the effectiveness of the particular strategies. Hence, establishing a structured supplier evaluation process could enrich the appropriate supply risk management.

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