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„Total Beta vs. Mayers' CAPM with non-marketable assets: analytical models for SME valuation"

verfasst von / submitted by
Daniel-Andreas Lepis

gemeinsam mit / together with
Philip Nagengast

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List of abbreviations

AIM	Alternative Investment Market, a small cap market in the United Kingdom
APV	Adjusted Present Value
AR	Autoregressive (model)
BPM	Butler Pinkerton Model
CAPM	Capital Asset Pricing Model
CML	Capital Market Line
CNY	Chinese Yuan (Renminbi)
CPT	Cumulative Prospects Theory
CSRP	Company-Specific Risk Premium
DAX30	Deutscher Aktienindex, the leading blue chip stock market index of Germany
DCF	Discounted Cash Flow
EBITDA	Earnings Before Interest, Taxes, Depreciation and Amortisation
EC	European Commission
EPS	Earnings per Share
EU	European Union
EUR	Euro
EUT	Expected Utility Theory
FAQ	Frequently Asked Questions
FCF	Free Cash Flow
GDP	Gross Domestic Product
HC	Human Capital
HGB	Handelsgesetzbuch (Commercial Code of Germany)
IAS	International Accounting Standards
IFRS	International Financial Reporting Standards
ITS	Interest Tax Shiedl
JPY	Japanese Yen

KfW	Kreditanstalt für Wiederaufbau, German development bank
LE	Large Enterprise
MIIT	Ministry of Industry and Information Technology (PRC)
MOF	Ministry of Finance (of the PRC)
MPT	Modern Portfolio Theory
NBS	National Bureau of Statistics (PRC)
NDRC	National Development and Reform Commission (PRC)
NOPAT	Net Operating Profit After Taxes
PEG	Price/Earnings to Growth Rate
PRC	People's Republic of China
ROE	Return on Equity
SLM	Sharpe-Lintner-Mossin, an alternative name of the CAPM
SME	Small and Medium-Sized Enterprises
TCOE	Total Cost of Equity
USA	United States of America
USITC	United States International Trade Commission
WACC	Weighted Average Cost of Capital

1. Introduction

With the global mergers and acquisitions activity at historical heights (Bloomberg, 2016), determining the value of potential to-be-acquired firms seems more important than ever. For large enterprises listed on stock markets, a relatively large amount of publicly available information and, more importantly, the company's market price (the market capitalisation) can be used in valuation. Relatively little (or often, none) of this information is available for smaller enterprises, making valuation difficult and costly. Moreover, a universally accepted pricing model that considers the peculiarities of small and medium-sized enterprises (SMEs) has yet to be developed.

   This insufficiency in valuation theory (and practice) is the point of departure for the present Thesis; if there is little to no possibility of determining the fair value of firms, then a fair market is impossible. Therefore, we believe that it is imperative to develop a method for SME valuation that fulfils the following criteria: firstly, it must be theoretically sound. Secondly, it must account for characteristics of SMEs that differ from those of large enterprises. And thirdly, it must strike an optimal balance between accuracy and practicability – an overly complicated model is not likely to find universal acceptance among lawmakers and practitioners, no matter how accurate it is.

1.1. Objectives and methods

The main objective of our work is finding a valuation method that satisfies the aforementioned criteria. For this purpose, we find it necessary to first analyse existing valuation techniques and, if possible, identify techniques designed especially for SME valuation, one of which may be the so-called Total Beta. Assuming that none of the existing approaches satisfies the three criteria above, we aim to introduce a new method that could do so. This method is based on the "modified model" of Mayers (1972) who extended the standard Capital Asset Pricing Model (CAPM) by the human capital of investors. Because of this CAPM affiliation, we feel that the standard CAPM itself must be thoroughly discussed, particularly with regard to its flaws, the way it used in the modified model and the reason why it cannot be used for SME valuation. Our aim is to

provide a comprehensive picture of this standard model and to justify the use of a CAPM-based method despite the serious shortcomings of the standard CAPM.

The approach of Mayers (1972) may seem to be a relatively simple analytical extension of the standard CAPM, but it actually foots on a number of complex concepts of financial theory. We will discuss the modified model and its underlying concepts in great detail in order to provide a thorough explanation for the reader. Human capital – the centrepiece of Mayers' concept – also has to be addressed carefully, while keeping in mind that it is difficult to quantify due to its highly subjective nature. One larger section will thus be devoted to discussing methods of operationalising human capital. Finally, we will test the modified model on a sample of SMEs in Germany. The choice fell on Germany for two reasons: firstly, it boasts a relatively high SME density in all major industries. And secondly, German SMEs have a high export ratio and cooperate closely with leading German multinationals. Owing to this, they are more likely to follow worldwide economic trends, which is useful for the empirical analysis, because those trends are largely homogeneous; heterogeneous trends within the sample might neutralise or amplify each other and thus distort the result.

1.2. Proposed structure

In order to achieve the aforementioned objectives, we will use a three-tier structure: tier 1 encompasses a theoretical analysis of existing valuation approaches. A special focus will lie on the Total Beta method since it has been developed particularly for SMEs. Tier two will be dedicated to the newly proposed method based on Mayers' modified model and the aforementioned discussions on the CAPM. Human-capital related topics will also be included here. Finally, the empirical testing of the proposed model and a subsequent discussion of the results will constitute tier three.

2. Small and medium-sized enterprises (SMEs)

We begin by providing a definition for our object of study, which is somewhat difficult in the case of small and medium-sized enterprises (SMEs), since a universally accepted definition has not been developed (and likely never will be). In the following, we present relevant regulations in some major economies and the economic impact of SMEs. The chapter is concluded by a brief introduction into SME valuation and its challenges.

2.1. Enterprise size and definitions of SMEs

As can be seen in the various definitions of SMEs we summarise below, enterprise size is not at all determined by the size of its physical assets such as facilities. The most commonly used traits for size determination are: staff size, turnover, value of gross assets and balance sheet total. In a number of definitions, the values of more than one trait are considered to classify enterprises into certain categories. The threshold values, as well as the categories vary across countries. In the following, we provide SME definitions as used by major world economies:

The European Commission (EC) defines medium-sized, small and micro companies. As this is merely a recommendation, each member state may use its own definitions, i.e. company categories, classification criteria and thresholds (EC, 2003):

Table 1: SME definition according to EU recommendation 2003/361 (EC, 2003)

Company category	Employees	Turnover	or	Balance sheet total
Medium-sized	< 250	≤ €50 mil		≤ €43 mil
Small	< 50	≤ €10 mil		≤ €10 mil
Micro	< 10	≤ €2 mil		≤ €2 mil

A further criterion of Recommendation 2003/361 is *independence*, i.e. the share of external shareholders of an SME should not exceed 25 per cent (Par. 2).

Selected EU member states have varying definitions for SMEs: while the French *loi de modernisation de l'economie* uses exactly the same definition as EC Recommendation 2003/361 (l'article 51), the German *Handelsgesetzbuch* (§ 267) differentiates between *kleine Kapitalgesellschaften* (small companies) and *mittelgroße Kapitalgesellschaften* (medium-sized companies); for a company to fall in either of these two categories, it has to fulfil two of three criteria – number of employees (less than 50 and 250, respectively), yearly turnover (below € 12 mil and € 40 mil, respectively) or balance total (€ 6 mil and € 20 mil, respectively). The law also mentions *Kleinstkapitalgesellschaften* with an annual average of 10 employees, a balance total of €350,000 or a yearly turnover of €700,000 (§ 267a HGB). The United Kingdom's *Companies Act 2006* uses the same categories with the same thresholds for the number of employees as the German law but with different thresholds for turnover and balance sheet total: not more than £6.5 million[1] and £3.26 million[2], respectively for a small company (s. 382) and £25.9 million[3] and £12.9 million[4], respectively, for a medium-sized company (s. 465).

In Japan, Article Two of the Small and Medium-Sized Enterprises Base Law (*Chūshōkigyō Gihonhō Dainijō*) considers the number of employees and the company's share capital. Although the definition varies across branches, even the "largest" SME considered by the law does not have more than 300 employees and a base capital of more than JPY 300 million[5].

In the United States, several governmental agencies issued SME definitions according to sector affiliation; while the number of employees must not exceed 500 in any case, the threshold for yearly revenues varies. The largest SME type considered has revenues of not more than $25 million (USITC, 2010)[6].

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[1] EUR 7.51 million.
[2] EUR 3.77 million.
[3] EUR 29.92 million.
[4] EUR 14.90 million.
[5] EUR 2.65 million.
[6] EUR 22.27 million.

The People's Republic of China (PRC[7]) has by far the most complicate view on SMEs of all the countries we consider. The Ministry of Industry and Information Technology (MIIT) together with the National Bureau of Statistics (NBS), the National Development and Reform Commission (NDRC) and the Ministry of Finance (MOF) issued a common Regulation on SME Size Classification (*Zhōngxiǎoqǐyè Huàxíng Biāozhǔn Guīdìng,* 2011) which uses the number of employees, revenue and total assets to classify enterprises into medium-sized, small and micro depending on sector affiliation (for fifteen different sectors) . However, the use of these classification criteria is not consistent. For some sectors, only two or just one of three is used. The largest possible number of employees for an SME in China is 2000 in the Information Transmission sector (*Xìnxī Chuánshūyè*). An enterprise could have a turnover of up to CNY 2 billion[8] and an asset total of CNY 1.2 billion[9] in and would still be considered an SME (MIIT, 2011).

Aside from these purely quantitative definitions, SMEs can also be defined qualitatively by three typical characteristics of such enterprises (Knop, 2009: 9):

1. Concentration of ownership, management, liability and risk in the hands of the entrepreneur;
2. Flat hierarchy – simple organisational structure with little delegation and informal and close contact between management and employees;
3. High local orientation – the entrepreneur is in personal contact with customers and suppliers, making the enterprise more responsive to customer's demands and/or the general market situation.

In this Thesis, we consider SMEs of all sizes and legal forms except *Aktiengesellschaften*. The sample will be discussed in Chapter 5.

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[7] In this Thesis, the PRC is defined as Mainland China without the Hong Kong Special Administrative Region and the Macao Special Administrative Region, as these regions have separate jurisdictions with legislations that differ greatly from that of Mainland China.

[8] EUR 267.04 million.

[9] EUR 160.22 million.

2.2. Economic impact of SMEs

Existing business and finance literature usually deals with large enterprises, because their formalised structures and processes facilitate the analysis (which is in many cases hardly possible for the often less transparent smaller companies). However, large enterprises constitute only a fraction of all enterprises; figures for Europe show that over 99% of all enterprises are SMEs (EC, 2003). Furthermore, SMEs contribute to up to two thirds of employment and to a little over half of the GDP in high-income countries. These figures are lower in medium- and low-income economies (Ayyagari, Beck & Demirguc-Kunt, 2003). The reason for these discrepancies lies in the development stage of institutions; SMEs tend to thrive in countries with a stronger regulatory environment, particularly with regard to the enforcement of property rights (Ayyagari et al, 2003).

Knop (2009: 12) summarises the contribution of SMEs to an economy from a macro-economic perspective: firstly, SMEs improve an economy's competitiveness, as they offer a broadly differentiated economic output. Secondly, SMEs are able to adapt faster and more flexibly to new circumstances, hence they act as a stabilising force in the economic cycle. And thirdly, SMEs are important contributors to local employment and council tax. Their 'localness' is a crucial factor in regional development; while large enterprises (LEs) usually need to be located in larger and denser regions to be innovative, this is not true for SMEs (Karlsson & Olsson, 1998). Therefore, SMEs can remain in their (in some cases more rural) home region without negative consequences and the region profits from the presence of such enterprises.

2.3. Difficulties in SME valuation

Other than many large enterprises, SMEs are relatively rarely listed on stock exchange markets. There are dedicated SME markets with easier listing criteria such as the AIM UK or the German Entry Standard Index of Deutsche Börse; however, only a fraction of all SMEs is actually listed on these markets (Caccavaio et al, 2012). And even among the listed ones, not few are in concentrated ownership, for example in the hands of a family (Kramer, 2000). The value of a company whose shares are not listed (and thus

sparsely or not at all traded by investors) is thus much harder to determine, as there is no consensus among numerous investors as to what the company in question is really worth. In addition, non-listed firms are not subject to strict transparency regulations as is the case with listed ones, meaning that periodical results, investments or balance sheet ratios are not publicly available. Without a market price and publicly available data, valuation becomes much trickier.

Even if data is available, the choice of the valuation method is also a factor that needs to be taken into consideration. Approaches which utilise stock market data (i.e. share prices) are not usable for the reasons mentioned above. Furthermore, if a model (e.g. the CAPM) predicts perfect diversification of investments of the investors/owners, it may not accurately account for the reality of SMEs, where there are usually few investors/owners, whose investments are likely not broadly diversified and largely comprised of the SME itself (Damodaran, 2002). Another issue is that standard valuation models, assuming an infinite lifetime[10] of enterprises, take into account a so-called residual value which significantly contributes to the final value of the enterprise. It has been shown, however, that most enterprises do not exist for more than 40 years (Hüttche, 2014). All this needs to be considered when choosing a valuation method.

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[10] Frühling (2009) shows that using the perpetuity formula with a reasonable after-tax discount rate (5-10%) actually implies an enterprise life cycle of 80-120 years.

3. Valuation of small and medium-sized enterprises

This chapter introduces techniques that can be used for SME valuation. After providing an overview of the most well-known methods, we will turn to an approach designed particularly for smaller enterprises – the so-called Total Beta. Following that, we introduce a new potential SME valuation method based on the work of Mayers (1972) that considers the human capital of the owners/managers in addition to market and firm data. The chapter is concluded by a discussion of the latter two approaches.

3.1. Well-known valuation methods

3.1.1. Book value method

A relatively simple way of determining a firm's value is taking its equity by subtracting the value of the liabilities from that of the assets. Although this information is readily available just by looking at the balance sheet, it merely provides a static, ex-post view on the firm's value. Even after the creation of the International Financial Reporting Standards (IFRS)/International Accounting Standards (IAS), with different accounting standards still being used worldwide, any balance-sheet-based valuation method suffers from a lack of comparability; this applies especially in the case of SME, as these will not be likely to follow IFRS/IAS in the foreseeable future. Any intangible assets are thus not captured when using the book value method. The more serious issue is that only historical costs are considered for book values. And even though the *adjusted* book value method tries to ameliorate this flaw by employing market prices/replacement costs, it sees just as little practical application as the simple book value (Dukes, Bowlin & Ma, 1998: 427).

3.1.2. Capitalisation of earnings

A more forward-looking approach involves earnings in the form of dividends and a capitalisation rate. The value of an ownership share is then determined by discounting expected future dividend payments: under perfect capital markets and certainty, the discount rate in question would be equivalent to the equilibrium one-period spot rate of

interest. In a real-world setting under uncertainty, ad-hoc adjustments are performed in order to account for risk. Since such adjustments are not performed in accordance with an established theory, they vary among practitioners (Gonedes & Dopuch, 1974:97). Hence, the trickiest part of this method is identifying an appropriate capitalisation rate, as different practitioners may consider different subjective factors when determining it (Dukes, Bowlin & Ma, 1998). Nevertheless, despite its relative simplicity, it is one of the most recognised valuation techniques for small and large enterprises alongside discounted cash-flow methods (Dukes, 2001).

3.1.3. Valuation multiples

Another relatively simple approach to enterprise valuation employs so-called multiples derived mostly from the income statement. Each company's set of multiples has little validity on its own, which is why a set of comparable firms has to be chosen, for which the selected multiples have to be calculated as well. The "comparability" of firms has to be stressed here, as the measures considered in valuation (e.g. earnings, cash-flows etc.) may differ greatly depending on factors like industry affiliation. Therefore, it is reasonable to choose firms of the same industry as the valuation target, ideally with a similar history of earnings growth in order to minimise valuation errors (Boatsman & Baskin, 1981)[11]. Regarding the choice of the multiples set itself, the industry affiliation is of less importance; it has been shown that there are no sets of "optimal multiples" for different industries (Liu, Nissim & Thomas, 2002). Commonly used multiples are based on measures such as equity or enterprise value, sales, EBITDA (earnings before interest, taxes, depreciation and amortisation). Forward measures, such as forecasts of the EPS (earnings per share) or PEG (price-earnings-growth) ratios may be used as well; indeed, these seem to perform better in predicting stock prices (Liu, Nissim & Thomas, 2002). The selected multiples are then calculated for each of the comparable firms. After calculating certain statistics (e.g. mean, median, maximum) for the multiples, a value

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[11] According to the findings of Boatsman and Baskin (1981), making a random choice of firms within the same industry results in relatively larger valuation errors than when firms with similar earnings growth histories are chosen.

range for the valuation target is estimated by multiplying these multiples with the target's accounting measures.

Valuation with multiples is certainly quite convenient and less complicated in implementation than valuation techniques relying on capitalisation or discount rates. The key weakness of this method lies in the need for comparable firms which may either be selected incorrectly or may distort the picture in case they are not fairly valued by the market or (in the worst case scenario) do not report correct values in the income statement. That said, the values of certain multiples, especially those based on the EBITDA, depend on the accounting standard (Hüttche, 2014), which would further reduce comparability in case of discrepancies. Furthermore, Alford (1992) found that the valuation accuracy decreases with the size of the firm. This is a particularly critical finding for our study, given that we consider the smallest category of firms. Another issue is that there are few traded comparable companies, which makes trading multiples difficult or impossible to obtain. Using transaction multiples instead of trading multiples may solve this issue. However, such data is unavailable for the majority of transactions involving SME. Given that plausible budget planning figures are usually not available either (Hüttche, 2014), forward measures like those mentioned above cannot be used in the case of SME. Therefore, with only the less accurate measures based on historical data left and considering the fact that multiples-based valuation is less accurate for small enterprises, the valuation results for SME should be interpreted with caution. They may provide a quick overview, but are not a replacement for thorough, in-depth valuation (such as discounted cash flow valuation).

3.1.4. Discounted cash flow (DCF)

Valuing a company by calculating the present value of (expected) future cash flows is somewhat trickier than using the aforementioned methods, as neither of its components – cash flows and discount rate – can be directly taken from the balance sheet and/or income statement. The DCF is thus one of the more complicated, albeit fundamentally more thorough approaches and most firms seem to see it as the most practical valuation method (Dukes, Bowlin & Ma, 1996).

All cash-flow-based valuation methods begin by estimating expected cash flows for a certain time window in the future, the so-called projection period. The basis for such estimates is the operational Free Cash Flow (FCF), which is calculated from the net operating profit after taxes (NOPAT) by adding depreciation and other non-cash changes and subtracting changes in the working capital and capital expenditure. After that, the procedure varies with the approach used:

1. *Weighted Average Cost of Capital (WACC) approach*

   The WACC-based DCF formula can be written as follows:

$$V = \sum_{t=1}^{T} \frac{FCF_t}{(1 + WACC)^t} + \frac{RV_T}{(1 + WACC)^T} \tag{1}$$

where V is the enterprise value, FCF is the expected free cash flow, $WACC$ is the discount rate and RV_T is the estimated residual value at the end of the cash flow projection period. The weighted average cost of capital is then determined as follows:[12]

$$WACC = R_E \frac{MV_E}{MV_E + MV_D} + R_D(1 - \tau)\frac{MV_D}{MV_E + MV_D} \tag{2}$$

where R_E is the cost of equity, R_D is the cost of debt, τ is the corporate tax rate and MV_E and MV_D are the market values of equity and debt, respectively. Here the challenging nature of the WACC approach becomes apparent: to calculate an appropriate discount rate, one has to use a formula, in which yet another variable is not directly observable – the cost of equity R_E. A common way of determining R_E is using the CAPM which provides the following estimate:

$$E(R_E) = R_f + \beta_E\big[E(R_M) - R_f\big] \tag{3}$$

where R_f is the risk-free rate, β_E is the levered equity beta and $\big[E(R_M) - R_f\big]$ is the market risk premium. β_E is also not directly observable but can be estimated from

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[12] This WACC formula assumes that the firm is financed only by equity and debt. It also incorporates tax effects.

historical data or even obtained from statistical databases[13]. As this is usually not possible for small enterprises, other beta estimation techniques (e.g. the Butler Pinkerton Model or Mayers' modified model) have to be used. We will discuss these in the following sections.

Determining the residual value RV_T poses yet another challenge in using the WACC approach. A commonly used way of estimating it is calculating the first FCF after the projection period by additionally accounting for the reinvestment rate k and the expected long-term growth rate g (Damodaran, 2012).[14] This yields an FCF of:

$$FCF_T = NOPAT_T(1 - k) \tag{4}$$

Taking into account the long-term growth rate g[15], we obtain:

$$RV_T = \frac{NOPAT_T(1 - k)(1 + g)}{WACC - g} \tag{5}$$

2. *Adjusted Present Value (APV) approach*

As the WACC is comprised of the cost of equity and the cost of debt including the debt ratio, this ratio would have to be constant. In case more flexibility is needed, the APV approach may provide better results, due to the fact that the present values of unlevered equity and of the interest tax shield (ITS) are calculated separately. In fact, after the long dominance of the WACC-based DCF, APV has gained increasing support in the 1990s (Jonikas, 1998). The APV further incorporates expected bankruptcy cost, thus yielding the value of the levered firm:

$$V = \frac{FCF_0(1 + g)}{R_U - g} + \tau D - \pi BC \tag{6}$$

where FCF_0 is the current Free Cash Flow, R_U is the cost of unlevered equity, tD the value of the interest tax shield on debt[16], π is the probability of default after

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[13] This applies to larger firms listed on stock exchanges whose data is readily available.

[14] $k = \frac{(Capital\ expenditures - Depreciation + \Delta WC)_t}{NOPAT_t}, for\ all\ t \geq T$

[15] $g = \frac{\Delta NOPAT_{t+1}}{NOPAT_t} = \frac{ROIC(Capital\ expenditures - Depreciation + \Delta WC)_t}{NOPAT_t} = ROIC \times k$

additional debt and BC is the bankruptcy cost. The cost of unlevered equity R_U can be estimated by using the CAPM:

$$R_U = R_f + \beta_U\big(R_M - R_f\big) \tag{7}$$

There are different ways of calculating the unlevered beta β_U (c.f. Harris & Pringle, 1985; Miles & Ezzell, 1980). As an example, we present the one suggested by Damodaran (2012):

$$\beta_L = \beta_U \left(1 + (1 - t)\frac{MV_D}{MV_E + MV_D}\right) \tag{8}$$

where β_L is the levered beta which is the current investment beta of the firm. By rearranging, we can compute the unlevered beta as:

$$\beta_U = \frac{\beta_L}{\left(1 + (1 - t)\dfrac{MV_D}{MV_E + MV_D}\right)} \tag{9}$$

What remains to be determined is the last component of the APV levered firm value formula – the present value of the bankruptcy cost πBC. With the corporate bond rating of the firm available, the probability of bankruptcy after additional debt π can be obtained from corporate default rates charts, such as the one published for the period from 1981-2014 by Standard & Poor's (2015). The bankruptcy cost BC itself cannot be directly observed which makes it necessary to rely on empirical data. A number of authors have attempted to estimate direct and indirect bankruptcy costs (e.g. Altman, 1984; Kwansa & Cho, 1995; Bris, Welch & Zhu, 2006), with results ranging from 2% to over 20% of total enterprise value.

In summary, an appropriate approach must be chosen for the valuation target according to its financing policy: the WACC for firms with set future debt *ratios* and the APV for firms with set future debt *amounts*. This choice is of crucial importance, as the two approaches yield "[…] thoroughly different values of firms (Kruschwitz & Löffler, 2006).

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[16] $ITS = \frac{t \times R_D \times D}{R_D} = tD$

DCF valuation is well established both among academics and practitioners. However, in the explanations above we showed that a relatively large amount of information and estimation is necessary, which means that the approach most likely works best for larger and listed firms. Hence, results for smaller enterprises may not be optimal, given that peculiarities of such firms (e.g. incomplete compliance with CAPM predictions, imperfect cash management, etc.) are often not accounted for in the standard frameworks (Hüttche, 2014).

3.2. Total Beta

The valuation methods discussed thus far have at least one thing in common – they are, for various reasons (see above), not by default optimised for SME valuation. One of these reasons is the CAPM prediction of perfectly diversified investments of the investors. It stands to reason to hypothesise that this prediction is likely being violated in most SME, where the owner often is the only investor who has limited options for diversification. Due to the lack of diversification, the total risk of the firm could be higher than the systematic risk predicted by the standard CAPM beta.

In order to tackle this issue, Damodaran (2002) proposed a new approach accounting for the under-diversification in small enterprises. In such a case, the investor not only faces the market risk as measured by the standard CAPM beta, but also additional firm-level risk depending on the degree of under-diversification. According to Damodaran, this requires an adjustment to the "market beta". The market beta is defined as follows[17]:

$$\beta_j = \frac{\rho_{jM}\sigma_j}{\sigma_M} \tag{10}$$

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[17] This expression can be *mathematically* derived from the standard CAPM beta definition $\beta_j = \frac{cov(j,M)}{\sigma_M^2}$; given that the correlation between individual stock returns and market returns is defined as $\rho_{jM} = \frac{cov(s,M)}{\sigma_j\sigma_M}$, we obtain the expression above by rearranging terms:

$\beta_j\sigma_M^2 = \rho_{jM}\sigma_j\sigma_M$

$\beta_j\sigma_M = \rho_{jM}\sigma_j$

$\beta_j = \frac{\rho_{jM}\sigma_j}{\sigma_M}$

where σ_j is the standard deviation (total risk) in the company's equity, σ_M is the standard deviation (total risk) in the market and ρ_{jM} is the correlation between the firm's equity (i.e. its stock) and the market (represented by an index). Damodaran (2002) then proceeds by dividing the market beta β_M by the correlation term ρ_{jM} in order to obtain the exposure to total enterprise risk σ_j:

$$\beta_j/\rho_{jM} = \frac{\sigma_j}{\sigma_M} \tag{11}$$

He calls this expression in which the two standard deviation terms are scaled against one another "total beta" which we denote as β_{Total}. Naturally, this total beta will be higher than (or equal to) the market beta β_j and it is positively correlated with the correlation between the firm's equity and the market index. Hence, according to Damodaran (2002), it is possible to measure the investors' exposure to the "total risk of the firm" rather than just the market risk. The argument seems to be that with a totally under-diversified investor, the correlation with the market ρ_{jM} becomes obsolete. However, as simple as this approach may seem, it is not uncontroversial. We summarise arguments of its supporters and opponents below.

3.2.1. Total Beta and the Butler Pinkerton model: an overview

Damodaran (2002) himself has not really further advanced the notion of the total beta[18]. Instead, Butler & Pinkerton (2006) used the concept to create their Butler Pinkerton Model for Company-Specific Risk (BPM), in which they define the total cost of equity for a single asset (TCOE) as follows:

$$TCOE = r_f + \beta_{Total}\left(R_M - R_f\right) \tag{12}$$

Or alternatively as:

$$TCOE = R_f + \beta_j\left(R_M - R_f\right) + SP + CSRP \tag{13}$$

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[18] Indeed, "Total Beta" is merely a new name for a measure called the "Beta quotient" developed by Camp & Eubank (1981).

where SP is a size premium and $CSRP$ is the company-specific risk premium. The CSRP is then defined as:

$$CSRP = (\beta_{Total} - \beta_j)(R_M - R_f) - SP \tag{14}$$

The $TCOE$ is not directly estimated for private companies, but for so-called "guideline comparables", i.e. the smallest of public companies (whose situation is assumed to be comparable to that of private firms) by using historical stock prices. The size premium SP has to be estimated as well[19] before the $CSRP$ can be calculated.

This basic methodology has remained unchanged. However, in an attempt to prove Total Beta's worth, Butler & Schurman (2011) have made some alterations. As a starting point, they define a two-asset portfolio with the following expected return:

$$R_P = wR_j + (1 - w)R_M \tag{15}$$

where w is invested into the small business j and $(1 - w)$ into the market portfolio. This expression bears a striking similarity to the capital market line (CML) portfolio, only that the risk-free return R_f has been replaced by R_j. The authors substitute the market price of risk by[20]:

$$\varphi = \frac{(R_M - R_f)}{\sigma_M} \tag{16}$$

and define λ as the percent share of the private firm's Total Beta that has not been eliminated by diversification in the two-asset portfolio to arrive at:

$$R_j = R_f + \sigma_j \lambda \varphi \tag{17}$$

In their opinion, this is "no different in concept from the CAPM equation" (Butler & Schurman, 2011:23), provided that their derived definition of the market beta as given in Equation (10) is used:

$$R_j = R_f + \rho_{jM} \frac{\sigma_j}{\sigma_M}(R_M - R_f) \tag{18}$$

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[19] The FAQ section of the "Butler Pinkerton Calculator" (#23 and #45) suggests several options to estimate the guideline comparable companies' size premium. http://www.bvmarketdata.com/

[20] Note that Butler & Schurman (2011) do not use the expected value operator in their notation.

By substituting φ from Equation (16) back into Equation (17) and rearranging terms they obtain:

$$R_j = R_f + \sigma_j \lambda \frac{(R_M - R_f)}{\sigma_M}$$

$$R_j = R_f + \lambda \frac{\sigma_j}{\sigma_M}(R_M - R_f) \qquad (19)$$

where $\lambda \frac{\sigma_j}{\sigma_M}$ is the so-called "private-company beta" which becomes Total Beta in case $\lambda = 1$, i.e. when the investor is entirely undiversified as they only hold a single asset – the private firm. Until the point of perfect diversification (where only the market asset is held), λ will be greater than ρ_{jM} of the market beta. The CML-like logic of Equation (17) is used again for the market asset (obviously without λ):

$$R_M = R_f + \sigma_M \varphi \qquad (20)$$

and for their initial two-asset portfolio:

$$R_P = R_f + \sigma_P \varphi \qquad (21)$$

Butler & Schurman (2011) proceed by substituting for R_f and R_M in Equation (15):

$$R_P = w(R_f + \sigma_j \lambda \varphi) + (1 - w)(R_f + \sigma_M \varphi)$$

$$R_P = R_f + w\sigma_j \lambda \varphi + (1 - w)\sigma_M \varphi \qquad (22)$$

Now that R_P is defined by Equations (21) and (22), the authors set these two equations equal. By algebraic reduction and rearrangement of terms they finally arrive at:

$$\lambda = \frac{(\sigma_P - (1 - w)\sigma_M)}{w\sigma_j} \qquad (23)$$

Hence, the "private-company beta" is defined as:

$$\text{Private company beta} = \frac{(\sigma_P - (1 - w)\sigma_M)}{w\sigma_j}\frac{\sigma_j}{\sigma_M} = \frac{(\sigma_P - (1 - w)\sigma_M)}{w\sigma_M} \qquad (24)$$

Butler & Pinkerton (2006; 2008 2009; 2009a), Butler & Schurman (2011), Butler, Schurman & Malec (2011), Butler (2012) and others have repeatedly defended the BPM as the appropriate method for valuing private firms whose investors have limited diversification options (or none at all). According to these authors, the BPM captures

the total risk stemming from such investments rather than only the market risk. However, Butler & Schurman (2011) admit that Total Beta is not applicable for valuation when the firm in question is about to go public, is acquired by a public firm or is invested in by the likes of private equity funds (Butler & Schurman, 2011: 26). In other words, its use is restricted to investors who cannot be presumed to hold well-diversified investments.

Indeed the main argument in favour of Total Beta and the BPM seems to be that of diversification; in the logic of BPM proponents, investors who invest a significant amount of their wealth into a private firm forfeit the possibility of optimal diversification and thus demand a premium in return. According to Butler & Schurman (2011:21-22), this lack of diversification has a direct impact on the pricing of company-specific risk, which necessitates certain adjustments to the CAPM in order to catch total risk (i.e. systematic and firm-specific) as priced in private in public capital markets. This diversification argument is reinforced by Butler (2012:38) who claims that demanding a risk premium merely for making riskier portfolio choices is not an issue;[21] after all, we are talking about *price-setters*, i.e. "[…] the relatively undiversified investor pool consisting of most potential business owners […]". We could summarise the above arguments as follows: the Total Beta approach considers the case of investors with a single asset in their portfolio (i.e. their stake in a private firm). Consequently, the *CSRP* measures the standalone risk of that asset, which in turn enables appropriate valuation with respect to the level of under-diversification of the investor.

3.2.2. Critical evaluation of the approach

Criticism of Total Beta and the BPM begins right at the point where Total Beta is formulated as $\beta_{Total} = \beta_j / \rho_{jM}$. A simple question presents itself: "Why divide by the correlation coefficient of all things?" (Kruschwitz & Löffler, 2014:265). None of the

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[21] In accordance with portfolio theory, Investors are compensated for the risk of their investments rather than for the risk borne as a result of the way these investments are held (Kasper, 2012). Butler (2012) asserts that this applies to price-takers but not to price-setters. However, only buyers are assumed to be price-setters.

proponents of the concept ever deal with this question. Indeed, Damodaran (2002) and earlier Camp & Eubank (1981) simply proposed this measure without really deriving it from anything. After Kasper (2008, 2009, 2010, etc.) repeatedly criticised Total Beta for being inconsistent with established financial theory, Schurman (2010) presented an attempt at "deriving" the BPM. At first glance, removing the correlation coefficient from the beta equation may seem simple and is algebraically correct. However, as von Helfenstein (2011:5) correctly remarks, statistical equations, such as the CAPM beta formula cannot be simply rearranged "[…] without the danger of losing the sense of the relationships they describe."[22] Unfortunately, aside from such incorrect manipulations of analytical expressions, the entire body of work on Total Beta and the BPM exhibits conflicts with financial theory and the arbitrary (and improper) use of portfolio theory concepts. For example, the CML framework is applied in a wrong manner: firstly, BPM proponents use it for single assets (even though it is designed to determine the expected return and risk of efficient portfolios). And secondly, their two-asset portfolio in Equation (15) is apparently supposed to look like the CML portfolio comprised of a riskless and a risky asset (or portfolio). However, that implies that the market portfolio asset in Equation (15) constitutes the zero-variance riskless asset (or portfolio) on the CML. Butler & Malec (2011:10) admit that it is indeed the CML they "used" and even claim to have used it appropriately. They do not seem to understand that their two-asset portfolio – a combination of the market portfolio and another risky investment (the private firm) – could not possibly lie on the CML which is by definition formed by combinations of the market portfolio and the risk-free asset. But we shall not go into greater detail at this point – Kasper (2008; 2009; 2010; 2012; 2013), von Helfenstein (2009; 2011; 2011a), Conn (2011) and others have provided in-depth analyses of the shortcomings of Total Beta.

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[22] While agreeing with the point of von Helfenstein, we would refer to the CAPM as an analytical expression rather than a "statistical equation". Statistical equations are used to characterise a statistical sample and/or to deduce properties of the underlying statistical population, which implies a strictly ex-post view. This is not compatible with the ex-ante perspective of the basic CAPM formulation. In our view, only Jensen's econometric "market model" could be described as a statistical equation.

There is an issue, however, we cannot but mention: Total Beta has never been subject to an *academic* debate. There has been a heated exchange of opinions in practitioner journals (most notably the *Business Valuation Review*), but to the best of our knowledge, not a single one of the renown scholarly journals (e.g. *Journal of Finance*, *Journal of Business*, etc.) has picked up the concept thus far. Confronted with the fact that Total Beta has not appeared in a peer-reviewed journal, Butler and others respond by this answer in the FAQ section (#5) of their website: "The calculation is dependent upon the CAPM, which, obviously, has been subject to significant academic debate". Are the BPM's authors suggesting that their technique is "peer-reviewed by association" here? What strikes us even more are the authors' responses to criticism, where they make claims such as "It is 'CAPM 101!'[sic]" or that there is "[…] overwhelming evidence that Total Beta (practically) and private-company Beta (theoretically) are correct calculations based on MPT […]"(Butler & Malec, 2011: 10). These two claims were made on the same page which makes us wonder whether Total Beta is, in the eyes of its proponents, based the CAPM or rather on MPT, and whether these proponents understand the difference between the two theories. More importantly, the problem with this line of argumentation is that the CAPM does not provide an appropriate rate of return for under-diversified investors bearing unsystematic risk. Accordingly, the market price of risk (which they substituted by φ in Equation (16)) is by default the rate to compensate systematic risk. The BPM proponents have not shown that it could be extended to non-systematic risk (Conn, 2011:11). Instead, they take this measure and see the use of it as evidence for their claim that Total Beta depends upon CAPM theory.

And this is not the only occasion on which Butler, Schurman or Malec simply make claims without providing proof. Aside from the aforementioned assertions about Total Beta being "CAPM 101", they frequently argue by statements like "Any corporate finance textbook will tell you that […]" (Butler & Malec, 2011) or by referring to "modern portfolio theory" without ever citing academic references. They do use numerical examples, but in these, they simply plug in numbers into their equations.

We would have liked to be more balanced in our critical evaluation of Total Beta. Unfortunately, its proponents provide critics with too many targets for criticism without

truly addressing problems in the theory. The fact that the BPM (now Butler Pinkerton Calculator) is proprietary and that its creators are not willing to grant full transparency (cf. Kasper, 2009[23]) does not add to the method's credibility in our view. In fact, this lack of transparency is most likely yet another reason why the BPM has failed to qualify for academic discourse.

3.3. Mayers' CAPM with human capital

In the previous few sections, we discussed a number of valuation techniques. It is clear that in the case of SME, a prediction of the CAPM – perfect diversification of investments – does not apply for the typical owner-manager of a small firm. Hence, the CAPM is not suitable for the valuation of such businesses. The proponents of Total Beta proposed an approach that seemingly ameliorates the under-diversification problem; however, with its grave shortcomings (of which some we discussed above), it does not seem to be a viable alternative after all.

At this point, we highlight a paper by Canefield, Kruschwitz & Löffler (2014)[24] who pointed out a model that surfaced in the 1970s but has not caught a great amount of attention. Mayers (1972) provided an attempt to extend the CAPM to non-marketable assets. His work has been referred to by a number of scholars (c.f. Black, Jensen and Scholes, 1972; Jagannathan and Wang, 1996), however, his model has not seen practical application, presumably due to the difficult operationalisation of the model's human capital component. It has the same aim as the standard CAPM, that is to provide security prices under uncertainty and an individual measure of risk, β_j, for the single asset, j. The only difference is that it calculates an equilibrium which is influenced by the non-marketable assets the investors hold. Mayers' intention was to move a step closer to "complete markets" in CAPM testing, as such tests usually were (and still are)

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[23] In Appendix D (p. 72) of the analysis by Kasper (2009) there is a letter from Pinkerton in which he firmly refuses to share data from his calculations.

[24] Canefield, Kruschwitz & Löffler (2014) showed that Total Beta violates the no-arbitrage principle.

conducted by using only stocks. As Mayers (1972:224) points out, "[...] investors also hold claims on probability distributions of income that are not marketable," and thus it seems reasonable for investors to consider such non-marketable assets for their portfolio choice as well. As we will see in section 4.3 the influence of the investors' non-marketable assets results in individually different weights for the single assets in equilibrium portfolios. Mayers regards human capital as "by far the most important of such claims", with government transfer payments, pensions or trust income being other examples (Mayers, 1972:224). We agree with this view of Mayers, for the ratio of capital-labour income is around 1:2 in industrialized economies (Piketty, 2014). By testing the standard CAPM based on the notion that investors reflect solely upon their capital market investments during their asset allocation decision-making, one simply ignores two thirds of the average income of citizens.

We consider this proposed model quite promising for SME valuation, given that it addresses the aforementioned critical shortcoming of the CAPM, namely that an appropriate rate of return cannot be determined for investments of under-diversified investors. In Mayers' model, investors hold investments in their own human capital which influences their portfolio choice. Compared to the totally under-diversified "Total Beta investor", such investors can be considered more diversified. Furthermore, if investors are rational and resourceful it can be assumed that they have (apart from their SME investment) also capital market investments. Given that they know very well the risks to which their SMEs are exposed, they would, as rational investors, diversify their capital market investment according to the risk of their SMEs. The SME investment can then be seen as an investment among many, with the exception that it is much less liquid, however most likely not illiquid. Investors in SMEs may therefore not be perfectly diversified but they certainly strive to optimally allocate their portfolio consisting of (i) the SME share, (ii) the capital market investment[25] and (iii) their

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[25] Mayers model allows only to integrate one additional asset class (such as human capital). Integrating all three aforementioned portfolio components in the analysis would require an analytical expansion of Mayers' model. Such an expansion is theoretically possible. We will restrict our analysis on the SME

human capital. It is clear that perfect diversification cannot be achieved in this context given that the human capital cannot be adapted to market changes and that the SME investment can be adapted only very slowly. We show in this Thesis that it is practically possible to quantify the human capital investment of SME investors.

Mayers' approach is basically a modification, or more specifically, an expansion of the traditional CAPM. Mayers simply added returns on human capital (and potentially, other non-marketable assets) and the correlation thereof with the market and with the marketable assets. Hence, the beta of this modified model is expanded accordingly. He then solves for the equilibrium price of assets and examines the individual investors' portfolio choice. A detailed derivation is provided in Chapter 4.

Mayers (1972) himself does not test his model empirically, but suggests a method of doing so: the model is transformed into the "market model" of Jensen (1969) which will also be shown in Chapter 4. The first to really test Mayers' modified model were Fama and Schwert (1977). They used a simple logic: since the only difference between the traditional CAPM and Mayers' model lies in the betas, estimating the discrepancy between the two risk measures could reveal the goodness of the modified model. It was found that the discrepancy between the two betas was negligible, which means that testing without including human capital would yield almost identical results. Fama and Schwert (1977) concluded by noting that Mayers' model could potentially be used to explain individual portfolio choices. However, Liberman (1980) who used 'individual' human capital (the previous study employed aggregated data) achieved largely similar results. According to the results, the inclusion of human capital did not matter for individual portfolio choices either, except in the case of self-employed or farm-related occupational classes. Both aggregate and individual human capital once again turned out to be largely uncorrelated with the market, meaning that it does not have a significant impact on the equilibrium relationship between risk and return.

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share and the human capital only, because we first want to examine if integrating human capital changes the beta factor significantly.

3.4. Discussion: Total Beta vs. Mayers' CAPM

We discussed our views on Total Beta above. In this last section of Chapter 3, we point out key differences between Total Beta and Mayers' CAPM. Furthermore, we explain why we, despite the empirical evidence against the latter, believe that the modified model of Mayers may be suitable for valuation purposes after all.

Several authors have shown that the Total Beta approach (and the BPM which is based on it) exhibits critical conflicts with financial theory (c.f. section 3.2.2). In practical valuation, using a method that does not have a basis in financial theory is perfectly acceptable (e.g. valuation multiples); however, if the BPM's proponents claim that their method is based on the CAPM or other concepts of economic theory, then their approach must be compliant with extant theories (e.g. the aforementioned CML framework). As discussed above, this is not the case. Worse yet, Total Beta has never been subject of an academic debate. Given that the sheer logic behind Total Beta appears to be flawed (Kruschwitz & Löffler, 2014) and that it seems to violate the no-arbitrage principle (Canefield, Kruschwitz & Löffler, 2014), it really is not an alternative we would consider.

At first glance, one may arrive at a similar conclusion for Mayers' modified model. Although it is, without a doubt, truly based on the CAPM and thus not in conflict with financial theory or logic, it has admittedly been an empirical failure. That said, these results of Fama & Schwert (1977) or Liberman (1980) were achieved for various classes of securities, but not for unlisted small enterprises. Additionally, and more importantly, the returns on human capital were represented simply by per-capita labour income. The authors of both studies pointed out that such a definition of human capital returns may be problematic. This does not mean that the results of Fama & Schwert and Liberman are not correct; in fact, they make perfect sense. In larger firms, where ownership and management are separated to a great extent, the human capital of individual investors probably does not have a significant impact on the overall success of the firm. However, it must also be pointed out that the data of these studies comes from a time before the

neoliberal revolution when wages in the USA were much more tied by collective agreements (Piketty, 2014). Using current German data, we obtain the following:
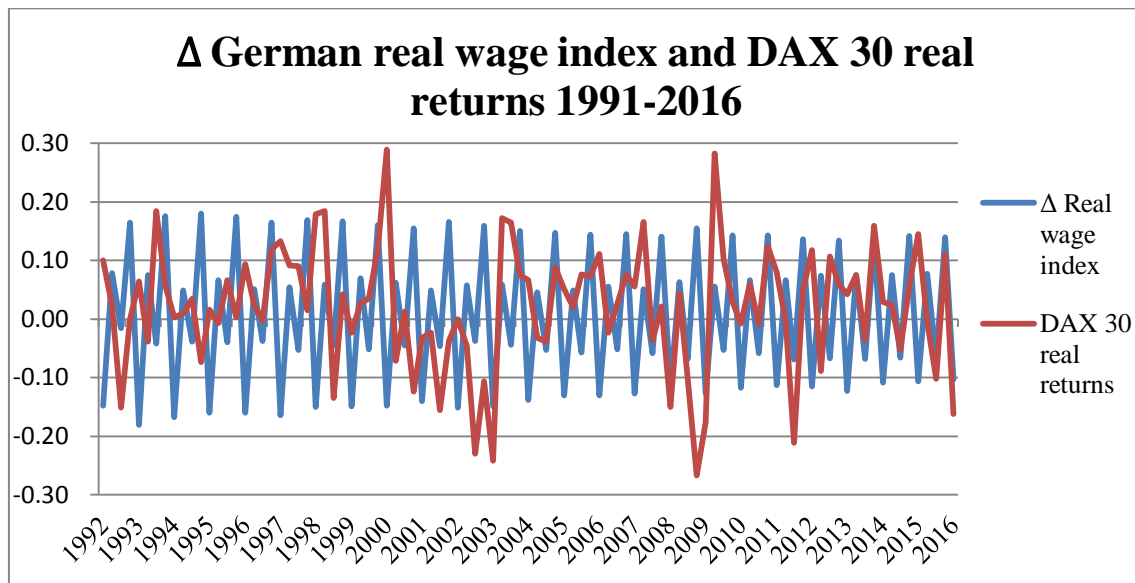
Figure 1: Δ German real wage index and DAX 30 real returns 1991-2016 (Source: Destatis, Yahoo! Finance)

Not surprisingly, Figure 1 does not reveal a strong correlation between the two time series with a correlation coefficient of a mere 0.028200155. However, it is not at all negligible; the findings of Weil (1994) indicate that a correlation of as little as 3 per cent suffices to produce meaningful results[26]. We observed nearly 3% even with unadjusted general labour income data. This leads us to two conclusions: firstly, if the methodologies of Fama & Schwert (1977) and Liberman (1980) were used with current data, results may differ greatly. Secondly, if labour income is adjusted to account for the reality of SMEs (see discussion below), the 3% mark could be significantly surpassed.

In our study, we use a different method of calculating returns on human capital as will be explained in detail in Chapters 4 and 5. Consequently, the correlation between human capital and the market may be larger, yielding a more significant beta of the modified model. In addition, we only consider small enterprises whose success is dependent on the human capital of the owner-manager to a much larger extent than in

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[26] For a detailed discussion of Weil's findings please refer to section 4.4.

larger firms. Given that our analysis revolves around such owner-managers in their own small firms, it can be argued that their human capital will be the driver of the firm's success. The objective of the next two chapters will thus be twofold: firstly, to show the derivation of Mayers' model and transform it into a testable version. And secondly, to employ a method of calculating the returns on human capital for testing the model on our sample of small firms in Germany. The ultimate goal is to show that Mayers' modified model could be suitable for valuing small enterprises with under-diversified investors and that human capital should be considered in valuation.

4. The proposed analytical model

4.1. The standard CAPM

As mentioned before, the proposed model in this Thesis is based on the work of Mayers (1972). In order to provide a better understanding of his modified model, we begin this chapter with an in-depth analysis of the standard CAPM which Mayers' model is based on.

4.1.1. Problems of the standard CAPM

Research suggests that CAPM has a number of empirical shortcomings (c.f. Basu, 1977; Banz, 1981; Bhandari, 1988; Rosenberg, Reid and Lanstein, 1985). For instance, Basu (1977) found that the earnings-price ratio has an impact on stock returns which the standard CAPM neglects. According to Bhandari (1988), firms with a high debt-equity ratio have returns which exceed the returns predicted by their market betas. Generally speaking, the researchers listed above all found empirical problems with the standard CAPM which are due to the fact that the model only reflects risk as a consequence of inter-firm dependencies while it neglects risk arising from differences in intra-firm accounting ratios . Fama and French (2004) concluded that the empirical implications of the standard CAPM could not be confirmed through empirical testing.

Another reason for the lack of empirical validity of the standard CAPM is the difficulty in finding a proxy for the market portfolio. The theoretical market portfolio includes all assets (i.e. all tradable claims on probability distributions of income), and according to Roll (1977), this makes it impossible to test the CAPM, since no proxy could represent all individual assets in the market. Three empirical contradictions to CAPM theory underline the notion that no adequate proxy has been found so far. Firstly, the intercept found in regression models often exceeds the risk-free rate of return in the respective market, and the average weighted beta coefficient of the single assets is smaller than the excess return on the market (c.f. Douglas, 1968; Blume and Friend, 1973; Fama and French, 1992). Secondly, the ratio between the beta factor and the excess return on the market is too 'flat' i.e. the predicted value of the slope of the

security market line was too low for all post-war periods tested by Black, Jensen and Scholes (1972; c.f. Stambaugh, 1982). Furthermore, empirical studies found that the beta factor does not comprise the total fluctuation of the returns and that the effects of the price-to-book ratio, company size or momentum (among others) cannot be explained by the standard CAPM (c.f. Fama and French 1992; 1996; Carhart, 1997). Moreover, the standard CAPM predicts identical risky portfolio weights for all investors[27]. We will see in Section 4.3 that the analytical framework of Mayers (1972) can circumvent this narrowing outcome of the CAPM.

Another reason for the poor empirical confirmation of the CAPM lies in the simple yet strict set of assumptions upon which it is based. One of these assumptions is the permanent marketability of all assets. Mayers (1972) points out that this is unrealistic and one should strive to analytically expand the formulation of the CAPM in order to let it account for assets that are not permanently or completely marketable.

We have seen that there are in principle two angles by which one can seek to improve the standard CAPM's empirical validity. Firstly, one can try to improve the proxy for the market portfolio by including more assets and additional types of assets in order to make the market portfolio more resembling of its theoretical foundation. Secondly, one can focus on the investor looking for factors that might influence their asset allocation in equilibrium without leaving the realms of the individual mean-variance trade-off that fuels the genesis of the capital market equilibrium. In section 4.1.5, we will briefly mention some extant approaches that aim to boost the standard CAPM's empirical validity. More importantly, in line with the research goal of the present Thesis, we will discuss Mayers' proposed extension in the context of "healing" the standard CAPM.

In this Thesis, we will test the model for SMEs in Germany with the investors' human capital as the non-marketable asset. Since we base our empirical analysis on Mayers' model, we will thoroughly show how it is derived, how it is transformed into

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[27] The risky part of any investor's portfolio is identical for all investors, since they all hold the market portfolio, i.e. Tobin's tangency portfolio on Markowitz' efficiency frontier (Markowitz, 1952a, Tobin, 1958).

an ex-post, empirically testable model, how it is different from the standard CAPM and finally, which conclusions can be drawn from it.

4.1.2. Jensen's derivation of the market model

A crucial step in the empirical verification of the modified model is transforming it into a testable version by applying the methodology of Jensen (1969). We will use Jensen's market model[28] for the testing and consider it fruitful to critically deliberate the empirical findings upon which it is based. Jensen devised this methodology reacting to Fama (1968) who had uncovered a series of flaws in the methodology used for testing the CAPM until then. We will subsequently show the problems discovered by Fama and the solution proposed by Jensen. The reason for this long-winded approach is that - for a better understanding of our own testing results - we feel it is necessary to address a critical assumption made by Jensen which is based on empirical findings by King (1966) and Blume (1968). This assumption is essential to Jensen's methodology and the empirical findings on which it is based seem poor to us. We will discuss the problems concerning the empirical validity of this critical assumption at the end of this section because it is complex and a proper discussion demands that it is first shown how it was used in the derivation of the improved market model.

The original version of the market model provided by Sharpe (1964) is based on a linear relationship between the one-period asset return and the market return.

$$R_i = \alpha_i + \beta_i R_M + \varepsilon_i; \quad i = 1, 2, \dots, N \tag{25}$$

The disturbance terms are assumed to have the following properties

$$E(\varepsilon_i) = 0; \qquad i = 1, 2, \dots, N \tag{26}$$

$$cov(\varepsilon_i, \varepsilon_j) = 0; \qquad i = 1, 2, \dots, N \quad i \neq j \tag{27}$$

$$cov(\varepsilon_i, R_M) = 0; \qquad i = 1, 2, \dots, N \tag{28}$$

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[28] To be precise, we will use an adapted version of Jensen's improved market model that accounts for non-marketable assets.

In these assumptions, Fama (1968) identifies two problems. Firstly, it is clear that in the context of the market model

$$R_M = \sum_{j=1}^{N} X_j \left[\alpha_j + \beta_j R_M + \varepsilon_j \right] {}^{[29]} \tag{29}$$

Since R_M contains error terms, (28) cannot hold. Also, (28) was used to derive the excess return of assets[30]

$$E(R_i) - R_F = [E(R_M) - R_F] \beta_i; \; i = 1,2,\dots,N \tag{30}$$

This expression is therefore incorrect in the context of the market model. Secondly, we can infer from (29) that

$$\sum_{j=1}^{N} X_j \alpha_j = 0$$

$$\sum_{j=1}^{N} X_j \beta_j = 1 \tag{31}$$

$$\sum_{j=1}^{N} X_j \varepsilon_j = 0.$$

The first two relations in (31) do not contradict the assumptions concerning the error terms (26) and (27). The error terms however, are assumed to be independent, which makes it impossible to constrain their weighted sum to zero (Fama, 1968).

Fama (1968a) pointed out that the influence of the aforementioned inadequacies of the market model had only a minor influence on the results of the market model. He shows that (30) is not unique. (30) was obtained by applying the market model to the general CAPM equation. Therefore one should be able to obtain (30) also by applying the market model to an equivalent expression of the general CAPM as used in obtaining

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[29] $R_M = \sum_{j=1}^{N} X_j R_j \,; \; R_j = \alpha_j + \beta_j R_M + \varepsilon_j \qquad i = 1,2,\dots N$

[30] $cov(R_i, R_M) = E\{(\beta_i[R_M - E(R_M)] + \varepsilon_i)(R_M - E(R_M))\} = \beta_i \sigma^2(R_M) + cov(\varepsilon_i, R_M) = \beta_i \sigma^2(R_M)$

Substituting $\beta_i \sigma^2(R_M)$ into the general CAPM relation $E(R_i) - R_F = \left[\frac{E(R_M) - R_F}{\sigma^2(R_M)} \right] cov(R_i, R_M)$

yields the excess returns of assets.

(30). In doing so, Fama (1968) obtains a different result [31] stemming from the contradictory assumptions:

$$E(R_i) - R_F = [E(R_M) - R_F] \left[\beta_i + \frac{X_i \sigma^2 \varepsilon_i}{\sigma^2 R_M} \right] \tag{32}$$

Citing King (1966) and Blume (1968) who found empirically that $\sigma^2(\varepsilon_i)$ and $\sigma^2(R_M)$ were of roughly the same size, Fama (1968) concludes that (32) is approximately equivalent to (30) since the weight of each company in the market portfolio is very small (assuming that a sufficient number of firms is included in the sample).

Because of the structural flaws of the CAPM testing model, Fama proposes an alternative market model which does not suffer from the problems shown above. The improved market model (which is also used by Mayers, 1972) is specified as follows

$$R_i = \alpha_i + \beta_i r_M + \varepsilon_i; \qquad i = 1, 2, \dots, N. \tag{33}$$

In this model, the return on the market portfolio (R_M) is substituted by (r_M), a market factor influencing all assets with the following assumptions[32]

$$E(\varepsilon_i) = 0; \qquad\qquad i = 1, 2, \dots, N \tag{34}$$

$$cov(\varepsilon_i, \varepsilon_j) = 0; \qquad\qquad i = 1, 2, \dots, N \quad i \neq j \tag{35}$$

$$cov(\varepsilon_i, r_M) = 0; \qquad\qquad i = 1, 2, \dots, N. \tag{36}$$

The return on the market portfolio is now

$$R_M = \sum_{j=1}^{N} X_j R_j = \sum_{j=1}^{N} X_j [\alpha_i + \beta_i r_M + \varepsilon_i]. \tag{37}$$

The risk premium on asset i is

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[31] $E(R_i) - R_F = \left[\frac{E(R_M) - R_F}{\sigma^2(R_M)} \right] cov(R_i, R_M) \equiv E(R_i) - R_F = \left[\frac{E(R_M) - R_F}{\sigma^2(R_M)} \right] \sum_{j=1}^{N} X_j \, cov(R_j, R_i) \equiv$
$\left[\frac{E(R_M) - R_F}{\sigma^2(R_M)} \right] \left\{ \beta_i \sum_{j=1}^{N} X_j \, \beta_j \sigma^2(R_M) + X_i \sigma^2(\varepsilon_i) \right\}$ since from $E(\varepsilon_i) = 0$ it follows that $E(\beta_j R_M) \equiv R_M$ and $E(R_i) \equiv \beta_i R_M$ for further details concerning the derivation procedure see Appendix A. Given that $\sum_{j=1}^{N} X_j \, \beta_j = 1$ we obtain $(R_i) - R_F = \left[\frac{E(R_M) - R_F}{\sigma^2(R_M)} \right] [\beta_i \sigma^2(R_M) + X_i \sigma^2(\varepsilon_i)]$ (32).

[32] The parameters have the following properties: $E(r_M) = 0$, β_i is a constant and r_M and ε_i are normally distributed random variables (c.f. Jensen 1969, Fama 1968).

$$E(R_i) - R_F = \left[\frac{E(R_M)-R_F}{\sigma^2(R_M)}\right] X_j cov(R_j, R_i) =$$

$$\left[\frac{E(R_M)-R_F}{\sigma^2(R_M)}\right] \sum_{j=1}^{N} X_j E\{(\beta_j[r_M - E(r_M)] + \varepsilon_j)(\beta_i[r_M - E(r_M)] + \varepsilon_i\} = \quad (38)$$

$$\left[\frac{E(R_M)-R_F}{\sigma^2(R_M)}\right] \left\{\beta_i \sum_{j=1}^{N} X_j \beta_j \sigma^2(r_M) + X_i \sigma^2(\varepsilon_i)\right\}.$$

Again, we see that that $\sigma^2(\varepsilon_i)$ and $\sigma^2(R_M)$ become negligibly small due to the smallness of the single firm's share in the market portfolio, which makes the difference between the improved market model and the flawed version discussed earlier very small – at least with a sufficiently high number of firms included in the market portfolio. Applying the empirical evidence of King (1966) and Blume (1968) and scaling r_M in (37) in order to have $\sum_{j=1}^{N} X_j \alpha_j = 0$ and $\sum_{j=1}^{N} X_j \beta_j = 1$, we obtain the following relationship for the variance of the market return in the improved market model[33]

$$\sigma^2(R_M) = \sigma^2(r_M) + \sum_{j=1}^{N} X_j^2 \sigma^2(\varepsilon_i) \quad (39)$$

$X_j^2 \sigma^2(\varepsilon_i)$ is neglibly small relative to the variance of the market factor resulting in an equivalence of the variance of the market returns in the market model and in the standard CAPM. However, Fama (1968) did not explicitly show the equivalence of the β_j in the standard CAPM and the b_j (notation of Jensen), respectively the β_i (notation of Fama). The procedure to obtain this result will subsequently be shown in more detail, for it paves the way to understanding the logic on which the CAPM has been tested since Fama and Jensen devised this approach in the 70s. This equivalence is the cornerstone of Mayers' (1972) proposed testing procedure. It is important to have acceptable certainty about this point. The procedure of Jensen (1969) who analytically showed this equivalence will be shown below.

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[33] $R_M = \sum_{j=1}^{N} X_j[\alpha_i + \beta_i r_M + \varepsilon_i]$ Fama (1968) is equivalent to $R_M = \sum_{j=1}^{N} X_j E(R_j) + \sum_{j=1}^{N} X_j \beta_j r_M + \sum_{j=1}^{N} X_j(\varepsilon_j)$ Jensen (1969). Since normality of returns is assumed $E(R_j)$ and α_j have the same properties, i.e. they are unique for each security and their sum is zero.

In the first part of his market model derivation procedure, Jensen (1969) duplicates Fama's (1968) result of the variance equivalence shown above. However, he stated the expression for the return of the single asset, and consequently also of the market return somewhat differently than Fama (1968) in (37)

$$R_j = E(R_j) + b_j \pi + \varepsilon_j; \qquad\qquad j = 1,2,\dots,N \qquad (40)$$

$$R_M = \sum_j x_j E(R_j) + \sum_j x_j b_j \pi + \sum_j x_j \varepsilon_j; \qquad j = 1,2,\dots,N \qquad (41)$$

This has no impact on the comparability of the two approaches since the changed notations still represent constants.

Jensen (1969) then continues the work of Fama (1968) by stating the standard CAPM in terms of the market model in a form taken over from Lintner (1965, Equation [37]).

$$E\left[R_j \middle| E(R_M), \left[\frac{b_j \sigma^2(\pi) + x_j \sigma^2(\varepsilon_j)}{\sigma^2(R_M)}\right]\right] = R_F + [E(R_M) - R_F]\left[\frac{b_j \sigma^2(\pi) + x_j \sigma^2(\varepsilon_j)}{\sigma^2(R_M)}\right] \qquad (42)$$

This term is also equivalent to our equation (38) provided by Fama (1968) in the process of showing the contradictions in the flawed testable version of the CAPM. The only difference is that here, the standard CAPM has been combined with the improved market model where π stands for the market factor. This market factor is unobservable and therefore problematic. The aim must be to eliminate this market factor from (42). Again, empirical findings of King (1966) and Blume (1968) are used to achieve this elimination. They found that the market factor can explain roughly 50 per cent of the variation in individual security returns. The variance of the individual security can be shown to be $\sigma^2(R_j) = b_j^2 \sigma^2(\pi) + \sigma^2(\varepsilon_i)$[34].

Blume (1968) remarked that this ratio might be declining for the more recent part of his sample which spanned data of only 63 US-securities from 1927 to 1960. As far as we know, no further testing on this ratio and possible trends has been conducted in order to see if there indeed is a trend or if the ratio between $\sigma^2(\pi)$ and $\sigma^2(\varepsilon_i)$ remained at 50 per cent.

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[34] $cov(R_j, R_j) = cov\{[E(R_j) + b_i \pi + \varepsilon_i][E(R_j) + b_i \pi + \varepsilon_i]\} = b_j^2 \sigma^2(\pi) + \sigma^2(\varepsilon_i) = \sigma^2(R_j)$

The term $\frac{b_j\sigma^2(\pi)+x_j\sigma^2(\varepsilon_j)}{\sigma^2(R_M)}$ from Equation (42) measures the systematic risk in the market model. It can be simplified by substituting $\sigma^2(\pi)$ with $\sigma^2(R_M)$[35].

$$\beta_{2j} \cong \left[\frac{b_j\sigma^2(R_M)+x_j\sigma^2(\varepsilon_j)}{\sigma^2(R_M)}\right] = b_j + \frac{x_j\sigma^2(\varepsilon_j)}{\sigma^2(R_M)} \; ; \; z_j = \frac{x_j\sigma^2(\varepsilon_j)}{\sigma^2(R_M)} \tag{43}$$

The aim is now to depict (40) as an ex-post relationship in which all variables are measurable (Jensen, 1969). Equation (42) is substituted for $E(R_j)$ in (40).

$$R_j = R_F + [E(R_M) - R_F]\beta_{2j} + b_j\pi + \varepsilon_j \tag{44}$$

$$R_j = R_F + E(R_M)\beta_{2j} - R_F\beta_{2j} + b_j\pi + \varepsilon_j \tag{45}$$

$$R_j = R_F(1 - \beta_{2j}) + \beta_{2j}E(R_M) + b_j\pi + \varepsilon_j \tag{46}$$

In order to eliminate the unobservable market factor, the term $z_j\pi + \beta_{2j}\sum_i x_i\varepsilon_i$ is added and subtracted on the RHS of (46). Since β_{2j} is equivalent to $b_j + z_j$, (46) can be reduced to

$$R_j = R_F(1 - \beta_{2j}) + R_M\beta_{2j} - z_j\pi - \beta_{2j}\sum_i x_i\varepsilon_i + \varepsilon_j \tag{47}$$

As it is assumed that $E(\varepsilon_j) = 0$, $\beta_{2j}\sum_i x_i\varepsilon_i$ is zero and $z_j\pi$ is negligible, for x_j will be decreasing with increasing sample size, while $\sigma^2(\varepsilon_j)$ is on average assumed to be equivalent to the variance of the market factor and therefore stable, (47) is reduced to

$$R_j = R_F(1 - \beta_{2j}) + R_M\beta_{2j} + \varepsilon_j. \tag{48}$$

At this point, it is necessary to go back one step. The assumption that the term $x_i\varepsilon_i$ is zero implies that with a market factor of zero, Equation 49 would give a market return that is essentially equal to the expected market return, which is unlikely to be observed in reality. To prevent this, the expected return on the single asset needs to be conditioned on R_M and β_{2j}:.

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[35] C.f. the final result of Fama in (39) where r_m stands for the market factor π in the notation of Jensen (1969).

$$E(R_j|R_M,\beta_{2j}) \cong R_F(1-\beta_{2j}) + R_M\beta_{2j} \tag{50}$$

The equation above is crucial for empirical testing of any hypothesis involving the CAPM. The CAPM itself states the expected return on the single asset conditional on the expected ex-ante return on the market portfolio. In order to be able to test the CAPM, we need the expected return on the single asset conditional on ex-post realisations of the market portfolio.

Before finally accepting this result, we need to examine the accuracy of (50).

$$\delta_{2j} = R_j - \left[R_F(1-\beta_{2j}) + R_M\beta_{2j}\right] \tag{51}$$

From (47) we infer that

$$\delta_{2j} = z_j\pi - \beta_{2j}\sum_i x_i\varepsilon_i + \varepsilon_j. \tag{52}$$

The first two terms of (52) were shown to be negligible before, which confirms the accuracy of (50). Based on (52), the equivalence of β_j in the standard CAPM and b_j becomes clear. If the difference between the expected return on the single asset and the actual return on this asset is ε_j, then the β_{2j} in the market model must be equal to the beta factor in the standard CAPM.

$$\beta_{2j} = \frac{b_j\sigma^2(R_M) + x_j\sigma^2(\varepsilon_j)}{\sigma^2(R_M)} = \frac{cov(R_j,R_M)}{\sigma^2(R_M)} \cong b_j + \frac{x_j\sigma^2(\varepsilon_j)}{\sigma^2(R_M)} \tag{53}$$

With $x_j\sigma^2(\varepsilon_j)$ being minor we get $\frac{cov(R_j,R_M)}{\sigma^2(R_M)} \cong b_j$.

As we saw in this section, the findings of King (1966) and Blume (1968) concerning the market factor are essential for the transformation of the standard ex-ante CAPM in an ex-post market model. We mentioned at the beginning of this paragraph that we see problems with these findings. Blume's work is a dissertation that has never been published. Although we trust that it has been cited correctly we see two major problems concerning the applicability of Blume's findings today. Firstly, the size of his sample is small, spanning 251 securities. Secondly, the time horizon from 1927 to 1960 reflects market conditions which are fundamentally different from today's conditions. Also, we hold that is not ideal to compare post- and pre-war data. The role of the state and the

structure of the economy in the US have undergone very severe metamorphoses in this period (Heikkinen & Kanto, 2000).

Why this is problematic becomes clearer when we look at what the market factor actually is. It is seen as an unobservable and unknown factor which influences all assets in the market. This is very interesting in the context of the CAPM since it is based on covariations between asset returns. A certain amount of the correlations deduced from these covariations may then be spurious reflecting only the joint dependency of assets on the market factor. It is likely that the aforementioned changes of state and economy are also reflected in the market factor. The influence of the market factor on the variability of stock returns should therefore only be determined empirically within periods of somewhat comparable market conditions. This view is supported by the finding of Knif (1989). He showed that β can change over time and therefore should be re-determined after major market changes. When β changes, it stands to reason that also the market factor changes. On the one hand, β is influenced by the variability of stock returns, and on the other hand, the market factor is assumed to explain a significant share of said variability of stock returns. Therefore, it is reasonable to assume that with changing β also the market factor might have changed.

While Blume (1968) used regression analysis, King (1966) used factor analysis. We see factor analysis as a more suited technique with regard to estimating the impact of the market factor on stock returns. It is designed to determine the underlying influence of unobservable variables on observed correlated variables. Unfortunately, King (1966) used a sample of only 63 securities, spanning the same time horizon (1927 to 1960) as did the sample of Blume (1966). King found that 44% to 53% of the variability of stock returns can be explained by the market factor. However, he also found that around 10% of this variability is explained by industry affiliation. Jensen (1968) and Fama (1968) have neglected this as minor but we are not that sure. The effects of mixed industry affiliations of firms in the sample could have a distorting impact on the testing results. In order to determine this, one would have to examine each industry separately (it is for this reason that we only consider firms of a single industry in Chapter 5). Damodaran (2011) agrees with this view and argues in favour of sector betas. Regardless of the

strength of the mixed industry effect, we find it surprising how easily Jensen (1968) and Fama (1968) applied empirical findings of feeble generalizability without pointing out that further empirical work needed to be done in order to have certainty about the validity of the market model. As far as we know, the equality of the beta in the market model and the beta in the standard CAPM has never been questioned. With regard to the assumptions in the derivation of said equality, we would consider further empirical work on these assumptions useful. Although we use the market model of Jensen (1969) (since there is no existing alternative), considering its shortcomings, we would want to clarify the following questions: (i) how strong is the influence of industry affiliation for different industries, (ii) what percentage of the variability of stock returns is explained by the market factor. If the influence of the market factor were significantly different, the market model would have to be adapted analytically. Examining these points could be the objective of further research.

4.1.3. The Gaussian market model

Mayers (1972) in the tradition of Jensen refers to his market model as the "Gaussian Market Model". "Gaussian" refers to the findings of Mandelbrot (1963) and Fama (1965) that the distribution of stock returns seem to be stable, having a finite mean but infinite variance. The normal or Gaussian distribution is a special case of the stable class of distributions and it is the only one in the class with a finite variance. It is imperative to assume that stock returns actually do follow a distribution with finite variance as long as the standard CAPM and consequently also the market model derived from it is not extended to account also for non-finite variances of stock distributions. The more widespread approach until today is not extending the market model to non-finite variances analytically, although Jensen (1969) has provided a framework to do so. Mayers (1972), too, contented in the notion of Fama (1965, 1968) that the results obtained for finite variances can be generalised to non-finite variances with only a minor loss of validity. Expressed in non-mathematical terms, the reason for the assumed infinite variance of the distribution of stock returns is that with increasing distance from

the mean, the probability for arbitrarily high values in the tails of the distribution of stock returns decreases slower than the squared distance of these values from the mean.

This becomes clearer when one looks at the definition of variance for a continuous distribution where the random variable x has the density function $f(x)$

$$Var\ (x) = \int_{-\infty}^{\infty} (x - \mu)^2\ f(x)\ dx \qquad (54)$$

If the function value $f(x)$ decreases slower than the squared distance of x from the mean $(x - \mu)^2$ increases, the variance will become infinite. When Mandelbrot (1963) became aware of this property of the distributions of stock returns, arbitrarily high values in the tails were already a problem in his data. We hold that the erratic nature of stock returns with eventually arbitrarily large values in the tails of the distribution is likely to have become more pronounced over time due to the increased use of derivatives[36]. In this context it would be interesting to compare results generated by an approach assuming a normal distribution of stock returns and the adapted framework offered by Jensen which accounts for infinite variance. In fact, it may be added here that the dependency of the mean-variance approach on the normal distribution of stock returns is even more significant. Tobin (1958, 1965) showed that the distributions of the single asset returns and of any portfolio constructed from these assets need to be of the same form when applying the standard CAPM. Only stable distributions fulfil this condition. They are stable under addition or self-decomposable, meaning that a weighted sum of random variables will have the same distribution as these variables themselves. After all, it seems that only the normal distribution as a special case of the family of stable distributions is suited for the mean-variance approach. As mentioned above, Jensen (1969) based on Fama (1968) showed analytically how to extend the framework at least to other stable distributions of stock returns with infinite variance.

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[36] Derivatives allow to incur risk over-proportionate to the capital invested, which means that gains or losses are theoretically unlimited and that the realization of stock returns in the tails become more extreme.

The discussion of this demonstration would go beyond the scope of this Thesis, however, we must point out that the demonstration heavily relies on the empirical findings of King (1966) and Blume (1968) on which we reflected critically before.

4.1.4. Problems with expected utility theory (EUT)

In the previous sections, we have discussed problems concerning the empirical validity of the CAPM. Due to the empirical aim of the present Thesis, we consider it most relevant to concentrate on problems, which relate to the ex-post validity of CAPM testing results. However, there are two general problems (one being the empirical validity) of the CAPM, and since we use the CAPM, we find it mandatory to at least briefly address the other set of problems which have been attested to the CAPM (c.f. Allais, 1953; Markowitz 1952; Tversky & Kahneman, 1992). Interestingly, the empirical problems are considered to be the less severe of the two (Levy, 2010). Empirical problems, no matter how severe they may be, cannot *per se* cause rejection of a theoretically correctly derived model. The aforementioned authors on the other hand all pointed out to contradictions in the theoretical foundation of the CAPM. Allais (1953) found that there are decision situations in which investor preferences, when applying expected utility theory (EUT), are not stable (Allais Paradox). Markowitz (1952) criticised the assumption of risk aversion, claiming that investor preferences (i.e. utility functions) must contain risk-averse as well as risk-seeking segments[37]. Baumol (1963) and Leshno and Levy (2002) showed that there are pairs of outcomes for which the mean-variance decision rule delivers ambiguous preferences. The consequence of this is that the mean-variance decision rule on which the CAPM is based is not a necessary but only a sufficient investment decision criterion. In terms of CAPM theory this means that segments of the efficient frontier are not an efficient investment for investors. This effect is challenging for the CAPM especially in the case where the market portfolio is located on such a segment of the efficient frontier.

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[37] This finding challenges the mean-variance decision rule of investors.

Tversky & Kahneman (1992) showed numerous examples of human decision-makers behaving in accordance with several situation-dependent heuristics and biases and thereby in contradiction to EUT[38]. The findings of Kahneman and Tversky (1979) and Tversky and Kahneman (1992) suggest that the CAPM may be invalid because the CAPM only holds if EUT holds, given that the CAPM is based on the mean-variance decision rule which is derived within EUT. To sum up, they applied behavioural psychology to EUT and the result was that EUT does not conform to the insights into human behaviour from behavioural psychology. There are two ways to respond to the aforementioned findings, either one can try to prove that the mean-variance decision rule and EUT actually do hold in a CAPM setting, or one accepts this criticism (c.f. Levy, 2010). The first approach seems futile to us since the critique concerning the mean-variance decision rule and EUT is convincing and generally accepted. In this context it should suffice to mention that Kahneman won the Nobel Memorial Prize in Economic Sciences in 2002 for Cumulative Prospect Theory (CPT), which is his alternative to EUT. The latter approach by Levy (2010) accepts and addresses the critique on the theoretical validity of the CAPM. Levy (2010) shows that the CAPM can be modified in a way that preserves its' theoretical validity when EUT is abandoned in favour of CPT. Levy (2010) also shows that the problem addressed by Allais (1953), Baumol (1963), Leshno and Levy (2002) can be overcome theoretically (i.e. by modifying the CAPM). The procedure of Levy (2010) replaces the risk-aversion paradigm with first order stochastic dominance. We cannot discuss Levy's procedure in its' entirety, the point we want to make here is that the CAPM can be theoretically modified in order to account for its theoretical shortcomings. Levy's results indicate that these modifications are dependent on the context in which the CAPM is used. For instance, the modifications in a setting where the investors are primarily engaged in their own family businesses might be different from the modifications necessary in a context of mainly institutional investors.

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[38] Examples are the representativeness and the availability heuristic, the anchoring effect and biases such as overconfidence or illusory correlation.

Our final remark on the theoretical criticism of the CAPM is that we see no alternative to the CAPM when it comes to modelling the investment decision process of investors. All other approaches for the determination of a company's risk are simply statistical ex-post descriptions for company specific risk. An example for this is the value-at-risk technique. If one strives to capture the causal factors of company specific risk, one has to use a model that tries to simulate how investors assess this risk. In fact, on capital markets risk can ultimately be regarded as a consequence of investor decision-making. Therefore, we find it more correct to use a model that simulates this process rather than a technique that assumes that the risk of the past is the key to predict future risk.

In this Thesis, the different critiques and the context dependent solutions to these critiques as proposed by Levy (2010) will not be addressed with regard to Mayers' CAPM. We hold that before doing so it is necessary to first show that Mayers' CAPM delivers results different from those of the standard CAPM. This point will be empirically examined in Chapter 5.

4.1.5. Attempts to ameliorate the flaws of the standard CAPM

Many attempts have been made to heal the standard CAPM from its empirical shortcomings. Early attempts to improve the empirical validity of the model were undertaken by Merton (1973) with the inter-temporal CAPM and by Black (1972) with the zero-beta CAPM. In the inter-temporal CAPM investors optimise consumption over a period of time while consumption is assumed to stem from security returns exclusively[39]. Empirical findings suggest that the zero-beta CAPM, which does not assume the existence of a risk-free asset, is empirically more robust than the standard CAPM (c.f. Fama and Mac Beth, 1974; Köseoğlu and Mercangöz, 2013). We will address the implication of the zero-beta model for our empirical work in Section 4.4.

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[39] The assumption that consumption is funded by security returns exclusively renders this model unsuited for the context of SMEs where it is reasonable to assume that investors often work in the companies in which they are invested.

Mayers' approach is fundamentally different from the aforementioned ones. It is an analytical framework which is intended to enable other researchers to integrate any type of non-marketable asset an investor may hold in the capital market equilibrium calculation. It is important to stress here that the market portfolios calculated by Mayers model do not include non-marketable assets. The non-marketable assets only influence the individual "market portfolios". Mayers himself did not explicate how to quantify these non-marketable assets, and probably as a consequence of that, his model was seldom tested [40]. In general, there are many reliable empirical findings on the distribution and quantity of non-marketable assets on the aggregate level of countries (c.f. Piketty, 2014). On the individual level, there are only very few. It is difficult to financially quantify or merely assess non-marketable assets on an individual level given their private nature. We presume that simply nobody wanted to test with data that is difficult or impossible to acquire, especially in the 1970s. Judging from the extant literature, this attitude has not changed. We try to fill this gap in the literature by testing Mayers' modified model with individualised human capital income.

4.2. Extending the standard CAPM by human capital

Following Mayers (1972), we consider it helpful to give a brief overview on the formulation of the traditional CAPM. The classic SLM model, which is widely referred to as the CAPM[41], states that the expected return of a risky asset is composed of a risk-free return and a risk premium. The magnitude of the risk premium for the single asset is determined by the covariance of the single asset's return with the returns on all other assets.

$$E(\rho_j) = r + \left[\frac{E\rho_M - r}{\sigma^2(\rho_M)}\right] cov(\rho_j, \rho_M); \quad i = 1, 2, \dots, N \qquad (55)$$

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[40] To the best of our knowledge the only one who have tested Mayers' model were Fama & Schwert (1977) and Liberman (1980).

[41] We refer to it as the standard CAPM in order to distinguish it from Mayers' model, to which we will also refer to as the modified model.

E is the expectation operator, ρ_j is the random return on asset j at the end of the period, ρ_M is the random return on the market portfolio at the end of the period, r is the risk free rate of return which Mayers incorrectly defined as one plus the risk-free rate of return. The term $\left[\frac{E\rho_M - r}{\sigma^2(\rho_M)}\right]$ can be interpreted as the market price for one unit of risk. The term $cov(\rho_j, \rho_M)$ is the systematic risk of asset j, which cannot be eliminated through diversification.

An important assumption of the standard CAPM is that the market consists solely of totally liquid, i.e. marketable assets. If that is the case, all investors have the same investment opportunities. The risky share of the investors' portfolio is assumed to be identical for all investors. They hold the market portfolio which is composed of all assets where the individual weight of the asset is the result of an optimisation of the mean-variance ratio of the portfolio. We mentioned previously that Roll (1977) believed the market portfolio in the sense of the CAPM as being unrepresentable by a proxy since all individual assets would have to be included in the sample. From a theoretical viewpoint, this is a very intuitive argument. However, Stambaugh (1982) found that extending the market portfolio with additional marketable asset classes other than stock did not significantly change the test results. Of course, this does not mean a priori that an integration of human capital as a non-marketable asset would also have insignificant effects. The same is true for other non-marketable assets such as claims to state transfers, pensions and benefits from trusts. Thus it is unclear if the market portfolio in empirical tests should mirror the investment universe as comprehensively as possible (including other marketable assets such as debenture bonds, consumer goods or real estate), or if stock suffices, since there are controversial findings concerning this issue (Fama and French, 2004).

Mayers (1972) defines human capital as cumulative wealth resulting from investments in oneself in addition to the human wealth one was endowed with by one's parents. Returns to human capital are seen as wages and salaries (ibid.). In any case, one can assume that every investor possesses a certain amount of human capital which generates volatile cash flows. However, the volatility of cash flows induced by human

capital can be discussed. Labour income is the form in which human capital is widely assumed to manifest itself financially (Le, Gibson, and Oxley, 2003). For the majority of the population we can assume that labour has the purpose to secure human subsistence, especially when we consider the severe concentration of capital in the industrialised countries (Piketty, 2014). Minimum wages on a national level and collective wage agreements for single branches are often justified by the society's perceived necessity to uncouple wages from business cycles in order to secure the sustaining quality of labour. It follows that we can expect a low volatility for labour income that is subject to the aforementioned regulations with smooth it over business cycles and economic crises. On the other hand, it also follows that we can expect a certain volatility of labour incomes that are not subject to cash-flow-smoothing regulations.

4.3. The derivation of Mayers' modified model

Mayers' (1972) modified model is firstly concerned with searching for asset prices under uncertainty which also reflect the income vector of investors for non-marketable assets. These are subsequently compared to the prices derived from the standard CAPM. Then the systematic risk also reflecting the influence of non-marketable assets is derived on an ex-post basis according to Jensen (1969). This is made possible by transforming the ex-ante view of the CAPM into an ex-post view, which is based on Jensen's (1969) and Fama's (1968) proof that $\beta_j \cong b_j$, where b_j is a constant originating from ordinary-least-squares estimation. The proof was derived thoroughly in section 4.1.2.

The result is a depiction of the risk of marketable asset j, i.e. β_j, which consists of exclusively observable components (c.f. Equation (70)). The composition of systematic risk β_j becomes clearly differentiable. The part of the systematic risk that results from the dependency of one firm's return on the other firms' returns and the part of the risk that is attributable to the dependency on the return from non-marketable assets, such as human capital, become quantifiable. According to Mayers (1972), this depiction of β_j

delivers a possible explanation for the results of Black, Jensen und Scholes (1972), which show that contrary to the standard CAPM's prediction, not every investor holds the market portfolio.

The assumptions on which Mayers' (1972) modified model is based are identical to those of the standard CAPM restricted to marketable assets. That is, it is assumed that all individuals are risk-averse single-period expected utility maximisers. The utility function G_i (E_i, V_i), being subject to the budget constraint

$$W_j = \sum_{j=1}^{n} X_{i,j} P_j - d_i; \quad i = 1,2,\dots,N \tag{56}$$

is maximised with $X_{i,j}$ and d_i as decision variables using a Lagrange function.

$$\begin{array}{c} \max \\ X_{i,j}, d_i \end{array} G_i(E_i V_i) \tag{57}$$

$$L = G_i(E_i V_i) + \lambda_i \left[W_i - \sum_j X_{i,j} P_j + d_i \right] \tag{58}$$

Where $X_{i,j}$ is the share of firm j held by investor i, d_i is the net debt of investor i, W_j is the marketable wealth of investor i at the beginning of the period, E_i is the expected return of the investor i's portfolio at the end of the period including investor i's return on human capital. P_j is the market value of the firm j at the beginning of the period.

$$E_i = \sum_{j=1}^{n} X_{i,j} E(R_j) + E(R_i^H) - r d_i \tag{59}$$

R_j is the cash flow that is paid to the owners of the firm j at the end of each period, R_i^H is a random variable that stands for the return on nonmarketable assets of the investor i which is received at the end of the period. The risk-free rate of return r corresponds to 1+i, where i is the risk-free rate of return of one period. Depending on the context, the variable i in Mayers' (1972) modified model stands for both the one-period risk free rate of return and for the investor. Despite this confusing double usage of i, for the sake of better comparability, we follow Mayers' notation in this Thesis. The variance of the ith investor's portfolio in the modified model consists of the variance of risky securities and of the variance of the return on human capital.

$$V_i = \sum_{j=1}^{n} \sum_{k=1}^{n} X_{ij} X_{ik} \sigma_{jk} + \sigma^2(R_i^H) + 2 \sum_{j=1}^{n} X_{ij} \operatorname{cov}(R_i^H, R_j) \tag{60}$$

In case $j = k$, the variable σ_{jk} corresponds to the variance of R_j. In case $j \neq k$, σ_{jk} corresponds to the covariance between R_j and R_k.

The marginal rate of substitution between risk and return for the investor i is derived from the Lagrangian equation for the maximisation problem described above:

$$\frac{\partial V_i}{\partial E_i} = \frac{2\left[\sum_k x_{ik}\sigma_{jk} + cov\left(R_i^H, R_j\right)\right]}{E\left(R_j\right) - rP_j} \tag{61}$$

In the capital market equilibrium, $\frac{\partial V_i}{\partial E_i}$ has to be equal for all assets j.

$$\frac{\left[\sum_k x_{ik}\sigma_{jk} + cov\left(R_i^H, R_j\right)\right]}{E\left(R_j\right) - rP_j} = \frac{\left[\sum_k X_{ik}\sigma_{tk} + cov(R_i^H, R_t)\right]}{E\left(R_t\right) - rP_t} \tag{62}$$

If that were not the case, at least one investor would not be acting rationally by treating at least one asset with bias.

From (62) one can infer an expected price of asset j in equilibrium which contains a price for each unit of risk, or in other words, the additional return per additional unit of risk as well as the systematic risk of asset j where the systematic risk of the single asset also accounts for human capital[42].

$$E(\rho_j) = r + \frac{E(\rho_m) - r}{P_M \sigma^2(\rho_M) + cov(\rho_M, R_H)} \times \left[P_M cov(\rho_j, \rho_M) + cov(\rho_j, R_H)\right] \tag{63}$$

The price per unit of risk is $\left[\frac{E(\rho_m) - r}{P_M \sigma^2(\rho_M, R_H) + cov(\rho_M, R_H)}\right]$ and the systematic risk of return on asset j is $\left[P_M cov(\rho_j, \rho_M) + cov(\rho_j, R_H)\right]$. At this point, Mayers (1972) presents a very interesting conclusion: as long as the market price per unit of risk is smaller than the market price per unit of risk in the standard CAPM $\frac{E(\rho_M) - r}{\sigma^2(\rho_M)}$, it follows that the market price per unit of risk in the standard CAPM is too high. The reason for this is that the modified model considers the relationship between the return on assets and the cash flow from human capital. The price per unit of risk can of course be higher than in the standard CAPM, namely in case when the covariance between the market return and the

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[42] For the derivation of (63) from (62) please refer to Appendix B.

cash flow from human capital is negative. The difference H_j between the systematic risk of asset j in the modified model and the risk of asset j in the standard CAPM shall be defined as $H_j = \frac{cov\,(\rho_j, R_H)}{P_M}$ [43]. Nevertheless, the systematic risk in (63) is equivalent to the comprehension of systematic risk in the standard CAPM. The systematic risk in the modified model also represents the dependency of the return on asset j on the returns on all other assets (including human capital).

It is necessary to point out that the 'missing assets model'[44], a version of the CAPM in which R_E stands for all non-marketable assets, as does R_H in the modified model, is equivalent to (63) and to the modified model per se. Therefore, it is obsolete to differentiate between non-marketable assets that are included in the model, be it human resources or other non-marketable assets. The effect of including such assets can be described with the modified model.

As previously mentioned, the problem with the modified model is that it is difficult to verify it empirically. Mayers (1972) approaches this problem by transforming his modified model into Jensen's (1969) market model. Firstly, he shows that the two models are equivalent[45]. Secondly, the returns on marketable and non-marketable assets are expressed in terms of the market model. These can then be inserted into β_j of the modified model, which enables its empirical verification.

The market model of Jensen (1969) for risky assets is postulated

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[43] Notation $\left[P_M cov\,(\rho_j, \rho_M) + cov\,(\rho_j, R_H)\right]$ and $\left[cov\,(\rho_j, \rho_M) + \frac{cov\,(\rho_j, R_H)}{P_M}\right]$ are equivalent. They could be rewritten as $\left[cov\,(\rho_j, R_M) + cov\,(\rho_j, R_H)\right]$ and $\left[cov\,(\rho_j, \rho_M) + cov\,(\rho_j, \rho_H)\right]$.

[44] The 'missing assets model' is a version of the CAPM that describes the effect of omitted assets in the investment universe (bonds, real estate, etc.). It comes from an unpublished manuscript by Black and Jensen (1972) and has only been referred to by Jensen, Black & Scholes (1972) and by Mayers (1972). The latter shows that his modified model is equivalent to the 'missing assets' version, if human capital is seen as one the missing assets.

[45] We showed and criticised the procedure that shows the equivalence of the market model and the standard CAPM in Section 4.1.2

$$\rho_j = E(\rho_j) + b_j\pi_1 + e_j \,, \tag{64}$$

where b_j is a constant, π_1 is a market factor, and e_j is a random variable. It is assumed

that $E(\pi) = 0; E(e_j) = 0 \,\forall_j; E(e_j\pi_1) = 0; E(e_ie_j) = \begin{cases} 0 & \forall_{j\neq i} \\ \sigma^2 & \forall_{j=i} \end{cases}$. The share of asset j in

the market portfolio equals the price of asset j in proportion to aggregate asset prices P_M.

The market return is the weighted average of the aggregate return. In order to transform

the single asset's return ρ_j into the market return ρ_M, the market factor π_1 is scaled in

order to eliminate b_j:

$$\rho_M = E(\rho_M) + \pi_1 + \sum_j X_j e_j \tag{65}$$

The same transformation logic is applied to transform the return on non-marketable

assets:

$$\rho_i^H = E(\rho_i^H) + b_i\pi_2 + z_i \tag{66}$$

where the assumptions for $b_i\pi_2$ and z_j are the same as the assumptions for $b_j\pi_1$ and e_j in

(64). The return on all non-marketable assets (in equilibrium) is derived applying the

same logic as in the previous transformations:

$$\rho_H = E(\rho_H) + \pi_2 + \sum_i Y_i z_i \tag{67}$$

where Y_i represents the value of the non-marketable wealth of investor i at the beginning

of the period relative to the total non-marketable wealth of all investors.

The equations (64), (65) and (67) are now inserted into the following expression of β_j,

which is implicitly contained in (63):

$$\beta_j = \left[\frac{P_M cov\left(\rho_j, \rho_M\right) + cov\left(\rho_j, R_H\right)}{P_M\sigma^2(\rho_M) + cov\left(\rho_M, R_H\right)}\right] \tag{68}$$

By applying the different possible notations of covariance and variance with subsequent

simplification under consideration of the assumptions regarding the variables in (64)

and (65), we can write (68) in a form that is entirely composed of observable terms. We

shall now look at how the actual objective of the insertions in (68) has been achieved,

which is the elimination of the unobservable market factors π_1 and π_2. The insertions in (68) deliver the following result:

$$\beta_j = \left[\frac{P_M \left[b_j \sigma^2(\pi_1) + X_j \sigma^2(e_j) \right] + P_H \left[b_j cov(\pi_1, \pi_2) + cov(e_j, \pi_2) + cov(\sum_i Y_i z_i, e_j) + b_j cov(\sum_i Y_i z_i, \pi_i) \right]}{P_M \left[\sigma^2(\pi_1) + \sum_j X_j^2 \sigma^2(e_j) \right] + P_H \left[cov(\pi_1, \pi_2) + cov\left(\sum_j X_j e_j, \pi_2 \right) + cov(\sum_i Y_i z_i, \pi_1) + cov(\sum_i Y_i z_i, \sum_j X_j e_i) \right]} \right] \quad (69)$$

For a better understanding of this interim result, one should bear in mind that $cov(\rho_j, R_H) = P_H cov(\rho_j, \rho_H)$ and $cov(\rho_M, R_H) = P_H cov(\rho_{jM}, \rho_H)$. The terms $\sum_j X_j^2 \sigma^2(e_j)$ and $X_j \sigma^2(e_j)$ decrease with $j \to n$ since X_j represents the fractional value of asset j on the market. As a consequence of this, one can assume $\sigma^2(\rho_M) \cong \sigma^2(\pi)$. This point becomes clearer when one considers that π is defined as $[\rho_M - E(\rho_M)]$. Consequently, the difference between the real-world return on the market portfolio and the estimated value is e_j. Although Mayers (1972) claimed that it was necessary to show that $\pi = [\rho_M - E(\rho_M)] = X_j(e_j)$, he did not actually prove this relation. What is more, neither Fama (1968) nor Jensen (1969), of whom Mayers claims to have the result, show it explicitly either. In fact only Jensen (1969) showed that the difference $[\rho_j - E(\rho_j)] = e_j$ (compare Equations (51) and (52) in Section 4.1.2). This implicitly means that $[\rho_M - E(\rho_M)]$ actually does equal $X_j(e_j)$. However, if one is able to show that $[\rho_j - E(\rho_j)] = e_j$, one has already proven that $\beta_j \cong b_j$ for the standard CAPM. During the derivation of the market model, Mayers (1972, [28]-[33]) evokes the impression that the actual objective of this procedure (showing that $\beta_j \cong b_j$) was just an interim step in the derivation of the market model. Therefore, we advise not to follow Mayers' approach in the derivation of the market model. Albeit it is not incorrect, it is misleading and this is one of the reasons for which we decided to show this derivation in more detail in Section 4.1.2.

It is further assumed that z_i and e_j are independent, conform to the normal distribution and have expected means of zero. This allows us to express (69) as

$$\beta_j \cong b_j + \left[\frac{cov\,(e_j, R_H)}{P_M \sigma^2(\rho_M) + cov\,(\rho_M, R_H)} \right] \quad (70)$$

The first part of this term b_j, is the systematic risk of the return on asset j based on asset j's dependency on the returns of all other assets that are represented in the market portfolio. The second part of (70) is now only the systematic risk of the return on asset j based on asset j's dependency on the aggregated returns of all non-marketable assets. This finding is very interesting. We see that although the non-marketable assets are not themselves included in the market portfolio [46] they do in equilibrium influence the return on the single assets since they increase or decrease demand for single assets based on $\text{cov}(R_i^H, R_j)$. In other words, integrating non-marketable assets changes the equilibrium price of asset j because not every investor holds an identical 'market portfolio' anymore [47]. They chose their portfolios by adjusting the market portfolio from the standard CAPM based on their cash flows from non-marketable assets. The question that arises now is how these individually different portfolios are structured exactly.

To show this, it is convenient to transform the modified model into matrix notation in order to determine the share that the investor holds in each firm. Again, the utility function $G_i \, (E_i, V_i)$, being subject to the budget constraint W_j, is maximised with respect to X_i and d_i.

In matrix notation, the expected return, the variance of investor i's portfolio and the wealth of investor i are expressed as follows:

$$E_i = X_i' E(R) + E(R_i^H) - rd_i \tag{71}$$

$$V_i = X_i' Z X_i + \sigma^2(R_i^H) + 2X_i' Z_i^H \tag{72}$$

$$W_i = X_i' P - d_i \tag{73}$$

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[46] This would not per se be impossible to model, one can express human capital in terms of a mixture of marketable assets and financially quantifiable investments in oneself which have expected pay-offs and therefore also a present value (c.f. Hugget and Kaplan 2011; 2016).

[47] See Equation (74), which is the end result of Mayers' modified model. From Equation (63) it is clear that the modified model (and thus also the resulting individual portfolio) only differs from the standard CAPM if $cov \, (\rho_j, R_H)$ is different from zero.

X_i is a vector containing the investments of investor i in all firms, $E(R)$ is a vector containing the expected cash flows of assets j, Z is the variance-covariance matrix of the random cash flows of the marketable assets, Z_i^H is a vector containing the covariances of assets j with the income from non-marketable assets for investor i, P is a vector containing the market values of the n firms. Solving the Lagrange equation for X_i one obtains a term with individually different components. In the standard CAPM, we would have obtained a term with uniform components apart from a scalar, which expresses the different amounts invested in the uniform market portfolio held by every investor. Now the shares held by the investors are expressed as

$$X_i \cong \frac{1}{2}\left(\frac{\partial V_i}{\partial E_i}\right) Z^{-1} \left\{[E(R) - rP] - 2\left(\frac{\partial E_i}{\partial V_i}\right) Z_i^H\right\}. \qquad (74)$$

Before the risk premium is multiplied with the marginal rate of substitution and with the variance-covariance matrix of the random cash flows of the marketable assets we see an individualised adjustment of the risk premium. This adjustment is achieved through a diminution of the risk premium by the inverse of the marginal rate of substitution which is weighted with the vector containing the covariances of assets j with the income from non-marketable assets for investor i.

With (74), the way in which the risk adjustment works is expressed formally for the first time. The higher the covariance of an asset with the income from non-marketable assets, the higher the reduction of the risk premium will be. This means that X_i will be different from the standard CAPM's market portfolio if Z_i^H (which is equivalent to $cov\,(\rho_j, R_H)$, c.f. Equation (63)) is different from zero. Equation (74) constitutes a model that allows the exact prediction of individual portfolio allocation (based on individual utility functions and individual holdings in non-marketable assets) in the case of risky assets. It should be noted that the degree of risk aversion influences the adjustment of the risk premium. An increase in risk aversion will increase the term $\left(\frac{\partial E_i}{\partial V_i}\right)$, as the investor will demand a higher expected return for accepting an additional unit of risk. The result is an increase of the risk premium for assets that covary negatively with R_i^H while assets that covary positively with R_i^H will have a lower risk

premium. In other words: what is quantified in (74) is the individual valuation an investor undertakes in order to realise their personal hedging against fluctuations in personal wealth. One final remark on the diversification in Mayers' modified model has to be made. The standard CAPM predicts perfect diversification, meaning that everyone holds the market portfolio containing only systematic risk (except in the case when an investor only holds the risk-free asset). In Mayers' modified model, perfect diversification has to be interpreted differently: it still means that investors hold portfolios containing only systematic risk. However, these portfolios are now individually different, since the modified model considers individual exogenous income on human capital. Therefore, one could say that the modified model predicts individual perfect diversification rather than uniform perfect diversification in the sense of the standard CAPM.

4.4. Necessary assumptions for the application of the modified model

The standard CAPM as well as the CAPM with non-marketable assets are both directly derived from the marginal rate of substitution between risk and return (c.f. Equation (62)). Any equilibrium price of assets is therefore influenced by the marginal rate of substitutions of investors.

One legitimate question that comes up in the context of extending the standard CAPM to non-marketable assets is: Are asset prices in equilibrium influenced at all by the extension? And if so, how are they influenced? These questions can be addressed empirically (c.f. Fama and Schwert, 1977 and Liberman, 1980) as well as analytically.

In order to question these findings and in order to understand what they imply (and what they do not imply) for our own empirical results, we have to address the influence of market extensions on equilibrium prices analytically from a utility theory perspective. Especially, we want to know what general properties the utility functions of investors must have in a world where market extensions change equilibrium prices of assets. In this context, we highlight the findings of Mayers (1976) and Weil (1994). Mayers (1976) analysed how extending the standard CAPM to non-marketable assets would influence

the asset prices in equilibrium. The starting point of his analysis is the price per unit of

risk $\frac{E(R_M) - rP_M}{\sigma^2(R_M) + cov\,(R_H, R_M)} = \lambda_M$. We mentioned before that the market prices of assets in

equilibrium are derived from the marginal rate of substitution between risk and return.

In the notation of λ_M this is not directly visible. Mayers (1976) presents an alternative

formulation for the market price per unit of risk as $\lambda_M = \frac{2}{L} \overline{\frac{\partial \mu}{\partial V}}$ where L is the number of

investors in the market and $\overline{\frac{\partial \mu}{\partial V}}$ is the harmonic mean of the marginal rate of substitution

between risk and return[48]. The way in which λ_M will change, when extending a market

to non-marketable assets, allows us to make statements about the asset prices in

equilibrium. The marginal rate of substitution between risk and return can also be

derived by differentiating the utility functions of the investors[49]. In fact, for the purpose

of understanding empirical results, it would not make sense to examine market

extension effects on $\frac{E(R_M) - rP_M}{\sigma^2(R_M) + cov\,(R_H, R_M)}$, since this term is the result of optimising the

investor's problem which does not reflect individual utilities but only a risk-neutral risk-

return maximisation. $\frac{\partial \mu}{\partial V}$ as shown in Footnote 49 is the investors' marginal rate of

substitution of return for risk based on their individual utility function. If we

differentiate $\frac{\partial \mu}{\partial V}$ with respect to μ and V, we obtain the direction of change of the slope

of the investors indifference curves while investors adapt their portfolios to the

existence of non-marketable assets (Mayers, 1976)[50]. From the signs of $\frac{\partial}{\partial \mu}\frac{\partial \mu}{\partial V}$ and $\frac{\partial}{\partial V}\frac{\partial \mu}{\partial V}$

one can state the effect of the extension on the average risk attitude of investors and on

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[48] Please note that this relationship is derived from the Lagrange Equation (57) and not yet from the investors' utility function.

[49] $max\,E[u(w)]$ where wealth w is a random variable. It follows that $\frac{\partial \mu}{\partial V} = -\frac{1}{2}\frac{E[u''(w)]}{E[u'(w)]}$ which has the dimension of the Arrow-Pratt measure of absolute risk version with $u'(w) > 0$ and $u''(w) < 0$ assuming non-satiation of wants and risk aversion.

[50] For a proof of this please refer to Appendix C.

the asset prices. Given that $\frac{\partial \mu}{\partial V}$ is an expression for the market price per unit of risk we can then infer the change of assets prices when non-marketable assets are considered. Mayers (1976) finds that $\frac{\partial}{\partial \mu}\frac{\partial \mu}{\partial V} \leq 0$ (with $\frac{\partial}{\partial \mu}\frac{\partial \mu}{\partial V} < 0$ in the case of decreasing absolute risk aversion and $\frac{\partial}{\partial \mu}\frac{\partial \mu}{\partial V} = 0$[51] with constant absolute risk aversion). We learn that in order to see an effect of the market extension, we must not have investors with constant absolute risk aversion. Empirical findings as well as intuition imply that absolute risk aversion is not constant but decreasing (c.f. Arrow, 1965; Mayers, 1976) which is in accordance with the finding that $\frac{\partial}{\partial \mu}\frac{\partial \mu}{\partial V} \leq 0$. Now it is important to see how $\frac{\partial}{\partial V}\frac{\partial \mu}{\partial V}$ behaves. Mayers (1976) shows analytically that $\frac{\partial}{\partial V}\frac{\partial \mu}{\partial V}$ is strictly positive when a constant relative risk aversion coefficient hovering around 1 is assumed for the average investor[52]. The sign has to be strictly positive since otherwise, with the established finding that $\frac{\partial}{\partial \mu}\frac{\partial \mu}{\partial V} < 0$, we would have a situation where the change in the slope of the investors' indifference curves would be negative with increasing variance and also negative with an increasing mean of wealth. In other words, we would have a situation where an increase of wealth and an increase of variance have the same effect on the investors' risk attitudes. This would violate the assumption of risk aversion. In summary, we have found that unambiguous and empirically meaningful statements on the effect of market extensions can only be made when investors show constant relative risk aversion. It should be noted here that constant relative risk aversion implies decreasing absolute risk aversion. It follows that the influence of a market extension on the equilibrium price of assets is that in an extended market we have higher prices for assets. This becomes clear by remembering that with the sign of $\frac{\partial}{\partial V}\frac{\partial \mu}{\partial V}$ we have examined how the

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[51] Constant absolute risk aversion implies linear indifference curves Mayers (1976) for $\frac{\partial \mu}{\partial V} =$
$-\frac{1}{2}\frac{E[u''(w)]}{E[u'(w)]} = -\frac{1}{2}\frac{E[-r_a u'(w)]}{E[u'(w)]} = \frac{1}{2}r_a$ where r_a is the Arrow-Pratt measure of absolute risk aversion.

[52] Please refer to Appendix D.

market price per unit of risk changes when the investors consider also non-marketable assets. Since this sign is positive, from Equation B3 in Appendix B[53], we can infer higher prices in the extended market[54]. This finding is very intuitive: when investors have more wealth (which is the case when non-marketable assets are considered), under the assumption of decreasing absolute risk aversion, they will demand less return for a given level of risk.

We have established that only in the case of constant relative risk aversion will the equilibrium price be affected by human capital. Since the case of constant absolute risk aversion contradicts observed behaviour of investors, we must assume constant relative risk aversion with decreasing absolute risk aversion as a consequence.

The necessary assumption of decreasing absolute risk aversion in the context of the modified model is the focus of the analysis of Weil (1994). Empirical work by Mankiw and Zelde (1989) shows that stock is mostly held by wealthy investors who also have large labour incomes[55]. Piketty (2014) asserts that both capital and labour income are in an accelerating process of concentration in the industrialised world. In absolute numbers, this trend is strongest for the Anglo-Saxon world, however in Western Europe the situation is qualitatively identical. In this situation, it is reasonable to assume that the average absolute risk aversion is becoming smaller as the aforementioned trend continues. How does this affect the modified model? When investors are rich and when a considerable and growing percentage of their wealth is in the form of human capital,

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[53] $P_j = \frac{1}{r}\left\{E(R_j) - \left[\frac{E(R_M) - rP_M}{\sigma^2(R_M) + cov\,(R_H, R_M)}\right] \times \left[cov\,(R_j, R_M) + cov\,(R_j, R_H)\right]\right\}$.

[54] $\frac{\partial}{\partial v}\frac{\partial \mu}{\partial v} > 0$ means that with increasing variance the slope of the indifference curve changes positively with respect to the previous slope in the non-extended market. This implies a smaller risk aversion in the extended market and therefore a lower market price per unit of risk, which in turn increases P_j.

[55] Consider also that labour incomes make up two thirds of total incomes in industrialised countries (Weil,1994; Piketty, 2014). It is therefore not surprising that investors also have large labour incomes. Piketty (2014) also points out that the labour income of investors is likely to continue to grow disproportionally due to the increasing cost of an education that is sufficient to gain large labour incomes from.

we would have to expect lower risk premiums by the modified model in comparison to the standard CAPM[56]. This would not be problematic had the standard CAPM estimated risk premiums that are too high[57]. We are now in a dilemma: obviously the standard CAPM does not deliver correct asset values and investor portfolios, however it does, on average, deliver correct historical risk premiums and volatilities of stock markets. We saw that improving firm values and investor portfolios would distort the risk premiums and volatilities. Weil (1994) found that this effect can be circumvented by assuming a sufficiently large covariation between human capital returns and stock returns. He also shows how the necessary magnitude of the covariance can be determined. In the following, we want to examine Weil's procedure, for it finalises and substantiates a set of assumptions we have to make for the investors in our sample if we want to apply the modified model[58]. Also, we want to show the equation which allows us to infer the necessary magnitude of $cov\,(\rho_j, R_H)$. We have already seen in Equation (62) that the beta factors of the standard and of the modified CAPM will only differ if $cov\,(\rho_j, R_H)$ is different from zero. Now we will see that $cov\,(\rho_j, R_H)$ has to be positive when considering the utility functions of investors. Weil (1994) shows this by setting up a simple two-period model of investor utility with power utility functions that represent constant relative risk aversion. Consumption takes place in the future, i.e. in the second period. Consumers have total wealth W of which fraction θ is tradable while fraction $1 - \theta$ is not. R_W is the return on total wealth, R_M the return on marketable wealth and R_H is the return on human capital. Consumption is given by

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[56] Relative to the standard CAPM, investors are richer in the modified model and therefore, under the assumption of decreasing absolute risk aversion, less risk averse, which results in lower risk premiums.

[57] The standard CAPM explains the historical average risk premium and volatility of stock markets well (c.f. Mehra and Prescott, 1985; Weil 1994). Please note that this does not at all imply that the standard CAPM also delivers correct asset values and investor portfolios.

[58] In light of empirical findings concerning the concentration of wealth, these assumption are constant relative risk aversion and therefore power utility functions and a sufficiently large covariation of human capital and stock returns. The assumptions necessary for creating a market model were discussed in Section 4.1.2.

$$C = R_W W \text{ where}$$

$$R_W = \theta\, R_M + (1 - \theta)R_H$$

(75)

The share R_M of W, which is invested in the market, is subdivided in a riskless and a risky investment. The return on the riskless investment is denoted by R_0 and the return on the risky investment by R_1. In the case where an investor invests share α in the riskless asset, R_M is denoted by

$$R_M = \alpha R_1 + (1 - \alpha)R_0$$

(76)

Consumption can then be expressed as

$$C = \{\theta[\alpha R_1 + (1 - \alpha)R_0] + (1 - \theta)R_H\}W.$$

(77)

All investors have the power utility function $u(C) = \frac{C^{1-\gamma}}{1-\gamma}$. Constant relative risk aversion γ ensures that this model is aligned with the findings of Mayers (1976) which we presented above. In this model of consumption, the decision variable is α, since θ cannot be influenced by the investor (at least not in the short run). Investors will therefore

$$\max_\alpha Eu(C) \text{ subject to}$$

$$(1)\ \ C = \{\theta[\alpha R_1 + (1 - \alpha)R_0] + (1 - \theta)R_H\}W$$

$$(2)\ C \geq 0$$

(78)

Solving the first order condition for α yields the approximate solution[59]

$$\alpha = \frac{1}{\theta}\left[\frac{Er_1}{\gamma\sigma_{11}} + \frac{1}{2\gamma}\right] + \left(1 - \frac{1}{\theta}\right)\frac{\sigma_{1H}}{\sigma_{11}}$$

(79)

The change in the risky investment of an investor can now be distinguished depending on (i) their total wealth and its composition and (ii) the covariance between the risky asset and the non-marketable asset. We shall now try to isolate the constellation in which α does change when non-marketable assets are considered by the investor. In the case where θ equals one, it is clear that we will see no difference in α

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[59] The equations are now log-linearised and it is assumed that r_1 and r_H are jointly normal (the lower case letters denote the logarithm) with variance-covariance matrix $\begin{pmatrix} \sigma_{11} & \sigma_{1H} \\ \sigma_{1H} & \sigma_{HH} \end{pmatrix}$ (Weil, 1994).

with respect to the standard CAPM. With θ smaller than one, the difference in α will obviously depend on the magnitude of σ_{1H} which must not be zero. However, we also see that not every positive correlation $\frac{\sigma_{1H}}{\sigma_{11}}$ can suffice to induce differences in α. The magnitude of the decline in θ and the magnitude of σ_{1H} determine the effect on α. In the case where σ_{1H} is zero and where non-marketable assets are suddenly considered, the share of risky assets in the investors' portfolio will increase[60]. When the second term in the above equation for α is not zero, the situation is as follows: α decreases with increasing $\frac{\sigma_{1H}}{\sigma_{11}}$ and it increases with decreasing θ. We have seen the rationale of α increasing with decreasing θ but what is the logic behind the decrease of α with increasing $\frac{\sigma_{1H}}{\sigma_{11}}$? Ultimately, the investor is interested in maximising the utility of consumption. With increasing $\frac{\sigma_{1H}}{\sigma_{11}}$ the risk of the total wealth becomes bigger when non-marketable assets are considered. It follows that the consumption stream appears less steady to the investor. In order to reduce that, they can only reduce the risky investment, since the non-marketable assets are fixed (at least when human capital is the non-marketable asset). Which of the two effects prevails depends, of course, on the magnitudes of $\frac{\sigma_{1H}}{\sigma_{11}}$ and θ but also on the coefficient of constant relative risk aversion γ. We assume that the coefficient γ does not change when non-marketable assets are considered because utility functions are defined over all possible consumption levels. From the evidence of Mehra and Prescott (1985), an average value of 2 can be estimated for γ. The procedure that leads to this estimation is as follows: In the model of Weil (1994), equilibrium excess returns and risk can be equated as

$$Er_1 - r_0 + \frac{1}{2}\sigma_{11} = \gamma[\theta\alpha\sigma_{11} + (1 - \theta)\sigma_{1H}] \tag{80}$$

Mehra and Prescott (1985) found that the historical excess returns were 6% and the historic risk was 3% in the 20th century. In the case where θ equals 1 (i.e. the case

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[60] The reason being that with increasing wealth considered in asset allocation, decreasing absolute risk aversion will lead to a higher acceptance of market risk.

where non-marketable assets are not considered) and where α is assumed to be 100% in equilibrium[61], we see that γ needs to be 2 in order to explain the historical data. When θ decreases, we can easily deduce the necessary magnitude of σ_{1H}. That is because for a labour share of 2/3 in total income, we would need a covariance between the risky assets and the labour income of 3%. This insight will help us to put our own empirical results presented in Chapter 5 in perspective.

4.5. Human capital

The key component of the proposed model is human capital or more precisely, the returns thereof. Mayers himself apparently had only salaries and wages in mind. Consequently, his definition of human capital encompasses both the accrual of investments in oneself and the human wealth received from the parents (Mayers, 1972:245). Fama & Schwert (1977) largely follow this definition by using labour income statistics for their tests. However, they admit that using gross income per capita is not unproblematic; while it captures quantitative changes in the labour force, qualitative changes (e.g. attained level of education) are not accounted for. Nevertheless, they believe that the result – i.e. the negligible difference between the traditional CAPM and Mayers' model – is robust irrespective of how human capital is defined (Fama & Schwert, 1977:121). Liberman (1980) did not challenge this definition either.

We believe that more thought should be given to this issue. Especially in our case of small firms and their owner-managers, using average wages and salaries of the entire labour force may not be correct; for instance, a mere 1.5% of Germans aged 18-64 founded a firm in 2015, of which 3/4 were new enterprises. The *Kreditanstalt für Wiederaufbau* (KfW) expects a further decline in these figures (KfW, 2015). Naturally, only a fraction of all firms ever "appears on the radar" by becoming a valuation target or

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[61] An α value of 100% implies the applicaation of the zero-beta model. Given that we pointed out before that the zero-beta model was empirically favoured in comparison to the standard CAPM we find its' application in this empirical context acceptable.

at least by being obliged by law to file annual accounts[62]. Therefore, our group of potential investors is very small and very specific. This is where we depart from the world of Mayers in which anyone in the work force is a potential investor making portfolio choices based on the returns on their human capital. But then again it was shown that "nonfarm salaried workers appear to be able to ignore their human capital when constructing their portfolios […]" (Liberman, 1980:188). As mentioned above, we do not challenge this very much reasonable view; employees with an income protected against stock market fluctuations by collective wage agreements, minimum wages, transfers etc. are most likely able to ignore their human capital.

Our investors typically do not earn a regular wage or salary. Hence, their returns on human capital reflect their potential ability to achieve income in the labour market. We hypothesise that this potential is best represented by management salaries; a successful entrepreneur (in our sense one that has filed their annual accounts in the last five years) can surely be seen as a corporate manager. Moreover, management salaries are not subject of collective agreements and often include incentives (pecuniary or other (Weil, 1994)) dependent on the firm's performance. In addition, empirical data suggests that the stock market tends to be held by wealthier individuals (Weil, 1994; Piketty, 2014). Hence, we believe that management salaries better reflect the intuition that owner-managers with a "more valuable" human capital are more likely to lead a successful enterprise. This would imply a higher correlation between the returns on human capital and marketable assets than in Figure 1 which would in turn result in a more significant difference between the beta of the proposed model and the standard CAPM beta.

There are other approaches to the determination of human capital and returns thereof as well. The more peculiar methods (i.e. those not simply using earnings data or discounted values thereof) are summarised in the following subsections.

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[62] In Germany, unlimited liability companies (such as *Einzelunternehmen*) with an annual turnover of less than EUR 600,000.00 or a profit of less than EUR 60,000.00 are exempt from the obligation to file annual accounts (§ 241a HGB)

4.5.1. Investments in human capital (Athreya et al, 2015)

Athreya, Ionescu and Neelakantan (2015) proposed an investment decision model that captures both investments in capital markets and human capital. We found the human capital part quite peculiar, and thus mention it here in brief.

In the highly complex model of Athreya et al (2015), the motion of human capital in time is defined as follows:

$$h_{t+1} = h_t(1 - \delta) + H(a, h_t, l_t) \tag{81}$$

where h_t is the stock of human capital at time t, δ is the depreciation rate of human capital[63], and $H(a, h_t, l_t)$ is the human capital production function as proposed by Ben-Porath (1967). This production function is given as

$$H(a, h, l) = a(h, l)^{\alpha}; \qquad \alpha \in (0,1) \tag{82}$$

where a is the agents immutable learning ability, l is the fraction of available time put into human capital production and α is the elasticity of the production function. With this information, Athreya et al (2015) proceed by defining labour income of agent i (i.e. the return on human capital) as the product of a deterministic and a stochastic factor:

$$log(y_{it}) = G(w_t, h_t, l_t) + z_{it}[64] \tag{83}$$

The deterministic component $G(w_t, h_t, l_t)$ includes the human capital rental rate $w_t = (1 + g)^{t-1}$, while l_t stands for $1 - l_t$ which is the time spent on the labour market. The stochastic factor is given by

$$z_{it} = u_{it} + \epsilon_{it}; \quad \epsilon_{it} \sim iid\ N(0, \sigma_{\epsilon}^2) \tag{84}$$

ϵ_{it} is an idiosyncratic temporary shock and $u_{it} = \rho u_{i,t-1} + v_{it}$ is the persistent shock following an AR(1) process with $v_{it} \sim N(0, \sigma_v^2)$.

The formulae above are but an illustrative excerpt from the model of Athreya et al (2015). The authors further discuss social transfers and pension income to finally

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[63] The model assumes that investments in human capital decrease over the life time, resulting in an (empirically observed) decrease in average real earnings at the end of the working life cycle.

[64] There is heavy autocorrelation in earnings data (Parsons, 1978), which makes it necessary to employ a log function whenever such data is researched.

formulate a complex agent decision problem involving also risky and riskless assets. It involves a large amount of parameters not only for asset markets or human capital, but very importantly, for the initial distribution of assets, ability and the stock of human capital. These parameters are, in part, adopted from previous studies and mapped to historical data, so that the model replicates the evolution of life-cycle mean earnings, the ratio of mean to median earnings and the Gini coefficient of earnings.

We consider this model peculiar for several reasons: firstly, it maps the evolution of the stock of human capital with a growth rate and a depreciation rate. In addition, it allows for investments in human capital (i.e. the expansion thereof) due to the existence of a human capital production function. Secondly, the model considers earnings shocks instead of simply using earnings data. Thirdly, it accurately simulates reality by also incorporating transfers and pension income. And fourthly, Athreya et al (2015) calibrated the model so that it can replicate historical US data.

The reason we chose not to use their model concerns its practicability – it would constitute an enormous effort to calibrate the model for a German setting. Athreya et al (2015) could rely on a substantial body of work which made the greater part of their parameters available with relative ease. In addition, even with all the parameters readily available, the calibration alone would clearly exceed the scope of this Thesis. We believe that this highly realistic approach definitely warrants future research.

4.5.2. Individual human capital values and returns (Huggett & Kaplan, 2016)

The model suggested by Huggett & Kaplan (2016) may well be the most complicated method of calculating human capital values and returns. The amount of input considered is comparable to that of Athreya et al (2015), but the key difference is, that Huggett & Kaplan (2016) rely heavily on econometrically simulated processes[65] rather than on parameters quantifiable with data from extant research. Moreover, they innovate the long-term earnings modelling process by splitting up the human capital into a stock and

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[65] A good example is the decomposition of earnings into an aggregate and an idiosyncratic component as shown below.

a bond component which are implicitly contained in the value of future earnings[66] and a residual-value component orthogonal[67] to the previous two. The gross return to human capital is defined as the sum of next period's (next year's age) value and earnings divided by this period's (current age) human capital value:

$$R_{j+1}^h = \frac{v_{j+1} + e_{j+1}}{v_j} \tag{85}$$

The human capital value is defined as the discounted value of future earnings:

$$v_j(z^j) \equiv E\left[\sum_{k=j+1}^{J} m_{j,k} e_k \big| z^j \right] \tag{86}$$

where $z^j = (z_1, \dots, z_j) \in Z^j$ are shock histories mapped into consumption[68]. The agent's stochastic discount factor $m_{j,k}$ with conditional probability $P(z^k|z^j)$ given by

$$m_{j,k}(z^k) = \frac{\partial U(c^*)/\partial c_k(z^k)}{\partial U(c^*)/\partial c_j(z^j)} \frac{1}{P(z^k|z^j)} \tag{87}$$

is the marginal rate of substitution between consumption goods in period j and in an extra period k, respectively. It is the solution to the agent's decision problem

max $U(c)$ subject to

(3) $c_j + \sum_{i \in I} a_{j+1}^i = \sum_{i \in I} a_j^i R_j^i + e_j$ and $c_j \geq 0, \forall j$ $\qquad$ (88)

(4) $a_{j+1}^i = 0, \forall I$

in which lifetime utility $U(c)$ from consumption plan $c = (c_1, \dots, c_J)$ is maximised. In every period, the agent can choose between consumption c_j and saving $\sum_{i \in I} a_{j+1}^i$ with exogenous earnings e_j and financial assets brought into the period $\sum_{i \in I} a_j^i R_j^i$ forming the

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[66] This means that, according to Huggett & Kaplan (2016), earnings behave differently at different stages of the working life cycle – after retirement, for instance, most agents consume mostly risk-free pension income, which results in a largely "bond-like" human capital.

[67] When earnings are decomposed in this way, the residual-value component is independent of the asset returns (i.e. bond + stock returns, see below).

[68] Consumption at age j is defined as $c_j : Z^j \rightarrow R_+^1$ and all variables considered are functions of shocks z_j.

resources available for each period (age). The value of the latter depends on the savings a_j^i invested in each asset and on the return $R_j^i > 0$ thereof. Earnings e_j consist of an idiosyncratic component simulating an agent's individual effects on their earning process[69] and of an aggregate component of earnings and stock returns[70]. We shall not go into greater detail here.

The value of human capital at age j can alternatively be stated as

$$v_j = E[m_{j,j+1}(v_{j+1} + e_{j+1})|z^j].$$ (89)

Other than in Equation (86) where the expected value of all future earnings is considered (much like in a dividend discount model), the value of human capital at age j v_j is stated in terms of the discounted value of next period's human capital stock v_j and earnings e_{j+1}. Huggett & Kaplan (2016) use this formula as a starting point to further decompose human capital values for each period into a stock, bond and orthogonal component to simulate the behaviour of earnings processes over the life cycle:

$$v_j = E[m_{j,j+1}(v_{j+1} + e_{j+1})|z^j] = E\left[m_{j,j+1}\left(\sum_{i \in I} a_j^i R_{j+1}^i + \epsilon_{j+1}\right)\Big|z^j\right]$$ (90)

where the asset returns $a_j^i R_{j+1}^i$ are decomposed into expected bond returns $a_j^b E[m_{j,j+1}R_{j+1}^b|z^j]$ and expected stock returns $a_j^s E[m_{j,j+1}R_{j+1}^s|z^j]$.

We highlight two important findings of Huggett & Kaplan (2016). Firstly, the values of human capital calculated by their model are far below those calculated simply by discounting earnings with the risk-free interest rate. Given the existence of idiosyncratic shocks (e.g. unemployment or disability) in the earnings data which have a lasting effect on earnings and thus on human capital values, this is not a surprising finding. Moreover, the individual stochastic discount factor shows a negative covariance with these shocks,

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[69] These effects are: a common age effect, an individual-specific fixed effect, a persistent component and a transitory component. They are modelled by means of different distributions of varying moments.

[70] The aggregate component is defined as a first-order vector autoregression (VAR(1)) for $\Delta y_t = (\Delta u_t^1 \, \log R_t^s)'$, where u_t^1 are labour earnings and R_t^s are stock returns: $\Delta y_t = \gamma + \Gamma \Delta y_{t-1} + \varepsilon_t$.

which makes perfect sense: if there is a downward shock in the earnings, the discount factor increases, resulting in a lower present value of human capital. Or, from another perspective, lower earnings result in less saving, which in turn reduces available resources for future periods. Secondly, the stock component implicitly contained in future earnings may seem quite large at 35 per cent; however, it is caused, among others, by retirement benefits positively correlated to the level of average earnings or a positive conditional correlation between stock returns and the aggregate component of individual earnings (Huggett & Kaplan, 2016:2). This share appears more realistic than the 50 per cent calculated by Benzoni et al (2007).

Huggett & Kaplan (2016) offer a complicated albeit highly realistic approach to calculating human capital and returns thereof. Much like in the case of Athreya et al (2015), their model is fitted to US earnings data and thus not generalizable. In addition, although it may offer more realism than the already quite realistic model of Athreya et al (2015), it is also much more complicated, which, from a valuator's point of view, is a rather serious drawback. It is questionable whether such increased complexity is outweighed by more precise results. Future research could compare the two studies to establish which of the models is more suitable.

4.5.3. Other approaches to measuring human capital

There are in principle three main groups of methods of calculating human capital – income-based, cost-based and education-stock-based approaches (Le, Gibson & Oxley, 2003). The former two are alternatively referred to as prospective and retrospective (Dagum & Slottje, 2000). Two income-based approaches were presented in the previous sections. In line with the generally accepted notion that thorough valuation must be based on expected future returns (as in the DCF, for instance), we would not consider retrospective methods, as these merely present an ex-post image. As for the methods using the stock of education, it should be noted that these merely proxy human capital by attained education in years, which may provide an insight into how large a country's stock of human capital is in terms of person-years, but not in terms of value. An exception here is Judson (2002) who calculates the costs of primary, secondary and

tertiary education and then weights them according to their costs to obtain dollar values of human capital. Nevertheless, it is still a backward-looking cost-based method.

The method proposed by Dagum & Slottje (2000) is, at least according to the authors, a compromise between prospective and retrospective approaches. They first estimate human capital on a personal level by defining it as a latent variable standardised for each household in their survey data from 1983. Through exponential transformation with partial least squares regression, the latent variable is transformed into a monetary unit, which is then divided by the share of household i's income on total population income to yield an average value. In a second step, human capital is estimated on a macroeconomic level in terms of average earnings per household: in the model of Dagum & Slottje (2000), expected earnings at a certain age correspond to population average earnings at the same age. They increase yearly by the productivity growth rate[71], are discounted at a given discount rate[72] and depend on the survival probability[73]. Finally, these average earnings are divided by the average value of individual human capital calculated earlier and multiplied by the latent-value human capital to yield the HC value for the ith sample observation.

We found this interesting integrative approach of Dagum & Slottje (2000) worth mentioning, given that it is also quite present in the human capital literature. However, we would not use it because it largely depends on the interest rate. Huggett & Kaplan (2016) have shown that discounting at the risk-free rate without accounting for shocks yields greatly overestimated human capital values. We agree with this view. Especially in times with prolonged low-interest spells, such a model would calculate higher human capital values even if human capital was in fact deteriorating.

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[71] Dagum & Slottje (2000) assume an annual productivity growth of 3% for ages 20-29, 2% for ages 30-54, 1% for ages 54-65 and 0% for ages above 65. It is not explained where these rates come from.

[72] The study uses two rates: 6% and 8% which are probably supposed to mirror the risk-free rate.

[73] The survival probability is obtained from mortality tables for the US population.

5. Empirical testing

5.1. The data

5.1.1. Firm and market data

As outlined before, we focus on SMEs in Germany. Recalling the discussion on the market factor in Section 4.1.2, in the effort of eliminating industry affiliation effects, we decided to test for a single industry only. The choice fell on the automotive industry (SIC 371), since we expected that it would provide a sufficient amount of samples given its size and importance in Germany. Regrettably, due to the nature of our human capital data[74], the statistical population had to be narrowed down to firms with less than 100 employees. This left us with 379 companies. Of these, all firms without complete balance sheet and income statement data for the 2009-2014 period had to be eliminated. A final adjustment was removing firms without owner-managers (i.e. without at least one shareholder in the top management), leaving 99 companies. For this final sample of 99 companies, we obtained their equity, net income (i.e. earnings after taxes), turnover and shareholder stakes for 2009-2014 from the *Hoppenstedt Company Database*.

Choosing a suitable market portfolio proxy was challenging. Had the objective been choosing a proxy that is representative for the given market (in our case, Germany) in the most general sense, we would have opted for the DAX 30, the blue chip index of Germany. However, we hold that the market portfolio should mirror our sample as best as possible; if any of our sample companies went public, it would not become a constituent of the DAX 30 in many years to come, regardless of its success. Small firms aiming to be listed are most likely to do so in a dedicated market with more relaxed listing criteria. In the case of Germany, this is the *entry standard* with over a hundred firms, of which the 30 with the highest trading volumes constitute the Entry Standard Index which we selected as our market portfolio proxy.

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[74] For firms with more than 100 employees, the smallest category offered in the survey questionnaire is 100-500 employees, while the upper bound for medium-sized enterprises is 250 employees.

5.1.2. The investors' human capital

We mentioned some methods of calculating human capital returns in the previous chapter. Of these, we found the one by Athreya et al (2015) most promising. However, with the necessary base data not being available for Germany, we find it necessary to employ the method based on managerial wages as outlined below in this section.

It was quite challenging to obtain highly differentiated[75] income data of managerial staff (particularly of the variable wage component) dependent on (i) firm size in terms of the number of employees, (ii) industry affiliation and (iii) firm performance.

The reason for these criteria is our aspiration to determine an idiosyncratically simulated human capital income for each owner-manager of the sample. Our proxy for this purpose is labour income of executives that matches the firms of our sample with respect to criteria (i-iii). The data was provided by the German executives' association *Die Führungskräfte* from their comprehensive annual executives' survey.

With the variance, mean and the sizes of fixed and variable wage components obtained from this data, we are able to calculate a human capital income[76] for each one of our sample's owner-managers. It was shown in previous sections that Mayers' model can only deliver results that differ from those given by the standard CAPM if there is a (sufficient) covariance between the returns on human capital and stock returns[77]. In light of this fact, we consider it necessary to model human capital income in the way explained in Section 5.2. If we ignored the individual differences in the variable wage components, and simply assigned peer group means to each shareholder, we would obscure the true covariance.

It is the variable wage component that is of crucial importance here, as it is most likely to be tied to the firm's performance. As mentioned before, it is reasonable to

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[75] By "differentiated" we mean a sufficiently high data granularity to be able to obtain the variance and the distribution of managerial income.

[76] "The returns to human capital we are talking about are wages and salaries." (Mayers, 1972: 245).

[77] As our sample is comprised of non-listed firms, we use returns on equity as a replacement for stock returns (c.f. Section 5.2.1).

assume that the financial performance of an SME's owner-manager's work and the overall success of the SME are linked to a much larger extent than in large enterprises. After all, the owner-manager can only distribute profit if one is achieved (that includes profits of previous periods carried forward, which enables the smoothing of profit distribution). Even if the owner-manager is employed in their own SME, all other wages payable are senior to the contractual wage of the owner-manager, meaning that their wage is still dependent on firm performance. That said, in the top-level management data we use, firm performance has a lower weight in the variable wage component than it could be assumed for owner-managers working in their own firm. In fact, the owner-manager's remuneration can be seen as virtually variable-only (particularly in smaller firms). Therefore, we assume that if we model the human capital income with this data, the covariance between returns on the owner-managers' human capital and the returns on equity will still be underestimated. At the same time, the underestimation would be even more pronounced without modelling.

5.2. Methodology

In the derivation of Mayers' modified model in Section 4.3 it was shown that the modified model beta could be calculated by Equation (70):

$$\beta_j \cong b_j + \left[\frac{cov\,(e_j, R_H)}{P_M \sigma^2(\rho_M) + cov\,(\rho_M, R_H)} \right]$$

All the variables of Equation (70) can be obtained from our data as will be shown in the following subsections: in Section 5.2.1, we explain how ρ_M and ρ_j (which is necessary to determine b_j) are obtained, and in Section 5.2.2, the modelling of R_H is elaborated on. P_M is the average value of the equity of all firms.

5.2.1. ROE as a proxy for investor and market returns

In the standard CAPM (and thus also in Mayers' modified model), investor returns are represented by profits on the shares of stock they hold. For unlisted firms such as our sample of SMEs of the automotive industry, investor returns have to be proxied by a different measure. We hypothesise that the return on equity (ROE) is the most suitable

alternative; after all, investors in small firms are also equity holders with a claim on the ROE. The difference to investors in listed firms is that the ROE is an ex-post measure, since the book value of equity does not include future expectations as opposed to stock market prices. Furthermore, the market for equity stakes in small firms is far less liquid than any stock market, meaning that trading profits are limited or unachievable.

Using a proxy for investor and market returns may seem controversial, but there is no other way to employ CAPM-based models for the valuation of unlisted firms. In fact, our method is not at all in conflict with CAPM theory, although not having returns containing future expectations admittedly is a drawback in valuation. Nevertheless, we assert that using a proxy for returns is still superior to not using a CAPM-based approach and relying on methods such as valuation multiples; even the biggest (in terms of equity) firms of our sample are at least 3-4 times smaller than the smallest firms of the Entry Standard Index (most firms of this index are actually large enterprises). The meaningfulness of using valuation multiples with such discrepancies between the compared firms is thus questionable. For our ROE method it is of course necessary to also calculate the 'index ROE' with the firms' ROE weighted according to their weight in the index, because calculating the covariance between the sample firms' ROE and the return on the Entry Standard Index would not produce meaningful results.

5.2.2. Operationalisation of human capital

The returns on human capital are divided into a fixed and a variable wage component. Because of the problems with individualising the fixed wage component described in the previous section, we use the fixed wage component mean from the survey for companies with less than 100 employees.

The variable wage component is dependent on the realisations of various performance evaluation factors, which are determined for each firm. It is therefore unique for each of our firms. To measure this performance, we rank the firms in our sample following the

criteria used in determining the practically unlimited variable wage component [78] according to the survey of *Die Führungskräfte*. These criteria are: general personal performance evaluation (10% on average), profit of the company (16% on average), personal achievements in relation to objectives previously agreed upon (50% on average), company turnover (7% on average), operating income (9% on average), other unspecified criteria (8% on average). Of these factors the profit of the company, the company turnover and the operational income are known for the firms of our sample. We normalise the known factors to 1 and rank the firms by their performance with regard to these weighted factors. Unfortunately we have to ignore the factors general personal performance evaluation and personal achievements in relation to objectives previously agreed upon. These factors cannot be empirically determined since they are of a sensitive nature and it is unlikely that a sufficient number of executives would share this information. In our special case of owner-manager executives, however, this is less dramatic than it might seem in the first place. Owner-managers of SMEs are unlikely to negotiate milestones for personal achievements with superiors. Also, their performance is not evaluated. Their income, i.e. the profit of the firm is the most important factor for determining their performance, since they bear the economic risk and make strategic decisions. Having ranked our sample firms in the way explained above, we match the list of firms with the density function of the variable wage component from the survey. The result is an individual variable wage for each owner-manager. Finally, we obtain the individual total human capital return by adding the fixed wage component.

5.2.3. Estimating standard CAPM betas

Following Fama & Schwert (1977) and Liberman (1980), we estimate b_j from Equation (70) (i.e. the beta of the standard CAPM) as the slope coefficient from a linear regression, where the return of firm j (ρ_j) is regressed on the market return (ρ_M). For this purpose, ρ_j is expressed as a $j \times year$ matrix and ρ_M as a vector. Since it has been

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[78] The survey differentiates between contracts with capped variable wage components and those that are linearly linked to the company's profit, much like in the case of the owner-manager of SMEs

found in the literature that the same results are achieved when testing with excess returns and unadjusted returns (c.f. Gangemi et al, 2000), we use unadjusted returns. We did not find evidence for autocorrelation, heteroscedasticity or non-normality of the error terms in our model.

As expected, the individual regressions mostly had a low R^2. The reason is twofold: firstly, we only had very few observations. And secondly, most of our sample firms were too different in size from the market portfolio firms, resulting in a low explanatory power of the variance in market portfolio returns in most cases. However, this finding is not surprising, given that the whole logic of Mayers' approach lies in improving the standard CAPM beta (b_j) by adding terms as shown in Equation (70). We can thus take the individual slope coefficients together with the error terms of each firm (e_j) and proceed to the final step by implementing Equation (70).

5.3. Empirical results

Equation 70 returns modified model betas for every firm of our sample (c.f. Appendix E). We compare the standard CAPM beta with the beta of the modified model. The difference between the two risk-measures is far below 1% for all firms[79]. Our modified models' beta is analytically equivalent to the β_j^* defined by Fama and Schwert (1977) and Liberman (1980). Both expressions are an implementation of the modified models' beta. The different notations are due to different data sets. Fama and Schwert (1977) and Liberman (1980) use aggregate labour statistics data which (not surprisingly) suffers from autocorrelation and a lack of stationarity. Our human capital data did not exhibit autocorrelation while being stationary (also not surprising since our data was calculated based on the success of related firms – a measure which is naturally less probable to show trends). Our results therefore indicate, in accordance with the aforementioned authors, that the integration of human capital for non-traded SMEs has no effect on the

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[79] The average difference between the two measures is 0.000045%. The biggest difference in the sample is 0.0022%.

relative risk of firms. When we look at the components of Equation 70 (c.f. Appendix E) it is obvious that the miniscule difference can be explained by the relative magnitude of the average book value of equity of the Entry Standard Index compared to the equity of the sample firms. In the following section, we will discuss probable theoretical explanations for this result.

6. Summary and Discussion

The objective of this Thesis was to explore existing approaches and to examine a new, promising approach for the valuation of SMEs. We began by assessing the difficulties in the valuation of such enterprises, which we saw to be due to the limited amount of public expectations focused on them. Then we examined existing valuation methods in the context of their usability for SMEs. Of these methods, the DCF method was found to be widely used and accepted for most firms. However, we identified two problems with regard to SME valuation: on the one hand it is especially difficult to forecast future cash flows. The reasons for this are their often undiversified activities and the limited amount of publicly available analyses regarding single SMEs. On the other hand, it is difficult to determine a suitable discount rate, which is a problem we address in particular. Both the APV method and the WACC method require a cost of equity estimated from the CAPM. However, in an SME setting, the CAPM cannot provide a suitable rate of return due to the non-existence of stock returns and the assumed under-diversification of investors. As we showed in Section 3.2, the second issue could not be ameliorated by Total Beta either, since it, among other reasons, exhibits critical conflicts with financial theory (e.g. misuse of the CML).

If the DCF is to be used in SME valuation the aforementioned issues have to be addressed. Given the obvious inadequacy of the Total Beta approach in this context we proposed an alternate method, which is based on CAPM. With regard to the non-existence of stock returns, we proposed to use ROEs instead. The controversiality and inevitability of using this strategy was discussed in Section 5.2.1. The more severe problem of under-diversification was dealt with by integrating the investors' human capital in an attempt to simulate a possible diversification strategy of investors. We introduced some methods of calculating human capital in Section 4.5, two of which we found theoretically suitable. However, due to the unavailability of necessary data for Germany, these approaches could not be used. Instead, we devised a method of our own that utilises management wages of firms comparable to our sample companies.

Through empirical testing, we did not find evidence for significant differences between the betas of the standard CAPM and of the modified model. Consequently, the approach of Mayers (1972) proved ineffective for our sample. We discuss possible reasons for this below. However, it must be pointed out first that with the evidence provided by Weil (1994), it can be argued that Mayers' reasoning is theoretically valid on the aggregate level of the economy; in Section 4.4, we examined how strong the correlation between human capital and firm returns would have to be (on the aggregate level of the US-economy in the second half of the 20th century) in order to be able to expect an influence on the modified model's beta. The relevant variables in this analysis were θ (the share of investor income from capital market investments), r_1 (the historic average returns of the firms themselves), r_0 (the historic average risk-free rate) and γ (the coefficient of constant relative risk aversion). Since γ is static with a value of 2 (c.f. section 4.4), we could theoretically use the relation $Er_1 - r_0 + \frac{1}{2}\sigma_{11} = \gamma[\theta\alpha\sigma_{11} + (1-\theta)\sigma_{1H}]$ to determine reasons for the non-significance of the difference between β_j and β_j^* in our sample. It would be interesting to see for which thresholds of θ, σ_{1H} and their combinations we would then theoretically obtain significant differences between β_j and β_j^*. However, we would have only been able to determine σ_{1H} for our sample had we used another methodology for calculating human capital. σ_{1H} refers to the covariance between firm and human capital *returns*. We only calculated R_h which is (according to Weil, 1994) the cash flow derived from an unknown stock of human capital. Athreya et al (2015) furnished a methodology (c.f. section 4.5.1) which allows the calculation of this stock of human capital. We are therefore left without a theoretically sound, quantifiable model for finding possible causes for our insignificant differences between β_j and β_j^*. As stated before, one potential explanation lies in the strongly differing equity values of the Entry Standard Index and our sample firms. We thus recommend the implementation of an alternate market proxy in possible future research on the findings of Mayers (1972). At least for the German market, this market proxy would have to be constructed, since there simply is no existing index more resembling of SMEs in Germany than the Entry Standard Index (in terms of equity,

turnover and employees). Such a construction would have to exploit balance sheet and income statement data of comparable SMEs. The question of how such a market proxy could be constructed exactly might also be the objective of future research.

With the results shown in the previous section, we can confirm the findings of Fama & Schwert (1977) and Liberman (1980) who found that human capital did not play a significant role in investment decisions and could thus be omitted in CAPM testing. Even though the results of the latter indicated that human capital could matter in the case of small and medium-sized enterprises, we did not find any evidence for this in our sample of small automotive firms in Germany; it seems that SME owners can, too, ignore their human capital when making their individual portfolio choices. Therefore, it can once again be concluded that Mayers' modified model does not provide additional explanatory power compared to the standard CAPM.

This finding may seem surprising, as one would intuitively argue that human capital most likely matters for owner-managers of SMEs (as did we, particularly in Section 4.5). Two explanations come to mind here: firstly, SME owner-managers might not be aware of the value of their human capital (and returns thereof) or attach little importance to it when making investment decisions. And secondly, the job market may not be (or at least perceived to be) as liquid and frictionless as we implicitly assumed by stating returns on human capital as potential earnings of SME owner-managers in the executives market. In other words, such owner-managers might believe that they could not achieve incomes comparable to those we assumed. Or, they do not even consider such an "opportunity cost" of their work because they are strongly emotionally attached to their firms. Either way, our results indicate that they ignore their human capital.

The main implication (of both a theoretical and practical nature) of our work is that the SME valuation puzzle remains unsolved: the standard CAPM cannot provide an appropriate rate of return, Total Beta is not an alternative due to its flaws and Mayers' modified model does not deliver significantly different results from the CAPM, at least not in our setting. However, much like Liberman (1980), we hypothesise that a multi-

period model (e.g. the intertemporal CAPM proposed by Merton (1973)) could deliver different results[80]. More importantly, the crucial part of Mayers' modified model – the returns on human capital – could be calculated differently, ideally by models such as those of Athreya et al (2015) or Huggett & Kaplan (2016). Future research should be devoted to exploring the performance of these models and their viability in different markets. Lastly, the model of Mayers (1972) could be analytically expanded as discussed in Section 3.3, meaning that the stake held in the SME would become an individual asset aside from human capital and market portfolio investments. All these measures may lead to results different from our own. For the time being, however, we can only make two conclusions: firstly, Mayers' modified model does not provide significantly different results than the CAPM and secondly, there is still no appropriate model for calculating the rate of return for under-diversified investors.

One final remark should be made on the term "small and medium-sized enterprises": in Section 2.1, we provided a number of definitions that varied quite strongly from country to country. The German *Mittelstand* went unmentioned, because it is not a legal term and because its scope exceeds that of the legally defined medium-sized enterprises by quite a margin: while the *Mittelstand* includes firms with up to 500 employees and a yearly turnover of EUR 50 million (IfM, 2016), the *mittelgroße Kapitalgesellschaft* defined in German law cannot exceed 250 employees and a turnover of EUR 50 million. In retrospect, we feel that the term "privately-held firm" may have been more appropriate particularly in the context of the empirical testing.

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[80] Following the argumentation of Liberman (1980) that human capital is a multiperiod commitment, it might be more appropriate to use a multiperiod CAPM, such as those proposed by Jensen (1969) and Merton (1973).

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Appendix A

$$\left[\frac{E(R_M) - R_F}{\sigma^2(R_M)}\right] = \lambda \; ; \; \lambda \sum_{j=1}^{N} X_j \, cov(R_j, R_i)$$

Show that

$$\lambda \sum_{j=1}^{N} X_j \, cov(R_j, R_i) \equiv \lambda \left\{ \beta_i \sum_{j=1}^{N} X_j \, \text{ß}_j \sigma^2(R_M) + X_i \sigma^2(\varepsilon_i) \right\} :$$

$cov(R_j, R_M) = \beta \sigma^2(R_M)$, since

$$\beta_j = \frac{cov(R_j, R_M)}{\sigma^2(R_M)} \tag{A1}$$

$$\sum_{j=1}^{N} X_j \, cov(R_j, R_i) = \beta_i \sum_{j=1}^{N} X_j \, cov(R_j, R_M) + X_i \sigma^2(\varepsilon_i) \tag{A2}$$

$$\sum_{j=1}^{N} X_j \, cov(R_j, R_M) = \sigma^2(R_M) \tag{A3}$$

$$\beta_i \sigma^2(R_M) = cov(R_j, R_M) = \sum_{j=1}^{N} X_j \, cov(R_j, R_M) \tag{A4}$$

From (A4) we could induce that

$$\lambda \sum_{j=1}^{N} X_j \, cov(R_j, R_i) \equiv \lambda \left\{ \beta_i \sum_{j=1}^{N} X_j \, \text{ß}_j \sigma^2(R_M) \right\} \tag{A5}$$

However, the correct equivalence is

$$\lambda \sum_{j=1}^{N} X_j \, cov(R_j, R_i) \equiv \lambda \left\{ \beta_i \sum_{j=1}^{N} X_j \, \text{ß}_j \sigma^2(R_M) + X_i \sigma^2(\varepsilon_i) \right\} \tag{A6}$$

Where does the term $X_i \sigma^2(\varepsilon_i)$ come from? In the case when $j=i$ in $\lambda \sum_{j=1}^{N} X_j \, cov(R_j, R_i)$, we could write

$$X_i cov(R_j, \beta_j R_M) \text{ which is equivalent to} \tag{A7}$$

$$X_i E\big[\big(R_i - E\ (R_j)\big)\big(\beta_j R_M - E\ (\beta_j R_M)\big)\big] \tag{A8}$$

$\beta_j R_M - E\ (\beta_j R_M)$ is equivalent to the error term ε_i since $E(\beta_j R_M)$ is equivalent to R_M.

It follows that 6 can be written as

$$X_i E\big[\big(R_i - E\ (R_j)\big)(\varepsilon_i)\big] \tag{A9}$$

$E(R_i - E\ (R_j))$ is also equivalent to the error term and we see that 6 reduces to

$X_i \sigma^2(\varepsilon_i)$.

Appendix B

Finding the expression

$$E(\rho_j) = r + \frac{E(\rho_m) - r}{P_M \sigma^2(\rho_M) + cov\,(\rho_M, R_H)} \times \left[P_M cov\,(\rho_j, \rho_M) + cov\,(\rho_j, R_H) \right] \quad \text{(B1)}$$

based on the relationship

$$\frac{\left[\sum_k x_{ik} \sigma_{jk} + cov\left(R_i^H, R_j\right) \right]}{E\left(R_j\right) - rP_j} = \frac{\left[\sum_k x_{ik} \sigma_{tk} + cov\left(R_i^H, R_t\right) \right]}{E(R_t) - rP_t} \quad \text{(B2)}$$

$\sum_i X_{ij} = 1$, for all assets are held in market equilibrium. The marginal rate of substitution between risk and return is the same for all assets and this equality is not altered if B2 is summed over all investors (then representing the market equilibrium).

$$\frac{\left[\sum_k \sigma_{jk} + \sum_i cov\left(R_i^H, R_j\right) \right]}{E\left(R_j\right) - rP_j} = \frac{\left[\sum_k \sigma_{tk} + \sum_i cov\left(R_i^H, R_t\right) \right]}{E(R_t) - rP_t} \quad \text{(B3)}$$

The MRS between return and risk will be the same for the market as for the individual asset. It is interesting to see that already here it becomes clear how the equilibrium is influenced by $\sum_i cov\,(R_i^H, R_t)$ which reflects the covariation of the single assets return with the aggregate human capital income of all investors. One should not be misled by the fact that the MRS between return and risk will be the same for the market as for every individual asset. Considering that R_i^H is different for each investor, we cannot conclude that an identical MRS as described above leads to identical market portfolios. Albeit the MRS between risk and return is identical for all investors in equilibrium, they all have different human capital incomes at the end of the period. With the assumed rationality of the investor in mind, intuition lets us expect individually different portfolios in equilibrium already at this point. When the MRS between risk and return is fixed for all investors and when every investor a priori bears a certain amount of risk from their human capital income, then it is intuitive to assume that the individual portfolio will reflect the risk born a priori. We will see later in this section that this is indeed true.

Equation B1 can be solved for P_j:

$$P_j = \frac{1}{r}\left\{E(R_j) - \left[\frac{E(R_t) - rP_t}{\sum_k \sigma_{tk} + \sum_i cov\,(R_i^H, R_t)}\right] \times \left[\sum_k \sigma_{jk} + \sum_i cov\,(R_i^H, R_j)\right]\right\} \quad \text{(B4)}$$

The term $\frac{E(R_t) - rP_t}{\sum_k \sigma_{tk} + \sum_i cov\,(R_i^H, R_t)}$ is the marginal rate of substitution between return and risk.

It can be summed over all t without changing its value as can be done with the terms in Equation B1. In doing so, one should consider the following relations:

$$\sum_t E(R_t) = E(R_M) \tag{B5}$$

$$\sum_t P_t = P_M \tag{B6}$$

$$\sigma^2(R_M) = \sum_t \sum_k \sigma_{tk} \tag{B7}$$

$$cov\,(R_H, R_M) = \sum_t \sum_i cov\,(R_i^H, R_t) \tag{B8}$$

$$cov\,(R_j, R_M) = \sum_k \sigma_{jk} \tag{B9}$$

$$cov\,(R_j, R_H) = \sum_i cov\,(R_i^H, R_j) \tag{B10}$$

Applying these relations, Equation B1 can be expressed as

$$P_j = \frac{1}{r}\left\{E(R_j) - \left[\frac{E(R_M) - rP_M}{\sigma^2(R_M) + cov\,(R_H, R_M)}\right] \times \left[cov\,(R_j, R_M) + cov\,(R_j, R_H)\right]\right\}. \quad \text{(B11)}$$

For a better comparability with the classic notation of the result of the standard CAPM, Equation B10 will be transformed in an expression for the expected return of asset j. Before this is done, the terms in Equation B10 deserve some scrutiny. Especially $\frac{E(R_M) - rP_M}{\sigma^2(R_M) + cov\,(R_H, R_M)}$ is important; it represents the price an investor is willing to pay for a unit of risk. The systematic risk of the single traded asset is now $cov\,(R_j, R_M) + cov\,(R_j, R_H)$. We learn that the value of the firm is now determined by (i) the covariation of the firm's returns with the returns of all other firms and (ii) with the covariation of the single firm's returns with the total income of investors from non-marketable assets.

Exploiting the following equalities will enable us to transform Equation B10 into an expression for the expected return of asset j.

$$P_j = \frac{1}{r}\left\{E(R_j) - \left[\frac{E(R_M) - rP_M}{\sigma^2(R_M) + cov\,(R_H, R_M)}\right] \times \left[cov\,(R_j, R_M) + cov\,(R_j, R_H)\right]\right\}. \tag{B12}$$

$$P_j = \frac{1}{r}\left\{E(R_j) - \left[\frac{P_M E((\rho_M) - r)}{P_M(P_M \sigma^2(\rho_M) + cov\,(R_H, \rho_M))}\right] \times \left[P_M cov\,(R_j, \rho_M) + cov\,(R_j, R_H)\right]\right\}. \tag{B13}$$

Multiply equation B13 with $\frac{r}{(P_j)}$:

$$r = \left\{E(\rho_j) - \left[\frac{P_M E((\rho_M) - r)}{P_M(P_M \sigma^2(\rho_M) + cov\,(R_H, \rho_M))}\right] \times \left[P_M cov\,(\rho_j, \rho_M) + cov\,(\rho_j, R_H)\right]\right\}. \tag{B14}$$

$$r = \left\{E(\rho_j) - \left[\frac{P_M E((\rho_M) - r)}{P_M(P_M \sigma^2(\rho_M) + cov\,(R_H, \rho_M))}\right] \times \left[P_M cov\,(\rho_j, \rho_M) + cov\,(\rho_j, R_H)\right]\right\}. \tag{B15}$$

$$E(\rho_j) = r + \left[\frac{E(\rho_M) - r}{\sigma^2(\rho_M) + cov\,(\rho_M, R_H)}\right] \times \left[P_M cov\,(\rho_j, \rho_M) + cov\,(\rho_j, R_H)\right] \tag{B16}$$

Appendix C

We claimed that when differentiating $\frac{\partial \mu}{\partial V} = -\frac{1}{2}\frac{E[u''(w)]}{E[u'(w)]}$ with respect to μ and V one obtains the direction of change of the slope of the investors' indifference curves while investors adapt their portfolios to the existence of non-marketable assets. In other words, we claimed to obtain the change in the market price per unit of risk when extending the market. To comprehend this, one has to look at $-\frac{1}{2}\frac{E[u''(w)]}{E[u'(w)]}$ which has the dimension of the Arrow-Pratt measure of absolute risk aversion. This measure can be derived by simulating the very scenario an investor faces when a market is suddenly extended by previously non-marketable assets. It should be noted here that a market extension where all assets become marketable (also referred to as de-segmentation of the market in the literature) is strictly analogous to the case where non-marketable assets become part of the investors considerations in portfolio allocation as analysed by Mayers in 1972 (c.f. Mayers, 1976). Thence, when we speak of market extension, we also refer to the case described by Mayers' modified model. When a market is extended in the sense that minor markets that previously have been open to only a limited number of persons (in the modified model of Mayers (1972) this number is one since only the investors themselves have access to their own human capital) the investors face the question how much they would be willing to pay for the elimination of the zero-mean risk G. This question is expressed analytically as $E[u(W + G)] = u(W + E(G) - \pi)$ where W is the previously non-marketable wealth (human capital would be a perfect fit here, for it can be seen as innate and therefore immutable) and $\tilde{G}$ the zero-mean risk of the major market to which access has never been restricted. π is the risk premium or the market price per unit of risk. When this equation is solved for π by applying Taylor's series approximation one obtains the Arrow-Pratt measure of absolute risk aversion (Arrow, 1965).

Appendix D

Mayers (1976) shows analytically that $\frac{\partial}{\partial V}\frac{\partial \mu}{\partial V}$ is strictly positive when a constant relative

risk aversion coefficient hovering around 1 is assumed for the average investor. $\frac{\partial}{\partial V}\frac{\partial \mu}{\partial V}$

can be written as follows:

$$- \frac{E\left[U'(w)\right]\left[U''(w)\,w\,(w-\bar{w}) + E\left[U'(w)\right]E\left[U'(w)\,(w-\bar{w})\right] - E\left[U'(w)w\right]E\left[U''(w)\,(w-\bar{w})\right]\right]}{(2\,\sigma_w^2\,E[U'(w)])^2} \quad (D1)$$

By inserting the term $r^* = \frac{U''(w)w}{U'(w)}$, which is the constant relative risk aversion, into the

numerator, Equation D1 becomes positive when r^* is positive and equal to or less than

one, since all other signs of the components of the numerator apart from r^* are known.

The resulting term is

$$- E\left[U'(w)\right]\left[U'(w)\,r^*\,w(w-\bar{w}) + E\left[U'(w)\right]E\left[U'(w)\,(w-\bar{w})\right] - E\left[U'(w)\,w\right]E\left[U''(w)\,(w-\bar{w})\right]\right] \quad (D2)$$

It shall not be shown here how Equation D2 is determined as positive with the

assumption stated above. The reader is advised to refer to Mayers (1976: 10) directly.

Appendix E

$$\beta_j \cong b_j + \left[\frac{cov\ (e_j, R_H)}{P_M \sigma^2(\rho_M) + cov\ (\rho_M, R_H)}\right]$$

Firm	Beta Final	Standard CAPM Beta	$cov\ (e_j, R_H)$	$cov\ (\rho_M, R_H)$	Average Firm Equity
1	0,04231624	0,04231575	11,73	160,72	3.941.668,83
2	-0,02204982	-0,02207729	653,60	-162,30	7.552.606,50
3	0,80854744	0,80852710	484,04	877,71	1.771.352,33
4	0,10326442	0,10322340	976,13	290,57	7.686.927,50
5	-0,04489002	-0,04489862	204,74	664,48	3.315.126,00
6	0,08091487	0,08089211	541,61	305,02	1.210.608,79
7	-0,04097526	-0,04097715	45,01	-681,94	2.853.730,35
8	-0,08540635	-0,08542242	382,45	401,24	3.610.723,67
9	-3,10393576	-3,10414000	4.860,17	7,43	2.197.930,89
10	-0,05877819	-0,05880640	671,23	304,34	2.088.151,00
11	1,17882857	1,17882384	112,61	259,83	2.296.760,09
12	-0,31186632	-0,31189570	698,99	-910,53	5.281.217,01
13	0,25623442	0,25617817	1.338,72	1.425,15	2.547.797,00
14	0,17560014	0,17560084	-16,67	669,29	3.490.626,17
15	1,14043729	1,14038900	1.149,14	1.981,34	3.599.564,00
16	-0,37534643	-0,37541940	1.736,41	-679,38	10.995.000,00
17	-0,44253555	-0,44254040	115,35	-322,71	3.479.315,33
18	3,76233442	3,76211340	5.259,52	816,29	-267.392,33
19	8,24890067	8,24869900	4.799,06	656,14	383.817,61
20	0,09243873	0,09220923	5.461,25	511,91	3.312.476,67
21	-6,76010805	-6,76039300	6.780,65	-548,74	678.948,50
22	-0,07380891	-0,07420166	9.346,06	420,10	705.820,00
23	2,97118046	2,97117100	225,22	1.448,68	2.079.289,17
24	0,27391123	0,27391380	-61,19	-318,69	601.184,90
25	0,00466537	0,00466572	-8,20	236,76	5.108.787,00
26	0,01128967	0,01129371	-96,09	569,62	1.786.139,96
27	0,67376220	0,67376130	21,53	6,40	1.727.330,51
28	0,01392411	0,01390869	367,02	842,65	3.641.903,50
29	-0,08086847	-0,08086713	-31,88	43,11	3.514.373,50
30	-1,65554633	-1,65559763	1.220,71	-1.548,78	3.389.921,17
31	0,99293617	0,99292490	268,26	1.468,42	4.129.284,62
32	-0,36954233	-0,36953770	-110,13	-693,61	2.753.132,23
33	0,05821784	0,05819303	590,46	444,37	6.052.060,00
34	-0,82981099	-0,82981860	181,01	-797,91	6.347.016,00

35	-0,13143688	-0,13147257	849,32	-503,24	2.058.490,17
36	45,57589985	45,56816400	184.085,63	291,59	-195.038,47
37	-0,12371262	-0,12376233	1.182,83	-525,61	2.210.958,17
38	-1,59113958	-1,59121552	1.807,12	-438,42	2.513.173,50
39	0,78080564	0,78077342	766,70	804,58	2.816.802,33
40	14,70745592	14,70722518	5.490,89	410,85	22.665,83
41	-1,08545690	-1,08549730	961,39	-311,83	2.099.425,73
42	-0,27234569	-0,27235183	146,14	-315,02	2.170.355,31
43	0,54296115	0,54291600	1.074,31	-538,63	806.914,12
44	0,51233250	0,51233670	-99,84	-172,64	2.609.801,83
45	0,03001499	0,03001488	2,57	297,69	6.159.189,83
46	-0,69421314	-0,69508690	20.792,95	1.091,49	423.966,33
47	0,25113869	0,25112266	381,49	599,78	6.195.882,50
48	-0,03242929	-0,03242843	-20,47	231,75	3.071.614,33
49	-0,62859628	-0,62863090	823,91	313,31	9.673.511,67
50	0,41078121	0,41076020	500,06	206,04	1.180.398,83
51	-0,94967086	-0,94966190	-213,30	-15,56	21.810.280,86
52	-0,36722125	-0,36721390	-174,80	-753,75	2.565.540,84
53	0,02385394	0,02385463	-16,45	-235,06	1.370.231,00
54	-0,08554774	-0,08567063	2.924,31	45,03	7.168.215,67
55	0,01623539	0,01621900	390,03	27,42	2.929.809,83
56	0,44346430	0,44346650	-52,41	71,02	2.718.534,17
57	-0,21172852	-0,21173030	42,30	-35,85	3.860.216,50
58	0,50904367	0,50902650	408,57	19,79	2.573.951,17
59	-0,05901514	-0,05902145	150,19	454,32	5.111.024,33
60	-0,35300545	-0,35300500	-10,74	499,21	2.874.096,17
61	0,74822925	0,74805433	4.162,67	1.325,36	1.488.644,50
62	0,32804334	0,32803040	307,96	1.061,58	6.390.013,00
63	-0,38838759	-0,38840658	451,81	1.231,13	2.133.896,83
64	0,84375621	0,84375580	9,72	1.247,64	4.566.436,00
65	0,16204489	0,16204940	-107,39	689,60	6.730.962,33
66	-0,29699796	-0,29701524	411,08	-58,41	3.571.074,50
67	-1,72449302	-1,72448863	-104,51	472,60	1.853.408,67
68	-23,90890166	-23,90936000	10.906,42	-721,32	52.674,00
69	0,05071355	0,05071513	-37,55	361,47	7.876.949,33
70	0,34381215	0,34380130	258,18	499,20	2.063.383,67
71	-1,58469463	-1,58490200	4.934,58	-386,49	1.243.567,33
72	3,51329953	3,51328870	257,81	1.546,22	2.589.318,83
73	0,15171521	0,15170690	197,66	135,54	2.941.139,83
74	-0,88943994	-0,88916840	-6.461,72	369,73	465.503,67
75	-0,47653973	-0,47661260	1.733,97	-216,52	11.075.688,67

76	0,37790033	0,37789399	150,85	1.008,80	8.108.483,83
77	0,08958302	0,08958484	-43,39	-314,63	2.417.697,83
78	0,17622330	0,17622644	-74,77	545,45	7.559.960,17
79	10,89210845	10,89289000	-18.599,47	2.155,20	915.580,17
80	3,60024755	3,60025411	-156,18	1.462,68	2.091.987,83
81	0,14295693	0,14291970	885,95	365,54	11.564.175,17
82	0,03677462	0,03677296	39,41	731,28	8.948.140,17
83	0,24925897	0,24924310	377,54	365,17	4.852.897,67
84	-6,31427041	-6,31736670	73.680,88	351,90	-430.143,17
85	-2,36072518	-2,36074180	395,58	-989,73	4.547.384,83
86	0,15954006	0,15958080	-969,51	332,18	852.457,17
87	-0,31337224	-0,31337573	83,01	-535,22	1.872.951,33
88	0,19249056	0,19247500	370,27	27,61	6.838.876,17
89	-0,23873075	-0,23870670	-572,39	78,01	2.429.737,67
90	0,17307187	0,17308406	-290,04	789,86	2.267.080,00
91	0,54668374	0,54666050	553,06	1.215,39	2.422.783,33
92	1,13924997	1,13924810	44,43	1.488,02	2.381.244,00
93	-0,41218729	-0,41218330	-94,88	121,63	1.802.258,00
94	0,31346573	0,31346801	-54,26	1.182,95	3.407.820,33
95	-0,02149180	-0,02151001	433,38	826,63	7.988.754,50
96	-0,19850903	-0,19859020	1.931,47	-115,23	8.754.938,00
97	-0,21327631	-0,21326935	-165,62	140,91	3.202.638,33
98	-0,15888217	-0,15888143	-17,66	-330,04	8.943.770,83
99	0,74642790	0,74643400	-145,20	309,21	8.306.729,00

Variance ρ_M	P_M per Year	
0,011870969	2009	506.539.126,00
	2010	588.296.846,00
	2011	677.269.936,00
	2012	1.167.805.239,00
	2013	3.166.508.096,00
	2014	5.920.971.863,00
	Average P_M	2.004.565.184,33

Abstract (English)

At present there is no standard valuation method for small and medium-sized enterprises (SMEs). This Thesis explores a possible valuation technique for such firms. A controversially debated approach to valuing small firms is the Butler Pinkerton Model (BPM) which is based on the "Total Beta". In line with extant literature, we find this approach inappropriate for its intended purpose. Hence, we devised a different method drawing upon the "modified model" of Mayers (1972) which incorporates the human capital of investors. Since the modified model is an analytical extension of the CAPM, we thoroughly discuss the latter mainly with regard to its shortcomings and in the context of SME valuation. We find that despite the flaws and the fact that the standard CAPM cannot provide an appropriate rate of return for under-diversified investors in SMEs, CAPM-based valuation approaches are the most suitable for SME valuation. The central component of Mayers' "modified model", i.e. the investors' human capital, is difficult to operationalise due to its subjective nature. In this Thesis, a proxy based on individual management wages is used to represent returns on human capital. The unavailability of CAPM returns for our sample of unlisted German SMEs is overcome by using returns on equity (ROE) of both the sample firms and the benchmark firms from the German Entry Standard Index. We found insignificant differences between the standard CAPM beta and the modified model's beta. Our approach could be used in future research with a constructed market proxy which would have to be more resembling of our sample SME's. In such a setting we speculate that significant differences in betas may exist. For the moment, however, we conclude that SME investors ignore their human capital when making investment decisions.

Abstract (Deutsch)

Zur Zeit gibt es keine standardisierte Methode zur Bewertung von kleinen und mittleren Unternehmen (KMU). Die vorliegende Arbeit untersucht eine mögliche Bewertungsmethode für KMU. Ein kontrovers diskutierter Ansatz zur Bewertung solcher Unternehmen ist das Butler Pinkerton Model (BPM), welches auf dem „Total-Beta-Konzept" basiert. In Übereinstimmung mit der Literatur finden wir diesen Ansatz ungeeignet für sein angestrebtes Ziel. Daher entwickeln wir in Anlehnung an das „modifizierte Modell" Mayers' (1972) eine andere Methode, welche das Humankapital der Investoren berücksichtigt. Da das modifizierte Modell eine analytische Erweiterung des CAPM ist, diskutieren wir Letzteres vor allem in Hinsicht auf seine Mängel und im Kontext der KMU-Bewertung. Trotz seiner Defizite und der Unfähigkeit des CAPM, eine geeignete Rendite für unterdiversifizierte Investoren zu liefern, stellen wir fest, dass CAPM-basierte Ansätze im Rahmen der KMU Bewertung am besten geeignet sind. Die wesentliche Komponente des modifizierten Modells – das Humankapital der Investoren – ist aufgrund seiner Subjektivität schwer zu operationalisieren. In dieser Arbeit verwenden wir einen auf Managementgehältern basierenden Proxy zur Darstellung des Humankapitalertrages. Ein weiteres Problem ist die Nichtexistenz von in CAPM-Berechnungen verwendeten Kapitalmarktrenditen bei unserer Stichprobe nichtgelisteter deutscher KMU. Dies kompensieren wir durch die Verwendung der Eigenkapitalrentabilitäten (ROE) für Unternehmen unserer Stichprobe als auch für die Firmen des Entry Standard Index, welcher unseren Marktportfolioproxy darstellt. Wir fanden vernachlässigbare Differenzen zwischen den Standard-CAPM-Betas und den Betas des modifizierten Modells. Unser Ansatz könnte in zukünftiger Forschung mit einem konstruierten Marktproxy verwendet werden. Wir mutmaßen, dass bei einer derartig geänderten Vorgehensweise signifikante Unterschiede zwischen den Betas gefunden werden könnten. Gegenwärtig schlussfolgern wir jedoch, dass Investoren bei KMU-Anlageentscheidungen ihr Humankapital ignorieren.