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„Overcoming the Divine Coincidence in a baseline New Keynesian Model"

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1 Introduction

Dynamic stochastic equilibrium (DSGE) models have become the new paradigm for the analysis of business cycles. Among those, New Keynesian models are especially popular. They provide both a description of the transmission mechanism of monetary policy and a framework for an evaluation of that policy.

New Keynesian models have their origin in the Real Business Cycle (RBC) and the first wave of New Keynesian research. Both research movements emerged during the 1980's, subsequently to Lucas's critique of macroeconometric policy. Lucas pointed out that models that do not explicitly describe the decisions of all economic agents should not be used for policy evaluation as any change in policy would also systematically alter the structure of these models (Lucas, 1976, p. 41). In the spirit of the critique, both branches tried to underpin macroeconomic theory with microfoundations, that is, with a microeconomic analysis of rational economic agents' behavior.

The first wave of New Keynesian economics emphasized the role of price and wage stickiness and the resulting non-neutrality of monetary policy (see e.g. Akerlof and Yellen (1985)). New Keynesian economists aimed on the inclusion of everyday observed pricing or wage setting practices and investigated how these could account for the aggregate price level and real economic activity (Goodfriend and King, 1997, p. 250). The resulting models, however, were often static and not derived from explicit dynamic optimization problems that firms and households were facing (Galí, 2008, p. 5).

By contrast, RBC theory established the analysis of aggregate macroeconomic variables derived from individual intertemporal optimization problems in a stochastic environment (see e.g. Prescott (1986)). Both households' and firms' decisions were explicitly modeled and combined into a general equilibrium where quantities and prices are simultaneously determined. Under the assumption that the underlying decision calculus remained unchanged, the RBC framework made it possible to compare alternative policies on the basis of their effects on economic welfare. Additionally, a strong emphasis was put on the investigation of the effects of shocks on the business cycle and on the quantitative aspects of macroeconomic modeling, as reflected in the central role of calibration, simulation and evaluation of the models (Galí, 2008, p. 2). However, RBC models abstracted from monetary and financial factors and treated monetary policy as mostly neutral. This approach prevented the application of RBC models by central banks. Furthermore, it is in stark contrast to the widely held belief that monetary authorities are able to influence output or employment developments at least in the short run (Galí, 2008, p. 5). Not for nothing, decisions by the FED ore ECB are closely monitored by all economic agents.[1]

To tackle these shortcomings, New Keynesian models emerged as a confluence of both movements. Goodfriend and King (1997) call this the "New Neoclassical Synthesis". New Keynesian models adopt the RBC methodology of rational actors facing an intertemporal optimization problem and combine it with assumptions about nominal rigidities and monopolistic competition. The resulting framework is characterized by monetary non-neutrality, so that monetary policy can be analyzed, and explicit microfoundations allowing an evaluation of alternative policies without being subject to the Lucas critique. For these reasons, New Keynesian models are appealing for both academic research and central banks.

A great variety of models can be found in the literature ranging from small-scale to high-scale versions. Many of them have at their core a baseline model consisting of Calvo price staggering on the supply side, an Euler equation on the demand side and a Taylor rule that describes the policy of the monetary authority. Despite its appealing features mentioned above, the baseline model is subject to some controversy. The main reason is the New Keynesian Phillips curve (NKPC), a central element of the baseline model, which explains inflation as caused by expected inflation and the output gap. In the baseline model, the NKPC has the following representation:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa (y_t - y_t^f) \tag{1.1}$$

where π_t is inflation, y_t is (log) output, y_t^f is (log) natural output and $(y_t - y_t^f)$ is the output gap. Equation (1.1) implies a stable relationship between inflation

[1] However, it is difficult to estimate the degree of non-neutrality of monetary policy. Usually a central bank adjusts the interest rate as reaction to some economic development. A simple correlation analysis between interest rate and other economic variables like GDP is therefore not possible. See Galí (2008, p. 7) for a discussion.

and output. For the case of zero inflation, output would even remain equal to its natural level. In other words, there exists no trade-off between the stabilization of output and inflation. Therefore, a central bank can kill two birds with one stone: By fully stabilizing the price level, the output remains stable around its natural level. Blanchard and Galí (2007, p. 2) call this exceptional property the "divine coincidence".[2]

In practice, however, central banks face significant trade-offs between output and inflation stabilization (Galí, 2008, p. 96). They do not pursue a policy of strict inflation targeting but attach some weight to output gap fluctuations. It is therefore desirable to modify the baseline New Keynesian model in a way, so that the divine coincidence disappears and a meaningful trade-off between inflation and output stabilization emerges. This master thesis introduces three extension to the baseline model that aim on the overcoming of the divine coincidence. In particular, the NKPC is expanded by an additional distortion shock and two different staggered wage setting mechanisms are implemented: real wage and nominal wage rigidities. In the model comparison part of this thesis, it is investigated, if additional wage setting mechanisms improve the empirical fit of the baseline model and which of these mechanisms closer resembles the observed data dynamics. The answer to these questions can be helpful for the design of more complex New Keynesian models. For the purpose of model comparison, a Bayesian estimation is conducted and the Bayes factor is employed to quantify the empirical fit of the models.

The structure of the thesis is as follows: In section 2 the New Keynesian baseline model is introduced. In section 3 the divine coincidence is discussed and the extensions to the baseline model are introduced. In section 4 the models are estimated and the results are discussed. Section 5 concludes.

[2]The concept of the divine coincidence additionally requires a stable relation between output gap and *welfare-relevant* output gap, so that a stabilization of the output gap is desirable. See chapter 3 of this thesis for further details.

2 The Baseline New Keynesian Model

This section introduces the baseline New Keynesian model. It corresponds to the model by Galí (2008) and is modified such that the equilibrium conditions coincide with the model by Rabanal and Rubio-Ramirez (2005).[3] All mathematical derivations are done in the appendix and both main sources are not explicitly cited.

2.1 General assumptions

The baseline model relies on some simplifying assumptions. First, a cashless economy is assumed. Money can be thought as a unit of account in which prices, wages, and bond payoffs are stated but not as an asset. Therefore, households cannot gain utility from holding money and money does not show up in the households' utility function or budget constraint. Second, there is no capital and no investment. The whole output has to be consumed by households, i.e. the market clearing condition $Y_t = C_t$ holds at any time. Third, fiscal policy does not play a role in the economy.

2.2 Households

The economy is populated by a unit interval of identical and infinitely-lived households.[4] The households supply labor to final good producing firms, receive income and use the income to buy a consumption good. Income that is not consumed is saved by holding one-period government bonds. The households derive utility from consumption and disutility from working. The utility function of the households is assumed to be time separable and separable between consumption and labor and is

[3]The only difference is an additional preference shifter shock in the households' utility function. The additional shock is needed during the estimation procedure. See section 4.1.

[4]The assumption of a continuum $[0, 1]$ of identical households implies that per-household variables coincide with aggregate variables and the concept of a representative household can be used.

parametrized as

$$U(C_t, L_t) = \begin{cases} G_t \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\phi}}{1+\phi} & \text{for } \sigma \neq 1 \\ \\ G_t \log C_t - \frac{L_t^{1+\phi}}{1+\phi} & \text{for } \sigma = 1, \end{cases} \tag{2.1}$$

where C_t is consumption, L_t is the households' labor supply, G_t a stochastic preference factor, $\sigma > 0$ the inverse elasticity of intertemporal substitution and $\phi > 0$ the inverse elasticity of labor supply with respect to real wages.

The representative household seeks to maximize the expected infinite stream of utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t),$$

where β is the discount or preference factor that satisfies $0 < \beta < 1$. The representative household faces the the budget constraint

$$P_t C_t + B_t \leq B_{t-1}(1 + i_{t-1}) + W_t L_t + D_t.$$

Consumption at a price P_t and bond holdings B_t cannot exceed income received either from last period bond holdings that pay a nominal interest rate i_t, nominal labor income $W_t L_t$ or nominal profits D_t.[5] In each period t, the household maximizes its stream of utility with respect to consumption, labor supply and bond holdings subject to the budget constraint. This yields the following first-order conditions (see appendix A.1.1):

$$\frac{W_t}{P_t} = \frac{L_t^{\phi}}{G_t C_t^{-\sigma}} = -\frac{U_L}{U_C} \equiv MRS_t \tag{2.2}$$

$$G_t \frac{C_t^{-\sigma}}{P_t} = \beta(1 + i_t) E_t \left(G_{t+1} \frac{C_{t+1}^{-\sigma}}{P_{t+1}} \right). \tag{2.3}$$

Equation (2.2) is the labor supply equation linking the marginal rate of substitution between leisure and consumption to the real wage. Equation (2.3) is the

[5]Nominal profits show up in the budget constraint as the households are the owners of firms with monopoly power.

Euler equation representing a condition for the optimal allocation of intertemporal consumption. The preference shifter G_t has an influence on both intra- and intertemporal household choices.

Denoting logarithms by lower-case letters, the labor supply equation can be approximated by a Taylor expansion around the steady state as:

$$w_t - p_t = \phi l_t - g_t + \sigma c_t = mrs_t \tag{2.4}$$

and the Euler equation can be log-linearized around a steady state with constant inflation and consumption growth as (see appendix A.1.1):

$$c_t = E_t c_{t+1} - (1/\sigma)(i_t - E_t \pi_{t+1} + E_t g_{t+1} - g_t - \rho), \tag{2.5}$$

where $\rho = -\log(\beta)$ and $\pi_t = p_t - p_{t-1}$ denotes the inflation rate between period t and $t-1$.

2.3 Firms

There are two different kinds of firms in the economy: Final and intermediate good producing firms. The final consumption good Y_t is produced by fully competitive firms which use a range $[0, 1]$ of intermediate goods $Y_t(i)$ as input. Following Erceg et al. (2000), it is assumed that the final good producers aggregate intermediate goods in the same proportions as the households would choose. Formally, these firms use the following constant elasticity of substitution and constant returns to scale production function à la Dixit and Stiglitz (1977):

$$Y_t = \left[\int_0^1 Y_t(i)^{(\eta_p - 1)/\eta_p} \mathrm{d}i \right]^{\eta_p/(\eta_p - 1)},$$

where $\eta_p > 1$ is the elasticity of substitution between any two intermediate goods. Final good producers take the price $P_t(i)$ for the good $Y_t(i)$ as given and produce output at minimal cost. As shown in appendix A.1.2, this implies an aggregated price index

$$P_t = \left[\int_0^1 P_t(i)^{1-\eta_p}\mathrm{d}i\right]^{1/(1-\eta_p)} \tag{2.6}$$

and the demand function for intermediate good $Y_t(i)$ produced by intermediate firm i takes the form

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\eta_p} Y_t. \tag{2.7}$$

The demand for a differentiated good depends positively on total production and negatively on its relative price.

The intermediate goods producers have monopoly power and use labor supplied by the households as only production input. Each intermediate firm i produces a differentiated good $Y_t(i)$ with the same decreasing returns to scale production function

$$Y_t(i) = A_t L_t(i)^{1-\alpha}, \tag{2.8}$$

where A_t is the random productivity of labor in period t, assumed to be common to all firms. Cost minimization at a given wage implies the nominal marginal cost of each intermediate firm (see appendix A.1.3):

$$MC_t^n(i) = \frac{1}{(1-\alpha)}W_t\left(\frac{1}{A_t}\right)^{1/(1-\alpha)}\left[\left(\frac{P_t(i)}{P_t}\right)^{-\eta_p} Y_t\right]^{\alpha/(1-\alpha)}. \tag{2.9}$$

Due to the assumption of decreasing returns to scale, each intermediate firm exhibits different marginal cost depending on the charged price $P_t(i)$. For $\alpha = 0$ this expression simplifies considerably and marginal cost are equal across all intermediate firms.

The individual firm's marginal cost function can also be expressed by the economy's average marginal cost.[6] As labor is the only production input, the latter is defined by

$$MC_t^n = \frac{W_t}{MPL_t} = \frac{W_t}{(1-\alpha)A_t}\left(\frac{Y_t}{A_t}\right)^{\alpha/(1-\alpha)} \tag{2.10}$$

[6] The calculation of average marginal cost already needs the insight that the average production function is $Y_t = A_t L_t^{1-\alpha}$. This is shown in section 2.4. The marginal product of labor is then $MPL_t = A_t(1-\alpha)L_t^{-\alpha}$.

where MPL_t is the marginal product of labor. Equation (2.9) can therefore be written as

$$MC_t^n(i) = MC_t^n \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\eta_p \alpha}{1-\alpha}}. \tag{2.11}$$

In order to introduce price rigidities, a staggered price setting mechanism à la Calvo is used (Calvo, 1983). It is assumed that only a fraction $(1-\theta_p)$ of the intermediate producers can adjust their prices in each period. This assumption implies that a firm which recently adjusted its price cannot change it with probability θ_p^k within the next k periods. When a firm is allowed to reset its price in period t, it faces the following dynamic optimization problem:

$$\begin{aligned}
\max_{P_t(i)} \quad & \sum_{k=0}^{\infty} \theta_p^k E_t \left[Q_{t,t+k} \left(P_t(i) Y_{t+k|t}(i) - \Psi_{t+k}(Y_{t+k|t}(i))\right)\right] \\
\text{s.t.} \quad & Y_{t+k|t}(i) = \left(\frac{P_t(i)}{P_{t+k}}\right)^{-\eta_p} Y_{t+k}, \quad k = 0, 1, 2, ...
\end{aligned}$$

Here, $Q_{t,t+k} = \beta^k (G_{t+k}/G_t)(C_{t+k}/C_t)^{-\sigma}(P_t/P_{t+k})$ is the stochastic discount factor obtained from the household's Euler equation, $\Psi_t(\cdot)$ is the firm's cost function and $Y_{t+k|t}(i)$ is output of firm i that last set its price in period t. A firm that resets its price in period t takes expected future levels of consumption and prices and the probability θ_p^k, that it may not reset the price in the next k periods, into account when it chooses $P_t(i)$. As all price-resetting firms act identically, they choose the same price P_t^* and the index i is dropped in the following equations. The first order condition of the maximization problem takes the form (see appendix A.1.4):

$$\sum_{k=0}^{\infty} \theta_p^k E_t[Q_{t,t+k} Y_{t+k|t}(P_t^* - \mathcal{M}_p MC_{t+k|t}^n)] = 0, \tag{2.12}$$

where $\mathcal{M}_p \equiv \eta_p/(\eta_p - 1)$ and

$$MC_{t+k|t}^n = MC_{t+k}^n \left(P_t^*/P_{t+k}\right)^{\frac{-\eta_p \alpha}{(1-\alpha)}} \tag{2.13}$$

denotes the nominal marginal cost of a firm in period $t+k$ which last reset its price in period t. This expression follows directly from equation (2.11).

For the limiting case that all firms are allowed to reset their prices, i.e. $\theta_p = 0$, equation (2.12) reduces to

$$P_t^* = \mathcal{M}_p MC_t^n. \tag{2.14}$$

Due to the monopoly power, a firm would charge a constant markup $\mathcal{M}_p$ on its nominal marginal cost if price rigidities were absent. Galí (2008) calls $\mathcal{M}_p$ the "desired or frictionless markup" on marginal cost. The markup is constant given the assumption of a time invariant elasticity of substitution η_p. As a consequence of monopoly power, production will be inefficiently low.

Next, equation (2.12) is log-linearized around the non-stochastic zero inflation steady state which is characterized by constancy of prices. Therefore, $P_t^*/P_{t-k} = P_t^*/P_t = P_t^*/P_{t+k} = 1$ implying $Y_{t+k|t} = Y_{t+k} = Y$. Consequently, $C_{t+k} = C$ due to market clearing and $Q_{t,t+k} = \beta^k$. Furthermore, for real marginal cost, it holds that $MC^r_{t+k|t} = MC^r_{t+k} = MC^r = \frac{1}{\mathcal{M}_p}$, as all intermediate firms choose the same price and produce the same amount of output. As shown in appendix A.1.5, a first order Taylor expansion around that steady state yields:

$$p_t^* = \mu^p + (1-\theta_p\beta)E_t\sum_{k=0}^{\infty}\theta_p^k\beta^k(mc^r_{t+k|t} + p_{t+k}). \tag{2.15}$$

Reoptimizing firms choose a price corresponding to the logarithm of the desired markup $\mu^p = \log\mathcal{M}_p$ plus a weighted sum of their current and expected nominal marginal cost. In other words, firms adjust their prices according to current and future discounted cost conditions.

2.4 Aggregated conditions

For the derivation of the symmetric equilibrium two expressions for aggregated prices and the relationship between aggregated output and aggregated labor are needed (the algebra is done in appendix A.1.6). First, the aggregate price dynamics in the model economy can be written as:

$$\Pi_t^{1-\eta_p} = \theta_p + (1-\theta_p)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\eta_p}, \tag{2.16}$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation between period $t-1$ and t. For the case of zero inflation, i.e. $\Pi_t = 1$, it follows that $P_t^* = P_{t-1} = P_t$. This equation can be log-linearized around that zero inflation steady state:

$$\pi_t = (1-\theta_p)(p_t^* - p_{t-1}), \tag{2.17}$$

which shows that inflation is caused by the deviation of the current optimal price, chosen by the reoptimizing firms, from last period's average price.

Second, the link between aggregated output and aggregated labor can be expressed as:

$$L_t = s_t \left(\frac{Y_t}{A_t}\right)^{1/(1-\alpha)}, \tag{2.18}$$

where $s_t = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\eta_p}{(1-\alpha)}} \mathrm{d}i$ represents 'the resource costs due to relative price dispersion with long-run inflation' (Ascari and Merkl, 2009, p. 430). A higher s_t requires a higher level of labor to produce a given amount of output. However, $\log(s_t)$ is equal to zero up to a first-order and (2.18) can be approximated as:

$$l_t = \frac{1}{1-\alpha}(y_t - a_t). \tag{2.19}$$

2.5 The non-policy block of the baseline model

In this section, the New Keynesian Phillips curve and the New Keynesian IS-curve are derived. The NKPC is stated both in terms of marginal cost and in terms of the output gap. The former representation is used in the empirical part of this thesis as it facilitates the extension of the baseline model, while the latter is used to illustrate the phenomenon of the divine coincidence.

Before the New Keynesian Phillips curve is derived, a logarithmic expression of the intermediates good producers (real) marginal cost function (2.13) is needed:

$$mc_{t+k|t}^r = mc_{t+k}^r - \frac{\eta_p \alpha}{1-\alpha}(p_t^* - p_{t+k}) \tag{2.20}$$

Now, the log-linearized optimal pricing function (2.15) is combined with (2.20). In appendix A.1.7 it is shown that the resulting expression is:

$$p_t^* - p_{t-1} = \beta\theta_p E_t(p_{t+1}^* - p_t) + (1-\beta\theta_p)\Theta\,\widehat{mc_t}^r + \pi_t, \tag{2.21}$$

where $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\eta_p} \leq 1$. Combining (2.21) with (2.17) yields the New Keynesian Phillips curve (NKPC) expressed in terms of deviations of real marginal costs from its steady state value:[7]

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_p \widehat{mc}_t^r, \tag{2.22}$$

where $\lambda_p = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}\Theta$ is strictly decreasing in the index of price stickiness θ_p, in the measure of decreasing returns α and in the demand elasticity η_p. The New Keynesian Phillips curve relates current inflation with expected inflation and real marginal cost. Due to the forward-looking behavior of the intermediate producers, which base their pricing decisions on the evolution of marginal cost and also consider the possibility of being bound to a price, the NKPC is also forward-looking and cost-dependent.

In order to illustrate the divine coincidence, the New Keynesian Phillips curve is written in terms of the output gap, which is defined as the difference between actual output and its flexible (or natural) level, $y_t - y_t^f$. The flexible level of output y_t^f is the equilibrium level of output that would occur in absence of price rigidities ($\theta_p = 0$), i.e. when prices are fully flexible. Equation (2.14) implies that, under this scenario, all firms would choose the same price which is a constant markup over nominal marginal cost.[8] Consequently, the logarithm of real marginal cost would be time-invariant and equal to $mc^r = \log(\frac{1}{\mathcal{M}_p}) = -\mu^p$.

Independent of the nature of price setting, an expression for average real marginal cost can be obtained from equation (2.10) combined with the aggregate production

[7]Here and in the following, hatted variables denote log-deviations from steady state levels.

[8]Note, that the zero inflation steady state and the flexible output equilibrium are quite similar but not the same. In both scenarios prices and real marginal cost of intermediate firms coincide. However, the flexible equilibrium is still hit by production or preference shocks that are absent in a non-stochastic steady state.

function (2.19) and the marginal rate of substitution (2.4):

$$\begin{aligned} mc_t^r &= w_t - p_t - mpl_t \\ &= \sigma y_t + \phi l_t - g_t - (y_t - l_t) - \log(1-\alpha) \\ &= (\sigma + \frac{\phi+\alpha}{1-\alpha})y_t - \frac{1+\phi}{1-\alpha}a_t - g_t - \log(1-\alpha). \end{aligned} \tag{2.23}$$

This relation must also hold in the flexible price equilibrium where real marginal cost is constant:

$$mc^r = (\sigma + \frac{\phi+\alpha}{1-\alpha})y_t^f - \frac{1+\phi}{1-\alpha}a_t - g_t - \log(1-\alpha). \tag{2.24}$$

As marginal cost is constant, the flexible rate of output y_t^f absorbs both technology and preference shocks. Subtracting (2.24) from (2.23) yields:

$$\widehat{mc}_t^r = (\sigma + \frac{\phi+\alpha}{1-\alpha})(y_t - y_t^f).$$

This expression can be substituted into equation (2.22) yielding the New Keynesian Phillips curve in terms of the output gap:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_p \widetilde{y}_t, \tag{2.25}$$

where $\kappa_p = \lambda_p \left(\sigma + \frac{\phi+\alpha}{1-\alpha}\right)$ and $\widetilde{y}_t = y_t - y_t^f$ is the output gap. This representation of the NKPC is used in the next chapter to illustrate the phenomenon of the divine coincidence. Before doing so, the second key equation, the New Keynesian IS-curve, is stated. It can be obtained, after applying the (log) market clearing condition $y_t = c_t$, by rewriting the Euler equation (2.5) in terms of the output gap:

$$\widetilde{y}_t = E_t \widetilde{y}_{t+1} - (1/\sigma)(r_t - r_t^f), \tag{2.26}$$

where $r_t = i_t - E_t \pi_{t+1}$ is the real interest rate according to the Fisher identity and r_t^f is the natural rate of interest, i.e. the real interest rate that would occur in a flexible price equilibrium. It is given by:

$$r_t^f = \rho - E_t g_{t+1} + g_t + \sigma(E_t y_{t+1}^f - y_t^f) \tag{2.27}$$

Note, that r_t^f is exogenous, as it is solely determined by both shock processes g_t and a_t via the flexible level of output.

Equations (2.25) and (2.26) form the non-policy block of the baseline New Keynesian model. The NKPC determines inflation given the output gap, while the IS-curve determines the output gap given the deviation of the real interest rate from its natural counterpart. The real interest rate is, in turn, determined by expected inflation and the nominal interest rate. In order to close the model, an appropriate interest rate rule is needed.[9] Usually a Taylor rule or an exogenous path for the the money supply are specified to explain the interest rate. This is left to the empirical part of this thesis, where also a summary of the equilibrium conditions is provided.

[9]This shows that the equilibrium cannot be determined independently of monetary policy. Thus, monetary policy is non-neutral in the baseline New Keynesian model.

3 The Divine Coincidence

There are two slightly different definitions of the divine coincidence in the literature. Walsh (2010, p. 349) describes the divine coincidence as a feature of the baseline version of the New Keynesian Model that allows keeping inflation equal to zero "by keeping current and expected future output equal to the flexible-price equilibrium level." This property can be seen by solving the NKPC (2.25) forwards:

$$\pi_t = \kappa_p \sum_{k=0}^{\infty} \beta^k E_t \widetilde{y}_{t+k}$$

If $E_t \widetilde{y}_{t+k} = 0$ for all k, inflation remains equal to zero or, alternatively, if $\pi_t = 0$ the output gap is asymptotically closed. That is, the central bank does not face a trade-off between both stabilizing inflation and output gap. Price stability implies a stable level of output and vice versa.

This definition of the divine coincidence, however, is somewhat sloppy. It refers only to the stable relation between inflation and the output gap and does not indicate the desirability of a stabilization of both variables. Usually a central bank should try to stabilize output not around its natural but around its *efficient* or *pareto-optimal* level. The latter refers to the equilibrium output that would arise in absence of any rigidities *and* monopolistic competition, whereas the natural level of output is inefficient due to the monopoly power of the firms. In this regard, the definition by Walsh does not capture "the divine" of the New Keynesian Phillips curve.

The more precise definition of the divine coincidence stems from Blanchard and Galí (2007, p. 40): Stabilizing inflation not only stabilizes the output gap but also the *welfare-relevant output gap*, $(y_t - y_t^e)$, which is the distance between actual output and its efficient level. This means that a policymaker needs not consider both objectives when conducting the optimal policy. It is sufficient to achieve either price stability or a stable welfare-relevant output gap as the divine coincidence ensures that both objectives are attained simultaneously.

In presence of the divine coincidence, "strict inflation targeting" would be the optimal policy for a central bank (Galí, 2015, p. 127). In practice, however, "inflation targeting is never "strict" but always "flexible", in the sense that all inflation-targeting central banks not only aim at stabilizing inflation [...] but also put some weight on stabilizing the real economy" (Svensson, 2011, p. 1239). This view reflects on the one hand the trade-off monetary authorities are confronted with in reality, on the other hand it shows that policies that seek to fully stabilize inflation are rather unrealistic. Therefore, the baseline model should be modified in a way such that the divine coincidence disappears and a meaningful trade-off between both inflation and output stabilization arises.

The remainder of this chapter is structured as follows. First, the source of the divine coincidence is discussed and it is shown that the divine coincidence occurs in the baseline model. Then, three extensions that aim at overcoming the divine coincidence are introduced: Adding a cost-push shock to the NKPC and implementing real or nominal wage rigidities.

3.1 The Divine Coincidence in the baseline model

Blanchard and Galí (2007, p. 36) point out that the occurrence of the divine coincidence is linked with the fact that the gap between the natural level of output y_t^f and the efficient level of output y_t^e is constant over time and invariant to shocks. To illustrate this property it is helpful to rewrite the NKPC in terms of the welfare-relevant output gap using the identity $\widetilde{y}_t \equiv (y_t - y_t^e) + (y_t^e - y_t^f)$:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_p \widetilde{x}_t + \kappa_p \delta_t \tag{3.1}$$

where $\widetilde{x}_t = y_t - y_t^e$ is the welfare-relevant output gap and $\delta_t = y_t^e - y_t^f$ is the gap between the natural level of output and its efficient counterpart. As soon as δ_t is constant, the relation between inflation and welfare-relevant output gap is stable and the divine coincidence emerges.[10]

[10] There are also examples in the literature where the gap δ is zero. Equality between efficient and flexible output can be achieved, if a subsidy is introduced that compensates the inefficiency resulting from market power of the intermediate firms. In such a model, a zero inflation policy automatically leads to an efficient level of output. For an example, see Erceg et al. (2000) or Galí (2008, Chapter 4).

In the following it is shown that the divine coincidence emerges in the baseline version introduced in chapter 2. This is done in an analogous way to Blanchard and Galí (2007): First, the efficient and the flexible price equilibria of the baseline model are derived. Then the gap between the natural level and the efficient level of output is calculated which turned out to be actually constant.

3.1.1 Efficient equilibrium

In order to obtain the efficient equilibrium, perfect competition in goods and labor markets and absence of price rigidities are assumed. Note that, under this scenario, all intermediate firms are price takers and charge the same price. For this reason, the firm-specific index i can be omitted and the individual production functions correspond to the economy's average production function. From the log form of nominal average marginal cost (2.10) the nominal wage is determined as:

$$w_t^n = mc_t^n + mpl_t$$

or equivalently in real terms (as $p_t = mc_t^n$ under perfect competition):

$$\omega_t = mpl_t = \log(1-\alpha) + y_t - l_t,$$

where $\omega_t = \log(\frac{W_t}{P_t})$ is the real wage and the second equality makes use of the production function (2.19). From the households' labor supply equation (2.4) one obtains:

$$\omega_t = mrs_t = \phi l_t + \sigma y_t - g_t,$$

where the market clearing condition $c_t = y_t$ has been applied. Combining both expressions yields the efficient equilibrium level of employment:

$$(1+\phi)l_t^e = (1-\sigma)y_t^e + g_t + \log(1-\alpha), \tag{3.2}$$

which depends positively on the output and the preference shifter g_t. Note that in case of a log-utility function, i.e. $\sigma = 1$, fluctuations in efficient employment would solely be determined by g_t. Combining (3.2) with the production function (2.19) yields the efficient level of output:

$$\left(1 - \frac{(1-\alpha)(1-\sigma)}{1+\phi}\right) y_t^e = a_t + \frac{1-\alpha}{1+\phi}\left(g_t + \log(1-\alpha)\right), \tag{3.3}$$

which depends positively on the two shock processes, a_t and g_t.

3.1.2 Flexible price equilibrium

Next, the assumption of a perfectly competitive goods market is dropped but prices are still flexible. Firms act as monopolists and charge an additional mark-up on nominal marginal costs. From (2.14) one obtains $p_t = mc_t^n + \mu^p$. Similarly to efficient case one has for the firms side:

$$\omega_t = mpl_t - \mu^p = \log(1-\alpha) + y_t - l_t - \mu^p.$$

The households' labor supply equation is not affected by monopolistic competition and remains unchanged. The flexible level of employment l_t^f is therefore determined as:

$$(1+\phi)l_t^f = (1-\sigma)y_t^f + g_t + \log(1-\alpha) - \mu^p \tag{3.4}$$

and the corresponding flexible level of output is (this expression is same as (2.24)):

$$\left(1 - \frac{(1-\alpha)(1-\sigma)}{1+\phi}\right) y_t^f = a_t + \frac{1-\alpha}{1+\phi}\left(g_t + \log(1-\alpha) - \mu^p\right). \tag{3.5}$$

Again, output depends on both shocks and varies over time. Firms produce less output than in the efficient case due to monopolistic competition.

Now, the gap between efficient and flexible level of output can be calculated. Taking the difference of (3.3) and (3.5) yields:

$$y_t^e - y_t^f = \frac{(1-\alpha)\mu^p}{(1+\phi) - (1-\alpha)(1-\sigma)} \equiv \delta. \tag{3.6}$$

The gap δ between efficient and flexible level of output is constant over time (which is indicated by the omission of the time index t) and invariant to shocks. As shown above, the constancy of δ implies that stabilizing inflation is equivalent with a stabilization of the welfare-relevant output gap. Therefore, the divine coincidence emerges in the baseline New Keynesian model.

3.2 Overcoming the Divine Coincidence

In order to make the baseline model more realistic, the divine coincidence has to be overcome. In general, there are two options to break up the divine coincidence: Destabilizing either the exact relation between inflation and output gap or drive a time-varying wedge between efficient and flexible output. For these purposes, three extensions to the baseline model are introduced. The first approach expands the NKPC by a cost-push shock as it was done by Clarida et al. (2001), Canova and Ferroni (2011) or Smets and Wouters (2003). The second and third approach extend the baseline model by two alternative staggered wage setting mechanisms: real wage rigidities following Blanchard and Galí (2007) as well as nominal wage rigidities following Erceg et al. (2000).

3.2.1 Cost-push shocks

Considering the NKPC in terms of the welfare relevant output gap (3.1), the most obvious approach to eliminate the divine coincidence is generating a time-varying gap δ_t. A potential implementation is to assume a time varying elasticity of substitution between two intermediate goods, $\eta_{p,t}$. Following Canova and Ferroni (2011, p. 1) it now is assumed that

$$\eta_{p,t} = \eta_p \exp\left(\frac{1-\eta_p}{\eta_p}\epsilon_t^{cp}\right), \quad \eta_p > 1,$$

where ϵ_t^{cp} is an independent and identically distributed normal shock. If prices are flexible, the associated desired price markup in period t on marginal cost is not constant any longer and equal to:

$$\mu_t^p = \log\left(\frac{\eta_{p,t}}{\eta_{p,t}-1}\right). \tag{3.7}$$

It is important to note that a time-varying elasticity of substitution only affects the natural level of output while its efficient level remains unchanged as firms do not charge a markup in that case. Thus, fluctuations in the gap δ_t arise:

$$\delta_t = \frac{(1-\alpha)\mu_t^p}{(1+\phi)-(1-\alpha)(1-\sigma)} \tag{3.8}$$

and the divine coincidence no longer holds.

In appendix A.1.8 it is shown that the implementation of a time-varying markup yields following NKPC:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_p(\widehat{mc}_t^r + \epsilon_t^{cp}). \tag{3.9}$$

Inflation is determined by expected inflation and marginal costs that now are hit by an exogenous shock parameter. For this reason, the shock is referred to as cost-push shock.

The implementation of a cost-push shock, in order to overcome the divine coincidence, is not entirely satisfactory. Indeed, the cost-push shock breaks up the otherwise constant relation between natural and efficient level of output and removes the divine coincidence, but only with respect to the shock itself. The divine coincidence still holds with respect to all other shocks of the model. Blanchard and Galí (2007, p. 50) illustrate this limitation for the case of an oil price shock:[11] They point out that, even in presence of a cost-push shock, "the model still implies that keeping inflation constant in the face of increase in the price of oil" would be "the right policy". Therefore, the authors suggest the introduction of real wage rigidities which permanently destabilize the gap δ_t.

[11] In the baseline model, an oil price shock is not explicitly implemented. However, the effects are similar to a technology shock. Both affect the production function of the intermediate producers. See Goodfriend and King (1997, p. 272).

3.2.2 Real wage rigidities

In order to introduce real wage rigidities, an additional real imperfection in the labor market is assumed. In particular, a new wage setting mechanism is implemented that allows only for a partial wage adjustment. Following Blanchard and Galí (2007, p. 41), the labor supply equation (2.4) of the baseline model is modified as follows:

$$w_t - p_t = \gamma(w_{t-1} - p_{t-1}) + (1-\gamma)mrs_t \tag{3.10}$$

where $\gamma \in [0,1]$ is the measure of real wage rigidities. Households can only partly adjust their real wages to the marginal rate of substitution as they would do in absence of rigidities. An important assumption underlying equation (3.10) is that only the flexible price equilibrium is affected by real wage rigidities while the efficient equilibrium remains unchanged. This holds as the sluggish wage adjustment is assumed to be the result of distortions rather than preferences (Blanchard and Galí, 2007, p. 41). By definition, distortions do not have an effect on the efficient equilibrium.

Next, the influence of real wage rigidities on the gap between efficient and flexible level of output is examined. To facilitate the derivation of the gap δ_t, a log-utility function is assumed, that is $\sigma = 1$.[12] The efficient level of output is still given by (3.3), where σ is set to 1:

$$y_t^e = a_t + \frac{1-\alpha}{1+\phi}\left(g_t + \log(1-\alpha)\right). \tag{3.11}$$

Under flexible pricing, monopolistic competition, a log-utility function and real wage rigidities one has for the wage setting side:

$$\begin{aligned} \omega_t &= \gamma\omega_{t-1} + (1-\gamma)(\phi l_t + y_t - g_t) \\ &= \gamma\omega_{t-1} + (1-\gamma)((a_t - \alpha l_t) + (1+\phi)l_t - g_t) \end{aligned} \tag{3.12}$$

[12] An attempt to calculate the gap δ_t for the general case of the utility function turned out to be too messy for a reasonable presentation in this thesis.

and, as before, from the firms' side:

$$\begin{aligned}\omega_t &= \log(1-\alpha) + y_t - l_t - \mu^p \\ &= a_t - \alpha l_t - \log(1-\alpha) - \mu^p \end{aligned} \tag{3.13}$$

Combining (3.12) and (3.13) yields the flexible level of employment:

$$(\alpha\gamma+(1-\gamma)(1+\phi))l_t^f = (1-\gamma)(g_t+\log(1-\alpha)-\mu^p)+\gamma(a_t-a_{t-1})+\alpha\gamma l_{t-1}^f, \tag{3.14}$$

which now depends on the change of technology and the lagged level of employment. As shown in appendix A.1.9, the flexible level of employment can be used to derive following formulation of the gap between efficient and flexible output:

$$y_t^f - y_t^e + \delta = \Upsilon\,(y_{t-1}^f - y_{t-1}^e + \delta) + (1-\alpha)\Upsilon\,(\frac{1}{\alpha}\Delta a_t - \frac{1}{1+\phi}\Delta g_t) \tag{3.15}$$

where $\Upsilon \equiv \alpha\gamma/(\alpha\gamma + (1-\gamma)(1+\phi)) \in [0,1]$ and $\delta = \mu^p(1-\alpha)/(1+\phi)$ is the time invariant gap defined in equation (3.6) in case of a log-utility function. Equation (3.15) shows that the gap between efficient and flexible output is not constant any longer. Instead it is subjected to both technology and preference shocks. This is a key difference to the effects of a cost-push shock that also destabilizes the gap δ but only with respect to the shock itself.

In appendix A.1.10 it is shown that the NKPC in terms of the output gap has the form:

$$\pi_t = \beta E_t\pi_{t+1} + \frac{\lambda_p}{(1-\gamma L)(1-\alpha)}[(1+\phi)(1-\gamma)(y_t - y_t^f) + \alpha\gamma(\Delta y_t - \Delta y_t^f)] \tag{3.16}$$

where L is the lag operator. In presence of real wage rigidities, inflation depends on both current level and change of the output gap. Although the relation between output gap and inflation is more complex compared to the baseline model, it remains stable and a constant rate of inflation implies a constant output gap. However, a stabilization of the output gap is not longer desirable as the the gap between flexible and efficient output is not constant any longer. Therefore, the divine coincidence has been overcome by the introduction of real wage rigidities.

The real wage setting mechanism introduced above has the drawback that it is not explicitly derived from actions of economic agents. It is not clear why wage staggering should arise exactly in this form. For this reason, another extension is introduced where wage setting is explicitly based on decisions of the households.

3.2.3 Nominal wage rigidities

This section mainly refers to Galí (2008, Chapter 6) and Erceg et al. (2000) (EHL for short). Again, both source are not explicitly cited. Furthermore, most derivations are left out as they are very similar to the baseline model.

As in the real wage case, nominal wage rigidities represent an imperfection in the labor market. As the name implies, the nominal wage is subject to a sluggish adjustment. Following EHL, the labor market is modeled as a duplicate of the intermediate goods market of the baseline model. In particular, there is a continuum of monopolistically competitive households, now explicitly indexed by $j \in [0,1]$, that are allowed to set their wages. Each household is specialized in a differentiated labor service $N_t(j)$. As in the model by EHL, there is a representative labor aggregator (the counterpart of the final good producers), who combines labor services of the households in the same proportions as intermediate firms would choose. This assumption allows the intermediary firms' optimal behavior to be the same as in the baseline model. A further important assumption is full consumption risk sharing across households, as it ensures that all households face same optimization problem and choose identical wages and consumption amounts.[13]

The labor aggregator combines the labor index as:

$$L_t = \left[\int_0^1 N_t(j)^{(\eta_w-1)/\eta_w} \mathrm{d}j\right]^{\eta_w/(\eta_w-1)}, \tag{3.17}$$

[13]The assumption of full risk sharing requires the existence of complete bond markets which allow the households to perfectly hedge against idiosyncratic risks. Due to risk aversion, implied by the concave utility function, consumption is then identical across households in every period (Erceg et al., 2000, p. 288).

where $\eta_w > 1$ represents the elasticity of substitution among labor services.[14] The labor aggregator takes each household's wage rate $W_t(j)$ as given and "produces" aggregate labor at minimal cost. This implies an aggregate wage index of:

$$W_t = \left[\int_0^1 W_t(j)^{1-\eta_w} \mathrm{d}j\right]^{1/(1-\eta_w)}. \tag{3.18}$$

The aggregator's demand for labor services of household j, which also reflects the total demand by the intermediate producers for this type of labor, is given by:

$$N_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\eta_w} L_t. \tag{3.19}$$

As in the baseline model, the household j seeks to maximize the expected infinite stream of utility. However, households now have monopoly power and are allowed to set (nominal) wages. Additionally, in order to introduce nominal wage rigidities, the Calvo price setting rule is applied on the wage decisions of households. It is assumed that only a fraction $(1 - \theta_w)$ of the households can adjust its wage in each period while the rest has to keep the wage unchanged. Under these assumptions, a household will choose W_t^* in order to maximize:

$$E_t \sum_{t=k}^{\infty} (\theta_w \beta)^k U(C_{t+k|t}, N_{t+k|t}) \tag{3.20}$$

subject to the constraints:

$$P_{t+k}C_{t+k|t} + B_{t+k|t} \leq B_{t+k-1|t}(1 + i_{t+k-1}) + W_t^* N_{t+k|t} + D_{t+k} \tag{3.21}$$

[14]Note, that a household's labor supply is now denoted by $N(j)$ instead of L in the baseline model. This notation is introduced to clarify the difference between a household's supply and a firm's demand of labor. The latter is still denoted by $L_t(i)$ and refers to the aggregate labor index L under nominal wage rigidities.

and

$$N_{t+k|t} = \left(\frac{W_t^*}{W_{t+k}}\right)^{-\eta_w} L_{t+k}, \quad k = 0, 1, 2, ... \tag{3.22}$$

where $C_{t+k|t}$, $B_{t+k|t}$ and $N_{t+k|t}$ denote consumption, bond holdings and labor in period $t+k$ given the household last set its wage in period t. The associated optimality condition is:

$$E_t \sum_{k=0}^{\infty} (\theta_w \beta)^k \left[U_C(C_{t+k|t}, N_{t+k|t}) N_{t+k|t} \left(\frac{W_t^*}{P_{t+k}} - \mathcal{M}_w MRS_{t+k|t} \right) \right] = 0, \tag{3.23}$$

where $\mathcal{M}_w = \frac{\eta_w}{\eta_w - 1}$ and $MRS_{t+k|t} = -\frac{U_N(C_{t+k|t}, N_{t+k|t})}{U_C(C_{t+k|t}, N_{t+k|t})}$ is the marginal rate of substitution between consumption and labor in period $t+k$ given a last wage adjustment in period t.

In absence of wage rigidities, i.e. $\theta_w = 0$, all households choose the same wage and (3.23) collapses to:

$$\frac{W_t^*}{P_t} = \frac{W_t}{P_t} = \mathcal{M}_w MRS_{t|t}. \tag{3.24}$$

Due to monopoly power, households charge a markup $\mathcal{M}_w$ on their marginal rate of substitution. Furthermore, in the zero wage inflation steady state all households choose the same wage. Hence, it holds that $W^*/P = W/P = \mathcal{M}_w MRS$.

Next, equation (3.23) is log-linearized around that steady state, yielding following the wage setting rule:

$$w_t^* = \mu_w + (1 - \theta_w \beta) E_t \sum_{k=0}^{\infty} (\theta_w \beta)^k (mrs_{t+k|t} + p_{t+k}), \tag{3.25}$$

where $\mu_w = \log \mathcal{M}_w$. Households charge a nominal wage corresponding to the fixed markup μ_w plus a weighted sum which is increasing in expected price rises and in the marginal rate of substitution.

Before a NKPC-type equation for wage inflation can be derived, expressions for the the economy's aggregate wage dynamics and the marginal rate of substitution $mrs_{t+k|t}$ are needed. Both reasoning and derivation of the former are very similar to the aggregate price dynamics of the baseline model (see appendix A.1.6). The log-linearized aggregate wage is given by:

$$w_t = (1-\theta_w)w_t^* + \theta_w w_{t-1} \tag{3.26}$$

or alternatively:

$$\pi_t^w = (1-\theta_w)(w_t^* - w_{t-1}) \tag{3.27}$$

where $\pi_w = w_t - w_{t-1}$ denotes wage inflation, i.e. the share of inflation caused by sticky wages.[15] Wage inflation is driven by the distance of optimal wage from last period's average wage.

The household-specific marginal rate of substitution $mrs_{t+k|t}$ can be written in terms of economy's average marginal rate of substitution (2.4). Given that the utility function takes the same form as in the baseline model, one obtains:

$$\begin{aligned} mrs_{t+k|t} &= \phi n_{t+k|t} - g_{t+k} - \sigma c_{t+k|t} \\ &= \phi n_{t+k|t} - g_{t+k} - \sigma c_{t+k} \\ &= mrs_{t+k} + \phi(n_{t+k|t} - l_{t+k}) \\ &= mrs_{t+k} - \phi\eta_w(w_t^* - w_{t+k}). \end{aligned} \tag{3.28}$$

Note that in the second step, the assumption of a complete bond market has been used which implies that the consumption choices of households are independent of wage history, i.e. $c_{t+k|t} = c_{t+k}, \forall k$. The last step uses the households' labor demand constraint (3.22).

As shown in appendix A.1.11, combining (3.25) with (3.28) and using the results from (3.26) and (3.27) yields the wage inflation equation:

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \lambda_w[\widehat{mrs}_t - (\widehat{w}_t - \widehat{p}_t)] \tag{3.29}$$

where $\lambda_w = (1-\theta_w)(1-\beta\theta_w)/[\theta_w(1+\phi\eta_w)]$. Wage inflation is determined by expected

[15]Now, wage inflation π_t^w and price inflation π_t^p are explicitly distinguished.

wage inflation as well as by the distance between the marginal rate of substitution and the real wage. If the real wage is below the marginal rate of substitution, households will increase their nominal wage and generate positive inflation. Note that this equation has an analogous form to the NKPC in terms of of marginal cost. It replaces the labor supply equation (2.4) of the baseline model. The households' Euler equation and all optimality conditions of the firms remain the same as in the baseline model.

To asses the effects of this alternative wage setting rule on the divine coincidence, both NKPC and wage infaltion equation (3.29) are written in terms of the output and real wage gap. The latter is the distance between real wage and its natural (or flexible) counterpart:

$$\widetilde{\omega}_t = \omega_t - \omega_t^f. \tag{3.30}$$

Both natural real wage and natural output refer to the equilibrium outcome in absence of price *and* wage rigidities.

In appendix A.1.12 it is shown that price and wage inflation under nominal wage rigidities can be written as:

$$\pi_t^p = \beta E_t \pi_{t+1}^p + \kappa_p \widetilde{y}_t + \lambda_p \widetilde{\omega}_t \tag{3.31}$$

and

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \kappa_w \widetilde{y}_t - \lambda_w \widetilde{\omega}_t \tag{3.32}$$

where $\kappa_w = \lambda_w(\sigma + \frac{\phi}{1-\alpha})$, $\kappa_p = \frac{\alpha\lambda_p}{1-\alpha}$ and λ_p as specified in the baseline model. In presence of nominal wage rigidities, the economy's inflation is determined by price inflation as well as wage inflation. Note that both price and wage inflation depend on the output gap and the real wage gap. Thus, the exact relation between output gap and inflation is resolved and the divine coincidence does not emerge any longer. This approach is in contrast to the two other extensions that aimed on the destabilization of the gap δ between efficient and flexible output.[16]

[16]For future papers, it would also be interesting to investigate, how the nominal wage setting mechanism effects the gap δ_t.

4 Empirical analysis and model comparison

In this section, the three extensions of the baseline model are estimated using a Bayesian approach. Additionally it is investigated, which model fits best to the data. For that purpose, the Bayes factor, obtained from the log marginal likelihood of the models, is employed.

The outline of this section is as follows: First, the model equilibrium conditions are summarized and the Bayesian estimation strategy is sketched. Then, the data sample and the specifications of the priors are described. Finally, the estimation results are presented and discussed.

4.1 Log-linearized equilibrium

Before proceeding, the equilibrium conditions are summarized and a Taylor rule is specified. In order to stay consistent to Rabanal and Rubio-Ramirez (2005), all model variables are now stated in log-deviations from their steady state values instead of log-levels.[17] The log-linearized equilibrium is:

$$\widehat{y}_t = E_t\widehat{y}_{t+1} - 1/\sigma(\widehat{i}_t - E_t\widehat{\pi}_{t+1} + E_t\widehat{g}_{t+1} - \widehat{g}_t) \tag{4.1}$$

$$\widehat{w}_t - \widehat{p}_t = \widehat{mrs}_t \tag{4.2}$$

$$\widehat{mrs}_t = \sigma\widehat{y}_t + \phi\widehat{l}_t - \widehat{g}_t \tag{4.3}$$

$$\widehat{y}_t = \widehat{a}_t + (1-\alpha)\widehat{l}_t \tag{4.4}$$

$$\widehat{mc}_t = \widehat{w}_t - \widehat{p}_t + \widehat{l}_t - \widehat{y}_t \tag{4.5}$$

$$\widehat{\pi}^p_t = \beta E_t\widehat{\pi}^p_{t+1} + \lambda_p(\widehat{mc}_t + \epsilon^{cp}_t), \tag{4.6}$$

where $\lambda_p = \frac{(1-\theta_p)(1-\beta\theta_p)(1-\alpha)}{\theta_p(1-\alpha+\alpha\eta_p)}$. In presence of real wage rigidities, (4.2) is replaced by:

$$\widehat{w}_t - \widehat{p}_t = \gamma(\widehat{w}_{t-1} - \widehat{p}_{t-1}) + (1-\gamma)\widehat{mrs}_t \tag{4.2'}$$

[17] The only difference between equations in log-levels and in log-deviations is that in the latter all constant terms drop out. In the three models, only the Euler equation and the marginal cost function are concerned. Note further that in the zero inflation steady state $\pi = \log(\Pi) = \log(1) = 0$, which implies $\widehat{\pi}_t = \pi_t$.

and in presence of nominal wage rigidities, (4.2) is replaced by:

$$\widehat{\pi}_t^w = \beta E_t \widehat{\pi}_{t+1}^w + \lambda_w [\widehat{mrs}_t - (\widehat{w}_t - \widehat{p}_t)], \tag{4.2”}$$

where $\lambda_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\phi\eta_w)}$. The Taylor rule is specified as in Rabanal and Rubio-Ramirez (2005):

$$\widehat{i}_t = \rho_i \widehat{i}_{t-1} + (1-\rho_i)[h_\pi \widehat{\pi}_t^p + h_y \widehat{y}_t] + \epsilon_t^z, \tag{4.7}$$

where $\rho_i \in [0,1]$ is an interest rate smoothing parameter and ϵ_t^z is a monetary policy shock. The central bank reacts to deviations of price inflation and output from their steady state values. The parameters h_π and h_y determine the magnitude of the reaction.

As in Rabanal and Rubio-Ramirez (2005, p. 1154) the real wage evolution is defined as:

$$\widehat{w}_t - \widehat{p}_t = \widehat{w}_{t-1} - \widehat{p}_{t-1} + \widehat{\pi}_t^w - \widehat{\pi}_t^p. \tag{4.8}$$

This expression is needed in order to close the model. Finally, the technology and preference shocks are specified as AR(1) processes:

$$\widehat{a}_t = \rho_a \widehat{a}_{t-1} + \epsilon_t^a \tag{4.9}$$

$$\widehat{g}_t = \rho_g \widehat{g}_{t-1} + \epsilon_t^g \tag{4.10}$$

where ρ_a and $\rho_g \in [0,1)$. Further it is assumed that ϵ_t^i follows a normal $(0, \sigma_i^2)$ distribution, for $i = a, g, z, cp$, and that all ϵ_t^i are uncorrelated with each other.

Note that in above equilibrium system, the baseline model refers now to the model including a cost-push shock. To avoid stochastic singularity, it is necessary that the number of shocks is equal the number of observables (An and Schorfheide, 2007, p. 124). As the models are estimated using time series for output, real wage, inflation and interest rate, four shocks are needed. Hence, the model extended by a cost-push shock serves as “new” baseline model.

4.2 Estimation strategy

The models are estimated using a Bayesian approach. Unlike classical statistics, prior knowledge about parameter values can be incorporated in form of prior distributions into the estimation process. Confrontation to the data leads then to a revision of the parameters' probabilities, expressed by their posterior distribution.

The estimation of the posterior distribution needs the evaluation of a model's likelihood function and a specification of the prior distribution. More formally, define $M = \{\text{Base, RWR, NWR}\}$ as the set of the three models and $\vartheta = (\theta_p, \theta_w, \gamma, h_\pi, h_y, \sigma, \phi, \rho_i, \rho_a, \rho_g, \eta_p, \eta_w, \alpha, \beta, \sigma_a, \sigma_g, \sigma_{cp}, \sigma_z)'$ as the vector of the models' parameters. Then, the likelihood function of model $m \in M$ is $\mathcal{L}(\mathcal{Y}_t|\vartheta_m, m)$, where $\mathcal{Y}_t$ is the used data set.[18] For a linear model, the likelihood function can be evaluated using the Kalman filter. Let p_0 be the prior joint probability density function chosen on available prior knowledge about the parameters. Then, weighting the obtained likelihood function by the priors yields the unnormalized posterior distribution p_1 of the parameters (Gelman et al., 2014, p. 7):

$$p_1(\vartheta_m|\mathcal{Y}_t, m) \propto p_0(\vartheta_m|m)\mathcal{L}(\mathcal{Y}_t|\vartheta_m, m), \tag{4.11}$$

Equation (4.11) shows that the choice of the prior distribution is important as it affects the shape of the posterior distribution.

As there is no closed-form solution for the posterior distribution (Rabanal and Rubio-Ramirez, 2005, p. 1158), a Random Walk Metropolis-Hastings (RWMH) algorithm has to be applied to generate draws from the posterior distribution of ϑ_m. These draws can then be used to calculate the mean and variance of the parameters. For further details, see An and Schorfheide (2007, p. 131).

For model comparison, the marginal likelihood of each model m is calculated. The marginal likelihood is equal to the integral of the likelihood function weighted by the priors over the parameter space. Again, there is no closed-form solution and the marginal likelihood has to be computed numerically. This is done using Geweke's

[18] The parameter vector ϑ_m differs across the three models.

harmonic mean estimator.[19] With the help of the marginal likelihood the Bayes factor can be derived. It is calculated as the ratio of the marginal likelihoods of two models (Gelman et al., 2014, p. 183):

$$BF = \frac{\int p_0(\vartheta_1|m_1)\mathcal{L}(\mathcal{Y}_t|\vartheta_1, m_1)\mathrm{d}\vartheta_1}{\int p_0(\vartheta_2|m_2)\mathcal{L}(\mathcal{Y}_t|\vartheta_2, m_2)\mathrm{d}\vartheta_2}. \tag{4.12}$$

The Bayes factor "quantifies the relative probability of the observed data" under each model (Ly et al., 2016, p. 22). A Bayes factor larger than 1 indicates that the observed data $\mathcal{Y}_t$ more likely occurred under model m_1 and that model m_1 is preferred over model m_2 (Ly et al., 2016, p. 22).

Using the Bayes factor as a model comparison tool has the advantage that, even in case of misspecified and/or non-nested models, the Bayes factor is a consistent selection device of the best model (Fernandez-Villaverde and Rubio-Ramirez, 2004, p. 159).

In Dynare, all just mentioned steps are already implemented and executed automatically. After entering the model and the priors, a mode finding routine and corresponding start values for the parameters have to be specified. The posterior mode is required for the initialization of the RWMH algorithm. In this thesis, Chris Sims' *csminwel.m* is used for mode finding. Additionally, a scale parameter, determining the jumping distribution in RWMH algorithm, has to be set (Griffoli, 2013, p. 51). The scale parameter is chosen such that the acceptance ratio of draws form the posterior distribution is between 0.25 and 0.33. With these specifications, 200000 draws from the posterior distribution were generated for each model. Appendix A.4 shows an example of the Dynare code employed to estimate the baseline model and the transition of parameters from prior to posterior distributions during estimation.

4.3 Data

For the estimation of the models' parameters, quarterly US time series data for real output, inflation, real wage and interest rate are used. To guarantee the best

[19] See An and Schorfheide (2007, p. 146) for further details.

possible data consistency, the time series of output, inflation and wage stem from the same source, the Bureau of Labor Statistics. In particular, for real output the series "Nonfarm business sector: Real Output", for inflation the series " Nonfarm Business Sector: Implicit Price Deflator" and for real wages the series "Nonfarm Business Sector: Real Compensation Per Hour" are employed. The time series for the interest rate is the federal funds rate published by the Board of Governors of the Federal Reserve System. Two sample periods are considered: First, a sample ranging from 1982Q4 to 2001Q4 reflecting the choice by Rabanal and Rubio-Ramirez (2005). Additionally, as a robustness check, the sample is extended, ranging from 1982Q4 to 2007Q4. A longer time horizon was not possible, as the FED lowered the federal funds rate close to zero in the course of the financial crisis and kept it close to zero until 2016. That behavior by the FED is hard to harmonize with a standard Taylor rule.

All observed variables have to be transformed such that they coincide with the models' variables. That is, all observables have to be written as log-deviations from their steady-state values.[20] Therefore, all variables are logarithmized and demeaned. Additionally, as the models do not include a trend, the series for output and wage are detrended using a one-sided HP filter. An accurate description of all time series, their transformations and the specification of the measurement equations can be found in appendix A.2.

4.4 Priors

The specifications of the priors follow mostly the description in Rabanal and Rubio-Ramirez (2005, p. 1156). The second column of Table 1 shows the chosen priors for the estimated parameters.

The priors of the index of price and wage stickiness, θ_p and θ_w, were transformed in order to keep their values between 0 and 1. The transformed parameters can be interpreted as the average duration of prices and wages. They are assumed to follow a gamma distribution with mean 3 and standard deviation 1.42 for prices, and mean

[20] Pfeifer (2015) provides a comprehensive introduction to the specification of observation equations.

Table 1: Prior and posterior distributions for the parameters.
Sample: 1982Q4-2001Q4

	Prior distribution		Posterior distribution		
			Base	RWR	NWR
		Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)
$\frac{1}{1-\theta_p}$	$\mathrm{Gamma}(2,1)+1$	3.00 (1.42)	9.30 (1.25)	10.00 (1.11)	10.21 (1.67)
$\frac{1}{1-\theta_w}$	$\mathrm{Gamma}(3,1)+1$	4.00 (1.71)	-	-	3.63 (0.61)
γ	$\mathrm{Uniform}[0,1)$	0.50 (0.28)	-	0.93 (0.05)	-
h_π	$\mathrm{Normal}(1.5, 0.25)$	1.50 (0.25)	1.59 (0.20)	1.84 (0.20)	1.76 (0.21)
h_y	$\mathrm{Normal}(0.125, 0.125)$	0.125 (0.125)	0.16 (0.05)	0.28 (0.07)	0.30 (0.07)
σ	$\mathrm{Gamma}(2, 1.25)$	2.50 (1.76)	6.40 (1.36)	8.10 (2.51)	7.39 (1.94)
ϕ	$\mathrm{Normal}(1, 0.5)$	1 (0.5)	0.90 (0.23)	1.10 (0.38)	1.49 (0.39)
ρ_i	$\mathrm{Uniform}[0,1)$	0.5 (0.28)	0.76 (0.03)	0.86 (0.02)	0.86 (0.02)
ρ_a	$\mathrm{Uniform}[0,1)$	0.5 (0.28)	0.89 (0.03)	0.28 (0.16)	0.27 (0.13)
ρ_g	$\mathrm{Uniform}[0,1)$	0.5 (0.28)	0.88 (0.03)	0.92 (0.03)	0.91 (0.03)
σ_a	$\mathrm{Uniform}[0,1)$	0.5 (0.28)	0.008 (0.13)	0.069 (3.29)	0.220 (8.16)
σ_g	$\mathrm{Uniform}[0,1)$	0.5 (0.28)	0.042 (0.008)	0.058 (0.015)	0.054 (0.017)
σ_{cp}	$\mathrm{Uniform}[0,1.5)$	0.75 (0.43)	0.91 (0.186)	1.167 (0.226)	1.103 (0.317)
σ_z	$\mathrm{Uniform}[0,1)$	0.5 (0.28)	0.0015 (0.0002)	0.0012 (0.0001)	0.0012 (0.0001)
$\log(\widehat{L})$			1242.2	1276.1	1284.9

4 and standard deviation 1.71 for wages, reflecting informal microeconomic evidence by Taylor (Rabanal and Rubio-Ramirez, 2005, p. 1158) and a prior belief of more rigid wages compared to prices. The degree of real wage rigidities, γ, is uniformly distributed on the interval $[0, 1)$. The uniform distribution was chosen as no prior knowledge about this parameter was available. The Taylor rule coefficients, h_π and h_y, are normally distributed with means 1.5 and 0.125, respectively, which are Taylor's original estimates (Rabanal and Rubio-Ramirez, 2005, p. 1156). Additionally,

the support for h_π is limited to values greater than 1 to guarantee equilibrium determinacy (Galí, 2008, p. 128). The inverse elasticity of intertemporal substitution, σ, follows a gamma distribution with mean 2.5 and standard deviation 1.76. Further, the inverse elasticity of labor supply, ϕ, is assumed to be normally distributed with mean 1 and standard deviation 0.5. For the interest rate smoothing parameter, ρ_i, the persistence of technology and preference shocks, ρ_a and ρ_g, and the standard deviations of the shocks except the cost-push shock, a uniform distribution on $[0, 1)$ is assumed, reflecting again no prior knowledge about the parameter values. Following Lombardi and Nicoletti (2011, p. 19), the standard deviation of the cost-push shock, σ_{cp}, is assumed to be uniformly distributed on the slightly larger interval $[0, 1.5)$. This deviation from the specification of Rabanal and Rubio-Ramirez, who also used a uniform $[0, 1)$ distribution, has been introduced as the estimation using the extended sample required a support greater than 1 for σ_{cp}.[21]

Finally, the parameters α, β, η_p and η_w are set to fixed values. Without incorporation of capital into the models, an estimation of α and β is difficult. For this reason it is assumed that $\alpha = 0.36$ and $\beta = 0.99$. The parameters η_p and η_w cannot be estimated due to identification problems. The parameter η_p only shows up in the NKPC together with θ_p. Similarly, η_w and θ_w appear only in the wage inflation equation. Thus, both η_p and η_w cannot be identified separately and their values are fixed at 6 in correspondence to Rabanal and Rubio-Ramirez (2005).

4.5 Findings

In this section, findings of several estimation experiments are presented. First, difficulties that occurred in the estimation procedure are outlined. Then, results for the main and the extended sample are presented and discussed.

[21] Lombardi and Nicoletti (2011, p. 19), who used the same baseline model and the same data set (1960Q1-2001Q4), employed a uniform distribution on the interval $[0, 1.5)$ for σ_{cp} as the cost-push shock "is known to contribute to inflation dynamics with quite large movements". Other authors assume inverse gamma distributions which have a support of $(0, \infty)$ but peak in regions close to zero (see, e.g. Smets and Wouters (2003) or An and Schorfheide (2007)). In order to maintain comparability to Rabanal and Rubio-Ramirez, the first option was chosen.

4.5.1 Estimation difficulties

In their paper "Comparing New Keynesian models of the business cycle: A Bayesian approach", Rabanal and Rubio-Ramirez (2005) examine the empirical properties of several New Keynesian models. Among others they estimated the baseline and nominal wage rigidity model presented in this thesis using a Bayesian approach. Therefore, it immediately suggested itself to choose their settings as a starting point for estimation. Originally, Rabanal and Rubio-Ramirez used above mentioned prior distribution and time series ranging from 1960Q1 to 2001Q4. With these specifications, however, the estimation of the models' parameters was not possible. The estimation of the NWR model yielded the Taylor rule coefficient h_π to be on the lower boundary, i.e equal to 1. A value for h_π smaller or equal to 1, however, violates the Taylor principle and leads to equilibrium indeterminacy (Galí, 2008, p. 22). Without further assumptions about the priors of the other parameters, the NWR model could not be estimated. Rabanal and Rubio-Ramirez (2005, p. 1161) also report values "extremely close to one" for h_π, however always slightly larger than 1 and therefore estimable.

To obtain more stable results, Rabanal and Rubio-Ramirez restricted their sample in a second estimation to the period 1982Q4-2001Q4, which is characterized by relatively constant monetary policy by the FED (Christensen and Dib, 2008, p. 163) and allows for a "safe" estimation of h_π. Likewise, the next experiment was estimating the models with the help of the reduced data sample, while the prior specification remained unchanged. This experiment was successful, the results are presented in Table 3 in appendix A.3. Except for a higher estimated standard deviation of the cost push-shock and lower values of the persistence of the technology shock in the nominal wage rigidity model, the obtained estimates were similar to the findings by Rabanal and Rubio-Ramirez.

However, the original prior specification did not permit an estimation using the extended data set ranging from 1982Q4 to 2007Q4. The posterior mode of σ_{cp} was always larger than one and out of the domain of the uniform [0,1) distribution, regardless of the choice of the other parameters' start values. Thus, the RWMH algorithm could not be initialized.

A feasible solution was increasing the support for σ_{cp} to the interval $[0, 1.5)$ (see also section 4.4). The results with these specifications of the priors and the data samples are presented in subsequent section.

4.5.2 Main sample

Columns 3 - 6 of Table 1 show the means and standard deviations of the estimated parameters of each model for the sample 1982Q4-2001Q4.

The average duration of prices in the baseline model is 9.3 quarters, which seems to be quite high compared to the assumed prior distribution. For both RWR and NWR even higher values around 10 arise, corresponding to values for θ_p of approximately 0.9. The average duration of wages is fixed to be one in the baseline and the RWR model. In the NWR model, that parameter is estimated to be 3.63 quarters and hence much lower than the average duration of prices. This applies despite a higher prior mean for wages rigidities. The degree of real wage rigidities is estimated to be at 0.93 in the RWR model, showing that current wages are mainly determined by last quarter's wage. The Taylor Rule coefficients are 1.59 and 0.16 in the baseline model and slightly higher in both the RWR and NWR model. The inverse elasticity of intertemporal substitution, σ, is in the range between 6.40 and 8.10 corresponding to an intertemporal substitution elasticity between 0.16 and 0.19. The elasticity of labor supply, ϕ^{-1}, is with an estimate of 1.1 higher in the baseline model than in the wage rigidity models. In absence of wage rigidities, the marginal rate of substitution is equal to the real wage. Hence, a larger value of this elasticity accounts for the fluctuations of the observed real wage (Rabanal and Rubio-Ramirez, 2005, p. 1159). The interest rate smoothing parameter is 0.86 in both RWR and NWR model and slightly lower in the baseline model. The same holds true for the persistence parameter of the preference shock. The persistence parameter of the technology shock, however, is much higher in the baseline model compared to the wage rigidity models. This observation suggests that a large part of the technology shock persistence is absorbed by the introduction of wage rigidities. The estimated prior means for the standard deviations of the shocks are quite different from each other. While monetary policy and preference shifter shocks show only small amplitudes in all models, the estimates for the standard deviations of the technology and cost-push shocks are diverse. The technology shock volatility is almost zero in the baseline model and increases in the RWR and NWR models with estimates of 0.069 and 0.22, respectively. Even larger is the volatility of the cost-push shock. Its standard deviation is estimated to be 0.91 in the baseline and around 1.15 in the wage rigidity models.

The last row of Table 1 reports the marginal likelihood of each model. The marginal likelihood for the RWR model is 1276.1, which translates into a Bayes factor of $e^{1276-1242} = e^{34}$ with respect to the baseline model.[22] For the NWR model, a Bayes factor of e^{43} is obtained. Hence, the Bayes factor clearly prefers the wage rigidity models over the baseline model and the data set provides very strong evidence against flexible wages.[23] A comparison of the marginal likelihoods of the NWR and the RWR model results in a Bayes factor of e^{8} in favor of the NWR model. Therefore, the nominal wage setting mechanism seems to fit best to the data of the main sample.

4.5.3 Extended sample

In a next step, in order to asses the robustness of above results, the sample is extended to 2007Q4. Table 2 in appendix A.3 reports the estimation results.

Noteworthy findings are: The average duration of prices in the baseline model has increased and is now on the same level as the estimates obtained from the wage rigidity models. Similarly, the standard deviation of the cost-push shock in the baseline model adjusts upwards to the other models' values. Another differences to the main sample is that the elasticity of labor supply, ϕ^{-1}, dropped in the wage rigidity models, while it remained constant in the baseline model. The other parameters are quite robust to the extension of the sample in all three models, in particular both NWR and RWR model exhibit very similar estimation results.

The marginal likelihood and the Bayes factor show qualitatively the same results as in the main sample. Wage rigidity models are preferred over the baseline model and

[22]The exponential needs to be taken as the marginal likelihood is reported in logs.

[23]According to Jeffreys (1961, p. 432) a Bayes factor greater than 100 is considered as decisive evidence for model m_1 over m_2.

the NWR model has the better empirical fit compared to the RWR model.

4.5.4 Discussion of the results

All in all, the parameter estimates are different between baseline and wage rigidity models, while the estimation results of RWR and NWR models turn out to be very similar in all estimations, including the estimation with the uniform $[0, 1)$ prior for σ_{cp}. On the one hand, one could have expected more diverse parameter estimates caused by two different wage setting rules. On the other hand this observation is not too surprising as two related mechanisms have been implemented that both prevent the real wage to adjust immediately.[24] Hence, it can be concluded, that in terms of parameter estimates both wage rigidity models are as good as the other.

Turning to the Bayes factor, a clear pattern is observable. In all estimations, the introduction of wage rigidities enhances the fit to the data. It could be argued that richer models automatically rank better than more sparse models. However, as Rabanal and Rubio-Ramirez (2005, p. 1161) point out "richer models have many more hyperparamters, and the Bayes factor discriminates against them". Hence, employing the Bayes factor for model comparison automatically takes the model size into account, with a preference for parsimonious modeling. It can be concluded that introducing wage rigidities not only overcomes the divine coincidence but also improves the empirical fit of the baseline model.

Regarding the Bayes factor of the wage rigidity models, the nominal outperforms the real wage rigidity model. There is decisive evidence in favor of the nominal wage rigidity model in all estimations. Hence, the dynamics of the nominal wage rigidity model closer resembles the observed data.

The Bayesian approach, however, is not flawless. Due to the difficulties mentioned in section 4.5.1, the estimation using the main sample was conducted twice, for $\sigma_{cp} \in [0, 1)$ and $\sigma_{cp} \in [0, 1.5)$. Comparing Table 1 and Table 3 of appendix A.3, one striking difference can be observed. Under the prior specification with $\sigma_{cp} \in [0, 1)$,

[24] Nominal wage rigidities in combination with price rigidities lead to sticky real wages. The impulse response functions of the models in appendix A.3 show that real wage dynamics after a technology shock are similar in both models but more pronounced in the NWR model.

estimates of the average price duration are in the range 8.4 to 8.9 periods and estimates of σ_{cp} are between 0.75 and 0.83. For $\sigma_{cp} \in [0, 1.5)$, the values of average price duration increase in the wage rigidity models to approximately 10 periods and estimates for σ_{cp} jump to values above 1.10. As all other priors were held completely identical, these difference are solely caused by changes of the prior distribution of σ_{cp}. That observation suggests a close relationship between θ_p and σ_{cp}. Indeed, by considering the New Keynesian Phillips curve (4.6), it becomes evident that higher values of estimated σ_{cp} require lower values of λ_p which, in turn, go hand in hand with higher values for θ_p. This relationship implies that a truncation of the support for σ_{cp} has a direct influence on the estimates of the average distribution of prices and hence on θ_p (Rabanal and Rubio-Ramirez, 2005, p. 1156).

The variation of the estimates highlights the sensitivity of the estimates to the specification of prior distributions in a Bayesian estimation approach. The inclusion of prior knowledge has an undeniable appeal but it also potentially confounds estimation results. This seems to be especially problematic, if no prior knowledge is available and the researcher has to rely on uninformative (but influential) priors that cannot be justified by other empirical findings.

5 Conclusion

This master thesis introduced three possibilities to overcome the divine coincidence in a baseline New Keynesian model. From a methodological point of view, the easiest way to tackle the divine coincidence is the introduction of real wage rigidities, as long as it comes in form of an ad-hoc rule. This simplification, however, is at cost of a lack of microfoundations. Hence, the nominal wage setting mechanism, derived from explicit decisions of households, is preferable. The introduction of a cost-push shock has to treated separately from the other approaches, as the divine coincidence is removed only with respect to the shock itself. Nevertheless, using a cost-push shock is reasonable, as it accounts for permanently changing cost conditions the producers are exposed to.

The Bayesian estimation revealed that the introduction of wage rigidities, not only made the divine coincidence to disappear but also improved the empirical fit of the baseline model. Both wage setting mechanisms yielded similar parameter estimates, which also were robust to an extension of the data sample. The nominal rigidity model outperformed the real wage model in terms of the empirical fit.

Hence, both theoretical considerations and empirical findings suggest, that the nominal wage rigidity mechanism is the best option to overcome the divine coincidence.

A Appendix

A.1 Mathematical Appendix

A.1.1 The Household's optimization problem

The representative household wants to maximize the infinite stream of utility subject to its budget constraint. The household chooses quantities of consumption, labor and bond holdings. Take the Lagrangian:

$$\Lambda = E_0 \sum_{t=0}^{\infty} \beta^t \left(G_t \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{bL_t^{1+\phi}}{1+\phi} \right) + \lambda_t (B_{t-1}(1+i_{t-1}) + W_t L_t + D_t - P_t C_t - B_t)$$

The FOC's are:

$$\frac{\partial \Lambda}{\partial C_t} = \beta^t G_t C_t^{-\sigma} - \lambda_t P_t \qquad = 0 \tag{A.1}$$

$$\frac{\partial \Lambda}{\partial L_t} = -\beta^t L_t^{\phi} + \lambda_t W_t \qquad = 0 \tag{A.2}$$

$$\frac{\partial \Lambda}{\partial B_t} = -\lambda_t + E_t(\lambda_{t+1})(1+i_t) \qquad = 0 \tag{A.3}$$

Shifting the first equation by one period and combining with the third equation yields the Euler equation:

$$\begin{aligned}
&\beta^t G_t C_t^{-\sigma} = \lambda_t P_t \Longleftrightarrow \beta^{t+1} G_{t+1} C_{t+1}^{-\sigma} = \lambda_{t+1} P_{t+1} \\
\Leftrightarrow &\beta^t G_t \frac{C_t^{-\sigma}}{P_t} = \beta^{t+1}(1+i_t) E_t \left(G_{t+1} \frac{C_{t+1}^{-\sigma}}{P_{t+1}} \right) \\
\Leftrightarrow &G_t \frac{C_t^{-\sigma}}{P_t} = \beta(1+i_t) E_t \left(G_{t+1} \frac{C_{t+1}^{-\sigma}}{P_{t+1}} \right)
\end{aligned}$$

For the labor supply equation combine the first and the second FOC:

$$\beta^t \frac{G_t C_t^{-\sigma}}{P_t} W_t = \beta^t L_t^{\phi}$$
$$\frac{W_t}{P_t} = \frac{L_t^{\phi}}{G_t C_t^{-\sigma}}$$

For the log-linearizion we rewrite the Euler equation to:

$$1 = E_t\left(e^{\log(\beta)+\log(1+i_t)+\Delta g_{t+1}-\sigma\Delta c_{t+1}-\pi_{t+1}}\right) = E_t\left(e^{-\rho+i_t+\Delta g_{t+1}-\sigma\Delta c_{t+1}-\pi_{t+1}}\right)$$

where $-\rho = \log(\beta)$. Note that $\log(1+i_t) \approx i_t$ for i close to zero. In a steady state with constant consumption growth $\gamma_c = \Delta c$, inflation π, interest rate i and zero shocks we have:

$$1 = e^{-\rho+i-\sigma\gamma_c-\pi} \Longrightarrow \sigma\gamma_c = -\rho + i - \pi$$

A first order Taylor approximation[25] around this steady state yields:

$$\begin{aligned} 1 &\approx E_t\left(1 + (i_t - i) + \Delta g_{t+1} - \sigma(\Delta c_{t+1} - \gamma_c) - (\pi_{t+1} - \pi)\right) \\ &= E_t\left(1 + (i_t - i) + \Delta g_{t+1} - \sigma\Delta c_{t+1} - \rho + i - \pi - (\pi_{t+1} - \pi)\right) \end{aligned}$$

and after rearranging:

$$c_t = E_t c_{t+1} - (1/\sigma)(i_t - E_t\pi_{t+1} + E_t g_{t+1} - g_t - \rho)$$

[25]For a multivariate function $f(x,y)$, the first-order Taylor expansion around a point $(\bar{x},\bar{y})$ is: $f(x,y) \approx f(\bar{x},\bar{y}) + f_x(\bar{x},\bar{y})(x-\bar{x}) + f_y(\bar{x},\bar{y})(y-\bar{y})$ where $f_{x,y}$ corresponds to the partial derivative with respect to x and y, respectively.

A.1.2 The final good producers optimization problem

The final good firms produce any output Y_t at minimal cost and take the price $P_t(i)$ for each differentiated good $Y_t(i)$ as given. They face the following optimization problem:

$$\min_{Y_t(i)} \quad \int_0^1 P_t(i)Y_t(i)\mathrm{d}i \qquad \text{s.t.} \qquad Y_t = \left[\int_0^1 Y_t(i)^{(\eta_p-1)/\eta_p}\mathrm{d}i\right]^{\eta_p/(\eta_p-1)}$$

The Lagrangian takes the form:

$$\Lambda = \int_0^1 P_t(i)Y_t(i)\mathrm{d}i + \lambda_t\left(Y_t - \left[\int_0^1 Y_t(i)^{(\eta_p-1)/\eta_p}\mathrm{d}i\right]^{\eta_p/(\eta_p-1)}\right)$$

The related FOC is:

$$\frac{\partial\Lambda}{\partial Y_t(i)} = P_t(i) - \lambda_t\frac{\eta_p}{(\eta_p-1)}Y_t^{1/\eta_p}\frac{(\eta_p-1)}{\eta_p}Y_t(i)^{-1/\eta_p} = 0$$

The Lagrangian multiplier λ_t equals the marginal cost of a final good producer(Walsh, 2010, p. 332). Solving this expression for $Y_t(i)$ yields the demand function for intermediated goods:

$$Y_t(i) = \left(\frac{P_t(i)}{\lambda_t}\right)^{-\eta_p} Y_t = \lambda_t^{\eta_p}P_t(i)^{-\eta_p}Y_t$$

This expression can be substituted into the final firms' production function:

$$Y_t = \left[\int_0^1 (\lambda_t^{\eta_p}P_t(i)^{-\eta_p}Y_t)^{(\eta_p-1)/\eta_p}\mathrm{d}i\right]^{\eta_p/(\eta_p-1)}$$

Solving the former for λ_t yields the expression for the aggregated price index P_t:

$$\lambda_t = \left[\frac{1}{\int_0^1 P_t(i)^{1-\eta_p}\mathrm{d}i}\right]^{1/(\eta_p-1)} = \left[\int_0^1 P_t(i)^{1-\eta_p}\mathrm{d}i\right]^{1/(1-\eta_p)}$$

Note, as the final good producers operate in a fully competitive market, final output is sold at a price equal to marginal cost. Therefore, it holds that $\lambda_t = P_t$.

A.1.3 Marginal cost of intermediate producers

As for the final good producers, firms minimize cost at a given factor price and technology constraint. Again, the Lagrangian multiplier reflects the nominal marginal cost of a intermediate firm:

$$\Lambda = W_t L_t(i) + \lambda_t (Y_t(i) - A_t L_t(i)^{1-\alpha})$$

The FOC is:

$$\frac{\partial \Lambda}{\partial L_t(i)} = W_t - \lambda_t (1-\alpha) A_t L_t(i)^{-\alpha} = 0$$

Solving for λ_t and eliminating $L_t(i)$ with the help of the production function yields

$$\lambda_t = \frac{W_t}{(1-\alpha)A_t} L_t(i)^{\alpha} = \frac{W_t}{(1-\alpha)} \left(\frac{1}{A_t}\right)^{1/(1-\alpha)} Y_t(i)^{\alpha/(1-\alpha)}$$

Using the intermediate firms' demand schedule for $Y_t(i)$ (2.7), one gets the expression for the nominal marginal cost of a firm i:

$$MC_t^n(i) = \frac{W_t}{(1-\alpha)} \left(\frac{1}{A_t}\right)^{1/(1-\alpha)} \left[\left(\frac{P_t(i)}{P_t}\right)^{-\eta_p} Y_t\right]^{\alpha/(1-\alpha)}$$

A.1.4 First order condition of intermediate producers

Using the demand function to eliminate $Y_{t+k|t}(i)$, the optimization problem can be written as an unconstrained one:

$$\max_{P_t(i)} \quad \sum_{k=0}^{\infty} \theta_p^k E_t \left[Q_{t,t+k} \left(P_t(i) \left(\frac{P_t(i)}{P_{t+k}}\right)^{-\eta_p} Y_{t+k} - \Psi_{t+k} \left(\left(\frac{P_t(i)}{P_{t+k}}\right)^{-\eta_p} Y_{t+k} \right) \right) \right]$$

Differentiating this expression with respect to $P_t(i)$ and equaling zero yields:

$$\sum_{k=0}^{\infty}\theta_p^k E_t\left[Q_{t,t+k}\left(\left(\frac{P_t(i)}{P_{t+k}}\right)^{-\eta_p}Y_{t+k}-\eta_p\left(\frac{P_t(i)}{P_{t+k}}\right)^{-\eta_p-1}\frac{P_t(i)}{P_{t+k}}Y_{t+k}\right.\right.$$
$$\left.\left.+\,\eta_p\Psi'_{t+k}\left(Y_{t+k|t}(i)\right)\left(\frac{P_t(i)}{P_{t+k}}\right)^{-\eta_p-1}Y_{t+k}\frac{1}{P_{t+k}}\right)\right]=0$$

Substituting back $Y_{t+k|t}(i)$ and rearranging:

$$\sum_{k=0}^{\infty}\theta_p^k E_t\left[Q_{t,t+k}Y_{t+k|t}(i)\left((1-\eta_p)+\eta_p\Psi'_{t+k}\left(Y_{t+k|t}(i)\right)\frac{1}{P_t(i)}\right)\right]=0$$

Multiplying by $P_t(i)/(1-\eta)$ and using the fact that the derivative of the cost function is the marginal cost, i.e. $\Psi'_{t+k}\left(Y_{t+k|t}(i)\right)=MC^n_{t+k|t}(i)$ it follows:

$$\sum_{k=0}^{\infty}\theta_p^k E_t\left[Q_{t,t+k}Y_{t+k|t}(i)\left(P_t^*(i)-\frac{\eta_p}{\eta_p-1}MC^n_{t+k|t}(i)\right)\right]=0$$

where

$$\begin{aligned}MC^n_{t+k|t} &= \frac{W_{t+k}}{(1-\alpha)}\left(\frac{1}{A_{t+k}}\right)^{1/(1-\alpha)}\left[\left(\frac{P_t^*(i)}{P_{t+k}}\right)^{-\eta_p}Y_{t+k}\right]^{\alpha/(1-\alpha)}\\ &= MC^n_{t+k}\left(\frac{P_t^*(i)}{P_{t+k}}\right)^{-\eta_p\alpha/(1-\alpha)}\end{aligned}$$

is the nominal marginal cost for an intermediate firm that last reset its price in period t.

A.1.5 Log-linearization of the Optimal Pricing Condition

The optimal pricing condition is now log linearized around the zero inflation steady state. A similar proof can be found in Bergholt (2012, p.14). For this purpose, a first order Taylor approximation is applied.

Solving the firms' optimality condition for P_t^* yields:

$$P_t^* = \mathcal{M} \frac{E_t \sum_{k=0}^{\infty} \theta_p^k Q_{t,t+k} P_{t+k}^{1+\eta_p} Y_{t+k} MC_{t+k|t}^r}{E_t \sum_{k=0}^{\infty} \theta_p^k Q_{t,t+k} P_{t+k}^{\eta_p} Y_{t+k}}$$

which can be rewritten as:

$$\frac{P_t^*}{P_{t-1}} E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k G_{t+k} C_{t+k}^{1-\sigma} P_{t+k}^{\eta_p - 1} = \mathcal{M} \frac{1}{P_{t-1}} E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k G_{t+k} C_{t+k}^{1-\sigma} P_{t+k}^{\eta_p} MC_{t+k|t}^r$$

where both sides were divided by P_{t-1} and the definition of the stochastic discount factor $Q_{t,t+k} = \beta^k (G_{t+k}/G_t)(C_{t+k}/C_t)^{-\sigma}(P_t/P_{t+k})$ together with the market clearing condition $Y_{t+k} = C_{t+k}$ have been used. That condition must hold as there is no capital and no investment. Hence, all produced goods must be consumed in the same period.

Now, do a first order Taylor expansion around the zero inflation steady state. This yields for the LHS of above expression:

$$\begin{aligned}
&\frac{P_t^*}{P_{t-1}} E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k G_{t+k} C_{t+k}^{1-\sigma} P_{t+k}^{\eta_p - 1} \\
&\approx \sum_{k=0}^{\infty} \theta_p^k \beta^k G C^{1-\sigma} P^{\eta_p - 1} + E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k C^{1-\sigma} P^{\eta_p - 1} (G_{t+k} - G) \\
&+ E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k G (1-\sigma) C^{-\sigma} P^{\eta_p - 1} (C_{t+k} - C) + \frac{1}{P} E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k G C^{1-\sigma} P^{\eta_p - 1} (P_t^* - P) \\
&- \frac{P}{P^2} E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k G C^{1-\sigma} P^{\eta_p - 1} (P_{t-1} - P) + E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k G C^{1-\sigma} (\eta_p - 1) P^{\eta_p - 2} (P_{t+k} - P) \\
&= G\, C^{1-\sigma} P^{\eta_p - 1} E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k \Big(1 + \widehat{g}_{t+k} + (1-\sigma)\widehat{c}_{t+k} + \widehat{p_t^*} + \widehat{p}_{t-1} + (\eta_p - 1)\widehat{p}_{t+k} \Big)
\end{aligned}$$

where variables with a hat on top denote log deviations from steady state. Applying the same steps for the RHS yiels:

$$
\begin{aligned}
&\mathcal{M}\frac{1}{P_{t-1}}E_t\sum_{k=0}^{\infty}\theta_p^k\beta^k G_{t+k}C_{t+k}^{1-\sigma}P_{t+k}^{\eta_p}MC_{t+k|t}^r \\
&\approx \mathcal{M}\,G\,C^{1-\sigma}P^{\eta_p-1}MC^r\,E_t\sum_{k=0}^{\infty}\theta_p^k\beta^k\left(1+\widehat{g}_{t+k}+(1-\sigma)\widehat{c}_{t+k}+\widehat{p}_{t-1}+\eta_p\widehat{p}_{t+k}+\widehat{mc}_{t+k|t}^r\right) \\
&= G\,C^{1-\sigma}P^{\eta_p-1}E_t\sum_{k=0}^{\infty}\theta_p^k\beta^k\left(1+\widehat{g}_{t+k}+(1-\sigma)\widehat{c}_{t+k}+\widehat{p}_{t-1}+\eta_p\widehat{p}_{t+k}+\widehat{mc}_{t+k|t}^r\right)
\end{aligned}
$$

as $MC^r = 1/\mathcal{M}$ in steady state. Equating LHS and RHS expressions and simplifying:

$$
E_t\sum_{k=0}^{\infty}\theta_p^k\beta^k\left(\widehat{p_t^*}-\widehat{p}_{t+k}\right) = E_t\sum_{k=0}^{\infty}\theta_p^k\beta^k\widehat{mc}_{t+k|t}^r
$$

Note, that $\widehat{p_t^*} = p_t^* - p$, $\widehat{p}_{t+k} = p_{t+k} - p$ and $\widehat{mc}_{t+k|t}^r = mc_{t+k|t}^r - mc = mc_{t+k|t} + \mu^p$, so one can write:

$$
\sum_{k=0}^{\infty}\theta_p^k\beta^k\left(p_t^*-p_{t+k}\right) = E_t\sum_{k=0}^{\infty}\theta_p^k\beta^k\left(mc_{t+k|t}^r+\mu^p\right)
$$

Rearranging and applying the properties of a geometric series to both sides:

$$
\frac{p_t^*}{(1-\theta_p\beta)} = \frac{\mu^p}{(1-\theta_p\beta)} + E_t\sum_{k=0}^{\infty}\theta_p^k\beta^k\left(mc_{t+k|t}^r+p_{t+k}\right)
$$

and finally:

$$
p_t^* = \mu^p + (1-\theta_p\beta)E_t\sum_{k=0}^{\infty}\theta_p^k\beta^k(mc_{t+k|t}^r+p_{t+k})
$$

A.1.6 Aggregate price dynamics and relationship between aggregated output and labor

The aggregate price in the economy is

$$P_t = \left[\int_0^1 P_t(i)^{1-\eta_p} \mathrm{d}i \right]^{1/(1-\eta_p)}$$

A constant fraction of firms $1 - \theta_p$ chooses the optimal price in period t while the fraction θ_p keeps the price of the last period. As all firms that set a new price face the same optimization problem, they choose the same price P_t^*. Thus, the integral can be split into two parts:

$$\begin{aligned}
P_t &= \left[\int_0^1 \left((1-\theta_p) P_t^{*1-\eta_p} + \theta_p P_{t-1}^{1-\eta_p} \right) \mathrm{d}i \right]^{1/(1-\eta_p)} \\
&= \left[\int_0^{1-\theta_p} P_t^{*1-\eta_p} \mathrm{d}i + \int_{1-\theta_p}^1 P_{t-1}^{1-\eta_p} \mathrm{d}i \right]^{1/(1-\eta_p)} \\
&= \left[(1-\theta_p) P_t^{*1-\eta_p} + \theta_p P_{t-1}^{1-\eta_p} \right]^{1/(1-\eta_p)}
\end{aligned}$$

Exponentiating both sides by $(1 - \eta_p)$ and dividing both sides by $P_{t-1}^{1-\eta_p}$,

$$\Pi_t^{1-\eta_p} = \theta_p + (1-\theta_p) \left(\frac{P_t^*}{P_{t-1}} \right)^{1-\eta_p}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$. For the log-linearization do a first order Taylor expansion around the zero inflation steady state (i.e. $\Pi = 1$):

$$\begin{aligned}
&\Pi^{1-\eta_p} + (1-\eta_p)\Pi^{-\eta} (\Pi_t - \Pi) \\
&= \theta_p + (1-\theta_p) \left(\frac{P}{P} \right)^{1-\eta_p} + (1-\theta_p)(1-\eta_p) \left(\frac{P}{P} \right)^{-\eta_p} \left(\frac{1}{P}(P_t^* - P) - \frac{P}{P^2}(P_{t-1} - P) \right) \\
&= 1 + (1-\theta_p)(1-\eta_p)(p_t^* - p_{t-1})
\end{aligned}$$

which can be simplified to:

$\pi_t = (1-\theta_p)(p_t^* - p_{t-1})$.

For the relationship between aggregated output and labor consider the labor market clearing condition:

$$L_t = \int_0^1 L_t(i)\mathrm{d}i$$

By using the intermediate goods production function we get:

$$\begin{aligned} L_t &= \int_0^1 \left(\frac{Y_t(i)}{A_t}\right)^{1/(1-\alpha)} \mathrm{d}i \\ &= \left(\frac{Y_t}{A_t}\right)^{1/(1-\alpha)} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\eta_p/(1-\alpha)} \mathrm{d}i \\ &= s_t \left(\frac{Y_t}{A_t}\right)^{1/(1-\alpha)} \end{aligned}$$

where the second equality follows from the demand function (2.7). Taking the log yields:

$$l_t = \frac{1}{1-\alpha}(y_t - a_t) + d_t$$

where $d_t = \log(s_t) = \log\left(\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\eta_p/(1-\alpha)} \mathrm{d}i\right)$. Now, we have to show that d_t is zero up to a first order approximation (a similar proof can be found in Bergholt

(2012, p. 19)). First, from the aggregate price index (2.6) we get:

$$
\begin{aligned}
P_t &= \left[\int_0^1 P_t(i)^{1-\eta_p}\mathrm{d}i\right]^{1/(1-\eta_p)} \\
1 &= \left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\eta_p}\mathrm{d}i\right]^{1/(1-\eta_p)} = \left[\int_0^1 e^{(1-\eta_p)(p_t(i)-p_t)}\mathrm{d}i\right]^{1/(1-\eta_p)} \\
\Rightarrow 1 &= \int_0^1 e^{(1-\eta_p)(p_t(i)-p_t)}\mathrm{d}i
\end{aligned}
$$

A second order approximation around the zero inflation steady state gives:

$$
\begin{aligned}
1 &\approx \int_0^1 \Big[e^0 + e^0(1-\eta_p)\left[(p_t(i)-p)-(p_t-p)\right] \\
&+ \frac{(1-\eta_p)^2}{2}\left[e^0(p_t(i)-p)^2 - 2e^0(p_t(i)-p)(p_t-p) + e^0(p_t-p)^2\right]\Big]\mathrm{d}i \\
&= 1 + (1-\eta_p)\int_0^1 (p_t(i)-p_t)\mathrm{d}i + \frac{(1-\eta_p)^2}{2}\int_0^1 (p_t(i)-p_t)^2\mathrm{d}i
\end{aligned}
$$

Solving for p_t yields:

$$
p_t \approx \int_0^1 p_t(i)\mathrm{d}i + \frac{(1-\eta_p)}{2}\int_0^1 (p_t(i)-p_t)^2\mathrm{d}i
$$

Next,

$$
\begin{aligned}
&\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\eta_p}{1-\alpha}} \mathrm{d}i \\
&= \int_0^1 e^{-\frac{\eta_p}{1-\alpha}(p_t(i)-p_t)} \mathrm{d}i \\
&\approx \int_0^1 \Bigg[e^0 - e^0 \frac{\eta_p}{1-\alpha}\left[(p_t(i)-p)-(p_t-p)\right] \\
&+ \frac{1}{2}\left(\frac{\eta_p}{1-\alpha}\right)^2 \left[e^0(p_t(i)-p)^2 - 2e^0(p_t(i)-p)(p_t-p) + e^0(p_t-p)^2\right]\Bigg] \mathrm{d}i \\
&= 1 - \frac{\eta_p}{1-\alpha}\int_0^1 (p_t(i)-p_t)\mathrm{d}i + \frac{1}{2}\left(\frac{\eta_p}{1-\alpha}\right)^2 \int_0^1 (p_t(i)-p_t)^2 \mathrm{d}i \\
&= 1 + \frac{\eta_p}{1-\alpha} p_t - \frac{\eta_p}{1-\alpha}\int_0^1 p_t(i)\mathrm{d}i + \frac{1}{2}\left(\frac{\eta_p}{1-\alpha}\right)^2 \int_0^1 (p_t(i)-p_t)^2 \mathrm{d}i
\end{aligned}
$$

Now, the foregoing approximation of p_t is inserted:

$$
\begin{aligned}
&\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\eta_p}{1-\alpha}} \mathrm{d}i \\
&\approx 1 + \frac{\eta_p}{1-\alpha}\left[\int_0^1 p_t(i)\mathrm{d}i + \frac{(1-\eta_p)}{2}\int_0^1 (p_t(i)-p_t)^2 \mathrm{d}i\right] - \frac{\eta_p}{1-\alpha}\int_0^1 p_t(i)\mathrm{d}i \\
&+ \frac{1}{2}\left(\frac{\eta_p}{1-\alpha}\right)^2 \int_0^1 (p_t(i)-p_t)^2 \mathrm{d}i \\
&= 1 + \left[\frac{\eta_p(1-\eta_p)}{2(1-\alpha)} + \frac{{\eta_p}^2}{2(1-\alpha)^2}\right]\int_0^1 (p_t(i)-p_t)^2 \mathrm{d}i \\
&= 1 + \frac{\eta_p(1-\alpha+\alpha\eta_p)}{2(1-\alpha)^2}\int_0^1 (p_t(i)-p_t)^2 \mathrm{d}i > 1
\end{aligned}
$$

There are two conclusions from this calculation: $s_t = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\eta_p}{1-\alpha}} \mathrm{d}i$ is bound below at one and $s_t \approx 1$ up to a first order approximation. This implies that $d_t = \log(s_t) = 0$ and we finally have:

$$
l_t = \frac{1}{1-\alpha}(y_t - a_t).
$$

A.1.7 Derivation of the New Keynesian Phillips curve

First, recall the intermediate firms' nominal marginal cost function (2.11). From that equation we get an expression of a intermediate firm's real marginal cost function:

$$MC^r_{t+k|t} = MC^r_{t+k} \left(\frac{P^*_t}{P_{t+k}} \right)^{-\eta_p \alpha/(1-\alpha)}$$

and in logarithmic form:

$$mc^r_{t+k|t} = mc^r_{t+k} - \frac{\eta_p \alpha}{1-\alpha}(p^*_t - p_{t+k}) \tag{A.4}$$

Starting point of the derivation of the New Keynesian Phillips curve is the optimal price setting equation (2.15):

$$p^*_t = \mu^p + (1-\theta_p \beta) E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k (mc^r_{t+k|t} + p_{t+k})$$

Rearranging yields (using $\mu^p = -mc^r$):

$$p^*_t = (1-\theta_p \beta) E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k (mc^r_{t+k|t} - mc^r + p_{t+k})$$

Now, use above marginal cost function (A.4) the to get rid of $mc^r_{t+k|t}$:

$$\begin{aligned} p^*_t &= (1-\theta_p \beta) E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k \left(mc^r_{t+k} - \frac{\alpha \eta_p}{1-\alpha}(p^*_t - p_{t+k}) - mc^r + p_{t+k} \right) \\ &= -\frac{\alpha \eta_p}{1-\alpha} p^*_t + (1-\theta_p \beta) E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k \left(\widehat{mc}^r_{t+k} - \frac{1-\alpha+\alpha\eta_p}{1-\alpha} p_{t+k} \right) \end{aligned}$$

Rearranging and subtracting p_{t-1} on both sides:

$$\begin{aligned}
p_t^* - p_{t-1} &= (1-\theta_p\beta)E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k \left(\frac{1-\alpha}{1-\alpha+\alpha\eta_p} \widehat{mc}_{t+k}^r + p_{t+k} - p_{t-1} \right) \\
&= (1-\theta_p\beta)E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k \left(\Theta\, \widehat{mc}_{t+k}^r + p_{t+k} - p_{t-1} \right) \\
&= (1-\theta_p\beta)E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k \Theta\, \widehat{mc}_{t+k}^r + (1-\theta_p\beta)E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k (p_{t+k} - p_{t-1})
\end{aligned}$$

Now, consider the second sum. It can be written as:

$$\begin{aligned}
&(1-\theta_p\beta)E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k (p_{t+k} - p_{t-1}) = (1-\theta_p\beta)E_t[\theta_p^0\beta^0(p_t - p_{t-1}) + \\
&+ \theta_p^1\beta^1(p_{t+1} - p_t + p_t - p_{t-1}) + \theta_p^2\beta^2(p_{t+2} - p_{t+1} - p_{t+1} - p_t + p_t - p_{t-1}) + ...] \\
&= (1-\theta_p\beta)E_t[\theta_p^0\beta^0\pi_t + \theta_p^1\beta^1(\pi_{t+1} + \pi_t) + \theta_p^2\beta^2(\pi_{t+2} + \pi_{t+1} + \pi_t) + ...] \\
&= E_t[\theta_p^0\beta^0\pi_t + \theta_p^1\beta^1(\pi_{t+1} + \pi_t) + \theta_p^2\beta^2(\pi_{t+2} + \pi_{t+1} + \pi_t) + ...] - \\
&- E_t[\theta_p^1\beta^1\pi_t + \theta_p^2\beta^2(\pi_{t+1} + \pi_t) + \theta_p^3\beta^3(\pi_{t+2} + \pi_{t+1} + \pi_t) + ...] \\
&= E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k \pi_{t+k}
\end{aligned}$$

This expression can be used in above equation:

$$p_t^* - p_{t-1} = (1-\theta_p\beta)\Theta\, E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k \widehat{mc}_{t+k}^r + E_t \sum_{k=0}^{\infty} \theta_p^k \beta^k \pi_{t+k}$$

Now, this equation is written recursively to eliminate the sum operator. First, take out the terms for $k = 0$:

$$\begin{aligned}
p_t^* - p_{t-1} = (1-\theta_p\beta)\Theta\, E_t \sum_{k=1}^{\infty} \theta_p^k \beta^k \widehat{mc}_{t+k}^r + E_t \sum_{k=1}^{\infty} \theta_p^k \beta^k \pi_{t+k} + \\
+ (1-\theta_p\beta)\Theta\, \widehat{mc}_t^r + \pi_t
\end{aligned}$$

Then, shift the sum operators by one period backwards:

$$
\begin{aligned}
p_t^* - p_{t-1} &= \theta_p\beta\left[(1-\theta_p\beta)\Theta\, E_t\sum_{j=0}^{\infty}\theta_p^j\beta^j\widehat{mc}^r_{t+j+1} + E_t\sum_{j=0}^{\infty}\theta_p^j\beta^j\pi_{t+j+1}\right] + \\
&\quad + (1-\theta_p\beta)\Theta\,\widehat{mc}^r_t + \pi_t \\
&= \theta_p\beta E_t(p^*_{t+1} - p_t) + (1-\theta_p\beta)\Theta\,\widehat{mc}^r_t + \pi_t
\end{aligned}
$$

Combining with (2.17) yields the New Keynesian Phillips Curve:

$$
\pi_t = \beta E_t\pi_{t+1} + \lambda_p\widehat{mc}^r_t
$$

A.1.8 Implementing a cost-push shock

If intermediate firms face a time-varying elasticity of substitution $\eta_{p,t}$, their first order pricing condition becomes:

$$
\begin{aligned}
&\sum_{k=0}^{\infty}\theta_p^k E_t\left[Q_{t,t+k}Y_{t+k|t}(i)\left(1-\eta_{p,t+k}+\eta_{p,t+k}MC^r_{t+k|t}\frac{P_{t+k}}{P_t(i)}\right)\right] \\
=&\sum_{k=0}^{\infty}\theta_p^k E_t\left[Q_{t,t+k}Y_{t+k|t}(i)\left(1-\eta_{p,t+k}+\eta_{p,t+k}MC^r_{t+k}\left(\frac{P_{t+k}}{P_t(i)}\right)^{\frac{\alpha-1-\alpha\eta_{t+k}}{1-\alpha}}\right)\right] = 0
\end{aligned}
$$

Using the definitions of the stochastic discount factor and of the firms' demand schedule and applying the same steps as in A.1.5, yields following log-linear expression:

$$
\sum_{k=0}^{\infty}\theta_p^k\beta^k E_t\left(\frac{\eta_p}{1-\eta_p}\widehat{\eta}_{p,t+k} + \frac{\alpha-1-\alpha\eta_p}{1-\alpha}\widehat{p}_t^* - \frac{\alpha-1-\alpha\eta_p}{1-\alpha}\widehat{p}_{t+k} + \widehat{mc}^r_{t+k}\right) = 0
$$

Note that $\widehat{\eta}_{p,t} = \log(\eta_{p,t}) - \log(\eta_p) = \frac{1-\eta_p}{\eta_p}\epsilon_t^{cp}$. Therefore, we have:

$$
\sum_{k=0}^{\infty}\theta_p^k\beta^k E_t\left(\epsilon^{cp}_{t+k} + \frac{\alpha-1-\alpha\eta_p}{1-\alpha}\widehat{p}_t^* - \frac{\alpha-1-\alpha\eta_p}{1-\alpha}\widehat{p}_{t+k} + \widehat{mc}^r_{t+k}\right) = 0
$$

Solving for p_t^* yields:

$$p_t^* = (1-\theta_p\beta)E_t\sum_{k=0}^{\infty}\theta_p^k\beta^k\left(\frac{1-\alpha}{1-\alpha+\alpha\eta_p}(\widehat{mc}_{t+k}^r+\epsilon_{t+k}^{cp})+p_{t+k}\right)$$

Applying the steps of section A.1.7 yields the NKPC with a cost-push shock:

$$\pi_t = \beta E_t\pi_{t+1} + \lambda_p(\widehat{mc}_t^r + \epsilon_t^{cp}) \tag{A.5}$$

A.1.9 Derivation of the gap δ in case of real wage rigidities

As discussed in the main text, the derivation is done only for the case of a log-utility function, so $\sigma = 1$. Starting point is the flexible level of employment:

$$(\alpha\gamma+(1-\gamma)(1+\phi))l_t^f = (1-\gamma)(g_t+\log(1-\alpha)-\mu^p)+\gamma(a_t-a_{t-1})+\alpha\gamma l_{t-1}^f$$

or with $\Upsilon \equiv \alpha\gamma/(\alpha\gamma+(1-\gamma)(1+\phi))$:

$$l_t^f = \frac{(1-\gamma)}{\alpha\gamma+(1-\gamma)(1+\phi)}(g_t+\log(1-\alpha)-\mu^p)+\frac{\Upsilon}{\alpha}(a_t-a_{t-1})+\Upsilon\, l_{t-1}^f$$

Subtract l_t^e on both sides, add $\Upsilon\, l_{t-1}^e - \Upsilon\, l_{t-1}^e$ on the RHS and use $l_t^f - l_t^e = (y_t^f - y_t^e)/(1-\alpha)$:

$$\frac{y_t^f-y_t^e}{1-\alpha} = \Upsilon\left(\frac{y_{t-1}^f-y_{t-1}^e}{1-\alpha}\right)+\frac{\Upsilon}{\alpha}\Delta a_t - l_t^e+\Upsilon\, l_{t-1}^e+\frac{(1-\gamma)}{\alpha\gamma+(1-\gamma)(1+\phi)}(g_t+\log(1-\alpha)-\mu^p)$$

Multiply by $(1-\alpha)$ and add $\delta = \mu^p(1-\alpha)/(1+\phi)$ on both sides:

$$\begin{aligned} y_t^f - y_t^e + \delta &= \Upsilon\,(y_{t-1}^f - y_{t-1}^e+\delta)+(1-\Upsilon)\frac{\mu^p(1-\alpha)}{(1+\phi)} \\ &+ (1-\alpha)\left[\frac{\Upsilon}{\alpha}\Delta a_t - l_t^e+\Upsilon\, l_{t-1}^e+\frac{(1-\gamma)}{\alpha\gamma+(1-\gamma)(1+\phi)}(g_t+\log(1-\alpha)-\mu^p)\right]\end{aligned}$$

Plugging in the both current and lagged expressions for the efficient level of employment (3.2) and rearranging yields:

$$y_t^f - y_t^e + \delta = \Upsilon\,(y_{t-1}^f - y_{t-1}^e + \delta) + (1-\alpha)\Upsilon\,(\frac{1}{\alpha}\Delta a_t - \frac{1}{1+\phi}\Delta g_t)$$

A.1.10 Derivation of the NKPC under real wage rigidities

This derivation is based on Blanchard and Galí (2007, p. 35). The derivation by Blanchard and Galí seems to contain a typo that complicates reproducing of the derivation but does not alter the final representation of the NKPC. In presence of RWR we have following wage setting mechanism:

$$\omega_t = \gamma(\omega_{t-1}) + (1-\gamma)mrs_t$$

The deviation of marginal cost from its steady state value can be written as (see also equation 2.23):

$$mc_t + \mu^p = \omega_t - mpl_t + \mu^p$$

Combining both expressions yields:

$$\begin{aligned} mc_t + \mu^p &= \gamma(\omega_{t-1} + \mu^p) + (1-\gamma)mrs_t - mpl_t + (1-\gamma)\mu^p \\ &= \gamma(mc_{t-1} + \mu^p) - mpl_t + mpl_{t-1} + (1-\gamma)mrs_t + (1-\gamma)\mu^p \end{aligned}$$

Plugging in the expression $mrs_t = \phi l_t - g_t + y_t$ and $mpl_t = y_t - l_t + \log(1-\alpha)$ yields after some rearranging:

$$\begin{aligned} mc_t + \mu^p = {} & \gamma(mc_{t-1} + \mu^p) - \gamma\Delta a_t + \alpha\gamma\Delta l_t + (1+\phi)(1-\gamma)l_t \\ & + (1-\gamma)(\mu^p - \log(1-\alpha) - g_t) \end{aligned}$$

In immediately above equation, Blanchard and Galí left out μ^p in the last term. Under flexible prices it holds that $mc = -\mu^p$:

$$0 = -\gamma \Delta a_t + \alpha\gamma\Delta l_t^f + (1+\phi)(1-\gamma)l_t^f + (1-\gamma)(\mu^p - \log(1-\alpha) - g_t)$$

Subtracting the former from the latter equation yields:

$$mc_t + \mu^p = \gamma(mc_{t-1} + \mu^p) + (1+\phi)(1-\gamma)(l_t - l_t^f) + \alpha\gamma(\Delta l_t - \Delta l_t^f)$$

From the production function we have $l_t - l_t^f = (y_t - y_t^f)/(1-\alpha)$. Additionally make use of the lag operator L:

$$mc_t + \mu^p = \gamma L(mc_t + \mu^p) + (1+\phi)(1-\gamma)(1-\alpha)^{-1}(y_t - y_t^f) + \alpha\gamma(1-\alpha)^{-1}(\Delta y_t - \Delta y_t^f)$$

Solving for $mc_t + \mu^p$ and combining the resulting expression with the NKPC (2.22) yields the equation in the main text:

$$\pi_t = \beta E_t \pi_{t+1} + \frac{\lambda_p}{(1-\gamma L)(1-\alpha)}[(1+\phi)(1-\gamma)(y_t - y_t^f) + \alpha\gamma(\Delta y_t - \Delta y_t^f)]$$

A.1.11 Derivation of the wage inflation equation under nominal wage rigidities

Combining (3.25) with (3.28) yields:

$$w_t^* = \frac{1-\beta\theta_w}{1+\eta_w\phi} E_t \sum_{k=0}^{\infty} (\theta_w\beta)^k (\mu^w + mrs_{t+k} + \eta_w\phi w_{t+k} + p_{t+k})$$

Defining the economy's average log wage up as $\mu_t^w = (w_t - p_t) - mrs_t$ this can be written as:

$$
\begin{aligned}
w_t^* &= \frac{1-\beta\theta_w}{1+\eta_w\phi} E_t \sum_{k=0}^{\infty} (\theta_w\beta)^k (\mu^w - \mu_{t+k}^w + (1+\eta_w\phi) w_{t+k}) \\
&= \frac{1-\beta\theta_w}{1+\eta_w\phi} E_t \sum_{k=1}^{\infty} (\theta_w\beta)^k [(1+\eta_w\phi) w_{t+k} - \widehat{\mu}_{t+k}^w] + (1-\beta\theta_w)(w_t - \frac{\widehat{\mu}_t^w}{1+\eta_w\phi}) \\
&= \frac{1-\beta\theta_w}{1+\eta_w\phi} E_t \sum_{j=0}^{\infty} (\theta_w\beta)^{j+1} [(1+\eta_w\phi) w_{t+j+1} - \widehat{\mu}_{t+j+1}^w] + (1-\beta\theta_w)(w_t - \frac{\widehat{\mu}_t^w}{1+\eta_w\phi}) \\
&= \beta\theta_w E_t w_{t+1}^* + (1-\beta\theta_w)(w_t - \frac{\widehat{\mu}_t^w}{1+\eta_w\phi})
\end{aligned}
$$

This expression can be combined with $w_t = (1-\theta_w)w_t^* + \theta_w w_{t-1}$ (3.26):

$$
\begin{aligned}
w_t &= \theta_w w_{t-1} + (1-\theta_w)\left[\beta\theta_w E_t w_{t+1}^* + (1-\beta\theta_w)(w_t - \frac{\widehat{\mu}_t^w}{1+\eta_w\phi})\right] \\
&= \theta_w w_{t-1} + (1-\theta_w)\left[\beta\theta_w E_t (w_{t+1}^* - w_t) + w_t - \frac{1-\beta\theta_w}{1+\eta_w\phi}\widehat{\mu}_t^w\right] \\
\Leftrightarrow \quad & \theta_w (w_t - w_{t-1}) = \beta\theta_w (1-\theta_w) E_t (w_{t+1}^* - w_t) - \frac{(1-\beta\theta_w)(1-\theta_w)}{1+\eta_w\phi}\widehat{\mu}_t^w
\end{aligned}
$$

Make now use of $\pi_t^w = (1-\theta_w)(w_t^* - w_{t-1})$ (3.27) and divide by θ_p:

$$
\pi_t^w = \beta E_t \pi_{t+1}^w - \lambda_w \widehat{\mu}_t^w \tag{A.6}
$$

Finally, note that $\widehat{\mu}_t^w = \mu_t^w - \mu^w = -mrs_t + (w_t - p_t) + mrs - (w-p) = -[\widehat{mrs}_t - (\widehat{w}_t - \widehat{p}_t)]$. Therefore, the wage inflation equation can be written as:

$$
\pi_t^w = \beta E_t \pi_{t+1}^w + \lambda_w [\widehat{mrs}_t - (\widehat{w}_t - \widehat{p}_t)] \tag{A.7}
$$

A.1.12 Derivation of the wage and price inflation equation in terms of the output gap under nominal wage rigidities

Note again, that flexible variables - denoted by a superscript f - refer to equilibrium values that occur in absence of any rigidity (price and wage). We have for the log deviation of marginal cost from its steady state:

$$\begin{aligned}
\widehat{mc}_t^r &= mc_t^r - mc^r = \omega_t - mpl_t - \omega_t^f + mpl_t^f \\
&= \omega_t - (a_t + \log(1-\alpha) - \alpha l_t) - \omega_t^f + (a_t + \log(1-\alpha) - \alpha l_t^f) \\
&= \widetilde{\omega}_t - (a_t + (1-\alpha) l_t - l_t) + (a_t + (1-\alpha) l_t^f - l_t^f) \\
&= \widetilde{\omega}_t - (y_t - l_t) + (y_t^f - l_t^f) \\
&= \widetilde{\omega}_t - (y_t - \frac{y_t - a_t}{1-\alpha} - y_t^f + \frac{y_t^f - a_t}{1-\alpha}) \\
&= \widetilde{\omega}_t - (\widetilde{y}_t - \frac{1}{1-\alpha}\widetilde{y}_t) = \widetilde{\omega}_t + \frac{\alpha}{1-\alpha}\widetilde{y}_t
\end{aligned}$$

Combining this with the standard NKPC:

$$\pi_t^p = \beta E_t \pi_{t+1}^w + \lambda_p \widehat{mc}_t^r = \pi_t = \beta E_t \pi_{t+1} + \lambda_p (\widetilde{\omega}_t + \frac{\alpha}{1-\alpha}\widetilde{y}_t)$$

For the wage inflation equation we need an alternative expression for the desired wage markup. Start with the definition of $\widehat{\mu}_t^w$:

$$\begin{aligned}
\widehat{\mu}_t^w &= \mu_t^w - \mu^w = \omega_t - mrs_t - \omega_t^f + mrs_t^f \\
&= \widetilde{\omega}_t - (\phi l_t - g_t + \sigma c_t) + (\phi l_t^f - g_t + \sigma c_t^f) \\
&= \widetilde{\omega}_t - (\phi l_t - g_t + \sigma y_t) + (\phi l_t^f - g_t + \sigma y_t^f) \\
&= \widetilde{\omega}_t - \sigma \widetilde{y}_t - \phi (l_t - l_t^f) \\
&= \widetilde{\omega}_t - (\sigma + \frac{\phi}{1-\alpha})\widetilde{y}_t
\end{aligned}$$

Combining this expression with the wage inflation equation A.6 yields the equation in the main text:

$$\pi_t^w = \beta E_t \pi_{t+1}^w - \lambda_w \widehat{\mu}_t^w = \beta E_t \pi_{t+1}^w - \lambda_w (\widetilde{\omega}_t - (\sigma + \frac{\phi}{1-\alpha})\widetilde{y}_t)$$

A.2 Data Appendix

This section describes the time series used for estimation. All variables are in quarterly frequency.

A.2.1 Original data

- **Nonfarm business sector: Real Output** published by the Bureau of labor statistics. Obtained from the FRED homepage.
  - FRED label: OUTNFB
  - Units: Index 2009=100, Seasonally Adjusted
- **Nonfarm Business Sector: Implicit Price Deflator** published by the Bureau of labor statistics. Obtained from the FRED homepage.
  - FRED label: IPDNBS
  - Units: Index 2009=100, Seasonally Adjusted
- **Nonfarm Business Sector: Real Compensation Per Hour** published by the Bureau of labor statistics. Obtained from the FRED homepage.
  - FRED label: COMPRNFB
  - Units: Index 2009=100, Seasonally Adjusted
- **Effective Federal Funds Rate** published by the Board of Governors of the Federal Reserve System (US). Obtained from the FRED homepage.
  - FRED label: FF
  - Units: Percent, Not Seasonally Adjusted

A.2.2 Transformed data

- $y_{obs} = f_{hpfilter}(\log(OUTNFB))$
- $rw_{obs} = f_{hpfilter}(\log(COMPRNFB))$
- $\pi_{obs} = \log(IPDNBS) - \log(mean(IPDNBS))$
- $i_{obs} = \log(1 + \frac{FF}{400}) - \log(mean(1 + \frac{FF}{400}))$

The function $f_{hpfilter}$ is a one-sided HP-filter. It was written for Matlab by Alexander Meyer-Gohde and can be obtained via `https://ideas.repec.org/c/dge/qmrbcd/181.html`. It returns the trend and cycle component of a time series. Here, the cycle component was used.

A.2.3 Measurement equations

The measurement equations, linking observed to model variables, are simply specified as:

- $y_{obs} = \widehat{y}_t$
- $rw_{obs} = \widehat{w}_t - \widehat{p}_t$
- $\pi_{obs} = \widehat{\pi}_t^p$
- $i_{obs} = \widehat{i}_t$

A.3 Further estimation results and impulse response functions

Table 2: Prior and posterior distributions for the parameters.
Sample: 1982Q4-2007Q4

	Prior distribution		Posterior distribution		
			Base	RWR	NWR
		Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)
$\frac{1}{1-\theta_p}$	$\text{Gamma}(2,1)+1$	3.00 (1.42)	10.88 (1.69)	10.45 (0.91)	10.79 (1.88)
$\frac{1}{1-\theta_w}$	$\text{Gamma}(3,1)+1$	4.00 (1.71)	-	-	3.76 (0.60)
γ	$\text{Uniform}[0,1)$	0.50 (0.28)	-	0.96 (0.06)	-
h_π	$\text{Normal}(1.5,0.25)$	1.50 (0.25)	1.53 (0.20)	1.90 (0.20)	1.79 (0.21)
h_y	$\text{Normal}(0.125,0.125)$	0.125 (0.125)	0.21 (0.06)	0.34 (0.07)	0.35 (0.08)
σ	$\text{Gamma}(2,1.25)$	2.50 (1.76)	5.27 (2.12)	7.99 (2.41)	7.49 (2.69)
ϕ	$\text{Normal}(1,0.5)$	1 (0.5)	0.94 (0.21)	1.49 (0.36)	1.80 (0.39)
ρ_i	$\text{Uniform}[0,1)$	0.5 (0.28)	0.80 (0.02)	0.89 (0.01)	0.89 (0.01)
ρ_a	$\text{Uniform}[0,1)$	0.5 (0.28)	0.90 (0.02)	0.12 (0.19)	0.26 (0.12)
ρ_g	$\text{Uniform}[0,1)$	0.5 (0.28)	0.86 (0.03)	0.89 (0.03)	0.89 (0.03)
σ_a	$\text{Uniform}[0,1)$	0.5 (0.28)	0.007 (0.001)	0.093 (0.018)	0.26 (0.108)
σ_g	$\text{Uniform}[0,1)$	0.5 (0.28)	0.034 (0.012)	0.054 (0.014)	0.518 (0.015)
σ_{cp}	$\text{Uniform}[0,1.5)$	0.75 (0.43)	1.294 (0.363)	1.299 (0.190)	1.254 (0.448)
σ_z	$\text{Uniform}[0,1)$	0.5 (0.28)	0.002 (0.0001)	0.001 (0.0001)	0.001 (0.0001)
$\log(\widehat{L})$			1628.6	1679.4	1691.6

Table 3: Prior and posterior distributions for the parameters with $\sigma_{cp} \in [0, 1)$. Sample: 1982Q4-2001Q4

	Prior distribution		Posterior distribution		
			Base	RWR	NWR
		Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)
$\frac{1}{1-\theta_p}$	Gamma$(2, 1) + 1$	3.00 (1.42)	8.51 (0.98)	8.44 (0.79)	8.90 (0.78)
$\frac{1}{1-\theta_w}$	Gamma$(3, 1) + 1$	4.00 (1.71)	-	-	3.24 (0.41)
γ	Uniform$[0, 1)$	0.50 (0.28)	-	0.89 (0.07)	-
h_π	Normal$(1.5, 0.25)$	1.50 (0.25)	1.61 (0.19)	1.84 (0.20)	1.78 (0.21)
h_y	Normal$(0.125, 0.125)$	0.125 (0.125)	0.17 (0.05)	0.27 (0.07)	0.30 (0.07)
σ	Gamma$(2, 1.25)$	2.50 (1.76)	6.48 (1.90)	8.03 (2.65)	7.39 (2.38)
ϕ	Normal$(1, 0.5)$	1 (0.5)	0.95 (0.23)	1.16 (0.35)	1.53 (0.37)
ρ_i	Uniform$[0, 1)$	0.5 (0.28)	0.76 (0.03)	0.85 (0.02)	0.86 (0.02)
ρ_a	Uniform$[0, 1)$	0.5 (0.28)	0.89 (0.03)	0.41 (0.18)	0.29 (0.13)
ρ_g	Uniform$[0, 1)$	0.5 (0.28)	0.88 (0.03)	0.91 (0.03)	0.90 (0.03)
σ_a	Uniform$[0, 1)$	0.5 (0.28)	0.008 (0.002)	0.034 (0.012)	0.172 (0.062)
σ_g	Uniform$[0, 1)$	0.5 (0.28)	0.042 (0.011)	0.056 (0.016)	0.054 (0.014)
σ_{cp}	Uniform$[0, 1)$	0.5 (0.28)	0.749 (0.157)	0.817 (0.132)	0.831 (0.125)
σ_z	Uniform$[0, 1)$	0.5 (0.28)	0.002 (0.0001)	0.001 (0.0001)	0.001 (0.0001)
$\log(\widehat{L})$			1248.4	1274.5	1284.3

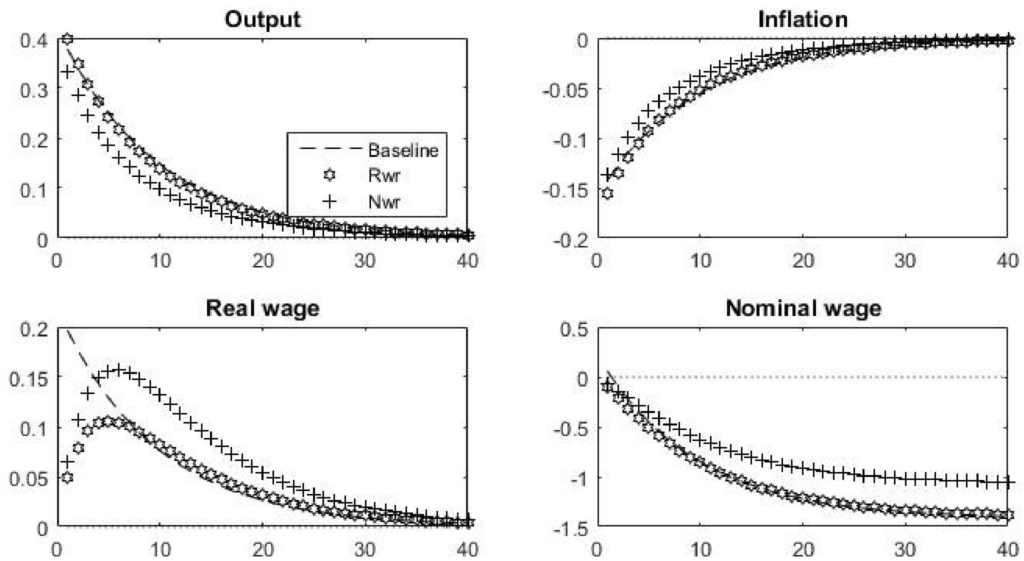

Figure 1: Impulse response to a 0.25 points standard deviation technology shock

A.4 Dynare code and figures

```
//Baseline New Keynesian Model

var y      (long_name='output')
    pi     (long_name='inflation')
    i      (long_name='nominal interest rate')
    winf      (long_name='wage inflation')
    rw     (long_name='real wage')
    mc     (long_name='real marginal cost')
    mrs    (long_name='marginal rate of substitution')
    l      (long_name='labor')
    a      (long_name='AR(1) technology shock process')
    g      (long_name='AR(1) preference shock process')
    y_obs (long_name='observed output')
    pi_obs(long_name='observed inflation')
```

```
    rw_obs (long_name='observed real wage')
    i_obs (long_name='observed interest rate')
    ;

varexo
    eps_a (long_name='technology shock')
    eps_z (long_name='monetary policy shock')
    eps_g (long_name='preference shock')
    eps_cp(long_name='cost-push shock')
    ;

parameters
    bbeta    (long_name='discount factor')
    ssigma   (long_name='inverse elasticity of
      intertemporal substitution')
    pphi     (long_name='inverse elasticity of labor supply')
    aalpha  (long_name='capital share')
    eta_p   (long_name='demand elasticity between goods')
    theta_p (long_name='Calvo parameter firms')
    h_pi    (long_name='inflation reaction Taylor Rule')
    h_y     (long_name='output reaction Taylor Rule')
    rho_i   (long_name='interest rate smoothing')
    rho_a   (long_name='persistence technology shock')
    rho_g   (long_name='persistence preference shock')
    theta_p_trans
    ;

//parametrization

bbeta = 0.99;
ssigma= 1;
pphi = 1;
```

```
aalpha = 0.36;
eta_p = 6;
theta_p = 1.1;
h_pi = 1.5;
h_y = 0.5/4;
rho_i = 0.2;
rho_a = 0.9;
rho_g = 0.9;

//Log-linearized model in terms of deviations from
 zero inflation steady state

model(linear);

//composite parameters
//#theta_p=1-1/theta_p_trans;
#lambda_p = (1-theta_p)*(1-bbeta*theta_p)*
(1-aalpha)/(theta_p*(1+aalpha*(eta_p-1)));

//1. euler equation
y=y(+1)-1/ssigma*(i-pi(+1)+g(+1)-g);

//2. labor supply equation
rw=mrs;

//3. marginal rate of substitution
mrs=pphi*l+ssigma*y-g;

//4. production function
y=a+(1-aalpha)*l;

//5. marginal cost function
```

```
mc=rw+l-y;

//6. New Keynesian Phillips curve
pi=bbeta*pi(+1)+lambda_p*(mc+eps_cp);

//7. Taylor rule
i= rho_i*i(-1)+(1-rho_i)*(h_pi*pi+h_y*y)+eps_z;

//8. AR(1)Technology shock
a=rho_a*a(-1)+eps_a;

//9. AR(1)Preference shock
g=rho_g*g(-1)+eps_g;

//10. Real wage evolution
rw=rw(-1)+winf-pi;

//11. measurement equations
y=y_obs;
rw=rw_obs;
pi=pi_obs;
i=i_obs;
end;

//define shock variances
shocks;
var eps_a;
stderr 0.25;
var eps_z;
stderr 0.25;
var eps_g;
stderr 0.25;
```

```
var eps_cp;
stderr 0.25;
end;

//steady state equals 0 due to log-linear model
steady;
check;

//priors and starting values for baseline model
estimated_params;
theta_p_trans,7,,,GAMMA_PDF,3,1.42;
h_pi,1.8,1.001,,NORMAL_PDF,1.5,0.25;
h_y,0.25,,,NORMAL_PDF,0.125,0.125;
ssigma,7.3,,,GAMMA_PDF,2,1.25;
pphi,0.60,,,NORMAL_PDF,1,0.5;
rho_i,0.64,,0.9999,UNIFORM_PDF,,,0,1;
rho_a,0.75,,0.9999,UNIFORM_PDF,,,0,1;
rho_g,0.75,,0.9999,UNIFORM_PDF,,,0,1;
stderr eps_a,0.115,0,.9999,UNIFORM_PDF,,,0,1;
stderr eps_g,0.2338,0,.9999,UNIFORM_PDF,,,0,1;
stderr eps_cp,0.732,0,1.5,UNIFORM_PDF,,,0,1.5;
stderr eps_z,0.012,0,.9999,UNIFORM_PDF,,,0,1;
end;

//declare observed variables
varobs pi_obs i_obs y_obs rw_obs;

//specify estimation, mode_compute=4 corresponds to csminwel.m
estimation(datafile=rabanal1982,plot_priors=1,first_obs=1,
mode_compute=4,mode_check,mh_replic=200000,mh_nblocks=2,
mh_jscale=0.59,mh_drop=0.1,nograph);
```

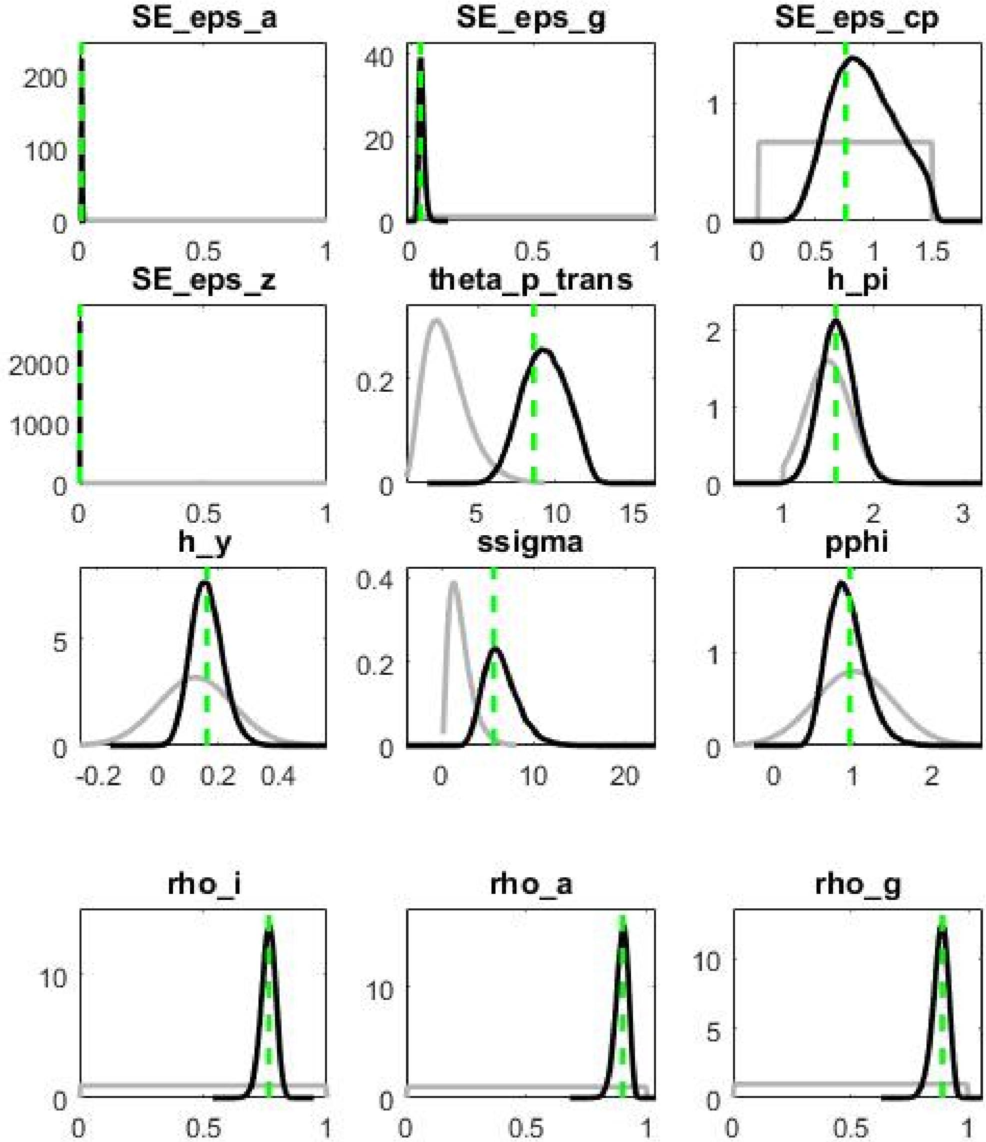

Figure 2: Prior (grey) and posterior (black) distributions of the baseline model. The vertical dashed line is the posterior mode.

B Abstract

Abstract

This master's thesis introduces a baseline New Keynesian model which is at the core of many medium- to high-scale versions. It is shown that the baseline model suffers from the presence of the "divine coincidence". This feature allows a simultaneous stabilization of inflation and welfare-relevant output gap. In practice, however, most central banks are confronted with a trade-off between inflation and output gap stabilization. The baseline model is therefore extended by three mechanisms that aim on the overcoming of the divine coincidence: Real and nominal wage rigidities as well as cost-push shocks. These extensions are compared using a Bayesian estimation approach and it is investigated which model fits best to observed data samples. The main results are: The fit of the baseline model can be improved by the introduction of wage rigidities and the nominal wage setting mechanism outperforms the real wage mechanism in terms of the empirical fit.

Zusammenfassung

Diese Masterarbeit stellt eine Basisversion der Neukeynesianischen Modelle vor, welches als Grundgerüst für zahlreiche Erweiterungen dient. Es wird gezeigt, dass ein Phänomen, welches als "divine coincide" bekannt ist, im Basismodell auftritt. Dieses Phänomen erlaubt eine gleichzeitige Stabilisierung der Inflation und der wohlfahrt-relevanten Produktionslücke. In der Praxis sehen sich die meisten Zentralbanken mit einem Zielkonflikt zwischen der Stabilisierung von Inflation und Produktionslücke konfrontiert. Daher wird das Basismodell um drei Mechanismen erweitert, die auf die Überwindung des "divine coincidence" abzielen: Reale und nominale Lohnrigiditäten und sogenannte "cost-push" Schocks. Diese Erweiterungen werden mit Hilfe einer Bayesianischen Schätzung verglichen und es wird untersucht,

welche Erweiterung am besten mit beobachtbaren Daten übereinstimmt. Die Hauptresultate sind: Das Basismodell passt nach der Erweiterung um Lohnrigiditäten besser zu den Daten und der nominale übertrifft den realen Lohnsetzungsmechanismus.

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