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GERMAN

By

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ABSTRACT

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To translate *at least* into German, several expressions can be used – among them *mindestens, immerhin* and *wenigstens*, which will be discussed in this MA-Thesis. I am going to show that this diversity of possible translations is not just due to a broad stylistic variety, but rather corresponds to different interpretations of *at least*. While I adopt an analysis for *mindestens* from the literature on *at least*, I propose an independent account on the meaning and interpretation of *immerhin*. Assuming that *mindestens*, like *at least*, has a deontic/epistemic modal base which is anchored in its lexical content (cf. Geurts & Nouwen 2007), I suppose that *immerhin* has a circumstantial modal base. I argue that the difference among *mindestens*'s and *immerhin*'s modal bases is responsible for their distinct semantic interpretations. My analysis not only has implications for the German *at least*-kind expressions, but also for *at least* itself – I argue that, according to the data I discuss here and the specific analysis I give for *immerhin*, we actually should assume two distinct lexical entries for *at least*: One that corresponds to *mindestens* (deontic/epistemic modal base), and one that corresponds to *immerhin* (circumstantial modal base). Evidence for this claim comes from another German *at least*-kind expression, *wenigstens*, which behaves very much like *at least* in this respect.

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CHAPTER 1

INTRODUCTION

The outline of this MA-Thesis is going to deal with several issues that will be relevant for the approaches from the literature concerning *at least* (to be discussed in chapter 2), as well as for my own analysis (given in chapter 3) of German *at least*-kind expressions. In the first section of this chapter, I will illustrate some main points of the Generalized Quantifier Theory (GQT, for short) and give a brief explanation of why GQT (alone) doesn't suffice to capture the meaning of *at least*. The second section is going to deal with a proper classification (according to Nouwen 2010) among the class of numeral modifiers to which *at least* belongs. Additionally, I will discuss four puzzles from Geurts & Nouwen (2007) in this section, concerning the issue why *at least n* and *more than n-1* are not interdefinable. In the last section of this chapter, I will introduce and explain important assumptions about the nature of scales in natural language, and degrees. Furthermore, a mechanism to generate scalar implicatures will be discussed which has been applied to independent accounts in the literature on *at least* quite often.

1.1 The Generalized Quantifier Theory and its Shortcomings

The theory of generalized quantifiers can be traced back to Mostowski (1957), where it was claimed that not every quantifier (in natural language) can be defined in terms of $\forall$ and $\exists$, which correspond to first-order properties. Barwise & Cooper's (1981) influential article builds on this insight and develops an interpretational basis for quantified noun phrases (QNPs or simply QPs, for short) along the lines of Montague's (1974) PTQ (= proper treatment of quantification in ordinary English).

The analysis of noun phrases has achieved a considerable profit from interpreting them as generalized quantifiers with the structure [Det N]. The meaning of such an NP can be

analyzed as a second-order predicate – i.e. a predicate which is true of a property of N if the (set of) particular N(s) has that property (cf. Barwise & Cooper 1981) – combining with the verb. Determiners are interpreted as a relation between the meaning of a nominal and a verbal predicate (both denote sets of individuals), hence, quantificational determiners denote a relation between two sets. Those sets are provided via the two arguments – called restrictor and nuclear scope – that the quantificational determiner takes. The Generalized Quantifier Theory treats the relation between sets as a semantic primitive (of quantification). Therefore, it's not necessary to split up an (complex) expression denoting such a relation – for instance, *more than a half* – into its compositional sub-parts in terms of GQT. Nevertheless, within the past decades of research, more and more evidence arises that not every determiner should or actually can be properly analyzed that way either. The next two subsections will give a brief description of two problems arising under the Generalized Quantifier Theory.

1.1.1 An insufficient Distinction for complex Expressions

GQT doesn't distinguish between two morphologically or morphosyntactically distinct expressions (or, more specifically: determiners) as long as they define the same relation between two sets of individuals. So, according to GQT, there's no difference among the following expressions:

(1)   a. $[\![no]\!] = 1$ iff $A \cap B = \varnothing$ (from Hackl 2009: 66, ex. (3a))

        b. $[\![zero]\!] = 1$ iff $A \cap B = \varnothing$ (from Hackl 2009: 66, ex. (3b))

        c. $[\![fewer\ than\ 1]\!] = 1$ iff $A \cap B = \varnothing$ (from Hackl 2009: 66, ex. (3c))

But that means that, for instance, *no* could be defined as any of the set-relations in (2):

(2)   a. $[\![no]\!] = 1$ iff $A \cap B = \varnothing$ (from Hackl 2009: 66, ex. (4a))

        b. $[\![no]\!] = 1$ iff $|A \cap B| = 0$ (from Hackl 2009: 66, ex. (4b))

        c. $[\![no]\!] = 1$ iff $|A \cap B| < 1$ (from Hackl 2009: 66, ex. (4c))

Especially (2-c) appears very counterintuitive. Imagine a context where a bunch of people came to the opening of a new gay bar downtown. During the opening evening and night, 1000 guests were encountered, of which eight were female. That is, the proportion of women was 0,8%. Although this is only a small amount, one wouldn't say either (3-b) nor (3-a) in such a context, but rather utter something like (3-c).

(3)   a.   No female guests attended the opening party.

       b.   Zero (percent of the) guests were female.

       c.   Fewer than one percent of the guests were female.

The differences among the sentences in (3) suggest that there should be a way to distinguish between *no, zero* and *fewer than one* – But within GQT, there is none, at least not in terms of Barwise & Cooper's (1981) initial attempt.

1.1.2   An insufficient Distinction between bare and modified Numerals

The classical GQ Theory proposes the following interpretations for modified numerals:[1]

(4)   a. $[\![\text{exactly three}]\!] = \lambda P \lambda Q [\#(P \cap Q) = 3]$ (from Krifka 1999: 2, ex. (3d))

       b. $[\![\text{more than three}]\!] = \lambda P \lambda Q [\#(P \cap Q) > 3]$ (from Krifka 1999: 1, ex. (3a))

       c. $[\![\text{at least three}]\!] = \lambda P \lambda Q [\#(P \cap Q) \geq 3]$ (from Krifka 1999: 2, ex. (3b))

These can serve the right results for classical examples in which an NP (or, according to more recent strategies of analyzing a [Det N] structure: a DP) combines with a V, but as soon as we consider more abstract examples, it becomes clear that this cannot be the whole story. For instance, GQT makes no (analytical) difference between a bare numeral and a numeral plus what is nowadays called a numeral modifier, such as *at least*. This inadequacy is due to the fact that Barwise & Cooper (1981) declare a meaning for numerals such that n's

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[1]I only quote the interpretations of those expressions which are relevant for this MA-Thesis. For an extensive account cf. Krifka 1999.

value can be interpreted as exactly or more than n^2 – That's actually something to be ruled out be the Gricean Maxim of Quantity (cf. Grice 1975), but if we assume such a treatment of numerals, the meaning of a bare numeral n cannot be distinguished from the meaning of *at least n*. They'll both end up having the same truth conditions, namely, allowing the cardinality of X (a set of individuals) to be exactly or more than n. But notably, *at least n* doesn't trigger the same scalar implicature(s) as the bare numeral n does – The dialog in (5-a) is fine, the one in (5-b) turns out deviant, because *at least* doesn't implicate that the number of getting-undressed strippers didn't exceed three:

(5)   a.   J: Three strippers got undressed at James' bachelor farewell party. M: No, four! Remember the one who showed up late, wearing a garish green leotard.

      b.   J: At least three strippers got undressed at James' bachelor farewell party. M: *No, four! Remember the one who showed up late, wearing a garish green leotard.

Furthermore, we cannot apply the same strategy (as with bare numerals) to cancel the implicature.

(6)   a.   Three strippers got undressed, perhaps even four.

      b.   ?At least three strippers got undressed, perhaps even four.

It can be tempting to propose then, that *at least* doesn't refer either to the same Horn-scale as a bare numeral does, or no Horn-scale at all. Such an assumption turns out problematic, though – Because if we consider numerals to form a Horn-scale, they should form a Horn-scale in every considerable syntactic environment. This would predict that, for instance, *at least three* ought to have a stronger expression as its alternative – something like *at least four* (cf. Krifka 1999). Since the speaker of the utterance under discussion did not make use of a stronger expression, it has to be assumed that he/she is uncertain whether this alternative would hold in a given context.

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[2] That is, the numerals receives an *at least n*-reading, not an *exactly n*-reading.

Another challenge presents itself if we consider focus taking influence on the meaning of numeral modifiers.[3] Different accents yield different readings, i.e. the truth of a sentence heavily relies on which constituent (or part of a constituent) bears focus:

(7)   a.   At least [eight]$_F$ pupils showed up in a pink leopardskin on Monday morning.

        b.   At least eight [pupils]$_F$ showed up in a pink leopardskin on Monday morning.

While (7-a) is interpreted as stating that eight or more pupils wore a pink leopardskin for school, (7-b) would also be compatible with a situation in which eight pupils and some of the teachers have a strange idea of latest fashion.

The previous subsections have shown that, for obvious reasons, GQT doesn't suffice to capture *at least*'s (and some other quantifier's) meaning and interpretational mechanisms, or rather, mechanisms taking influence on *at least*'s interpretation. In chapter 2, I will discuss different accounts on (capturing the meaning of) *at least* from the literature of the past decades.

1.2   Towards a Classification of Numeral Modifiers

The main kinds of numeral modifiers this MA-Thesis will be concerned with are so-called comparative modifiers, like *more than n*, and superlative modifiers, like *at least n*. The former term is (as to my knowledge) initially attributed to Hackl (2001), who used it in accordance with his analysis treating *more than n* like other comparative expressions (e.g. gradable adjectives). The latter term is due to a morphological similarity – Because it's only *at least* and *at most* where one can find the same superlative suffix *-(e)st* commonly used to create an adjective's superlative form. Consider, for instance

(8)   the loud-*est* goldfinch, the bigg-*est* boor, the great-*est* danger of collapse, the small-*est* tapeworm, the sweet-*est* semolina dumpling etc.

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[3]The interpretation of quantifiers like *every* isn't affected by accent and syntactic distribution the way numeral modifiers are, according to Krifka (1999).

among those adjectives which need an obligatory *most* to form their superlative anyway

(9)   the *most* dangerous infectious disease, the *most* shameless allusion, the *most* graceful death, the *most* depraved nonsense etc.[4]

Along the lines of the GQ Theory, there was nothing more to the semantics of *fewer than/more than* and *at least/at most* than the numerical relations $>/<$ and $\geq / \leq$ respectively. Within the past decades, many different accounts have shown that things aren't as simple as that – cf. Krifka (1999) which will be discussed in section 2.1, Hackl (2001), Takahashi (2006) for comparative modifiers and Geurts & Nouwen (2007), Büring (2008), Nouwen (2010), among many others to be discussed in chapter 2, for superlative modifiers.

1.2.1   Class A and Class B numeral Modifiers

Particularly in Nouwen (2010), a distinction between Class A and Class B numeral modifiers is introduced. Elements of the former class establish a relation between the numeral and a specific cardinality, while elements of the latter class just place a bound with respect to the cardinality of individuals having some specific property. Comparative modifiers belong to Class A, superlative ones to Class B.[5] That is, Class A modifiers express an exclusive or strict ordering with regard to the numeral they modify, whilst Class B modifiers express an inclusive or non-strict ordering with regard to the numeral they modify. As will become apparent in section 2.5, this might turn out as a crucial difference.

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[4]Since I will give an analysis of them in chapter 3, I want to say a few words about the German equivalences to *at least*: The most common option to express this meaning in German is by the use of *mindestens*, which actually doesn't share the German superlative morphology anymore (the German superlative ending is only expressed by the suffix *-en* instead of *-ens*). But, the origin of *mindestens* can be traced back to *zum Mindesten* – that is, *mindestens* is its genetive form. There's a notable parallel to *at least*, namely the insertion of a preposition and it's there where one could, back then, find the superlative ending *-en* still used for German superlative adjective forms nowadays. In German, however, there also exist some other expressions declaring *at least*'s meaning – They will be discussed at length in chapter 3 of this MA-Thesis.

[5]Since we'll focus on these two kinds of expressions, i.e. *more than* and essentially *at least*, I won't give a more detailed organization of all the elements that belong to one or the other class here, but cf. Nouwen (2010), Schwarz et al. (2012) for a more extensive account and discussion.

There's something more to be said about the kind of bound superlative modifiers set for the numeral in question – *At least n* places a lower-bound, i.e. scalar alternatives to (*at least*) *n* are either *n* or greater than *n*. Contrary, *at most n* places an upper boundary, hence, scalar alternatives to (*at most*) *n* are either *n* or fewer than *n*.[6] This, however, is a crude generalization, since there are some (syntactic) environments turning the ≥-relation for *at least* around, i.e. reversing it to ≤.

(10)   In   der Nacht wird es auf einigen Bergen   bis auf mindestens 700
           during the night will it on some   mountain.PL to up ATLEAST   700
           Meter   hinab schneien.
           meter.PL down snow
           Meaning: Snowfall down to 700 meters or less than 700 meters above sea level

Maybe those cases are only marginal, but still, it can be considered as another piece of evidence that one shouldn't interpret numeral modifiers solely in terms of the mathematical relations illustrated above.

Nouwen (2010) ties the distinction between comparative and superlative modifiers to their prevailing ability of expressing definite amounts. While elements of Class A are acceptable in examples like (11), Class B elements are not.

(11)   a.   A hexagon has fewer than eleven sides.   (from Nouwen 2010: 3, ex. (2))

           b. #A hexagon has at most ten sides.   (from Nouwen 2010: 3, ex. (3a))

Both (11-a) and (11-b) are equally uninformative, and both property-denoting VPs pick out objects with $n \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ sides, naively speaking. So, from a semantic point of view, we shouldn't expect any differences between the two. But, as we can already see from (11), there actually are differences between comparative and superlative modifiers. As Nouwen (2010) claims, superlative modifiers turn out deviant in contexts where one is talking about a definite amount of something, whereas comparative quantifiers are fine in

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[6]Although not explicitly mentioned in Nouwen (2010), we could also claim a similar principle for comparative modifiers – *More than n* would henceforth set a lower-bound on $n+1$, whilst *fewer than n* would set an upper-bound on n-1.

such contexts. Imagine that A, B and C are talking about the amounts of memory their laptops have – A's laptop has 1GB of memory, B's laptop has 2GB and C's laptop has 512MB of memory. According to Nouwen (2010), B can say (12-a) to A and (12-b) to C:

(12) a. My laptop has less than 2GB of memory. (from Nouwen 2010: 4, ex. (6))

   b. My laptop has more than 512MB of memory. (from Nouwen 2010: 4, ex. (7))

What B does in (12) is comparing the definite amount of memory his laptop has to the definite amount of memory A's (in (12-a)) and C's (in (12-b)) laptops have. 2GB and 512MB respectively serve as contrast to the amount of memory of B's laptop, i.e. 1GB. To contrast definite amounts is something that a speaker can do well if he or she uses a comparative modifier like *more/fewer than*, but, as Nouwen (2010) states, utterances of this kind are infelicitous with superlative modifiers like *at least/at most*:

(13) I know exactly how much memory my laptop has...

   a. and it is {#at most / #maximally / #up to} 2GB.

   (from Nouwen 2010: 4, ex. (8a))

   b. and it is {#at least / #minimally} 512GB. (from Nouwen 2010: 4, ex. (8b))

Contrary, superlative modifiers fit constructions very well when the object(s) under discussion are not supposed to refer to a definite amount, but rather to a range of amounts.

(14) a. Computers of this kind have {at least / minimally} 512MB of memory.

   (from Nouwen 2010: 4, ex. (9a))

The speaker of (14) doesn't intend to ascribe a specific amount of memory to each of the computers he or she presents to the hearer of (14) – He or she rather rather wants to express the fact that the amount of memory varies from computer to computer, but all of them have either exactly 512MB of memory or more than 512MB of memory. Hence, 512MB serves as a lower-bound on the range of amounts of memory those computers have.

Another argument Nouwen (2010) gives to show that superlative modifiers tend to avoid

to be linked to definite amount relies on the fact that a speaker wouldn't utter a sentence involving a class B modifier if he or she had a definite amount in mind.

(15)   Jasper invited maximally 50 people to his party. #43, to be precise.

<div align="right">(from Nouwen 2010: 5, ex. (11))</div>

Contrary, if we assume that 50 is the amount of people one has to invite to his or her party to count as a cool guy or girl, (16) is fine in a context where A and B are mocking about Jasper (intending to convey that Jasper is not a cool guy).

(16)   Jasper invited fewer than 50 people to his party. 43, to be precise.

<div align="right">(from Nouwen 2010: 5, ex. (i) in footnote 4)</div>

While I do agree with Nouwen's (2010) judgements concerning the examples given so far in this subsection, I think that we can also create contexts in which both types of modifiers are equally fine although a definite amount is stated in the continuation. Just consider:

(17)   *Context: I threw a dress-up-in-a-double-breasted-suit-party last Saturday to celebrate Emmanuel Macron's electoral victory. Due to this joyful occasion, I got sloshed with Gin Tonic. On Monday morning at the office, my colleague Genevieve (who couldn't attend) asks me about the party. Amongst other things, I say: It was great...*

    a.   More than 30 people came to my party. 34, if I remember it correctly.

    b.   At least 30 people came to my party. 34, if I remember it correctly.

But in the end, it seems questionable whether we're really talking about a definite amount in (17) with respect to the given context, i.e. considering the fact that I was completely sloshed and probably unable to count the amount of people who attended my party correctly, due to my constitution. A continuation of the form *34, to be precise* is, however, only compatible with the sentence containing *more than*.

The crucial point of Nouwen's (2010) classification is this: Superlative quantifiers are not compatible with the speaker having a specific or definite amount in mind. Comparative

quantifiers can be, if the definite amount the speaker has in mind serves as a compared-to/contrasted-with amount with regard to the numeral modified by *more than/fewer than* in the actual utterance.

Since the Class A/Class B distinction in terms of Nouwen (2010) also contains several expressions I won't take into consideration in this MA-Thesis, I will never refer to *at least/at most* and *more than/fewer than* as Class B and Class A modifiers here, but rather stick to the fact that we're able to distinguish between their prevailing nature/behaviour by means of classifying them as comparative and superlative modifiers.

1.2.2 Four famous Puzzles

In their 2007 paper, Geurts & Nouwen list four puzzles that arise if we compare comparative modifiers such as *more than* with superlative modifiers such as *at least*. Because it appears only at first sight that we can replace superlative ones through their comparative counterparts, as in:

(18) a. At least n A are B. $\iff$ More than n - 1 A are B.

   b. At most n A are B. $\iff$ Fewer than $n + 1$ A are B.

(both from Geurts & Nouwen 2007: 533)

If the examples in (18) would express a true equivalence, we'd expect the following sentences in (19-a) to correspond to one another, as well as those in (19-b).

(19) a. Francis had at least three Bloody Marys. $\iff$ Francis had more than two Bloody Marys. (modified from Geurts & Nouwen 2007: 534, ex. (1))

   b. Francis had at most three Bloody Marys. $\iff$ Francis had fewer than four Bloody Marys. (modified from Geurts & Nouwen 2007: 534, ex. (2))

At first sight, the correspondence seems to hold, but if we dig a little deeper, things get more complicated than the simple mathematical relations $\leq/\geq$, $</>$. Furthermore, if it's just a

matter of an one-to-one equivalence between, for instance, *at least n* and *more than n-1*, then why would a language actually need both expressions? Digging deeper is what we'll do next.

In what follows, I will examine Geurts & Nouwen's (2007) puzzles concerning four phenomena which question this commonly assumed symmetry between superlative modifiers and their comparative counterparts.

Specificity. One way to show that *at least n* and *more than n-1* do not express exactly the same meaning is by making those n things or entities specific. Consider:

(20)   a.   Last night at the speed-dating-event, Carol dismissed at least two guys, namely

        Frank and Fred.   (modified from Geurts & Nouwen 2007: 534, ex. (3a))

      b.   ?Last night at the speed-dating event, Carol dismissed more than one guy, namely

        Frank and Fred.   (modified from Geurts & Nouwen 2007: 534, ex. (3b))

Obviously, the two modifiers aren't interdefinable in this example. With regard to the question how this contrast arises, Geurts & Nouwen (2007) suggest that the *namely*-parts of the sentences in (20) get licensed differently. According to Geurts & Nouwen (2007), *at least* contains an existential expression, which allows for a specific construction, which furthermore can be identified – I.e., the superlative modifier opens up a space for specificity via [$_{DP}$ two guys], which gets filled via the *namely*-construction, identifying the two previously mentioned individuals.

Assuming that this analysis is correct, we can not apply it to the comparative counterpart, *more than* – Not only because *one guy* has the wrong cardinality (that is, *one* instead of *two*), but also because an indefinite, embedded under a comparative quantifier, never opens up space for specific construals. Consequently, the *namely*-part must be licensed by something else than the given antecedent construction, or a specific element within. Geurts & Nouwen (2007) suggest that the antecedent clauses in (20) have to be understood as implying that

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an unspecific number, i.e. some, people/guys, were dismissed by Carol. The *namely*-part restricts this number to two guys, who can be made explicit in the remaining sentence – as far as it goes for superlative modifiers. So, why may an indefinite expression have a specific reading in this case, but not in the case with the comparative modifier *more than*? Geurts & Nouwen (2007) try to explain this fact in terms of argument-types. Since superlative modifiers allow various types as their arguments – a fact that will be shown at length throughout this MA-Thesis –, they should also allow those to have a specific interpretation. Therefore, it's the less stringent restriction regarding the type of argument *at least/at most* are taking, that constructions containing superlative modifiers can have a specific interpretation while constructions containing their comparative counterparts *more/fewer than* cannot.

Inference Patterns. Another proof of the inequivalence of comparative and superlative modifier interpretations can be given in terms of inferences. If it were as simple as in (18-a) and (18-b), respectively, we would expect to find corresponding inference patterns. But consider

(21)   (*premise*) Beryl had three sherries.   (from Geurts & Nouwen 2007: 535, ex. (6))

         a.   Beryl had more than two sherries.   (from Geurts & Nouwen 2007: 535, ex. (7a))

         b.   Beryl had at least three sherries.   (from Geurts & Nouwen 2007: 535, ex. (7b))

in which (21-a) can be said to follow from (21)'s premise, whereas (21-b) does not so – or at least in a less obvious manner than in (21-a). Instead, the sentence containing *at least* conveys that it may also be the case that Beryl had more than (just) three sherries.

Distributional Restrictions. Speaking in terms of quantity, superlative modifiers can occur in a wider range of constructions than their comparative counterparts:

(22)   a.   Ede had three glasses of Riesling-Sylvaner, {at least/at most/*more than/*fewer

               than}.   (modified from Geurts & Nouwen 2007: 535, ex. (9a))

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b. {At least/*More than}, Ede had three glasses of Müller-Thurgau.[7]

<div align="right">(modified from Geurts & Nouwen 2007: 535, ex. (9b))</div>

c. Charlotte takes care of {at most/*fewer than} every second member of the Alcoholics Anonymous who asks her.

<div align="right">(modified from Geurts & Nouwen 2007: 535, ex. (9c))</div>

d. Charlotte takes care of {at least/?more than} Fred and Frank.

<div align="right">(modified from Geurts & Nouwen 2007: 535, ex. (9d))</div>

Only (but crucially) in certain cases, comparative modifiers are perfectly fine, whereas superlative ones are not. Specifically, *at least/at most* turn out deviant in the scope of downward-entailing expressions[8] and some existential quantifiers, like *often* or *occasionally.*

(23) a. Ede didn't have {?at least/?at most/more than/fewer than} three glasses of Riesling-Sylvaner. (modified from Geurts & Nouwen 2007: 535, ex. (10a))

b. Few of the guests had {?at least/?at most/more than/fewer than} three Bloody Marys. (modified from Geurts & Nouwen 2007: 535, ex. (10b))

c. {All/Most/?About five/?None} of the girls had at least/at most three martinis.

<div align="right">(from Geurts & Nouwen 2007: 535, ex. (11a))</div>

d. Before going to bed, Betty {always/usually/?often/?occasionally/*never} has at least/at most three martinis. (from Geurts & Nouwen 2007: 535, ex. (11b))

The answer to the question of why superlative modifiers appear quite freely in general and more restricted in special cases can be given in terms of Geurts & Nouwen's (2007) treatment of *at least* and *at most* as modal expressions.[9] The use of (epistemic) modals actually is

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[7]This example might have another reading under which the sentence conveys that Ede having three glasses of wine was the least he could/should do. So far, this ambiguity doesn't concern us, but as will be shown later in this MA-Thesis, the difference between one and the other reading may be crucial for the overall interpretation of the sentence.

[8]Hence, we might be tempted to assume that superlative modifiers are PPIs, but this is not quite right, cf. Geurts & Nouwen 2007: 535.

[9]A detailed account will be given in section 2.3.

restricted in several ways (cf. Fintel and Iatridou 2003, Tancredi 2005), among them if the (epistemic) modal is outscoped by negation. This also turns out true for superlative modifiers.

(24)   a.   Ede might not have had three glasses of Müller-Thurgau.

                                 (modified from Geurts & Nouwen 2007: 548, ex. (58a))

        b. ?Ede didn't have perhaps three glasses of Müller-Thurgau.

                                   (modified from Geurts & Nouwen 2007: 548, ex. (58b))

        c.   Ede didn't have {?at least/?at most/more than/fewer than} three glasses of Müller-Thurgau.   (modified from Geurts & Nouwen 2007: 548, ex. (58c))

Hence, if we assume *at least* and *at most* to (lexically, as Geurts & Nouwen do in their 2007 article) involve modality of an epistemic force, we can account for (24-c)'s infelicity.

Missing Readings. Concerning the last puzzle given in their article, Geurts & Nouwen (2007) claim that comparative modifiers raise ambiguities in certain constructions, whereas their superlative counterparts do not.

(25)   a.   You may have at most four beers.   (from Geurts & Nouwen 2007: 536, ex. (12a))

        b.   You may have fewer than five beers.   (from Geurts & Nouwen 2007: 536, ex. (12b))

According to Geurts & Nouwen (2007), (25-a) conveys that the hearer is allowed to have four beers or less than four beers, but under no circumstances may he or she have more than four beers. (25-b) expresses the same meaning, but also has a weaker reading, under which the hearer may have less than five beers, but at the same time, may also have more than four beers.

While I do not considers those judgements as wrong, I just think that the examples under discussion are kind of tricky – Imagine what would happen if we replace *at most* in (25-a) by *at least*:

(26)   You may have at least four beers.

What does (26) tell us? Well, strictly speaking, that the hearer is allowed to have four beers or more than four beers, but no less than four. But, at least according to my own judgements, (26) also has another reading, under which the hearer is allowed to have four beers or more, but also a smaller amount of alcohol. It might be due to world knowledge – i.e. that it is by no means a requirement (apart from party games or whatsoever) that an individual drinks a certain amount of beer – and in this particular example, the other reading may be a little hard to get, but the crucial point is this: (26) might as well give rise to an ambiguity, despite from what Geurts & Nouwen (2007) state.

But this seems to be a complicated issue with far-reaching consequences, which can be seen more clearly once we've gained some insight in the generation of (scalar) alternatives. In the next section, we'll try to find out more about implicature generation and scales in general.

1.3   Scales, Degrees and Implicature

This section subsumes important insights to scales in natural language, degrees as elements for linguistic theory and a well-known mechanism to generate scalar implicatures. All of these issues will be relevant to the discussion of different accounts on *at least*, given in chapter 2.

1.3.1   The Nature of Scales: Solt 2013

In Solt (2013), a nice summary of how and why scales are important for linguistic theory is given. Because scales will also be of some importance to the analysis in chapter 3, I will give a brief outline of the nature of scales in this subsection.

According to Kennedy (2007), a scale S consists of three components, listed below.

(27) $S = \langle D, \succ, DIM \rangle$, where

        D is a set of degrees,

$\succ$ is an ordering relation on that set, and

DIM is a dimension of measurement (from Solt 2013: 2, ex. (2))

Degrees are connected to individuals or entities (having this degree of a specific measure) through a measurement function μ. Therefore, μ_S is a function that assigns an individual or entity x to a degree on the prevailing scale S. This degree represents x's measure with respect to the dimension DIM. For instance, *tall* can be taken as a linguistic element that assigns individuals or entities to their specific height.

(28) $[\![tall]\!] = \lambda d\lambda x.\mu_{\text{HEIGHT}}(x) \geq d$ (from Solt 2013: 3, ex. (4))

   Meaning: The denotation of *tall* is the characteristic function from (a set of) degrees to the characteristic function of a set of individuals such that the measure function μ for height assigns x a degree (equal to) d (i.e., x's specific height)

For reasons of clarification, let's take a look at an example.

(29)   a.   Rudolf is 20cm taller than Mary.

   b. $\mu_{\text{HEIGHT}}(Rudolf) \geq \mu_{\text{HEIGHT}}(Mary) + 20\text{cm}$

(modified from Solt 2013: 3, ex. (5))

   Meaning: The degree of *Rudolf*'s height (assigned via the measure function μ for height) equals the degree of *Mary*'s height (assigned via the measure function μ for height) plus 20cm

The degree argument can be saturated by comparative or superlative morphology – as in (29-a) – or by a degree modifier such as *very, too*. If neither of the two appear in a sentence that involves reference to a scale (such as *Rudolf is tall*, for instance), a null degree morpheme *pos* is assumed, introducing a standard comparison.

But (27) seems kind of underspecified regarding the specific structure of a scale, i.e. what it looks like or may/must not look like. This question is closely related to the actual origin of scales. (Among others) von Stechow (1984) assumes that scales are a kind of abstrac-

tion which language refers to, to be able to compare things. Others, like Kennedy (2007), rather describe degrees as abstract representations of measurement as such. In turn, Krifka (1989) claims degrees to be numbers, based on the assumption that degrees are some sort of primitive components, existing independently of the individuals or entities whose measure they denote. Being a semantic primitive means assigning degrees a separate semantic type, $\langle d \rangle$. A very influential view regarding scale structures goes back to Cresswell (1977), who assumed degrees to be construed as equivalence classes of individuals/entities.

Solt (2013) distinguishes between three kinds of structures for scales – ordinal, ratio and interval scales. To what kind of scale-structure a sentences refers to depends on the sentence's construal. Sentences like (30-a) refer to an ordinal scale – That is, a linearly ordered set of (measure) points. Crucially, such a scale-structure is only applicable if degrees assign numerical values in a way that preserves the specific ordering of the prevailing scale. Hence, an ordinal scale is insufficient for (30-b) and (30-c).

(30)   a.   Rudolf is 178cm tall. Mary is 158cm tall.   refers to: ORDINAL SCALE

       b.   Rudolf is 20cm taller than Mary.   refers to: INTERVAL SCALE

       c.   Lola is twice as tall as Ben.   refers to: RATIO SCALE

Let's consider (30-c), because we've been already dealing with (30-b) and the structure of a scale for (30-a) above. (30-c) refers to a so-called ratio scale. In this example, measurement would work in two steps: First, taking the specific height of Lola and the specific height of Ben together (concatenation) – one might think of this operation as placing one above the other, yielding the sum of their individual heights – and then applying a mathematical relation, such as, for (30-c), addition. This procedure can be applied to many physical dimensions, e.g. weight or width, but the mathematical relation doesn't always have to be addition. Take the dimension of temperature, for instance. If I have two bowls of soup having fifty degrees Celsius and I pour them together, I won't get a bowl of scalding hot soup – But rather just one bowl of soup having fifty degrees Celsius. Other dimensions, such

as intelligence or beauty, are pretty hard to capture on a scale.

Besides the structure of a scale, what other kinds of variation can occur? Well, there might be variation with regard to each of the three components a scale consists of, given in (27) – Degrees, ordering relations and the dimension of measurement. Above, we took degrees to be certain points on a scale. But couldn't they also be understood as intervals or dimensions? Kennedy (2001) establishes a system of degrees as intervals, for instance, in which he distinguishes between so-called positive grades (intervals extending from the scale's starting point to their mid-point) and so-called negative grades (intervals extending from a mid-point to infinity). Kennedy's (2001) model comes with the advantage that it can exclude cross-polar anomalies, like

(31)   *Rudolf is taller than Mary is short.   (modified from Solt 2013: 12, ex. (23))

Those two kinds of degrees, i.e. being tall and being short, must not be compared under Kennedy's (2001) account, which is quite a nice result anyway. But further, intervals can also be treated as sets of degrees-as-points, cf. Büring (2007), Beck (2011).

Another possible variation concerning degrees is whether they have to be dense or discrete (cf. Fox & Hackl's 2006 view on that issue, that degrees should always be linked to density). Dense degrees, rather than discrete ones, work out pretty well for a wide range of dimensions of measurement, such as the cardinalities of numbers.[10] Fox & Hackl (2006) can explain a range of phenomena involving a maximality operator. A further, closely related aspect with regard to degrees is granularity of the scale (cf. Krifka 2007). Take, (32), for instance. (32-a) has an approximate reading, giving the hearer to understand that the numerical value mentioned in the utterance, i.e. 100, doesn't have to match the actual number of journalists who did attend the press conference. Contrary, (32-b) has a precise reading, for which

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[10]A question immediately arising is, whether we should take numbers as a kind of default measurement. Strictly speaking, are we talking about numerals anyway or rather about numerical values? So far, there's no common agreement about this issue in the literature (as far as I know). Nevertheless, a property like beauty (which can be subject to comparative constructions, hence – probably – refers to a scale) cannot fully be measured in terms of numerical values or numerals.

we wouldn't assume that the numeral mentioned by the speaker doesn't match the actual numerical value of journalists at the press conference.

(32)   a.   There were 100 journalists at the press conference held by the ÖVP.

        (= APPROXIMATE VALUE)

        b.   There were 99 journalists at the press conference held by the ÖVP.

        (= PRECISE VALUE)

That is, *100* in (32-a) denotes an approximate value rather than an exact value (let's say, for instance, if the speaker being a journalist himself didn't count how many workmates of his' were around, but rather only noticed that there have been many of them) – Contrary to (32-b) which (usually) lacks a reading that states an approximate number rather than a precise one. So far, no agreement has been reached in the literature as to whether granularity is a semantic or pragmatic phenomenon.

Turning to the next component of scales, the ordering relation $\succ$, a possible parameter to variation is its directionality. That is, whether a bi-directional inference as in (33) is possible or not.

(33)   Rudolf is taller than Mary $\Longleftrightarrow$ Mary is shorter than Rudolf

Because there are some gradable expressions which don't allow for such a bi-directionality.

(34)   Stephanie is more sorrowful than Franz Josef $\nLeftrightarrow$ Franz Josef is happier than Stephanie

In a context where Stephanie and Franz Josef just received the message that her husband/his son, Rudolf, shot himself and his lover Mary – with Stephanie showing more signs of devastation than Rudolf's father Franz Josef – it would be kind of weird to conclude that Franz Josef is in a better mood than Stephanie.

Variation concerning $\succ$ can also occur in terms of ordering strength. In a word, scales do not have to be ordered so strictly, at least not along the precise lines of the mathematical

operator $\succ$. That is, we don't have to have an ordering like $a \succ b$, we could also have one like $a \succ b + \epsilon$ – Such an ordering is called a semi-ordering, according to Luce (1956).

The last scale component we haven't been considering so far is DIM. The dimension makes the clearest distinction between different kinds of scales. So, does it make sense to claim that there are certain subtypes (occurring occasionally) to which grammar is sensitive? We've seen some possibilities, like numerical values, already. Dimensions referring to numerical values work differently than scales without a dimension of measurement which refers to numerals. Let's take a look at (34) again. There we said that, contrary to, for instance, *tall/short*, there's no such bi-directional inference for *happy/sad*. Linking those pairs to the dimension of measurement, we can refer to *tall/short* as an evaluative dimension, whilst *happy/sad* is an unevaluative dimension.

The crucial difference between semantic scales we've discussed in this subsection and pragmatic scales (the latter to be taken into consideration in the last subsection of this section) is this: Pragmatic scales, i.e. so-called Horn-scales (cf. Horn 1972), arise through and are formed by certain expressions, such as quantificational elements. Semantic scales, though, are part of the domain which provides a content for those expressions. The adjectival Horn-scale for *hot* could look like $\langle cold, lukewarm, warm, hot, scalding \rangle$. Stating *The water is hot* yields the pragmatic inference that the water is neither cold, lukewarm, warm or scalding. The semantic (part of the) scale provides the specific ordering of the elements on the Horn-scale – *hot* refers to a higher temperature value than *warm*, a lower temperature value than *scalding* and so on and so forth. For things to become clearer, especially with regard to pragmatic scalar implicatures, I'll summarize the main insights of an influential paper by Sauerland (2004) on that topic after discussing further issues concerning degrees along the lines of Hackl (2001, 2009).

1.3.2 A brief Note on Degrees: Hackl 2001, 2009

The research field of degree semantics is mainly concerned with degree expressions such as gradable adjectives (e.g. *tall, small* and *taller than, smaller than* respectively) or number words. Besides the common basic semantic types $\langle e \rangle$ (for individuals) and $\langle t \rangle$ (for truth values), degree semantics identifies another basic type, $\langle d \rangle$ for degrees (as was already mentioned in the previous subsection). Numerals – a special case of degrees – are supposed to be arranged on a scale according to an arrangement relation $<$.

According to Cresswell (1977), a gradable adjective like *tall* expresses a relation between an individual/an entity and a certain degree. We can formalize this relation as

(35) $[\![\text{tall}]\!] = \lambda d \lambda x.\ \text{tall}(x,\ d)$

or, as Hackl (2001) puts it:

(36) $[\![\text{tall}]\!] = \lambda d \lambda x.x$ is d-long   (from Hackl 2001: 24, ex. (23a))

        Meaning: The denotation of *tall* is the characteristic function from (a set of) degrees to the characteristic function of a set of individuals/entitities

Degree-scales are downward-monotone. That is, if an individual/an entity has a certain degree of, let's say, height, this individual/entity also has every lower degree of height. If for instance an individual called Oliver is 172cm tall, he's also 171cm, 170cm, 169cm etc. tall. Hence, if

(37)   Oliver is 172cm tall.

is true, it is also true that Oliver is 171cm, 170cm, 169cm etc. tall, for every degree smaller (or: lower on the height-scale on which the degrees are ordered) than 172cm. Hackl (2001) assumes *172cm* in examples like (37) to be taken as a measure phrase that refers to a certain degree, i.e. 172cm, on the scale.

I want to end this subsection with a few remarks on Hackl's (2001) analysis of comparative

constructions, since his assumptions will be of some relevance for Nouwen's (2010) proposal (to be discussed in section 2.4). Hackl (2001) gives the paraphrases (38-b) and (38-c) for the sentence (38-a).

(38) a. John is taller than six feet.   (from Hackl 2001: 23, ex. (22a))

   b. 'John's height exceeds 6'.'   (from Hackl 2001: 23, ex. (22b))

   c. 'There is a degree d such that John is tall to that degree and d is greater than 6 feet.'   (from Hackl 2001: 23, ex. (22c))

In (38-a) which contains the comparative form of the adjective *tall*, John's height – although not explicitly mentioned in terms of measurement – is compared to *six feet*, due to the *than*-part of the sentence. Hackl (2001) claims that the paraphrase in (38-c) is more accurate with respect to the actual syntax of (38-a). Furthermore, Hackl (2001) detects three crucial components of the comparative construction:

(39) a. A gradable predicate or degree function expressed by *tall*

   b. An expression referring to a degree that provides the standard of comparison, i.e. *six feet*

   c. A comparative relation   (from Hackl 2001: 23)

Another important (morphological) element from (38-a) is the morpheme *-er*, used to create an adjective's comparative form. Hackl (2001) assumes that *-er* introduces a degree quantifier and a comparative relation. The degree quantifier is base-generated in an argument position of the adjective *tall* and then moves up to a higher node, leaving a trace of the same type (i.e. type $\langle d \rangle$) behind which is bound by the λ-operator. Movement of the degree quantifier is essential for the analysis, since it creates a derived (degree) predicate.[11] That is, we arrive at the following LF for (38-a):

(40) [-er than 6 feet]$_1$ [John is d_1-tall]   (from Hackl 2001: 25, ex. (25b))

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[11]For independent motivation of such a step and an introduction to the underlying mechanisms, cf. Heim & Kratzer (1998).

The morpheme for comparatives, *-er*, is assumed to involve a quantificational force – In the literature, this quantificational force has been claimed to be existential, universal or maximal, depending on the prevailing analytical proposal. Hackl (2001) adopts Heim's (2000) maximality operator for his purposes.

(41)   Definition: Maximality$_\leq$

$\text{max} = \lambda D_{\langle d,t \rangle}$. the unique d such that $D(d) = 1$ & $\forall d'[D(d') = 1 \to d' \leq d]$

<div align="right">(from Hackl 2001: 27, ex. (27))</div>

Since, for the case of *at least*, we're dealing with superlative morphology – that is, the superlative morpheme *-(e)st* instead of the comparative morpheme *-er* for adjectives – I want to mention a few remarks on that too. In Hackl (2009), it is assumed that *-est* is (just as its comparative counterpart) also a degree quantifier. A covert variable C, that restricts the degree quantifier, is responsible for the availability of a comparison class. Hackl (2009) formalizes his attempt as follows:

(42) $[\![-\text{EST}]\!](C)(D)(x) = 1$ iff $\forall y \in C[y \neq x \to \text{max}\{d:D(d)(x) = 1\} >$

$\text{max}\{d:D(d)(y) = 1\}]$

Meaning: [-EST C] combines with a degree property D of type $\langle d, \langle e,t \rangle \rangle$ and yields a predicate that is true of a given x if x has the degree property to a higher degree than any alternative to x in C (from Hackl 2009: 79)

Note that the comparison class introduced by C has to have at least two members which are distinct of each other. With regard to an interpretable LF, [-EST C] has to move out of its base-position (that is, an AP adjacent to its modified NP), just like it was assumed for comparative *-er*.[12]

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[12]Although the two movement operations of comparative *-er* and superlative [-EST C] yield the same effect – an interpretable LF-structure –, there's one important difference between the movement of *-er* and the movement of [-EST C]: Sentences containing superlatives or rather superlative adjective forms usually have two distinct readings – First, an absolute reading, for which [-EST C] moves to a position inside the DP that contains [$_{NP}$ AP NP]. Second, a relative reading, for which [-EST C] moves to a higher, VP-external

Since I will briefly repeat some of Hackl's (2001) assumptions illustrated in this subsection as soon as they will become relevant for the analysis (hence, in 2.4 where I will discuss Nouwen's 2010 account), I want to leave it at that for now. In the next subsection, I will summarize the main ideas of Sauerland's (2004) mechanism to generate scalar implicatures.

1.3.3 A Mechanism to generate Implicatures: Sauerland 2004

In his 2004 paper, Sauerland introduces an account of implicature generation. Since it has been adopted to plenty of those analysis we'll discuss in chapter 2, I'll give a brief and simplified summary of the Sauerland's (2004) main points and strategies.

Starting out from problems with the generation of adequate implicatures arising for disjunctive sentences, Sauerland (2004) proposes that the relevant scale for disjunction does not only consist of $\langle or, and \rangle$, but rather contains the prevailing disjuncts as well. That is, for a sentence like (43-a), we receive the set of alternatives in (43-b).

(43)   a.   Bob smokes or drinks.

        b. $\langle s \vee d, s, d, s \wedge d \rangle$

A scale for alternatives arising through disjunction can hence be thought of as a set containing the disjunction and the conjunction of the disjuncts, as well as the prevailing disjuncts themselves. But it's not only this particular scale which poses problems with regard to implicatures – Furthermore, we cannot solve those problems in terms of additional scale members. Consider a Horn-scale for quantificational expressions, for instance.

(44) $\langle every/all, most, many, some, few \rangle$

Horn (1972) assumes that scalar implicatures come into play through a syntactic substitution

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position (for instance, a VP-adjunct position). These assumptions are based on a movement-account for superlatives that claims that one or the other reading arises depending on [-EST C]'s landing site (cf. Szabolsci 1986, Heim 1995, or more recent evidence from Syrian Arabic in Hallman 2016). Since this MA-Thesis isn't concerned with the derivation of the absolute and relative reading of superlatives, the reader is referred to the given references for an extensive discussion of this topic.

mechanism. The generation mechanism for implicatures proposed by Gazdar (1979) involves two steps: First, the scalar element from the actual utterance gets replaced with the next logically stronger alternative from the prevailing scale. Second, the sentence is negated.

(45)   Some chemists like the smell of nitric acid.

      ⤳ Not all chemists like the smell of nitric acid.

Putting syntactic issues aside, the main complication behind Gazdar's (1979) strategy of computing implicatures comes from embedding the scalar term under negation. Gazdar's (1979) theory doesn't predict any implicatures for such sentences. For sentences in which the scalar expression occurs in the scope of another logical operator – for instance, disjunction – implicatures are only predicted for scalar alternatives of the logical operator (if there are any). That is, (46-a) has the implicature in (46-b) according to Gazdar's (1979) account, while it should have the implicature given in (46-c) with respect to the scalar term *some* occuring embedded under disjunction.

(46)   a.   Kai had the broccoli or some of the peas last night.

                                      (from Sauerland 2004: 370, ex. (5))

      b.   ⤳ Kai didn't have the broccoli and some of the peas last night

                                        (from Sauerland 2004: 370, ex. (6))

      c.   ⤳ Kai did not have all of the peas last night.   (from Sauerland 2004: 370, ex. (8))

Coming back to the problems with sentences involving negation and the generation of an adequate implicature for such examples, Sauerland (2004) states that negation reverses the ordering on the scale in question (cf. Atlas & Levinson 1981, Horn 1989). Hence, to yield a stronger assertion, one has to replace the scalar expression with an alternative lower on the scale, rather than replacing it by a higher element and let the two occurrences of negation cancel each other out. But that would imply that we'd have to figure out whether a scalar element occurs in an upward-, or downward entailing context each time, testing which re-

placement yields the stronger assertion then.

Sauerland (2004) claims the following conditions for an expression to be such that it can be replaced by another expression from the same scale:

(47)   A sentence ψ is a one-step scalar alternative of ϕ iff

    a. $\phi \neq \psi$ (from Sauerland 2004: 374: ex. (15a))

    b.   there are scalar expressions α and α' which both occur on the same scale C such that ψ is the result of replacing one occurrence of α in ϕ with α'

                                                                        (from Sauerland 2004: 374, ex. (15b))

(48)   A sentence ψ is a scalar alternative of ϕ if there is a sequence $(\phi_0, ..., \phi_n)$ with $n \geq 0$ and $\phi_0 = \phi$ and $\phi_n = \psi$ such that, for all i with $1 \leq i \leq n$, ϕ_i is a one-step scalar alternative of ϕ_{i-1} (from Sauerland 2004: 374, ex. (16))

A scalar alternative only comes about as an implicature if it's logically stronger than the assertion.

(49) $\neg\psi$' is a scalar implicature of ψ iff

    a. ψ' is a scalar alternative of ψ (from Sauerland 2004: 374, ex. (17a))

    b. ψ' entails ψ (from Sauerland 2004: 374, ex. (17b))

    c. ψ does not entail ψ'   (from Sauerland 2004: 374, ex. (17c))

Beyond the mechanism's outline sketched above, we'll be mainly concerned with the following two computations:[13]

(50)   a.   If $\psi \in$ ScalAlt(ϕ) and $\psi \Rightarrow \phi$ and not $\phi \Rightarrow \psi$, then $\neg K\psi$ is a primary implicature of ϕ (from Sauerland 2004: 383, ex. (42))

       b.   If $\neg K\psi$ is a primary implicature of ϕ and $K\neg\psi$ is consistent with the conjunction of ϕ and all primary implicatures of ϕ, then $K\neg\psi$ is a secondary implicature of ϕ (from Sauerland 2004: 383, ex. (43))

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[13]K is to be read as *The speaker is certain that...*

That is, Sauerland (2004) distinguishes between implicatures arising from the Gricean Maxim of Quantity, i.e. (50-a), stating that the speaker is uncertain that ψ; and implicatures which were originally taken to be of some importance for disjunction, (50-b), stating that the speaker is certain that not ψ.

1.4 Interim Summary

We touched upon various issues in this first chapter. First of all (section 1.1), it was shown why Generalized Quantifier Theory (GQT) doesn't suffice to explain *at least*'s meaning – In terms of truth conditions, GQT makes no difference between NPs with bare numerals, such as *three strippers*, and NPs with more complex expressions, like *at least three strippers*. According to GQT, both *three* and *at least three* may have more than three elements in the intersection of the denotation of their verbal and nominal predicate (each denoting a set of individuals). With respect to *at least*'s syntactic distribution – hence, *at least*'s ability to combine with various syntactic/semantic types of expressions – the quantificational status of *at least* is furthermore at least questionable. In section 1.2, a classification along the lines of Nouwen (2010) for *at least* among the class of numeral modifiers was given. On the basis of four puzzles from Geurts & Nouwen (2007), I illustrated why the superlative numeral modifier *at least n* and its comparative counterpart *more than n-1* are not interdefinable expressions. Finally, in the last section of this introductory chapter, essential components and structural differences among semantic scales were illustrated (1.3.1). I briefly explained some important mechanism concerning degrees in section 1.3.2, and introduced an implicature generation mechanism from Sauerland (2004) in section 1.3.3.

In the next chapter, I will review and discuss different accounts on *at least* from the literature. Especially Büring's (2008) and Geurts & Nouwen's (2007) attempts will be of some importance for my own analysis of German *at least*-kind expressions in chapter 3 of this MA-Thesis.

CHAPTER 2

ANALYSIS OF *AT LEAST* FROM THE LITERATURE

In this chapter, I will give an overview of several different accounts worked out within the past decades, paying particular attention to the prevailing analysis of *at least*'s meaning, and the alternatives/inferences that arise from *at least*-sentences, respectively. That is, I won't go into those parts of the attempts concerning *at most* and other upper-bounded quantifiers.[1] As far as possible, I will discuss some attempts' pros and cons and whenever possible, I will give own examples instead of just quoting those one can find in the literature anyway.[2] We'll start with Krifka's (1999)[3] article, resuming the discussion in section 1.1.

2.1 Krifka 1999

Facing the problems an interpretation of numeral modifiers in terms of the GQ Theory brings up, Krifka (1999) proposes a new analysis. Krifka (1999) suggests that a possible way to interpret *at least* is as a kind of numeral adjective, which denotes a sum of individuals. The crucial point here is, that Krifka (1999) doesn't assume a set of individuals like {John, Mary, Hubert, Brunhilde, ...}, but rather supposes that the individuals under discussion are atoms

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[1]Sometimes in the relevant literature (in Büring 2008, for instance), it is stated that one can't define *at least* without considering *at most*. While this might turn out true for some cases, it will serve for the analysis given in chapter 3 to concentrate on the *at least*'s interpretation. Or rather, I will cite relevant aspects of *at most*'s interpretation if needed for the one of *at least* – But please note that this MA-Thesis is not supposed to be about numeral modifiers in general, but will be truly dealing with *at least* and three closely related German expressions (so far, as to my knowledge, not discussed in the literature at the time of writing).

[2]I will only replace the initial examples if my own examples don't intervene with respect to the intended interpretation/meaning/analysis. Just to make sure – It's not that I consider my own examples to be better or anything like that, I just appreciate a (hopefully) welcome change, so I will use *modified from...* to indicate that I actually made a change, which might be either a slight modification like changing names/letters or a more radical change like replacing the initial sentence from the literature with another one, considering to maintain the point the author(s) wants to make.

[3]Since I only have an early version of Krifka's (1999) article, the numbering of pages and examples probably differs from the numbering in the final version.

of a sum individual, forming a singleton set. But such a procedure raises several complications, as Krifka (1999) points out. It would, for instance, predict that an *at least*-sentence like (1-a) evokes alternatives being such that we end up with the same truth conditions as for a bare numeral sentence like (1-d).

(1) a. At least three Jacobins recited at least nine Hail Mary.

(modified from Krifka 1999: 10, ex. (33a))

b. Meaning: $\exists x,y[\geq 3(x) \wedge$ JACOBINS(x) $\wedge \geq 9(y) \wedge$ HAILMARY(y) $\wedge$ RECITE(x,y)]

(modified from Krifka 1999: 10, ex. (33b))

c. Alternatives: $\{\exists x,y[\geq n(x) \wedge$ JACOBINS(x) $\wedge \geq m(y) \wedge$ HAILMARY(y) $\wedge$ RECITE(x,y)] $\mid$ n,m $\in$ N$\}$ (modified from Krifka 1999: 11, ex. (33c))

d. Three Jacobins recited nine Hail Mary.

Another phenomenon standing in the way of analyzing *at least* as a numeral adjective is that the alternatives associated with *at least* don't have to be ordered in terms of logical/semantic strength. While we can assume an arrangement in relation to semantic strength for examples in which *at least* combines with a numeral, we're faced with an ordering unrelated to the specific strength of the alternative expressions in an example like

(2) Mary is at least an associate professor (perhaps even a full professor).

(from Krifka 1999: 11, ex. (34))

Alternatives of the first clause: $\langle$\{Mary is an assistant professor\}, \{Mary is an associate professor\}, \{Mary is a full professor\}, ...$\rangle$

To account for the fact that alternatives don't have to be ranked by their semantic strength, Krifka (1999) cites the definition in (3), which shows a partial ordering relation – $\leq$ – among the alternatives. The set of alternatives is to be taken as the field of $\leq$, and can be derived from the partial ordering relation.

(3) Field($\leq$) = $\{$x $\mid \exists y[x \leq y \vee y \leq x]\}$ (from Krifka 1999: 11, ex.(35))

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To my mind, we might of course not call one alternative semantically stronger than the other in (2), but there is at least some kind of rank relation between the alternative expressions. But what about sentences which do not seem to impose any kind of specific arrangement at all?

(4)   At least Bonnie and Clyde were arrested (perhaps also Billy the Kid).

Possible alternatives for the first clause: ⟨{Bonnie and Clyde and Billy the Kid were arrested}, {Bonnie and Clyde and Django were arrested}, {Bonnie and Clyde and Turbot Tom were arrested}, ...⟩

For cases such as (4), Krifka (1999) assumes a part relation of sum individuals for the alternatives, $\leq_i$ – It says that Bonnie is a part of the sum individual consisting of Bonnie and Clyde, which, in turn, is a part of the sum individual consisting of Bonnie, Clyde and Billy the Kid and so on and so forth.

If we assume that the alternatives are arranged in one way or the other, they should also project in this certain order. Expressions not introducing a proper ordering relation of the induced alternatives are to be thought of as forming a pair with their proper meaning. Krifka (1999) names them 'standard alternatives' – The alternative set also contains the proper meaning of the expression, in addition to the proper meaning pair. Krifka (1999) gives a rule for projection (= (5)), an example of how it works is given below.

(5)   If $[[\alpha\ \beta]] = f([[\alpha]], [[\beta]])$,

then $[[\alpha\ \beta]]_A = \{\langle f(X, Y), f(X', Y')\rangle \mid \langle X, X'\rangle \in [[\alpha]]_A$ and $\langle Y, Y'\rangle \in [[\beta]]_A\}$

(from Krifka 1999: 8, ex. (29))

(6)   a.   At least [three]$_F$ strippers got undressed.   (modified from Krifka 1999: 11, ex. (38))

b. $[[three_F]] = \lambda P\lambda x[3(x) \wedge P(x)]$

$[[three_F]]_A = \{\langle\lambda P\lambda x[n(x) \wedge P(x)], \lambda P\lambda x[m(x) \wedge P(x)]\rangle \mid n \leq_N m\}$

(modified from Krifka 1999: 11, ex. (39a))

c. $[\![\text{strippers}]\!] = \text{STRIPPERS}$

$[\![\text{strippers}]\!]_A = \{\text{STRIPPERS}, \langle\text{STRIPPERS}, \text{STRIPPERS}\rangle\}$

(the standard alternatives)   (modified from Krifka 1999: 11, ex. (40b))

Applying *at least* to α at this level of interpretation gives us a new meaning, which consists of the union of α's alternatives – I.e., we end up with a singleton-set, no new alternatives are projected. The definition of this special kind of union operation is given in (7). Since we want to get the same effect for the semantic union of the elements of a sentence if the elements' meanings are functions, Krifka (1999) adopts the generalized join operation in (8) from Keenan & Faltz (1985). An example of how this works out is stated below the definitions.

(7) $[\![\text{at least } \alpha]\!] = \bigcup \{P \mid \langle[\![\alpha]\!], P\rangle \in [\![\alpha]\!]_A\}$

      $[\![\text{at least } \alpha]\!]_A = $ the standard alternatives   (from Krifka 1999: 12, ex. (41))

(8)   Join Operation

    a.   If Φ, Ψ are sentences (of type $\langle t\rangle$), then $[\![\Phi]\!] \cup [\![\Psi]\!] = [\![\Phi \vee \Psi]\!]$

    b.   If α, β are expressions of type $\langle\sigma, \tau\rangle$, then $[\![\alpha]\!] \cup [\![\beta]\!] = \lambda X[[\![\alpha]\!](X) \cup [\![\beta]\!](X)]$

    c.   If S is a set of meanings of a type that can be conjoined by $\cup$,

       then $\bigcup$ S is the result of conjoining all the elements of S by $\cup$

                                (from Krifka 1999: 12, ex. (42))

(9) $[\![\text{at least } [\text{three}_F \text{ strippers}]]\!]$

    $= \bigcup \{P \mid \langle\lambda x[3(x) \wedge \text{STRIPPERS}(x)], P\rangle \in$

    $\{\langle\lambda x[n(x) \wedge \text{STRIPPERS}(x)], \lambda x[m(x) \wedge \text{STRIPPERS}(x)]\rangle \mid n \leq_N m\}\}$

    $= \bigcup \{\lambda x[m(x) \wedge \text{STRIPPERS}(x)] \mid 3 \leq_N m\}$

    $= \lambda x [\geq 3(x) \wedge \text{STRIPPERS}(x)]$

    $[\![\text{at least } [\text{three}_F \text{ strippers}]]\!]_A = \{\langle\lambda x[\geq 3(x) \wedge \text{STRIPPERS}(x)],$

    $\lambda x[\geq 3(x) \wedge \text{STRIPPERS}(x)]\rangle\}$ (modified from Krifka 1999: 12, ex. (39b))

Crucially, the meaning of a sentence like (6-a) therefore differs from the meaning of the same sentence with a bare numeral in that no alternatives arise. The interpretation we get under the numeral modifier must somehow be built in its semantics, according to Krifka (1999). But what would happen if we assume focus on *strippers*, to make somebody understand that maybe even some of the guests got naked? How can we account for this meaning? Krifka (1999) does so by assuming the part relation for individuals, $\leq_i$, also for sets, defined as:

(10)   If P and Q are sets, then P $\leq_i$ Q iff $\forall x \in P \ \exists y \in Q[x \leq_i y]$

<div align="right">(from Krifka 1999: 14, ex. (48))</div>

So, we end up analyzing (11-a) as in (11-b). The crucial point here is: Krifka (1999) takes *three strippers* $\leq_i$ *three strippers and a guest*, that is, he assumes *three strippers* to be an individual part of *three strippers and a guest*. But this alternative is 'used' by the numeral modifier and gets eliminated then, i.e. it doesn't project.

(11)   a.   At least [three strippers]$_F$ got naked.

            Intended meaning: Three strippers and perhaps *n* guest(s) got naked.

         b. $[\![[[\varnothing$ at least [three strippers]$_F]_1$ [t_1 got naked]$]\!]\!]$

                $= \exists x \exists Q[\lambda y[3(y) \wedge \text{STRIPPERS}(x)] \leq_i Q \wedge Q(y) \wedge \text{GOT-NAKED}(y)]$

                $[\![[[\varnothing$ at least [three strippers]$_F]_1$ [t_1 got naked]$]\!]\!]_A =$ the standard alternatives

<div align="right">(modified from Krifka 1999: 14, ex. (49b'))</div>

To account for the following analysis, I have to briefly sketch Krifka's (1999) principle of cumulativity, because it is needed for Krifka's (1999) treatment of constructions in which *at least* combines with another syntactic/semantic type of expression than a numeral. Cumulativity is a particular way to interpret verbal predicates. Take an intransitive verb α, for instance – If α gets applied to two individuals x and x', it may also be applied to the sum of those individuals. Krifka depicts a sum individual by the symbol $\oplus$. Hence, if a verb like *sleep* is true of *Peter* and true of *Paul*, it will also be true of *Peter* and *Paul*:

(12)   Peter is sleeping and Paul is sleeping.

        → Peter and Paul are sleeping.

Transitive predicates behave more or less in the same way – i.e., a transitive predicate β applying to x and y, furthermore to x' and y', ends up applying to their sum x⊕x' and y⊕y'. (13) can be considered as true if Peter is drinking vodka and Paul is drinking eggnog.[4]

(13)   Peter and Paul drink vodka and eggnog.

Generalizing this principle of ⊕, Krifka (1999) defines cumulativity as in (14).

(14)   An n-place predicate R is cumulative iff the following holds:

        If $R(x_1, ... x_n)$ and $R(x_1', ..., x_n')$, then $R(x_1 \oplus x_1', ..., x_n \oplus x_n')$.

                                                        (from Krifka 1999: 6, ex. (21))

Let's consider an example with *at least* now – specifically, one with focus on the NP:

(15)   At least [Fritz and Gerald]$_F$ drink.

Given the principle of cumulativity and the part relation on individuals, we arrive at:

(16)   a. $[\![_{NP}$ Fritz and Gerald$]_F]\!] =$ FRITZ⊕GERALD

              $[\![_{NP}$ Fritz and Gerald$]_F]\!]_A = \{\langle$ x, y$\rangle \mid x \leq_i y\}$

                                                        (modified from Krifka 1999: 13, ex. (46b))

        b.   type-shifting from type $\langle$e$\rangle$, individuals, to type $\langle\langle$e, t$\rangle$, t$\rangle$, quantifiers – This step is essential because Krifka's ⋃ is not defined for elements of type $\langle$e$\rangle$

              $[\![_{NP}$ Fritz and Gerald$]_F]\!] = \lambda P[P($FRITZ⊕GERALD$)]$

              $[\![_{NP}$ Fritz and Gerald$]_F]\!] = \{\langle$ $\lambda P[P(x)], \lambda P[P(y)]\rangle \mid x \leq_i y\}$

                                                        (modified from Krifka 1999: 13, ex. (46c))

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[4]The following example has other readings as well, but Krifka (1999) assumes that sum formation isn't order-sensitive and substantiates this claim by the observation that only under the intended reading, a continuation of the form *More specifically, x V-s x' and y V-s y'* is possible.

c. ⟦at least [Fritz and Gerald]$_F$⟧

$= \bigcup \{P \mid \langle\lambda P[P(\text{FRITZ}\oplus\text{GERALD})], P\rangle \in \{\langle\lambda P[P(x)], \lambda P[P(y)]\rangle \mid x \leq_i y\}$

$= \bigcup \{\lambda P[P(y)] \mid \text{FRITZ}\oplus\text{GERALD} \leq_i y]$

$= \lambda P \exists y[\text{FRITZ}\oplus\text{GERALD} \leq_i y \wedge P(y)]$

$=$ the standard alternatives   (modified from Krifka 1999: 13, ex. (46d))

As Krifka (1999) notes, the meaning we get is such that it states that a sum individual –
containing Fritz and Gerald in this particular example – drinks. The question how other
individuals become a member of this sum individual, i.e. what kind of algorithm or what so
ever should be used to get them in, remains open.

Krifka's (1999) treatment of *at least*-sentences that give rise to alternatives that are organized
via a hierarchical relation gives us a desirable outcome. Consider the example

(17)   Mary is at least [an associate professor]$_F$.   (from Krifka 1999: 14, ex. (51a))

again. (17) says that Mary is either an associate professor or some higher rank. Notice
that she cannot be both at the same time. Krifka (1999) gives the following set of ordered
alternatives to the focused expression in (17)

(18)   ⟦[an associate professor]$_F$⟧$_A$

$= \{\langle\text{ASSIST.PROF, ASSIST.PROF}\rangle, \langle\text{ASSIST.PROF, ASSOC.PROF}\rangle, \langle\text{ASSIST.PROF, FULLPROF}\rangle,$

$\langle\text{ASSOC.PROF, ASSOC.PROF}\rangle, \langle\text{ASSOC.PROF, FULLPROF}\rangle, \langle\text{FULLPROF, FULLPROF}\rangle\}$

$= \leq_{\text{PROF}}$ (from Krifka 1999: 14, ex. (51b))

and finally arrives at

(19) $\exists Q[\text{ASSOC.PROF} \leq_{\text{PROF}} Q \wedge Q(\text{MARY})]$ (from Krifka 1999: 14, ex. (51c))

for (17), a seemingly nice result, although I consider the $\leq_{\text{PROF}}$ relation as a little bit
problematic. Maybe it should be replaced with a more general mechanism that suffices to
deal with the prevailing relation.

2.2 Büring 2008

Büring (2008) claims that the meaning of *at least* can be properly captured as *exactly n or more than n*. His assumption is mainly based on the observation, that *n, more than n* and *at least n* all evoke different implicatures, if any.

(20)   a.   Paul has four guitars. ↝ Paul does not have more than four guitars.

<div align="right">(from Büring 2008: 1, ex. (2))</div>

        b.   Paul has more than three guitars. ↝ NO IMPLICATURES

<div align="right">(from Büring 2008: 1, ex. (3))</div>

        c.   Paul has at least four guitars. ↝ Paul may have more than four guitars.

<div align="right">(from Büring 2008: 1, ex. (4))</div>

Therefore, (20-c) in its literal meaning can be analyzed as:

(21)   Paul has exactly four guitars OR Paul has more than four guitars.

<div align="right">(from Büring 2008: 2, ex. (6))</div>

or, generally speaking, we can treat the superlative modifiers like

(22)   at least n p q = exactly n p q OR n' p q, where $n < n'$ (from Büring 2008: 2, ex. (5))

a combination of the meaning of *exactly n* and *more than n*. This definition involves one crucial element – disjunction. According to Grice (1975), a speaker uses disjunction if he or she isn't sure about whether p or q is true. Since the literal meaning of the *at least*-sentence in (21) involves disjunction, we can conclude that by uttering (20-c), the speaker wants to convey that he or she isn't certain that Paul owns exactly four guitars and he or she isn't certain that Paul owns more than four guitars. But only one of the two parts – that is, either *Paul has exactly four guitars* or *Paul has more than four guitars* – can actually be the case at the same time in the actual world. Büring (2008) draws an inference to the implicature typically associated with disjunction, and slightly modifies it to capture the meaning just

stated for *at least.*

(23)   Global Implicature Schema: If a speaker utters *p or q*, it is implied that (i) in all of

the speakers doxastic alternatives $q \lor p$, and (ii-a) in some of the speaker's doxastic

alternative *p*, and (ii-b) in some *q*.   (from Büring 2008: 2-3, ex. (8/10))

On the part of the speaker, uttering a sentence containing *at least n* comes along with being

uncertain about whether *exactly n* or *more than n* would hold in a given situation/context.

According to the global implicature schema, we therefore get the implicature shown in (24-b)

for the *at least*-sentence in (20-c), repeated here:

(24)   Paul has at least four guitars.

a.   lit.: Paul has exactly four guitars or Paul has more than four guitars

(from Büring 2008: 3, ex. (11a))

b.   ↝ the speaker is certain that Paul has four guitars, and considers it possible

that Paul has exactly four guitars, and considers it possible that Paul has more

than four guitars   (from Büring 2008: 3, ex. (11b))

When combining with a modal expression, Büring (2008) furthermore identifies two readings

for *at least* which he calls the authoritative and the speaker insecurity reading. In (25-a) and

(25-b), I illustrate the two readings with examples, also giving the relevant implicatures:

(25)   a.   (*Context: Sam and Harry want to beat the world record in baking the biggest*

cream gateau. So far, the record is held by a french confectioner, Pierre Briece,

who baked a cream gateau with three meters in diameter in 1926. Being aware

of this fact, Sam and Harry say to each other:) Our cream gateau must be at

least 4meters in diameter.

↝ the challenger cream gateau must be (at least) 4meters in diameter AND it

is allowed to be exactly 4meters in diameter AND it is allowed to be more than

4meters in diameter (for Sam and Harry would still be the new recordholders

then) (= AUTHORITATIVE READING)

b. (*Context: MaryAnn and Peggy Sue walk down Broadway in Nashville, noticing a poster announcing a Brad Paisley concert next week at Ryman's. The girls are big fans, but the name of the location makes them wonder whether they will be able to afford two tickets – because for them, there's too much month at the end of the money and maybe they'd only get some leftover tickets at the overpriced black market. Pessimistic Peggy Sue says:*) To attend this concert, we'll have to pay at least 300$ each.

⤳ to attend the Brad Paisley concert at the Ryman theatre they have to pay (at least) 300$ each AND perhaps they have to pay exactly 300$ each AND perhaps they have to pay more than that (= SPEAKER INSECURITY READING)

While the implicature given for (25-b) is already pretty close to the implicature schema given above in (24-b), (25-a) involves deontic possibilities rather than epistemic or doxastic ones, due to the occurrence of the epistemic modal *must*. This doesn't always have to be the case – A pragmatic bias, materialized by the context, aside, we could also get both readings for *at least + must*.

(26)   Nurse Lilly has to x-ray at least three patients.

(modified from Büring 2008: 4, ex. (15))

a.   It has to be the case that Lilly x-rays three or more patients. (= AUTHORITATIVE READING) (modified from Büring 2008: 4, ex. (15a))

b.   Three or more is such that Lilly has to x-ray that many patients. (= SPEAKER INSECURITY READING) (modified from Büring 2008: 4, ex. (15b))

The crucial difference between these two readings for (26) comes from *at least*'s scope, which is below the modal expression in (26-a) and above the modal in (26-b). Whereas Krifka (1999) in his analysis (which did not touch upon examples in which *at least* combines with a modal) tried to prevent the superlative modifier's meaning from scoping at all, we have to

define a meaning for *at least n* that permits scope ambiguities caused by modal expressions. Büring (2008) defines the following meaning for *at least*:

(27)   for any set D of number/degrees, $[\![\text{at least n}]\!](D) = 1$ iff $n = max(d) \vee 3 > max(D)$

(from Büring 2007: 5, ex. (16))

Meaning: For any set D of numbers/degrees, applying D to the denotation of *at least n* will be true iff n is the highest number/degree of D or n is greater than the highest number/degree of D

Büring (2008) furthermore assumes that the superlative modifier is generated as a degree phrase specifier to an AP, which then moves to a propositional position, arriving at the following LFs for (26):

(28)   a.   Lilly has to [[at least three] [λd [t_{Lilly} x-ray d-many patients]]]

$\square[3 = max(\lambda$d. Lilly x-rays d-many patients) $\vee$

$3 > max(\lambda$d. Lilly x-rays d-many patients)]

Meaning: in every permitted world, the maximum number of patients Lilly x-rays is greater than or equal to three   (modified from Büring 2008: 5, ex. (18))

        b.   [at least three] [λd [Lilly has to [t_{Lilly} x-ray d-many patients]]]

$3 = max(\lambda$d. $\square[$Lilly x-rays d-many patients]) $\vee$

$3 > max(\lambda$d. $\square[$Lilly x-rays d-many patients])

Meaning: the maximum number such that in every permitted world, Lilly x-rays that many (or more) patients is greater than or equal to three

(modified from Büring 2008: 5, ex. (19))

The derivation of the speaker insecurity reading, i.e. (28-b) is straightforward, but a further step is needed to derive the desired implicatures for (28-a) – Because in the latter example, the disjunction is embedded under the (universal) modal. For cases like this, Büring (2008) uses the following local implicature scheme:

(29)   Local Implicature Schema, simplified: $\forall w \in R[p(w) \lor q(w)] \rightsquigarrow \exists w \in R[p(w)]$ and $\exists w$

$\in R[q(w)]$ (from Büring 2008: 6, ex. (21))

If we're not dealing with such a kind of embedding, R in (29) should be interpreted as set

of doxastic alternatives w, following the classical schema from (23) – I.e., according to an

utterance *p or q*, the hearer can deduce that either $q'(w) = 1$ or $p'(w) = 1$.[5] But if we do,

like in (25-a), R rather turns out as set of deontically accessible worlds, hence $\square$. Therefore,

we get the following meaning:

(30)   literally: in all deonticall accessible worlds, the challenger cream gateau is 4meters

in diameter or more $\rightsquigarrow$ in some deontically accessible worlds, the challenger cream

gateau is exactly 4meters in diameter and in some deontically accessible worlds it is

more than 4meters in diameter   (modified from Büring 2008: 6, ex. (22))

Summing up our observations, we have to move *at least* out of the modal's scope to get the

speaker insecurity reading, because otherwise only the local implicature schema in (29) can

apply instead of the global one, given in (23). So, if raising of the superlative quantifier is

blocked for independent reasons, we'd expect the speaker insecurity reading to vanish – and

that's exactly what happens.

(31)   a.   Jana had to pay at least 500$ illicitly to the ambassador to get her visa.

= AUTHORITATIVE or SPEAKER INSECURITY READING

(modified from Büring 2008: 7, ex. (24a))

b.   It was required that Jana paid at least 500$ to the ambassador to get her visa.

= AUTHORITATIVE READING only   (modified from Büring 2008: 7, ex. (24b))

This raises the question whether the authoritative reading can also be blocked in particular

constructions, and this is indeed the case. As Büring (2008) argues, the authoritative reading

arises because the disjunction is 'locked up' below the universal modal expression – Hence,

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[5]p', q' are the propositions expressed by p and q, respectively.

the implicature can only apply locally. This predicts that we should not get an authoritative reading with a possibility modal, which is not a universal.

(32)   a.   The challenger cream gateau can be at least 4meters in diameter.

b.   It is permitted to order at least three scoops of strawberry icecream.

So, (32-a) is a rather odd instruction to somebody who strives for a new world record, (32-b) turns out a little weird for pragmatic reasons alone. Unfortunately, Büring (2008) doesn't go into that topic any deeper, so for now, we'll have to leave it at that.

Turning to occurrences of *at least* with other types of expressions than numerals, Büring (2008) proposes an account along the lines of Krifka (1999) – I.e., some elements come with a scale naturally, others get them via focus and some elements might not be associated with a specific scale at all. The last-mentioned ones then evoke a trivial scale, or, in Krifka's (1999) terms, the standard alternatives.

Büring (2008) assumes pointwise combination of scalar implicatures of the immediate constituents of *at least* to form (scalar) alternatives to more complex expressions. Here are some examples:

(33)   a. $[\![\text{four}]\!]_A = \{\langle 1, 2\rangle, \langle 1, 3\rangle, \langle 2, 3\rangle, \langle 1, 4\rangle, \langle 2, 4\rangle, \langle 3, 4\rangle, \langle 1, 5\rangle, \langle 2, 5\rangle, \langle 3, 5\rangle,$
$\langle 4, 5\rangle...\}$ (from Büring 2008: 9, ex. (28a))

b. $[\![\text{Mary}_F]\!]_A = \{\langle\text{Mary, Mary}\oplus\text{Sue}\rangle, \langle\text{Mary, Mary}\oplus\text{Pete}\rangle, \langle\text{Mary, Mary}\oplus\text{Pete}\oplus\text{Karen}\rangle,$
$\langle\text{Mary}\oplus\text{Pete, Mary}\oplus\text{Pete}\oplus\text{Karen}\rangle, ...\}$ (from Büring 2008: 9, ex. (28b))

c. $[\![\text{second year student}]\!]_A = \{\langle\text{first year student, second year student}\rangle, \langle\text{second}$
year student, third year student$\rangle, ...\}$ (from Büring 2008: 9, ex. (28c))

To properly fix the meaning of *at least*, we further have to do two things: First, defining the set of scalar alternatives ABOVE(E) – where E might be any expression – which are higher than the meaning of A. Second, defining that *at least A* ought to be true if either $[\![A]\!]$ is true, but none of the higher alternatives – which captures the meaning of *exactly A* –, or one of

the higher alternatives is true – which furthermore captures the meaning of *(or) more than A –*, just as given in our implicature schema.

(34)   a.   for any expression E, ABOVE(E) $=_{\text{def}} \cup \{O' \mid \langle[\![E]\!], O' \in [\![E]\!]_A\}$

<div align="right">(from Büring 2008: 9, ex. (29))</div>

   b.   (for any q of type $\langle s, t \rangle$), $[\![\text{at least q}]\!] = [[\![q]\!] - \cup(\text{ABOVE}(q))] \vee \cup(\text{ABOVE}(q))$

<div align="right">(from Büring 2008: 9, ex. (30))</div>

Let's look at an example for things to become more illustrative.

(35)   a.   Helmut admires at least Brad Paisley.

   b. $[\![$at least [Helmut admires [Brad Paisley]$_F$]$]\!]$ = [H admires Brad Paisley and not Brad Paisley and Kenny Chesney, Tim McGraw and Gary Allan, or Brad Paisley, Kenny Chesney and Tim McGraw] OR [H admires Brad Paisley and Kenny Chesney, or Brad Paisley and Gary Allan, or Brad Paisley and Kenny Chesney and Tim McGraw, or ...]

   ≡ Helmut admires only Brad Paisley, or he admires Brad Paisley and someone else

<div align="right">(modified from Büring 2007: 10, ex. (32))</div>

The main points of Büring's (2008) analysis, repeated briefly, are: *At least* can be treated as a disjunction operator over scalar alternatives. Doing so enables us to pin down and explain not only simple examples with numerals and certain other expressions (combining with the superlative modifier), but also more complex sentences involving modals, for which two readings were diagnosed. An open question might be, how *at least* is supposed to trigger its implicatures, because we can't just apply the Gricean idea of attaching this implicature to the expression *or* here. And what about this mysterious *may* in the implicature part of (20-c)? Since it is a modal expression, shouldn't we somehow take it into consideration as well, when it comes to defining a possible meaning of *at least*? That's exactly what Geurts & Nouwen (2007) do, to whose paper I will turn in the next section.

2.3   Geurts & Nouwen 2007

Geurts & Nouwen (2007) start out from the fact that comparative and superlative modifiers behave differently, and they assume this difference to be anchored on the modifiers' lexical content. To plumb the depths of what specifically distinguishes class A numeral modifiers from class B numeral modifiers (cf. section 1.2.1), they list and discuss four puzzles – specificity, inference patterns, distributional restrictions and missing readings. Because I already attended to those in section 1.2.2, I will get into Geurts & Nouwen's (2007) analysis directly here.

2.3.1   Superlative Modifiers as modal Expressions

In the outline of their proposed analysis, Geurts & Nouwen (2007) present an example-pair like the following:

(36)   a.   More than three Stalinist soldiers deserted.

        b.   At least three Stalinist soldiers deserted.

Example (36-a) just says that the number of Stalinist soldiers who deserted exceeds three, but (36-b) seems to express two kinds of things: First, that there's a set of Stalinist soldiers such that each of them deserted – The cardinality of that set is *three*. Second, according to the interpretation of (36-b), possibly more than three Stalinist soldiers deserted. Geurts & Nouwen (2007) claim that these meaning-parts can be represented properly if we treat superlative modifiers as modal expressions. Contrary, comparative modifiers are not treated as modal expressions. Hence, (36-a)'s meaning can be illustrated as in (37-a), whereas (36-b) receives the interpretation shown in (37-b):

(37)   a. $\exists x[\text{soldier}(x) \wedge \#x > 2 \wedge \text{desert}(x)]$

               Meaning: there exists a group x such that the members of this group are soldiers, the cardinality of x being greater than two and each member of x deserted

b.　□ ∃x[soldier(x) ∧ 3(x) ∧ desert(x)] ∧ ◇ ∃x[soldier(x) ∧ #x >3 ∧ desert(x)]

　　Meaning: there must exist a group of three soldiers who deserted (first conjunct)

　　and the number of deserted soldiers may exceed three (second conjunct)

Since *less/fewer than* paired with *at most* obey the same interpretational principle (but in the other direction, hence <3), we can determine two properties of superlative modifiers – They construe some sort of cutoff point in claiming that there must be a group of ABs which has the cardinality n, and furthermore, they allow the existence of AB-groups with a cardinality beyond that cutoff point. That's what Geurts & Nouwen (2007) call the primary and secondary component of *at least* and *at most*. For the time being, they assume these components to be a part of the lexical meaning of superlative modifiers. According to this assumption, superlative modifiers can be understood as modal expressions, because they state that something MUST be the case (for instance, that the cardinality of x is n), while something else (for instance, a higher number of n in the given context) MAY be the case. The question is, how scalar implicatures come into play. Geurts & Nouwen (2007) more or less follow the ideas of Krifka (1999) here, i.e. SIs come about via focus on an expression α, leading to a set of alternatives to α's denotation, $[\![\alpha]\!]$.

(38)　a.　[Lotta]$_F$ drank instant coffee.

　　　　Meaning: Rather than somebody else among a set of possible individuals in a given context, Lotta drank instant coffee.

　　b.　[Four]$_F$ nuns were dancing.

　　c.　[Four$_F$ nuns] were dancing.

For natural numbers, we end up with a sequence like, for instance for (38-b), $\lambda x.6(x)$ ▷ $\lambda x.5(x)$ ▷ $\lambda x.4(x)$ ▷ ... .[6] According to Geurts & Nouwen (2007), there's no entailment

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[6]The ▷ sign is used to indicate the precedence relation.

relation between the alternatives, though. Hence, a group with the propery $\lambda x.4(x)$ doesn't automatically also have the property $\lambda x.n(x)$. The alternative members are rather scalar inasmuch as the group with the cardinality of four also has a subgroup with the cardinalities 3, 2, 1.

Defining alternatives for (38-c) is more or less straightforward now – Notably, Geurts & Nouwen (2007) employ the type-shifting rule of existential closure for cases like this, triggered by a covert element at the relevant level, to transform a predicate into an existential quantifier.[7] So, after [four$_F$ nuns] has undergone existential closure in (39-a), it combines with the predicate *were dancing*, reaching a scale like in (39-b).

(39) a. $\lambda P\exists x[6(x) \wedge nun(x) \wedge P(x)] \triangleright$

   $\lambda P\exists x[5(x) \wedge nun(x) \wedge P(x)] \triangleright$

   $\lambda P\exists x[4(x) \wedge nun(x) \wedge P(x)] \triangleright ...$

          (modified from Geurts & Nouwen 2007: 539, ex. (27))

 b. $... \exists x[6(x) \wedge nun(x) \wedge dance(x)] \triangleright$

   $\exists x[5(x) \wedge nun(x) \wedge dance(x)] \triangleright$

   $\exists x[4(x) \wedge nun(x) \wedge dance(x)] \triangleright ...$

          (modified from Geurts & Nouwen 2007: 539, ex. (28))

The scalar implicature a sentence like (38-c) has can then be cited by ruling out those alternatives which are higher (and, in this particular case, also logically stronger) than the literal meaning, because this, in turn, says that rather than any other number of possibly dancing individuals in the given context, four nuns were dancing.

(40) $\neg \exists x[5(x) \wedge nun(x) \wedge dance(x)] \wedge \neg \exists x[6(x) \wedge nun(x) \wedge dance(x)] \wedge ...$

          (modified from Geurts & Nouwen 2007: 539, ex. (29))

One major advantage of Krifka's (1999) account of scalar implicatures (which originally

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[7]The main motivation behind this step is to highlight the contrast between existential and predicative constructions.

goes back to Rooth 1985) is that scales aren't tied to a sentential level, but can also be related to constituents. That makes it possible to assume non-sentential operators whose interpretation depends on the current scale, which, in turn, is useful for the analysis of some focus particles like *only* and *even*. Geurts & Nouwen (2007) suppose that the interpretation of scalar modifiers involves scales at constituent-level, too.

So, with regard to superlative (numeral) modifiers, Geurts & Nouwen (2007) claim that those operate on scales which are essentially determined by their arguments. But they differ from natural numbers and comparative modifiers in two respects: Superlative numeral modifiers carry a modal meaning, as already mentioned above, and their arguments don't necessarily have to be first-order predicates – They rather combine with various argument-types that can be of any BOOLEAN type. To be precise, type $\langle t \rangle$ (propositional arguments) or $\langle a, t \rangle$ (predicative arguments), where a can be any type. This leads us to two distinct semantic entries for *at least*:

(41)   a.   If α is of type t, then $[\![$at least $\alpha]\!] = \Box\alpha \wedge \exists\beta[\beta \triangleright \alpha \wedge \Diamond\beta]$

(from Geurts & Nouwen 2007: 543, ex. (44a))

   b.   If α is of type $\langle a,\ t \rangle$, then $[\![$at least $\alpha]\!] = \lambda X[\Box\alpha(X) \wedge \exists\beta[\beta \triangleright \alpha \wedge \Diamond\beta(X)]]$

(from Geurts & Nouwen 2007: 543, ex. (44b))

If α and β are not members of the same entailment scale, (41-b) is too strong. Since this MA-Thesis will, especially in chapter 3, be concerned with such cases, let's take a look at Geurts & Nouwen's (2007) accommodation for non-entailment scales.

(42) $[\![$at least $\alpha]\!] = \exists\beta[\beta \trianglerighteq \alpha \wedge \Box\beta] \wedge \exists\beta[\beta \triangleright \alpha \wedge \Diamond\beta]$

(from Geurts and Nouwen 2007: 558, ex. (13) of the Appendix)

This gives us the following result for a sentence in which *at least* combines with an argument of type $\langle a, t \rangle$, which in turn refers to a scale organized by hierarchical (military) ranks:

(43)   a.   Hans is at least a major.

b.   □[major(h) ∨ lieutenant-colonel(h) ∨ general(h) ∨ ...] ∧ ◇[lieutenant-colonel(h)

   ∨ general(h) ∨ ...]

In terms of semantics, this result seems desirable.

Coming back to entailment scales, (41-b) provides us with an analysis of scalar adjectives

such as *hot*.

(44)   a.   The soup is at least warm.

   b.   □warm(s) ∧ ◇[hot(s) ∨ scalding(s)]

   Meaning: The speaker is certain that the soup must be warm and considers it

   possible that the soup is even hot or scalding

   c. λx.scalding(x) ▷ λx.hot(x) ▷ λx.warm(x) ▷ λx.lukewarm(x) ▷ λx.cold(x) ▷ ...

Referring to a temperature scale, applying ⟦at least⟧ to ⟦warm$_F$⟧ selects those properties

which outrank λx.warm(x) and gives us those properties an entity has iff it is warm or more

than warm, i.e. hot or scalding.

Geurts & Nouwen's (2007) analysis also works out for cases where a superlative modifier

combines with a numeral expression in, say, object position:

(45)   a.   Agatha took at least four$_F$ laxatives.

   b.   □∃x[4(x) ∧ laxative(x) ∧ took(a, x)] ∧

   ◇∃x[#x >4 ∧ laxative(x) ∧ took(a, x)]

   Meaning: The speaker is certain that there exists a group of four laxatives

each of which was taken by Agatha, and furthermore considers it possible that Agatha took more than four laxatives.

Since applying existential closure after the object NP combined with *at least* results in a contradictory meaning[8], Geurts & Nouwen (2007) suggest that it applies before *four laxatives* combines with the superlative modifier. Deriving things this way, we get the interpretation in (45-b).

2.3.2 Modal Concord

The term 'modal concord' was (as far as I know) introduced by Geurts & Huitink (2006), following so-called negative concord. In languages in which negative concord phenomena appear – such as (the Bavarian dialect of) German, or French –, sentences that contain double negation are ambiguous between a concord reading and a compositional reading:

(46)   Maria hat kein Buch nicht gelesen.
       Maria has no   book not   read
       a.   Concord reading: lit. 'Maria didn't read any book.'

            ≈ There is no x such that x is a book and Maria read x

       b.   Compositional reading: 'Maria read every book.'

Under the concord reading in (46-a), both NEG-elements, *kein* and *nicht* are interpreted such that they express a single denial of the fact that Maria read a book. Concord interpretations are not compositional. Under the compositional reading in (46-b), (46) implies that Maria read every book, because the two elements expressing negation cancel out.

Based on previous observations (cf. Halliday 1970, Lyons 1977 or more recent attempts like Papafragou 2000 or the (only) theoretical account by Geurts & Huitink 2006), Geurts & Nouwen (2007) claim that modal expressions can be subject to concord phenomena, too. In their article, they present data from Dutch like

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[8]The sentence would then be interpreted as stating that there exists a group consisting of four members being laxatives, and at the same time, that this group may consist of more than four members.

(47)   Hij moet zeker   in Brussel zijn.
        he   must certainly in Brussels be

    a.   Compositional reading: 'I suppose he has to be in Brussels.' (□□A)

    b.   Concord reading: 'He (definitely) has to be in Brussels.' (□A)

(from Geurts & Nouwen 2007: 550, ex. (66))

where a deontic and an epistemic modal operator is present in the sentence. But under the concord reading illustrated in (47-b), (47) is interpreted as if only one modal expression would occur. Due to the fact that *must* and *certainly* express the same modal force, i.e. necessity, such a concord interpretation is possible – Otherwise, the displayed modal concord phenomenon wouldn't occur for (47), as Geurts & Nouwen (2007) argue.

Hence, one restriction for modal concord to occur is that the modal expressions in question have to be of the same modal force. So, we already have to face one major complication, because Geurts & Nouwen's (2007) definition of *at least* involves two different modal forces. But the main information of the superlative modifier, we could claim, is conveyed in the first conjunct of the given definition, repeated here in a more generalized variant:

(48)   At least n A are B.

        $\Box\exists x[A(x) \wedge n(x) \wedge B(x)] \wedge \Diamond\exists x[A(x) \wedge \#x > n \wedge B(x)]$

(from Geurts & Nouwen 2007: 551, ex. (69a))

Hence, we'd assume *at least* to be involved in modal concord phenomena (only) if the competing modal expression is one of necessity, i.e. a deontic one. Let's examine this, starting with the obvious case.

(49)   Julie must have at least two sips of poison.

    a.   Modal concord reading:

        $\Box\exists x[2(x) \wedge \text{sips-of-poison}(x) \wedge \text{have}(j, x)] \wedge$

        $\Diamond\exists x[\#x > 2 \wedge \text{sips-of-poison}(x) \wedge \text{have}(j, x)]$

(modified from Geurts & Nouwen 2007: 551, ex. (70a))

Meaning: Julie is called upon to bring about a state of affairs in which she has two sips of poison, and she is allowed to bring about a state of affairs in which she has more than two sips of poison.

b. Compositional reading:

□ □ ∃x[2(x) ∧ sips-of-poison(x) ∧ have(j, x)] ∧

◇ □ ∃x[#x >2 ∧ sips-of-poison(x) ∧ have(j, x)]

<div align="right">(modified from Geurts & Nouwen 2007: 551, ex. (70b))</div>

Meaning: For all the speaker knows, it must be the case that Julie has to bring it about that Julie has two sips of poison, and it may be the case that Julie has to bring it about that she has more than two sips of poison.

The crucial difference between (49-a) and (49-b) arises through the outer modal in each conjunct of the latter, causing an epistemic interpretation. According to Geurts & Nouwen (2007), the reading in (49-b) might be harder to obtain, but it's available in principle. What happens if we change the modal's force?

(50)   Julie may have at least two sips of poison.

a. Concord reading: none

b. Compositional reading:

□ ◇ ∃x[2(x) ∧ sips-of-poison(x) ∧ have(j, x)] ∧

◇ ◇ ∃x[#x >2 ∧ sips-of-poison(x) ∧ have(j, x)]

Meaning: For all the speaker knows, Julie is allowed to bring it about that she has two sips of poison and it may be the case that Julie is allowed to bring it about that she has more than two sips of poison.

<div align="right">(modified from Geurts & Nouwen 2007: 552, ex. (71b))</div>

According to Geurts & Nouwen (2007), no modal concord reading arises for (50), because the modal's force doesn't match the force of the superlative modifier's first conjunct in (48). Geurts & Nouwen (2007) summarize their observations in the following 'rules of engagement':

(51)   a.   If a superlative modifier combines with a modal expression whose force matches that of its primary operator, the two modals may fuse to yield a concord reading, which is preferred, ceteris paribus, to a compositional construal.

        b.   If there is no such match, there is no concord reading.

        c.   The modal operators introduced by a superlative modifier are epistemic by default, and therefore generally tend to take wide scope.

<div align="right">(from Geurts and Nouwen 2007: 551)</div>

Summing up, Geurts & Nouwen (2007) treat superlative modifiers such as *at least* as modal expressions. Their meanings consist of two parts or conjuncts, the first one headed by a deontic modal operator, taking the focused expression (and the XPs it combines with) as its main argument – the second one headed by a modal operator of possibility, taking alternatives (and the XPs they combine with) as its argument. The two conjuncts are combined via logical $\land$. Comparative modifiers such as *more than* lack this modal interpretation, hence Geurts & Nouwen (2007) assume that this might be the crucial difference between superlative and comparative modifiers, lexically.

Unfortunately, the modal analysis goes wrong if the superlative modifier shows up in an embedded environment:

(52)   If Betty had at least three martinis, she must have been drunk.

<div align="right">(from Geurts & Nouwen 2007: 554, ex. (79))</div>

    Meaning: # If it must be the case that Betty had three martinis and it may be that she had more than three, then she must have been drunk

Geurts & Nouwen (2007) end their discussion of specific differences between comparative and superlative modifiers with a remark concerning adjectives and adverbials in their comparative and superlative use. Maybe all kinds of comparison should involve the same underlying mechanism – which eventually shouldn't be based on a notion of degree, but rather on comparing alternatives to one another.

2.4   Nouwen 2010

The analysis of this article mainly builds on Hackl's (2001) semantics for modified numerals, which was originally designed to work for constructions containing a comparative modifier such as *more than/fewer than n*. Because it is inevitable to know the outline of Hackl (2001) to further understand the account Nouwen (2010) presents in his paper, I will first give a brief introduction to Hackl's (2001) attempt.

2.4.1   Hackl (2001) Semantics for comparative Modifiers

Due to works of the past decades (among them Krifka 1999), it has become clear that we should not think of comparative modifiers in terms of the GQ-style determiner denotations. The semantics of the new definitions according to Hackl (2001) are those of comparative constructions in general. Cardinalities are treated as certain kinds of degrees.

(53)   a.   GQ definition of comparative modifiers

$[\![$more than 10$]\!] = \lambda P.\lambda Q.\exists x[\#x{>}10 \& P(x) \& Q(x)]$

$[\![$fewer than 10$]\!] = \lambda P.\lambda Q.\neg\exists x[\#x \geq 10 \& P(x) \& Q(x)]$

(from Nouwen 2010: 6, ex. (12))

b.   Definition according to Hackl's (2001) arguments

$[\![$more than 10$]\!] = \lambda M.max_n(M(n)){>}10$

$[\![$fewer than 10$]\!] = \lambda M.max_n(M(n)){<}10$ (from Nouwen 2010: 6, ex. (13))

The definitions of (53-b) are analogous to those of comparative structures of the form $[\![$-er than $d]\!]$ – $\lambda M.max_{d'}(M(d')){>}d$ – M is a degree predicate, *max* is to be understood as a maximality operator, applying to the degree predicate. Hackl (2001) assumes that an argument-DP, containing a modified numeral, must come with a silent counting quantifier *many*, which he defines along the following lines:

(54)   a. $[\![$many$]\!] = \lambda n\lambda P\lambda Q.\exists x[\#x = n \& P(x) \& Q(x)]$ (from Nouwen 2010: 7, ex. (15))

b.　10 sushis $\leadsto$ [$_{\text{DP}}$ [10 many] sushis] $\hfill$ (from Nouwen 2010: 7, ex. (16))

Sticking to this framework, the modified numeral of type $\langle d \rangle$ functions as the argument of the silent *many*-quantifier, which is of type $\langle d, \langle \langle et \rangle, \langle \langle et \rangle, t \rangle \rangle \rangle$. Because *more than/fewer than* denotes a degree quantifier, not a degree constant, we might end up in a type clash. To avoid such a type mismatch, we should assume that the modified numeral moves from its initial position, leaving a degree trace behind and creating a degree property (cf. section 1.3.2). We then arrive at the simple truth conditions we were longing for.

(55)　　The angora cat ate fewer than ten sushis.

　　　a.　[[fewer than ten] [λn [the angora cat ate [[n many] sushis]]]]

$\hfill$ (modified from Nouwen 2010: 7, ex. (18))

　　　b.　$[\lambda M.max_{\text{n}}(M(n)){<}10](\lambda n.\exists x[\#x = n\,\&\ \text{sushi}(x)\ \&\ \text{ate}(c,\,x)])$

　　　　　$= max_{\text{n}}(\exists x[\#x = n\,\&\ \text{sushi}(x)\ \&\ \text{ate}(c,\,x)]) < 10$

$\hfill$ (modified from Nouwen 2010: 7, ex. (19))

Because modified numerals take scope, we should expect scope ambiguities, which we actually get – But only for non-upward-entailing quantifiers like *fewer than*.[9]

(56)　　(Kelly Willis has to sing three songs.) Taylor Swift is required to sing fewer than three songs. $\hfill$ (modified from Nouwen 2010: 8, ex. (20))

　　　a.　Reading$_1$ = Taylor shouldn't sing more than three songs

　　　　　LF: [require [[fewer than 3] [λn [Taylor sing n-many songs]]]]

　　　　　$\Box[max_{\text{n}}(\exists x[\#x = n\,\&\ \text{song}(x)\ \&\ \text{sing}(t,\,x)]) < 3]$

$\hfill$ (modified from Nouwen 2010: 8, ex. (21))

　　　b.　Reading$_2$ = The minimal number of songs Taylor should sing is fewer than three

　　　　　LF: [[fewer than 3] [λn [require [Taylor sing n-many songs]]]]

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[9]For a proof, compare (56) to something like *(Brad Paisley has to sing three songs.) Dwight Yoakam has to sing more than three songs.* No such ambiguity as in (56-a) and (56-b), respectively, arises for this sentence.

$$max_n(\Box \exists x[\#x = n \ \& \ song(x) \ \& \ sing(t, \ x)]) < 3$$

In (56-a), the modal takes wide scope, whereas in (56-b), the maximality operator does take wide scope, i.e. to get the intended reading for the latter, the comparative modifier has to be moved out of its position (as we can see in (56-b)'s LF). Quite the same happens under existential modals, i.e. we get an upper-bound reading on the one hand, and a very weak reading on the other hand, stating that whatever is below the value of the numeral is what is allowed. Notably, nothing is said about a possible boundary to values greater than the numeral.

(57)   The BVB is allowed to bring fewer than five substitutes.[10]

    a. $Reading_1$ = The BVB shouldn't bring more than five substitutes

    b. $Reading_2$ = It's OK if the BVB brings four or fewer substitutes (and it might also be OK if the BVB brings more)

Assuming that comparative modifiers are no more than comparative constructions as such, this analysis proves appropriate. The question immediately arising is, whether superlative modifiers behave the same, but with $\leq/\geq$ instead of $</>$. If we consider modifying expressions like *maximally n*, there is some evidence that a maximality operator should be existing in the semantics of superlative modifiers. But for at least one reason, it has to be different from the semantics of their comparative counterparts. The crucial point here are cases with existential modals – While comparative modifiers do not define an upper boundary, as in (57-b), superlative ones always do. Plus, the latter tend to be resistent to weak readings.

(58)   a.   The BVB is allowed to bring fewer than five substitutes. But more is fine too.

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[10]The relevant LFs of the readings given above match those of (56), except that *require* gets replaced by *allowed*, and $\Box$ gets replaced by $\Diamond$.

b. The BVB is allowed to bring {up to/at most/maximally} five substitutes. #But more is fine too. (modified from Nouwen 2010: 10, ex. (28))

The BVB will bring maximally five substitutes conveys that the speaker isn't well-informed about the actual amount of substitutes the BVB will bring. There's no way to draw such a conclusion with *The BVB is allowed to bring maximally five substitutes*, because this utterance rather intends to say that the speaker knows very well what is allowed for the BVB and what is not.

2.4.2 Two kinds of *many*

For his further analysis, Nouwen (2010) proposes to assume not only one, but two silent counting quantifiers. He therefore renames Hackl's (2001) *many* to $many_1$ – It stands for the weak/lower-bounded/existential meaning. Hence, there's also a $many_2$ in Nouwen's (2010) terms, standing for the strong/doubly-bound meaning. Put differently, we can think of this distinction like that: There are two ways to interpret a numeral like *nine* – Either as *exactly nine* or as *nine or/and more*. The weak reading (≥ 9) arises from the weak semantics of $many_1$, whereas the strong reading ($= 9$) is the result of $many_2$. This leads us to the following distinct definitions:

(59) a. $[\![many_1]\!] = \lambda n \lambda P \lambda Q.\exists x[\#x = n \& P(x) \& Q(x)]$

b. $[\![many_2]\!] = \lambda n \lambda P \lambda Q.\exists! x[\#x = n \& P(x) \& Q(x)]$

(from Nouwen 2010: 11, ex. (32))

The $\exists! x$-part in $\exists! x[\#x = n \& P(x)]$ expresses the fact that there is exactly one x such that... – Hence, if x ranges over a group of individuals, this part of the formula will be verified by assigning a maximal group with the property P to x, with n being the particular cardinality of that group. Every smaller group wouldn't be the unique group with the property P. If we assume, for instance, a set {a, b, c, d} as our domain, x has to be four, since we have four elements in this domain, which all have the property P.

Distinguishing one *many* from the other comes with the considerable advantage of explaining the ambiguity of bare numerals.[11] The denotation of the degree predicate now crucially depends on which *many* we use.

(60)   Linda knits eight cardigans.   (modified from Nouwen 2010: 12, ex. (34))

LF: [MOD n [λd [Linda knit d many$_{1/2}$ cardigans]]]

    a.   applying *many*$_1$: $\lambda d.\exists x.[\#x = d$ & cardigan(x) & knit$(l, x)]$

        denoting $\{1, 2, 3, 4, 5, 6, 7, 8\}$ (modified from Nouwen 2010: 12, ex. (35))

    b.   applying *many*$_2$: $\lambda d.\exists! x[\#x = d$ & cardigan(x) & knit$(l, x)]$

        denoting $\{8\}$ (modified from Nouwen 2010: 12, ex. (36))

In (60-b) we get a singleton set, i.e. the unique group of cardigans such that Linda knits them – being the only unique group with that cardinality. *many*$_2$ therefore denotes singleton sets, which contain the maximum value of the denotation of *many*$_1$'s degree predicate. This analysis can also properly be applied to examples involving modals. But since this MA-Thesis is mainly concerned with *at least*, the question arises whether we can also handle such examples. So let's straightforwardly turn to these.

First of all, it seems notable that in the case of *at least*, we're not dealing with a maximum, but rather with a minimum. Therefore, we need a minimality operator,

(61) $[\![\text{MOD}_\text{B}]\!] = \lambda d.\lambda M.min_\text{n}(M(n)) = d$ (from Nouwen 2010: 20, ex. (59))

which happens to be sensitive to the distinction between *many*$_1$ and *many*$_2$. Superficially, (61) gets us into some trouble. Imagine that Linda knits eight cardigans, as above. For a degree predicate [λd. Linda knit d many$_{1/2}$ cardigans], applying *many*$_1$ at LF will give us a minimal degree that equals one – Actually, as Nouwen (2010) notes in his article, no matter how many cardigans Linda knits, as long as she does this, the minimal degree will always be one. Applying *many*$_2$, though, we get the singleton set $\{8\}$. Whilst these predictions

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[11] As Nouwen (2010) notes, it also doesn't boycott the analysis of comparative modifiers, since we don't have to deal with such an ambiguity in those cases.

might not be too bad for modifier-less sentences, wouldn't it turn out quite contradictory to something like the following?

(62)   Linda knits minimally eight cardigans.   (modified from Nouwen 2010: 20, ex. (60))

Indeed, Nouwen (2010) claims that we would have to reject the $many_1$ interpretation, because it could never come out as true of (62). But, on top of that, we should also neglect the $many_2$ interpretation as well, because it doesn't describe the certain state of affairs in (62). Rather, the $many_2$ denotation gives us a meaning which states that Linda knits exactly eight cardigans. According to Nouwen (2010)'s attempt, this reading is blocked by the bare numeral interpretation – That is, because the same sentence without the modifier would tell us the same (in terms of informativity), the $many_2$ reading is not allowed to arise.[12] The only way out of this dilemma is to assume an existential modal operator being involved in the $many_2$-interpretation.[13]

(63) $min_d(\Diamond \exists! x[\#x = d \ \& \ \text{knit}(l, x) \ \& \ \text{cardigan}(x)]) = 8$

(modified from Nouwen 2010: 21, ex. (61b))

Meaning: The minimal number for which it is considered possible that Linda knits exactly that many cardigans is 8 ⤳ It is not considered possible that Linda knits fewer than eight cardigans.

While (63) already looks pretty good, we have no direct evidence for the appearance of the existential modal in it so far – Unfortunately, Nouwen (2010) neither has any. We have to be

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[12]Nouwen explains this via markedness of a sentence. A bare numeral sentence is unmarked, whilst a sentence containing a numeral modifier represents the marked form. If it happens to be the case (because of a specific strategy of interpreting this particular sentence) that the marked form has the same meaning as the unmarked one, the unmarked interpretation for the marked sentence is blocked.

[13]The $many_1$ interpretation cannot be saved anyway, because we always get a minimal degree equal to one, as mentioned above. The only effect we would gain from applying the existential modal operator would therefore be that the meaning would change to something like saying: The minimal degree such that Linda knits d cardigans and for which it is considered possible that she does so, is, once more, one – But this again turns out pretty contradictory to every basic intuition one might have about (62). So, we should apparently refuse the $many_1$ interpretation in such cases.

contended with the fact that the superlative modifiers' modal flavour has such an effect, it seems. Although this can be regarded as very unsatisfactory, we're henceforth at least able to derive a more or less appropriate reading for (62). Unfortunately, too, this is the end of the success story already. Because, if we're dealing with a sentence like

(64)   Linda should knit minimally eight cardigans.

(modified from Nouwen 2010: 22, ex. (64))

we get four possible LFs, none of which fits the intended meaning of (64).

(65) $\Box$ > min:

The minimum n such that Linda will knit n cardigans should be eight

a. $\Box[min_n(\exists x[\#x = n \text{ \& } \text{cardigan}(x) \text{ \& } \text{knit}(l, x)]) = 8 \perp$

(modified from Nouwen 2010: 22, ex. (65a))

b. $\Box[min_n(\exists! x[\#x = n \text{ \& } \text{cardigan}(x) \text{ \& } \text{knit}(l, x)]) = 8]$

(modified from Nouwen 2010: 22, ex. (65b))

Before turning to the remaining possible LFs, let's briefly discuss (65). The $many_1$-interpretation, given in (65-a), turns out to be a contradiction – If Linda knits eight cardigans, there ought to be a singleton set containing a cardigan knitted by Linda. Hence, the minimum amount of cardigans Linda knits is either one or zero (in case she decided not to give a damn about knitting anymore). The crucial point is, it can never be eight – A problem we've already been faced with above. Under the $many_2$ interpretation, too, we have to deal with the same problematic issue again: Since there's just a single n such that Linda knits exactly n cardigans, this equals the meaning of a bare numeral – Consequently, this meaning is blocked according to Nouwen's (2010) assumptions. The remaining LFs confront us with the very same inadequacies. I will report them here, however.

(66)   min > $\Box$:

The minimum n such that Linda should knit n cardigans is eight

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a. $min_n(\Box\exists x[\#x = n \ \& \ \mathrm{cardigan}(x) \ \& \ \mathrm{knit}(l, \ x)]) = 8$

<div align="right">(modified from Nouwen 2010: 22, ex. (66a))</div>

b. $min_n(\Box\exists! x[\#x = n \ \& \ \mathrm{cardigan}(x) \ \& \ \mathrm{knit}(l, \ x)]) = 8$

<div align="right">(modified from Nouwen 2010: 22, ex. (66b))</div>

Apparently, Nouwen's (2010) analysis fails for such sentences. But we might fix the given analysis by a better understanding of a certain property of modals, which Nouwen (2010) generalizes as: Universal modal operators are interpreted as operators with existential modal force when minimality is at stake. In other words, we should paraphrase our awkward example as *Eight is the smallest number of cardigans Linda is allowed to knit*. Again, we lack independent motivation for this step, but it might come with some benefit.

2.4.3 Some minimal Requirements

Apparently, Nouwen's (2010) analysis hinges on an accurate paraphrase of sentences containing numeral modifiers and modal expressions. Intuitively, the following sentences have the same meaning:

(67)   a.   The minimum number of tricks Bello needs to perform to please his owner is ten. (modified from Nouwen 2010: 25, ex. (74))

       b.   Bello needs to perform minimally ten tricks to please his owner.

<div align="right">(modified from Nouwen 2010: 25, ex. (75))</div>

Let's assume that Bello performing ten or more tricks will make his owner happy, otherwise – i.e. if Bello performs less than ten tricks –, his owner won't give him any credit. A default analysis of goal-oriented modality concerning utterances of the form *to q, x needs to r* implies that such constructions are true iff r is true of every world in which q is true as well. The conclusion to draw for our case at hand is that first, we should have a look at possible worlds.[14]

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[14]Only those worlds yielding the truth value 1 are listed here.

(68) a. In all goal worlds: $\exists x[\#x = 10$ & trick(x) & perform$(b, x)]$

b. In all goal worlds: $\exists x[\#x = 9$ & trick(x) & perform$(b, x)]$

c. In all goal worlds: $\exists x[\#x = 1$ & trick(x) & perform$(b, x)]$

d. In some (but not all) goal worlds: $\exists x[\#x = 11$ & trick(x) & perform$(b, x)]$

e. In some (but not all) goal worlds: $\exists x[\#x = 12$ & trick(x) & perform$(b, x)]$

f. In no goal world: $\neg\exists x[$trick(x) & perform$(b, x)]$

(modified from Nouwen 2010: 26, ex. (76))

Whilst (69-a) and (69-b) are true, (69-c) is predicted to be false, because there are some worlds in which Bello only performs ten tricks and still pleases his owner by doing so:

(69) a. To please his owner, Bello needs to perform ten tricks.

(modified from Nouwen 2010: 26, ex. (77a))

b. To please his owner, Bello needs to perform a trick.

(modified from Nouwen 2010: 26, ex. (77b))

c. To please his owner, Bello needs to perform eleven tricks.

(modified from Nouwen 2010: 26, ex. (78))

As soon as we place a bound on what Bello is required to do to please his owner, we run into trouble, because the theory, then, makes wrong predictions. Only if we interpret (70-a) as (70-b), we can save the truth by virtue of (68-c).

(70) a. The minimum number of tricks Bello needs to perform, to please his owner, is one. (modified from Nouwen 2010: 26, ex. (79))

b. $min_n[$In all goal worlds: $\exists x[\#x = n$ & trick(x) & perform$(b, x)]] = 1$

(modified from Nouwen 2010: 26, ex. (80))

Generally, theories predict that an utterance of the form *the minimum requirement to q is r* will always be false – assuming that there is an entailment scale S, and r is a proposition on this scale, then r is a minimum requirement for the goal proposition q – unless r is the

minimum proposition of *S*. But that would be kind of devastating, because then, we could never express minimal requirements, because they always correspond to the absolute minimum. Seemingly, under this assumption, we won't solve the problem in terms of Nouwen's (2010) framework.

Maybe what goes wrong here hinges on an existential interpretation of the sentences, i.e. that we're talking about *at least* how many tricks Bello performs? The alternative at hand would be to interpret the number of tricks Bello performs in terms of $many_2$. But this step doesn't make it any better, as Nouwen (2010) notes himself, because we would only arrive at exact numbers (singleton sets) of tricks Bello performs – And this is, once more, not what the sentence initially is meant to express. I don't want to carry out the complications in detail, but the crucial point is that we wouldn't get any *n* such that Bello performs exactly *n* tricks in all goal worlds. In a word, the problem doesn't get solved in Nouwen's (2010) article.

But at least, we can derive another LF for (64) in terms of the analysis recited above. Instead of (66-b), the following interpretation gives us the correct meaning:

(71) $min_n(\Diamond\exists!x[\#x = n \text{ \& } \text{cardigan}(x) \text{ \& } \text{knit}(l, x)]) = 8$

<div align="right">(modified from Nouwen 2010: 28, ex. (85))</div>

From all we've seen, it still seems quite out of the blue to replace the universal modal with an existential one. Taking phenomena concerning disjunction into consideration, the combination of an universal modal with a minimality operator apparently yields an existential semantics in general, cf. Zimmermann (2000), Geurts (2005), Aloni (2007).

Summing up, Nouwen (2010) tried to develop a new framework, based on the insights of Hackl (2001), to treat numeral modifiers as degree quantifiers, but so far, many issues remain open.

2.5 Kennedy 2015

Nouwen (2010) makes his contribution to the article we'll consider now – On the one hand, Kennedy (2015) adopts Nouwen's (2010) distinction between Class A (comparative) and Class B (superlative) modifiers (cf. section 1.2.1). On the other hand, Kennedy (2015) discusses Nouwen's (2010) analysis and tries to make some improvement.

According to Kennedy (2015), one fundamental distinction related to the modifier type's meaning is responsible for so-called ignorance inferences to arise or not to arise.

(72) a. Class A modifiers express exclusive (strict) orderings relative to the modified
          numeral.   (from Kennedy 2015: 3, ex. (4a))

      b. Class B modifiers express inclusive (non-strict) orderings relative to the modified
          numeral.   (from Kennedy 2015: 3, ex. (4b))

Kennedy (2015) assumes that ignorance inferences arise for superlative modifiers – whilst they do not for comparative ones – because numerals (occurring with the modifiers in question, i.e. *at least* or *at most*) are thought to be less informative than their relevant alternatives. That is, *at least n* is not as informative as *(exactly) n*. To get a better picture of what Kennedy (2015) intends to say, consider the following example:

(73) (*Context: We're on a plane to Amsterdam. During his acquainting us with the safety
       regulations on this airplane, the steward says:*)

      This airplane has six emergency exits.   (from Kennedy 2015: 2, ex. (1))

Taking the steward seriously, passengers like us will – according to his utterance – assume that the plane they're on has exactly six emergency exits. Now, if we find out by chance during the flight that the airplane has five or seven emergency exits, we'd probably assume that the steward was wrong – Merely on the basis of his statement, though, there'd be no reason for us to think so. If the steward would want to point out the fairly high or low safety standards of the plane we're on, he could say something like the following:

(74)   *Unlike an airplane you'd fly with if you had booked your flight at AUA, ...*

    a.   This airplane has more than six emergency exits.

<div align="right">(from Kennedy 2015: 2, ex. (2a))</div>

    b.   This airplane has fewer than six emergency exits.

<div align="right">(from Kennedy 2015: 2, ex. (2b))</div>

Passengers would probably still be fine with that. But we would start questioning the steward's mental condition if he'd make an utterance as in (75-a) or (75-b).

(75)   a.   This airplane hast at least six emergency exits.   (from Kennedy 2015: 2, ex. (3a))

    b.   This airplane has at most six emergency exits.

<div align="right">(from Kennedy 2015: 2, ex. (3b))</div>

Unlike the above examples, the sentences in (75) give the passengers to understand that the steward isn't familiar with the exact number of emergency exits – The only thing he knows is a lower-, or an upper-bound. The question immediately arising is, why solely (75-a) and (75-b) might make us feel like we're suffering from a stomach disorder, while (74) doesn't. I.e. the question is: Why do only superlative modifiers express insecurity of the steward regarding an exact number of emergency exits? Because, superficially speaking, both kinds of numeral modifiers – i.e. comparative and superlative ones – can be considered equally uninformative concerning an exact amount of what so ever. So, we could either assume that superlative modifiers have an epistemic component or effect which their comparative counterparts lack, or we could think of the modifiers' semantic and pragmatic mechanisms as being different for each class, causing ignorance inferences or not doing so.

2.5.1   Unmodified Numerals and their Analysis

Intuitively, an unmodified numeral n can be interpreted as *exactly n* or *at least n*, depending (amongst other things) on the context.

(76)   a.   *Context: Carrie wants to make a list of the names and ages of all her lovers.*

          Ayden was exactly thirty-five years old when Carrie met him.

          C: Do you remember how old Ayden was?

          M: Ayden was thirty-five.

    b.   *Context: To buy alcohol legally at the drug store, you have to be 21 years old.*

          20 year old Cynthia and 19 year old Frank plan to have a booz up party this

          weekend. Their friend Hubert is exactly 22 years old.

          F: Who could get us some Vodka Tonic?

          C: Frank is 21!

For a long time, unmodified numerals were interpreted along the lines of Horn (1972), whose definition implies that unmodified numerals assert a lower-bound – i.e. *at least n* – and can implicate an upper-bound – i.e. *at most n* –, together yielding an exact interpretation of the numeral. Ever since then, a considerable number of theoretical and practical studies shed a different light on that topic, this *exactly n* interpretation being a matter of semantic content rather than a matter of implicature. So, there are two options to take: We could interpret unmodified numerals as quantificational determiners with either an *exact* or *at least* kind of meaning, or we could analyze them as singular expressions, denoting a numeral, i.e. objects of type $\langle d \rangle$.

Let's have a look at some evidence against the pragmatic-based interpretation in terms of Horn (1972). First of all, we'd like to prove that the *exactly n*-reading is not due to any implicature effects. Expressions occurring in downward-montone contexts usually don't trigger implicatures. Horn's (1972) pragmatic account therefore predicts that the *exact*-interpretation should disappear in such an environment.

(77)   *Context: Three different groups of taxpayers can be distinguished according to how many exemptions they are allowed to claim on their tax returns, where the minimum number allowed by law is zero and the maximum number allowed by lax is four. Group*

A contains individuals who are not allowed to claim any exceptions at all, Group B contains individuals who are not allowed to claim more than two exemptions, and Group C contains individuals who are allowed to claim all four exemptions. The society in which these individuals live is exemption-maximizing, but law-abiding, so everyone in Group A claims zero exemptions, everyone in Group B claims exactly two exemptions, and everyone in Group C claims exactly for exemptions.

a. No individual who was allowed to claim two exemptions claimed four.

b. No individual who was allowed to claim some exemptions claimed four.

(from Kennedy 2015: 18, ex. (36))

Because persons of Group C intuitively belong to the ⟦individual who was allowed to claim some exemptions⟧-set, (77-b) is judged as incorrect. Hence, the scalar implicature triggered by *some* (that is, *not all*), disappears in *no*'s restrictor. Now, if the *exact*-interpretation of *two* was a scalar implicature as well, it should vanish in (77-a) too. Our ⟦individual who was allowed to claim some exemptions⟧-set should contain every individual of Group B and C, and (77-a) should be wrong. This pragmatic prediction is not applicable to this case, then.[15] Preceding further, we should try to find an answer as to whether the *exact* reading is the only one being derived semantically. I.e. is there any independent evidence for the *at least*-interpretation? Kennedy (2015) argues that, for one thing, the continuation in the following example should actually be odd if the numeral solely had an *exact*-interpretation.

(78)   Carrie has had six lovers, if not seven.   (modified from Kennedy 2015: 19, ex. (37a))

But (78) is actually fine. Furthermore, (79) can be judged as true.

(79)   *Context: Driving test. Whoever answers ten or more questions correctly will get his/her driver's license. Curd, Knut and Eric took the test. Curd gave eleven, Knut*

---

[15]According to Kennedy (2015), it's also possible to force a reading for (77-b), only quantifying over Group B individuals by putting stress on *some*. But the crucial point is, that we get this reading for (77-a) without any special prosody.

gave thirteen and Eric gave ten correct answers. The examiner says:

Everyone answered ten questions correctly, so everyone will get their license.

Hence, both the *exactly n* and the *at least n* interpretation of an unmodified numeral should be derived in terms of semantics rather than in terms of pragmatics. But how are we supposed to analyze those distinct readings? Let's start with the former, i.e. the *exactly n*-interpretation.

According to Kennedy (2015), unmodified numerals denote quantifiers over degrees, therefore, they are of type $\langle\langle d, t\rangle\ t\rangle$. *max(D)* is the highest number of set *D*.

(80) $\llbracket \text{three} \rrbracket = \lambda D_{\langle d,\ t\rangle}.max\{n \mid D(n)\} = 3$

three is true for a property of degrees iff the highest number which satisfies that property is three.[16]   (from Kennedy 2015: 15, ex. (29))

Numerals with an *exact* interpretation must therefore somehow be applied to a predicate of type $\langle d, t\rangle$, a set of degrees. For (81), the set would be $\{n \mid \text{Kim took } n \text{ classes}\}$.

(81)   Kim took three classes.   (from Kennedy 2015: 15, ex. (30a))

How we arrive at this set remains open in Kennedy's (2015) article. One possibility would be the following: The predicate gets derived through quantifier raising. Bringing the numeral and the NP together is what MANY[17] does, then. These steps yield a type-mismatch inside of the raised DP, which we can resolve through movement of *three*. We therefore get the following interpretable structure:

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[16]This analysis is inspired by Frege's (1980 [1884]) claim that *three* matches for a property of individuals if the amount of individuals for whom this property is true of, is three. But Kennedy's (2015) account differs from Frege's (1980 [1884]) inasmuch as Kennedy (2015) takes unmodified numerals to be degree expressions and takes numerals in general to be degrees.

[17]Remember section 2.4
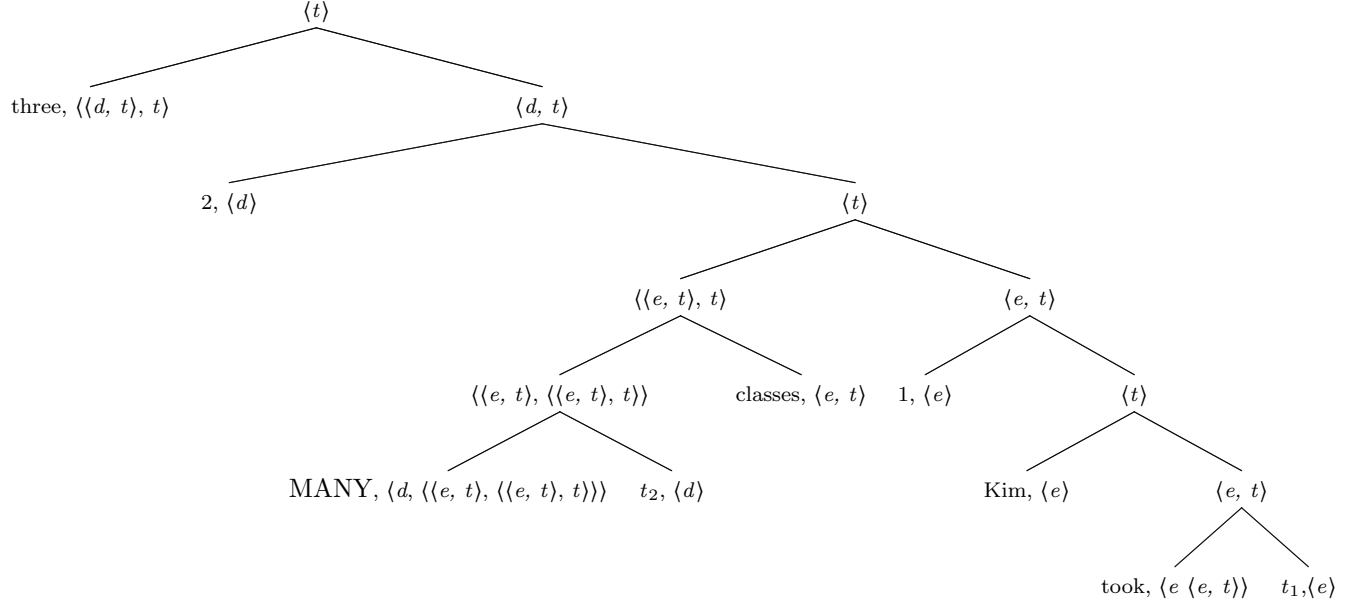
The predicate we're looking for is therefore of type $\langle d, t \rangle$.

(82) $[\lambda D_{\langle d, t \rangle}.\ \max(D) = 3](\lambda n.\exists x_{\mathrm{e}}[\mathrm{classes}(x) \wedge \mathrm{take}(x)(\mathrm{Kim}) \wedge \#(x) = n])$

$= \max(\lambda n.\exists x_{\mathrm{e}}[\mathrm{classes}(x) \wedge \mathrm{take}(x)(\mathrm{Kim}) \wedge \#(x) = n]) = 3$

three is the highest number n such that there are n classes that Kim took

To derive the *at least*-reading, Kennedy (2015) assumes an ambiguity in meaning for unmodified numerals – i.e. they can either be of type $\langle\langle d, t \rangle, t \rangle$ as above, or unmodified numerals have a meaning of type $\langle d \rangle$. In a word, an unmodified numeral n can also just denote a number.

(83) $[\![\mathrm{three}_d]\!] = 3$

This would give us an alternative structure, which we can interpret without movement of the degree expression at LF-level.

66

$\langle t \rangle$

$\langle \langle e,t \rangle, t \rangle$ $\langle e,t \rangle$

$\langle \langle e,t \rangle, \langle \langle e,t \rangle, t \rangle \rangle$ classes, $\langle e,t \rangle$ 1, $\langle e \rangle$ $\langle t \rangle$

MANY, $\langle d, \langle \langle e,t \rangle, \langle \langle e,t \rangle, t \rangle \rangle \rangle$ three, $\langle d \rangle$ Kim, $\langle e \rangle$ $\langle e,t \rangle$

took, $\langle e, \langle e,t \rangle \rangle$ t_1, $\langle e \rangle$
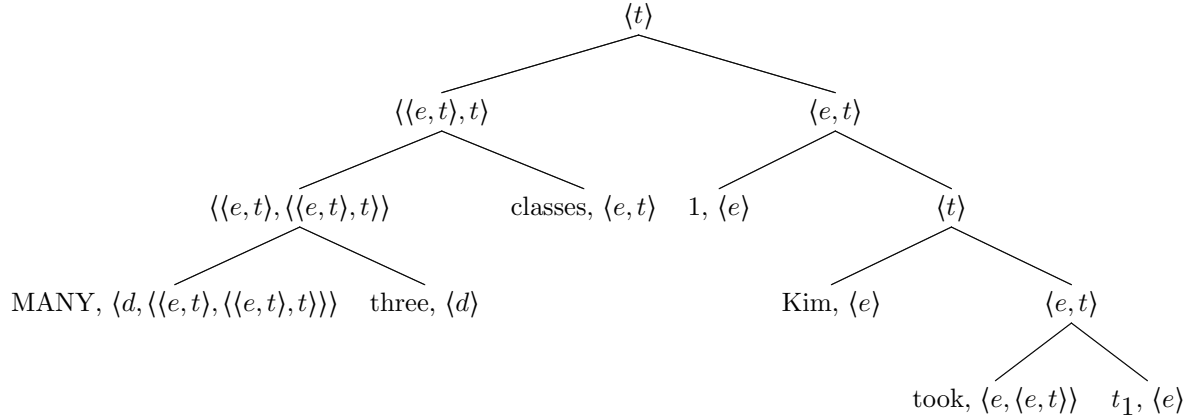
$= \exists x_e[\text{classes}(x) \wedge \text{take}(x)(\text{Kim}) \wedge \#(x) = 3]$

Truth conditions: There are three classes such that Kim took them.

Note that those truth conditions given above do not rule out the possibility that Kim might have taken more than three classes.

Despite of the fact that the analysis so far works out pretty well, it would still be very unsatisfactory to assume two lexical entries for every numeral to account for both readings (the *exact* and the *at least*-reading of an unmodified numeral). But according to Kennedy (2015), this rather costly step isn't necessary anyway, because we can derive the meaning in (83) via two independently motivated type shifts – illustrated in (84) – plus the quantifier meaning of unmodified numerals from (80).

(84)   For any type α:

     a. $\text{BE} = \lambda Q_{\langle \langle \alpha, t \rangle, t \rangle}.\lambda x_\alpha.\ Q(\lambda y_\alpha.y = x)$[18]   (from Kennedy 2015: 19, ex. (38a))

     b. $\text{IOTA} = \lambda P_{\langle \alpha, t \rangle}.\iota x_\alpha.P(x)$

          $= \lambda P_{\langle \alpha, t \rangle}.$ the unique x with the property *P(x)*

---

[18]A full account on how the BE-operator works can be found in Partee 1987. Just to give a brief sketch: BE gets applied to a (generalized) quantifier which denotes a set of sets. BE seeks for all singleton sets inside this set of sets and collects their members to form a new set. Using BE in Kennedy's (2015) analysis can be motivated independently, because BE eneables indefinite DPs in a predicative position to get a meaning of type $\langle e,t \rangle$, cf. Partee 1987

So, for unmodified numerals, we arrive at a predicate being satisfied by exactly one numeral, (85-a). Applying IOTA gives us, in the case at hand, the number three, (85-b).

(85)   a. $\text{BE}(\llbracket \text{three} \rrbracket) = \text{BE}(\lambda D_{\langle d,t \rangle}.\ \max(D) = 3)$

                $= [\lambda Q_{\langle \langle d,t \rangle, t \rangle}.\lambda x_d.Q(\lambda y_d.y = x)]\lambda D_{\langle d,t \rangle}.\ \max(D) = 3$

                $= \lambda x_d[\lambda D_{\langle d,t \rangle}.\ \max(D) = 3](\lambda y_d.y = x)$

                $= \lambda x_d[\max(\lambda y_d.y = x) = 3]$

                $= \lambda x_d[x = 3]$ (modified from Kennedy 2015: 20, ex. (39))

    b. $\text{IOTA}(\text{BE}(\llbracket \text{three} \rrbracket))$[19]

                $= \text{IOTA}(\lambda x_d[x = 3])$

                $= \iota x_d[x = 3]$

                $= 3$

So far we've seen how unmodified numerals can yield an *exact-*, or an *at least*-interpretation. Embedded under a modal, though, we get systematic *at least-*, or *at most*-readings, i.e. unmodified numerals get either a lower-bounded or an upper-bounded reading.

(86)   Embedding under universal modals

    a.   Henry needs to own three Harley Davidsons to become a member of Hell's Angels.

    b.   Klaus must provide three psychiatric reports.

    c.   Charles is required to call his probation officer three times a day.

                                  (modified from Kennedy 2015: 16, ex. (31))

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[19]A potential problem of this step might be that we have no independent evidence for a phonologically empty IOTA-operator (contrary to BE). Another crucial point to find a way out of is, that applying BE and IOTA successively isn't properly possible in every construction – For instance, we'd get unwanted (and furthermore impossible) readings for sentences containing downward-monotone quantifiers. Hence, something like *[No drunkard]$_i$ came in. He$_i$ smashed the furniture.* would mean that the unique person not being a drunkard came in and smashed the furniture.

(87)   Embedding under existential modals

    a.   Susan can have a flirt with three guys within an evening without taking anyone of those poor guys home.

    b.   Little Joel is permitted to have three sips of rum.

    c.   Noel is allowed to own three portable firearms.

<div align="right">(modified from Kennedy 2015: 16, ex. (32))</div>

The examples in (86) and (87) suggest that we get an *at least*-reading of the unmodified numeral if it is embedded under a universal modal on the one hand, and we get an *at most*-reading if the unmodified numeral is embedded under an existential modal on the other hand. According to Kennedy's (2015) Fregean-style analysis, we can explain these patterns as an interaction between the unmodified numeral's scope and the modal's scope.

Before turning to modified numerals, I'll give a brief discussion for examples similar to those above, and their readings. Let's start with a case with embedding under a universal modal.

(88)   Kim is required to take three classes.   (from Kennedy 2015: 16, ex. (33))

    a. $\Box[max\{n \mid \exists x[\text{take}(x)(\text{Kim}) \wedge \text{classes}(x) \wedge \#(x) = n]\} = 3]$

<div align="right">(from Kennedy 2015: 16, ex. (33a))</div>

    Meaning: Every world is such that the highest number of classes, such that Kim took them in this particular world, is three. (= *EXACT*-INTERPRETATION)

    b. $max\{n \mid \Box[\exists x[\text{take}(x)(\text{Kim}) \wedge \text{classes}(x) \wedge \#(x) = n]]\} = 3$

<div align="right">(from Kennedy 2015: 16, ex. (33b))</div>

    Meaning: The highest number n such that there's a plurality of classes of at least the size n in every world, such that Kim took those classes, is three.

    (= *AT LEAST*-INTERPRETATION)

Changing scope relations between the unmodified numeral and the modal changes the meaning of the sentence, as we can see from (88-a) and (88-b)'s different meanings given below the formula. We get an equal ambiguity if the unmodified numeral is embedded under an

<div align="center">69</div>

existential modal:

(89)   Kim is allowed to take three classes.   (from Kennedy 2015: 17, ex. (34))

    a. $\Diamond[max\{n \mid \exists x[\text{take}(x)(\text{Kim}) \wedge \text{classes}(x) \wedge \#(x) = n]\} = 3]$

                                                  (from Kennedy 2015: 17, ex. (34a))

        Meaning: There is a possible world in which the highest number of classes taken by Kim is three (= WEAK READING)

    b. $max\{n \mid \Diamond[\exists x[\text{take}(x)(\text{Kim}) \wedge \text{classes}(x) \wedge \#(x) = n]]\} = 3$

                                                  (from Kennedy 2015: 17, ex. (34b))

        Meaning: The highest n such that there is a possible world in which Kim takes at least n classes, is three (= STRONG READING)[20]

2.5.2 Modified Numerals and their Analysis

Just like unmodified numerals under their *exact*-interpretation, Kennedy (2015) assumes that modified numerals are quantifiers of type $\langle\langle d, t\rangle, t\rangle$. To get the intended meaning, we need a type $\langle d \rangle$ interpretation for the numeral, which was also needed to derive the *at least*-reading of an unmodified numeral. Kennedy (2015) gives the following definitions:

(90)   a. $[\![\text{more than}]\!] = \lambda n \lambda P_{\langle d, t\rangle}.max\{n \mid P(n)\} > n$ (from Kennedy 2015: 21, ex. (41a))

    b. $[\![\text{at least}]\!] = \lambda m \lambda P_{\langle d, t\rangle}.max\{n \mid P(n)\} \geq m$ (from Kennedy 2015: 22, ex. (42a))

What distinguishes comparative modifiers from superlative modifiers is, according to Kennedy (2015), that the latter introduce partial orderings while the former introduce total orderings. Kennedy's (2015) account of ignorance implications is based on Büring (2008, cf. section 2.2) and Cummins & Katsos (2010) – Ignorance inferences are a kind of conversational implicature, because the semantic content of the sentence in question is less informative than potential alternatives. To derive those alternatives, Kennedy (2015) uses Sauerland's

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[20]Note that the sentence in (89) is false under this interpretation if there's a world in which Kim takes more than three classes.

(2004) model of quantity implicature calculation. Because I already went through Sauerland's (2004) principles in section 1.3.3, I'll straightforwardly present the analysis it leads Kennedy (2015) to.[21]

Kennedy (2015) draws his major attention to the interactions occurring between modified numerals and universal/existential modals. We don't have to take epistemic modals into consideration, because (unmodified as well as modified) numerals cannot take scope over such a kind of modal for (so far) unknown reasons. Contrary to Nouwen (2010), Kennedy (2015) doesn't assume a minimality operator, i.e. he also handles cases involving *at least* in terms of the maximality operator. Kennedy's (2015) analysis neither relies on the (so far unmotivated) insertion of an epistemic modal, as Nouwen's (2010) did. This should avoid some of the complications Nouwen (2010) had to go through.

Again, let's first look at examples with universal modal operators, like (91), which has two different interpretations, one of them triggering an insecurity inference.

(91)   Lucy is required to register for at least three classes.

(modified from Kennedy 2015: 26, ex. (48))

    a.   Reading$_1$ = The minimal number of required classes is exactly three

    b.   Reading$_2$ = The minimal number of required classes is at least three (and the speaker is uncertain about the exact number of required classes)

The prevailing LF should give us some more information about these two readings:

(92)   a.   at least three > required

        $\max(\lambda n. \square\, [\exists x[\mathrm{reg}(x)(\mathrm{lucy}) \wedge \mathrm{classes}(x) \wedge \#(x) = n]]) \geq 3$

(modified from Kennedy 2015: 26, ex. (48a))

        Meaning: The highest number n such that in every deontically accessible world, Lucy registers for n classes, is three

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[21]One crucial point to mention, though, is that Kennedy (2015) doesn't assume that modified numerals form a classical Horn-scale. Rather, alternatives of, for instance, *at least*, contain an *exact* and a *more than* variant. I will come back to this point later within this analysis.

≈ The minimal number of classes required is at least three

b. required > at least

$\square[\max(\lambda n.\exists x[\text{reg}(x)(\text{lucy}) \wedge \text{classes}(x) \wedge \#(x) = n]) \geq 3]$

(modified from Kennedy 2015: 26, ex. (48b))

Meaning: In every deontically accessible world, the highest number n such that lucy registers for n classes, is at least three

≈ The minimal number of classes required is at least three

Regarding their truth conditions, the two LFs of the sentence in (91) are equivalent. The crucial difference between them are their prevailing scalar alternatives. Pragmatically speaking, (92-a) behaves like a sentence without a modal expression. Its alternatives, according to the Horn-scale Kennedy takes *at least* to refer to (cf. footnote 20 above), are *exactly three* and *more than three*. Therefore, we get the following primary implicatures for (92-a):

(93) $\{\neg K(max(\square) = 3), \neg K(max(\square) > 3)\}$ (from Kennedy 2015: 27, ex. (49))

a. $\neg K(max(\square) = 3)$: The speaker isn't certain that the minimal number of required classes is exactly three.

b. $\neg K(max(\square) > 3)$: The speaker isn't certain that the minimal number of required classes is more than three.

Hence, we can conclude that the speaker isn't certain about the minimal number of classes Lucy is required to take. Note that we cannot strengthen these primary implicatures without contradicting the assertion – So, no secondary implicatures can arise for (92-a).[22] In turn, we get primary implicatures for (92-b), as well as secondary implicatures.

(94) Primary Implicatures

$\{\neg K(\square(max = 3)), \neg K(\square(max > 3))\}$ (from Kennedy 2015: 27, ex. (51))

a. $\neg K(\square(max = 3))$: The speaker isn't certain that it is required for Lucy to

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[22]This interpretation of (91) matches what Büring (2008) calls the 'speaker insecurity' reading. We always arrive at that meaning whenever a superlative modifier outscopes a necessity or possibility modal.

register for three classes

b. $\neg K(\square(max> 3))$: The speaker isn't certain that it is required for Lucy to register for more than three classes

(95) Secondary Implicatures

$\{K(\neg \square (max = 3)), K(\neg \square (max> 3))\}$ (from Kennedy 2015: 27, ex. (52))

a. $K(\neg \square (max = 3))$: The speaker is certain that it's not required for Lucy to register for exactly three classes

b. $K(\neg \square (max> 3))$: The speaker is certain that it's not required for Lucy to register for more than three classes.

According to (94) and (95) respectively, we can conclude that either enrollment in exactly three classes or enrollment in more than three classes is allowed – I.e., there's no speaker insecurity inference for this interpretation of (91) under the given scope relations from (92-b), but rather a kind of Free-Choice-implicature.[23]

It's hardly surprising now that combining a superlative modifier with an existential modal also yields two readings, depending on scopal relations. But the analysis of such cases turns out a little more complicated.

(96) Lucy is allowed to register for at least three classes.

(modified from Kennedy 2015: 29, ex. (57))

a. Reading_1 = The maximum number of classes for which it is allowed to register in those classes is at least three (and the speaker is uncertain about the exact number)

b. Reading_2 = It's allowed to register for three classes or more than three classes

We can see that constructions involving existential modals are slightly different from the

___

[23]Remember that Büring (2008) calls this interpretation the 'authoritative reading'. One of the achievements of Kennedy's (2015) analysis therefore is, that we now can account for those two readings, stated in Büring's (2008) article, in a semantic fashion too. Although, of course, the implicature generation mechanism is based on pragmatics.

paraphrases in (96) already. Let's look at the relevant LFs, classified by the scopal relation their prevailing structure has:

(97)   a.   at least > allowed

$$max\{n \mid \Diamond[\exists x[\text{register}(x)(\text{lucy}) \wedge \text{classes}(x) \wedge \#(x) = n]]\} \geq 3$$

(modified from Kennedy 2015: 29, ex. (57a))

Meaning: The highest number of allowed, registered-for classes is three or more (does not entail that it is forbidden to register in one or two classes)

        b.   allowed > at least

$$\Diamond[max\{n \mid \exists x[\text{register}(x)(\text{lucy}) \wedge \text{classes}(x) \wedge \#(x) = n]\} \geq 3]$$

(modified from Kennedy 2015: 29, ex. (57b))

Meaning: There is a possible world in which Lucy registers for three or more classes

The particular scope resolution in (97-a) yields, again, an ignorance inference, which can be derived analogously to the example with an universal modal. The more striking example turns out to be (97-b) – Its scalar alternatives are once more *exactly three* and *more than three*, which both entail the expressed proposition. Henceforth, we receive primary implicatures of the following form.

(98) $\{\neg K(\Diamond(max = 3)), \neg K(\Diamond(max> 3))\}$ (from Kennedy 2015: 29, ex. (59))

        a. $\neg K(\Diamond(max = 3))$: The speaker is not certain that there's a possible world in which Lucy registers for exactly three classes

        b. $\neg K(\Diamond(max> 3))$: The speaker is not certain that there's a possible world in which Lucy registers for more than three classes

According to Kennedy (2015), this version of the sentence under discussion does not trigger an ignorance inference. To derive this prediction via our primary implicatures in (98), we would have to strengthen them. But the problem is, that Sauerland (2004) wouldn't predict

secondary implicatures, because they would – together with the primary implicatures – contradict the assertion.[24]

One might get into the very same trouble with Free-Choice-Disjunction. Consider, for instance, (99) – The implicatures we'd be longing for, (99-a) and (99-b), cannot be derived under the present account of Sauerland (2004).

(99)   Jim is allowed to kiss Mary or Sue.

$\diamond$(Jim kisses Mary $\lor$ Jim kisses Sue)

    a. $\diamond$ Jim kisses Mary

    b. $\diamond$ Jim kisses Sue

A possible solution for our example with the modified numeral is to assume exhaustified versions of the disjuncts, rather than the bare disjuncts. Instead of the regular scalar alternatives for (97-b), we ought to use the following ones:

(100)   a.   (*exactly three*) $\land$ $\neg$(*more than three*): It is allowed for Lucy to register for exactly three classes, but no more than three

    b.   (*more than three*) $\land$ $\neg$(*exactly three*): It is allowed for Lucy to register for more than three classes, but it is not allowed for her to register for exactly three classes

With this alternative set, we can now derive secondary implicatures.

(101) $\{K\neg(\diamond(max = 3) \land \neg \diamond (max >3)), K\neg(\diamond(max >3) \land \neg \diamond (max = 3))\}$

(from Kennedy 2015: 35, ex. (71))

    a. $K(\neg$(100-a)): The speaker is certain that it is allowed for Lucy to register for more than three classes

    b. $K(\neg$(100-b)): The speaker is certain that it is allowed for Lucy to register for

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[24]As Kennedy (2015) states, our primary implicatures already contradict the assertion – As to my mind, that's something one can agree or disagree with.

exactly three classes

Taking the implicatures from (101) and the assertion (96) together, we can conclude that both enrollment in exactly three classes, as well as enrollment in more than three classes is allowed for Lucy. Interestingly, this is a very weak meaning, defining neither a minimum, nor a maximum amount of classes Lucy is allowed to register for. Whilst this kind of reading is not very informative, Kennedy (2015) gives the following evidence for its existence.

(102)   Previously in Germany, students were allowed to take at least five years to complete their Magister's diploma, the basic university degree. But now, Germany has adopted the Anglo-Saxon style of bachelor's and master's degrees. The bachelor's degree is designed to take three years to complete, the master's, a further two years.   (from Kennedy 2015: 35, ex. (72))

Hence, under the Magister's diploma, students were allowed to take at least five years to complete, while now, they have to finish their studies within five years.

Summing up, Kennedy's (2015) analysis of *at least* gives us the correct predictions with regard to scopal ambiguities with superlative numeral modifiers and universal modals. For superlative numeral modifiers in combination with existential modals, in turn, the predictions aren't obviously incorrect, but one of the two readings expresses a very weak meaning. In a way, however, this weak meaning shouldn't be considered as an unwanted effect due to Kennedy's (2015) treatment of *at least*+existential modal constructions. Because, in the end, doesn't it resemble the meaning we ought to be longing for? Put differently, if an individual is allowed to x at least n y, he/she should (in principle) be allowed to x less than n y as well, otherwise (as to my mind) one is supposed to say that he/she is required to x at least n y. That is, we would have to change the modal's force.

2.6 Schwarz 2016

An essential observation already made in the course of the previous sections is, that numerals modified by *at least* do not generate upper-bounding (scalar) implicatures. So, contrary to (103-a), (103-b) triggers the implicature that Al might have ordered more than just two chainsaws.[25]

(103) a. Al ordered two chainsaws.   (modified from Schwarz 2016: 2, ex. (2))

      b. Al ordered at least two chainsaws.   (modified from Schwarz 2016: 2, ex. (1))

The fact that (103-b) has this reading while (103-a) doesn't can be attributed to an ignorance implication, stating that the speaker isn't certain about the exact number of chainsaws ordered by Al.[26] For the upcoming analysis, we need to distinguish between the following types of inferences:[27]

(104) Quality inference: $\Box\alpha$

      Primary (Quantity) inference: $\neg\,\Box\,\alpha$

      Secodary (Quantity) inference: $\Box\neg\alpha$

      Inference of possibility: $\neg\,\Box\,\neg\alpha$

      Competence inference: $\Box\alpha \vee \Box\neg\alpha$

      Ignorance inference: $\neg\,\Box\,\alpha \wedge \neg\,\Box\,\neg\alpha$

Remember that the secondary inference is a strengthened version of the primary inference, which can only be drawn if it (i.e. the secondary inference) won't contradict the assertion. Furthermore, $\Box\neg\alpha$ is equivalent to the conjunction of the primary inference $\neg\,\Box\,\alpha$ and $\Box\alpha \vee \Box\neg\alpha$, suggesting that the speaker is competent with regard to α.

There's one more thing to say in advance – Namely, how we can derive an adequate set of

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[25]Given that we take numerals to denote an exact number rather than to set a lower-bound meaning.

[26]Remember that we said with Büring (2008) that an ignorance inference may be explained in terms of conversational implicature.

[27]Where α is a proposition and $\Box$ is to be read as *The speaker is certain that...*.

alternatives for modified numerals. So far, we got to know two default options, to be briefly repeated here. The first way of computing alternatives works along the lines of Horn (1972), following the idea that a quantity implicature gets derived through a syntactic mechanism of substitution. Alternatives are generated at the level of logical form by replacing a certain lexical element with another lexical element. This kind of replacement isn't arbitrary, of course, but rather linked to the specific members of the prevailing Horn-scale. Hence, for *two* we get a set $\{[n,...)\}_{n \geq 1}$, that is, a possibly infinite set of alternatives to $[2,...)$. In section 2.1 we assumed with Krifka (1999) that *at least* takes influence on the alternative set by blocking the projection of alternatives in its scope. Therefore, we could explain the missing upper-bounding implicature, but this assumption also came with some disadvantages. Contrary to this and similar analysis, Mayr (2013) claimed that *at least* itself is a Horn-scale member, as well as the numeral it modifies.

(105)   a.   Horn-scale for numerals: {one, two, three, four, five, ...}

b.   Horn-scale for *at least*: {at least, exactly, more than}

Replacing *at least* with *more than*, we arrive at the intended alternative $[3,...)$, replacing the superlative modifier with *exactly*, we get $[2]$. Crucially, as soon as we apply the syntactic substitution mechanism, this analysis might yield an ungrammatical result.

(106)   a.   Al is at least allowed to order three chainsaws.

b.   *Al is more than allowed to order three chainsaws.

The main complication with (106-b) can straightforwardly be explained – *more than* does neither share *at least*'s (broader) syntactic distribution, nor the superlative modifier's ability to combine with numerals (or other expressions) over a distance. Another problem which will become apparent in the next subsection is that, under the analysis Schwarz (2016) makes use of, we derive the proposition for (106-a) that Al is only (meaning exclusively) allowed to order three chainsaws and, at the same time, is allowed to order more than three chainsaws

– a plain contradiction.

So, using only one scale to derive an adequate set of alternatives doesn't work out so well. Plus, there's no salient scalar element that could replace *more than* in examples like (106-b). We can solve the former problem through making use of two separate scales at the same time. Since the Horn-scale for numerals covers the meaning of *more than n*, we can remove the comparative modifier from *at least*'s Horn-scale in (105-b). Assuming that substitutions of different scales may derivationally combine, we get a broader set of alternatives for sentences containing *at least*. We'll see the consequences of this step in the next subsection. Before doing so, there's only one problem remaining, or, strictly speaking, one element in (105-b) remaining which won't survive syntactic substitution – *exactly*, like *more than*, neither fits the syntactic environments where *at least* can occur. So, once more, applying the intended mechanism of syntactic substitution, we get an ungrammatical sentence. Schwarz (2016) fixes this inadequacy by replacing *exactly* with *only*.

(107)   Horn-scale for *at least*, revised: {at least, only}

With this new scale at hand, we will now move on to Schwarz's (2016) analysis – But it should be kept in mind that Schwarz's (2016) analysis initially assumes just one Horn-scale. In the following summary, I will make a note at the point from which on Schwarz (2016) uses two scales for his analysis.

2.6.1   The standard Recipe

The standard recipe (henceforth STR) is an elaboration of the Neo-Gricean account of Quantity implicature and can be subsumed under the following three basic definitions.

(108)   The Standard Recipe

    a. $0_p = \{\Box p\}$

       Singleton-set $\Box p$ (= The speaker believes that p)

    b. $1_{p,A} = 0_p \cup \{\neg\Box q: q \in A \wedge q \subset p\}$

The assumption $\neg\Box q$ gets added to 0_p for every q in the set of alternatives A that is semantically stronger than p – The result of this step is the set $1_{p,A}$

c. $2_{p,A} = 1_{p,A} \cup \{\Box\neg q:\ q \in 1_{p,A}\ \&\ \Box\neg q$ is consistent with $1_{p,A}$

The set $2_{p,A}$ is a result of adding the inference $\Box\neg q$ to $1_{p,A}$ for every assumption $\neg\Box q$ in $1_{p,A}$ with the proviso that it is compatible with $1_{p,A}$

(from Schwarz 2016: 4, ex. (3))

According to (108), the utterance of a sentence ϕ with the semantic content p will lead the hearer to add the quality inference of p to previously assumed states of belief of the speaker, and henceforth also add the primary quantity inferences of the semantic meanings of all the alternatives to ϕ being stronger than p. The hearer (of ϕ) can even widen the set of alternatives by adding a secondary inference of q for every primary inference (of q), if this is consistent with the quality inference and the quantity inference. An ignorance inference to an alternative which is stronger than the asserted meaning will be derived iff this alternative pairs with another strong alternative, together catching the meaning of the sentence. Such pairs of alternatives are called symmetric – Whether symmetry exists or not will be crucial for what's coming next.

Let's test the predictions of the STR with an unmodified numeral first.

(109)   David invited two table dancers.

Assuming with Horn (1972) that sentences with unmodified numerals define a lower-, but not an upper-bound – hence being semantically weak – (109) expresses the proposition that David invited more than one table dancer, represented as [2,...). Alternative meanings of (109) only vary with regard to the numeral. So, generally speaking, we get a set of alternatives $\{[n,...):$ n is a natural number$\}$, i.e. $\{[n,...)\}_{n\geq 1}$.[28]

(110)   ... ...

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[28]Stronger alternatives are placed above weaker ones. The asterisk marks the asserted meaning of the sentence under discussion.

$$[4\ ...)$$

$$[3\ 4\ ...)$$

$$[2\ 3\ 4\ ...)^*$$

$$[1\ 2\ 3\ 4\ ...)$$ (from Schwarz 2016: 6, ex. (4))

Based on this set of alternatives, we get the following result applying STR:

(111)   a. $0_{[2,...)} = \{\Box[2,...)\}$

      b. $1_{[2,...),\{[n,...)\}_{n\geq1}} = \{\Box[2,...)\} \cup \{\neg\Box[n,...)\}_{n\geq3}$

      c. $2_{[2,...),\{[n,...)\}_{n\geq1}} = \{\Box[2,...)\} \cup \{\neg\Box[n,...)\}_{n\geq3} \cup \{\Box\neg[n,...)\}_{n\geq3}$

(from Schwarz 2016: 7, ex. (5))

Via conjunction of all quantity inferences from set 2, i.e. $\Box\neg[3,...)$, with the quality inference $\Box[2,...)$, we arrive at $\Box[2]$ – The hearer assumes that the speaker of (109) wants to express that David invited two table dancers.[29] Because *at least*'s literal meaning and interpretation often gets related to disjunction (cf. Büring 2008 in section 2.2) and since Schwarz's (2016) account in one way or the other resembles Sauerland's (2004) analysis, let's also consider an example involving (inclusive) disjunction.

(112)   a.   Bingo stinks or Elviro stinks.

      b. $b \wedge e$

          b e

          $b \vee e^*$ (modified from Schwarz 2016: 7, ex. (7))

Schwarz (2016) assumes with Sauerland (2004) that the alternatives to a disjunctive sentence not only consists of the disjunction and conjunction of the two disjuncts, but also of the two disjuncts themselves, as we can see in (112-b). Applying STR, we get:

(113)   a. $0_{b \vee e} = \{\Box b \vee e\}$

      b. $1_{b \vee e, \{b \vee e, b, e, b \wedge e\}} = \{\Box b \vee e, \neg\Box b, \neg\Box\ e, \neg\Box b \wedge e\}$

---

[29]Deriving the meaning of (109) this way is often referred to as two-sided interpretation.

c. $2_{b\lor e,\{b\lor e,b,e,b\land e\}} = \{\Box b\lor e, \neg\Box b, \neg\Box e, \neg\Box b\land e, \Box\neg b\land e\}$

Note that set 2 contains two primary inferences $\neg\Box b$ and $\neg\Box e$ without containing their corresponding secondary inferences $\Box\neg b$ and $\Box\neg e$, because those secondary inferences would be inconsistent with set 1. Bringing primary inferences together with the quality inference $\Box b\lor e$, we can derive the inferences of possibility $\neg\Box\neg b$ and $\neg\Box\neg e$. Therefore, the latter (secondary) inferences turn out inconsistent with set 1 – we could also say that they don't pass the consistency check of the third step of our STR, (108) – and it's not predicted that the hearer will make such inferences. Just to point that out, (112-a) doesn't mean that the speaker believes neither Bingo nor Elviro to stink. It rather means that the speaker isn't certain whether Bingo stinks and isn't certain whether Elviro stinks either.

So, the STR gives us (more or less) the right result for a simple disjunctive sentence – but not for every disjunctive sentence. If the sentence under discussion contains three different disjuncts, the STR won't work properly.

(114)   b e z

    $b\lor e\lor z*$

According to the STR, we get the set of inferences in (115) for (114). Set 2 contains secondary inferences for each of the three alternatives not having a symmetric partner element and therefore not generating any insecurity inferences, which could have wiped out those secondary inferences.

(115)   a. $0_{b\lor e\lor z} = \{\Box b\lor e\lor z\}$

    b. $1_{b\lor e\lor z,\{b\lor e\lor z,b,e,z\}} = \{\Box b\lor e\lor z, \neg\Box b, \neg\Box e, \neg\Box z\}$

    c. $2_{b\lor e\lor z,\{b\lor e\lor z,b,e,z\}} = \{\Box b\lor e\lor z, \neg\Box b, \neg\Box e, \neg\Box z, \Box\neg b, \Box\neg e, \Box\neg z\}$

Because we get a set of inferences of possibility (which are inconsistent with subsets of 2)

from the primary inferences, set 2 as a whole turns out inconsistent. Even with disregard to these primary inferences, we cannot resolve this issue, because {□¬b, □¬e, □¬z} isn't consistent with the quality inference □b∨e∨z.

Besides how we can resolve this inadequate analysis, the more urgent question is what kind of predictions the STR makes for cases with *at least*. As already stated, *at least* triggers an ignorance inference which (according to the STR) arises through symmetry within the set of alternatives. Before we turn to the extended analysis – assuming two scales the superlative modifier operates on – I briefly want to sketch why an analysis along the lines of just one Horn-scale has to fail, at least for some cases. Hence, we now initially assume that *at least*, modifying a numeral, doesn't affect the meaning of the embedding sentence and furthermore, we'll assume that numerals have weak propositions not defining an upper-boundary. That is, (116-a) has the semantic meaning [2,...) and its alternatives are [2] and [3,...) respectively.

(116)   a.   Karl-Hubert denied at least two tax evasions.

b.   [2] [3 4 ...)

[2 3 4 ...)*   (from Schwarz 2016: 12, ex. (16))

Crucially, the alternatives given in (116-b) are not only stronger than [2,...), but also form a symmetrical pair – The disjunction of [2] and [3,...) follows from [2,...). Based on (116-b), we get:

(117)   a. $0_{[2,...)} = \{□[2,...)\}$

b. $1_{[2,...),\{[2,...),[2],[3,...)\}} = \{□[2,...), ¬□[2], ¬□[3,...)\}$

c. $2_{[2,...),\{[2,...),[2],[3,...)\}} = \{□[2,...), ¬□[2], ¬□[3,...)\}$

(from Schwarz 2016: 12, ex. (17))

Due to symmetry, we arrive at the ignorance inferences ¬□[2]∧¬□¬[2] and ¬□[3,...)∧¬□¬[3,...), respectively – The speaker isn't certain that Karl-Hubert denied two tax evasions and the speaker is not certain that Karl-Hubert denied more than two tax evasions. The STR

therefore explains two observations made earlier: (116-a) has no upper-bounding bottom line implicature ¬[3,...) being typical for unmodified numerals. (116-a) rather expresses that the speaker is not certain about the actual number of tax evasions denied by Karl-Hubert. But what happens if *at least* is embedded under an universal operator?

(118) a. Every former minister of finances denied at least two tax evasions.

  b. $\forall[2]$ $\forall[3\ \ 4\ \ ...)$

  $\forall[2\ \ 3\ \ 4\ \ ...)^*$ (from Schwarz 2016: 13, ex. (20))

(119) a. $0_{\forall[2,...)} = \{\square\forall[2,...)\}$

  b. $1_{\forall[2,...),\{\forall[2,...),\forall[2],\forall[3,...)\}} = \{\square\forall[2,...),\ \neg\square\forall[2],\ \neg\square\forall[3,...)\}$

  c. $2_{\forall[2,...),\{\forall[2,...),\forall[2],\forall[3,...)\}} = \{\square\forall[2,...),\ \neg\square\forall[2],\ \neg\square\forall[3,...),\ \square\neg\forall[2],\ \square\neg\forall[3,...)\}$

                    (from Schwarz 2016: 13, ex. (21))

With respect to $\forall[2,...)$, the propositions $\forall[2]$ and $\forall[3,...)$ aren't symmetric, i.e. their disjunction isn't entailed by $\forall[2,...)$. The latter would also turn out true in a situation in which some former ministers of finance denied two tax evasions and other denied more than two, which is quite a nice result. Notably, the universal operator abolishes the symmetry that would otherwise occur. Hence, no ignorance inferences can be derived for $\forall[2]$ and $\forall[3,...)$ from the propositions of set 1 and that is why set 2 contains the corresponding secondary inferences $\square\neg\forall[2]$ and $\square\neg\forall[3,...)$. In terms of the STR, we wouldn't get any ignorance inferences for (116-a), i.e. the sentence is consistent with the speaker being well-informed about the exact amount of tax evasions denied by each former minister of finances. Furthermore, the inference that not every former minister of finances denied exactly two tax evasions and that not every former minister of finances denied more than two tax evasions is predicted.

A straightforward advantage of referring to two scales instead of just one is that we can derive a broader set of alternatives. The set of propositions of (116-a) would therefore no longer be $\{[2,...), [2], [3,...)\}$, but a larger set $\{[n], [n+1,...)\}_{n\geq 1}$, illustrated below.

(120)   [3] [4 ...)

         [2] [3 4 ...)

         [1] [2 3 4 ...)*

         [1 2 3 4 ...)   (from Schwarz 2016: 18, ex. (30))

The set of those alternatives in (120) stronger than the asserted meaning [2,...) contains the set of strong alternatives from (116-b). Which inferences do we receive applying STR to this bigger set of alternatives?

(121)   a. $0_{[2,...)} = \{\Box[2,...)\}$

         b. $1_{[2,...),\{[2,...),[2],[3,...)\}} = \{\Box[2,...), \neg\Box[2], \neg\Box[3,...)\} \cup \{\neg\Box[n], \neg\Box[n+1,...)\}_{n\geq3}$

         c. $2_{[2,...),\{[2,...),[2],[3,...)\}} = \{\Box[2,...), \neg\Box[2], \neg\Box[3,...)\} \cup \{\Box\neg[n], \Box\neg[n+1,...)\}_{n\geq3}$

(from Schwarz 2016: 19, ex. (35))

The set of primary inferences contains the corresponding set from (119). Although, we receive a lot of alternatives not appearing in (119) – In (121), there's an additional primary inference for each additional alternative being stronger than [2,...). Out of this, the set $\{\neg\Box[n], \neg\Box[n+1,...)\}_{n\geq3}$ is built, which actually already follows from $\neg\Box[3,...)$. Contrary to (119), set 2 in (121) furthermore contains secondary inferences, i.e. the set $\{\Box\neg[n], \Box\neg[n+1,...)\}$, so once again, we have an additional secondary inference for each alternative being stronger than [2,...). Their appearance seems kind of contradictory to the possible secondary inferences $\Box\neg[2]$ and $\Box\neg[3,...)$ being absent. But those have to be missing, because none of the alternatives from $\{[n], [n+1,...)\}$ can form a symmetric pair with respect to [2,...), together with another alternative. Hence, STR derives secondary inferences since none of these alternatives is symmetric to another one.

Schwarz (2016) claims that, actually, only those alternatives should be taken into consideration which are truly relevant for a sentence's alternative meaning(s). Unfortunately, Schwarz (2016) doesn't develop such an account here. But he further tests some variations of the STR to derive the correct meaning for sentences containing the superlative modifier under

discussion. In what's coming, I'll give a quick review of how those adaptions might work and what predictions they make for *at least*.

2.6.2 Closure based Recipe

If we stick with Sauerland's (2004) analysis, a sentence containing three disjuncts doesn't only have the alternatives in (114), but a broader set of elements and their disjunctive pairings, respectively. These are given below in (122), as well as the sets of inferences we get applying STR, (123).

(122) b e z

bᐧe eᐧz bᐧz

bᐧeᐧz (modified from Schwarz 2016: 23, ex. (36))

(123) a. $0_{b\lor e\lor z} = \{\Box b\lor e\lor z\}$

  b. $1_{b\lor e\lor z,\{b\lor e\lor z,b\lor e,b\lor z,e\lor z,b,e,z\}}$
     $= \{\Box b\lor e\lor z, \neg\Box b\lor e, \neg\Box b\lor z, \neg\Box e\lor z, \neg\Box b, \neg\Box e, \neg\Box z\}$

  c. $2_{b\lor e\lor z,\{b\lor e\lor z,b\lor e,b\lor z,e\lor z,b,e,z\}}$
     $= \{\Box b\lor e\lor z, \neg\Box b\lor e, \neg\Box b\lor z, \neg\Box e\lor z, \neg\Box b, \neg\Box e, \neg\Box z\}$

(modified from Schwarz 2016: 23, ex. (37))

Contrary to what we've seen before, set 2 in (123) doesn't contain any secondary implicatures. The reason for this is that every alternative from (122) is one part of a symmetric pair, together expressing the meaning of b∨e∨z. Taken the implicature of quality and the primary implicature together, symmetry causes entailment of the negation of each possible secondary implicature. Put differently, an ignorance inference is generated for every alternative, hence, the inconsistency we were faced with before can be omitted. We'd actually need the very same effect for *at least*, ensuring us that no secondary inferences arise. So, the question is, what kind of STR-modifiction we would have to make to obtain this result.

An important observation in this context may be that the bigger set of alternatives, i.e.

(122), can be derived from the former, smaller set (114) through Closure under Disjunction (CUD, for short) – The generalized disjunction of every (non-empty) subset of (122) is itself an element of (122). Therefore, we replace set A from (108) with the new set CUD(A).

(124)   Closure based Recipe (CBR)

  a. $0_p = \{\Box p\}$

  b. $1_{p,A} = 0_p \cup \{\neg\Box q\colon q \in \text{CUD}(A) \wedge q \subset p\}$

  c. $2_{p,A} = 1_{p,A} \cup \{\Box\neg q\colon q \in 1_{p,A}$ & $\Box\neg q$ is consistent with $1_{p,A}\}$

<div align="right">(from Schwarz 2016: 24, ex. (39))</div>

The crucial difference for our example with *at least*, (116-a), under the CBR is that no secondary inferences arise. For instance, the set we arrive at, $\text{CUD}(\{[n]\}_{n\geq 1})$ contains not only [4,...) as one of its propositions, but also its symmetric partner [2,3]. In the very same way, [3] gets its symmetric partner [2,4...) and so on and so forth. Owing to this mechanism, CBR derives an ignorance inference for every alternative, i.e. no secondary inferences can be generated at all.

Unfortunately, we would have to make a fatal concession if we analyze (116-a) under CBR – Because every alternative would end up forming a symmetric pair with another alternative, (116-a) would express the meaning that the speaker is completely uncertain about the number of tax evasions denied by Karl-Hubert – i.e., the lower-bound would get lost in space – and this is not quite what (116-a) actually tells us. The example below illustrates that this truly cannot be the case.[30]

(125) a. #At least two members of the quintet were born in Germany. Exactly three

    were born in Canada.   (from Schwarz 2016: 26, ex. (41a))

  b. At least one member of the quintet was born in Germany. Exactly three were

---

[30]Schwarz (2016) notes that the only (empirically) provable ignorance inferences for *at least* are [n] and [n+1,...). Even though one can claim that no ignorance inferences are generated for very high numbers or infinitely many numbers, of course. However, since this is an article focusing on semantic theory rather than on experiments, I won't go into this topic here.

born in Canada. (from Schwarz 2016: 26, ex. (41b))

If *at least* would come with a meaning derived under the CBR, both examples in (125) should be odd. Since they're not, we should turn down this STR-modification and move on to the next.

2.6.3   Exhaustivity based Recipe

Instead of radically wiping out secondary inferences (and henceforth assumptions about the speaker's competence), the following modification is based on the thought that the hearer assigns the highest possible competence with regard to those alternatives which are consistent with the primary inferences to the speaker. A substantial step to take, then, will be to assume sets of states of beliefs rather than sets of inferences characterizing those beliefs. Those states of belief we're now assuming are to be thought of as propositions that can entail other propositions – like the asserted meaning of a sentence or its alternatives – or not (cf. Spector 2007). These assumptions are the starting point of the Exhaustivity based Recipe, henceforth EBR.

(126)   Exhaustivity based Recipe (EBR)

    a. $0_{p,u} = \{s \in u : s \subseteq p\}$

        u is defined as the states of beliefs for which the hearer considers it possible that the speaker has them – therefore, $0_{p,u}$ is the set of belief states in u entailing p; the hearer assumes that p is part of the speaker's states of belief

    b. $1_{p,A,u} = \{s \in 0_{p,u} : \neg\exists s' \in 0_{p,u}[POS_{s',A} \subset POS_{s,A}]\}$

        $POS_{s,A}$ is that set of alternatives s predicts to be true $\to \{q \in A : s \subseteq q\}$; $1_{p,A,u}$ only keeps those elements from $0_{p,u}$ which have a minimal positive yield in A, a primary inference for each strong alternative is added

    c. $2_{p,A,u} = \{s \in 0_{p,u} : \neg\exists s' \in 1_{p,A,u}[NEG_{s',A} \subset NEG_{s',A}]\}$

        $NEG_{s,A}$ is the set of those alternatives which s predicts to be false $\to \{q \in$

A: s ⊆ ¬q}; $2_{p,A,u}$ only keeps those elements from $1_{p,A,u}$ having a maximal

negative yield in A   (modified from Schwarz 2016: 29, ex. (43))

According to (126), we expect the main analytical differences in terms of what we receive for set 2. I will avoid to illustrate the analysis for sentences containing three disjuncts here and straightforwardly have a look at what happens with (116-a) if we assume the alternatives in (120).

(127) a. $0_{[2,...)} = \{s: s \subseteq [2,...)\}$

   b. $1_{[2,...),\{[n,...),[n]\}_{n \geq 1}} = \{s: s \subseteq [2,...) \ \& \ s \not\subseteq [2] \ \& \ s \not\subseteq [3,...)\}$

   c. $2_{[2,...),\{[n,...),[n]\}_{n \geq 1}} = \{s: s \subseteq [2,...) \ \& \ s \not\subseteq [2] \ \& \ s \not\subseteq [3,...) \ \& \ s \subset \neg[4,...)$ or $s \subset$
       $\neg[3,5,...)$ or $s \subset \neg[3,4,6,...)$ or...$\}$ (from Schwarz 2016: 31, ex. (47))

Set 1 picks out those states of belief from set 0 which have a minimal positive yield of $\{[1,...),$ $[2,...)\}$, i.e. set 1 is the set that entails $[2,...)$ – and therefore $[1,...)$, too – but nothing else. Note that there is symmetry in this particular set, hence, we get inferences of possibility. In a word, every state of belief from set 1 entails neither $[2]$ or $[3,...)$, nor $\neg[2]$ or $\neg[3,...)$. The maximal negative yields here are maximal subsets from the alternatives given in (110), which contain $[1]$ and a maximal subset of $\{[n], [n+1,...)\}_{n \geq 3}$, but do not contain $[1,...)$, $[2,...)$, $[2]$ and $[3,...)$. Set 2 therefore consists of belief states entailing $[2,...)$ – but not $[2]$ or $[3,...)$ – and from which an entailment of a proposition $\neg[4,...)$, $\neg[3,5,...)$, $\neg[3,4,6,...)$ etc. is possible. What EBR predicts is the following: A speaker uttering (116-a) will lead the hearer to maximize the speaker's competence by assuming that for an $n \geq 3$, the speaker believes that Karl-Hubert denied exactly two or exactly n tax evasions. This result is kind of the mirror-image of what we got under CBR. Neither one of those seems very likely to give us the intended meaning of a sentence containing the superlative modifier under discussion.

2.6.4 Innocent Exclusion based Recipe

At least, we found out two things due to the previous, unsuccessful analysis: First, the effects of STR with respect to primary inferences should be kept as is, to a great extent. Second, an adequate solution must not derive inferences basing on assumptions regarding the speaker's competence.

What Schwarz (2016) does in his final modification of STR is to adopt Fox's (2007) principle of Innocent Exclusion. This results in the desirable effect that a secondary inference gets added to a set iff that doesn't come with a restriction with respect to adding more secondary inferences which are consistent with the utterance in question as well. Each alternative that is symmetric to another alternative cannot be innocently excluded, because the secondary inference of such an alternative can't be an element of any possible set of possible inferences being consistent with set 1. But still, even not-symmetrical alternatives sometimes turn out not to be innocently excludable.

(128)   Innocent Exclusion based Recipe (IEBR)

    a. $0_p = \{\Box p\}$

    b. $1_{p,A} = 0_p \cup \{\neg\Box q: q \in A\ \&\ q \subset p\}$

    c. $2_{p,A} = 1_{p,A} \cup \{\Box\neg q: \neg\Box q \in 1_{p,A}\ \&\ q$ is innocently excludable relative to $1_{p,A}\}$

        p is innocently excludable iff $\Box\neg p$ is an element of every maximal subset of $\{\Box\neg q: \Box\neg q \in S\}$ being consistent with S.   (from Schwarz 2016: 35, ex. (53))

Like under STR, the hearer assumes the speaker's competence with regard to the alternatives being stronger than the asserted meaning per default. What distinguishes one attempt from the other is that IEBR kind of circumvents the derivation of inconsistent inferences.

Back to our example under discussion, (116-a). Under the IEBR, we can build consistent subsets of $\{\Box\neg[n], \Box\neg[n+1,...)\}_{n\geq 3}$ by picking out those elements entailing $\Box\neg[o]$ for every $o \geq 3$ – Henceforth, $[o]$ and $[o,...)$ are not innocently excludable for every $o \geq 3$. Put differently, none of the alternatives from $\{\Box\neg[n], \Box\neg[n+1,...)\}$ is innocently excludable. Means, we receive the

following results applying IEBR:

(129)   a. $0_{[2,...)} = \{\square[2,...)\}$

          b. $1_{[2,...),\{[n,...),[n]\}_{n\geq1}} = \{\square[2,...), \neg\square[2], \neg\square[3,...)\} \cup \{\neg\square[n], \neg\square[n+1,...)\}_{n\geq3}$

          c. $2_{[2,...),\{[n,...),[n]\}_{n\geq1}} = \{\square[2,...), \neg\square[2], \neg\square[3,...)\} \cup \{\neg\square[n], \neg\square[n+1,...)\}_{n\geq3}$

<div align="right">(from Schwarz 2016: 38, ex. (55))</div>

So, we can finally generate the desired ignorance inferences without deriving unwanted secondary inferences. The crucial difference between IEBR in (128) and the attempt under STR is their prevailing treatment of alternatives which are neither symmetric, nor innocently excludable. While STR derives secondary inferences for this kind of alternatives, IEBR does not. But notably, both accounts generate primary inferences for this type of alternatives – They don't get strengthened under IEBR, although (at least in theory), they could so due to the prevailing context, for instance, through specific assumptions by the hearer with regard to specific states of beliefs on part of the speaker.

Schwarz's (2016) analysis suggests that a Neo-Gricean account, trying to deal with *at least* and its ignorance inferences, should include the principle of innocent exclusion. Because, as we've seen, the derivation of unwanted secondary inferences gets restricted to those alternatives which, in the end, turn out as innocently excludable. As Schwarz (2016) notes himself, all these assumptions only apply to the cases just discussed, i.e. so far, we have no evidence that this principle would also work for cases involving modals, or other syntactic expressions (besides numerals) *at least* can combine with.

But then, the question immediately arising is, whether the underlying mechanisms of implicature/inference generation are the same for numerical as well as for other syntactic/semantic types of expressions. In the following chapter, I will try to move towards an answer to this and some related questions, such as, if *at least* really always comes with the meaning we have so far claimed it to bear.

CHAPTER 3

GERMAN *AT LEAST*-KIND EXPRESSIONS

This chapter is devoted to the interpretation and analysis of the German equivalent to *at least* and related German expressions. As we'll see within this chapter, there are several expressions a speaker of German can use to form a sentence which would (probably) end up being translated as an *at least*-sentence in English. But not every expression shares *at least*'s semantic behaviour according to the relevant literature we discussed in the previous chapter. In the following chapter, I want to highlight semantic differences among the German *at least*-kind expressions and propose an analysis for each of them. The detected semantic differences will also come with an implication for common analysis of *at least*.

The chapter is organized as follows: First, I will briefly discuss the interpretation of *mindestens*, the default expression to translate *at least* into German. Then, I will elaborate on an analysis for *immerhin* by illustrating in which syntactic and semantic respects *immerhin* differs from *mindestens* (and hence *at least*) and adjusting *mindestens*'s properties in order to fit for *immerhin*. The last section in this chapter is devoted to *wenigstens*, an element that takes the best of both – i.e. *mindestens* and *immerhin*'s – worlds.

Before we start, I have to introduce a notational convention I will use. I assume that each of the German expressions discussed in this chapter takes two arguments: The literal meaning of its syntactic sister, which – since *mindestens, immerhin* and *wenigstens* combine with a wide range of syntactic/semantic types – I will abbreviate as v and sometimes refer to it as 'value'. The other argument I assume *mindestens, immerhin* and *wenigstens* to take is a set of alternatives which is such that it contains alternatives that are of the same type as *mindestens/immerhin/wenigstens*'s syntactic sister. In the analysis, I occasionally switch between the terms 'alternative' and 'alternative value', but both refer to the alternatives from the set of alternatives *mindestens/immerhin/wenigstens* take as a second argument.

3.1 *mindestens*

I will begin my analysis of German expressions which are related to *at least* with the superlative modifier's most common translation into German – *mindestens*. In the first subsection, I want to show that *mindestens* actually shares *at least*'s ability to syntactically combine with a wide range of types of expressions, such as numerals (the obvious case), nouns, or adjectives. But unlike *at least*, *mindestens* does not occur in a sentence-initial position removed from its syntactic sister (to be discussed in 3.1.2). The third subsection is devoted to some basic semantic assumptions concerning the interpretation of *mindestens*.

3.1.1 Syntactic Considerations

The default option to translate *at least* into German is to use *mindestens*. *Mindestens* does not only share *at least*'s property of combining with expressions of many different syntactic/semantic types, it also gets interpreted along the lines of what we took to be *at least*'s meaning throughout the previous chapters.

So, just like *at least*, *mindestens* can be combined with numerals – like in (1-a) –, nouns or rather DPs/NPs – like in (1-b) –, or adjectives – as we can see in (1-c):

(1)   a.   Hans hat mindestens vier Morde   gestanden.
          Hans has ATLEAST   four murder.PL confessed
          'Hans confessed to having committed at least four murders.'

          Paraphrase, informally: Hans confessed to having committed four murders or

          Hans confessed to having committed more than four murders

      b.   Fred hat mindestens den   militärischen Rang eines Majors.
          Fred has ATLEAST   the.ACC military.ACC rank a.GEN major.GEN
          'Fred is at least a major.'

          Paraphrase, informally: Fred is a major or Fred is a lieutenant-colonel/general/etc.

      c.   Jörg war letzte Nacht mindestens betrunken.
          Jörg was last   night ATLEAST   drunk.

'Jörg was at least drunk last night.'

Paraphrase, informally: Jörg was drunk last night or Jörg was completely sloshed/etc. last night

From a syntactic point of view, *mindestens* also occurs in combination with PPs:

(2) Hilde ist mindestens auf den   Arlberg geklettert.
     Hilde is ATLEAST   on the.ACC Arlberg climbed
     'Hilde climbed at least up the Arlberg.'

     Paraphrase, informally: Hilde climbed up the Arlberg or Hilde climbed up more mountains than just the Arlberg

From an interpretational perspective, *mindestens* takes [$_{DP}$ den Arlberg] as its base in (2) (with normal intonation) to generate alternatives. Hence, *mindestens* can skip P^0 with respect to the constituent it actually modifies – Another property *mindestens* shares with its English equivalent *at least.*

3.1.2 Geurts & Nouwen (2007) revisited: Syntax

Before moving on to semantic considerations, I want make a note about *mindestens*' behaviour with regard to (overt) syntactic movement: *Mindestens* can occur postposed in a sentence, removed from its syntactic sister XP.

(3) Hans hat vier Morde   gestanden, mindestens.
     Hans has four murder.PL confessed   ATLEAST
     'Hans confessed to having committed four murders, at least.'

As the third line of (3) shows, this is also a possible syntactic position for *at least.* But at the same time, *mindestens* alone cannot occupy a sentence-initial position like *at least* can:

(4) a. At least it isn't raining.   (from Geurts & Nouwen 2007: 544, ex. (45a))

     b. *Mindestens regnet   es nicht.
         ATLEAST   rain.PROG it not
         intended, but unavailable: 'At least it isn't raining.'

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    c.   At least, Hans confessed to having committed four murders.

    d. *Mindestens hat Hans vier Morde   gestanden.
        ATLEAST   has Hans four murder.PL confessed
        intended, but unavailable: 'At least, Hans confessed to having committed four murders.'

Unlike *at least*, *mindestens* can only be moved to SpecCP (or whatever sentence-initial position above SpecTP we assume) within the whole XP it occurs adjacent to[1] – which, in turn, is impossible or at least a marked form for *at least*.

(5)   a. */?At least four murders, Hans confessed to having committed.

       b. Mindestens vier Morde   hat Hans gestanden.
          ATLEAST   four murder.PL has Hans confessed

In the following scheme, I marked all possible syntactic positions for *mindestens*.[2]

(6)   *mindestens'* possible syntactic positions:

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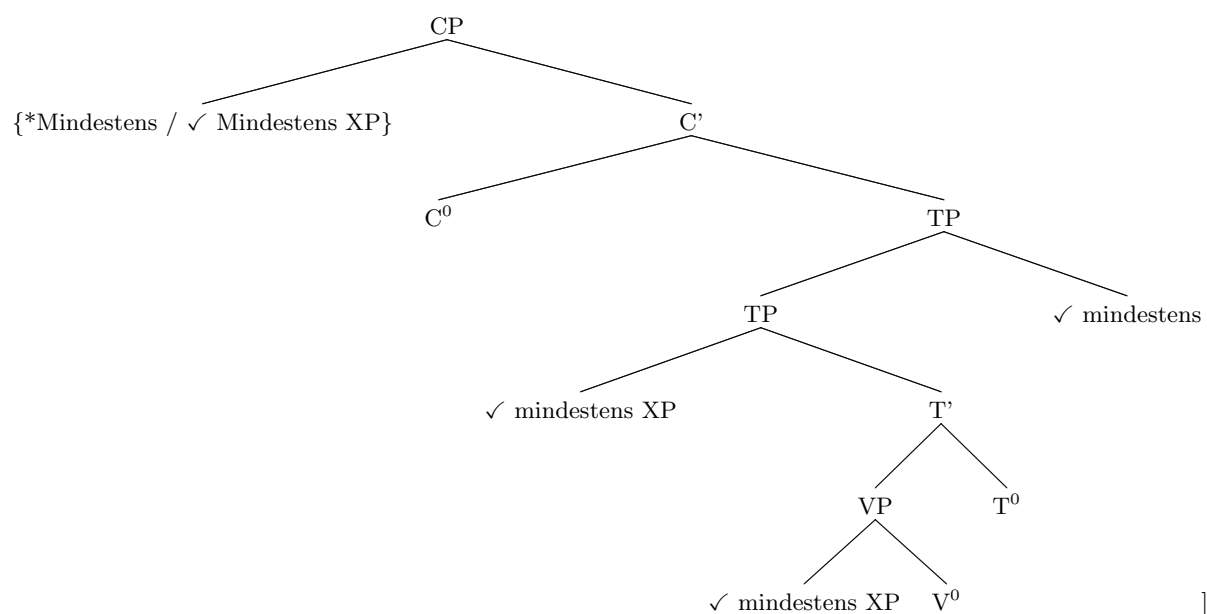
[1]Remember example (1-a) in which [$_{XP}$ mindestens vier Morde] occurs in its basic position, i.e. before it undergoes movement.

[2]Note that (6) doesn't mean that *mindestens* may occur in every marked position in one and the same sentence.

The syntactic positions *mindestens* can occupy will be of some relevance for the analysis in section 3.2 and section 3.3.

3.1.3 Semantic Considerations

With regard to the semantics of *mindestens*, we can assume a meaning parallel to *at least*. That means, if we try to analyze *mindestens*, we're faced with the same complications the accounts discussed in chapter 2 were.

In the remaining chapter, I will mainly adopt Büring's (2008) and Geurts & Nouwen's (2007) analysis for *at least* and apply it to *mindestens* and related German expressions. Specifically, I 'm taking *mindestens v* – where *v* expresses the value denoted by the expression *mindestens* combines with – to have the literal meaning 'exactly *v* or more than *v*'. This is in accordance with Büring's (2008) treatment of the implicature that arises for *at least*. I will also refer to Geurts & Nouwen's (2007) attempt in as much as *mindestens*, like *at least*, states that (for all the speaker knows) something (=v) must be the case, while something else (in relation to the former) might be the case as well. That is, I assume *mindestens* to evoke epistemic alternatives (i.e. alternatives to the literal meaning of *mindestens*' syntactic sister, which are of the same type and are also considered possible by the speaker), based on a deontic/epistemic modal base[3] of *mindestens*.

I consequently assume that, equal to *at least*, *mindestens* forces an epistemic reading. Hence, *mindestens v* may not be paraphrased as *exactly v* only. With Büring (2008), I consider *exactly v or more than v* to be a proper paraphrase for the meaning of *mindestens v*. A sentence like (1-a), repeated here for convenience,

(7)   a.   Hans hat mindestens vier Morde   gestanden.
           Hans has ATLEAST   four murder.PL confessed
           'Hans confessed to having committed at least four murder.'

       b.   epistemic alternatives: {four, five, six, seven, eight, nine, ten, ...}

---

[3]I'm furthermore assuming, with Geurts & Nouwen (2007), that this modal base is built into the lexical entry/content of *at least/mindestens*.

therefore is true iff Hans confessed that he committed exactly four murders or more than four murders (i.e. five, six, seven and so on), provided that [vier] is the focused element in (7). If, instead, [$_{\text{DP}}$ vier Morde] bears focus, (7) is true iff Hans confessed that he committed four murders (but no other crimes) or Hans confessed that he committed four murders plus some additional crimes (robberies, for instance).[4]

The alternatives in (7-b) arise because of focus on the one hand, and Quantity implicature, or rather speaker insecurity on the other hand. Uttering (7) is therefore not compatible with the speaker being well-informed about the exact number of murders Hans confessed.

In what follows, I am not going to try to elaborate on an appropriate analysis of my own for *mindestens*, but rather adopt the analysis mentioned above. Based on the assumptions made there for *at least*, plus some additional assumptions of my own, I want to examine the syntactic and semantic differences between *mindestens* and two closely related German expressions, namely *immerhin* and *wenigstens*.

3.2 *immerhin*

Contrary to *mindestens*, *immerhin* doesn't belong to any class of numeral modifiers. *Immerhin* can be rather classified as (concessive) adverb, or simply as a modifying particle. Nevertheless, *immerhin* stands in a close relationship to *mindestens* and yet to *at least*, as will become clear within this section.

I'll begin this section with an examination of syntactic and semantic properties that distinguish *immerhin* from *mindestens*. Afterwards, I will elaborate on a proper analysis for *immerhin*. I will end this section with the review of an example taken from Geurts & Nouwen (2007), for which I will argue that their analysis of *at least* isn't appropriate. Rather, I want to claim that *at least* in this specific example (that we will discuss in the last subsection of

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[4]Note that (7) can also have a third reading if [$_{\text{NP}}$ Morde] is focused. The sentence would be true then iff Hans confessed that he committed four murders or Hans confessed that he committed four murders plus (four) other kinds of crimes.

this section) should be analyzed along the lines of *immerhin*.

3.2.1 Syntactic Differences to *mindestens*

There are two essential syntactic properties *immerhin* shares with *mindestens*: First, *immerhin* can also combine with a variety of syntactic/semantic types of expressions as its syntactic sister. Actually, it's possible to replace *mindestens* with *immerhin* in every sentence of (1):

(8)   a.   Hans hat immerhin vier Morde   gestanden.
            Hans has IMMERHIN four murder.PL confessed
            ≈ '(At least,) Hans confessed to having committed four murders.'

        b.   Fred hat immerhin den   militärischen Rang eines Majors.
            Fred has IMMERHIN the.ACC military.ACC rank a.GEN major.GEN
            ≈ '(At least,) Fred is a major.'

        c.   Jörg war letzte Nacht immerhin betrunken.
            Jörg was last   night IMMERHIN drunk.
            ≈ '(At least,) Jörg was drunk last night.'

Substitution is possible, too, for the case of PPs, as we can see from the following grammatical example:

(9)   Hilde ist immerhin auf den   Arlberg geklettert.
        Hilde is IMMERHIN on the.ACC Arlberg climbed
        ≈ '(At least,) Hilde climbed up the Arlberg.'

The second syntactic property *immerhin* and *mindestens* have in common is their possible occurrence adjacent to the element they modify, as one can see from the examples in (8). *Immerhin* can further also be postposed in a sentence:

(10)   Hans hat vier Morde   gestanden, immerhin.
        Hans has four murder.PL confessed   IMMERHIN

But, crucially, *immerhin* can (contrary to *mindestens*) also occupy a sentence-initial position solitarily, that is, without the XP it modifies
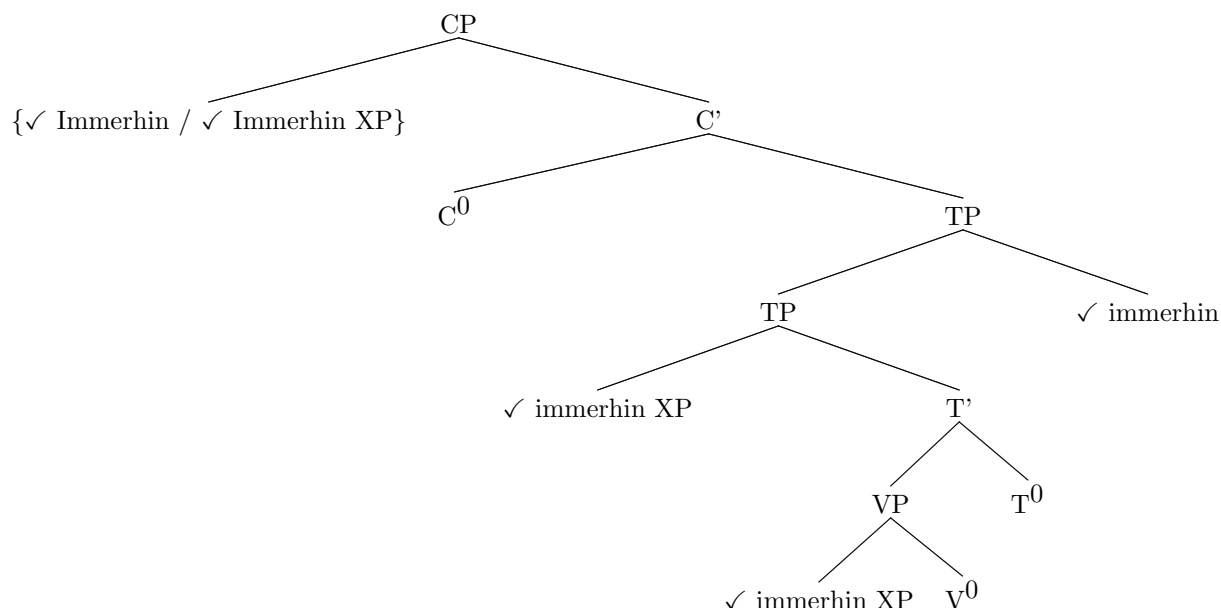
(11)   Immerhin hat Hans vier Morde   gestanden.
        IMMERHIN has Hans four murder.PL confessed

or (like *mindestens*) together with its syntactic sister XP:

(12)   Immerhin vier Morde   hat Hans gestanden.
         IMMERHIN four murder.PL has Hans confessed

We therefore arrive at the following scheme illustrating the possible syntactic positions of *immerhin*:

(13)   *immerhin*'s possible syntactic positions:

```
                              CP
                  ╱                      ╲
{✓ Immerhin / ✓ Immerhin XP}             C'
                              ╱                    ╲
                            C⁰                      TP
                                            ╱              ╲
                                          TP              ✓ immerhin
                                   ╱              ╲
                            ✓ immerhin XP          T'
                                              ╱        ╲
                                            VP          T⁰
                                          ╱    ╲
                                ✓ immerhin XP   V⁰
```

What can we derive from the difference between (6) and (13), i.e. from the fact that *immerhin* can be separated from its syntactic sister XP and move to a sentence-initial position, whereas *mindestens* cannot?

The answer to that question, I suppose, relies on focus, or rather possible argument types. On the one hand, *mindestens* cannot – in terms of syntax – take a propositional argument, as already shown in section 3.1.2. On the other hand, it is not possible for *mindestens* – in terms of semantics – to generate alternatives on the basis of a sentence's propositional meaning. So, besides the fact that a sentence like (14-a) isn't grammatical anyway, a (grammatical) sentence like (14-b) can not express the intended meaning given below, hence, (14-b) neither

conveys the (possible) epistemic alternatives in (14-c).[5]

(14) a. *Mindestens hat Hans vier Morde gestanden.
　　　ATLEAST　　has Hans four murder confessed
　　　intended, but unavailable: 'At least, Hans confessed to having committed four

　　　murders.'

　　b. Mindestens [vier Morde　　hat Hans gestanden]$_F$.
　　　ATLEAST　　four murder.PL has Hans confessed.
　　　intended, but unavailable: Hans confessed to having committed four murders or

　　　Hans confessed to having committed four murders and someone else confessed

　　　to having committed a crime as well

　　c. intended, but unavailable epistemic alternatives for (14-b):

　　　{Charles confessed to having committed adultery, Frank confessed to having

　　　committed six robberies, ...}

Unlike *mindestens*, *immerhin* is either syntactically compatible with a propositional argu-ment, as well as semantically compatible with focus on the sentence's propositional meaning:

(15) Immerhin hat Hans vier Morde　　gestanden.
　　IMMERHIN has Hans four murder.PL confessed
　　≈ '(At least,) Hans confessed to having committed four murders.'

Crucially, (15) only gives rise to the first part of the meaning that Geurts & Nouwen (2007) claim for *at least*. That is, (15) states that (for all the speaker knows) Hans confessed that he committed four murders – But (15) doesn't state that it is considered possible or likely that something else (like Charles confessing that he committed adultery) might be the case as well. Nevertheless, (15) does not have the same meaning as the corresponding bare numeral sentence (16).

(16) Hans hat vier Morde　　gestanden.
　　Hans has four murder.PL confessed
　　'Hans has confessed to having committed four murders.'

---

[5]No other syntactic position of *mindestens* would make the intended reading available either. Not even a post-sentential syntactic position of *mindestens* would give rise to the reading sketched in (14-b) above.

The specific meaning *immerhin* adds to a sentence will be discussed in the next subsection(s). What should be kept in mind is that *mindestens* and *immerhin* differ with respect to whether they can (syntactically) take a propositional argument and with respect to whether they are (semantically) compatible with focus on the sentence's propositional meaning.

This distinction between possible readings for *mindestens*-, and *immerhin*-sentences directly leads us to distinctions of semantic nature among the two elements under discussion.

3.2.2   Semantic Differences to *mindestens*

Immerhin's contribution to a sentence can be described as: 'Under the given circumstances (of the utterance), it is a considerable achievement that *v*' (or: 'the fact that *v* is the case is noteworthy') – where *v* is a placeholder for the element in *immerhin*'s focus, which can be either an ⟦X⟧, ⟦XP⟧ or the sentence's proposition. This description has two main implications. First, contextual information plays an important role for the interpretation of an *immerhin*-sentence – The utterance will have to be evaluated with respect to the given circumstances under which it is made. So, for instance, *mindestens vier* ('at least four') has the literal meaning given in (17-a) (cf. Büring 2008), whereas *immerhin vier* ('IMMERHIN four')'s meaning can be paraphrased as in (17-b):

(17)   a.   mindestens vier

lit. 'exactly four or more than four'

b.   immerhin vier

≈ Under the given circumstances (of uttering a sentence containing *immerhin vier*), *vier* expresses a considerable achievement/the fact that *vier* is the case is noteworthy with respect to any smaller amount than *vier* (hence: *three, two, one (, zero)*); but *vier* doesn't express the biggest possible achievement (for instance: *five, six, seven, ...*) under the given circumstances

c.   Therefore, *immerhin vier* ≉ 'exactly four or more than four',

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but rather 'four' and (17-b)

Stating that something, let's say (again) v, is a considerable, but not the biggest possible achievement requires two kinds of alternative elements: On the one hand, the set of those alternative elements compared to which v is a noteworthy amount (or fact, if v denotes a proposition) – For (17-b), this set would look like {(zero), one, two, three}. On the other hand, the set of those alternative elements that would count as an even bigger achievement than v – Hence, {five, six, seven, ...} for (17-b) above. Crucially, the alternatives of both sets describe states of affairs counterfactual to v, so, in our example above, to *vier*. I will apply (17-b) to an example in the next subsection for a further illustration of *immerhin*'s contribution to a sentence. For the purposes of the analysis that will be proposed, I will subsume both sets of alternatives under one set of counterfactual alternatives to avoid unnecessary complications.

The second implication is closely related to the former. *Immerhin* does not share *mindestens*'s deontic/epistemic modal base, since it doesn't give rise to epistemic alternatives. I want to claim here that *immerhin* rather comes with a circumstantial modal base, in accordance with the element's context-dependent interpretation.

3.2.3   A basic Analysis

As an illustration of *immerhin*'s semantic behaviour, let's look at two examples. In the first one, the focus of *immerhin* is assumed to be on [vier], whereas in the second one, focus on the sentence's proposition (i.e. *Hans confessed to having committed four murders*) is assumed. To force the intended readings, *immerhin* occurs adjacent to the focused element in (18) and in a sentence-initial position in (19).

(18)   Context: *At the police station. Hans is suspected of having killed and dismembered the corpses of six boy scout leaders, who spent the night in the forrest with their scout group. Hans denies having committed any murder – Although he actually is guilty*

of what he's accused of. Inspector Tory calls Inspector Bunberry, who is known to subject suspects to a merciless interrogation. But Hans is a tough nut to crack. After eight hours, Inspector Bunberry finally steps out of the questioning room. He tells Inspector Tory:

a. Hans hat immerhin [vier]$_F$ Morde gestanden.

b. given circumstances: Hans refuses to confess

c. *immerhin*'s contribution to the meaning of (18-a): Under the given circumstances (= (18-b)), it considerable that Hans has now confessed to having committed four (of six salient) murders

d. counterfactual alternatives: {zero, one, two, three, five, six (, ...)}[6]

Contrary to the corresponding *mindestens*-sentence in (7), (18) does not express speaker insecurity with regard to the exact number of murders Hans confessed. Every other amount of confessed murders therefore appears a counterfactual alternative to *four*, the actual amount. Considering the given circumstances that Hans doesn't want to confess the crimes he is suspected of, it is a noteworthy achievement (due to Inspector Bunberry's merciless interrogation) that after all, Hans confessed to having committed four murders.

Let's now have a look at the example where the whole proposition of the sentence is focused:

(19)   Context: *At the police station. Charles is suspected of having kidnapped four old ladies from a residential home of the elderly and taking them with him on a boat trip. Karl is suspected of having robbed those ladies. Hans is suspected of having killed those ladies. Last but not least, Frank is suspected of having dumped their corpses in the sea. All suspects deny the crimes they are accused of – Although they are guilty. Inspector Tory is out of his depth, and calls Inspector Bunberry for help,*

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[6]The set of counterfactual alternatives can also be expanded, i.e. it could also include numbers greater than six. If, for instance, we assume that Hans collapses due to Inspector Bunberry's method of interrogation and confesses that he has committed even a higher number of murders – murders he wasn't even suspected of by the police.

who is known to subject suspects to a merciless interrogation. After twelve hours,
Inspector Bunberry finally steps out of the questioning room and tells Inspector Tory:

a. Immerhin hat [Hans vier Morde gestanden]$_F$.

b. given circumstances: None of the suspects wants to confess

c. *immerhin*'s contribution to the meaning of (19-a): Under the given circumstances (= (19-b)), it is considerable that Hans has now confessed to having committed four murders (i.e. that at least one of the suspects confessed the crime(s) he is accused of)

d. counterfactual alternatives: {Charles confessed to kidnapping, Karl confessed to having robbed, Frank confessed to having dumped the corpses in the sea, ...}

Here, again, (19-a) does not express speaker insecurity. But this time, uttering (19-a) goes hand in hand with being well-informed about the fact that Hans confessed to having committed four murders, rather than being well-informed about the exact number of murder Hans finally confessed, as in (18-a) with focus on the numeral expression. Every other suspect confessing the individual crime he is accused of is a counterfactual alternative to *Hans confessed to having committed four murders*, i.e. the focused proposition of (19-a). Because every suspect initially refused to confess in the given context of (19), it is a considerable achievement (due to Inspector Bunberry's merciless interrogation again) that in the end, one of the suspects (= Hans) confessed. Like in (18-a), it's not the best achievement one can think of – because that would be every suspect confessing the crime he is accused of in (19) and Hans confessing the whole amount of murders he is accused of in (18) –, but still a noteworthy achievement under the given circumstances in (18-b) and (19-b), respectively.

3.2.4 Expanding the Theory: Propositional Goals and Common Grounds

I want to give some motivation for the assumptions made so far about the semantics of *immerhin* now. First of all, I will elaborate on a concept to properly interpret an *immerhin*-sentence. After that, I am going to recap the semantic behaviour of *mindestens* and how *immerhin* differs from it. In this recapitulation, I will interweave the additional new assumptions and point out the consequences they have for *immerhin*'s interpretation compared to *mindestens*'s interpretation.

Propositional Goal. So, here's the idea: An *immerhin*-sentence comes with a contextually defined variable, which can therefore vary with respect to the context. It acts as a propositional goal (henceforth p_g) that should be achieved according to a common ground. In an utterance containing the expression *immerhin v*, *v* indicates a certain degree of closeness to achieving p_g at the utterance time. Put differently, *immerhin v* proves the propositional goal to the degree expressed by *v*.

Take the previously discussed (18) (repeated here for convenience), for instance.

(20) Hans hat immerhin vier Morde   gestanden.
      Hans has IMMERHIN four murder.PL confessed

Common ground tells us that if somebody committed a crime, he or she should confess and take responsibility for what he or she has done, especially when he or she gets caught by the police. The propositional goal according to (18)'s context henceforth is: Hans confesses to having committed the amount of murders he actually committed. Assuming that Hans actually killed the six boy scout leaders, *immerhin vier* defines the degree to which p_g has been reached so far – I.e. Hans confessed to having committed four (of six salient) murders at the utterance time. *Immerhin*'s specific contribution to (18-a), paraphrased in (18-c) tells us that reaching this degree (= *four*) is a considerable achievement already with respect to the given circumstances (= Hans refuses to confess) and an intended satisfaction of p_g.

We can assume the same common ground as above for our previous example (19), where

Hans was only one of the suspects being subject to an interrogation.

(21)   Immerhin   hat Hans vier Morde   gestanden.
             IMMERHIN has Hans four murder.PL confessed

Since we were dealing with a different context for this example, the context-sensitive p_g changes. According to the fact that Hans is not the only suspect in the series of crimes related to each other in (19), the proposition goal can be defined as: All suspects confess to having committed the individual crimes they actually committed (and are accused of by the police now). *Immerhin hat Hans vier Morde gestanden* expresses the degree up to which p_g is satisfied at the specific time of uttering (19-a). Hence, it achieves the propositional goal to the degree of one suspect confessing the crime he committed, which, under the given circumstances (= All the suspects refuse to confess) is a considerable achievement.

Alternatives. Being a considerable, but not the biggest possible achievement under the given circumstances requires at least one alternative value to v – namely, the biggest possible achievement under the given circumstances, hence, what was defined as p_g above. That is, for *immerhin v*, v cannot be (equal to) the contextually defined propositional goal. Furthermore, to state that v is a considerable achievement also requires another alternative value to v which has to be such that it expresses a less considerable achievement (if any) than v with respect to p_g – For instance, {three} in our example (18). According to these assumptions, we're dealing with (at least) three propositions with regard to an *immerhin*-sentence. So, the informal idea behind this concept is that

(22)   for any three propositions p_1, p_2, p_g, $p_1 \leq_{pg} p_2$ iff if from p_1 one concludes that p_g, then one concludes p_g from p_2

That is: p_g is a propositional concept – a function from worlds to propositions –, e.g. *Hans confessed the amount of murders he actually committed* for our example (18). If Hans indeed committed six murders, a world in which Hans confessing to four murders entails p_g (i.e. a

world in which Hans committed exactly four murders) is closer to the actual world than a world in which Hans confesses to three murders, etc., hence:

(23) $p_1 \leq_{\text{pg,w}} p_2$ iff every world w' such that $p_2 \subseteq p_g(w')$ is at least as similar to w as any w'' such that $p_1 \subseteq p_g(w'')$

A restriction that applies to the set of *immerhin v*'s alternatives (and by the way also applies to the set of *mindestens*'s alternatives) is that the set of *mindestens/immerhin v*'s alternatives must not be the empty set or a singleton set. After a few notes on how to arrange *immerhin v*'s alternatives on a scale, I will come back to the issues just mentioned.

Scales. I suppose that *immerhin v*'s alternatives are arranged on a valuation scale in a teleological sense. The arrangement occurs with respect to the propositional goal, which defines the upper limit of that scale. Hence, the higher an element appears on the valuation scale, the more probable it is that p_g. Or, put differently, the higher an element appears, the more does it denote a proposition that would serve to satisfy the propositional goal. In (18-a), *immerhin vier* corresponds to the following scale:

(24)   Hans hat immerhin [vier]$_F$ Morde gestanden.
        counterfactual alternatives: {one, two, three, four, five, six(, seven, eight, nine, ...)}
        valuation scale: ⟨... two <three <**four** <five <six (<seven <eight <...)⟩

Since all of *immerhin vier*'s alternatives are counterfactual ones, *vier* marks the actual value (i.e. the actual amount of murders confessed by Hans) for the actual world on its scale (written in bold type). Assuming that Hans is only an occasional serial killer and that *six* defines the actual amount of murders ever committed by Hans, *six* marks the upper limit of the scale, hence, reaching *six* goes hand in hand with reaching the propositional goal – i.e. Hans confessing to having committed the number of murders he actually committed. Every element to the right hand side of the actual value *four* of confessed-by-Hans murders is such

that it denotes an alternative value closer to p_g, every element to the left hand side of the actual value *four* is such that it denotes an alternative value which makes reaching p_g less likely or probable.

An important note on (24): The scale's structure or rather the arrangement of the alternative elements matches the scale for the corresponding *mindestens*-sentence, i.e. (24) shows the same arrangement of alternatives as the classical Horn-scale for numerals.[7]

(25)   Hans hat mindestens vier Morde gestanden.

epistemic alternatives: {four, five, six, seven, eight, nine, ...}

Horn-scale for numerals: ⟨ ... <two <three <**four** <five <six <seven <eight <...⟩

Despite this fact, I want to claim that this is only a case of coincidental isomorphism. In a word, we're not dealing with the classical Horn-scale for numerals in (24) like in (25), but with an isomorphic valuation scale. For our example at hand, (18) in its given context, I suppose that the sequence of numerals stops at *six* (if *six* is the actual amount of murders Hans ever committed in his life), because *Hans confessed to having committed six murders* is equal to the propositional goal *Hans confesses to having committed the amount of murders he actually committed.* Every alternative greater than *six* can be said to be another counterfactual alternative in principle, but as to my mind, it is not really necessary to have them on the valuation scale for (18). Because, if Hans is suspected of six murders and he didn't commit any other amount of murders, then why should, for instance, *seven* or *eight* be an alternative we should take into consideration in our given context? The assumption that the valuation scale has a contextually defined end-point (hence, an upper limit) might not be incontrovertible, but since nothing in my analysis hinges on whether such an end-point exists or not, I will assume that if an alternative is such that it is equal (in meaning) to p_g, no higher alternatives appear on that scale.

The scale for (19-a) is slightly different, because we're dealing with focus on the whole

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[7]The element written in bold type, *four*, is the lower-bound *mindestens* places with regard to the (higher) epistemic alternatives.

sentential proposition of the *immerhin*-sentence here:

(26)   Immerhin hat [Hans vier Morde gestanden]$_F$.

counterfactual alternatives: {Charles confessed to kidnapping, Karl confessed to having robbed, Frank confessed to having dumped the corpses in the sea, ...}

valuation scale: ⟨**Hans confessed to murder** <Hans confessed to murder⊕Charles confessed to kidnapping <Hans confessed to murder⊕Charles confessed to kidnapping⊕Karl confessed to robbery <Hans confessed to murder⊕Charles confessed to kidnapping⊕Karl confessed to robbery⊕Frank confessed to having dumped the corpses in the sea⟩

The rightmost alternative element on the scale in (26) matches the propositional goal, i.e. every suspect confessing to having committed the individual crime he actually committed in the given context. The bold type element, in turn, reflects the actual value of crimes confessed by suspects in the actual world. Furthermore, again, the higher an alternative element appears on the scale, the more probable an achievement of p_g will be.

Presupposition and conversational Implicature. *Mindestens v* gives rise to a conversational implicature *exactly v or more than v* (cf. Büring 2008). *Immerhin v*, I suppose, gives rise to a conversational implicature which states that the given circumstances are such that v is a considerable achievement with respect to at least one alternative to v.[8] In a word, I assume that, what was formerly called *immerhin*'s contribution to a sentence, is a conversational implicature. Furthermore, I assume that *immerhin* triggers a presupposition – namely the presupposition that the value *immerhin* combines with may neither be a numerical expression that denotes the empty set (that is, *zero*), nor an expression equal to the uppermost element of a valuation scale, hence, the propositional goal. Evidence for the first part of the presupposition comes from the fact that the sentences in (27) seem to be a subliminal contradiction to the considerable-achievement-part of *immerhin*'s meaning.

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[8]If there's only one alternative to v, this other alternative value has to match the propositional goal, I supppose.

(27) a. (?)Hans hat immerhin null Morde   gestanden.
          Hans   has IMMERHIN zero murder.PL confessed

      b. Immerhin hat keiner der   Verdächtigen ein Verbrechen gestanden.
          IMMERHIN has none   the.GEN suspect.PL   a   crime   confessed
          ≈ 'At least, none of the suspects confessed to having committed a crime.'

According to my personal judgement, (27-a) is a marked sentence, but for other speakers,

this sentence might be fine – Hence, I marked it with a bracketed question sign. The crucial

point, however, is that (27-a) is not an utterance to be made in our context from (18), i.e. if

Hans is suspect of six cases of murder, subject to an interrogation and the amount of murders

confessed by Hans during this interrogation is the question under discussion. In a similar

way, (27-b) is an odd statement in our context from (19), i.e. if Hans is one of the suspects,

all the suspects are subject to an interrogation and the speaker wants to convey which of the

suspects he could persuade to confess the crime this suspect/these suspects is/are accused

of. The fact that (27-a) without *immerhin* (that is, the bare numeral sentence) would, in

turn, be a perfectly fine thing to say under the given circumstances of (18) further confirms

that the first part of the presupposition is indeed triggered by *immerhin*, and not due to a

specific contextual setting.

To also prove the second part of the presupposition given above, consider

(28) Hans hat immerhin sechs Morde   gestanden.
      Hans has IMMERHIN six   murder.PL confessed
      'At least, Hans confessed to having committed six murders.'

which is a grammatical sentence in principle, but a rather weird thing to say in the context

of (18), i.e. in a context where *six* murders is all there is to be confessed by Hans and the

amount of murders confessed by Hans is the question under discussion. An equally deviant

example under the assumed reading would be to say

(29) Hans hat immerhin alle   sechs Morde   gestanden.
      Hans has IMMERHIN all.ACC six   murder.PL confessed
      'At least, Hans confessed to having committed all six murders.'

which, too, is a perfectly grammatical sentence. Nevertheless, (29) is not compatible (in our context) with a reading in which focus is on *six*. Rather, (29) would be consistent with focus that has to be expanded to at least the whole constituent *all six murder*. (29) would then state that under the given circumstances, it is a considerable achievement that Hans confessed to having committed all the murders he actually committed – But crucially, the sentence then conveys that there is some other illegal thing, robbery for instance, that Hans committed but hasn't confessed yet. Or, assuming that *immerhin* takes the sentential proposition as its argument, (29) conveys the meaning that it is a considerable achievement that Hans confessed to having committed all the murders he actually committed, but there's something else to be done, for instance convincing Hans that he will repeat his confession in the court room, in order to lock Hans up.

Summing up, although *mindestens* and *immerhin* both serve as translations for *at least*, the two German elements differ from each other in certain respects. In the following subsection, I will discuss an implication of this approach (that more or less suggests itself) on the basis of an example from Geurts & Nouwen (2007).

3.2.5 Geurts & Nouwen (2007) revisited: Semantics

In the previous subsections, I showed that *mindestens* lacks two properties (closely related to each other) that Geurts & Nouwen (2007) claim for *at least*. First, *mindestens* cannot be removed from its syntactic sister XP and move to a sentence-initial position. Second, *mindestens* is not compatible with focus on the sentential proposition (taking the proposition as a basis for generating its alternatives). Geurts & Nouwen (2007) assume both properties for *at least* in their example:

(30)   a.   At least [it isn't raining]$_F$.   (from Geurts & Nouwen 2007: 544, ex. (45a))

        b. $\Box\neg\text{raining} \wedge \exists p[p \vartriangleright \neg\text{raining} \wedge \Diamond p]$ (from Geurts & Nouwen 2007: 544, ex. (45b))

According to their analysis, (30-a) states that the speaker is sure of the fact that it isn't

raining and he or she considers it possible that something 'better' than not-raining might be the case as well. In this subsection, I want to argue that (30-a) does not have the reading in (30-b). This issue stands in a close relationship with the differences among *mindestens* and *immerhin* we discussed in the previous subsections. Specifically, I will claim that *at least* in (30-a) corresponds to *immerhin* rather than to *mindestens*.

Before we proceed to the implications for German, I want to illustrate why I consider that Geurts & Nouwen (2007) are wrong about their treatment of (30-a). Since Geurts & Nouwen (2007) themselves state that what (30-a) means very much depends on the context, let's imagine a context in which a speaker would be likely to utter a sentence like (30-a):

(31) Context: *Robert and Julie want to spend a romantic holiday in the mountains. Unfortunately, the cable car which should get them up the mountain is closed, so they have to walk all the way up. The hut in which they wanted to spend the night just burned down the day before Robert and Julie arrived. So they have to spend the night outside – without food, without a covered place to sleep. Robert and Julie are freezing. Especially Julie is very disappointed about the misfired romantic holiday, but her optimistic friend Robert says:*

At least it isn't raining.

While I agree with Geurts & Nouwen's (2007) part of (30-a)'s analysis that claims that the speaker of (30-a) is sure of the fact that it isn't raining, I don't think that (30-a) further conveys that the speaker considers it possible that something better than not-raining might be the case as well. To illustrate the point I want to make, let's imagine some possible alternatives that would count as 'better' than no cloudburst taking place.

(32) {candle light dinner, a warm place to sleep, a bath in the hay, ...}

According to Geurts & Nouwen's (2007) treatment of (30-a), the alternatives given above in (32) ought to be epistemic ones – That is, alternatives that the speaker considers to possibly

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be the case (besides being sure about the fact that it isn't raining) as well. But, especially with regard to a context in which one is likely to utter (30-a), given above in (31), this is not a meaning (30-a) can convey. Put differently, assuming the context from (31), having a candle light dinner with Julie cannot be something that Robert considers to possibly be the case (as well). Rather, not-raining is all there is in (31), i.e. there are no epistemic alternatives to it. Hence, if there are any alternatives to the fact that it is not raining, they can only be counterfactual ones. As to my mind, that's even the crucial point about the 'all is not lost'-feeling that (30-a) conveys according to Geurts & Nouwen (2007). This 'all is not lost'-feeling just arises for (30-a) because not-raining is the only positive fact to claim in a context where (30-a) is likely to be uttered. In a word, *at least* in (30-a) cannot be analyzed along the lines of Geurts & Nouwen's (2007) account, that is, as a modal expression that states that something must be the case while something else (in relation to the former) might be the case (as well). But how are we supposed to analyze *at least* in this specific example, then? At this point, the previously discussed German expressions come into play.

The first property I denied for *mindestens* – i.e. occupying a sentence-initial position without its syntactic sister XP in a grammatical sentence – is not the crucial one here. Since we're talking about two different languages, variation regarding possible positions for an element may occur. What's important here is the fact that *mindestens* cannot take an entire propositional argument (as Geurts & Nouwen 2007 consider it possible for *at least* in their example), but only *immerhin* can.

In order to understand the point I want to make, let's look at the corresponding German translation(s) of (30-a).

(33) Context: as above in (31)

    a. *Mindestens regnet   es nicht.
       ATLEAST   rain.PROG it not
       intended, but unavailable: 'At least, it isn't raining.'

    b. Immerhin regnet   es nicht.
       IMMERHIN rain.PROG it not

'At least, it isn't raining.

c. given circumstances: Everything that could go wrong does go wrong in Robert and Julie's holiday

d. *immerhin*'s contribution to (33-b): Under the given circumstances (= (33-c)), it is noteworthy that it doesn't rain

e. (counterfactual / #epistemic) alternatives: {candle light dinner, a warm place to sleep, a bath in the hay, ...}

As mentioned before, (33-a) is not a grammatical sentence in German, but (33-b) is perfectly fine. Consequently, the 'better' alternatives cannot be epistemic ones, but only counterfactual ones – In a word, (33-b) is the proper way to translate (30-a) and hence, *at least* in (30-a) corresponds to *immerhin*, not to *mindestens*.

This claim has to two major implications: There might exist two kinds of *at least* – One corresponding to *mindestens* (let's call it *at least$_{epi}$*), i.e. having a deontic/epistemic modal base, giving rise to epistemic alternatives. This kind of *at least* can only occur adjacent to the XP or rather the element it modifies and it cannot appear sentence-initial without its syntactic sister XP. Furthermore, *at least$_{epi}$* cannot take propositional arguments as a basis for its epistemic alternatives, that is, it is not compatible with focus on the sentence's proposition. The other kind of *at least* (let's call it *at least$_{circ}$*) corresponds to *immerhin*, i.e. it has a circumstantial modal base, evoking counterfactual alternatives. *At least$_{circ}$* may appear in a sentence-initial position without its syntactic sister XP, and is compatible with a propositional argument as a basis for its counterfactual alternatives. Evidence for two different kinds of *at least* relies on the fact that in German, there are two separate expressions for one and the other kind.

The other implication is that, apparently, Geurts & Nouwen's (2007) analysis of their example given in (30) is not on the right track. On the one hand, Geurts & Nouwen (2007) themselves state, regarding their example, that the type of modality involved in the interpretation (of scalar modifiers in general) doesn't have to be an epistemic one. They furthermore

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even mention that (30-a)'s meaning depends very much on the context – which corresponds to the context-dependent evaluation of *immerhin*-sentences. But on the other hand, Geurts & Nouwen (2007) claim their analysis – i.e. the *mindestens*-kind of analysis – to be consistent with (30-a)'s 'all is not lost feeling' conveyed by this sentence.

So, I have to contradict Geurts & Nouwen (2007) in this respect. What they paraphrase as an 'all is not lost' feeling is, in a sense, exactly the contribution *immerhin* makes to a sentence. Just think about Robert and Julie again – Under the given circumstances, i.e. that everything that could go wrong in their holiday actually did go wrong, it is a considerable achievement (due to the weather, which, at least, does its best for Robert and Julie) that it isn't raining.

Further evidence for the claim that two different kinds of *at least* do exist comes from another German *at least*-kind expression, *wenigstens*. Actually, *wenigstens* can also be assumed to match *mindestens*'s semantic behaviour if it occupies a position adjacent to its syntactic sister constituent, and to match *immerhin*'s semantic behaviour if it occurs in a sentence-initial syntactic position solitarily.

3.3 *wenigstens*

This section is devoted to *wenigstens*, another element which can be used almost interchangeably with *mindestens* – and yet, *immerhin*.

3.3.1 One of two Kinds

Notably, *wenigstens* takes the best of both worlds, i.e. it can behave like *mindestens*, as well as like *immerhin* in a semantic and syntactic sense. The crucial point is that the prevailing interpretation of *wenigstens* apparently depends on the element's syntactic position – just like it does for *at least*.

In a word, *wenigstens* can occur adjacent to its syntactic sister XP

(34) Hans hat wenigstens   vier Morde   gestanden.
      Hans has WENIGSTENS four murder.PL confessed
      'Hans confessed to having committed at least four murders.'

as well as in a sentence-initial position:

(35) Wenigstens   hat Hans vier Morde   gestanden.
      WENIGSTENS has Hans four murder.PL confessed
      'At least, Hans confessed to having committed four murders.'

So far, *wenigstens*'s possible syntactic positions match those for *immerhin*.[9]

(36)   *wenigstens*'s possible syntactic positions:
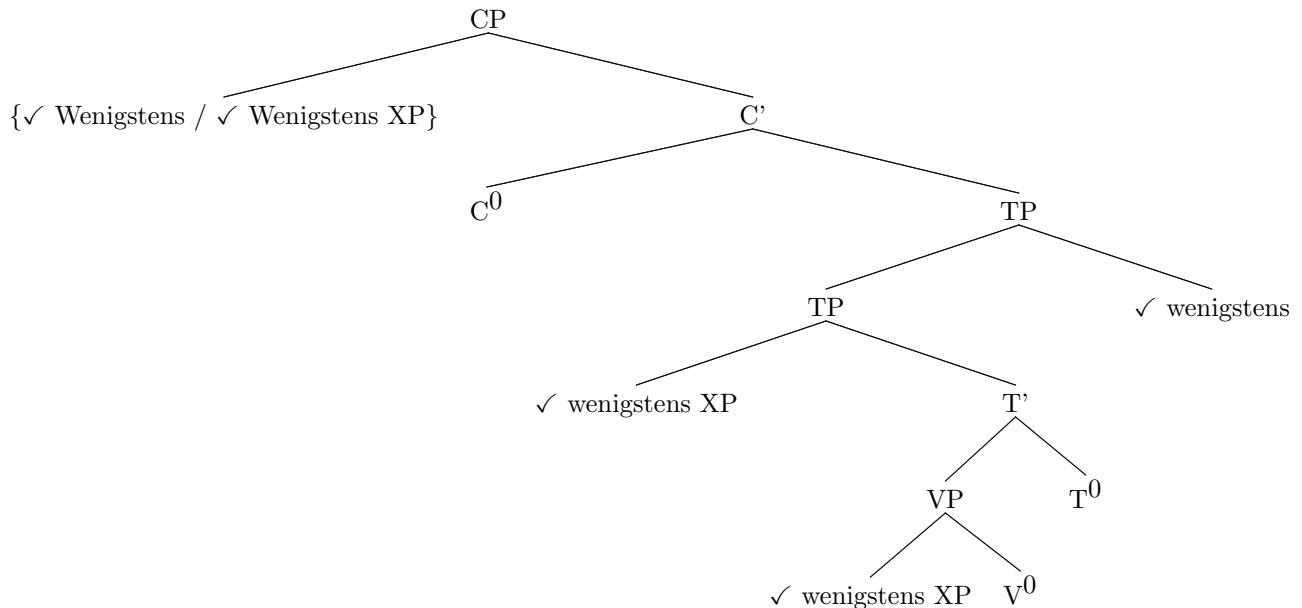
But contrary to *immerhin*, the syntactic position *wenigstens* occupies does not correspond to different focus relations – While *wenigstens* in a sentence-initial position (without tits syntactic sister constituent) is very likely to correspond to focus on the sentential proposition, *wenigstens* adjacent to its syntactic sister XP doesn't automatically correspond to focus on this element. Rather, the following is the case: If *wenigstens* occurs sentence-initial removed from its syntactic sister XP, *wenigstens*'s semantic behaviour matches the semantic behaviour

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[9]A postposed position for *wenigstens* is also possible, like it is for *mindestens* and *immerhin*. Since I did not touch upon those cases in the previous analysis, I also won't do so for the case of *wenigstens*.

of *immerhin*. If *wenigstens* occupies a sentence-initial position together with its sister constituent, *wenigstens*'s semantic behaviour matches *mindestens*'s semantic behaviour – note that this is the default case. With a special intonation or if the context explicitly forces it, *wenigstens* can also resemble *immerhin* in that syntactic position. If *wenigstens* appears together with its syntactic sister XP somewhere else in the syntactic structure – that is, in SpecTP or as an argument of the verb –, *wenigstens*'s semantic behaviour can have either the semantic behaviour of *mindestens* or the semantic behaviour of *immerhin*.

Consequently, (37-a) gives rise to epistemic alternatives, while (38-a) gives rise to counterfactual alternatives.

(37)   a.   Wenigstens [vier]$_F$ Morde hat Hans gestanden.

                ≈ Hans has confessed to having committed four murders or Hans has confessed to having committed more than four murders, i.e. five, six, seven etc.

    b.   epistemic alternatives: {four, five, six, seven, eight, nine, ...}

    c.   Horn-scale for numerals: ⟨... <two <three <**four** <five <six, <seven <...⟩

Wenigstens exactly matches the corresponding *mindestens*-sentence in (37-a): The sentence states that it must be the case that Hans confessed to having committed four murders and that it might be the case that Hans confessed to having committed more than four, that is, five, six, seven etc. murders. *Wenigstens* therefore evokes epistemic alternatives which can be arranged on a scale (I assumed a classical Horn-scale for numerals here again). Like *mindestens*, *wenigstens* in (37-a) places a lower-bound on that scale – Every alternative element to the right of this boundary, as well as the element written in bold type, *four* itself, is such that it makes the sentence true. Every alternative element to the left of the boldfaced element is entailed by *four*.

Assuming the context from (19), consider now:

(38)   a.   Wenigstens hat [Hans vier Morde gestanden]$_F$.

                ≈ Under the given circumstances (= All the suspects refuse to confess), it is

an (considerable) achievement that after all, one of the suspects confessed to

having committed the crime he actually has committed

b.   counterfactual alternatives: {Charles confessed to kidnapping, Karl confessed

to robbery, Frank confessed to having dumped the corpses in the sea, ...}

c.   valuation scale: ⟨**Hans confessed to murder** <Hans confessed to murder⊕Charles

confessed to kidnapping <Hans confessed to murder⊕Charles confessed to kidnapping⊕

Karl confessed to robbery <Hans confessed to murder⊕Charles confessed to

kidnapping⊕Karl confessed to robbery⊕Frank confessed to having dumped the

corpses in the sea⟩

In (38-a), *wenigstens* cannot take the literal meaning of *four* or *four murders* as its argument – It has to take the propositional argument. No variation appears for sentence-initial *wenigstens* without its syntactic sister XP in this respect.

The third case is the most tricky one: A sentence where *wenigstens* occurs adjacent to its sister in a SpecYP position, is ambiguous between a *mindestens*-kind of interpretation and an *immerhin*-kind of interpretation. (39) can therefore either mean that Hans confessed exactly four or more than four murders (= (39-a)), or that under the given circumstances it is a considerable achievement that Hans confessed four murders (= (39-b)).

(39)   Hans hat wenigstens vier Morde gestanden.

a.   ≈$_{mindestens}$ Hans has confessed to having committed at least four murders.

epistemic alternatives: {four, five, six, seven, eight, nine, ...}

Horn-scale for numerals: ⟨... <two <three <**four** <five <six <seven <...⟩

b.   ≈$_{immerhin}$ Under the given circumstances, it is an achievement that Hans finally

confessed to having committed four murders

counterfactual alternatives: {(zero,) one, two, three, four, five, six}

valuation scale: ⟨... two <three <**four** <five <six⟩

The question immediately arises how to distinguish between the two readings for cases like

(39). I don't have a definite answer to that. I suppose that the prevailing context in which the utterance appears probably forces one or the other reading. Or, put differently, whether the speaker can be considered to be well-informed (= *immerhin*-kind of meaning) or not (= *mindestens*-kind of meaning) might disambiguate such sentences. In our police station context from (18), Inspector Tory would probably interpret *wenigstens* in (39), uttered by Inspector Bunberry, along the lines of *immerhin*. Because Inspector Bunberry first of all is (at that point of time) the only person who is in the position to know how many murders Hans confessed. Furthermore, it is more likely that Inspector Bunberry wants to convey a meaning that has that *immerhin*-specific achievement-flavour, because he just spent eight hours questioning Hans about the crimes he is accused of – and, Hans initially refused to confess any murder at all.

But let's say that there's a corrupt detective at the police station, who secretly passes on strictly confidential information to curious journalists waiting outside the police station. And let's assume that this detective, call him Scott, heard Inspector Tory and Inspector Bunberry talking after Hans' interrogation. Scott isn't sure whether Inspector Bunberry said that Hans confessed four or five murders, so he says (39) to the journalists. Imagine further that the journalists know that the information they get from detective Scott isn't always something they can truly rely on. Such a context would probably tend to force the *mindestens*-kind of reading for (39).

In the end, there might be several aspects that come into play, such as prosodic cues or (as already mentioned) contextual information, for a proper disambiguation of a *wenigstens*-sentence in which the element under discussion appears in a SpecTP position or in a position inside the VP. For the analysis in the following subsection, however, the only crucial point is that both readings do exist.

3.3.2 All about that Base

I want to start this subsection with a summary of the syntactic distribution of the elements we analyzed so far, and their corresponding modal bases determining their semantic behaviour:

(40)   **Sentence-initial** position:

   ✓ {*immerhin, wenigstens*} → circumstantial modal base

   *mindestens

   ✓ *mindestens* XP → deontic/epistemic modal base

   ✓ *immerhin* XP → circumstantial modal base

   ✓ *wenigstens* XP → deontic/epistemic modal base OR (with special intonation / the context forcing this interpretation) circumstantial modal base

   TP/VP-internal position:

   ✓ {*mindestens* XP, *wenigstens* XP} → deontic/epistemic modal base

   ✓ {*immerhin* XP, *wenigstens* XP} → circumstantial modal base

The observations made in the previous subsection, together with the schematic illustration in (40) suggest two lexical entries for *wenigstens*: The *mindestens*-kind of *wenigstens*, call it *wenigstens$_{epi}$*, which has an epistemic modal base and cannot occur without its syntactic sister XP sentence-initially; and the *immerhin*-kind of *wenigstens*, call it *wenigstens$_{circ}$*, which has a circumstantial modal base and which can occur removed from its sister constituent in a sentence-initial position. Both types are compatible with any SpecYP position, as long as *wenigstens* occurs adjacent to its syntactic sister.

A deontic/epistemic modal base makes it impossible for the prevailing element (among those we discussed) to take a sentential proposition as its argument. Only elements with a circumstantial modal base may take propositional arguments.

Those insights taken together, *wenigstens$_{epi/circ}$* is, to some extent, a mirror-image of *at least*, for which I also claimed two different lexical entries *at least$_{epi}$* (= deontic/epistemic modal base) and *at least$_{circ}$* (= circumstantial modal base). But contrary to *wenigstens*, *at*

least cannot yield an *immerhin*-kind of interpretation if the prevailing constituent (that is, *at least* and its syntactic sister) occurs in a TP/VP-internal position. Hence, the modal base a modifier comes with takes influence on the semantic interpretation of the sentence, as well as on the type of argument the modifier can take, and it takes influence on possible syntactic positions of the modifier.

Finally, I have to point something out: Although *immerhin* and *wenigstens$_{circ}$* have the same kind of modal base, they are not one-to-one equivalents. They share the same semantic and syntactic analysis, but differ with respect to their contextual distribution. Put differently, not in every context in which *immerhin* is felicitous, *wenigstens$_{circ}$* is, too[10]:

(41)   Context: *At the court room. Hans is charged for the murders he committed. The public prosecutor argues for a life-sentence for Hans, but Hans' defense lawyer Köck wants to claim mitigating circumstances and ten years only. Köck tries to satisfy the judge by pointing out that Hans did cooperate with the police, after all. Köck says:*

   a.   Immerhin hat Hans vier Morde   gestanden.
        IMMERHIN has Hans four murder.PL confessed

   b.   ?Wenigstens hat Hans vier Morde   gestanden.
        WENIGSTENS has Hans four murder.PL confessed

In the context of (41), (41-b) would be an odd thing to say, whereas (41-a) would be perfectly fine in defense of Hans. Maybe this difference is due to the fact that *immerhin* is not ambiguous with regard to its modal base (and is therefore the more obvious choice to take here), maybe it's because *immerhin* conveys the achievement-meaning more directly that *wenigstens$_{circ}$* is kind of deviant in (41)'s context. For whatever reason (to be explored in future research on that topic), *immerhin* and *wenigstens$_{circ}$* cannot be used synonymously in every conceivable context. But at least for examples like (38-a), the correspondence holds.

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[10]Note that both (41-a) and (41-b) are grammatical sentences. The question sign in (41-b) is only supposed to indicate that the sentence is infelicitous in (41)'s context.

3.4 Interim Conclusion

German *at least*-kind expressions primarily differ from each other in one respect – their prevailing modal base, which determines their semantic interpretation. The most common option to translate *at least* into German, *mindestens*, shares the deontic/epistemic modal base Geurts & Nouwen (2007) assume for its English counterpart *at least*. But *mindestens* lacks *at least*'s ability to occur solely in a sentence-initial position. Superficially speaking, variation across languages may occur – but crucially, there's another German expression related to *at least* which has exactly this ability, namely *immerhin*. *Immerhin*, on the other hand, doesn't share *at least*'s modal base, but rather has a circumstantial base, yielding a reading that can be paraphrased as 'Under the given circumstances, it is a considerable achievement that *v*' rather than 'exactly *v* or more than *v*' (= *at least/mindestens*'s literal meaning) for *immerhin v*. Consequently, the alternatives *immerhin* evokes are not epistemic ones (like *at least/mindestens*'s), but counterfactual alternatives.

I developed an analysis for *immerhin*, assuming a contextually defined variable which acts as a propositional goal p_g that ought to be achieved (according to some common ground). A sentence containing *immerhin v* expresses a certain degree (provided by *v*) of closeness to achieving this propositional goal – Hence, *immerhin v* conveys that reaching this specific degree of closeness to p_g is a considerable achievement already, but not the biggest possible achievement (because that would be p_g itself). I supported this meaning for and analysis of *immerhin* with the fact that uttering *immerhin v* would be odd in case *v* is equal to the propositional goal or does not express any achievement at all.[11]

On the basis of these semantic differences between *mindestens* and *immerhin*, I illustrated that Geurts & Nouwen (2007) probably were on the wrong track with their analysis of sole *at least* in a sentence-initial position in one of their examples. Specifically, Geurts & Nouwen (2007) treated *at least* in *At least* [*it isn't raining*]$_\text{F}$ in terms of a deontic/epistemic modal base

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[11]I took this meaning of *immerhin v* to be a presupposition triggered by *immerhin* itself, but nothing hinges on this assumption.

(hence, like *mindestens*), which does not give us the right meaning here. According to my analysis of *immerhin*, I claimed that *at least* rather corresponds to *immerhin* in that sentence, and therefore should be analyzed differently than along the lines of Geurts & Nouwen's (2007) default analysis for *at least* – Syntactic issues aside, my assumption can be justified for mainly two reasons: Only *immerhin* is compatible with focus on a sentential proposition; and because every (context-dependent) alternative to not-raining is a counterfactual one, not an epistemic alternative. Raising this objection to Geurts & Nouwen's (2007) treatment of *at least* in the example under discussion on the basis of the data from German has one essential consequence – There might be two distinct lexical entries for *at least*: One of the *mindestens*-kind (deontic/epistemic modal base, epistemic alternatives), and one of the *immerhin*-kind (circumstantial modal base, counterfactual alternatives).

Evidence for two distinct lexical entries for *at least* comes, again, from German. Because there's another *at least*-kind expression, *wenigstens*, which just behaves like the English counterpart, i.e. it is ambiguous with respect to its modal base. If *wenigstens* solely occupies a sentence-initial position or the whole sentential proposition bears focus, *wenigstens* can only be interpreted along the lines of what I claimed as *immerhin*'s meaning. Otherwise, *wenigstens*'s interpretation can either match *mindestens* or *immerhin*. Hence, the prevailing modal base determines the element's semantic interpretation.

CHAPTER 4

GENERAL CONCLUSION AND FURTHER ISSUES

In this MA-Thesis, I discussed the meaning and interpretation of *at least* and related expressions in German – namely *mindestens, immerhin* and *wenigstens.*

In the first chapter, it was shown that there's more to *at least*'s interpretation than the simple ≥-relation the Generalized Quantifier Theory (GQT) assigned to it. I also illustrated further reasons why GQT alone doesn't suffice to capture *at least*'s meaning – Primarily because it's not possible to distinguish between a bare numeral *n* and *at least n* in terms of truth conditions under the GQT. The Generalized Quantifier Theory also doesn't account for different interpretations of *at least*-sentences which are due to different foci. In the second subsection of the introductory chapter, I gave a classification (according to Nouwen 2010) of *at least* among the class of numeral modifiers and pointed out why superlative *at least n* and its comparative counterpart *more than n-1* are not interchangable expressions on the basis of four puzzles from Geurts & Nouwen (2007). The last section of chapter 1 was devoted to some aspects which are relevant for an analysis of *at least*, such as possible structures of semantic scales and the components they are built on, the nature of degrees, and a mechanism for the generation of scalar implicatures from Sauerland (2004) which has been adopted in the literature quite often.

Chapter 2 contains the review and discussion of different accounts on *at least*, according to several articles from the past decades of research on that topic. Due to a lack of space, I could only give a discussion of some attempts, mainly those which provide a basis for my own analysis of German *at least*-kind expressions.

In the third chapter, I discussed three German expressions which are closely related to *at least – mindestens, immerhin* and *wenigstens –*, regarding syntactic and semantic issues. In the first section, I drew attention to *mindestens*, the default option to translate *at least*

into German. I assumed that *mindestens* shares *at least*'s literal meaning 'exactly v or more than v' (cf. Büring 2008) and can be interpreted as stating that the speaker is certain that v and considers it possible that something else related[1] to v is the case as well (cf. Geurts & Nouwen 2007). I detected a crucial difference between *mindestens* and *at least*: The former cannot occur solely (that is, removed from the XP adjacent to which *mindestens* is base-generated) in a sentence-initial position, and furthermore, *mindestens* is not compatible with focus on the whole sentential proposition.

In the second section of the third chapter, I discussed *immerhin*, which is not a numeral modifier, yet another possible translation of *at least* into German. I illustrated that *immerhin* occurs in the same syntactic environments as *mindestens*, but crucially may occur sentence-initial solely and is compatible with focus on the sentence's proposition. An *immerhin*-sentence, I assumed, only gives rise to the first part/conjunct of the reading that Geurts & Nouwen (2007) claimed for *at least* – That is, *immerhin v* only states that (for all the speaker knows), v is the case (and consequently, *immerhin* doesn't evoke epistemic alternatives like *mindestens* does, but counterfactual ones). Nevertheless, the meaning of a sentence containing *immerhin v* is not equivalent to the same sentence not containing *immerhin*. The specific meaning *immerhin* adds to a sentence can rather be paraphrased as 'Under the given circumstances (of the utterance), it is a considerable achievement that v'. Due to the fact that contextual information plays an important role for the interpretation of an *immerhin*-sentence, I proposed an analysis for this element which involves a contextually defined (and henceforth context-dependent) variable. This variable, I supposed, acts as a propositional goal p_g that should be achieved according to some common ground. I furthermore supposed that *immerhin* has a circumstantial modal base (not a deontic/epistemic one like *mindestens*).

According to my attempt, *immerhin v* mainly does two things: First of all, v expresses the degree to which the propositional goal has been achieved already (at utterance time); and

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[1]The relation between v and that 'something else' can be provided by a Horn-scale, a hierarchical sequence, entailment etc.

immerhin conveys that reaching this degree of closeness to p_g is a considerable achievement, although not the biggest possible achievement (because that would be p_g itself). *Immerhin* is therefore neither compatible with v expressing a degree that is equal to the propositional goal, nor with v expressing a degree that is equal to no achievement at all (or rather, what would count as an achievement with respect to the context). Evidence for this component of *immerhin*'s meaning, which I took to be a presupposition, comes from the fact that *immerhin v* would be an odd thing to say if v is either equal to p_g or equal to a non-achievement value (for instance, if we're dealing with numerals: *zero*). My analysis has the following implications: First of all, the set of *immerhin v*'s alternatives must not be the empty set, but rather has to be a set containing at least three propositions – A proposition p_1, judged as a less considerable achievement than v (or no achievement at all), a proposition p_2 containing v, and finally, a proposition equal to p_g. Second, the scale *immerhin* refers to is a valuation scale in a teleological sense – If this valuation scale has the same structure and/or contains the same elements as, for instance, a Horn-scale *mindestens* would refer to, this is only due to coincidental isomorphism.

Taken all the syntactic and semantic insights into *immerhin* together, I claimed that *at least* in Geurts & Nouwen's (2007) example *At least* [*it isn't raining*]$_F$ rather corresponds to *immerhin* than to *mindestens*. But that means that *at least* here cannot be interpreted along the lines of Geurts & Nouwen's (2007) default analysis for *at least*, which they applied to this example as well. I claimed that Geurts & Nouwen's (2007) treatment of this example is on the wrong track, and illustrated why their example does not convey that the speaker considers it possible that something 'better' than not-raining might be the case as well, but rather only states that the speaker is certain that it is not raining. Consequently, there are no epistemic alternatives for the sentential proposition of their example, but only counterfactual ones. The conclusion to draw from my contradiction to Geurts & Nouwen's (2007) judgement regarding the interpretation of this sentence is, that we might have to assume two distinct lexical entries of *at least*: One lexical entry (which I called *at least$_{epi}$*) that

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has a deontic/epistemic modal base, evokes epistemic alternatives, may not occur solely in a sentence-initial position and is not compatible with focus on the whole sentential proposition. The other lexical entry of *at least* (which I called *at least$_{circ}$*) has a circumstantial modal base, evokes counterfactual alternatives, may occur in a sentence-initial position solely and does well with focus on the sentence's proposition.

Evidence for the claim that an element can have two distinct modal bases (and hence: two lexical entries) comes from another German translation of *at least*, namely *wenigstens*. I gave a brief discussion of *wenigstens*'s different interpretations in the last section of chapter 3, highlighting the fact that the prevailing modal base is crucial for the element's syntactic, and especially for its semantic behaviour.

There are several issues I haven't touched upon within my own analysis of German *at least*-kind expressions in this MA-Thesis. Primarily, I did not discuss examples involving modals. Furthermore, I used contexts in which the propositional goal is more or less obvious or at least easily understandable. I also didn't take another crucial German element into consideration, which is also closely related to *at least – zumindest*. Although at first sight and according to my initial judgements, *zumindest* seems to behave quite similar to *wenigstens* (that is: two lexical entries because of two distinct modal bases), I think *zumindest* should be considered for further research on that topic, since there might be some differences between the two. Finally, as mentioned at the end of the last section in chapter 3, *immerhin* and *wenigstens$_{circ}$* are not one-to-one equivalents for so far unknown reasons – The latter can be infelicitous in certain contexts in which *immerhin* is perfectly fine. All those issues have to remain open for future work.

CHAPTER 5

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5.2 Abstract (German)

In dieser MA-Arbeit werden der englische numerale Modifizierer *at least*, sowie drei verwandte Ausdrücke aus dem Deutschen diskutiert – *mindestens, immerhin* und *wenigstens*. Meine Analyse von *mindestens* folgt im wesentlichen den Ansätzen zur Interpretation von *at least* aus der relevanten Fachliteratur. Ich nehme in dieser MA-Arbeit an, dass *mindestens v*s, wie *at least v*s, wörtliche Bedeutung als 'exakt *v* oder mehr als *v*' (cf. Büring 2008) paraphrasiert werden kann und ein Satz, der *mindestens v* enthält, besagt, dass der Sprecher sich sicher ist, dass *v* der Fall ist und es für möglich hält, dass etwas anderes in Relation zu *v* auch der Fall sein kann (cf. Geurts & Nouwen 2007). Weiters folge ich Geurts & Nouwen (2007) in der Annahme, dass *at least*, und somit auch *mindestens* eine (lexikalisch festgelegte) deontische/epistemische modale Basis hat.

Für meine Analyse zu *immerhin* gehe ich von einer kontextuell definierten und daher kontextabhängigen Variablen aus, die als eine Art propositionales Ziel fungiert, welches es zu erreichen gilt. *Immerhin v* drückt nach dieser Ansicht vor allem zwei Dinge aus: Zum einen stellt *v* den Grad dar, zu welchem das propositionale Ziel zum Zeitpunkt der Äusserung bereits erreicht wurde. Zum anderen besagt *immerhin*, dass das Erreichen dieses Grades unter den (kontext-)gegebenen Umständen eine nennenswerte Leistung darstellt, jedoch nicht die höchste oder grösstmögliche, denn diese wird vom propositionalen Ziel vorgestellt. *Immerhin* ist somit mit zwei Graden nicht kompatibel: Mit einem *v*, das dem propositionalen Ziel entspricht und mit einem *v*, das nicht in irgendeiner Form eine nachvollziehbare Leistung darstellt, wie zum Beispiel *null* in Fällen mit Numeralen. Ich nehme an, dass diese Restriktion eine Presupposition von *immerhin* ist und gehe ferner davon aus, dass *immerhin* über eine circumstantielle modale Basis verfügt.

Immerhin ruft demnach keine epistemischen Alternativen hervor (wie *mindestens* dies tut), sondern vielmehr kontrafaktuelle. Weiters referiert *immerhin* meinem in dieser MA-Arbeit vorgestellten Ansatz zufolge nicht auf eine Horn-Skala oder ähnliche Skalen, auf welche sich *mindestens* beziehen kann, sondern vielmehr auf eine Bewertungsskala in einem teleologischen Sinne.

Ich zeige in dieser MA-Arbeit zwei wesentliche Unterschiede zwischen *mindestens* und *immerhin* auf: Nur *immerhin* kann alleine an einer satzinitialen Position stehen und nur *immerhin* ist mit Fokus kompatibel, der sich über die gesamte sententiale Proposition erstreckt. Anhand eines Beispiels aus Geurts & Nouwen (2007) erläutere ich mögliche Konsequenzen dieser Beobachtung. Spezifisch gesprochen gehe ich davon aus, dass für *at least* zwei unterschiedliche lexikalische Einträge existieren – Einer mit einer deontischen/epistemischen modalen Basis, der epistemische Alternativen hervorruft und eine syntaktische Verteilung hat, welche jener von *mindestens* entspricht; und einer mit einer circumstantiellen modalen Basis, der kontrafaktuelle Alternativen evoziert und dessen syntaktische Distribution sich mit jener von *immerhin* deckt.

Evidenz für diese Annahme lässt sich anhand von *wenigstens* finden, das sich in dieser Hinsicht sehr ähnlich zu *at least* verhält und für das man demnach ebenfalls zwei divergente lexikalische Einträge annehmen muss. Die semantische Interpretation und syntaktische Distribution der hier besprochenen Elemente wird also vor allem von deren jeweiliger (lexikalisch festgelegten) modalen Basis beeinflusst.