MASTERARBEIT / MASTER’S THESIS

Titel der Masterarbeit / Title of the Master‘s Thesis

„A Comparative Study of Logistics Districting and Daily Vehicle Routing“

verfasst von / submitted by

Maximilian Waltenberger, BSc

angestrebter akademischer Grad / in partial fulfilment of the requirements for the degree of

Master of Science (MSc)

Wien, 2018 / Vienna 2018

Studienkennzahl lt. Studienblatt / degree programme code as it appears on the student record sheet:	A 066 915
Studienrichtung lt. Studienblatt / degree programme as it appears on the student record sheet:	Masterstudium Betriebswirtschaft
Betreut von / Supervisor:	o. Univ.-Prof. Dipl.-Ing. Dr. Richard Hartl
Mitbetreut von / Co-Supervisor:	

Contents

1 Introduction

During the last decade, e-commerce experienced a steady increase in either users and usage volume and became an indispensable part of today's commercial landscape. With rising e-commerce acceptance, simulatenously the amount of parcels to be shipped is increasing. In Austria six companies are competing in the small package delivery market, the biggest player, the Post AG reported an increase in parcel deliveries from 59 million pieces in 2011 to an all time high of 97 million in 2017 [15] [16]. This yearly increasing delivery volume leads to increased complexity in the underlying vehicle routing problems (VRPs). In sight of rising customer demands such as an increasing amount of return shipments and new offers in absence deliveries, the complexity rises even further. One promising way to tackle those difficulties is shaping the problem to include consistency.

Kovacs et al. [8] provided a comprehensive survey where they classified the literature according to three consistency features such as arrival time consistency, person-oriented consistency and delivery consistency. Consistency in VRPs adds several positives effects like extended drivers knowledge of the territory, the customers and the last meters from a drivers parking spot to the customers door.

Kovacs et al. identified three different approaches how consistency can be achieved. With a priori routing, through historical data, routes covering the possible customers are calculated and then adapted when the actual demand is known. Groër et al. [4] referred to a priori routes as template routes, these template routes where created following a simple principle: each pair of customers, served by the same vehicle, has to be served in the same order each day they both require service. They implemented those template routes to increase customer satisfaction through route consistency.

Consistency can also be achieved through demand stabilization. Haughton [5] presented a model in the freight delivery context where route consistency is achieved through delivering not the actual demand each time it is required but delivering the expected demand each day and therefore eliminating demand fluctuations completely.

This masters thesis concentrates on the third possibility of bringing consistency to VRPs, namely districting. Districting, in a vehicle routing context, is dividing a service territory into several contiguous, compact and balanced sub territories that are each served by one unchanging driver, where the routes within the sub territories are calculated daily. Therefore the daily optimization problem is broken down from one big complex VRP to K simple TSPs.

The methods described above typically lead to an increase in tour lengths. However, the positive effects can offset parts of the higher routing cost [6]. Other positive effects are low day-to-day computational effort, no costly daily communication of new routing plans to drivers, temporal flexibility in the planning section, higher customer and employee satisfaction through familiarity between customer and driver [28].

This masters thesis examines the applicability of logistics districting using real life data from the city of Vienna. A simple construction heuristic, two local search improvement heuristics, and a more advanced metaheuristic are presented for generating districting plans of increasingly higher quality. In contrast to much of the literature on districting for VRPs, our approach is based on optimizing over a large set of representative demand scenarios, rather than averages. In addition, the districting plans are validated also on a further set of evaluation scenarios. This fills a notable gap in the literature, because numerous districting approaches exist but most of them are not checked against future periods, although they are produced to be used in the future. Not only the results of the objective function are compared, but also compactness and balance in terms of differences in tour lengths. The practical applicability of the districting plans is checked by the time needed to complete the tours, which shall lie under legal work time limits. For each instance a detailed evaluation chart is given, including two diagrams depicting the balance indicators and additionally for each districting plan a figure with the districting plan drawn on the map of the city of Vienna is presented.

Based on this analysis, relevant observations are pointed out and a number of preliminary conclusions about the performance of the heuristics, application possibilities and possible improvements are drawn.

2 Literature Review

A large variety of districting problem owing to many different applications exist. They have some high-level aspects in common namely a territory is partitioned into contiguous, compact and balanced sub-territories. Due to the subjective nature of some of these common aspects and the variety of applications, numerous models have been proposed. In the following a short overview of the different categories of districting models are presented.

As with all the vehicle routing problems, districting problems can be modelled exact or heuristically but the majority of the literature is using heuristic approaches. M. Salazar-Aguilar et al. [17] developed an exact solution framework based on branch and bound and a cut generation strategy. Their objective was to create a districting plan with a given number of districts out of a set of city blocks where the districts are required to be compact, contiguous and balanced. They recommended their model only for medium-sized instances as the computational time rises exponentially with the size of the input. The same authors later extended their model to fit a bi-objective problem [18]. They proposed a method generating the optimal Pareto front for instances with up to 150 units and 6 territories. It shall be noted that exact models are only exact when applied to their input data and not when their districts are applied on future periods not yet known during the districting process. This stands in contradiction to the main idea of districting, where a districting plan is produced using past data to gain positive effects in the future such as lowered computational effort and positive consistency effects. So the classification *exact methods* can be seen misleading.

Many authors use some element of stochasticity in their input parameters to tackle assumed uncertainties. The used element of stochasticity varies. Haughland et al. [7] worked with independend stochastic customer demands and given customer locations. The approach of Novaes et al. [11] uses stochastic service times and customer demands with a given demand probability. Zhong et al. [29] worked with random customer locations and random customer demand on strategic level. Carlsson [1] described a method with unknown customer locations but a given probability density of customers over the area.

Typically a districting problem is formulated as a single-depot problem with one depot serving all the districts such as in the early work of Wong and Beasly [27]. They proposed a model where customers are clustered to districts based on the number of times they lied on the same route in historical periods. Districting problems can also be formulated as multi-depot problems. Carlsson [1] considered an uncapacitated problem where multiple depot locations are known and customer locations are unknown but a probability density is given. The model partitions the area in n contiguous sub-regions where n equals the number of depots.

The districting problem can also be varied by introducing various constraints. Schneider et al. [19] developed a districting approach using service territories and exclusion zones around the depot where the customers are not assigned to a service territory but assigned to drivers on a daily basis. They investigated the requirements for handling time window constraints and the influence of those on the performance of the algorithm. Zhong et al. [29] developed a comparable approach with cells, representing a number of customers serving as the minimum unit of separation, core areas and flex zones with vehicle utilization as a constraint. Salazar et al. [18] created a bi-objective programming model where dispersion and balancing the number of customers are objectives and connectivity and balancing to sales volume are constraints. The model of Novaes et al. [11] takes into account time and capacity constraints at the vehicle level. Their districting approach is an extension of their previous work by the use of a continuous approximation [12].

Several methods to create more or less sophisticated initial solutions from scratch exist. A main distinctive feature is if they build the districts either agglomerative, so the districts grow after each iteration or divisive, where the main area is split recursively into the districts. Haughland et al. [7] developed an agglomerative districting model where a set of arcs, consisting of all the possible arcs connecting nodes without intersecting each other, is computed. Iteratively nodes are chosen as seeds of new districts using an exclusion list based on the set of arcs and then these districts are filled with neighbours as long as adding does not violate any constraints. Most divisive approaches use a geometric partitioning method of the underlying service area. Novaes and Graciolli [12] presented a divisive approach where the service region is first divided into sectors with a polar coordinate system, centered at the depot then partitioned into rings and finally each ring is partitioned into districts. The model of Galvao et al. [2] begins with a previously determined ring-radial districting pattern and creates district contours with the use of a Voronoi diagrams. They approximated the expected travel distance by the density of points within the underlying area, multiplied with a route factor and incorporated vehicle time utilization and vehicle capacity utilization as constraints.

More recently hybrid methods have been proposed which are combining two or more well known metaheuristics. Gonzales et. al [3] presented a hybrid method which makes use of elements of the greedy randomised adaptive search procedure and tabu search and is able to solve large-scale instances. They are tackling a real life single depot pick up and delivery problem of a Mexican parcel company with the goal of dividing the service area in single driver delivery and collection zones that are balanced in terms of workload between districts and compact in geographic shape.

3 Problem Description

Various approaches for efficient districting have been developed in the past. Many of them have in common that in order to construct a districting plan, some kind of historical data is used. The districting plans are then built to perform best on the given historical data. Literature containing comparisons on the performance of districting plans in future periods e.g. periods that are not used in the construction process of the algorithm are very sparse. This work fills in the gap, it provides a comparison of the performance of districting plans in periods not used in the construction phase with those used to create them.

The research focus in this work lies on ride intensive services with low service time at the customers location such as small package delivery or meter reading. Routes begin and end at one fixed depot. The problem is modelled around a set of Census tracts (CTs), representing the minimum unit of separation. CTs contain a set of geographic units, each representing the location of a customer. Per period X customers, chosen randomly request service, the others do not have to be approached during that Period. The service time of customers requesting service differs from period to period.

The whole workload within a district has to be assigned to a single driver. Districts consist of several census tracts. The districts should be contiguous and compact. In this context, contiguity means that all CTs in a district can be served in a continuous tour without exiting the district. Compactness refers to the geographic shape of the district, which should not be "too spread out" or "too thin". In a given period, the travel time within a CT is approximated with the TSP tour visiting all the customers requesting service in that CT during that period. This data can be preprocessed prior to any further optimization. In order to calculate the travel distance between CTs, artificial geographic center points of the census tracts are used. Figure 1 shows Vienna separated by the CTs with their geographic center points.

To take into account the fact that a driver will not go to this artificial center point when entering the census tract from a neighbouring one, but rather going to the nearest customers location, the distances between neighbouring census tracts are halved. The routes within the census tracts are preprocessed for each period. Therefore the calculation of the total travel distance within a district is a two stage process:

1. Calculate the TSP tour over the artificial centers of all the census tracts within the districts, starting and ending at the depot.

2. Calculate one TSP tour through all the customer locations where service is requested within each census tract in the district and summarize them.

Summarizing the two components leads to the approximation of the total travel distance within the district at the given period. In order to calculate the total

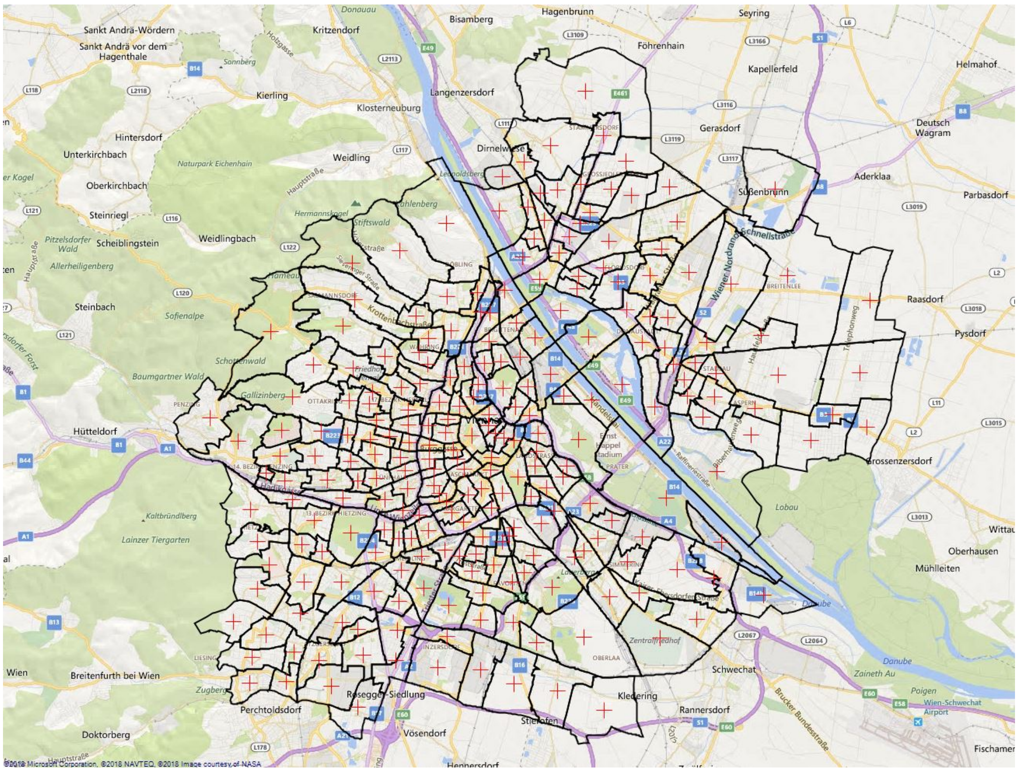


Figure 1: Vienna with CTs and CT centers

time needed to serve the district in the given period one last piece has to be added to the total travel distance:

3. Summarize the service time needed at all customers requesting service at the given period appearing within each census tract of the district.

With the data described above, the districting plans where generated. These districting plans where then assessed by their practical and legal applicability. Within the European Union, the working time is regulated by the Working Time Directive (2003/88/EC) [14] in general and for persons performing mobile road transport activities in directive (2002/15/EG) [13]. These directives limit the weekly average working time to 48 hours and the weekly maximum working time to 60 hours. In Austria, the working hours are generally capped more strictly. The working conditions in the shipping trade business are regulated by the collective agreement for employees in forwarding and logistics [25]. Within this collective agreement, the normal working hours are set to 8 hours per day and 40 hours per week but can be extended to a maximum working time per week of 48 hours and 10 hours per day when within the calculation period of 6 months the weekly normal working hours of 40 are met. Therefore the maximum overtime for a districting plan to be legally feasibly in Austria is 120 minutes. The cap of 40 working hours per week within the calculation period leads to average working minutes of 480 when a five workdays work week is supposed. Therefore a districting plan with an average worktime above 480 minutes is not practicably feasible in Austria. It shall be noted that the collective agreement for employees in forwarding and logistics also permits work weeks with 4 and 6 workdays in full time jobs [25].

Liu, Chang and Huang [9] stated that balanced workload between districts improve workers morale and the customer satisfaction. They proposed a model that achieved lower overtime and travel distance through better balancing. In this masters thesis, the contrariwise way was used, through lower overtime a better balanced districting plan is achieved. Balance was measured by the range between working times. The level of acceptable imbalances between the districts can not be stated in general but differs between companies. But generally speaking the more balanced the districting plan is, the better.

4 Model

The model used in this study is largely the same as the standard 3-index formulation of the VRP [24]. With some extra constraints for:

- objective function
- penalty function on route duration
- compactness
- contiguity

The same decision variables as in the standard 3-index formulation of the VRP are used, with customers being re-interpreted as CTs, so in this study routing is performed over CTs instead of customers.

Input Parameters are:

$T \leftarrow$ set of Periods
$V \leftarrow$ set of CTs
$K \leftarrow$ number of Vehicles = number of Tours = number of Districts
$S_i^t \leftarrow$ the TSP tour over the customers in CT i plus the sum service times at customers in CT i during period t
$c_{ij} \leftarrow$ travel time between the centers of CT i and j

$D \leftarrow$ maximum tour duration

Decision Variables are:

$x_{ij}^{kt} \leftarrow 1$ if tour k travels between customer i and j during period t
$y_i^{kt} \leftarrow 1$ if CT i is served by vehicle k during period k

Objective Function

$$\text{Minimize} \quad f(\mathbf{x}) = f_a(\mathbf{x}) + f_b(\mathbf{x}) + f_c(\mathbf{x}) \tag{1}$$

The objective function measures the cost associated with a districting plan. It consists of three parts:

1. $f_a = \sum_{t=1}^{T} \sum_{k \in K} \sum_{i \in V} y_i^{kt} S_i^t$: Sum of travel and service times within CTs summarized over all Periods T
2. $f_b = \sum_{t=1}^{T} \sum_{k \in K} \sum_{i \in V} \sum_{j \in V} x_{ij}^{kt} c_{ij}$: Sum of travel times between CTs summarized over all Periods T
3. $f_c = \sum_{t=1}^{T} \sum_{k \in K} 2o^{kt}$: The sum of the Overtime penalty over all Periods T

Four standard constraints are used to ensure that each tour begins and ends at the depot and each CT is visited exactly once:

$$\sum_{k=1}^{K} y_i^{kt} = 1 \qquad \forall \quad i \in V \setminus \{0\}\,; \quad t \in T \tag{2}$$

$$\sum_{k=1}^{K} \sum_{i=1}^{I} x_{0i}^{kt} = K \qquad \forall \quad t \in T \tag{3}$$

$$\sum_{k=1}^{K} \sum_{i=1}^{I} x_{i0}^{kt} = K \qquad \forall \quad t \in T \tag{4}$$

$$\sum_{j \in V} x_{ij}^{kt} = \sum_{j \in V} x_{ji}^{kt} = y_i^{kt} \qquad \forall \quad i \in V, \quad t \in T, \quad k = 1, ..., K, \tag{5}$$

Additionally a sub tour elimination constraint and two non negativity constraints are needed:

$$\sum_{i \in S} \sum_{j \in S} x_{ij}^{kt} \leq |S| - 1 \qquad \forall \quad S \subseteq V \setminus \{0\}, |S| \geq 2, t \in T, k = 1, ..., K, \tag{6}$$

$$y_i^{kt} \in \{0, 1\} \qquad \forall \quad i \in V, \quad t \in T \quad k = 1, ..., K, \tag{7}$$

$$x_{ij}^{kt} \in \{0, 1\} \qquad \forall \quad i, j \in V, \quad t \in T \quad k = 1, ..., K, \tag{8}$$

To take into account the overtime penalty, a non standard VRP variable has to be added to the model:

$o^{kt} \rightarrow$ overtime of route through district k during period t

Two additional overtime constraints are needed:

$$o^{kt} \geq \left(\sum_{i \in V} y_j^{kt} S_i + \sum_{i \in V} \sum_{j \in V} x_{ij}^{kt} c_{ij} \right) - D \qquad \forall \quad t \in T \quad k = 1, ..., K, \tag{9}$$

$$o^{kt} \geq 0 \qquad \forall \quad t \in T \quad k = 1, ..., K, \tag{10}$$

Overtime is defined as the difference between total tour duration and maximum tour duration in (9). Additionally the model contains two non standard constraints that are changing during the runtime every time a CT is added to a district. Therefore it is not possible to solve the problem to optimality with a MILP Solver.

1. Compactness constraint: The compactness of the districts may not fall below a certain lower bound. To measure compactness the formula $\dfrac{\sqrt{area}}{0.282 * perimeter}$ was used, where *area* stands for the summarized area of the CTs within the district and *perimeter* for the districts outside border lenght. It returns a Value between 0 and 1, where 1 stands for the most compact geographic shape, a circle [10].

2. Contiguity constraint: There must be a path that lies completely within the district connecting all the CTs.

The following table states the classic notation and the differences to the model used in this work.

VRP	model of this thesis
customer i	census tract (CT) i
x, y	x_{center}, y_{center} (center of census tract)
$x_{ij}^{k} \rightarrow$ tour k travels between customer i and j	$x_{ij}^{kt} \rightarrow$ decision to travel between CTs instead of customers in period t
$c_{ij} \rightarrow$ travel time between customer i and j	travel time between centers of CT i and j
$s_i \rightarrow$ service time	$S_i^t \rightarrow$ TSP tour over customers in CT i + sum of s_i^t over those customers in period t
D $\rightarrow$ max tour duration	unchanged
$y_{ik} \rightarrow$ customer i is served by vehicle k	$y_{ik}^t \rightarrow$ CT i is served by vehicle k in period t
K $\rightarrow$ number of Tours	K $\rightarrow$ number of Tours = number of vehicles = number of districts

Table 1: VRP vs this works model: Notation

Balanced workload between drivers is also desirable in principle, but it is not considered explicitly in the model because it was expected that through the overtime penalty on route duration, balanced districts are achieved and no further implementation of balancing in the model is needed. Because K represents the number of tours and both the number of trucks and the number of districts, a well balanced districting plan is characterised by roughly the same duration between the tours.

5 Methodology

In this work a districting method consisting of multiple phases is used:

1. Construction of an initial solution
2. Improvement through simple local search
3. Further improvement through a meta heuristic, namely large neighbourhood search (LNS)

For comparability reasons, all the three phases follow a deterministic paradigm.

5.1 Construction heuristic

As construction heuristic, a two stage agglomerative heuristic with the goal of assigning each census tract to a district was used.

- **The first phase**, the sequential districting phase begins with finding a CT for the first district and iteratively adding neighbouring CTs until either no CT can be added without violating the compactness or contiguity constraint or the stopping criteria is reached. Then the next district is opened with a new seed and filled the same way. This procedure continues until the number of districts equals the number of available trucks. Note that after the first phase is finished a feasible assignment is not guaranteed. Output is a districting plan with size equal to the fleet size and possibly a set of unassigned CTs.

- **The second phase**, the parallel districting phase finds in each iteration the combination of one district and one unassigned CT that increases the objective the least and merges them. This is repeated until all the CTs are assigned to a district.

With this two stage agglomerative heuristic it is possible to start with an sequential algorithm that iteratively searches for new suitable seed CTs. Therefore the seeds do not have to be preassigned before the districting starts and no random generator is needed. With the second stage it can be avoided that in the last iteration of the sequential districting all the unassigned CTs are assigned to the last district where compactness and contiguity would not be guaranteed.

Finding seed CTs: In order to find a seed census tract for a new district, several methods where used. These methods followed a strict ranking, only if the top ranking method was impossible, the next lower ranked method was used:

1. From the pool of all census tracts that lie on the border of the overall territory, the census tract with the highest compactness value is chosen as seed. Only possible once at the beginning when all the districts are empty.

2. From the pool of all CTs that lie on the border of the overall territory and are neighbours of at least one CT assigned to any district, the CT with the highest compactness value is chosen as seed. Only possible when there are unassigned census tracts left that share borders with the overall territory.

3. From the pool of all census tracts that share borders with any district, the one with the highest compactness value is chosen as seed.

The figures 2 (a) to (f) visualize the main steps of the construction procedure on a sample instance

- **Figure 2(a):** All the CTs marked by a black cross are possible seed customers for the first disrict, because they lie on the border of the overall territory. The blue CT is chosen because it is the most compact out of those. Now the district is ready to be filled with more CTs.

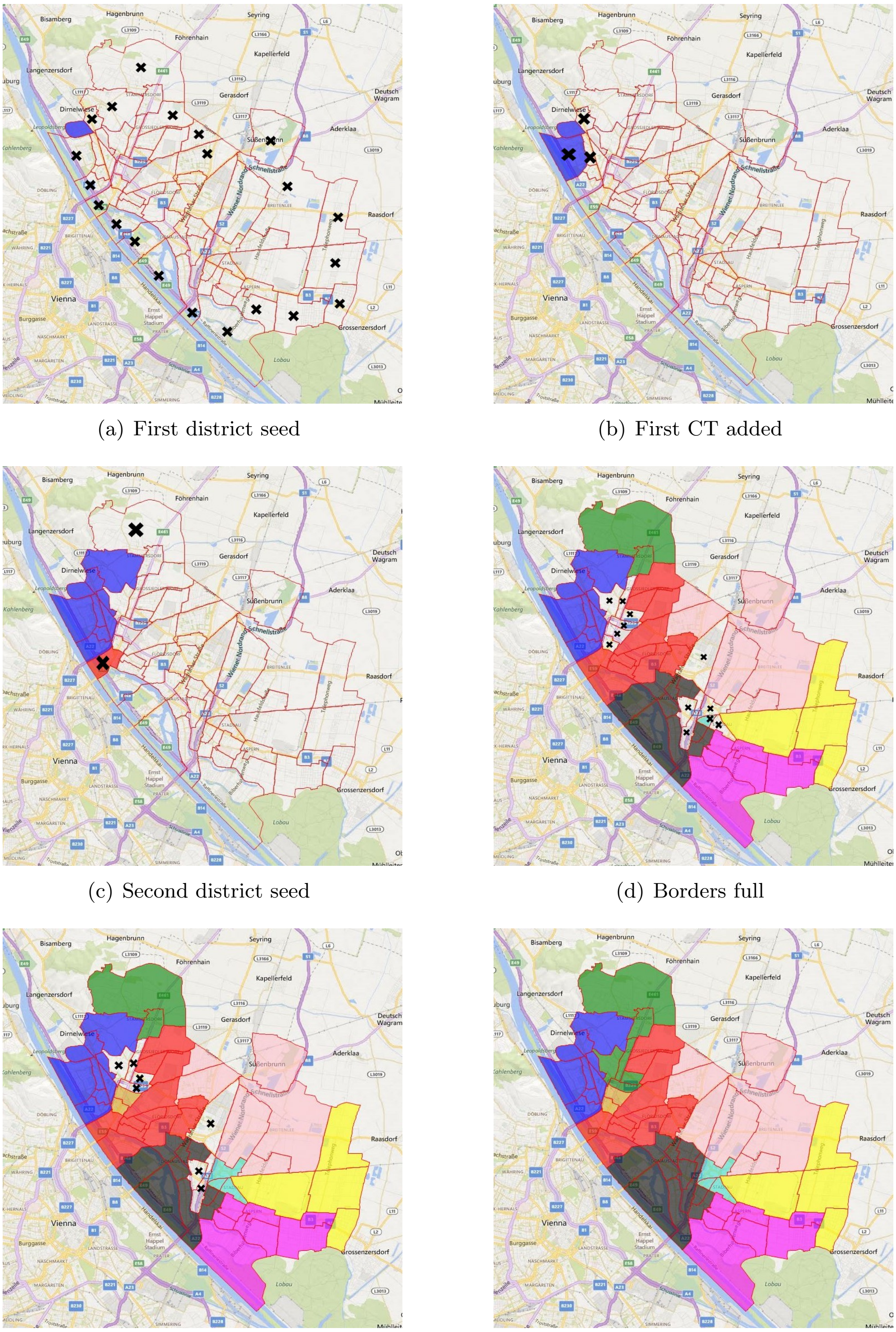
- **Figure 2(b):** The first CT to be added to the new district is chosen out of the marked neighbours of the seed customers. The CT that when added does not violate any constraints and increases the districts objective sum over all periods the least is added to the blue district. The possible new objective sums are calculated by adding the service time of the candidate CT to the district and inserting it via a cheapest insertion heuristic into the current districts route.

- **Figure 2(c):** At this stage, the first district, the blue one is closed because the stopping criterion is met. In this example, the stopping criterion is linked with the amount of workdays with overtime and is described in more detail in section 6. Before choosing a new seed customer, the optimal TSP route through the district is calculated and checked if the termination criterion is still met. Here it is still met, so a new seed customer is chosen out of the neighbouring CTs of the first district lying on the border of the overall territory. The most compact out of those two is chosen as seed of the red district.

- **Figure 2(d):** This figure shows the situation after the sixth districts termination criteria is met. All the border CTs are assigned, therefore the next seed is chosen out of all the seeds having neighbours that are assigned to a district. The turquoise CT is chosen as seed because it is the most compact.

- **Figure 2(e):** Here the last districts termination criteria is met. Now the parallel districting phase begins, where all the not yet assigned CTs, marked black are assigned to neighbouring districts by iteratively finding the combination of one district and one unassigned CT that increases the sum of the objectives over all periods the least and merging them.

- **Figure 2(f):** The final districting plan of the construction heuristic.



(a) First district seed

(b) First CT added

(c) Second district seed

(d) Borders full

(e) Sequential finished

(f) Districting finished

Figure 2: Example construction heuristic with candidates marked X

5.1.1 Pseudocode construction heuristic

Definition of variables and parameters:

U $\leftarrow$ the set of all CTs not yet assigned to any district.
D $\leftarrow$ Districting plan that partitions set U in K districts
K $\leftarrow$ the number of trucks and therefore the target number of districts.
border(C) $\leftarrow$ set of CTs in C sharing a border with any CT outside the set C or sharing a border with the overall territory.
neighbor(C) $\leftarrow$ all the CTs outside the set C sharing borders with any CT within the set C.
maxCompact(C) $\leftarrow$ the CT from the set C having the highest compactness value.
F(D) $\leftarrow$ objective function value of districting plan D
feasible $\leftarrow$ when u is added to d neither the stopping criterion is met nor any constraint is violated.

Algorithm 1: Construction Phase 1

Input: D $\leftarrow \emptyset$, U, K

Output: Incomplete Districting Plan D with K districts

```
Begin
    while |D| < K do
        if D is ∅ then
            N ← border(U)
        else
            N ← border(U) ∩ neighbor(border(D))
        if N is empty then
            N ← neighbor(border(D))
        u_best ← maxCompact(N) d ← ∅
        repeat
            d ← d ∪ u_best
            U ← U \ u_best
            δ_min ← ∞
            foreach u in border(d) do
                d ← d ∪ u
                D* ← D ∪ (d)
                δ ← F(D*) − F(D)
                if feasible & δ < δ_min then
                    δ_min ← δ
                    u_best ← u
        until δ_min < ∞
        D ← d
    return D
```

Algorithm 2: Construction Phase 2

Input: D $\leftarrow$ Phase 1 solution, U $\leftarrow$ residual after Phase 1, K

Output: Districting Plan D with K districts

Begin
    while $|U|$ *is not empty* **do**
        $\delta_{min} \leftarrow$ inf
        $u_{best} \leftarrow 0$
        $i_{best} \leftarrow 0$
        foreach $u \in U$ **do**
            $\delta_u \leftarrow \infty$
            foreach $d_i \in D$ **do**
                $d_i^* \leftarrow d_i \cup u$
                $D^* \leftarrow (D \setminus d_i) \cup d_i^*$
                $\delta_i^u \leftarrow F(D^*) - F(D)$ (compactness check, cost $= \infty$ otherwise)
                $i_{best}^u \leftarrow 0$
                if $\delta_i^u < \delta_u$ **then**
                    $\delta_u \leftarrow \delta_i^u$
                    $i_{best}^u \leftarrow i$
            if $\delta_u < \delta_{min}$ **then**
                $\delta_{min} \leftarrow \delta_u$
                $u_{best} \leftarrow u$
                $i_{best} \leftarrow i_{best}^u$
        if $u_{best} = 0$ **then**
            relax compactness target
        else
            $d_{i_{best}} \leftarrow d_{i_{best}} \cup u_{best}$
            $U \leftarrow U \setminus u_{best}$
    return D

5.2 Local Search Heuristics

As local search operators, Move and Swap have been chosen. Those where applied consecutively, Move on the initial solution provided by the construction heuristic and Swap on the Move improved solution. Both algorithms are performing Moves/Swaps following a first fit paradigm as long as there are permissible ones that reduce the objective function. Therefore the local search ends when a local optima is reached meaning no more improvement of the objective function is possible by LS.

5.2.1 Move operator

The move improvement algorithm tries to improve the objective function of a districting solution by consecutively moving one census tract from one district to a neighbouring one. This is repeated until no permissible moves can be found.

Prerequisites for a move to be permissible: Moving one census tract from a district to a neighbouring district is only allowed when following prerequisites are guaranteed:

1. The compactness of both the decreased and the increased district must not fall under the minimum compactness of all districts of the input districting solution.

2. The contiguity of both districts must be granted at all times. This implies that the CT to be moved has to lie on the border between the decreased and the increased district.

5.2.2 Swap operator

The swap algorithm tries to improve the objective function of a districting solution by exchanging one CT from one district with another CT in another district.

Prerequisites for a swap to be permissible:

1. The compactness of both districts must not fall under the minimum compactness of all districts of the input districting solution.

2. The contiguity of both districts must be granted at al times. Again this implies that the CT to be swapped has to lie on the border between the two modified districts.

5.3 LNS Metaheuristic

The LNS metaheuristic was first introduced by Shaw in 1997 [20]. He defined it as a metaheuristic that overcomes local optima by consecutively relaxing visits e.g. removing them from the initial tour until a given number of visits is relaxed. Finding the visits to relax is done by a certain *remove operator*, also called *destroy operator*. Afterwards the relaxed visits are put back into the tour at optimal cost or heuristically by a certain *repair operator*. If the so generated new tour is better than the initial tour, it gets accepted as the new initial tour. This procedure is repeated until a certain *stopping criterion*, like a predefined number of iterations is reached.

Because this work concentrates on the CT level, not customer visits are removed but CTs are removed from districts, this is done by 2 alternating destroy operators:

1. **Destroy Border:** All the border CTs of a chosen district are set unassigned. If all the CTs within the district lie on the border, a seed CT has to be chosen. In that case the CT with the least neighbourouring CTs from other districts is chosen as seed and remains in the district.

2. **Destroy Districts:** Two districts are destroyed by consecutively executing **Destroy Border** until the next execution of **Destroy Border** would result in empty districts. Therefore it is made sure that both districts still inhabit seed census tracts.

For comparability reasons, a deterministic approach in finding the districts to be destroyed was used. Each district is given a number from 1 to K that stays the same throughout the process. Both *destroy border* and *destroy districts* alternate through the list of districts. Algorithm 3 states the steps in choosing the destroy operator and the districts to be destroyed.

Algorithm 3: LNS choosing destroy operator

Begin
  $iter \leftarrow 1$
  i $\leftarrow$ 0
  while $iter \leq iter_{max}$ **do**
    $i \leftarrow mod\ K$
    if $iter\ mod\ 2 = 1$ **then**
      destroy border(i)
    else
      destroy district(i)
      $i \leftarrow i{+}1$
      $i \leftarrow i\ mod\ K$
      destroy district(i)
    $iter \leftarrow iter + 1$
    $i \leftarrow i + 1$

With this alternating destroy operators it is ensured that enough differing iterations are possible while determinability is ensured.

As repair operator, the second phase of the construction heuristic, the parallel agglomerative districting phase with consecutively Move and Swap improvement was used.

5.3.1 Pseudocode LNS Metaheuristic

Definition of variables and parameters:

U $\leftarrow$ the set of all CTs not yet assigned to any district
D $\leftarrow$ the set of Districts.
K $\leftarrow$ the number of trucks and therefore the target number of districts.
stoppingcriterion $\leftarrow$ some criterion like a predefined number of iterations.
repair(D^****)*** $\leftarrow$ perform Algorithm 2 with input D $\leftarrow D^*$ and U $\leftarrow D \setminus D^*$

Algorithm 4: LNS improvement

```
Input: D ← districting solution, U ← ∅, K , iterMax
Output: Improved Districting Plan D with K districts
Begin
    i ← 1
    iter ← 1
    while iter < iterMax do
        i ← i mod K
        if iter mod 2 = 1 then
            D* ← destroy border (D, d_i)
        else
            D* ← destroy district (D, d_i)
            i ← i+1
            i← i mod K
            D* ← destroy district (D, d_i)
        iter ← iter + 1
        i ← i + 1
        D* ← repair(D*)
        if F(D*) < F(D) then
            D ← D*
    return D
```

6 Numerical Study

In this study the interest lies in determining how ex ante districting plans perform over time. For this purpose instances with varying numbers of customers, vehicles, and city region have been generated.

For the purposes of this study, an instance consists of 100 ”training” scenarios representing historical demand realizations, and 100 evaluation scenarios representing future periods not yet known during the districting process. For each instance, a single districting plan is generated minimizing the total cost over all 100 training scenarios of the respective instance. Three plans of increasing quality are considered → construction solution, local search improved plan and LNS improved plan. The quality of these plans is then validated by applying them to the 100 associated evaluation scenarios.

The performance of the districting plans is further benchmarked against a policy of daily re-optimization, i.e. solving each scenario as an independent VRP with no geographic constraints regarding CTs or districts. These benchmark solutions were obtained using the hybrid genetic algorithm proposed by Vidal et al. [26].

One defining input variable having strong impacts on the districting solution and quality for any districting procedure is fleet size. The fleet size is typically given by the company requesting the districting service. Because in this work no physically existing firm is considered, the fleet size and therefore the amount of districts was derived from the VRP solutions of the underlying scenario. The maximum amount of tours the VRP solution needed to serve all the customers from the 100 training scenarios was used as fleet size. This conservative approach was needed because otherwise the construction heuristic would have to make use of very high amounts of overtime and the room for improvements would be very much limited.

6.1 Input Data

Because of the availability of accurate street point and citizen data, the city of Vienna has been chosen as subject of the study. Three .csv documents have been downloaded from www.data.gv.at, the open data site of the city government of Vienna:

- *ZAELBEZIRKOGD.csv* : This file contains various information regarding the 250 census tracts, most importantly: Longitude and Latitude parameters representing the geographic borders of each census tract, its circumference and its area [23].
- *vie_ 303.csv* : Here the population of each census tract is stated [22].
- *STRASSENGRAPHOGD.csv* : A street graph file that contains for every street within Vienna a number of Latitude and Longitude points [21].

For every citizen of Vienna a location has been produced by randomly choosing a street point within the census tracts. Each tract got as much locations as its population number indicates. With this distribution method, it was ensured that the distribution of customer locations over the city was comparable with the population density within the various parts of Vienna.

In the next step 10 000 citizens were chosen randomly from all the citizens of Vienna with their previously assigned location. These 10 000 citizens where representing the possible customers with their possible customer locations. With this pool of possible locations, the instances in the versions training and evaluation where produced, each containing 100 scenarios. Training and evaluation instances do not differ in their structure or the way they where constructed, their only difference lies in their application later on. Training instances are used by the districting algorithm to create the districting plan. Evaluation instances are used to ex post evaluate the districting plan. Another variation was introduced by varying the number of customers chosen as active per period. And finally the customers of each period where divided up by their location relatively to the Danube, resulting in two versions of each instance: Danube North and Danube South. This division was introduced because the Danube represents a broad obstacle within Vienna. Crossing by car is only possible on five bridges, therefore in most of the cases it does not make sense to create districts covering CTs on both sides. It also prevents problems arising from closed bridges due to accidents or renovation work. The total structure of the produced periods is shown in table 2.

1000 Customers	North	Training
		Evaluation
	South	Training
		Evaluation
2000 Customers	North	Training
		Evaluation
	South	Training
		Evaluation
3000 Customers	North	Training
		Evaluation
	South	Training
		Evaluation

Table 2: Input instance Tree

For each of the 6 instances with its two temporal manifestations shown in table 2, 100 scenarios where produced and each scenario is structured and constructed almost the same, the only difference lies in the number of customers set active per instance. Exemplary, in the following, the creation of an 1000 Customers instance is stated.

Out of the pool of 10 000 possible customer locations, randomly 1000 customers are picked to be equipped with a service time bigger than 0. The actual service time for each chosen customer is determined randomly between 5 and 15 minutes. The customers are split by their location relatively to the Danube, producing two instance files. North of the Danube lie 62 census tracts and south

188. A distance matrix from every chosen customer to every chosen customer and the depot is created by calculating the on street distance in meters, using the bing.com/maps algorithm. Because cost is throughout the study defined as minutes, the distance is divided by a supposed mean speed of 25 kilometres per hour and rounded to whole digits. Training and Evaluation are scenario sets, each scenario set can be considered an instance. So 6 instances are used, each consisting of 100 training and 100 evaluation scenarios.

The hereby generated scenario sets are then directly used for the VRP algorithm used to create the comparative data.

For the input data used by the districting algorithm, further processing took place. As in table 1 described, the districting algorithm uses an adapted version of service time s_i. There are two versions of service time used in this work:

- **Customer service time:** The time needed at each customer location.
- **Census tract service time:** The sum of all customer service times within the CT + the TSP tour between them which has been preprocessed for each of the 100 scenarios within each instance using the GUROBI optimization solver.

The districting algorithm works exclusively on census tract level. For the distance matrix, the geographic center point of each census tract was used. The distance between neighbouring census tracts was divided by two to accommodate for the fact that when driving from one district to the neighbouring one, typically not the center point, but the nearest customer is piloted to.

6.2 Results structure

The evaluation of results was split in two phases. First, the training phase where simulated input data from past periods was used to create the districting plans. Secondly these districting plans then where used in the evaluation phase on simulated future periods. Hereby the performance of the districting plans on periods that where not used when creating them was evaluated. As in subsection 6.1 described, three different input instances with 1000, 2000 and 3000 customers, each split at the Danube came to use. A daily optimized VRP solution was compared with the initial districting solution and the results of the improvement algorithms LS and LNS. Objectives, Penalties, Overtimes, Balance and Compactness changes where evaluated. Because in Evaluation instances, the same districting plan as in the training instances was used, the compactness parameters stay the same in both instances.

Displayed key figures

- **Objective:** Result of the objective function, so sum of travel times plus sum of service times plus sum of overtimes over all tours.
- **Total Penalty:** The total penalty that is included in the objective. Because penalty is a 100% surcharge on objectives, at each district within each period over 480, the total penalty also stands for the total amount of used minutes overtime.
- **Avrg. worktime:** Average working minutes per workday\tour.
- **# Tours with overtime:** Total amount of tours with tour duration over 480 minutes.
- **# Tours with overtime more than 120:** Total amount of tours with tour duration over 600 minutes.
- **Balance:** Balance is measured by how much the tours differ in their objective, the indicators are the maximum, average and minimum objective sums of the districts within the districting plan.
- **Compactness:** Maximum, average and minimum compactness of a single district within the districting plan.

Besides to the data categories described above, showing results on districting level, two charts displaying results on the tour level are presented:

Diagram 1: Number of tours per working minutes

In this diagram, the distribution of working minutes is plotted. Data values are rounded up to the nearest multiple of 10 minutes. The line represents the floating mean between neighbouring dots. Three colours where used to differentiate between the three districting solutions:

- **Blue:** Construction heuristic solution.
- **Red:** Local search solution
- **Green:** Large neighbourhood search solution.

For better a understanding, the diagram is explained by the example 1000 customers north, where 9 districts where created, thus 900 tours per instance:

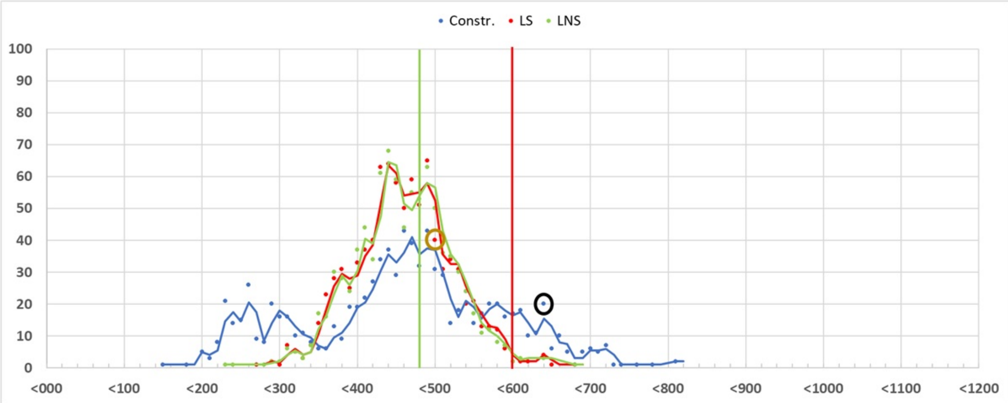


Figure 3: Example Diagram Distributions of Tour Lengths

- The red dot within the yellow circle gives us the following information: For the given instance, the districting plan found with the local search heuristic lead to 40 tours of length 490-500 minutes over the planning horizon.
- The blue dot within the black circle: For the given instance, the districting plan found with the construction heuristics solution lead to 20 tours of length 630-640 minutes over the planning horizon.

The green line divides the diagram between under 480 minutes and therefore tours finished within normal working time and over 480 minutes, tours finished in overtime. The dots between the green and the red line symbolize tours finished in over 8 hours and under 10 hours. Everything on the right of the red line is finished after more than 10 hours and therefore illegal by the Austrian collective agreement for employees in forwarding and logistics.

Diagram 2: Percent of workdays under certain amount of working minutes

This diagram shows how much percent of the total tours are finished under a certain work time. The same colour code as in diagram 1, described above is used. Again the example from the 1000 customers north instance:

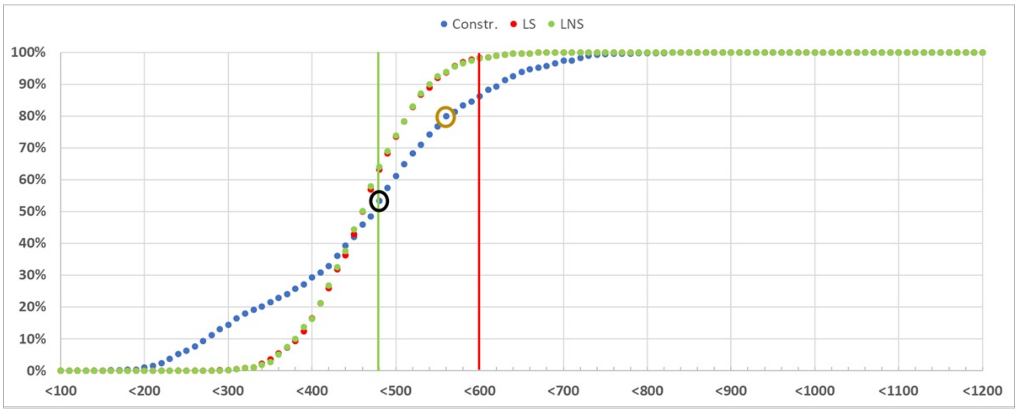


Figure 4: Example Diagram Cumulative Distributions of Tour Lengths

- The blue dot within the black circle: In the underlying instance with the construction heuristics solution, 53.3% of the tours are finished in less than 480 minutes.
- The blue dot within the yellow circle: In the underlying instance with the construction heuristics solution, 79.88% of the tours are finished in less than 560 minutes.

Again the diagram is divided at 480 minutes by a green line and at 600 minutes by a red line.

6.3 General trends

The construction heuristics performances was in most cases not practically applicable neither in the objective function performance, overtime measures or balance. But through LS and LNS it was possible in every instance to derive a districting plan with performance in useful ranges. Overall it was possible to decrease the objective of the construction heuristics solution by 2 to 12 percent through LS and LNS. Figure 5 shows the gap between VRP objective and the 12 instances with the three different types of districting plans compared to their ex ante counterpart using the formula: $\frac{(districting\, objective)-(VRP\, objective)}{(VRP\, objective)}$.

In bigger instances in terms of customers, the construction heuristic performs better. Generally the districting plans were able to hold their quality quite well when applied to the evaluation instances. The objective maximally increased by a low digit percentage and in one instance even decreased when comparing training with evaluation instances.

Although the stability of the districting plans from training to evaluation scenarios was quite smooth, the gap to the VRP solution was not. The range of the gap from the worst initial solution with 30.8% to the best LNS solution with 11.8% although is a quite significant improvement and shows the performance of these simple improvement algorithms. But they still are significantly much worse than the VRP solutions objective, which is ultimately no surprise since the VRP problem is very complex and as pointed out earlier districting plans are generally accepting some deterioration to gain other benefits. But with more sophisticated methods and models, there is still considerable room for improvements.

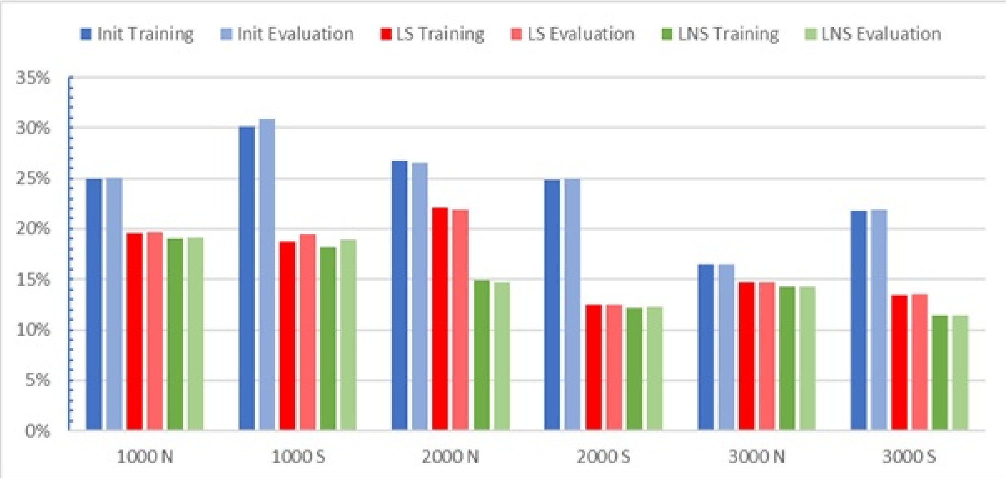


Figure 5: Gap to the VRP solutions objective

The critical average worktime of 480 minutes was exceeded slightly in four of the twelve instances, LS and LNS did not change the average tour length significantly, also virtually no difference between training and evaluation scenarios could be observed. But the total amount of overtime was reduced in all the instances by LS and consecutively LNS, therefore the many extremely low and high tour lengths approached the average. So throughout the instances, the construction heuristics solution could be increased drastically with LS in terms of balance with respect to variations in tour lengths. The balance difference between the LS and LNS solutions was also significant in most of the instances.

The most problematic boundary for the districting plans to be practically applicable in Austria turned out to be the number of days with overtime more than 120, so over 10 working hours. Figure 6 shows the percentage of tours with overtime bigger than 120. Although big improvements with LS and LNS could be observed, the percentages still lie in 50% of the instances above 10% of the total amount of tours. But extreme tour lengths have been eradicated by LNS, the tours with length above 600 minutes mostly lie slightly above the limit. Between training and evaluation again no significant difference could be observed.

The biggest differences between training and evaluation scenarios have been observed in terms of the variation of tour durations within an instance. In evaluation scenarios more spikes with extreme low or high amounts appear and they are generally less uniformly distributed around the target value of 480. This observation is discussed in more detail using a representative example instance in section 6.5.

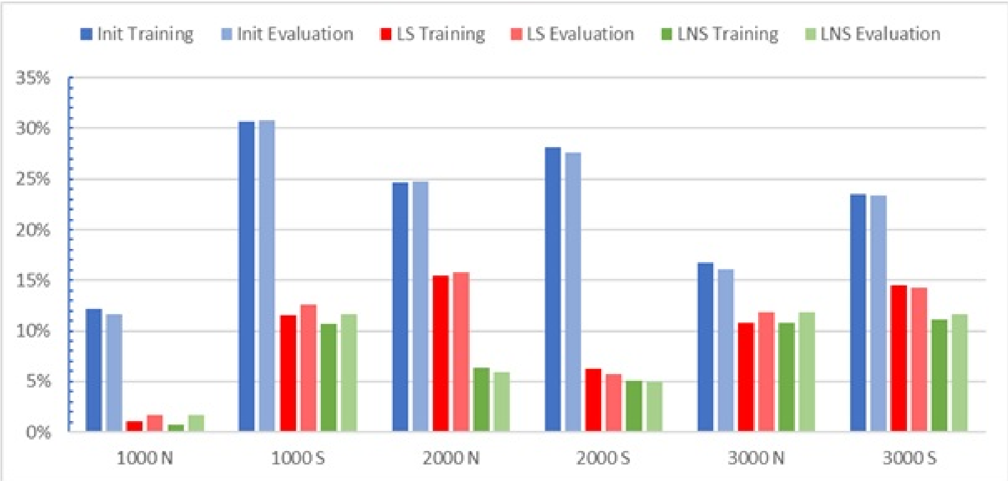


Figure 6: Overtime more than 120 minutes in % of total tours

6.4 Effects of customer numbers

As described in section 6.1 three different amounts of customers where used in the instances. Because customers are clustered into a predefined unchanging number of census tracts representing the minimum unit of separation, more customers mean higher operating expenses per CT. So the more customers, the more vehicles = districts are needed to serve the underlying area. Likewise, because the number of CTs remains constant, also the number of CTs per tour becomes lower. Since the relative changes from moving or swapping one CT are much larger when tours consist of only a small number of CTs, it becomes less likely that improving moves exist. Hence local optima are reached quickly, and are more difficult to escape from.

Nonetheless, because the number of customers is scaled linearly, the relative differences between the CTs remain approximately the same and significant improvements in the objective could be observed in all the instances.

6.5 Specific observations on an example instance

The data gained from the instances described in table 2 is provided in six training/evaluation comparison pages. In this subsection a representative instance is discussed in detail, the remaining are to be found in the appendix.

The instance to be discussed in detail has in total 2000 customers and lies north of the Danube. These customers are distributed among the 250 CTs, North of the Danube 62 CTs, and 496 customers are located. The VRP solution produced a maximum of 15 routes out of the 100 instances. Therefore 15 districts where created, leading to 1500 tours = workdays. All the gaps in the following are gaps between the summarized objective between all tours, so sum of travel times plus sum of service times plus sum of overtimes over the 1500 tours and the VRP solutions objective, using the formula: $\frac{(districting\, objective)-(VRP\, objective)}{(VRP\, objective)}$.

The gap of the construction solution to the reference solution amounted to 26.8% this gap could be reduced by LS to 22.1% and further through LNS to 14.97%. Overall, the improvement heuristics were thus able to reduce the gap to the reference solution by over 11 percentage points.

The strong performance of LNS can be observed more obvious when looking at the amount of used overtime. While LS reduced the overtime to 88 918.5 / 118 371.5 = 75.12% of the construction solutions objective, LNS reduced it further to 36.08%. So a reduction of 63.92% in overtime from construction to LNS was achieved.

This observation continues when looking at the number of days with overtime above the critical limit of 120 minutes. While the construction heuristic makes use of 692 of the 2000 tours with tour length above 600 minutes, LS reduces them to 232 tours. With LNS this amount could be drastically reduced to 96 tours above the limit which amounts for 4.8% of the total amount of tours within the instance. When looking at figure 7(a) it can be observed that the green LNS line flattens out much earlier than the other two, therefore the amount of tours with overtime above the limit of 600 minutes is reduced drastically by LNS. This is in sight of the total days with overtime interesting because the LS solution used 130 days less than LNS. Therefore when overtime appears, with the LNS solution the working time typically does not rise much above the normal working time. This stands in contrast to the LS solution and more to the construction heuristics solution which can be observed beautifully in figure 7(a) and in the amount of workdays with overtime more than 120.

In terms of balance, a steady decrease in range of tour durations between the tours throughout the three solutions was achieved. This can be observed in both diagrams. In figure 7(a) the blue construction heuristics line stretches from 150 minutes to an extreme of 1200 minutes e.g. 20 hours with a peak between 200 and 300 working minutes e.g. only half of the normal working time. The orange LS solutions line flattens the peak between 200 and 300 minutes and rises strongly between 400 and 500 minutes where the normal working time is located. Problematic is that the LS solution too needs too long to flatten out completely. It still makes use of unrealistic working times over 1000 minutes. As seen at the green line, LNS completely removes the peak between 200 and 300 minutes, it peaks around the normal working time and flattens out rapidly as overtime increases. In figure 7(b) an interesting behaviour can be observed. Regarding the 480 minutes line LS performs best with 63.5% of workdays under 480 minutes then LNS follows with a slight lead to construction of 54,6% to 53.7%. This ranking gets turned upside down. At the 600 minutes line LNS had 93% of workdays completed, LS 84.5% and construction 75.3%. Later on at 800 minutes working time the LS solution gets passed even by the construction line but because over 800 minutes working time is unacceptable either way it is not given much importance.

Figure 8 (a) is a good example why it is necessary to be able to relax the compactness constraint. The least compact district with a compactness value of 0.5272 is district 15 which consists of a single CT in the construction solution. As in subsection 5.1 described, the CT with the highest compactness value shall be chosen as seed, therefore all the other candidates had at this stage lower compactness than 0.5272. Because the compactness constraint is set to 0.55 the set of feasible seed candidates is empty, so without relaxing the constraint no CT could have been chosen as seed for the last district. In terms objective and balance the improvements can easily be seen from figure 8 (a) to (c). The three districts with the lowest objective at (a) are unsurprisingly the districts that inhabit only one CT (districts 7, 11 and 15). At (b) only one single CT district is left, at (c) every district inhabits at least two CTs.

From figures 8 (a) to (c) the limits of LS and the usefulness of LNS can be observed. The LS can only make small changes in the solution, and requires each change to be positive. It quickly becomes trapped in a local optimum that could be easily improved by allowing some intermediate changes that deteriorate the objective, but which then later lead to a better local optimum. On the other hand, the LNS is able to make larger changes in the solution which cannot be achieved through a chain of exclusively improving changes but which become positive overall. These general observations are true for every processed instance, for the particular example the steps leading to a local optimum are presented in detail in the following.

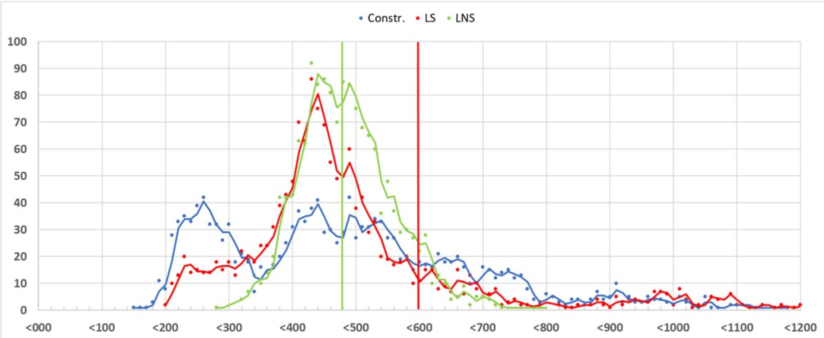
At figure 8(a) district 7 has the second lowest objective but moving and swapping with either the neighbouring CT from district 8 or 6 is impossible due to the contiguity constraint. District 8 has the highest objective of all the 15 districts, due to the compactness constraint it is not possible to move a CT to the single CT district 15 or to district 4. Moving a CT from 8 to 9 is impossible due to the contiguity constraint. Moving a CT from 8 to 14 would violate the compactness constraint of district 8, Swapping is not possible due to the contiguity constraint. The other two neighbouring districts of district 8 namely 5 and 6 both have quite high objectives in figure 8(a), only one move from 8 to 6 and a swap between 5 and 6 made sense. Because the CT moved from 8 to 6 has an exceptionally high service time, district 6 is in figure 8(b) the district with the highest objective, even more than district 8 in figure 8(a). With LNS the situation could be resolved, the objective of district 6 shrunk because the exceptionally big CT is now accompanied by only one other CT. District 7 took over the rest of district 6 and thus growing to a meaningful size. District 14 could incorporate two CTs from district 8 after the latter had been destroyed by the destroy district operator.

Between training and evaluation instances, the objectives gap only increased insignificantly by a maximum of 0.3%. The biggest differences between training and evaluation can be observed by comparing the figures 7(a) and 7(c). All the three lines are showing much more spikes, especially the blue construction line but also the LS line. All three lines are shifted slightly to the right, meaning more days with overtime bigger than 120 and in the cases of LS and LNS more days with working time bigger than 480 occurred. Nevertheless the differences between training and evaluation instances are quite small so the ex ante plan was indeed a good prediction of the future.

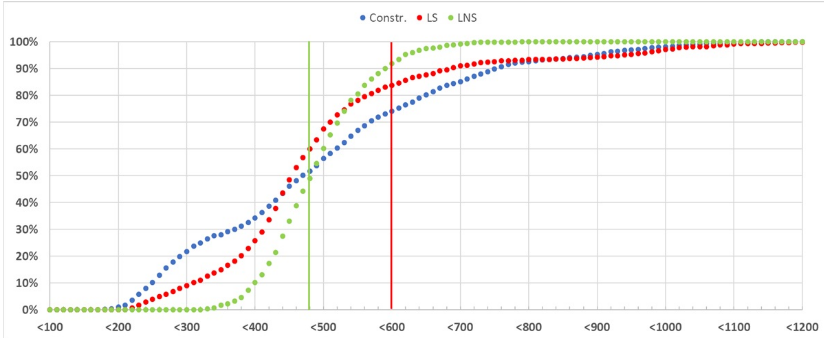
2000 Customers North: 1500 Workdays

Training

	Objective	Total Overtime	Avrg. Tour Length	# Tours with Overtime	# Tours with Overtime > 120	Workload Max	Workload Avrg	Workload Min	Compactness Max	Compactness Avrg	Compactness Min
VRP	660 848.5										
Constr.	837 988.0	118 371.5	479.74	692	369	140 004.0	55 865.9	22 455.0	0.6937	0.6206	0.5272
LS	806 810.0	118 371.5	478.59	547	232	149 526.0	53 787.3	24 068.0	0.7859	0.6590	0.5630
LNS	759 781.0	42 708.5	478.05	677	96	71 847.0	50 652.1	41 616.0	0.7859	0.6731	0.5688



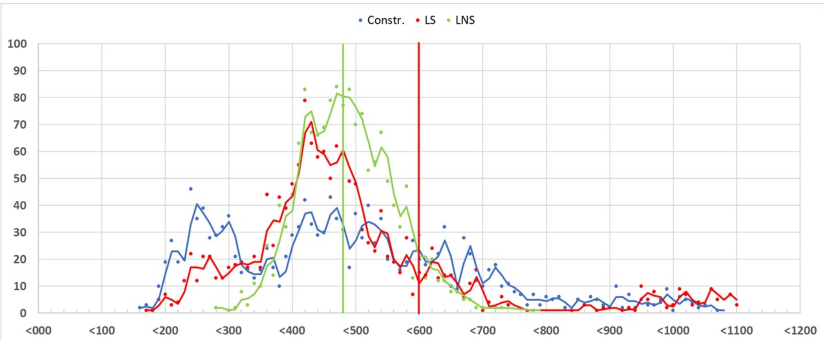
(a) 2000N Training Distributions of Tour Lengths



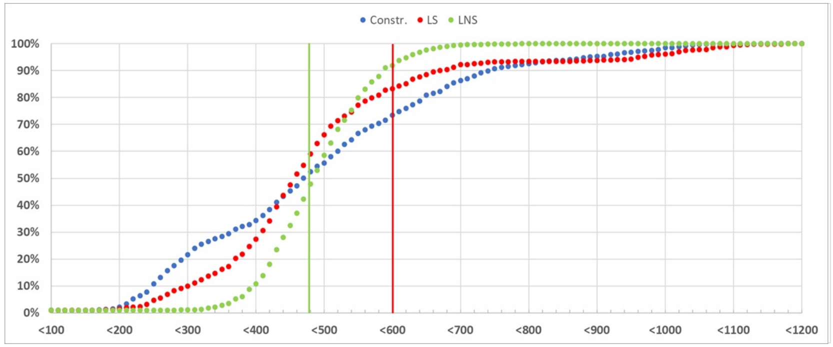
(b) 2000N Training Cumulative Distributions

Evaluation

	Objective	Total Overtime	Avrg. Tour Length	# Tours with Overtime	# Tours with Overtime > 120	Workload Max	Workload Avrg	Workload Min
VRP	662 157.0							
Constr.	840 365.5	118 544.0	481.21	689	375	138 930.0	56 024.4	22 608.0
LS	810 895.0	90 877.0	480.01	556	238	152 596.0	54 059.7	23 837.0
LNS	763 281.0	44 060.5	479.48	710	90	71 664.0	50 885.4	40 749.0

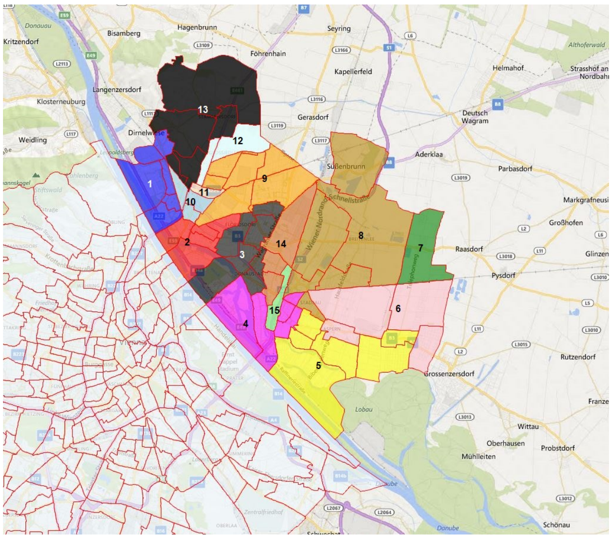


(c) 2000N Evaluation Distributions of Tour Lengths

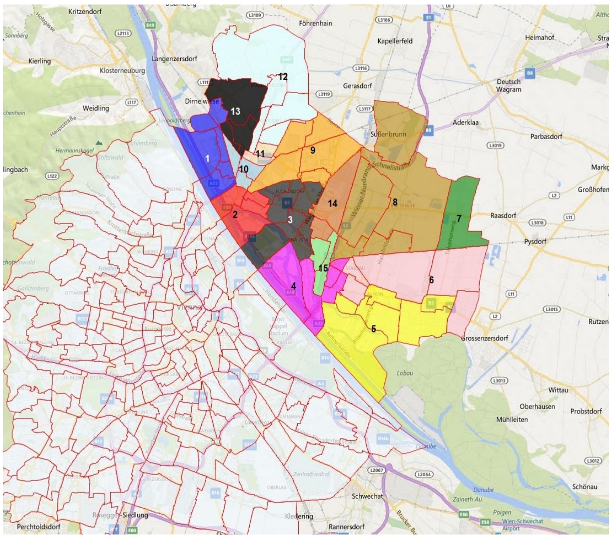


(d) 2000N Evaluation Cumulative Distributions

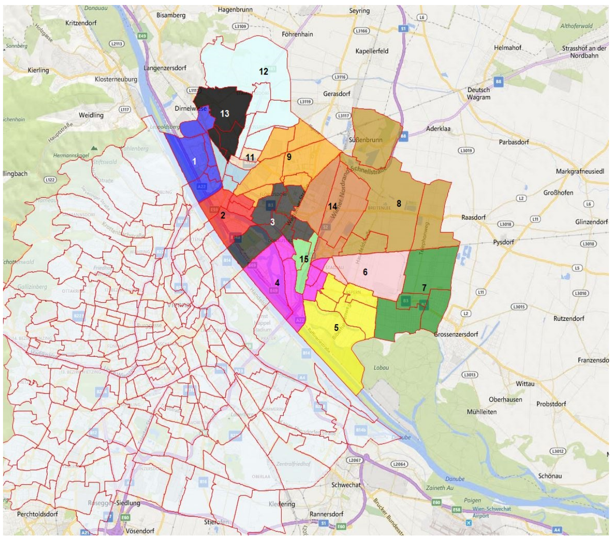
Figure 7: Tour Length Distributions 2000 Customers North



(a) Initial Districting



(b) Local Search Improved



(c) LNS Improved

Figure 8: Districting 2000 North

7 Conclusion

Districting in a logistics context has been proven to be a strong approach to reduce the daily routing effort and gain positive effects of consistency like customer to drivers acquaintance, increased drivers knowledge of his territory and the last-meters. On the other hand, through decreased flexibility, routing cost typically increases in comparison to daily optimization but it has been proven in the literature that the positive effects can offset parts of the higher routing cost. A districting process typically is based on some kind of historical data with its output to be used in future periods. Keeping this characteristic in mind it was surprising to see that most approaches in the literature where not checked against future periods. This work filled in the gap, by using training instances to create the districting plans and checking them by evaluation instances, representing future periods.

This masters thesis evaluated the performances of a simple construction heuristic, a local search improvement heuristic and a meta heuristic both between them and in comparison with daily optimization. These comparisons where then expanded on evaluation instances representing periods not yet known during the districting process. Not only the objectives function outcome was compared but also the legal applicability in Austria regarding working time restrictions was evaluated.

The power of even simple local search improvement methods such as Move and Swap and the ability to escape local optima by LNS was demonstrated. When modelling the problem it has been assumed that balanced districting plans in terms of differences between tour lengths will occur without explicit consideration in the model through the penalty function. It was shown that the penalty function for overtime did lead to an increase in balance. From the initial districting plans to the LS and the LNS plans, the balance indicators indeed did get better but when strict limits such as maximum work times are applied, an additional constraint could be useful. The differences between ex ante and ex post instances turned out to be insignificant in most cases and it was shown that it is legitimate to use plans created with historical data in future periods. This may change when the historic data is not as extensive or the range in possible service time is bigger. As input data, political census tracts have been chosen ensuring an realistic distribution of customers through the underlying service area. But because of their differences in compactness and population, those census tracts had significant impact on the districting process, so the output could change notably when the census tracts are created on logistic considerations instead of political ones.

The effects of the used amount of historical data during the districting process could be an interesting topic of future research. The same applies to the differences either in computational effort and performance in the objective between a classical districting problem and a clustered districting problem like the one in this masters thesis. Also the effects of more sophisticated improvement heuristics in contrast to the simple ones used in this work would present an interesting field of investigation.

8 Appendix

8.1 Abstract

Districting in a logistics context refers to a simplification of the complex vehicle routing problem where the underlying service territory is divided into several contiguous, compact and balanced sub territories that are each served by one unchanging driver. This masters thesis examined the applicability of logistics districting using real life data from the city of Vienna. The performance of districting plans with increasing quality created by a simple construction heuristic, a local search improvement heuristic and a metaheuristic has been evaluated both against each other and a reference daily optimized vehicle routing solution. A literature gap has been filled by carrying out comparisons not only on past data but also on ex post instances representing future periods not yet know during the districting process. Based on computational experiments the power of local search and large neighbourhood search was demonstrated and the differences between ex ante and ex post instances where observed to be insignificant. For each instance a detailed evaluation chart is given, including the districting plan drawn on the map of Vienna.

8.2 Zusammenfassung

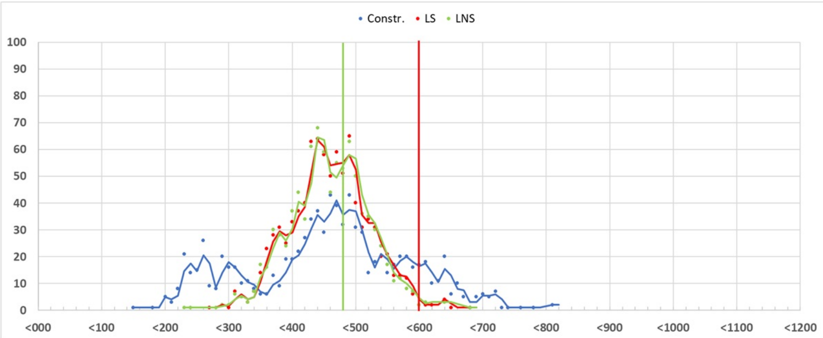
Districting in einem logistischen Kontext stellt eine Vereinfachung des komplexen Vehicle Routing Problems dar bei dem das zugrunde liegende Servicegebiet in mehrere zusammenhängende, kompakte und ausgewogene Untergebiete, welche von einem einzigen gleichbleibenden Fahrer betreut werden, aufgeteilt wird. Diese Masterarbeit untersucht die Anwendbarkeit von Districting unter Zuhilfenahme von echten Daten der Stadt Wien. Die Performance von Districting Plänen mit steigender Qualität, erstellt mithilfe einer simplen Konstruktionsheuristik, einer local search Heuristik und einer Metaheuristik wurden sowohl gegeneinander als auch mit einer täglich optimierten Vehicle Routing Lösung verglichen. Die Vergleiche wurden nicht nur aufgrund von Vergangenheitsdaten, sondern auch aufgrund von ex post Instanzen, welche Zukunftsdaten darstellen die während des Districting Prozesses noch nicht bekannt waren, durchgeführt. Dadurch konnte eine Literaturlücke geschlossen werden. Aufgrund von Computerexperimenten wurde die Stärke von local search und large neighbourhood search demonstriert und Unterschiede zwischen ex ante und ex post Instanzen stellten sich als unwesentlich heraus. Für jede Instanz wurde eine detaillierte Auswertungstabelle inklusive dem Districting Plan, gezeichnet auf der Karte der Stadt Wien, angegeben.

8.3 Results

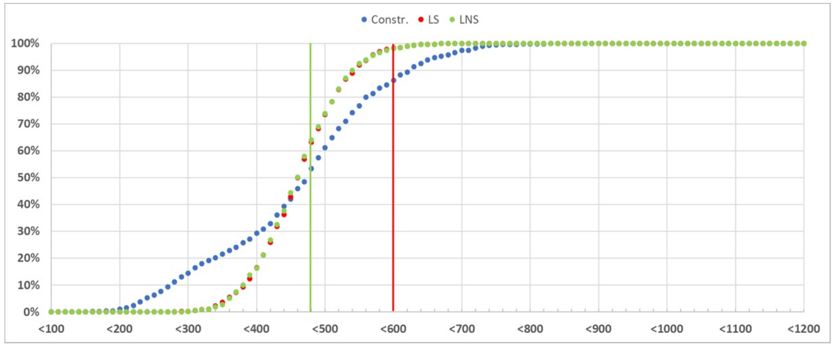
1000 Customers North: 900 Workdays

Training

	Objective	Total Overtime	Avrg. Tour Length	# Tours with Overtime	# Tours with Overtime > 120	Workload Max	Workload Avrg	Workload Min	Compactness Max	Compactness Avrg	Compactness Min
VRP	350 785.5										
Constr.	438 348.0	32 284.5	451.282	367	110	76 890.0	48 715.33	25 069.0	0.7772	0.6153	0.4883
LS	419 372.5	12 198.0	452.42	295	10	51 565.5	46 596.94	39 258.5	0.5863	0.5337	0.4894
LNS	417 671.5	11 442.5	451.37	300	7	51 565.5	46 407.94	39 051.0	0.6889	0.5638	0.4894



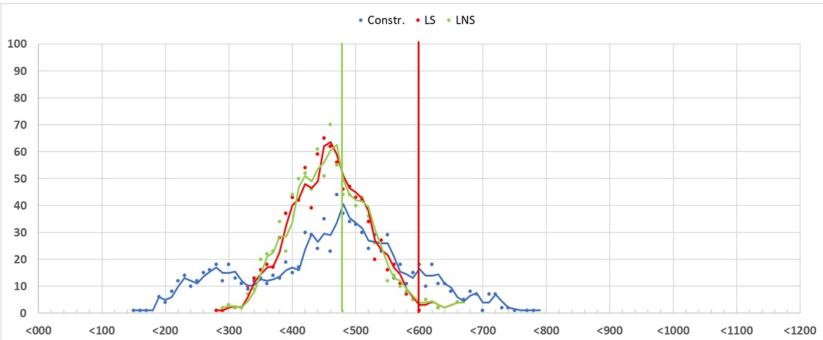
(a) 1000N Training Distributions of Tour Lengths



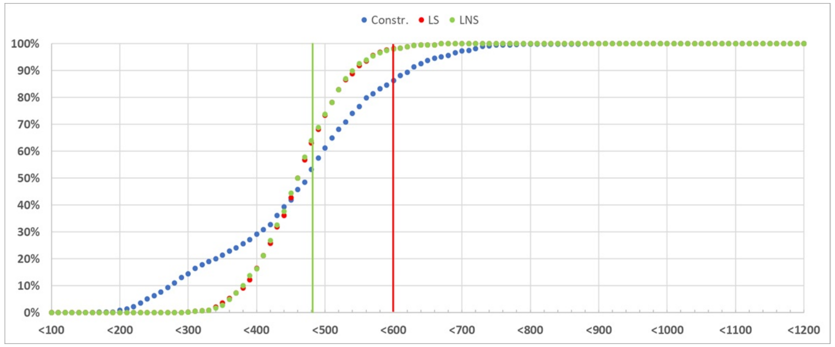
(b) 1000N Training Cumulative Distributions

Evaluation

VRP	350 545.5							
Constr.	437 939.5	32 683.5	450.28	380	105	78 411.0	48 659.94	24 156.0
LS	418 418.0	12 156.0	451.40	285	15	52 568.0	46 490.89	38 542.0
LNS	417 313.5	11 954.0	450.40	277	15	52 568.0	46 368.17	39 115.0

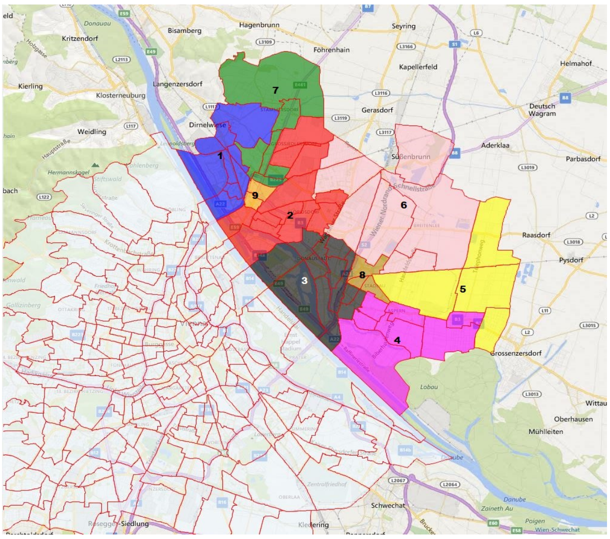


(c) 1000N Evaluation Distributions of Tour Lengths

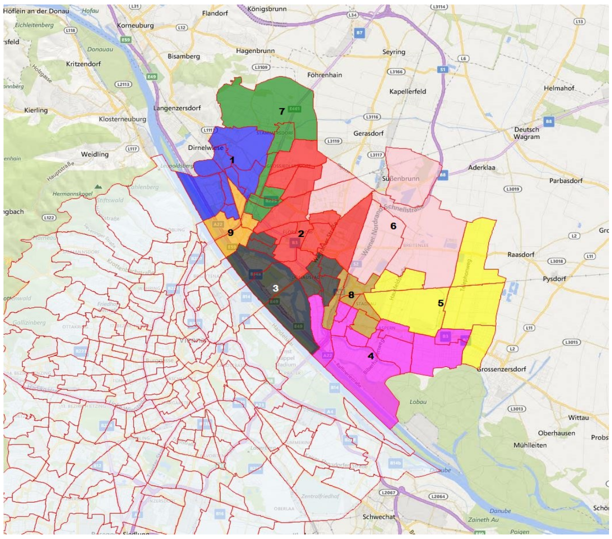


(d) 1000N Evaluation Cumulative Distributions

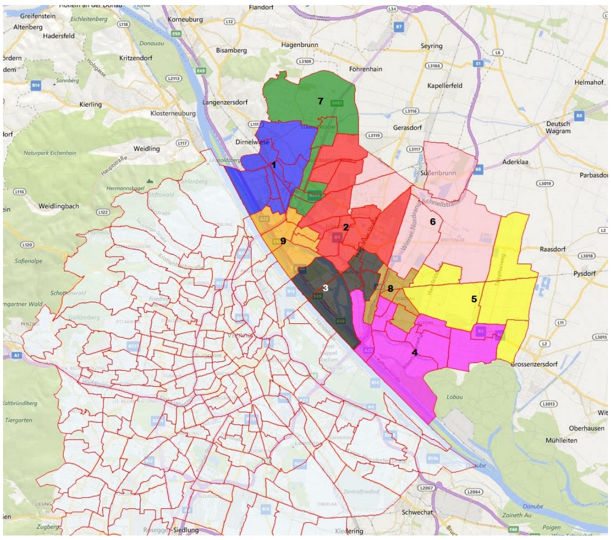
Figure 9: Tour Length Distributions 1000 Customers North



(a) Initial Districting



(b) Local Search Improved



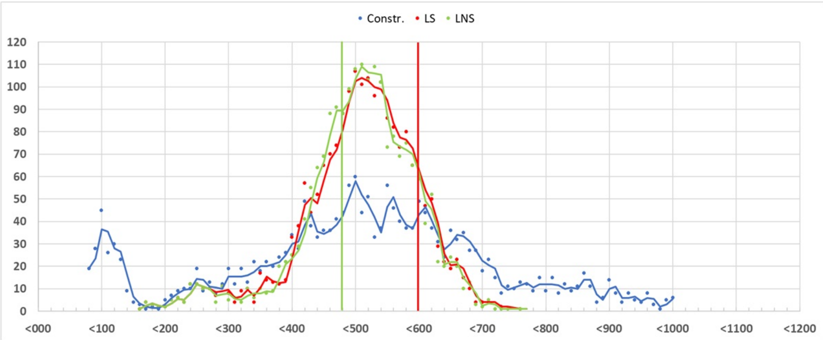
(c) LNS Improved

Figure 10: Districting 1000 North

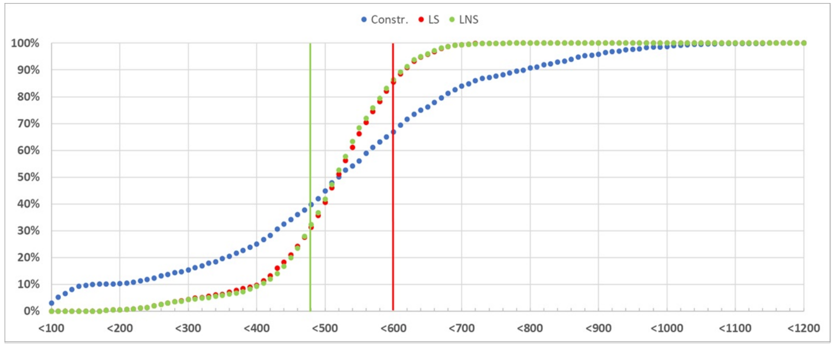
1000 Customers South: 2000 Workdays

Training

	Objective	Total Overtime	Avrg. Tour Length	# Tours with Overtime	# Tours with Overtime > 120	Workload Max	Workload Avrg	Workload Min	Compactness Max	Compactness Avrg	Compactness Min
VRP	918 020.0										
Constr.	1 194 407	189 080.5	502.66	1 158	613	131 372.0	59 720.35	9 473.0	0.7067	0.5912	0.4967
LS	1 090 212	92 659.5	498.78	1 280	230	66 462.0	54 510.6	24 911.0	0.7156	0.5614	0.5069
LNS	1 085 399.5	89 115.5	498.14	1 255	214	66 462	54 269.98	24 911	0.7407	0.5648	0.5069



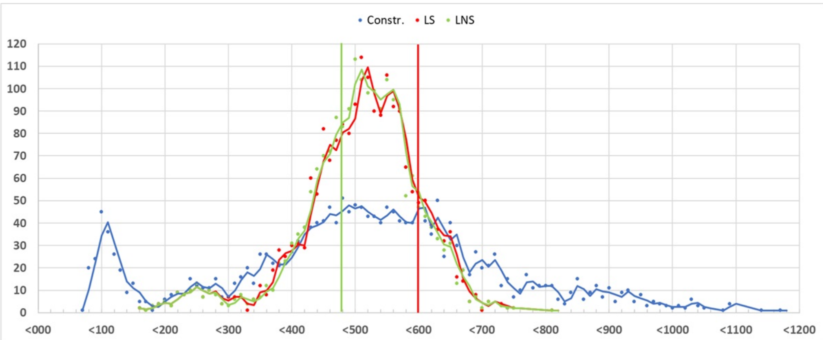
(a) 1000S Training Distributions of Tour Lengths



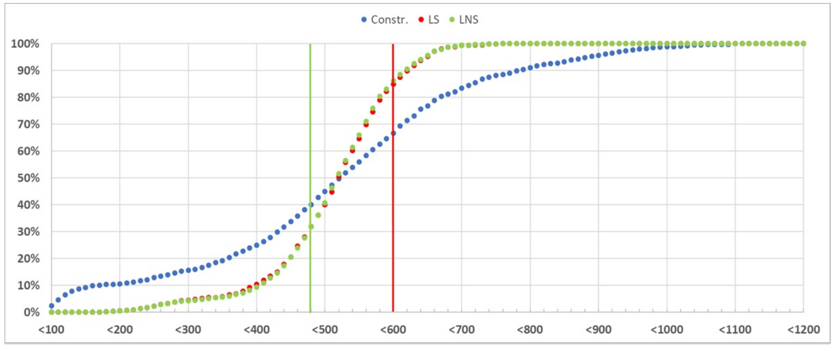
(b) 1000S Training Cumulative Distributions

Evaluation

	Objective	Total Overtime	Avrg. Tour Length	# Tours with Overtime	# Tours with Overtime > 120	Workload Max	Workload Avrg	Workload Min	Compactness Max	Compactness Avrg	Compactness Min
VRP	912 911.0										
Constr.	1 198 681.0	190 498.0	504.09	1 147	615	134 428.0	59 934.05	10 311			
LS	1 095 026.0	95 156.5	499.93	1 269	252	66 466.0	54 751.3	24 414.5			
LNS	1 090 850.0	92 239.5	499.31	1 267	232	66 466.0	54 542.5	24 414.5			

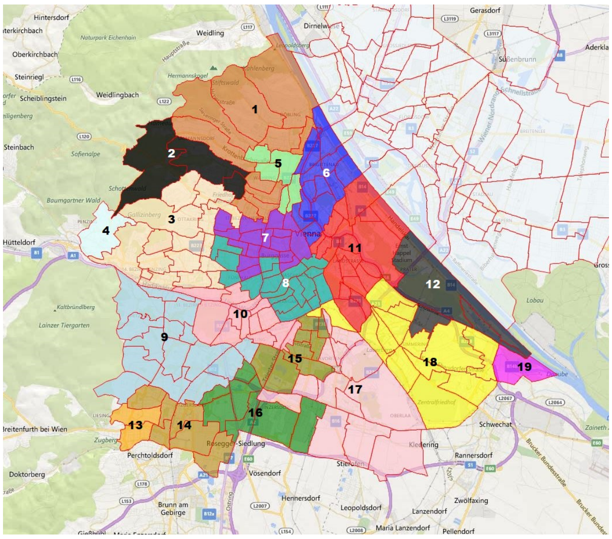


(c) 1000S Evaluation Distributions of Tour Lengths

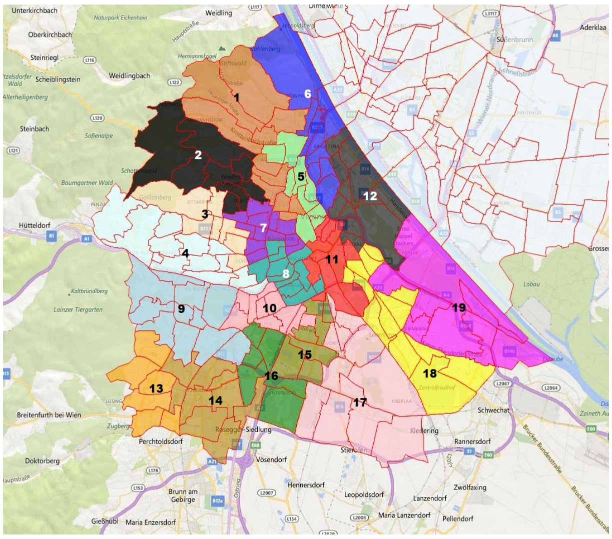


(d) 1000S Evaluation Cumulative Distributions

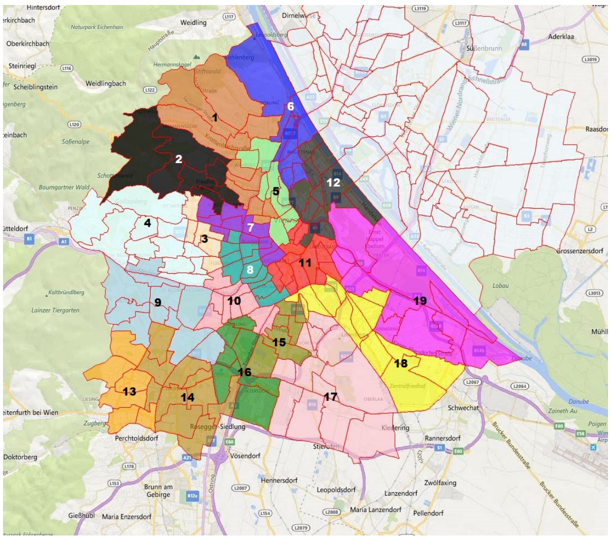
Figure 11: Tour Length Distributions 1000 Customers South



(a) Initial Districting



(b) Local Search Improved



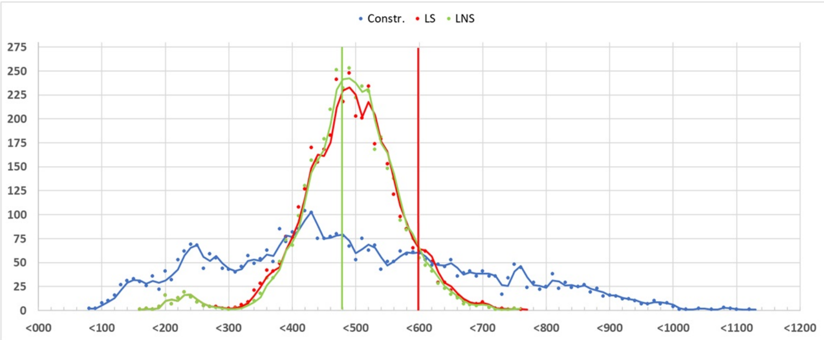
(c) LNS Improved

Figure 12: Districting 1000 South

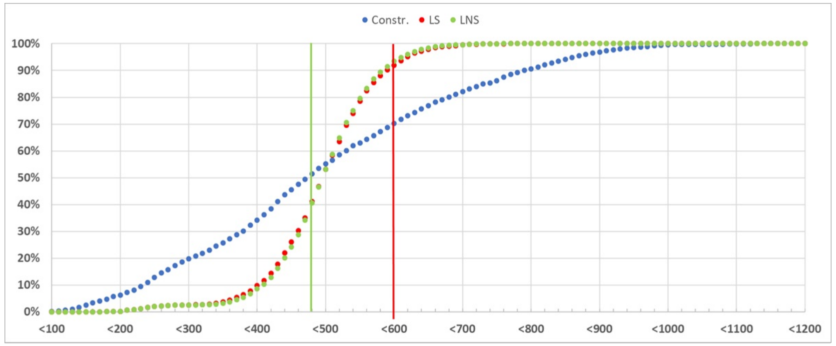
2000 Customers South: 3900 Workdays

Training

	Objective	Total Overtime	Avrg. Tour Length	# Tours with Overtime	# Tours with Overtime > 120	Workload Max	Workload Avrg	Workload Min	Compactness Max	Compactness Avrg	Compactness Min
VRP	1 781 991.0										
Constr.	2 225 804.5	335 760.5	484.62	1 808	1 096	135 952.0	57 071.91	13 408.0	0.8176	0.6124	0.4896
LS	2 003 775.0	122 443.5	482.39	2 058	246	69 893.0	51 378.85	22 199.0	0.7677	0.5739	0.4896
LNS	1 998 442.0	116 471.0	482.56	2 068	199	69 893.0	51 242.1	22 199.0	0.7703	0.5893	0.4896



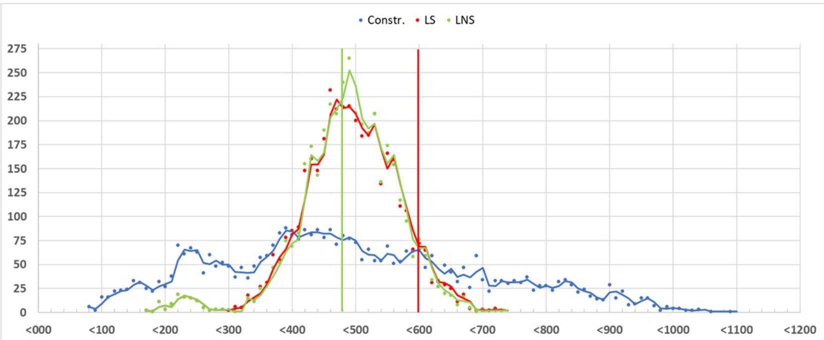
(a) 2000S Training Distributions of Tour Lengths



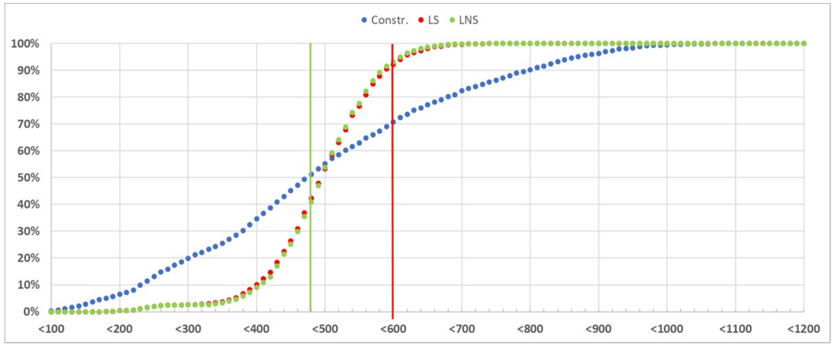
(b) 2000S Training Cumulative Distributions

Evaluation

	Objective	Total Overtime	Avrg. Tour Length	# Tours with Overtime	# Tours with Overtime > 120	Workload Max	Workload Avrg	Workload Min
VRP	1 780 622.0							
Constr.	2 223 974.5	335 841.5	484.14	1 822	1 076	136 962.0	57 024.99	12 789.0
LS	2 002 994.5	123 636.0	481.89	2 025	225	70 424.5	51 358.83	22 228.0
LNS	1 997 592.5	117 613.5	482.05	2 051	193	70 424.5	51 220.32	22 228.0

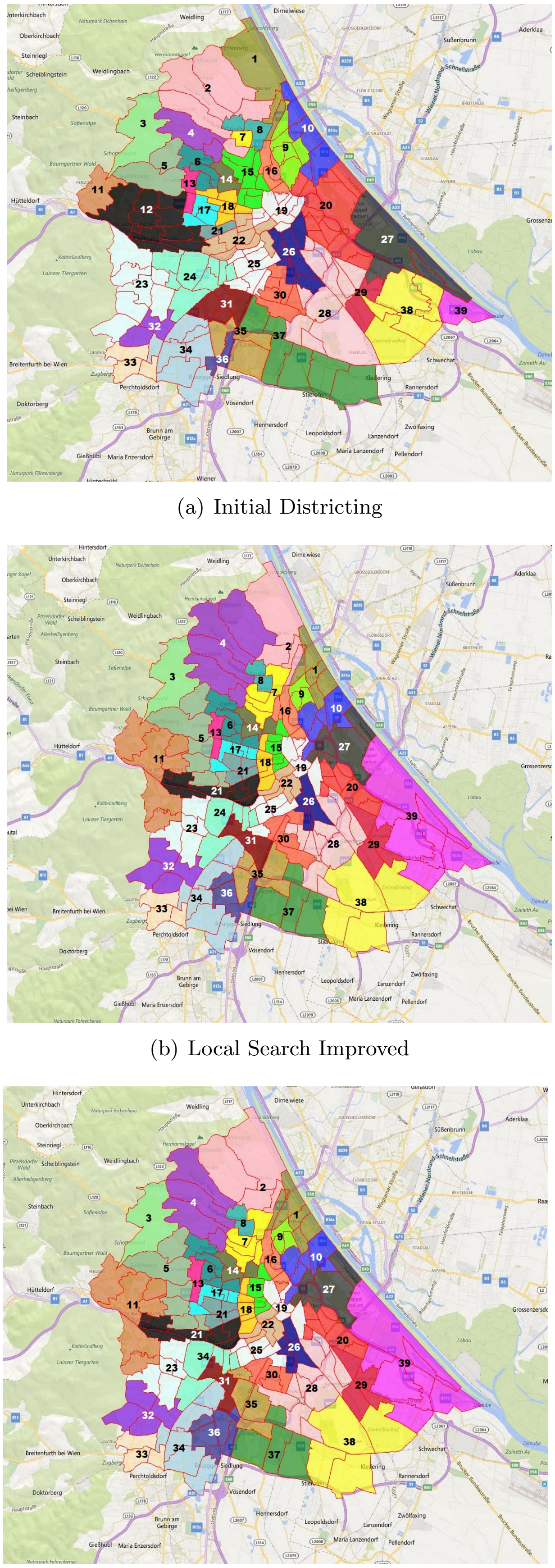


(c) 2000S Evaluation Distributions of Tour Lengths



(d) 2000S Evaluation Cumulative Distributions

Figure 13: Tour Length Distributions 2000 Customers South



(a) Initial Districting

(b) Local Search Improved

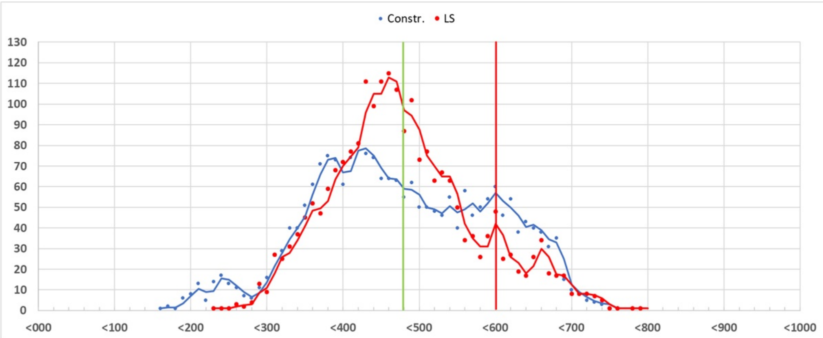
(c) LNS Improved

Figure 14: Districting 2000 South

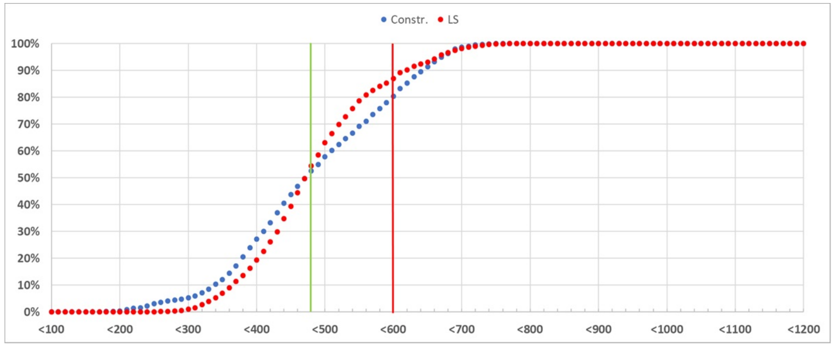
3000 Customers North: 2200 Workdays

Training

	Objective	Total Overtime	Avrg. Tour Length	# Tours with Overtime	# Tours with Overtime > 120	Workload Max	Workload Avrg	Workload Min	Compactness Max	Compactness Avrg	Compactness Min
VRP	972 180.0										
Constr.	1 133 022.0	98 189.0	470.38	990	367	81 144.0	51 501.0	22 905.0	0.7898	0.6698	0.5272
LS	1 111 491.5	74 411.5	471.4	911	238	84 030.0	50 522.34	33 386.0	0.7898	0.6555	0.5293
LNS	1 111 491.5	74 411.5	471.4	911	238	84 030.0	50 522.34	33 386.0	0.7898	0.6550	0.5293



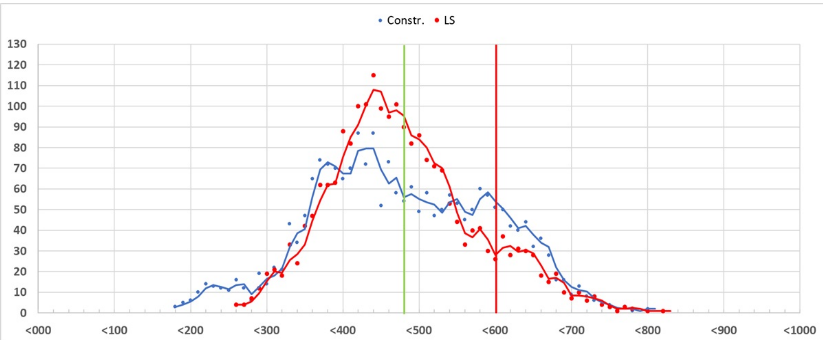
(a) 3000N Training Distributions of Tour Lengths



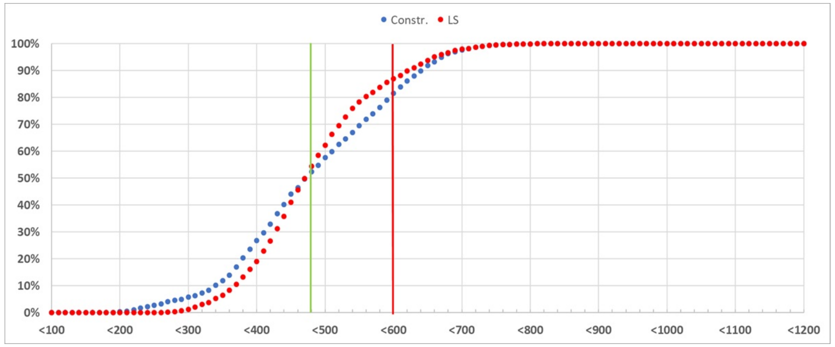
(b) 3000N Training Cumulative Distributions

Evaluation

VRP	972 307.0							
Constr.	1 133 031.0	97 916.5	470.51	989	354	82 882.0	51 501.41	22 978.0
LS	1 113 134.0	75 732.0	471.55	904	260	84 034.0	50 597.0	32 899.0
LNS	1 113 134.0	75 732.0	471.55	904	260	84 034.0	50 597.0	32 899.0



(c) 3000N Evaluation Distributions of Tour Lengths



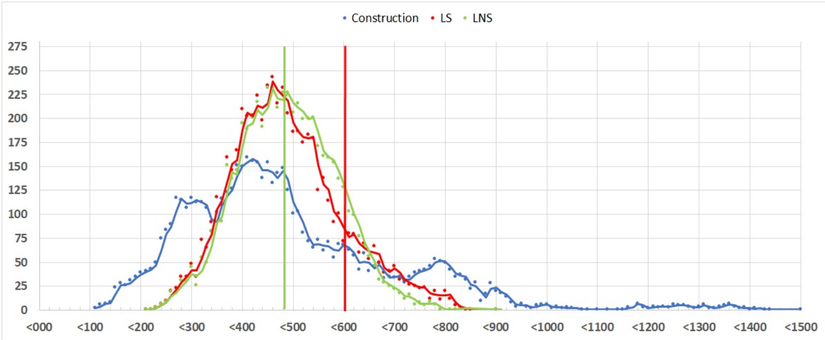
(d) 3000N Evaluation Cumulative Distributions

Figure 15: Tour Length Distributions 3000 Customers North

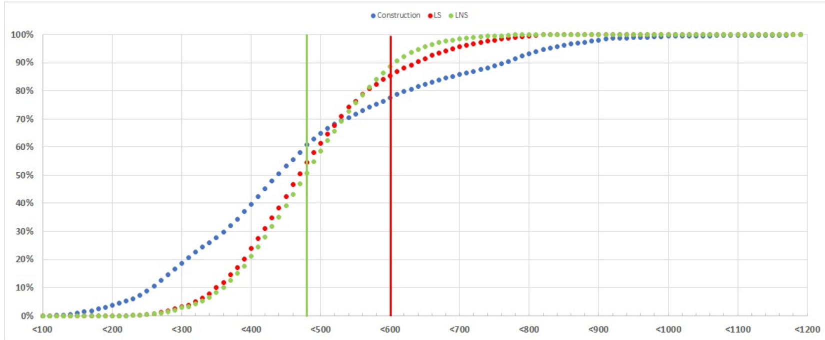
3000 Customers South: 5700 Workdays

Training

	Objective	Total Overtime	Avrg. Tour Length	# Tours with Overtime	# Tours with Overtime > 120	Workload Max	Workload Avrg	Workload Min	Compactness Max	Compactness Avrg	Compactness Min
VRP	2 635 612.0										
Constr.	3 209 494.0	461 023.5	482.19	2 271	1 337	207 374.0	56 306.9	15 685.0	0.8824	0.6521	0.4896
LS	2 990 961.5	250 109.5	480.85	2 579	825	97 767.0	52 473.0	28 169.0	0.8766	0.6058	0.4896
LNS	2 935 612	224 824.0	480.65	2 792	636	91 554.0	52 009.4	28 169.0	0.8766	0.6045	0.4896



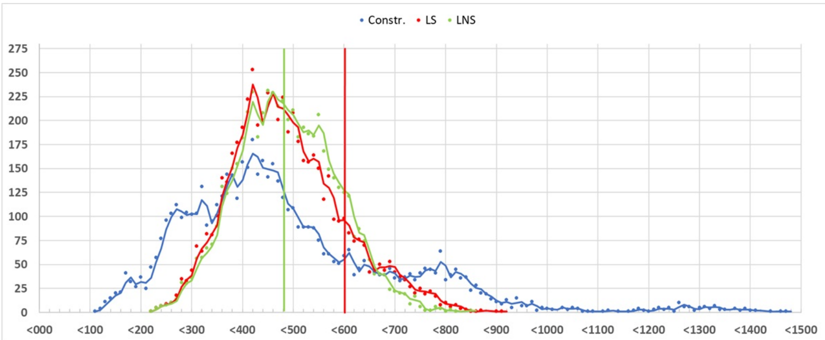
(a) 3000S Training Distributions of Tour Lengths



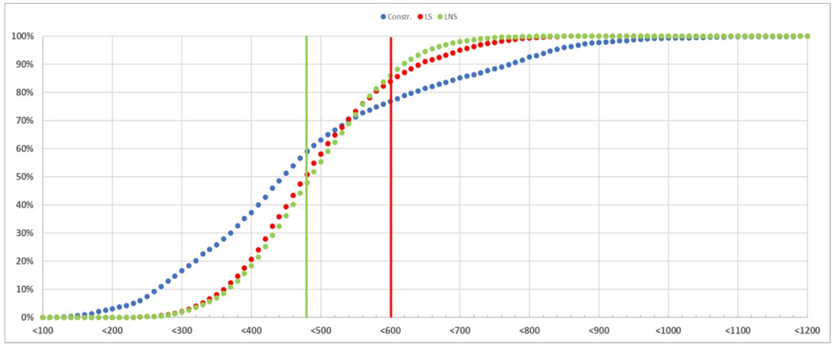
(b) 3000S Training Cumulative Distributions

Evaluation

VRP	2 633 306.0							
Constr.	3 207 187.5	461 838.5	481.64	2 263	1 334	207 836.0	56 226.5	15 487.0
LS	2 988 689.0	461 838.5	480.29	2 570	817	100 130.0	52 433.1	28 414.0
LNS	2 963 772.0	227 253.0	480.09	2 741	664	92 748.0	51 996.0	28 414.0

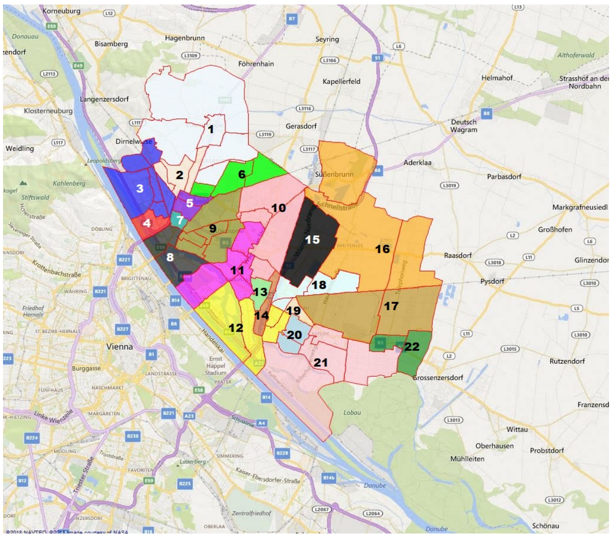


(c) 3000S Evaluation Distributions of Tour Lengths

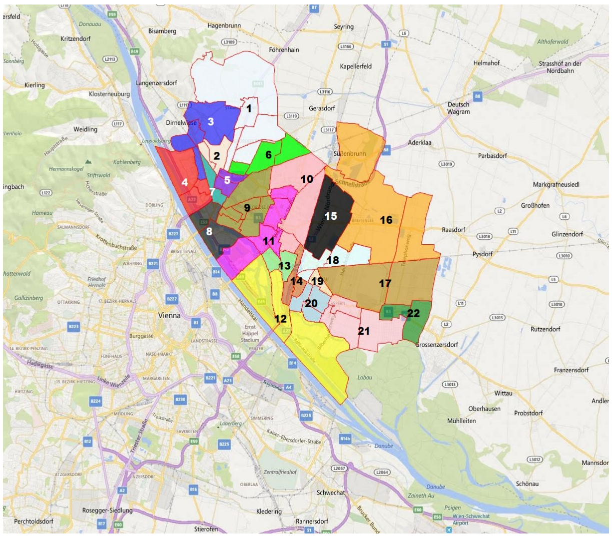


(d) 3000S Evaluation Cumulative Distributions

Figure 17: Tour Length Distributions 3000 Customers South

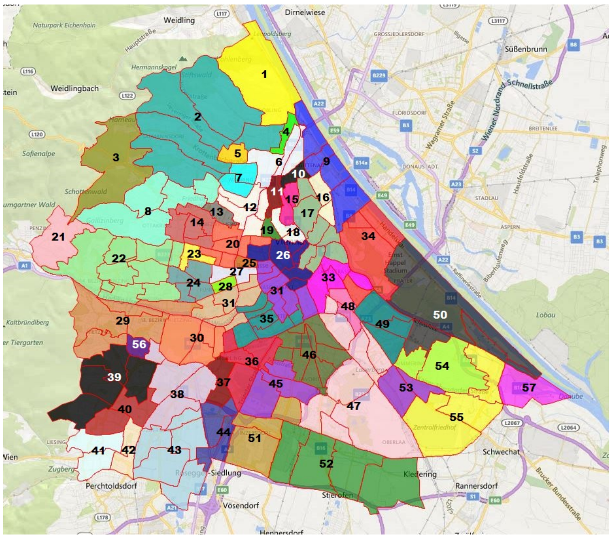


(a) Initial Districting

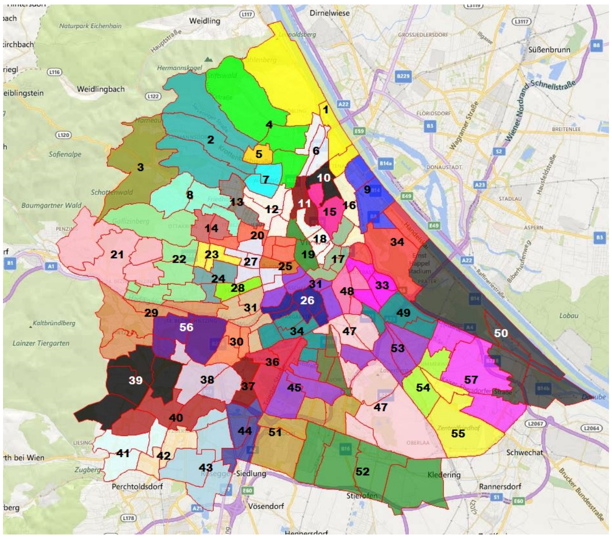


(b) Local Search Improved

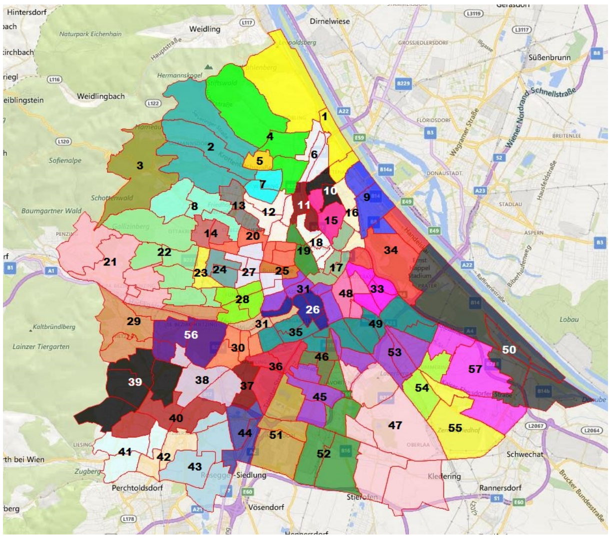
Figure 16: Districting 3000 North



(a) Initial Districting



(b) Local Search Improved



(c) LNS Improved

Figure 18: Districting 3000 South

List of Figures

List of Tables

References

[1] J. G. Carlsson. Dividing a territory among several vehicles. *INFORMS Journal on Computing*, 24(4):565–577, 2012.

[2] L. C. Galvão, A. G. Novaes, J. S. De Cursi, and J. C. Souza. A multiplicatively-weighted voronoi diagram approach to logistics districting. *Computers & Operations Research*, 33(1):93–114, 2006.

[3] R. G. González-Ramírez, N. R. Smith, R. G. Askin, J.-F. Camacho-Vallejo, and J. L. González-Velarde. A grasp-tabu heuristic approach to territory design for pickup and delivery operations for large-scale instances. *Mathematical Problems in Engineering*, 2017, 2017.

[4] C. Groër, B. Golden, and E. Wasil. The consistent vehicle routing problem. *Manufacturing & service operations management*, 11(4):630–643, 2009.

[5] M. A. Haughton. The performance of route modification and demand stabilization strategies in stochastic vehicle routing. *Transportation Research Part B: Methodological*, 32(8):551–566, 1998.

[6] M. A. Haughton. The efficacy of exclusive territory assignments to delivery vehicle drivers. *European Journal of Operational Research*, 184(1):24–38, 2008.

[7] D. Haugland, S. C. Ho, and G. Laporte. Designing delivery districts for the vehicle routing problem with stochastic demands. *European Journal of Operational Research*, 180(3):997–1010, 2007.

[8] A. A. Kovacs, B. L. Golden, R. F. Hartl, and S. N. Parragh. Vehicle routing problems in which consistency considerations are important: A survey. *Networks*, 64(3):192–213, 2014.

[9] C.-M. Liu, T.-C. Chang, and L.-F. Huang. Multi-objective heuristics for the vehicle routing problem. *International Journal of Operations Research*, 3(3):173–181, 2006.

[10] A. M. MacEachren. Compactness of geographic shape: Comparison and evaluation of measures. *Geografiska Annaler: Series B, Human Geography*, 67(1):53–67, 1985.

[11] A. G. Novaes, J. E. S. de Cursi, and O. D. Graciolli. A continuous approach to the design of physical distribution systems. *Computers & Operations Research*, 27(9):877–893, 2000.

[12] A. G. Novaes and O. D. Graciolli. Designing multi-vehicle delivery tours in a grid-cell format. *European Journal of Operational Research*, 119(3):613–634, 1999.

[13] E. Parliament. Directive 2002/15/ec of the european parliament and of the council of 11 march 2002 on the organisation of the working time of persons performing mobile road transport activities. *Official Journal of the European Union.*, 080:35–39, 2002.

[14] E. Parliament. Directive 2003/88/ec of the european parliament and of the council of 4 november 2003 concerning certain aspects of the organisation of working time. *Official Journal of the European Union.*, 299:9–19, 2003.

[15] PostAG. Geschäftsbericht post ag 2015. Downloaded from https://www.post.at/gb2015/de/gb/#/4; Accessed: 2018-09-11.

[16] PostAG. Geschäftsbericht post ag 2017. Downloaded from https://www.post.at/downloads/WEB_OePost_GB17_F_deutsch.pdf?1521625334; Accessed: 2018-09-11.

[17] M. A. Salazar-Aguilar, R. Z. Ríos-Mercado, and M. Cabrera-Ríos. New models for commercial territory design. *Networks and Spatial Economics*, 11(3):487–507, 2011.

[18] M. A. Salazar-Aguilar, R. Z. Ríos-Mercado, and J. L. González-Velarde. A bi-objective programming model for designing compact and balanced territories in commercial districting. *Transportation Research Part C: Emerging Technologies*, 19(5):885–895, 2011.

[19] M. Schneider, A. Stenger, F. Schwahn, and D. Vigo. Territory-based vehicle routing in the presence of time-window constraints. *Transportation Science*, 49(4):732–751, 2014.

[20] P. Shaw. A new local search algorithm providing high quality solutions to vehicle routing problems. *APES Group, Dept of Computer Science, University of Strathclyde, Glasgow, Scotland, UK*, 1997.

[21] StadtWien. Strassengraphogd.csv. Downloaded from https://www.data.gv.at/katalog/dataset/stadt-wien_straengraphwien/resource/2a264ff1-c053-4565-a95d-71d2c2eb8d69; Accessed: 2018-08-22.

[22] StadtWien. vie_303.csv. Downloaded from https://www.data.gv.at/katalog/dataset/stadt-wien_viebevlkerungsprognose2014bis2024wienerzhlbezirke1; Accessed: 2018-08-22.

[23] StadtWien. Zaelbezirkogd.csv. Downloaded from https://www.data.gv.at/katalog/dataset/stadt-wien_zhlbezirksgrenzenwien/resource/6946cce2-f4be-48e3-b345-b6a1f276a12e; Accessed: 2018-08-22.

[24] P. Toth and D. Vigo. *The vehicle routing problem*. Society for Industrial and Applied Mathematics, 2001.

[25] F. S. und Logistik [herausgebendes Organ]. *Kollektivvertrag für Angestellte in Spedition und Logistik*. WKO, Spedition, Logistik, [Wien], stand: 1. april 2017 edition, 2017.

[26] T. Vidal, T. G. Crainic, M. Gendreau, N. Lahrichi, and W. Rei. A hybrid genetic algorithm for multidepot and periodic vehicle routing problems. *Operations Research*, 60(3):611–624, 2012.

[27] K. Wong and J. E. Beasley. Vehicle routing using fixed delivery areas. *Omega*, 12(6):591–600, 1984.

[28] R. T. Wong. Vehicle routing for small package delivery and pickup services. In *The Vehicle Routing Problem: Latest Advances and New Challenges*, pages 475–485. Springer, 2008.

[29] H. Zhong, R. W. Hall, and M. Dessouky. Territory planning and vehicle dispatching with driver learning. *Transportation Science*, 41(1):74–89, 2007.