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„The effect of inclusion and exclusion in and from the German midcap index (MDAX) on stock's returns "

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Abstract (EN):

In this thesis we investigate the effects of inclusion and exclusion of a stock in and from the mid cap German index (MDAX). The results of inclusion and exclusion into and out of the index show mixed effects on stock returns. We find evidence in support of the anticipating-investors hypothesis, where in the case of stock promotions from the small cap German index into the mid-cap German index the cumulative abnormal returns of the stocks are significantly positive. On the other hand, we find significant positive cumulative abnormal returns in the case of stock demotions from the mid cap index into the small cap index, and negative cumulative abnormal returns for stock promotions into the big cap index from the mid cap index. The latter results are at odds with most but not all previous findings.

Keywords: inclusion, exclusion, abnormal returns, stock index, event-study, regression

Abstract (DE):

In dieser Masterarbeit untersuchen wir die Auswirkungen der Aufnahme und des Ausschlusses einer Aktie in und aus dem deutschen Index mit mittlerer Marktkapitalisierung (MDAX). Die Ergebnisse der Aufnahme und des Ausschlusses in und aus dem Index zeigen gemischte Auswirkungen auf die Aktienrenditen. Wir finden Evidenz für die antizipierende-Investoren-Hypothese. Im Falle von Aktienaufstiegen aus dem deutschen sind Index mit geringer Marktkapitalisierung in den deutschen Index mit mittlerer Marktkapitalisierung und die kumulierten anormalen Renditen der Aktien signifikant positiv. Im Gegensatz dazu finden wir signifikant positive kumulierte anormale Renditen im Falle von Aktienherabstufungen aus dem Index für mittlere Marktkapitalisierungen in den Index für kleine Marktkapitalisierungen und negative kumulierte anormale Renditen für Aktienaufstiege aus dem Index für mittlere Marktkapitalisierungen in den Index für große Marktkapitalisierungen. Die letztgenannten Ergebnisse stehen im Widerspruch zu den meisten, aber nicht allen früheren Studien.

1. Introduction

Market indices play an important role in finance. Since the introduction of the Dow Jones Industrial Average in May 26, 1896 up to today there are 70 times more indices than stocks (Authers, 2018).

The history of the family of German stock indices starts with the introduction of the DAX in July, 1988, followed by the introduction of the MDAX and SDAX in 1996 and 1999 respectively.

Methodologies of calculating indices do not differ much from each other, some available indices are calculated based on the weighted average method and some indices are calculated by using the free float market capitalization. The DAX family indices, the S&P 500, the Euro Stoxx 50 and the Rusell 2000 are based on the free float market capitalization method (Factsheet DAX, 2021).

Market indices are usually used to follow the movement of a certain market, industry, type of securities such as tech securities, health, semiconductor industry, financials etc. Indices in modern finance are used as benchmark tools in investment management for comparing investment opportunities.

In this thesis we will examine the effect of inclusion and exclusion of a stock into and from the mid-cap German index. The choice of the mid-cap index was not arbitrary. We chose to analyze the movement of stock indices into the MDAX for three reasons; 1) the MDAX sits in the middle of the German index family pyramid, hence the number of stocks moving into the MDAX is higher compared to the inclusions and exclusions into and from the DAX respectively, 2) there are already studies published on the big-cap German index (DAX) and 3) we hope to give a new perspective of the intra-index stock movement in the German DAX family.

We need to be careful with how we define an inclusion and exclusion in our case, because it has implications in our hypothesis testing.

The announcement day of an inclusion/exclusion precedes the effective event which is the official day a stock becomes part of the index. The inclusion and exclusion from the MDAX can have different meanings depending on the direction of a stock's movement in the DAX index family pyramid. If a stock is included into the MDAX it is either considered a promotion, where a stock is promoted from the small-cap German index upwards, or it is considered as a demotion where a stock is demoted into the MDAX from the big-cap German index.

We structure the reminder of this thesis as follows. In Section 2, we review the literature related to similar studies and results from studies conducted for the US stock market indices and other European indices. We discuss studies in favor of the hypotheses presented in Section 3 and against them. We will discuss the similarities and differences between those results and we will look at the methodologies used to conduct those studies.

In Section 3 of this thesis, we present the theoretical and economic hypotheses which serve as basis for the formulation of our null hypotheses and the test statistics we use in Section 5.

Section 4 of the thesis is dedicated to the data sources, the data format, data criteria and discussions involving the quality of the data involved in the empirical study.

In Section 5 we discuss the methodology we use to estimate the abnormal returns. We present two similar methods and models of estimating the abnormal returns, we define the events and the event lengths. The last subsections of Section 5 introduce the empirical null hypotheses along with the corresponding test statistics that we use to test for significance in abnormal returns.

Our results are presented in Section 6 where we discuss and interpret the outcomes of the test statistics for every corresponding event and event window lengths.

The final section is the conclusion and recommendation where we interpret the meaning of our results and how our results relate to the theoretical and economic hypotheses, we have set in Section 3. We make several suggestions for future research on the topic of the inclusion and exclusion of a stock in and from the MDAX and how future research can improve some of the difficulties we have faced.

2. Literature Review

The key component of determining the event effect on the returns of the stocks', is the construction of abnormal returns. Before defining abnormal returns, we initially need to define the actual returns and normal returns. Actual returns are defined as the difference between the price observed today and the price observed yesterday which is then divided by the price observed yesterday.

On the other hand, normal returns are derived by regressing the actual returns on the returns of a representative stock market index such as the US S&P 500, the German CDAX, or in the case of the Japanese stock market, the Nikkei 225 (Strong, 1992), (Bowman, 1983) and (MacKinlay, 1997). The normal returns model is also referred to as the market model in this thesis. Abnormal returns are defined as the difference between the actual returns and the normal returns (Brown & Warner, 1985). We discuss the issue and derivation of abnormal returns in detail in Section 5 of this thesis.

The earliest study concerning the effect of an event on stocks' returns can be attributed to (Fama, Fisher, Jensen, & Roll, 1969). The study involves the effect of stock splits on stocks' returns and whether the earnings of a stock are related to the abnormal return before the stock splits. It is concluded that as companies announce dividends the stock price increases as a result of the perceived robustness of the company and its management. Usually, stock splits happen in a period of an economic boom and stocks which have shown a good price performance in the boom cycle are more likely to split. We like to point out that the event study motivation is partly derived from (Fama, Fisher, Jensen, & Roll, 1969), however there are substantial differences in our work. While the above-mentioned study deals with the event of a stock split, the thesis is concerned with the inclusion and exclusion of a stock from an index. The work of (Fama, Fisher, Jensen, & Roll, 1969) is based on monthly data as opposed to daily returns utilized in this thesis.

Choosing a normal return generating process model is a big part of the event studies. Historically researchers have employed several models of the return generating process. Three different types of models are studied by (Brown & Warner, 1980), (Brown & Warner, 1985) and (Black, 1972); the mean adjusted returns model, the market adjusted returns model and the market and risk adjusted model. In (Brown & Warner, 1985) the authors perform an experiment of constructing over 200 samples that each contain 50 securities from the database of securities available in the Center of Research and Security Prices, and assign a random event date for every security selected in the sample. After the derivation of the abnormal returns the authors derive the standardized average abnormal returns for every point of time t in the event window by dividing them with their variance. An extra 0 to 2% return is added to the actual returns for the event day zero for every security, this is done with the intention to test how good the normal returns generating models are when compared to each other. Testing the power of every model is done by first testing how many times the null hypothesis of no abnormal returns is rejected, for every sample of 50 securities without the additional 0 to 2% return added to the actual returns. In this case the authors argue that on average there should not be any abnormal returns in the sample due to the random selection of the stocks and the random event-day assignment for every security. This is called Type I error, rejecting the null hypothesis when it is true. The null hypothesis of no abnormal return is tested again after the authors add the extra percent of return which makes the null hypothesis of no abnormal return not true. Notes are taken on how many times the null hypothesis is not rejected and classified as Type II error. The frequency of Type I and Type II errors is measured in (Brown & Warner, 1985), where it is concluded that the market model performs reasonably well compared to the other models used to generate the normal returns, the market model is more likely to detect stocks' abnormal returns in an event study methodology.

Even though the residual analysis of the market model is the most commonly used model to estimate the abnormal market returns, other methods such as the multivariate regression model have also been employed extensively. One of the most important works discussing the multivariate regression method dated back to the mid 1980s where (Thompson, 1985) discusses the multivariate regression method (MRM) and several tests statistics such as the Wald Statistic, the Lagrange Multiplier, the Likelihood Ratio and the F statistic. For the sake of brevity we will discuss the test statistics used in this thesis in Section 5 of this thesis. Common event periods such as earnings and dividend announcements are considered by (Thompson, 1985) and cross-sectional aggregation of the dummy coefficients is used along with linear restrictions to test different null hypotheses. Similar works concerning the properties of the multivariate market model with added dummy variables and the types of null hypothesis that can be tested in the framework of event studies have been done by (Karafiath, 1988), (Malatesta, 1986), (Binder J. J., 1985 (b)) and (Smith, Bradley, & Jarrell, 1986).

Estimating the multivariate regression model by the OLS method brings no efficiency in its estimation, the efficiency comes from accounting for heteroskedasticity and contemporaneous covariances of the residuals in the null hypothesis by estimating the variance-covariance matrix of the residuals as introduced by (Binder J. J., 1985).

A different method of employing the multivariate regression model is done by (Chou, 2004). The author performs the experiment in three steps. The first step involves running an OLS regression and deriving the estimated coefficients of the dummy variables, the estimated coefficient of the market returns, the OLS residuals and the test statistic (i.e., the Wald test statistic). In the second step the market residuals and the stock returns (ε_t, r_t) are sampled 1000 times with replacement and estimates of the model are calculated for every sample of residuals drawn. The third step is the calculation of the corresponding test statistic (i.e., the Wald test statistic) for every draw of the residuals and the calculated estimates of the model. In the fourth step the author calculates the percentage of the bootstrapped test statistic that is bigger than the statistic calculated in step 1. The bootstrap method applied to the residuals of the OLS developed by (Efron, 1979) performs better than the traditional methods as concluded by (Chou, 2004).

Another study that utilizes the bootstrapping method in event studies is conducted by (Kramer, 2001). The approach in this study is that of bootstrapping the mean adjusted t-statistics of the dummy variable coefficients and randomly sampling these mean adjusted t-stats N times, and defining Z-Statistics for each of the samples in order to finally create the empirical distribution of the bootstrapped Z-value and reject the null hypothesis at the chosen confidence interval. More precisely, if the bootstrapped Z-statistic at the 5^{th} percentile is bigger than the standardized Z-statistic from the OLS or the bootstrapped Z-statistic at the 95^{th} percentile is smaller than the standardized Z-statistic from the OLS regression then we reject the two tailed null hypothesis of no abnormal returns. A more detailed review of the types of bootstrap technique in event studies has been done by (Lefebvre, 2007).

In a study by (Gurel & Harris, 1986) investigating the price pressure hypothesis and the efficient market hypothesis, the authors conclude that changes in the S&P500 index cause fluctuations in the demand of the stock which are followed by an increase in price for the event of addition into the index. Negative returns are observed in the post announcement of the addition event period, which is what the price pressure hypothesis predicts.

Another similar study conducted by (Edmister, Graham, & Pirie, 1996) concerning the addition of stocks into the S&P500 is conducted by considering liquidity measures such as the ratio of the bid-ask spread to the price, trading volume and the open interest on trading futures on the stock. The conclusion is that the returns of the stock are positively affected after the effective day. However, in this thesis we are not able to use the liquidity measures of the bid and ask prices and prices of future contracts on the corresponding stocks due to the unavailability of the data. Hence, we use the simpler market model to estimate the effects of the events at hand.

The analysis of the price and volume related to additions and deletions in and from the S&P500 has also been conducted by (Lynch & Mendenhall, 1997) where they derive the abnormal returns by utilizing the market model, the authors analyze the abnormal returns between the time of after the announcement of the addition or deletion up to ten days after the actual (effective day) addition or deletion. The abnormal returns are positive for inclusions into the S&P500 and negative for exclusions from the said index, the study supports the price pressure hypothesis and the information hypothesis.

The effects of exclusion of stocks from the S&P500 of seven utility companies is studied by (Goetzmann & Garry, 1986), where they conclude that the event of a stock exiting the index has negative effects on the prices of the securities. The price drop is attributed to the future uncertainty in earnings and the future quality of information.

The short-term effects of addition into the index by studying the excess returns and the abnormal volume turnover is conducted by (Chen, Noronha, & Singal, 2004). This is a more recent study in comparison to the pre 2000s studies. The results obtained by employing the market model for calculating the abnormal volume turnover and excess returns provide different results to previous older studies. There exist positive statistically significant excess returns in the case of stocks included in the index. However, the negative excess returns are statistically insignificant, these results are inconsistent with the price pressure hypothesis of the older literature.

The literature we discussed above is related to the effect of the change in the index on the stocks' returns. However, there is evidence provided that the event of addition and deletion of a stock in an index can also affect the derivatives market. In a study concerning the effect of inclusion and exclusion from the S&P500, (Sui, 2004) studies the mean cumulative abnormal option volume in a period of ten days before the announcement of an inclusion or exclusion up to 20 days after the effective addition or deletion from the index. The author observes significant changes in the put option prices for the addition group and call option prices for the deletion group. The effect on the options' prices is positive for additions and negative for deletions. The effect on mean average abnormal volume options is also significantly positive for additions from the day of the announcement up to the day of the effective change. The change in average abnormal volume for put options for the exclusion event is also positive however, it is statistically insignificant. The findings of (Sui, 2004) are in line with the investor awareness hypothesis. Studies related to the German index family are not as common as studies concerning the US S&P 500 index. We hope that we can give new insights and a different perspective regarding the effect of the inclusion and exclusion from the German stock index. Overall, security markets across the world operate under the same conditions (with the exception of the Chinese stock market). There are however considerable differences between investors from different regions. The myth of the homogeneous investor has already been dispelled with the advance of behavioral finance. Despite the rich access to information through the help of the internet, investing differences still persist among investors from different parts of the world. A good summary concerning the biases in investments and the way they arise is discussed by (Bachmann, De Giorgi, & Hens, 2018).

3. Testable Research Hypotheses and Questions

3.1 Research Question

There are several ways to frame the research question of this thesis, since we want to study the effects of addition and deletion on a stock's returns, we can frame it as follows:

- How does the addition/removal to/from the German mid-cap index (MDAX) affect the added/removed equities' returns?

3.2 Testable Hypotheses

The Efficient Market Hypothesis: A market is called efficient if prices in the market fully reflect the available information in the market. Since the definition of efficient markets does not only apply to the securities' market, we can also use the analogy of the apple markets. If in a certain apple market all types of apples were priced according to their quality, then this market is called efficient. It would, however, be inefficient if a buyer or a seller possessed information about the quality of the apples sold that no one else has and use that information to generate arbitrage.

The same logic is applied to the securities market, as it is argued by (Malkiel & Fama, 1970) an efficient market incorporates all the available information in the security prices, Fama argues that market inefficiencies can potentially arise from the market conditions and frictions such as transaction costs.

Information Signaling Hypothesis: This hypothesis assumes that the event of inclusion (exclusion) of a stock into (from) an index is not an information free event and that the event contains information that can change investors perception of the stock, for example an inclusion into the S&P500 signals investors that the stock is low risk as argued by (Jain, 1987). It can also be argued that stocks that are included in the index can be perceived as companies with a good management which is prone to make the company perform better in the future.

Anticipating-Investors Hypothesis: Sophisticated investors might be able to anticipate index changes and take positions before the revision of the index takes place. Empirical evidence for this hypothesis come from the AEX Dutch Index (Doeswijk, 2005). The predictability of the DAX index composition changes is discussed by (Franz, 2020) where abnormal returns were present for all three German indices upon inclusion and exclusion.

4. Data

The data for the empirical research part of this thesis is downloaded from the Thompson Reuters Datastream (Datastream, Thompson Reuters, 2021) which is made available through the Vienna Data Center (Hautsch, 2021). The above-mentioned source contains data from 175 countries and 110 markets. The data categories available in the database range from equities, equity indices, bonds, constituent lists etc.

The data relevant for this thesis comprise the daily price data available in the time series format which fits the short-term effect on stocks' returns. The details concerning the properties of the time series data are discussed in Section 5 of this thesis.

We have collected daily equity prices of stocks that have been both, included and excluded from the German mid-cap index alias MDAX, the big-cap index DAX and the small-cap index SDAX. The rules of inclusion in and out from the index are clearly stated in the official rulebook.

The DAX index comprises the thirty biggest German companies by market capitalization in the German stock exchange, and it is considered the blue-chip German market index.

The MDAX is comprised of the sixty ranked below the thirty big-cap stocks in the DAX.

The SDAX is also a German stock market index which is composed of the seventy market-weighted small-capitalization equities ranked below the MDAX index.

We will use also the CDAX which is also a German market index that incorporates all the shares listed in the Frankfurt stock Exchange, General Standard and Prime Standard. We use this index as a proxy of the German market index.

The inclusion and exclusion of a certain stock from the index is based on four criteria: the fast exit and fast entry criteria done every quarter of the calendar year, and two semi-annual regular exit and regular entry criteria (Qontigo, 2021).

The fast exit and fast entry criteria are based on whether a certain equity meets the necessary size i.e., the free float market capitalization or the order book volume, which is a measure of liquidity, a stock enters or exits the index only if it fulfils the rules in Table 4.1.

For further and more detailed explanations one can check the official rulebook of the DB (Qontigo, 2021).

In this section, we shortly summarize Table 4.1 and the inclusion/exclusion rules for the MDAX index which can also be used to infer rules for the big and small cap indices.

Column number one in Table 4.1 is a list of the three German market indices and the exit and entry rules for each index listed below the corresponding index name.

Column one and its first five rows show the name of the big-cap index (DAX) followed by the four rules of inclusion and exclusion.

Column one and its rows six to ten show the name of the mid-cap index (MDAX) followed by the four rules of inclusion and exclusion. The same applies to column one and rows eleven to fifteen for the small-cap index (SDAX).

In the case of the fast and regular exit rule, a candidate stock of a corresponding market index represented in column number one that fulfills the criteria in column number two is replaced by an alternative candidate stock that fulfills the criteria in column number three.

In the case of the fast and regular entry rule, a candidate stock from one of the market indices represented in column number one that fulfills the criteria in column number two replaces an alternative candidate stock of the corresponding index that fulfills the criteria in column number three.

To illustrate how we read the table, we take the case of an MDAX candidate stock for the fast exit rule which fulfills the criterion in the intersection of column number two and row number seven, the candidate stock has to be ranked 105^{th} in both, the free float market capitalization and order book volume, i.e., (105/105).

The aforementioned candidate stock is subsequently substituted with an alternative candidate stock that fulfills one of the three criteria in the intersection of row seven and column number three (95/95, 95/100, 95/105), i.e., the alternative candidate stock has to be ranked 95th in free float market capitalization and, either 95th, 100th, or 105th in order book volume so that it substitutes the exiting candidate stock from the MDAX index. This rule is implemented quarterly throughout the calendar year as shown in Table 4.1, row seven and columns four to seven.

Columns four to seven represent the rule implementation months of the calendar year, the *x* symbol in every cell of the table under these four columns bears the meaning that the corresponding rule has been implemented for that month. The fast exit and entry rules apply to the regular revision months (March and September) and irregular revision months (June and December), and this is shown by the symbol *x* in the cells formed by the intersection of row seven and columns four to seven.

a) *Fast Exit:* As shown in Table 4.1, row seven and column two, if a company in the MDAX index is ranked in both the market capitalization (size) or the order book volume (liquidity) below place 105 then the company exits the index and it is replaced with an alternative company as shown in Table 4.1, row seven and column three, which fulfills the criteria of being ranked at least 95th in size and simultaneously either ranked the 95th, 100th or 105th in order book volume.
This rule is applied quarterly throughout the calendar year as depicted in Table 4.1, row seven and columns four to seven.

b) *Fast Entry:* The intersection cell of column number two and row number eight in Table 4.1 shows that a stock is included in the MDAX, if a company is ranked 85th or above in both market capitalization and order book volume (85/85), accordingly it replaces an alternative company that is ranked 95th or below in both market capitalization and order book volume as given in the intersection cell of row eight and column three (95/95).
This rule is applied quarterly throughout the calendar year as depicted in Table 4.1, row seven and columns four to seven.

c) *Regular Exit:* This rule is similar to the fast exit except for the fact that a company exits the mid-cap index when it is ranked 100th or below in both market capitalization and order book volume (100/100), and it is simultaneously replaced with an alternative company which is ranked 95th and above for both criteria (95/95).

The regular exit rule is applied semiannually, this is shown in the intersection of row nine with the month columns four (March) and six (September).

d) *Regular Entry:* This rule is similar to the fast entry rule with the only discernable difference that the candidate company enters the mid-cap index if it is ranked 90th or above in both FF MCap and OB criteria (90/90), and it simultaneously replaces a company that is ranked 95th or below in both criteria (95/95). The candidate stock criterion is shown in the intersection of column number two and row number ten, whereas the alternative candidate stock criterion lies in the intersection of column number three and row number ten. This rule, just like the regular exit rule, is implemented semiannually.

Table 4.1.: Overview of Rules

DAX®	Candidate rank FF MCap[16]/OB	Alternate candidate rank FF MCap/OB volume	Mar	Jun	Sep	Dec
Fast Exit	45/45	35/35; 35/40; 35/45	X	X	X	X
Fast Entry	25/25	35/35	X	X	X	X
Regular Exit	40/40	35/35	X		X	
Regular Entry	30/30	35/35	X		X	
MDAX®	Candidate rank FF MCap/OB volume	Alternate candidate rank FF MCap/OB volume	Mar	Jun	Sep	Dec
Fast Exit	105/105	95/95; 95/100; 95/105	X	X	X	X
Fast Entry	85/85	95/95	X	X	X	X
Regular Exit	100/100	95/95	X		X	
Regular Entry	90/90	95/95	X		X	
SDAX®	Candidate rank FF MCap/OB volume	Alternate candidate rank FF MCap/OB volume	Mar	Jun	Sep	Dec
Fast Exit	175/175	165/165; 165/170; 165/175	X	X	X	X
Fast Entry	155/155	165/165	X	X	X	X
Regular Exit	170/170	165/165	X		X	
Regular Entry	160/160	165/165	X		X	

Note: This table was created by Qontigo which is part of the Deutsche Börse Group, *Guide to the DAX equity indices (p. 32) February 25th 2021* (https://www.dax-indices.com/document/Resources/Guides/DAX_Equity_Indices.pdf)

It is also worth mentioning that, the quality of data affects the statistical inference of the research, however the quality of the Thompson Reuters Database platform is good for academic research.

A wide variety of research has been done based on the data made available in this platform, and a study conducted by (Ince & Porter, 2006) concludes that the use of non-equity data should be handled with care.

In a more recent and serious investigation on the quality of the data available in TRD concerning the German market by (Brückner, 2013), the author studies the prices, returns, volume, number of shares and many other important variables used in financial research. The author (Brückner, 2013) compares the data in the TRD platform with data obtained from the Frankfurt Stock Exchange and the equities listed therein, several serious issues have been discovered in the data quality and coverage prior to 1990, the author also finds a considerable mismatch in prices between the period of 1990 up to 2000.

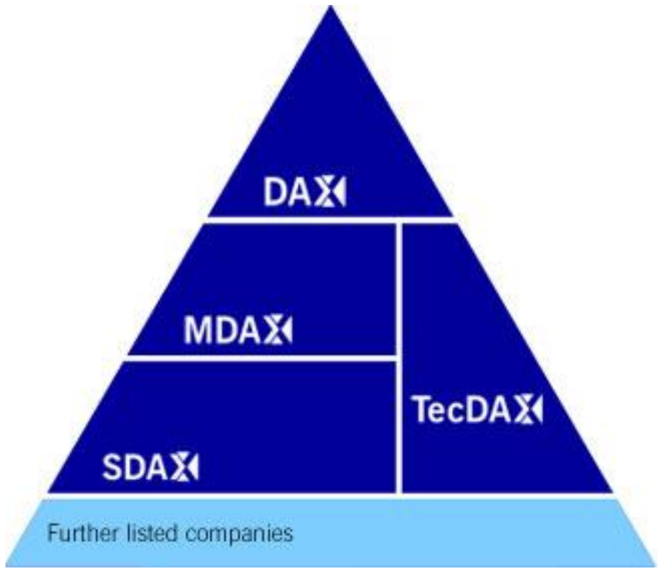
The author recommends, that data for the German market before 1990 should not be used due to its low quality (Brückner, 2013). Taking into account the above recommendations we will only use data available for the period 08/2001 – 01/2021 in order to avoid any problems. The necessary variables for our research will be the stock prices of the corresponding equities that make up the mid cap (MDAX) index, market prices on the German total Market index (CDAX) in a time series format, and the market index MDAX constituent symbols and mnemonics, we use these variables for the calculation of equities' returns and the CDAX index returns.

It is also worth noting that the CDAX prior to 01/1998 was only composed of stocks present in the Amtlicher Markt and the number of equities in the CDAX doubled from
1998 to 2000 after the inclusion of stocks from Geregelter Markt and Neuer Markt.

Considering the above measures, we have identified 107 removals from the MDAX and 100
inclusions. We have also identified that out of all the available removals from up to 12/2020, 19 of the removals were promotions into the top DAX30 index, 79 removals were
relegations to the bottom SDAX index. On the same note out of all inclusions 15 were relegations from the top DAX30 to the MDAX, 59 were promotions from the bottom SDAX
to the midcap MDAX, removals such as bankruptcies, mergers and take overs were excluded from the lists.

Figure 4.1

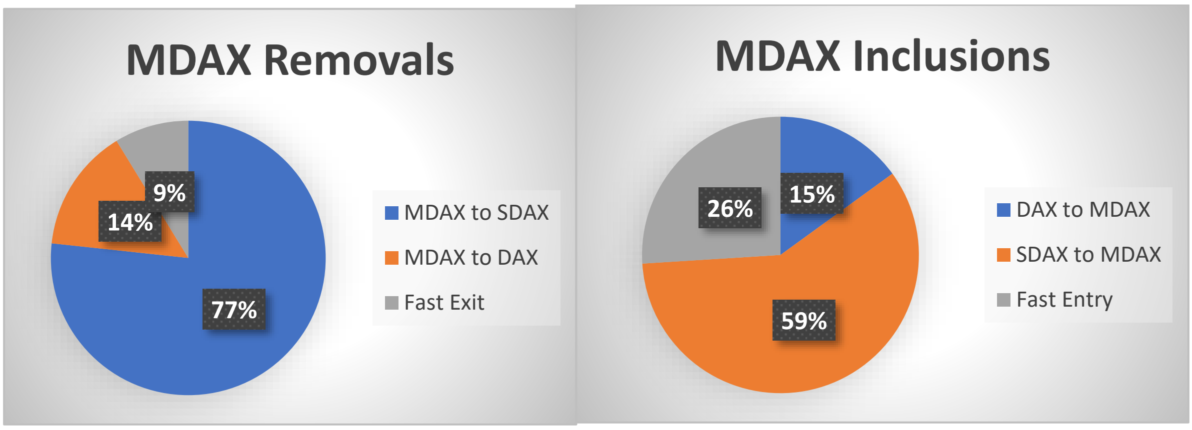
German Index Family



Note: This image was created by Deutsche Börse Group, *DAX-Index – Benchmark und Barometer für die deutsche Wirtschaft*, March 2021, (https://deutsche-boerse.com/dbg-de/media/deutsche-boerse-spotlights/spotlight/DAX-Index-Benchmark-und-Barometer-f-r-die-

Figure 4.2

Summary of stock inclusions/exclusion from/to the mid-cap index



As it can be seen from Figure 4.2, the majority of intra-index movements happens mainly between the mid-cap and small-cap, however the flow from the big-cap to the mid-cap and vice versa is also significant, fast entry seems to be a more common occurrence than fast exit.

5. Methodology

5.1 The Event Study Method

The earliest study on record using the event study method can be traced back to a highly cited article which has inspired decades long research, an article authored by (Fama, Fisher, Jensen, & Roll, 1969) which studies the question of the effect of corporate events on firms' returns/price performance.

In the work of (Fama, Fisher, Jensen, & Roll, 1969), the authors seek to answer the question whether the event of stock splits has a significant impact on a security's returns before and after the split. A list of theoretical hypotheses is tested to check for any anticipation effect on the stock's price performance and the event effect after the fact.

One of the main inspirations for this paper is the meticulous work of (MacKinlay, 1997), which extends the procedure of the event study method step by step and discusses the most commonly used models and techniques. In this thesis we will follow the steps laid out by (MacKinlay, 1997) as well.

There are several reasons for performing an event study, this thesis attempts to answer the question of the information content in or during an event and whether markets are efficient by using the hypotheses put forth in Section 3.

It is important that one is aware of the problems and sensitivities of this method, of which a good part consists in pinning down the event date. Misidentifications in the event date can lead to not being able to observe any significant price changes as concluded in a study, looking into mergers, done by (Dodd, 1980).

In this study we identify two types of events. The first event, the announcement date, is related to the information content and will attempt to answer hypotheses concerning information around the announcement day. The second event, the inclusion date, is the effective event when a stock is admitted or excluded from the mid-cap index, these events are clearly defined and taken from the official media outlets of the Deutsche Börse as mentioned in Section 4.

It seems that many event studies suffer from event clustering. Clustering in event studies in many cases can be common where more than one firm per event exits or enters the index. In this study and in the case of the German mid-cap index substantial clustering can be seen in the subsample of demotion from the MDAX to the lower/smaller market-cap index (SDAX) where in the same day nineteen firms are relegated. The same happens for the subsample of fast entry where twelve companies enter the mid-cap index at the same day, this can be seen in the official media outlet of DB (Qontigo, 2021). There are several remedies for this too. Two types of adjustments can be done, the first adjustment includes the model specification whereas the second one includes the appropriate test statistic.

Sample size in these type of studies poses a problem. In this study the sample size for stocks moving between the mid-cap index and small-cap index will not pose an issue, the number of firms in each subsample is above fifteen firms. However, the subsample fast exit has only nine firms, one can say that in this case the sample size could be said to pose a problem.

5.2 Event and Estimation window

The task of establishing an event window differs for different cases. Depending on the type of the study and data available the event length is different. In this thesis we consider five different event windows. In most cases long term event windows differ from short term event studies where the daily prices are utilized.

Figure 5.1: *Graphic illustration of the event timeline*

- The estimation window:

The estimation window is defined as the period or the number of days before the beginning of the event window denoted by t_b, and its length is defined as $L_E = t_n - t_N$. The blue rectangle in Figure 5.1 illustrates the estimation window.

The event window is the set of days before and after the event day, if we denote the event as t_e, then the event window is the number of days before t_e and after it, this case is depicted graphically in figure 5.1, where the set of pre-event days lies to the left of t_e and this set includes only the days between t_b and t_e. In contrast post-event days lie to the right of t_e and it is the set of days between t_e and T.

The pre-event period is usually referred to as the anticipation period, whereas the post-event period is referred to as the impact period.

The event timeline can be expressed in two ways. The first way of expressing the event timeline is in both positive and negative integer terms, where the event day takes the value of zero whereas the days prior to the event take negative values, and the days after the event day take positive integer values, i.e.,

Definition 1: $\{t_N, \ldots, t_n, t_b, \ldots, t_{e-1}\} \in \mathbb{Z}^- : N < n < b < -1$ and $\{t_{e+1}, \ldots, T\} \in \mathbb{Z}^+ : 1 < T$ and t_e is the event day where the subscript *e* stands for the event and the event day takes the value zero and $n = b-1$.

In this case the length of the estimation window is the difference between the first day in the event window and the first day in the event window.

To illustrate this, we take the length of the whole return timeline, and mark the event day as the day zero (i.e., $e = 0$), then the first day of the estimation window $t_N = -140$ and the first day of the event window is $t_b = -5$, then the length of the estimation window is $L_E = t_b - t_N = -5 - (-140) = 135$, where L_E is the length of the estimation window as depicted in Figure 5.1.

The second way of depicting the event windows in Figure 5.1 is by utilizing the positive integers as follows:

Definition 2: $\{t_N, \ldots, t_n, t_b, \ldots, t_e, \ldots, T\} \in \mathbb{Z}^+ : N < n < b < e < T : e$ is the event, $n = b-1$ and $N=1$.

The same logic applies to Definition 2. If we assume the length of the estimation window together with the length of the event window to be 155 such that $N = 1$, $b = 136$ then the length of the estimation window is $L_E = t_b - t_N = 136 - 1 = 135$.

- The event window:

The event window in this thesis is depicted in Figure 5.1 by the red rectangle where t_b signifies the beginning of the event window and T signifies the end of the event window.

The length of the pre-event (before t_e) period is defined as $L_B = t_e - t_b$, whereas the length of the post-event (after t_e) period is defined as $L_A = T - t_e$. By using L_A and L_B we define the length of the whole event window as $L_W = L_B + L_A = T - t_b$.

In this thesis the length of the whole period of the returns is 150 days where t_N is the first day of the estimation window and T is the last day of the event window.

If the window length (L_W) is 10 days, then, the last day of the event window is T = 150th day, and the first day of the event window is t_b = 140th day, i.e., $L_W = T - t_b = 150 - 140 = 10$.

The pre-event window L_B starts at t_b = 140 (day 140) and ends at t_e = 145 (day 145), i.e., $L_B = t_e - t_b = 145 - 140 = 5$ (days).

The post-event window L_A starts at t_e = 145 (day 145) and ends at T = 150 (day 150), i.e., $L_A = T - t_e = 150 - 145 = 5$ (days).

We denote the total length of the time series of the stock returns by $L_T = L_E + L_W$.

In this thesis we have decided to make use of three window lengths, the first type of window is set five days before ($L_B = 5$) the event which in Figure 5.1 is denoted as t_e. The second event window is five days after the event ($L_A = 5$). The third event window is the whole event window ($L_W = 10$) that starts at t_b = 140 and ends at T = 150, i.e., $L_W = T - t_b = 150 - 140 = 10$.

The empirical work in the thesis is done with the help of the programming language R and Excel, more specifically we utilize the 'systemfit' package built by (Henningsen & Hamann, 2019) which is specifically designed for running a system of equations at the same time where there is no need for running single regressions one by one. For more information on the use of the package one can visit the online platform of the authors and the white-paper of their project for more concise elaborations of the features of the algorithm.

5.3 Model Specifications

In this thesis we use two different approaches for estimating the market model. The justifications for using this model are that the model accounts for the general market returns when defining abnormal returns. More specifically, the model assumes a linear relationship between a security's returns and the market returns.

The actual returns are the simple returns of an equity defined in Equation 1.1; in contrast the normal returns are defined in Equation 2. These normal returns are calculated for every equity in the sample. The normal returns are calculated by regressing the actual equity returns (r_{it}) against the market returns (r_{mt}) throughout the estimation window (L_E), by following this procedure we have effectively estimated the intercept of Equation 1 to be α_i and the slope of Equation 1 is estimated as β_i.

The market model is applied to 7 different European common stocks in an early study by (Pogue & Solnik, 1974) where it is concluded that the Market efficiency hypothesis holds for big countries common stocks such as the United Kingdom, Germany, France and Italy. In this thesis we use the market return model which is the one discussed in (Brown & Warner, 1985). The market return model has been used as a benchmark in the works of (Fama, Fisher, Jensen, & Roll, 1969) in the effect of stock splits, (Thompson, 1985) compares different methods for firm events, (Karafiath, 1988) uses the market model in the framework of multivariate regression as well as (Binder J. J., 1985) discusses how the multivariate regression model is used in hypothesis testing and the advantages it brings over the residual analysis method.

- The traditional univariate market return model:

$$r_{it} = \alpha_i + \beta_i r_{mt} + \varepsilon_{it} \quad (1)$$

$$r_{it} = \frac{P_{it} - P_{i(t-1)}}{P_{i(t-1)}} \quad (1.1)$$

ε_{it} = IID error term s.t $E[\varepsilon_{it}] = 0$, $Var[\varepsilon_{it}] = \sigma_i$ and $COV[\varepsilon_{it,}\varepsilon_{is,}] = 0$.

β_i = estimated slope.

α_i = estimated constant.

r_{mt} = the market index returns.

The model in Equation (1) is the return generating process which will determine the normal returns of the i-th security.

The estimated market return model from Equation (1) takes the form:

$$E[r_{it}] = \alpha_i + \beta_i r_{mt} \qquad (2)$$

The error term (ε_{it}) is also assumed not to be correlated with the market index proxy r_{mt} and with the LHS variable r_{it}, equity returns.

5.3.1 The two-step market return model residual analysis

Before we define the steps of the method, we need to define the length of the vectors used for the estimation of returns. First, in the estimation window which starts from t_N to t_n, these returns form a vector of returns with length $L_E = t_n - t_N$. We also define the pre-event window length as the length of the vector formed by the returns in the event window defined by $L_B = t_e - t_b$, we define the post event window as the vector of length $L_A = T - t_e$, finally the whole event window which envelopes the event day, is defined by the length of $L_W = T - t_b$, a visual representation can be seen in Figure 5.1.

The first step involves calculating the realized returns of the equity over the estimation and event window. We use the simple returns of the equity as defined in Equation 1.1.

The second step in this method makes use of the estimation window defined by L_E to get the parameters of Equation (1) which will define the normal or predicted returns. In this step we use the data from t_N to t_n.

The assumptions of the model and remedies to deviations from those assumptions are explained in Section 5.5 when we lay out the test statistics.

5.3.2 The Multivariate regression model with dummy variables

The multivariate regression model (MRM) which involves the use of dummy variables to pick up the event impact during the event window has been extensively discussed across different publications in finance. The advantages of using the MRM method do not come from the estimation of the dummy variable coefficients and standard errors, since the results of the OLS estimation of the MRM model are the same as the estimations obtained by using Equation 1. However, (Binder J. J., 1985) argues that the advantages of the MRM model come from the incorporation of cross equation heteroskedasticity and contemporaneous correlation of the disturbances in the hypothesis testing. Briefly put, the coefficients of the dummies in the regressions are parameters that signify the abnormal returns inside the equation, the number of dummy variables in the equation is the same as the number of days in the event window used to calculate the abnormal returns, hence, the number of dummy variables will be different for the event post event window (L_A) and the entire event window (L_W).

The MRM method has been used by (Zellner, 1962) where he makes use of the Feasible Generalized least square method (FGLS) for running a number of regression equations simultaneously where the disturbance terms across equations are highly correlated. We also utilize the FGLS method in cases where we have clustering of event dates and see whether there is any improvement in estimation and statistical significance.

The difference between the traditional method and the MRM is that the parameters are estimated in one step by simply just running the regression over the whole time-line containing the returns. The other and more important difference between the traditional method is in the accounting for the difference in the variability of the disturbances and time dependance with one another of the disturbances in the system of equations. The MRM method mitigates these two issues by jointly testing the hypothesis that we will lay out in Section 5.5.

We shall now define the MRM precisely:

$$r_{it} = \alpha_i + \beta_i r_{mt} + \sum_{t \in [t_b, T]} \gamma_{it} D_{it} + \varepsilon_{it} \qquad (3)$$

r_{it} = the realized returns of security i at t.
α_i = intercept of the model.
r_{mt} = market index returns at t.
β_i = market index return parameter.
γ_{it} = the abnormal return of asset i on day t : t ϵ {t_b, ..., T} inside the event window, w takes values corresponding to the number of event days inside the event window.
$D_{it} = \begin{cases} 1, if\ t_b \leq t \leq t_a \\ 0, \quad otherwise \end{cases}$
ε_{it} = IID error term s.t $E[\varepsilon_{it}]$ = 0, $Var[\varepsilon_{it}] = \sigma_i$ and a covariance of zero among error terms.
$i = 1, \ldots, N\ and\ t = t_N, \ldots, T$

We will represent the matrix form of Equation 3 in Appendix F, sub-section (II) where we discuss the structure of the system of equations by partitioning the RHS variables in different matrices, however a more comprehensive expanded form of the above model is represented in Equation 4.

$$\begin{bmatrix} r_{1t} \\ r_{2t} \\ r_{3t} \\ \vdots \\ r_{nt} \end{bmatrix} = \begin{bmatrix} \alpha_1 + & \beta_1 r_{mt} + & \gamma_{10}D_{10} + & \gamma_{11}D_{11} + & \gamma_{12}D_{12} + & \gamma_{13}D_{13} + & \gamma_{14}D_{14} + \gamma_{15}D_{15} + & \varepsilon_{1t} \\ \alpha_2 + & \beta_2 r_{mt} + & \gamma_{20}D_{20} + & \gamma_{21}D_{21} + & \gamma_{22}D_{22} + & \gamma_{23}D_{23} + & \gamma_{24}D_{24} + \gamma_{25}D_{25} + & \varepsilon_{2t} \\ \alpha_3 + & \beta_3 r_{mt} + & \gamma_{30}D_{30} + & \gamma_{31}D_{31} + & \gamma_{32}D_{32} + & \gamma_{33}D_{33} + & \gamma_{34}D_{34} + \gamma_{35}D_{35} + & \varepsilon_{3t} \\ \vdots & & & & \vdots & & & \\ \alpha_n + & \beta_n r_{mt} + & \gamma_{n0}D_{n0} + & \gamma_{n1}D_{n1} + & \gamma_{n2}D_{n2} + & \gamma_{n3}D_{n3} + & \gamma_{n4}D_{n4} + \gamma_{n5}D_{n5} + & \varepsilon_{nt} \end{bmatrix} \qquad (4)$$

The Left-hand side $(r_{1t}, r_{2t}, \ldots, r_{nt})$ variables are vectors of security return data; they represent one return data point per every time (t) point, the same is valid for r_{mt}. On the other hand, the sequence of D_{it} variables are indicator vectors/dummies which take the value of one at a date that is an element of the event window set and zero in every other entry. Equation 4 is an expanded representation of the dummy market regression estimation, i.e., Equation 3, where the event window corresponds to the five days period after the event which is denoted by the number zero in the subscript of the estimated coefficient (γ) and the dummy variable (D), i.e. the event window in Equation 4 is the post event period L_A defined in Section 5.2.

The matrix notation of Equation 3 and Equation 4 is:

$$Y_i = X_i\beta_i + \varepsilon_i \qquad (4.1)$$

Y_i is a L_E x 1 vector where the elements of the vector are the daily returns of stock i at day t (r_{it}) and it represents the dependent left-hand side of Equation 3 and Equation 4.

X_i is a L_E x M_i (M_i is the number of variables or regressors in equation i) matrix where the first vector is the constant (α_i) in Equation 3 and Equation 4, the second vector is the daily mid-cap market index returns r_{mt} and n-2 vectors that represent dummy variables (D_{it}) for every day inside the event window where every vector consists of zeros and a single number one for the specific day inside the event window.

ε_i is a L_E x 1 vector representing the error terms of equation i.

The i individual equations in Equation 4.1 can be stacked and we get the following form:

$$Y_* = X_*\beta_* + \varepsilon_* \quad (4.2)$$

Similarly, the dimensions of the vector Y_* are N L_E x 1, X_* is a N L_E x M_* matrix composed of N X_i matrices where N is the number of equations in the system such that every equation represents the returns of a firm as expressed in Equation 3. The total number of regressors across all i equations is represented by $M_* = \sum_{i=1}^{N} M_i$, β_* is the coefficient stacked vector with dimensions M_*x1, and ε_* is the stacked error vector with dimensions N L_E x 1.

The system of equations corresponding to Equation 3 and Equation 4 will be run as a single equation where the coefficient estimates are defined as (Zellner, 1962):

$$\hat{\beta}_* = [X_*'(\Sigma^{-1} \otimes I)X_*]^{-1}X_*'(\Sigma^{-1} \otimes I)Y_* \qquad (4.3)$$

$$Cov[\hat{\beta}_*] = [X_*'(\Sigma^{-1} \otimes I)X_*]^{-1} \qquad (4.4)$$

The $\hat{\beta}_*$ in Equation 4.3 is a vector of estimated regression coefficients, X_* is a return and dummy variable stacked matrix composed of a vector whose inputs are a series of ones representing the constant α_i of the regression equation, a vector of the market return inputs and depending on the event length X contains six to eleven indicator vectors where the inputs are equal to zeros for non-event days and equal to one on the days inside the event window.

A more concrete and visual representation of Equation 4.1, Equation 4.2, the components of Equation 4.3 and Equation 4.4 can be found in Appendix F under Section II.i represented by Equations e and Equation e.1.

The $(\Sigma^{-1} \otimes I)$ represent the inverse covariance matrix which is a matrix of estimated variances from the first step OLS estimation, the covariance matrix under the normal OLS assumptions takes the form $(\Sigma \otimes I) = E({\varepsilon_{ij}}^2) = I_N$ since $(\sigma_i = \sigma_j \wedge \sigma_{ij} = 0;\ i \neq j)$ which means no correlation between error terms.

The operation $\otimes$, is the tensor product of the variance or the inverse covariance matrix with the identity matrix, this product in our case serves the purpose of constructing a bigger matrix which is made up of diagonal Σ^{-1} matrices, this product is used in the setting of multiple equations where the coefficients of all equations are estimated simultaneously.

The estimated variance matrix $(\Sigma^{-1} \otimes I)$ of Equation 4.4 on the other hand is taken from the weighted least square residuals of the system of equations, when the whole system is treated as one equation, the estimation of $(\Sigma^{-1} \otimes I)$ can be calculated as either one representative average variance for all equations $(\hat{\sigma}^2 I_{Nt}\ s.t.\ \hat{\sigma}^2 = E({\varepsilon_{it}}^2))$ or one can consider individual variances for each equation within the system where $(\Sigma \otimes I) \leftrightarrow (E({\varepsilon_{it}}^2) = \sigma_i^2\ s.t.\ \sigma_{ij} = 0\ \forall\ i \neq j)$, we use the first type of variance when running a system of regressions where many firms have a common event date, and the latter one for regressions where firms event dates do not overlap with each other.

The residual covariance matrix can be estimated in the traditional way where we only account for the number of observations without correcting for the degrees of freedom, another way is if we accounted for the number of degrees of freedom, this method gives us unbiased variance estimations, the denominator of the accounts for the number of observations as well as for the number of regressors, the formula can be seen in Appendix F, sub-section II.ii, Equation (i), this method was developed by (Zellner & Huang, 1962).

The covariance matrix is represented by the operation of the Kronecker Product which is another name for the tensor product that combines matrices or vectors of any orders as explained by (Pollock, 2013). Every element of the inverse variance matrix $\Sigma^{-1}$is multiplied with the identity matrix where a single block diagonal matrix is produced such that the elements in the diagonal are variance matrices of the corresponding equation.

The expanded matrix form of this model and its visual illustration can be found in Appendix F under sub-section (II.i), represented by Equation e, the OLS and SURE estimation of Equation 4.3 is drawn out in Appendix F under sub-section (II.ii). For an in-depth reading of the setting up, properties and types of methods of estimation discussed in this section can be found in (Greene W. H., 2017, pp. 326-371).

5.3.3 The Ordinary Least Squares (OLS) and the Seemingly Unrelated Regression (SURE) techniques

It is also worth noting the difference between the traditional OLS and the Seemingly Unrelated Regressions methods (SURE) developed by (Zellner, 1962). The OLS method can be used in overlapping event dates where several firms have the same event dates, this usually happens in regulatory events or the method can be used in cases where events are spread throughout different calendar periods, such events are stock split announcements, earnings announcements which can be different for different firms. This thesis is concerned with both types, in some cases we can see event clustering where several firms enter or exit the mid-cap index and in many other cases event dates are spread out over time. The clustering in the case of the MDAX is not as serious, with the exception of 11th of February 2003 where nineteen equities are relegated from the MDAX to the SDAX and the case of 5th of September 2018 where fourteen companies enter the MDAX index by the fast entry rules. In these two particular cases we can apply the SURE method and see the difference between the exit and entry effect on firms' returns by testing for the statistical difference between the two events, the empirical null hypothesis can be found in Section 5.5.

The significant difference between the SURE and OLS method stands in the way how the SURE accounts for the correlation between the error terms across equations by using the feasible generalized least square (FGLS) method.

The estimated values of the coefficients ($\hat{\beta}_*$) in Equation 4.3 for the OLS and SURE estimation are obtained differently. The OLS estimation method assumes that the covariance matrix (Σ) of the residuals is known and it is $\sigma^2 = E[\varepsilon_{it}^2]$, $\sigma_{ij} = 0 \; \forall \; i \neq j$. The SURE estimation method uses the residual's to consistently estimate the covariance matrix (Σ), where $\sigma_{ij} = \frac{\varepsilon_i' \varepsilon_j}{T}$ s.t. T is the number of observed returns that corresponds with the number of days in the time series that stretches from t_N to T as illustrated in Figure 5.1 in Section 5.2.

Unlike the OLS, the SURE method applies strictly to a system of equations. The estimated coefficients ($\hat{\beta}_{OLS}$) obtained from implementing the OLS method to the system of equations are the same as the estimated coefficients obtained by performing the OLS method to individual equations.

One of the assumptions of the OLS method is that the regressors and the disturbance terms of every single equations are uncorrelated (i.e. $E[\varepsilon_i' X_i] = 0 \ \forall \ i$), on the other hand the SURE method necessitates that all regressors and relevant disturbances are uncorrelated across all equations (i.e. $E[\varepsilon_i' X_j] = 0 \ \forall \ i,j$).

As explained by (Moon & Perron, 2006) the asymptotic distribution (T→∞) of the OLS estimators ($\hat{\beta}_{OLS}$) and the SURE estimators ($\hat{\beta}_{SURE}$) are different.

$$\sqrt{T}\left(\hat{\beta}_{OLS} - \beta\right) => N(0;[E(X_i X_i')]^{-1} E(X_i \Sigma X_i')[E(X_i X_i')]^{-1}) \qquad (4.5)$$

$$\sqrt{T}\left(\hat{\beta}_{SURE} - \beta\right) => N(0;[E(X_i \Sigma^{-1} X_i')]^{-1}) \qquad (4.6)$$

It can be deducted from Equations 4.5 and 4.6 that the SURE estimators have a smaller variance than the OLS estimators which is also consistent with the Gauss-Markov theorem.

5.4 Defining abnormal returns

In this section we will define and explain the calculation of abnormal returns for the residual analysis technique and dummy regression estimation by first starting with the traditional two-step residual model.

5.4.1 Abnormal Returns from the two-step market model

The derivation of abnormal returns using residual analysis has been explained in almost every article dealing with event studies. Precise and rigorous derivations of abnormal returns and the types of abnormal returns can be found in (Binder J. J., 1998) where a review of the history of event studies developed by different authors through the years has been carefully documented. The scrutiny falls on the types of statistics used to evaluate the abnormal returns and issues that might arise from them. In another similar earlier article (Bowman, 1983) does the same and explains in meticulous details the way an event study is conducted. On the same note (McWilliams & McWilliams, 2000) and (Pynnönen, 2005) comprehensively review the abnormal return calculations and their derivatives. The most significant work related to this analysis and the main source of many event studies can be attributed to what is consider to be the bible of Financial Econometrics is the work of (Campbell, Lo, & MacKinlay, 1997, pp. 149-180). This work sets the foundations of a well-structured procedure, a detailed explanation of the event study procedure and discussion on the impacts of an earning's announcement is added as an illustrative guide for the readers.

Abnormal returns are calculated as the difference between actual realized simple returns and normal returns which in the traditional way are calculated by the formula in Equation 2.

$$AR_{it} = r_{it} - (\alpha_i + \beta_i r_{mt}) \ s.t. \ t_{\mathrm{b}} \leq t \leq T \qquad (5)$$

α_i and β_i are the estimated parameters of Equation 1.

The assumption about the distribution of the abnormal returns is that they are normally distributed. However, it is reasonable to assume that estimated parameters are not completely independent and realized returns within the event window would have their own special event effect in them which can be either positive or negative. This is reasonable to assume this as study itself is looking for the effect of the day of the event on stocks' returns. Hence, the market model for the event window contains the effect variable within and the model fit in Equation (1) for the estimation window remains as it is whereas the return generating process for the event window will carry the event day effect in it:

$$r_{it} = \alpha_i + \tau_{it} + \beta_i r_{mt} + \varepsilon_{it} \quad s.t. \ t_{\mathrm{b}} \leq t \leq T \qquad (6)$$

Equation 6 represents the normal returns inside the event window, these normal returns are subtracted from the actual returns as we have depicted in equation 5. We can argue that if the event carries a special effect in itself then τ_{it} is the real effect of the day inside the event window. Theoretically, the normal returns outside the event window should be equal to the actual returns, such that there will be no abnormal returns in the absence of the event.

We can derive now the abnormal returns by substituting Equation 6 into Equation 5, s.t:

$$AR_{it} = \alpha_i + \tau_{it} + \beta_i r_{mt} + \varepsilon_{it} - (\alpha_i + \beta_i r_{mt}) = \tau_{it} + \varepsilon_{it} \qquad (7)$$

Then the true effect of the event can be estimated to be:

$$E(AR_{it}) = \tau_{it} \quad ; \quad Var(AR_{it}) = \frac{1}{L_E}\sum_{t\in[t_N,t_n]}(AR_{it})^2 \qquad (8)$$

The variance in Equation 8 is calculated by using the data of abnormal returns in the estimation window, the number of abnormal returns is equal to the number of days inside the estimation window which in Equation 8 is denoted by L_E.

Equation 8 can also serve to see how the traditional two-step residual estimation model can be interpreted as the MRM with dummies which is illustrated in Equation 3, essentially, we can simply estimate Equation 1 over the whole period length from t_e to T in Figure 5.1 and use dummy variables for every day in the event window, we will demonstrate this in the following sub-section pertaining to the MRM analysis.

In order to make meaningful statistical inference across sections and firms we need to define the cumulative abnormal returns (CAR), these abnormal returns are simply aggregations and cumulations across firms and time.

There are several ways of calculating cumulative abnormal returns. One type of cumulative abnormal returns are the simple event window cumulative abnormal returns defined as:

$$CAR_i = \sum_{t=t_b}^{t=T} AR_{it} \quad ; \quad Var(CAR_i) = Var(AR_{it})\left(L_w + \frac{{L_w}^2}{L_E} + \frac{\sum_{t\in[t_b,T]} r_{mt} - L_W(\bar{r}_m)^2}{\sum_{t\in[t_N,t_n]}(r_{mt} - \bar{r}_m)^2}\right) \qquad (10)$$

The second type of cumulative abnormal returns are the daily cumulative abnormal returns that we use to see the movement of the cumulative abnormal returns at every single consecutive day inside the event window by first cumulating the abnormal returns as defined through Equation 5 to Equation 8 of every single firm separately for each day within the event window and then cross-sectionally (across N firms) averaging the cumulative abnormal returns for the particular day inside the event window.

We proceed to define the daily cumulative abnormal returns and the average cross sectional cumulative abnormal returns as follows:

$$DCAR_{it} = \sum_{t=t-n}^{t=t} AR_{it} \;\; s.t.\; 0 \le n \le 11 \; and \; t_b \le t \le T, \qquad \overline{DCAR}_t = \frac{1}{N}\sum_{i=1}^{N} DCAR_{it} \qquad (11)$$

Cumulative abnormal returns in Equation 10 are calculated for every i firm in the sample of firms whereas the variance is the scaled AR_i variance for equity i and L_W and

L_E in the variance of CAR_i is the number of days in the event window and estimation window, respectively, the variance of CAR_i is adjusted for the serial correlation in the returns as argued by (Mikkelson & Partch, 1988).

The abnormal returns we have defined in this section will be used for the derivation of two types of standardized abnormal returns for the use of hypothesis testing in Section 5.5 of this thesis.

5.4.2 Abnormal Returns from the MRM Analysis

The multivariate regression model (MRM) takes the form of the market model from Equation 1 with added dummy variables which take the value of one in the case of the event day and zero otherwise.

We will demonstrate how abnormal returns are derived from the MRM form by using a single security case, which can be also applied to the whole system of equations.

The estimated dummy variables in the MRM setting do not affect β neither α, the difference between two slopes of the regression is the dummy coefficient γ_{it}, which captures the effect of a particular day in the event window on the returns of the stock.

Abnormal returns are defined as the difference between the normal returns defined by the market model through the event window, and actual returns calculated through the estimation window of length L_E.

We can derive the abnormal returns as follows:

$$AR_{it} = (\alpha_i + \gamma_{it} D_{it}) + \beta_i r_{mt} + \varepsilon_{it} - (\alpha_i + \beta_i r_{mt} + \varepsilon_{it}) = \gamma_{it} D_{it} \quad s.t. \; D_{it} = 1 \; if \; t_b \leq t \leq T \quad (13)$$

For a more detailed explanation on indicator-variable regressions one can refer to (Fox, 2016), where a whole chapter expands on the use of dummy-variable regression along with graphical illustration of the regression slopes.

The cumulative abnormal returns are defined as the sum of the estimated dummy coefficients throughout the event window and across firms (Karafiath, 1988) whereas the average abnormal returns are as well the sum of abnormal returns in the event window divided by the number of event days , it is also important to note that upon testing null hypothesis in the MRM setting there is no need for the standardization of the abnormal returns since the hypothesis tests account for cross sectional heteroskedasticity and autocorrelation of the error terms.

5.5 Null hypotheses and test statistics for significance

As the choice of test statistics is very important in every empirical work, we will be using a combination of different types of test statistics to account for different problems in the data. It is well established that usually stock returns are not normally distributed, also the problem of serial and cross correlation can be remedied by the choice of the test statistic.

5.5.1 Parametric Tests

The test statistics for significance involve the calculation of abnormal returns. These abnormal returns are simply the standardized form of the abnormal return, we have derived in Equations 5, 7 and 8.

The standardized abnormal returns take into account the variance of the abnormal returns as follows:

$$SAR_{it} = \frac{AR_{it}}{\sqrt{Var(AR_{it})}} \quad ;$$

$$Var(AR_{it}) = Var(AR_i)\left(1 + \frac{1}{L_E} + \frac{(r_{mt} - \bar{r}_m)^2}{\sum_{t_N}^{t_n}(r_{mt} - \bar{r}_m)^2}\right) \qquad (16)$$

$\boldsymbol{Var(AR_i)}$ is the variance of abnormal returns from the regression over the estimation window returns for firm i from Equation (8), L_E is the number of abnormal returns in the estimation window as calculated in Section 5.2, $\bar{r}_m$ is the average of the market returns in the estimation window (L_E). The variance in Equation (16) is adjusted for the market variance in the estimation window brought about by different exogenous factors, the standardized abnormal returns ($\boldsymbol{SAR_{it}}$) are adjusted for the out of sample estimation since the abnormal returns are estimated out of sample, a more detailed derivation and justification for this has been done by (Patell, 1976).

Equation (16.1) below defines the Cumulative Standardized abnormal returns where L_w stands for the event window length or the number of days in the event window as we have previously defined in Section 2.5.

$$CSAR_i = \frac{1}{\sqrt{L_W}} \sum_{t=t_b}^{t=T} SAR_{it} \quad ; \quad Var(CSAR_i) = N\frac{L_E - 2}{L_E - 4} \qquad (16.1)$$

5.5.1.1. Adjusted Patell Test statistic

The derivation of the Adjusted Patell Statistic (APS) has been derived by (Kolari & Pynnönen, 2010) which is in turn based on an older statistic developed to make inference on share earnings effect on stock prices by (Patell, 1976) where the author uses the standard market return model which we have utilized in this thesis as well as described in Equation (1). The Patell Test statistic (PS) is defined as:

$$T_{PS} = \sum_{i=1}^{N} \frac{CSAR_i}{\sqrt{Var(CSAR_i)}} \qquad (16.2)$$

In a setting of a system of equations where the effect of the event is being evaluated across firms and time, the APS test can serve as a useful tool where unlike the PS test which can be vulnerable to cross sectional correlation and result into rejection of the null hypothesis, the APS accounts for the cross correlation in abnormal returns.

The APS statistics is defined as follows:

$$T_{APS} = T_{PS}\sqrt{\frac{1}{1+(N-1)\bar{\rho}}} \qquad (17)$$

$\bar{\rho}$ in Equation (17) is the cross-sectional mean correlation as shown in Equation (17.3) of the estimation window (L_E) abnormal returns, in case of no correlation, as it can be seen from the formula the APS is simply the Patell S.

The average correlation of the sample abnormal returns is calculated in three steps. The first step involves the calculation of the Fisher's Z transformation as derived by (Fisher, 1921), the second step involves finding the average Fisher Z-transformation and the final third step is calculating the inverse of the average Fisher Z-Transformation. For a more detailed derivation of the Fisher's Z transformation properties one can also consult the work of (Hotelling, 1953).

The Fisher Z-transformation of the correlation coefficient (ρ_{ij}) of the abnormal returns of stock i and j is defined as:

$$z_{ij} = \frac{1}{2}\, ln\left(\frac{1+\rho_{ij}}{1-\rho_{ij}}\right) \qquad (17.1)$$

The Inverse Fisher Z-Transformation:

$$\rho_{ij} = \left(\frac{e^{2z_{ij}} - 1}{e^{2z_{ij}} + 1}\right) \quad (17.2) \qquad ; \qquad \bar{\rho} = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{i \neq j} \rho_{ij} \quad (17.3)$$

ρ_{ij} is the inverse Fisher Z transformation of the correlation coefficient of the abnormal returns of stock i and j.

The null Hypothesis tested is: $\mathbf{H_0^1}$: CSAR = 0 vs. $\mathbf{H_A^1}$: CSAR ≠ 0

The hypothesis here states that the cumulative standardized abnormal returns across the sample are zero. The standard deviation of the CSAR is shown as in Equation (16.1), the variance for CSAR is the scaled variance of the t statistics/SARs in the event window (Patell, 1976, p. 256) and (Kolari & Pynnönen, 2010, pp. 3999-4000), and the scaled CSAR have a unit variance.

The Patell test statistic in Equation c.1 in appendix F, sub-section (I.iii) as argued by (Kolari & Pynnönen, 2010, p. 4004) should not have an approximate N(0,1) distribution because of the cross correlation of abnormal returns, the distribution of that rest has a slightly greater variance N(0,1+(n-1) $\bar{\boldsymbol{\rho}}$).

To fix the deviation of the abnormal returns at a unit deviation, we standardize them and the adjusted Patell ($\boldsymbol{T_{APS}}$) has a distribution of N(0,1) as a result of the standardization.

5.5.1.2. Adjusted Standardized Cross-Sectional statistic

The Adjusted Standardized Cross-Sectional statistic (ACS) is a more refined form of the standardized cross-sectional statistic (SCS) explained in Appendix F, Equation (d.1).

The standardized cross-sectional statistic in Equation 17.4, is derived in such a way that the standard deviation in the denominator is defined as the sample (cross section) deviation of the standardized cumulative abnormal returns.

$$T_{CS} = \frac{\frac{1}{N}\sum_{i=1}^{N} SCAR_i}{\sqrt{\frac{\frac{1}{N-1}\sum_{i=1}^{N}(SCAR_i - \overline{SCAR})^2}{1-\bar{\rho}}}} \qquad (17.4)$$

Since the variance of the standardized abnormal returns is biased, the authors (Kolari & Pynnönen, 2010, pp. 4002-4003) correct for the correlation by including the average correlation of the estimation window (L_E) abnormal returns (AR_{it}).

The adjusted standardized cross-sectional statistics uses the standardized cumulative abnormal returns and their mean to test for the event day effect on the mean standardized cumulative returns which are defined as:

$$SCAR_i = \frac{CAR_i}{Var(CAR)_i} \quad ; \quad \overline{SCAR} = \frac{1}{N}\sum_{i=1}^{N} SCAR_i \quad ;$$

$$Var(SCAR) = \frac{1}{N-1}\sum_{i=1}^{N}(SCAR_i - \overline{SCAR})^2 \qquad (18)$$

The variance of cumulative abnormal returns, i.e., $Var(CAR)_i$, is defined in Equation 10.

The null Hypothesis tested in this case is $\mathbf{H_0^2}$: $\overline{\boldsymbol{SCAR}} = 0 \wedge \mathbf{H_A^2}$: $\overline{\boldsymbol{SCAR}} \neq 0$

The hypothesis states that the mean standardized cumulative abnormal returns are zero, the standardized cumulative abnormal returns would not have a mean of zero across the firm sample if the event had any real impact on the prices and returns of the firms' equities.

The ACS test accounts for the cross-sectional correlation ($\bar{\boldsymbol{\rho}}$) of the errors estimated through the estimation window for every firm. The estimated correlations then are aggregated from each firm and the average correlation of errors is calculated for all firms across the sample, a step-by-step explanation of the test is given by (Kolari & Pynnönen, 2010, pp. 3999 - 4001).

The ACS test is also standard normally distributed just like the Adjusted Patell Statistic.

$$T_{ACS} = T_{CS}\sqrt{\frac{1}{1+(N-1)\bar{\rho}}} \qquad (18.1)$$

5.5.2. The Non-Parametric GRANK – Z Test

This test is a non-parametric test which does not depend on the assumed distribution of the abnormal returns. Non-parametric tests are usually used when the distribution of the test has fat tails as it is usually the case when stock returns tend to be heavy tailed.

The Generalized Rank Test is a modified version of the rank test developed by (Corrado & Zivney, 1992). The modification of the rank test accounts for the cross-sectional correlation of returns, serial correlation and event induced volatility, the details of this test have been explored by (Kolari & Pynnönen, 2011).

The first step of using this non-parametric test is estimating the market model in Equation 1 and obtaining the estimates (i.e., α *and* β) in Equation 2.

The second step involves the calculation of abnormal returns as defined in Equation 5 for the whole period of available returns, this means that abnormal returns need to be calculated for both, the estimation window (L_E) and the event window (L_W).

The third step involves the calculation of standard abnormal returns (SAR) defined in Equation 16 and calculating the cumulative abnormal returns (CAR) within the event window as defined in Equation 10.

The fourth step is defining the standard cumulative abnormal returns (SCAR) as in equation 18 by dividing the cumulative returns (CAR) by its variance, the variance in this case is calculated as in (Campbell, Lo, & MacKinlay, 1997, pp. 160-161) which is also shown in Equation 10.

The fifth step in this procedure involves the standardization of the standard cumulative abnormal returns (SCAR) across the firm sample, this is done by using the cross-sectional variance of the standardized cumulative abnormal returns which has been defined in Equation 18. We use the same notation to define the cross sectional standardized cumulative returns as in (Kolari & Pynnönen, 2011, p. 955):

$$SCAR_i^* = \frac{SCAR_i}{Var(SCAR)_i} \quad (19.1)$$

The sixth step requires the definition of generalized standard abnormal returns (GSAR) as defined in (Kolari & Pynnönen, 2011, p. 955):

$$GSAR_{it} = \begin{cases} SCAR_i^* = \dfrac{SCAR_i}{Var(SCAR)} \; for\ t\ in\ the\ event\ window\ L_w \\ SAR_{it} = \dfrac{AR_{it}}{Var(AR_{it})} \; for\ t\ in\ the\ estimation\ window\ and\ event\ window\ L_E +\ L_w \end{cases} \tag{19.2}$$

The calculation of the cumulative returns within the event window is done for the purpose of considering the whole duration of the event as one point in time by adding the daily abnormal returns, this allows us to calculate the standardized cumulative returns within the event window and then re-standardize these same returns ($SCAR_i^*$) to account for the cross-sectional variance. This re-standardized return used in Equation (19.2) of the generalized standardized returns give us a single number for the whole cross section of standardized cumulative abnormal returns that in itself carries the effect of the event day on all firms in comparison to the $SCAR_i$ defined in formula 18 which carries in itself the effect of the firm event specific to the i-th firm.

We will use the same notation for the rank of abnormal returns as in (Kolari & Pynnönen, 2011). The event window of day lengths L_A, L_B and L_W are condensed into single cumulative event days since the calculation of the GSAR involves the cumulative abnormal returns (CAR). The cumulative event day can be considered as one observation per firm, the standardized abnormal ranks of the generalized standardized abnormal returns (GSAR) are defined as:

$$U_{it} = \frac{\text{rank}(GSAR_{it})}{L_E+2} - \frac{1}{2} \tag{20}$$

There is an extensive literature that deals with rank-order statistics and nonparametric statistical inference that dates back to (Siotani, 1956) where the author discusses the distribution of the largest and smallest values of a discrete random variable, the joint distribution of the largest and smallest values of the variable, the mean, variance of those values and an application of the order statistics for the binomial case. A more recent literature from (Gibbons & Subhabrata, 2014), (David & Nagaraja, 2004) and (Lehmann & D'Abrera, 1975) explores order-rank statistics issues and procedures in details.

The returns are ranked through the whole time period from t_N to T as depicted in Figure 5.1. If every daily return was different the highest rank would be 149 which is the length of the estimation and event window combined. In the case of two equal daily returns the mid-range is taken into account.

The theory of rank tests is based on a key assumption that the observed random variables follow certain continuous distribution functions, the implication of the continuous distribution of a random variable is that the probability of two outcomes having the same value is equal to zero. Another implicit assumption made about the observed random variable is that if a random variable follows a density function the rankings of the outcomes are complete, which in reality does not always hold true.

If two or more daily prices are equal in value, they make a set of groups and hence in this case the ranks cannot be well defined. A more detailed discussion in the treatment of ties can be found in the (Pratt & Gibbons, 2012) and (Sidak, Sen, & Hajek, 1999).

However, this assumption might fall short in practice where the outcome of two or more observations with the same value is quite possible. Taking into account the fact that this thesis deals with the analysis of the market and stock returns, the possibility of two equal consecutive prices is quite likely in situations such as the daily stock prices of thinly traded stocks of the small cap German index. Two different day stock prices might be the result of the fact that stock prices do not move in a continuous way. The stock price movement is limited to the tick size which is the minimum price difference that can exists between two consecutive bids which is in turn defined by the stock exchange.

$$\bar{U}_t = \frac{1}{N}\sum_{i=1}^{N} U_{it} \qquad (20.1)$$

The null hypothesis for this test is that there is no mean event effect. This is done by first accumulating the ranked returns through the event window for every firm and we get the cumulative ranked abnormal returns. We define the null hypothesis as in (Kolari & Pynnönen, 2011, p. 956)

$$\mathrm{H}_0^3\text{: } \mathrm{E}(\bar{U}_0) = 0 \wedge \mathrm{H}_\mathrm{A}^3\text{: } \mathrm{E}(\bar{U}_0) \neq 0$$

The null hypothesis here states that the mean cross-section demeaned generalized abnormal ranks of the generalized standardized abnormal returns are equal to zero, this hypothesis will be rejected if the demeaned ranked generalized abnormal returns for the event window are different from zero.

$$\mathrm{Z}_{\mathrm{GRANK}} = \frac{\mathrm{E}(\bar{U}_0)}{\sqrt{\mathrm{Var}(\bar{U}_0)}} = \sqrt{\frac{12\mathrm{N}(\mathrm{L_T}+1)}{\mathrm{L_T}-1}}\,\mathrm{E}(\bar{U}_0) \sim \mathrm{N}(0,1) \qquad (20.2)$$

The derivation of Formula 20.2 by (Kolari & Pynnönen, 2011, p. 956) is based on the fact that the variance of the ranks (i.e. rank($GSAR_{it}$)) is $\frac{L_T^2-1}{12}$. Hence the variance of the standardized ranks in Equation 20 is:

$$Var(U_{it}) = \frac{L_T - 1}{12(L_T + 1)} \qquad (20.3)$$

The Z statistic of the generalized ranked returns is assumed to have a standard normal distribution after the standardizations of the abnormal returns we have done above, as the number of days in the estimation window approaches infinity or grows large the $\mathbf{Z_{GRANK}}$ approaches the standard normal distribution.

The variance of the average standardized abnormal ranks as defined in Equation 20.1 is:

$$Var(\bar{U}_{it}) = \frac{L_T-1}{12N(L_T+1)} \qquad (20.4)$$

N is the number of firms standardized generalized ranked returns as defined in Equation 20 and L_T is the length of the returns times series as defined in Section 5.2.

5.6 Joint Hypothesis testing and restrictions on the MRM

In the setting of the multivariate regression model, we use the F-test (Greene W. H., 2018) which has the advantage of incorporating heteroskedasticity and error correlation itself.

The F test is a statistic of comparing two types of variances, since the F test involves testing the original model with a restricted model, it effectively addresses heteroskedasticity by carefully choosing the right null hypothesis. The F test has an F distribution under the null hypothesis with the number of degrees of freedom being the number of restrictions denoted by J and the number of equations denoted by N multiplied by the number of observations denoted by T minus the total number of estimated coefficients denoted by K , i.e., F ~ F (J, N·T-K).

The Joint Test Hypothesis starts with the linear Equation 4.2 in Sub-Section 5.3.2 where linear restrictions across equations take the following form:

$$r_{11}\beta_1 + r_{12}\beta_2 + \cdots + r_{1K}\beta_K = q_1$$

$$r_{21}\beta_1 + r_{22}\beta_2 + \cdots + r_{2K}\beta_K = q_2$$

$$\downarrow$$

$$r_{J1}\beta_1 + r_{J2}\beta_2 + \cdots + r_{JK}\beta_K = q_J$$

The above can be written in a Matrix and vector form as:

$$R\beta \quad = q \qquad (20.5)$$

Every single row in the matrix R is a JxK matrix, the vector β has Kx1 dimensions and q has Jx1 dimensions.

F-Test:

$$F \;= \frac{\frac{\left(R\hat{\beta}-q\right)'\left(R\left(X'(\hat{\Sigma}\otimes I)^{-1}X\right)^{-1}R'\right)^{-1}\left(R\hat{\beta}-q\right)}{J}}{\frac{\hat{\varepsilon}'(\hat{\Sigma}\otimes I)^{-1}\hat{\varepsilon}}{N\cdot T-K}} \sim \mathrm{F}\,(\mathrm{J}, N\cdot \mathrm{T}-\mathrm{K}) \qquad (21)$$

The capital letter R is the restriction matrix, q is a vector of zeros which is included for the sake of notational convenience since the null hypothesis does not always constrain the cross-equation coefficients to zero. The vector $\boldsymbol{\beta}$ represents K estimated coefficients, J is the number of restrictions and N is the number of equations. The notation is the same as in Equation 4.3 and Equation f in Appendix F, II.i.

The restriction in Equation 20.5 imposes J restrictions on K parameters for N different equations in the system of equations where each firms' returns are represented by Equation 3.

The F test is designed to test whether two population variances are equal. The formula in Equation 21 can be further simplified by simply writing the numerator of Equation 21 in terms of the restricted and unrestricted residuals of the system of Equation 3.

$$F = \frac{\frac{\varepsilon_{RS}'\varepsilon_{RS} - \varepsilon'\varepsilon}{J}}{\frac{\varepsilon'\varepsilon}{N\cdot T-K}} = \frac{\frac{SSE(\beta^{RS}) - SSE(\hat{\beta})}{J}}{\frac{SSE(\hat{\beta})}{N\cdot T-K}} \sim \mathrm{F}\,(\mathrm{J}, N\cdot \mathrm{T}-\mathrm{K}) \qquad (21.1)$$

The restricted residuals (ε_{RS}) are obtained by minimizing the sum of squared residuals with respect to the constraint imposed by the null hypothesis in Equation 20.5. The unrestricted residuals $\varepsilon'\varepsilon$ are simply obtained by minimizing the sum of squared residuals without the imposed constrained from Equation 20.5. In the setting of multiple regression equations as defined in Equation 3, the F test compares the variance of the restricted and unrestricted system of regression equations.

A more detailed derivation of Equation 21 and Equation 21.1 can be found in (Greene W. H., 2017) along with a practical example of the analysis of the production cost functions of over 145 American electricity generating companies. Another detailed derivation of Equations 21 and 21.1 can also be found in (Hallam, 2004 a) and (Hallam, 2004 b).

5.6.1 Null Hypothesis for the Multivariate Regression Model

H_0^4: The sum of the dummy coefficients is zero across the sample of N equations and over the event window, this represents the joint cumulative abnormal returns (CAR).

$$\sum_{i=1}^{N}\sum_{t=T_2}^{T_3}\gamma_{it} = 0$$

The null hypothesis H_0^4 is similar to the null hypothesis H_0^1, the difference between H_0^4 hypothesis and hypothesis H_0^1 is in the cumulation of the standardized abnormal returns for the calculation of the cumulative standardized abnormal returns (CSAR) in H_0^1. The second difference is in the choice of the test statistics, we use the F-Test to test for significance in the case of hypothesis H_0^4 whereas we employ the adjusted Patell-Test statistic to test for significance in the cumulative standardized abnormal returns in hypothesis H_0^1.

H_0^5: The dummy coefficients/abnormal returns are jointly equal to zero across firms and event window.

$$\gamma_{1,1} = \gamma_{1,2} = \gamma_{1,3} = \ldots\ldots = \gamma_{N,T_3} = 0$$

H_0^6: Abnormal returns are all equal to each other, this hypothesis tests whether the event has an equal/same effect across all firms through the event window.

$$\gamma_{1,1} = \gamma_{1,2} = \gamma_{1,3} = \ldots\ldots = \gamma_{N,T_3}$$

The mathematical representation of these null hypothesis can be found in Appendix F, sub-section (II.iii).

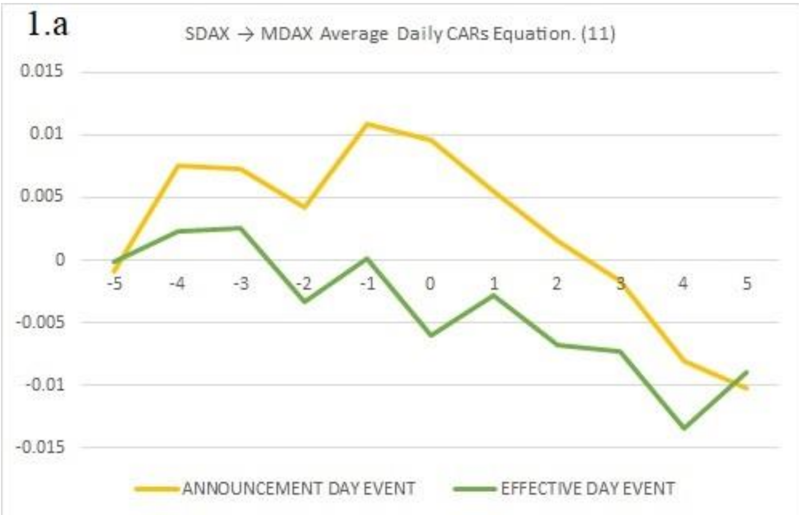
6. Results

In this section we present the results for all four subsamples. We do so by constructing two types of tables for every subsample. We use one table to represent the results from the univariate market model (see Section 5.3, Equation 1) and, we use another table to present the results from the multivariate regression model (see Section 5.3, Equation 3).

We create graphical representations of the average cross-sectional cumulative abnormal returns as defined in Equation (11) to show the movement of cross-sectional average cumulative abnormal returns of every sample around the effective and announcement event windows.

6.1. Promotion (inclusion), SDAX to MDAX

6.1.1 SDAX to MDAX univariate model results



Cross-sectional average cumulative abnormal returns as defined in Equation (11) for the period five days before and after the announcement event day (yellow line) and effective event day (green line) of stocks that are promoted into the MDAX from the smaller SDAX index.

There is a slight increase in average cumulative abnormal returns (CAR) from the fifth day before the effective event up to the third day before the effective event for stocks that moved from the small-cap German index into the mid-cap German index. The green curve in Graph 1.a. depicting the effective event day shows a steady decline in the average cumulative abnormal returns from the third day before the effective event up to the last day after the effective event. This type of negative behavior is at odds with findings from the US stock market. Lynch & Mendenhall, 1997 show that the inclusion into the S&P500 has a positive effect on a stock's returns.

The same positive effect of an inclusion into the S&P500 is also observed by (Chen, Noronha, & Singal, 2004).

The announcement event for the subsample of stocks promoted into the MDAX from the SDAX in Graph 1.a. has a different impact on the cumulative abnormal returns as opposed to the effective event, stocks moving into the mid-cap index show positive CARs in the anticipation period of five days before the announcement event, and a reversal (decline) in CARs after the event day marked with the number zero. The positive CARs in the anticipation period of five days before the event up to the event day zero which is in line with the anticipating-investors hypothesis as reported also by (Doeswijk, 2005). The stock price reaction after the announcement event day zero is negative which is also in line with the price pressure hypothesis of (Gurel & Harris, 1986).

Table 6.1.a: SDAX to MDAX subsample null hypothesis test results for the univariate market model.

Ann. Event	N	CSAR	AP. Z-score	p-value	$\overline{SCAR}$	AC. Z-Score	p-value	$\overline{U}_0$	GR. Z-score	p-value
L_W	59	-0.617	-0.076	0.939	-0.006	-0.041	0.967	0.0372	0.9979	0.3183
Sign.				x			x			x
L_A	59	-13.884	-1.720	0.086	-0.229	-1.490	0.136	-0.0150	-0.4023	0.6874
Sign.				*			x			x
L_B	59	16.408	2.032	0.042	0.274	2.058	0.040	0.1168	3.1318	0.0017
Sign.				**			**			***
Eff. Event	**N**	**CSAR**	**Pat. Z-score**	**p-value**	**$\overline{SCAR}$**	**BMP. Z-Score**	**p-value**	**$\overline{U}_0$**	**Grank Z-score**	**p-value**
L_W	61	-4.234	-0.498	0.618	-0.070	-0.458	0.323	-0.00747	-0.2036	0.8387
Sign.				x			x			x
L_A	61	-5.342	-0.629	0.530	-0.089	-0.788	0.431	-0.03361	-0.9158	0.3598
Sign.				x			x			x
L_B	61	-6.871	-0.808	0.419	-0.113	-0.699	0.485	-0.01354	-0.3691	0.7121
Sign.				x			x			x

Table 6.1.a. exhibits the results for stocks that move from the SDAX to the MDAX for the null hypothesis tests from Section 5.5 concerning the traditional univariate market return model, see Equation (1) in Section 5.3. The abbreviation Ann. Event and Eff. Event stands for Announcement Day Event and Effective Day Event.

x – no significance, * - significance at the 10% level, ** - significance at the 5% level and *** - significance at the 1% level

Column number one of Table 6.1.a. shows the event lengths for the two events, i.e., the announcement event (Ann. Event) and the effective event (Eff. Event), for which the univariate market model method is used.

The second column marked by N is the number of firms in the sample, the third, sixth and ninth columns show respectively the cumulative standardized abnormal returns (CSAR) as defined in Equation 16.1, the average standardized cumulative abnormal returns ($\overline{SCAR}$) as defined in Equation 18 and the average standardized abnormal ranks ($\overline{U}_0$) of the generalized standardized abnormal returns (GSAR) during the event period defined in Equation 19.2 and Equation 20.

Columns four, seven and ten show the Z-scores of the Adjusted-Patell test (AP. Z-score) defined in Equation 17, the Adjusted Standardized Cross-Sectional statistic (AB. Z-score) as defined in Equation 18.1 and the Generalized-Rank test (GR. Z-score) as defined in Equation 20.2. Columns number five, eight and eleven represent the p-values correspondingly.

Rows marked with the abbreviation Sign. stand for the significance of the corresponding Z-score in the rows above them.

Our results for all three measure types of abnormal returns show no significant deviations from zero on the effective event day, i.e., the day of promotion or the day of inclusion into the MDAX of a stock, the cross section cumulative standardized abnormal returns (CSAR), the cross section average standardized cumulative abnormal returns ($\overline{SCAR}$) and the demeaned standardized abnormal ranks are all negative, however, the results are insignificant for all event window lengths.

The announcement day event shows significant positive cumulative standardized abnormal returns (CSAR) of 16.4 and significant standardized cumulative abnormal returns (SCAR) of 0.27 for the anticipation window of five days before the event (L_B), both significant at the 5% level. The average demeaned standardized abnormal ranks across the sample of 0,11 is highly significant at the 1% significance level.

The post event window period (L_A), also shows significant negative CSAR at the 10% confidence interval, along with some almost significant ($\overline{SCAR}$) whose p-value is very close to the 10% confidence interval.

The univariate market model appears to support the anticipating-investors hypothesis which is in line with findings by (Doeswijk, 2005), (Fernandes & Mergulhão, 2016) and also in line with previous findings on the DAX index family done by (Franz, 2020).

We also find some support regarding the price pressure hypothesis after looking first at the reaction of cumulative abnormal returns, CSAR, $\overline{SCAR}$ and the average demeaned abnormal ranks before the anouncement event (L_B) where the CARs are positive and a price reversal for the post announcement event window (L_A) where the CARs, CSAR and $\overline{SCAR}$ are negative.

6.1.2. SDAX to MDAX multivariate model results

The second part of the results is utilizing the F statistic of the Wald test to test for significance in the dummy coefficients (see section 5.5.3.).

Table 6.1.b. illustrates the significance of the dummy coefficients by performing an F test on the model after we applied the necessary restrictions.

Table 6.1.b. is divided into two parts; the first upper part depicts the results of the F test values and their corresponding p-values obtained from the OLS regression and, the second part depicts the F values of the F test statistic from the seemingly unrelated regression method (see section 5.3).

The first row of the table shows the three different null hypothesis we constructed (see Section 5.6.1), the null hypotheses on the first row are applied to both event types and all the corresponding event window lengths depicted in column one of the table 6.1.a.

Table 6.1.b: SDAX to MDAX subsample null hypothesis test results for the multivariate market model

	$H_0^4: \sum_{i=1}^{N}\sum_{t=T_2}^{T_3} \gamma_{it} = 0$		$H_0^5: \gamma_{1,1} = \ldots = \gamma_{N,T_N} = 0$		$H_0^6: \gamma_{1,1} = \ldots = \gamma_{N,T_N}$	
			OLS - Method			
Ann. Event	F-value	p-value	F-value	p-value	F-value	p-value
L_W	0,0491	0,824 x	1,1636	0,0034 ***	1,141	0,009 ***
L_A	3,075	0,079 *	1,53	0,001 ***	1,52	0,001 ***
L_B	5,37	0,02 **	0,83	0,98 x	0,79	0,99 x
Eff. Event	F-value	p-value	F-value	p-value	F-value	p-value
L_W	1,84	0,17 x	1,24	0,001 ***	1,23	0,001 ***
L_A	1,22	0,26 x	1,113	0,073 *	1,1	0,097 *
L_B	0,005	0,93 x	1,35	0,001 ***	1,36	0,001 ***
			SUR - Method			
Ann. Event	F-value	p-value	F-value	p-value	F-value	p-value
L_W	0.0755	0.7836 x	2,210	0,001 ***	2,165	0,001 ***
L_A	2,779	0,095 *	2,82	0,001 ***	2,81	0,001 ***
L_B	4,68	0,03 **	1,55	0,001***	1,52	0,001***
Eff. Event	F-value	p-value	F-value	p-value	F-value	p-value
L_W	1,62	0,2 x	2,25	0,001 ***	2,24	0,001 ***
L_A	1,15	0,28 x	1,99	0,001 ***	1,95	0,001 ***
L_B	0,007	0,93 x	2,23	0,001 ***	2,25	0,001 ***

Table 6.1.b. exhibits the results of the F test statistic as defined in Equation (21) and Equation (21.1) for the OLS and SURE regression methods. Results on the table are shown for all three window lengths and for both event types (i.e., the announcement event and the effective event). x – no significance, * - significance at the 10% level, ** - significance at the 5% level and *** - significance at the 1% level

The results from the multivariate regression method are not so different from the results we obtained by running the traditional univariate market model.

Hypothesis four (H_0^4) implies that the sum of the dummy coefficients across all firms is jointly equal to zero, this hypothesis is the equivalent to hypothesis H_0^1 that tests for significance in the cross-sectional cumulative standardized abnormal returns.

The F values are insignificant for all three efficient event window lengths for the null hypothesis (H_0^4) regardless of the method of estimation of the model (i.e., OLS and SURE), these results are in accordance with the previous results from the univariate market model regression as shown in Table 6.1.a, even though hypothesis (H_0^5) tells us that there are some significant abnormal returns, they cancel out in the forecast window (event window).

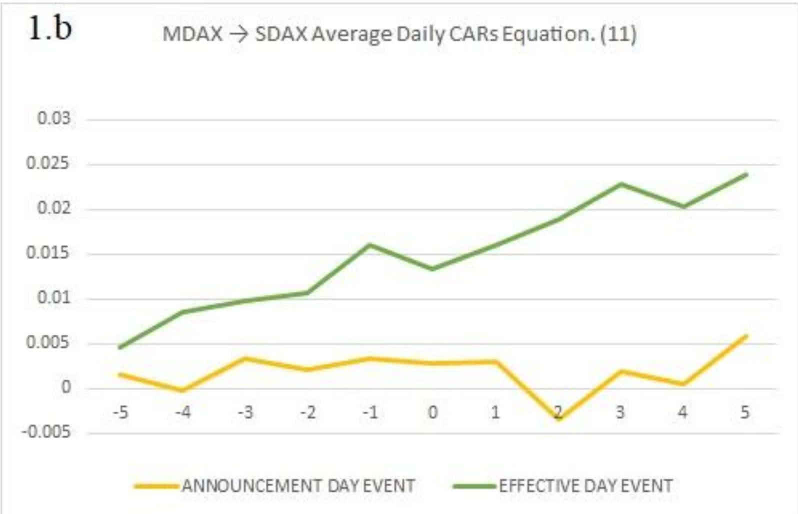
The announcement day event and the three window lengths associated with it are in accord with the previous results from the univariate regression method of the market model.

The anticipatory window (L_B) and the post event window (L_A) cumulative abnormal returns are significant at the 5% and 10% level accordingly, even though, hypothesis number five (H_0^5) is not rejected for the anticipatory event window, and jointly the abnormal returns are not significant from zero, the cumulative abnormal returns remain significant and do not cancel out.

6.2. Demotion (exclusion), MDAX to SDAX

6.2.1. MDAX to SDAX univariate model results

To get a better picture of the demotion of a stock from the mid-cap index (MDAX) into the small cap index (SDAX) we can take a look at Graph 1.b. The cross-sectional average cumulative abnormal returns as defined in Equation (11) and their movement around the effective and announcement event days are depicted by the green and yellow lines for both events respectively.



Cross-sectional average cumulative abnormal returns as defined in Equation (11) five days before and after the announcement event day and the effective event day for subsamples MDAX to SDAX. The vertical axis depicts the cross-sectional average cumulative abnormal returns, and the horizontal axis depicts the days around the event day labeled with the number zero.

Average cumulative abnormal returns for the anticipatory event window (L_B) are constant and positive, however, there is an upward (positive) movement of the average CARs 2 days after the event.

The demotion effect is stronger for the effective event day where we can see an upward movement of the average daily CARs throughout the whole event window.

Table 6.2.a: MDAX to SDAX subsample null hypothesis test results for the univariate market model.

Ann. Event	N	CSAR	AP. Z-score	p-value	$\overline{SCAR}$	AC. Z-Score	p-value	$\overline{U_0}$	GR. Z-score	p-value
L_W	79	6.707	0.708	0.479	0.081	0.756	0.450	0.058	1.796	0.072
Sign.				x			x			*
L_A	79	2.492	0.263	0.792	0.032	0.267	0.789	0.017	0.519	0.604
Sign.				x			x			x
L_B	79	3.397	0.359	0.720	0.041	0.447	0.655	0.019	0.598	0.550
Sign.				x			x			x
Eff. Event	N	CSAR	Pat. Z-score	p-value	$\overline{SCAR}$	BMP. Z-Score	p-value	$\overline{U_0}$	Grank Z-score	p-value
L_W	79	19.377	2.173	0.030	0.236	2.110	0.035	0.115	3.559	0.000
Sign.				**			**			***
L_A	79	4.730	0.531	0.596	0.055	0.515	0.607	0.048	1.492	0.136
Sign.				x			x			x
L_B	79	18.679	2.095	0.036	0.231	1.993	0.046	0.087	2.709	0.007
Sign.				**			**			***

Table 6.2.a. exhibits the results for stocks that move from the MDAX to the SDAX for the null hypothesis tests from Section 5.5 concerning the traditional univariate market return model, see equation (1) in Section 5.3. The abbreviation Ann. Event and Eff. Event stands for Announcement Day Event and Effective Day Event.

x – no significance, * - significance at the 10% level, ** - significance at the 5% level and *** - significance at the 1% level

The cumulative standardized abnormal returns (CSAR) defined in Equation 16.1, the average standardized cumulative abnormal returns ($\overline{SCAR}$) defined in Equation 18 and the average abnormal ranks ($\overline{\boldsymbol{U}}_{\mathbf{0}}$) defined in Equation 20.1 are all positive for the announcement event and for all window lengths. With the notable exception of the significant average abnormal ranks ($\overline{\boldsymbol{U}}_{\mathbf{0}}$) for the whole event window (L_W), the Z-scores of the cumulative standardized abnormal returns (CSAR), the average standardized cumulative abnormal returns ($\overline{SCAR}$) and the average standardized abnormal ranks for the anticipatory event window L_B and the after announcement event window L_A are all insignificant.

Overall, the announcement of the demotion (deletion) has no effect on the returns of the stocks. These results are similar to a study done by (Chen, Noronha, & Singal, 2004) related to the United States S&P500 index changes.

The effective event in our case has a positive significant impact (i.e., the null hypothesis number one, two and three from section 5.5.1 are rejected) on the CSAR, $\overline{SCAR}$ and $\overline{U}_0$ especially, for the pre inclusion window (L_B) and the combined length of the pre-inclusion and after inclusion window (L_W). These results are in odds with (Lynch & Mendenhall, 1997), (Chen, Noronha, & Singal, 2004) and a study done on the S&P SmallCap 600 index of the US market by (Shankar & Miller, 2006), however our results are in agreement with the conclusions from the MDAX index study conducted by (Steiner & Heinke, 1997).

Another study run by (Franz, 2020) concludes that there are positive post announcement day abnormal returns for the MDAX demotions, however those results are statistically insignificant, a similar conclusion is derived by (Vainikka, 2021) which shows that the effect of deletion from the DAX shows contradictory stock return effect to the studies conducted in previous literature from the US stock market indices.

6.2.2. MDAX to SDAX multivariate model results

Table 6.2.b: MDAX to SDAX subsample null hypothesis test results for the multivariate market model.

	$H_0^4: \sum_{i=1}^{N}\sum_{t=T_2}^{T_3} \gamma_{it} = 0$		$H_0^5: \gamma_{1,1} = \ldots = \gamma_{N,T_N} = 0$		$H_0^6: \gamma_{1,1} = \ldots = \gamma_{N,T_N}$	
OLS - Method						
Ann. Event	F-value	p-value	F-value	p-value	F-value	p-value
L_W	0,054	0,8163 x	0,9354	0,9051 x	0,9312	0,9182 x
L_A	< 0,0001	0,99 x	0,9738	0,6476 x	0,9674	0,6822 x
L_B	0,46	0,5 x	0,83	0,99 x	0,83	0,99 x
Eff. Event	F-value	p-value	F-value	p-value	F-value	p-value
L_W	2,42	0,12 x	1,27	0,001 ***	1,265	0,001 ***
L_A	1,17	0,28 x	1,06	0,18 x	1,064	0,17 x
L_B	2,987	0,084 *	1,457	0,001 ***	1,45	0,001 ***
SUR - Method						
Ann. Event	F-value	p-value	F-value	p-value	F-value	p-value
L_W	0,0691	0,79 x	2,527	0,001 ***	2,4797	0,001 ***
L_A	0,0018	0,966 x	2,4827	0,001 ***	2,4173	0,001 ***
L_B	0,45	0,5 x	1,87	0,001***	1,85	0,001 ***
Eff. Event	F-value	p-value	F-value	p-value	F-value	p-value
L_W	1,84	0,17 x	3,43	0,001 ***	3,37	0,001 ***
L_A	1,27	0,26 x	2,677	0,001 ***	2,6	0,001 ***
L_B	2,17	0,14 x	4,036	0,001 ***	3,99	0,001 ***

Table 6.2.b. exhibits the results of the F test statistic for the OLS and SURE regression methods, results on the table are shown for all three window lengths and for both event types (i.e., announcement event and effective event): x – no significance, * - significance at the 10% level, ** - significance at the 5% level and *** - significance at the 1% level

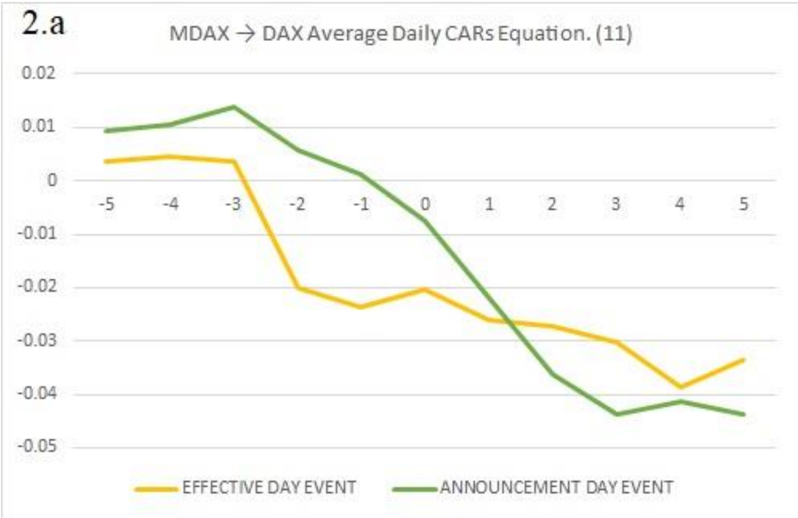
The announcement of the exclusion from the MDAX does not have a significant effect on the abnormal returns. Hypothesis number four (H_0^4) could not be rejected for both methods (i.e., OLS and SURE) of estimation, this implies that the dummy coefficients representing the cumulative abnormal returns cancel each other in all chosen event period lengths.

However, the null hypothesis number five (H_0^5) shows that some abnormal returns are significantly different from zero when we utilize the SURE method which accounts for residual correlation between equations.

The only significant results we observe in Table 6.2.b are for the pre-effective event day window (L_B) of the OLS estimation where the F-test value is significant at the 10% level, however this result is not significant when we account for the residual cross-equation correlation by utilizing the SURE estimation method.

The multivariate regression model and its OLS estimation results (Table 6.2.b) provide us with very similar results to the univariate market model from Table 6.1.b, the overall effective event window (L_W) gives us insignificant F-test statistic values, however, this can be attributed to the choice of the test statistic, in the case of the univariate market model we utilize the cross-section cumulative standardized abnormal returns (CSAR) as defined in Equation 16.1 coupled with the Adjusted-Patell test as defined in Equation 17 to test for their significance, we also use the average standardized abnormal returns ($\overline{SCAR}$) as shown in Equation 18 in conjunction with the Adjusted Standardized Cross-Sectional statistic test as defined in Equation 18.1. On the flip side, in the multivariate regression setting we use the cumulative abnormal returns as defined in Equation (14) in Section 5.4 and the F-test to check for any significance in those cumulative abnormal returns throughout all three event windows.

6.3. Promotion (exclusion), MDAX to DAX



Cross-sectional average cumulative abnormal returns as defined in Equation (11) five days before and after the announcement event day and the effective event day for subsamples MDAX to DAX. The vertical axis depicts the cross-sectional average cumulative abnormal returns, and the horizontal axis depicts the days around the event day labeled with the number zero.

The cross-sectional average cumulative abnormal returns for the promotion of a stock from the mid-cap index into the big-cap index in both event cases (i.e., announcement and effective event) are decreasing as depicted by the green and yellow line in Graph 2.a. The return reaction is stronger around the announcement event compared to the reaction of returns around the effective event.

Table 6.3.a: MDAX to DAX subsample null hypothesis test results for the univariate market model.

Ann. Event	N	CSAR	AP. Z-score	p-value	$\overline{SCAR}$	AC. Z-Score	p-value	$\overline{U_0}$	GR. Z-score	p-value
L_W	19	-10.019	-2.244	0.025	-0.511	-2.335	0.020	-0.111	-1.687	0.092
Sign.				**			**			*
L_A	19	-15.695	-3.516	0.000	-0.812	-4.518	0.000	-0.249	-3.792	0.000
Sign.				***			***			***
L_B	19	-1.502	-0.336	0.737	-0.081	-0.370	0.711	0.036	0.549	0.583
Sign.				x			x			x
Eff. Event	N	CSAR	Pat. Z-score	p-value	$\overline{SCAR}$	BMP. Z-Score	p-value	$\overline{U_0}$	Grank Z-score	p-value
L_W	19	-7.808	-1.571	0.116	-0.397	-2.036	0.042	-0.120	-1.824	0.068
Sign.				x			**			*
L_A	19	-2.772	-0.558	0.577	-0.147	-0.739	0.460	-0.023	-0.343	0.732
Sign.				x			x			x
L_B	19	-6.741	-1.356	0.175	-0.348	-1.731	0.083	-0.099	-1.509	0.131
Sign.				x			*			x

Table 6.3.a. exhibits the results for stocks that move from the MDAX to the DAX for the null hypothesis tests from Section 5.5 concerning the traditional univariate market return model, see equation (1) in Section 5.3. The abbreviation Ann. Event and Eff. Event stands for Announcement Day Event and Effective Day Event. x – no significance, * - significance at the 10% level, ** - significance at the 5% level and *** - significance at the 1% level

6.3.1. MDAX to DAX univariate model results

The event of announcement has an overall negative impact on all three types of measures of abnormal returns. The overall event window around the announcement day (L_W) exhibits significant negative cross-sectional cumulative standardized abnormal returns (CSAR) and cross-sectional average standardized cumulative abnormal returns ($\overline{SCAR}$) at the 5% confidence interval. The cross-sectional average demeaned ranked returns ($\bar{U}_0$) are negative as well and significant at the 10% confidence interval.

The announcement day results contradict the previous findings by (Chen, Noronha, & Singal, 2004), (Fernandes & Mergulhão, 2016) and (Lynch & Mendenhall, 1997) from the US stock market concerning the S&P 500 index and the British FTSE 100 index.

However, our findings for the promotion into the DAX index are in accordance with the results by (Vainikka, 2021), (Franz, 2020), (Bennett, Stulz, & Wang, 2020) and (Steiner & Heinke, 1997).

The effective event shows no significant cross-sectional cumulative abnormal returns (CSAR) for any of the defined event lengths, but the effective event shows significant negative cross-sectional average standardized cumulative returns ($\overline{SCAR}$) at the 5% confidence interval for the overall event window (L_W), and significant negative cross-sectional average demeaned ranked abnormal returns at the 10% confidence interval. These results demonstrate the power of the Adjusted Standardized Cross-Sectional statistic and the Generalized Rank Z test. These tests show that the cross-sectional cumulative standardized abnormal returns (CSAR) are not significant because the corresponding Adjusted-Patell test assumes that the standardized abnormal returns have the same variance; nevertheless, the Adjusted Standardized Cross-Sectional statistic and the Generalized Rank Z test account for the event induced volatility. We also want to mention that the MDAX to DAX subsample suffers from the small sample size.

6.3.2. MDAX to DAX multivariate model results

Table 6.3.b: MDAX to DAX subsample null hypothesis test results for the multivariate market model.

	$H_0^4: \sum_{i=1}^{N} \sum_{t=T_2}^{T_3} \gamma_{it} = 0$		$H_0^5: \gamma_{1,1} = \ldots = \gamma_{N,T_N} = 0$		$H_0^6: \gamma_{1,1} = \ldots = \gamma_{N,T_N}$	
OLS - Method						
Ann. Event	F-value	p-value	F-value	p-value	F-value	p-value
L_W	4,041	0,044 **	1,1500	0,07 *	1,08	0,21 x
L_A	13,088	0,000 ***	1,24	0,041 **	1,123	0,18 x
L_B	0,328	0,566 x	1,202	0,075 *	1,196	0,085 *
Eff. Event	F-value	p-value	F-value	p-value	F-value	p-value
L_W	5,405	0,02 **	0,83	0,99 x	0,794	0,98 x
L_A	1,51	0,22 x	0,72	0,98 x	0,718	0,98 x
L_B	4,56	0,032 **	1,06	0,29 x	1,006	0,46 x
SUR - Method						
Ann. Event	F-value	p-value	F-value	p-value	F-value	p-value
L_W	4,13	0,042 **	1,340	0,000 ***	1,31	0,003 ***
L_A	13,126	0,000 ***	1,43	0,002 ***	1,298	0,022 *
L_B	0,3	0,58 x	1,477	0,000 ***	1,45	0,001 ***
Eff. Event	F-value	p-value	F-value	p-value	F-value	p-value
L_W	4,22	0,04 **	0,92	0,77 x	0,91	0,79 x
L_A	1,065	0,3 x	0,83	0,89 x	0,84	0,88 x
L_B	3,68	0,055 *	1,116	0,19 x	1,103	0,22 x

Table 6.3.b. exhibits the results of the F test statistic for the OLS and SURE regression methods, results on the table are shown for all three window lengths and for both event types (i.e., announcement event and effective event):
x – no significance, * - significance at the 10% level, ** - significance at the 5% level and *** - significance at the 1% level

The significance of the F test values for the announcement event and all three window intervals resembles the results from the univariate market model, hypothesis number four (H_0^4) which test for the cross-sectional cumulative abnormal returns, the overall announcement event window (L_W) and the after-announcement event window (L_A) show that the cumulative abnormal returns are significant at the 5% and 1% confidence interval for both methods of estimation (i.e., the SURE and the OLS methods).

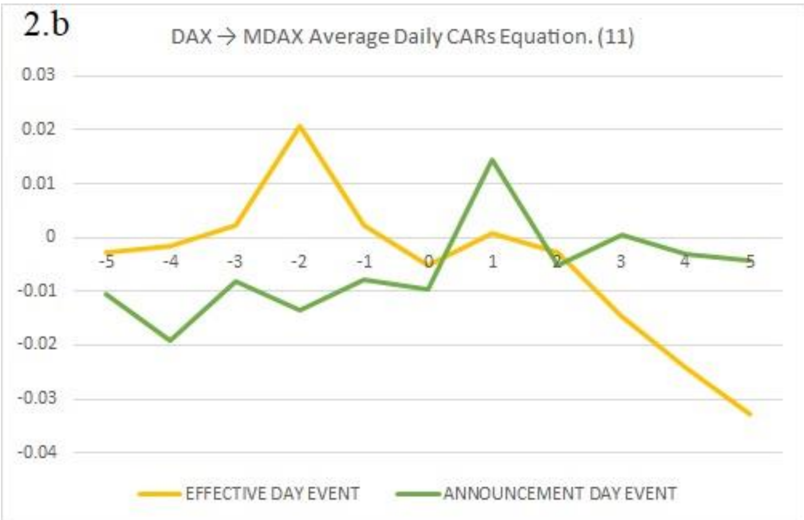
The overall event window (L_W) and the post announcement event window show homogeneity in the returns for their respective window lengths, however the seemingly unrelated regression estimation (SURE) rejects the hypothesis of cross-sectional abnormal return homogeneity. This is the result of the SURE method accounting for residual covariance such that it improves the estimation of the dummy coefficients that represent our abnormal returns.

The cumulative abnormal returns for the effective event are significant at the 5% confidence interval for the overall event window length (L_W) and the pre-promotion event window (L_B) for the OLS estimation method, on the other hand, the SURE estimation results differ in significance only for the pre-promotion event window (L_B) with a significant F-statistic value at the 10% confidence interval.

Null hypothesis number five concerning the windows around the effective event show no significant F-values for neither of the estimation methods (SURE and OLS), however even though the joint null hypothesis could not be rejected, we can see that the sum of the coefficients is significant, these results are in line with the results from the univariate market model where the cross-sectional average standardized cumulative abnormal returns ($\overline{SCAR}$) are significant for the same window lengths (see Table 6.3.a).

The homogeneity of abnormal returns inside the event window hypothesis (i.e., hypothesis number six) is also not rejected for the effective event and the corresponding window lengths, this implies that the effective event has a similar impact on the abnormal returns of the sample firms.

6.4. Demotion (inclusion), DAX to MDAX



Cross-sectional average cumulative abnormal returns as defined in Equation (11) five days before and after the announcement event day and the effective event day for subsamples DAX to MDAX. The vertical axis depicts the cross-sectional average cumulative abnormal returns, and the horizontal axis depicts the days around the event day labeled with the number zero.

6.4.1. DAX to MDAX univariate model results

The demotion of stocks into the mid-cap index from the big-cap index has different effects for the announcement event and effective event.

The cross-sectional average cumulative abnormal returns around the announcement event in graph 2.b depicted by the green line are negative in the anticipatory period (L_B), the announcement of demotion into the MDAX pushes the prices of the securities down, the same results have been observed for the US S&P500 by (Jain, 1987), (Brown & Warner, Measuring security price performance, 1980) and (Lynch & Mendenhall, 1997). There is also a stark increase in the abnormal returns starting at exactly the event day of announcement marked by zero which marks the start of the post announcement event window (L_A). This price reversal is consistent with the price pressure hypothesis, this type of price reversal has also been observed by (Elliott & Warr, 2003).

The effective event as depicted in Graph 2.b shows the opposite effect on cross-sectional daily average CARs as defined in Equation 11, unlike the announcement price movement where the anticipatory period showed negative cross-sectional average CARs, the pre-effective window (L_B) shows positive cross-sectional average CARs.

Graphical representations of the average CARs around both event windows are useful however it is hard to determine whether the event had any significant impact on the returns of the securities' prices.

Table 6.4.a: DAX to MDAX subsample null hypothesis test results for the univariate market model.

Ann. Event	N	CSAR	AP. Z-score	p-value	$\overline{SCAR}$	AC. Z-Score	p-value	$\overline{U_0}$	GR. Z-score	p-value
L_W	15	-1.005	-0.251	0.802	-0.068	-0.238	0.812	-0.032	-0.437	0.662
Sign.				x			x			x
L_A	15	3.128	0.780	0.435	0.201	0.849	0.396	0.050	0.676	0.499
Sign.				x			x			x
L_B	15	-5.005	-1.248	0.212	-0.329	-1.058	0.290	-0.059	-0.798	0.425
Sign.				x			x			x
Eff. Event	**N**	**CSAR**	**Pat. Z-score**	**p-value**	**$\overline{SCAR}$**	**BMP. Z-Score**	**p-value**	**$\overline{U_0}$**	**Grank Z-score**	**p-value**
L_W	15	-2.968	-0.761	0.447	-0.189	-0.840	0.401	-0.021	-0.284	0.777
Sign.				x			x			x
L_A	15	-7.673	-1.967	0.049	-0.489	-2.227	0.026	-0.152	-2.059	0.039
Sign.				*			**			**
L_B	15	3.094	0.793	0.428	0.204	0.657	0.511	0.027	0.364	0.716
Sign.				x			x			x

Table 6.4.a. exhibits the results for stocks that move from the DAX to the MDAX for the null hypothesis tests from Section 5.5 concerning the traditional univariate market return model, see Equation (1) in Section 5.3. The abbreviation Ann. Event and Eff. Event stands for Announcement Day Event and Effective Day Event. x – no significance, * - significance at the 10% level, ** - significance at the 5% level and *** - significance at the 1% level

The announcement event measured by the cross-sectional cumulative standardized abnormal returns (CSAR) defined in Equation 16.1, the cross-sectional average standardized cumulative abnormal returns ($\overline{SCAR}$) defined in Equation 18 and the cross-sectional average ranked abnormal returns ($\overline{U}_0$) defined in Equation 20.1 shows no significant effect for none of the test statistics associated with the aforementioned abnormal returns and for none of the event lengths.

The lower half-part of table 6.4.a shows that only the post-exclusion event window (L_A) shows negative cross-sectional cumulative standardized abnormal returns at the 10% confidence interval. We observe significant negative cross-sectional standardized cumulative abnormal returns and negative cross-sectional demeaned ranked abnormal returns. This result is in accord with other findings from studies in the US S&P 500 and Europe such as the FTSE 100 and the AEX Dutch index.

6.4.2. DAX to MDAX multivariate model results

Table 6.4.b: DAX to MDAX subsample null hypothesis test results for the multivariate market model.

	$H_0^4: \sum_{i=1}^{N}\sum_{t=T_2}^{T_3} \gamma_{it} = 0$		$H_0^5: \gamma_{1,1} = \ldots = \gamma_{N,T_N} = 0$		$H_0^6: \gamma_{1,1} = \ldots = \gamma_{N,T_N}$	
	OLS - Method					
Ann. Event	F-value	p-value	F-value	p-value	F-value	p-value
L_W	<0,0001	0,995 x	1,1540	0,095 *	1,15	0,1 *
L_A	0,235	0,627 x	1,46	0,003 ***	1,51	0,002 ***
L_B	0,052	0,82 x	0,839	0,85 x	0,807	0,89 x
Eff. Event	F-value	p-value	F-value	p-value	F-value	p-value
L_W	0,51	0,47 x	1,524	0,000 ***	1,465	0,000 ***
L_A	4,001	0,000 ***	1,991	0,000 ***	1,965	0,000 ***
L_B	0,04	0,840 x	1,27	0,046 **	1,208	0,099 *
	SUR - Method					
Ann. Event	F-value	p-value	F-value	p-value	F-value	p-value
L_W	<0,0001	0,981 x	1,240	0,024 **	1,27	0,016 **
L_A	0,171	0,678 x	1,586	0,000 ***	1,655	0,000 ***
L_B	0,038	0,844 x	0,866	0,81 x	0,871	0,79 x
Eff. Event	F-value	p-value	F-value	p-value	F-value	p-value
L_W	0,56	0,45 x	1,641	0,000 ***	1,546	0,000 ***
L_A	4,087	0,000 ***	2,12	0,000 ***	2,052	0,000 ***
L_B	0,03	0,861 x	1,412	0,007 ***	1,336	0,024 **

Table 6.4.b. exhibits the results of the F test statistic for the OLS and SURE regression methods, results on the table are shown for all three window lengths and for both event types (i.e., announcement event and effective event):
x – no significance, * - significance at the 10% level, ** - significance at the 5% level and *** - significance at the 1% level

The announcement event cross-sectional cumulative abnormal returns for the multivariate regression model represented by hypothesis number four shows no significant F-statistic values for none of the window lengths regardless of the method of estimation, these results coincide with the results from the univariate market model. However, hypothesis number five which tests for joint significance of the dummy coefficients shows that jointly the cross-sectional abnormal returns are significant, hypothesis number six on the other hand shows that the announcement event did not have the same impact on all stocks' abnormal returns and the null hypothesis was not rejected, the interesting observation is that the anticipatory event window abnormal returns were jointly not significant as well.

The announcement event has no effect on abnormal returns, even though hypothesis number five is able to detect significant abnormal returns, for the overall event window (L_W) and the post announcement window (L_A), nevertheless, these abnormal returns cancel each other out upon cumulation.

The significant results of the F-statistic values for the effective event and their corresponding window lengths reflects the results from the univariate market model, the only significant F-statistic value in Table 6.4.b is that of the post-demotion event window (L_A) represented by an F-Value of 4.001 in the case of the OLS estimation and an F-value of 4.087 in the case of the SURE estimation, both values are significant at the 1% confidence interval.

Hypothesis number five implies that there are significant abnormal returns for all three effective event window lengths, even though these abnormal returns cancel themselves out upon accumulation for the overall effective event window (L_W) and the pre-demotion window (L_B).

Hypothesis number six also implies that the effective event does not have the same impact on all stock returns and, different stocks react differently to the event of demotion.

7. Conclusions and Recommendations

This thesis analyzed return data for stocks that entered and exited the mid-cap German index for a period of around twenty years. We see that the majority of stocks movements into and out of the mid-cap German index takes place between the small-cap German index and mid-cap German index. Over 77% of MDAX exits are demotions into the small-cap German index and 59% of the stocks included into the MDAX are promotions from the small-cap index.

The samples of exclusions from the MDAX into the big-cap German index and inclusions from the big-cap into the mid-cap German index are relatively small. This small number of firms might induce a small sample size bias. We considered correcting the small sample size bias by extending the timeline of our research back to the year of 1993. However, we finally decided to simply include price data only for the period of 2001-2020. This was done by considering the conclusions of (Ince & Porter, 2006) and (Brückner, 2013), namely that the quality of the Thompson Reuters Datastream platform suffers from mismatches in prices for the time period of 1990-2000. We recommend that future researches collect data from a more reliable source in order to alleviate the small sample size bias.

Our results in this thesis reveal mixed effects for both promotions and demotions into and out of the MDAX index.

The anticipatory period of the inclusions (promotions) into the MDAX from the small-cap market index (SDAX) show strong positive effects shown in Table 6.1.a. as measured by the cross-sectional cumulative standardized abnormal returns, the cross sectional average standardized cumulative abnormal returns and the average ranked abnormal returns. These results affirm previous findings and literature from the S&P 500 Index. Also this result is in line with the anticipating-investors hypothesis and the price pressure hypothesis and the information signaling hypothesis. On the other hand the behavior of the returns upon the announcement of the inclusion is at odds with the efficient market hypothesis (EMH).

The announcement of demotion into the MDAX from the big cap index shows negative abnormal returns for the overall window and the anticipatory window, this is in line with the price pressure hypothesis, the anticipating-investors hypothesis and at odds with the efficient market hypothesis. However, the results related to the announcement of demotion of a stock from the DAX into the MDAX are not statistically significant.

Evidence for negative effects from the demotion of a stock from the DAX into the MDAX only comes from the post-effective event day (L_A), nevertheless, we cannot confidently conclude that any of the theoretical hypothesis indicated in Section 3 are confirmed in this case. Moreover, proving the price pressure hypothesis in our case would require an appropriate control group, i.e., utilizing supplementary indices as in (Jain, 1987). In our case finding or constructing supplementary indices with the available data is infeasible.

We observe opposite negative results for the sample of stocks promoted into the DAX from the MDAX, as we already discussed in Section 6.3 we find no support for the efficient market hypothesis, the price pressure hypothesis or the anticipating-investors hypothesis, despite the fact that our results are in line with (Steiner & Heinke, 1997), (Vainikka, 2021), (Bennett, Stulz, & Wang, 2020) and (Franz, 2020). We want to point out that the model used in our study in Section 5.3 in Equations 1 and Equation 3 is a simplified form of return modeling.

Similarly, we observe positive statistically insignificant abnormal returns for stocks demoted into the small-cap index from the mid-cap index, which are at odds with previous findings from the S&P500, FTSE 100 and AEX indices.

We recommend future research concerning the question of the stock inclusion or exclusion in and from the MDAX to employ data from different platforms in order to reduce the errors from the quality of the data available at the Thompson Reuters Database. We also want to make a side note, that for future analysis in support of the price pressure hypothesis the volume effect needs to be analyzed as well.

For future research in this subject, we also propose the approach of bootstrapping as developed by (Efron, 1979). Two such studies have been conducted by (Chou, 2004) and (Kramer, 2001). The first approach is the bootstrapping of the residuals of the OLS equations as in the case of Chou, and the second approach is the bootstrapping of the standardized Z statistics as in the case of Kramer. These techniques can improve the results for the small samples of the stocks that move from the big-cap index (DAX) to the mid-cap index (MDAX) and vice versa.

8. Appendix F

I. Univariate Market Model Formulation

I.i. Vector model representation of Equation 1 in Section 5.3.

Observed returns in the estimation window for the i-th firms in the system of N equations is represented in a vector and matrix form as follows:

$$R_i = X_i\beta_i + \varepsilon_i \qquad (a)$$

$$\hat{\beta}_i = (X_i'X_i)^{-1}X_i'R_i \quad (a.1)$$

$$\hat{\sigma}^2_{\varepsilon_i} = \frac{\hat{\varepsilon}_i'\hat{\varepsilon}_i}{L_E - 2} \qquad (a.2)$$

The Abnormal returns as defined in Equation 5 in Section 5.3 can be represented in a vector-matrix form as follows:

$$AR_i = R_i - X_i\hat{\beta}_i \qquad (a.3)$$

The vector $R_i = [r_{it_N} \ldots . r_{it_n}]$ in Equation a and Equation $a.3$ consists of the stock returns for firm i. The matrix $X_i = [\iota\ R_m]$ is composed of a column of ones represented by ι and a column of the market returns represented by $R_m = [r_{mt_N} \ldots . . r_{mt_n}]$. The vectors $\beta_i = [\alpha_i\ \beta_i]$ and $\hat{\beta}_i = [\hat{\alpha}_i\ \hat{\beta}_i]$ in Equation a and Equation $a.3$ are the parameter vectors.

The derived variance of the abnormal returns given in this matrix-vector representation can be found in (Campbell, Lo, & MacKinlay, 1997, pp. 149-180).

II. MRM model Formulation

II.i. Vector Model Representation

The description of the model in this section follows the structure and logic depicted in several published papers such as (Thompson, 1985), (Karafiath, 1988) and (Zellner, 1962) which give identical representation of the model in matrix-vector form.

OLS of the system:

$$\underbrace{\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_N \end{bmatrix}}_{Y_*} = \underbrace{\begin{bmatrix} X_1 & 0 & \cdots & 0 \\ 0 & X_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & X_N \end{bmatrix}}_{X_*} \underbrace{\begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_N \end{bmatrix}}_{\beta_*} + \underbrace{\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_N \end{bmatrix}}_{\varepsilon_*} \quad \textbf{(e)}$$

$$X_i = \begin{bmatrix} 1 & r_{m1} & \cdots & 0 & 0 & 0 \\ 1 & r_{m2} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \cdots & 1 & 0 & 0 \\ \cdot & \cdot & \cdots & 0 & 1 & 0 \\ 1 & r_{mT} & \cdots & 0 & 0 & 1 \end{bmatrix}; \quad Y_i = \begin{bmatrix} r_{i1} \\ r_{i2} \\ \vdots \\ r_{iT} \end{bmatrix} \quad \beta_i = \begin{bmatrix} \alpha_{i1} \\ \beta_{im} \\ \gamma_{ib} \\ \vdots \\ \gamma_{iT} \end{bmatrix} \quad \varepsilon_i = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix} \quad \text{(e.1)}$$

$$\mathrm{E}(\varepsilon_i \varepsilon_i') = Var(\varepsilon_i) = \Sigma \otimes I_T \quad = \sigma^2 I_{NT} \quad \text{(f)}$$

The matrix X_* is composed of N smaller X_i matrices with dimension NTxM_* or it can be interpreted as a decomposition into smaller matrices which in turn these smaller X_i matrices are composed of a vector of the constant number one which corresponds to the intercept α_i and a vector of the observed market return data points of the independent variable r_m which are used for the estimation of β_i, its last column is composed of a vector of dummy variable vectors which corresponds to the estimation of the dummy coefficients γ_{it} which is also a vector of dummy coefficients which match with corresponding dummy vectors from the matrix X when the dot product operation is performed. Matrix I in the definition of variance in Equation f is a TxT identity matrix where T stands for the number of observed stock and market returns. The expected value of the error terms if zero i.e., $\mathrm{E}(\varepsilon_i) = 0$. The matrix Σ is the error covariance matrix of dimensions NxN which in the case of the OLS the diagonal of the matrix is the variances of the error terms ($i.e., \Sigma = [\sigma_{ii}] = \mathrm{E}[\varepsilon_{it}\varepsilon_{it}] = E[\varepsilon_{it}^2]$) and the off-diagonal elements are the covariances which are assumed to be zero (i.e., $\Sigma = [\sigma_{ij}] = \mathrm{E}[\varepsilon_{it}\varepsilon_{jt}] = 0$).

II.ii. SURE vs OLS Estimation

The variance-covariance matrix of the error terms for the OLS and GLS differ in the fact that the GLS method relaxes the assumption in Equation f into $Var(\varepsilon_i) = \Omega = \Sigma \otimes I_T$.

The non-spherical form of the error terms in the setting of the GLS is the key difference in the estimation of β_{OLS} and β_{GLS}.

The off-diagonal elements of the positive definite covariance matrix Ω in the setting of the GLS and FGLS are not equal to zero (i.e., $\Sigma = [\sigma_{ij}] = \mathrm{E}[\varepsilon_{it}\varepsilon_{jt}] \neq 0$).

The graphical representation of the variance-covariance equation is presented in Equation j of this section.

- *SURE by using FGLS*:

The non-spherical errors in the GLS setting can be transformed back into spherical form as follows:

There exist a non-singular matrix H such that $\Omega = \mathrm{HH}'$

$$\mathrm{Y} = \mathrm{X}\beta + \varepsilon \quad ; \quad \mathrm{E}(\varepsilon_i\varepsilon_i') = Var(\varepsilon_i) = \Omega = \Sigma \otimes I_T \qquad \text{(g)}$$

We multiply Equation g by the inverse of the H matrix:

$$H^{-1}Y = H^{-1}X\beta + H^{-1}\varepsilon$$

$$\mathrm{E}\left(H^{-1}\varepsilon_i\varepsilon_i' H^{-1'}\right) = H^{-1}Var(\varepsilon)H^{-1'} = \sigma^2 H^{-1}\Omega H^{-1'} = \sigma^2 H^{-1}\mathrm{HH}'H^{-1'}$$
$$= \sigma^2 I_{NT} \qquad \text{(g.1)}$$

$$\hat{\beta}_{GLS} = [X'H^{-1\prime}H^{-1}X]^{-1}X'H^{-1\prime}H^{-1}Y = [X'\Omega^{-1}X]^{-1}X'\Omega^{-1}Y$$
$$= [X'(\Sigma^{-1} \otimes I_N)X]^{-1}X'(\Sigma^{-1} \otimes I_N)Y \qquad \text{(g.2)}$$

The matrix form of Equation g.2 is represented by the first part $[X'(\Sigma^{-1} \otimes I_N)X]^{-1}$ which is the inverse variance covariance matrix of the residual terms from the OLS regression from Equation g.1, the second part $X'(\Sigma^{-1} \otimes I_N)Y$ represents the covariance between the dependent and independent LHS variable, a graphical representation of this is represented by (Greene W. H., 2017, pp. 293-305)

The matrix Σ^{-1}at this point is unknown and needs to be estimated. The way matrix Σ^{-1} is estimated is by applying the FGLS procedure which gives us Equation g.1. First we run Equation g which is simply the OLS method applied to the system of equations, then the second step is extracting the OLS residuals and using them as the estimated variance for the Σ matrix in the variance of Equation g which in turn its inverted form is used to estimate the GLS estimators of the system in Equation g.2, this technique has also been applied by (Zellner, 1962), where the author expounds the model in full details.

- The estimation of the variance and covariance of the residuals:

The unbiased variance estimator of the single equation OLS variance (ε_i^2) is $\frac{\hat{\varepsilon}_i'\hat{\varepsilon}_i}{T-M_i}$ where M_i is the number of estimated independent variables of equation i as explained in Section 5.3.

The derivation of the residual variance covariance is derived by (Zellner & Huang, 1962) by taking the following expectation:

$$E(\hat{\varepsilon}_i'\hat{\varepsilon}_j) = E(\hat{\varepsilon}_i'[I - X_i(X_i'X_i)^{-1}X_i'][I- X_j\left(X_j'X_j\right)^{-1}X_j']\hat{\varepsilon}_j)$$

$$= \sigma_{ij}\, tr\left[I - (X_iX_i'X_i)^{-1}X_i') - \left(X_j'X_j)^{-1}X_j'\right) + (X_i'X_i)^{-1}X_i')(X_j'X_j)^{-1}X_j')\right] \qquad \text{(g.3)}$$

$$= \sigma_{ij}\left[T - M_i - M_j + tr(X_i'X_i)^{-1}X_i')(X_j'X_j)^{-1}X_j')\right]$$

The term notation tr in Equation g.3 denotes the trace of the square matrices represented within the equation, the trace is defined as the sum of the diagonal entries in the square matrix. The square matrix in our case$(X_i'X_i)X_i'X_j\left(X_j'X_j\right)X_j'X_i$ is the square of the residual canonical correlation coefficients and $tr\left[(X_i'X_i)X_i'X_j\left(X_j'X_j\right)X_j'X_i\right]$ is the sum of squared canonical coefficients. For a more detailed look into the meaning of the derivation in Equation g.3 and canonical correlation analysis one can review the work of (Hooper, 1959) and (Zellner & Huang, 1962), the theoretical justifications concerning the estimation and derivation of the variance and covariance of the residuals is beyond the scope of this thesis.

From the derivation in Equation g.3 we can infer that the estimated residual covariance is:

$$\hat{\sigma}_{ij} = \frac{\hat{\varepsilon}_i' \hat{\varepsilon}_j}{T - M_i - M_j + tr\left[(X_i'X_i)X_i'X_j\left(X_j'X_j\right)X_j'X_i\right]} \qquad \text{(i)}$$

T is the number of observations per equation, M_i is the number of regressors in equation i. The detailed derivation of the above formula can be found in (Zellner & Huang, 1962, p. 309).

$$\Omega = \begin{bmatrix} \sigma_{11}I\ \sigma_{12}I.....\sigma_{1N}I \\ \sigma_{21}I\sigma_{22}I\ ....\sigma_{2n}I \\ \vdots \quad \vdots \quad \vdots \\ \sigma_{N1}I\sigma_{N2}I\ ....\sigma_{NN}I \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & .....\sigma_{1N} \\ \sigma_{21} & \sigma_{22} & ....\sigma_{2n} \\ \vdots & \vdots & \vdots \\ \sigma_{N1} & \sigma_{N2} & ....\sigma_{NN} \end{bmatrix} \otimes I_N = \Sigma \otimes I_N \qquad \text{(j)}$$

II.iii. Defining restrictions

- *Restriction Matrices for the F-test:*

Number of restricted coefficients per equation = v

Number of unrestricted coefficients = n

Number of coefficients in any i-th equation = n+v

Number of equations = N

$$\mathrm{H}_0^4\text{: } [0_1' \quad \iota_1' \quad \dots \quad 0_N' \quad \iota_N']_{1|((\mathrm{n+v})\cdot \mathrm{N})} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{n-1} \\ \beta_N \end{bmatrix}_{((\mathrm{n+v})\cdot \mathrm{N})|1} = 0$$

The numbers $\mathbf{0}_i'$ in the transpose vector represent themselves transpose vectors of unrestricted coefficients; the inputs of these vectors are zeros which are equal to the number of unrestricted coefficients in equation i, the Greek letter $\boldsymbol{\iota}_i'$ represents a vector of restrictions, the input of these vectors is the number one whose repetition is equal to the number of restricted coefficients, $\boldsymbol{\beta}_i$ is the vector of regression coefficients of i-th equation.

The subscript $1|((\mathrm{n}+\mathrm{v})\cdot\mathrm{N})$ in H_0^4 denotes the dimensions of the transpose restriction matrix which is composed of one row and $(\mathrm{n}+\mathrm{v})\cdot\mathrm{N}$ columns

$$\mathrm{H}_0^5\!:\begin{bmatrix} 0 & I & 0 & 0 & \dots & 0 \\ 0 & 0 & I & 0 & \dots & 0 \\ 0 & 0 & 0 & I & \dots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \cdots & I \end{bmatrix}_{(v\cdot N)|((n+v)\cdot N)} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \\ \beta_n \end{bmatrix}_{((\mathrm{n}+\mathrm{v})\cdot\mathrm{N})|1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{(v\cdot N)|1}$$

The Numbers 0 in the above matrix are zero matrices with dimensions v·n whereas every capital letter I is a square identity matrix where the number of ones in the diagonal represent the number of restricted coefficients v, β_i is a vector whose number of elements is the number of total estimated coefficients of the i-th equation in the system of N equations, the β_i vector is illustrated in Equation e.1 in this appendix.

The subscript $(v\cdot N)|((n+v)\cdot N)$ in H_0^5 denotes the dimensions of the restriction matrix. The restriction matrix is composed of $(v\cdot N)$ rows and $(n+v)\cdot N$ columns.

$$\mathrm{H}_0^6\!:\begin{bmatrix} 0 & I & -I & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & I & -I & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & I & -I & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \ddots & \ddots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & -I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I & -I \end{bmatrix}_{((N-1)\cdot v)|(n+v)\cdot N} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \\ \beta_{n-1} \\ \beta_n \end{bmatrix}_{((\mathrm{n}+\mathrm{v})\cdot\mathrm{N})|1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}_{((N-1)\cdot v)|1}$$

The numbers zero in the above matrix with dimensions ((N-1)·v)|(n+v)·N) represent zero matrices which the number of zero rows is equal to the number of restricted coefficients and the number of zero columns is equal to the number of unrestricted coefficients, the capital letter I is an identity matrix whose the number of rows and columns is equal to the number of restricted coefficients and the diagonal contains a string of the number one, the same holds for the negative identity matrix with the distinction that the diagonal is a string of the number negative one, the β_i stands for vectors made of the number of all variables in Equation i.

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